Gravitation, Causality, and Quantum Consistency

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12/01/2016 CosPA

- M.H. (arXiv:1610.03065)
- M.H., McCullen Sandora (in preparation)

Question:

What are the universal principles that underly general relativity?

Historical Assumptions/Principles (GR/EM)

- Special Relativity (Lorentz symmetry)
- Quantum Mechanics (Unitarity)
- Equivalence principle
- Minimal coupling
- Gravity is curvature
- Gravity is sourced by $T_{\mu\nu}$
- Electromagnetism is sourced by J_{μ}
- Gauge principle/symmetry
- Diffeomorphism symmetry
- General co-ordinate invariance
- Beauty
- Renormalizable

Modern Assumptions/Principles (GR/EM)

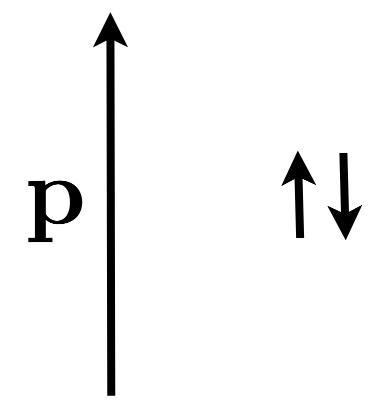
- Special Relativity (Lorentz symmetry)
- Quantum Mechanics (Unitarity)

- Effective Field Theory (Large distances)

Particles

- Representations of Lorentz group $s = 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \ldots$

- Massless particles

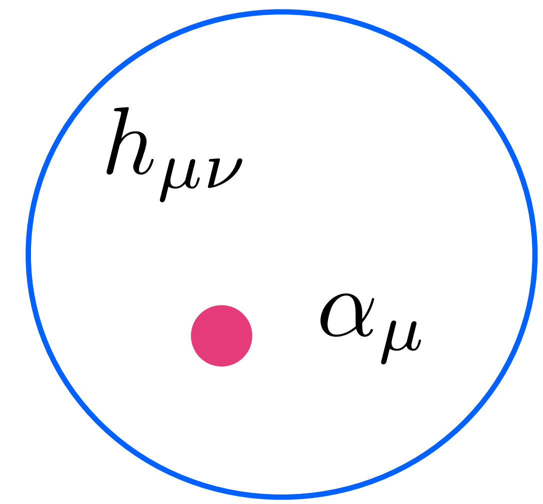


Number of polarizations $= 2$

- Embed into $h_{\mu\nu} \equiv h_{\mu\nu} + \partial_\mu \alpha_\nu + \partial_\nu \alpha_\mu$

Spin-2 Field

- Lorentz transformation



$$\boxed{h_{\mu\nu} \to \Lambda_\mu^\alpha \Lambda_\nu^\beta h_{\alpha\beta} + \partial_\mu \Omega_\nu + \partial_\nu \Omega_\mu}$$

So $h_{\mu\nu}$ is not a tensor

$$\boxed{R^{(L)}_{\mu\nu\rho\sigma} \equiv \frac{1}{2}\left(\partial_\rho \partial_\nu h_{\mu\sigma} + \partial_\sigma \partial_\mu h_{\nu\rho} - \partial_\sigma \partial_\nu h_{\mu\rho} - \partial_\rho \partial_\mu h_{\nu\sigma}\right)}$$

$$\to \Lambda_\mu^\alpha \, \Lambda_\nu^\beta \, \Lambda_\rho^\delta \, \Lambda_\sigma^\gamma \, R^{(L)}_{\alpha\beta\delta\gamma}$$

So $R^{(L)}_{\mu\nu\rho\sigma}$ is a tensor

Spin-2 Free Theory (Gravitons)

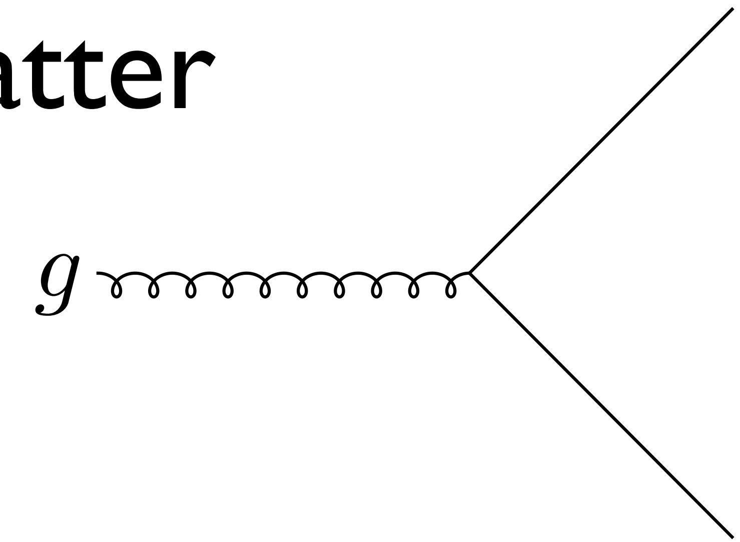
- Kinetic energy (free gravitons)

$$S_{kin} = \int d^4x \text{ ``}\frac{1}{2}(\partial h)^2\text{''}$$

$$S_{kin} = \int d^4x \left[\frac{1}{2}(\partial h)^2 - \frac{1}{2}(\partial h_{\alpha\beta})^2 + \partial_\mu h^{\mu\nu}\partial_\nu h - \partial_\mu h^{\rho\sigma}\partial_\rho h^\mu_\sigma\right]$$

- Kinetic term specified uniquely

Graviton-Coupling to Matter



A

$$\boxed{S_{int} = \int d^4x \sqrt{G_N}\, h_{\mu\nu}\, T_M^{\mu\nu} + \ldots}$$

B

$$\boxed{S_{int} = \int d^4x\, R^{(L)}_{\mu\nu\alpha\beta}\, \tilde{T}_M^{\mu\nu\alpha\beta}}$$

(Related: Wald 1986)

Theory A

- Dangerous; only gauge invariant to leading order

$$S_{int} = \int d^4x \sqrt{G_N}\, h_{\mu\nu}\, T_M^{\mu\nu} + \ldots$$

- Requires infinite tower of sub-leading corrections

$$\sim \sqrt{G_N}\, h_{\mu\nu}\, (T_M^{\mu\nu} + \text{``}(\partial h \partial h)^{\mu\nu}\text{''}) + G_N \left(\text{``}h^2 T'\text{''} + \text{``}h^2 (\partial h)^2\text{''}\right) + \ldots$$

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Resum $$\boxed{S = \int d^4x \sqrt{-g} \left[\frac{R}{16\pi G_N} + \mathcal{L}_M[\psi_i, g_{\mu\nu}] \right]}$$

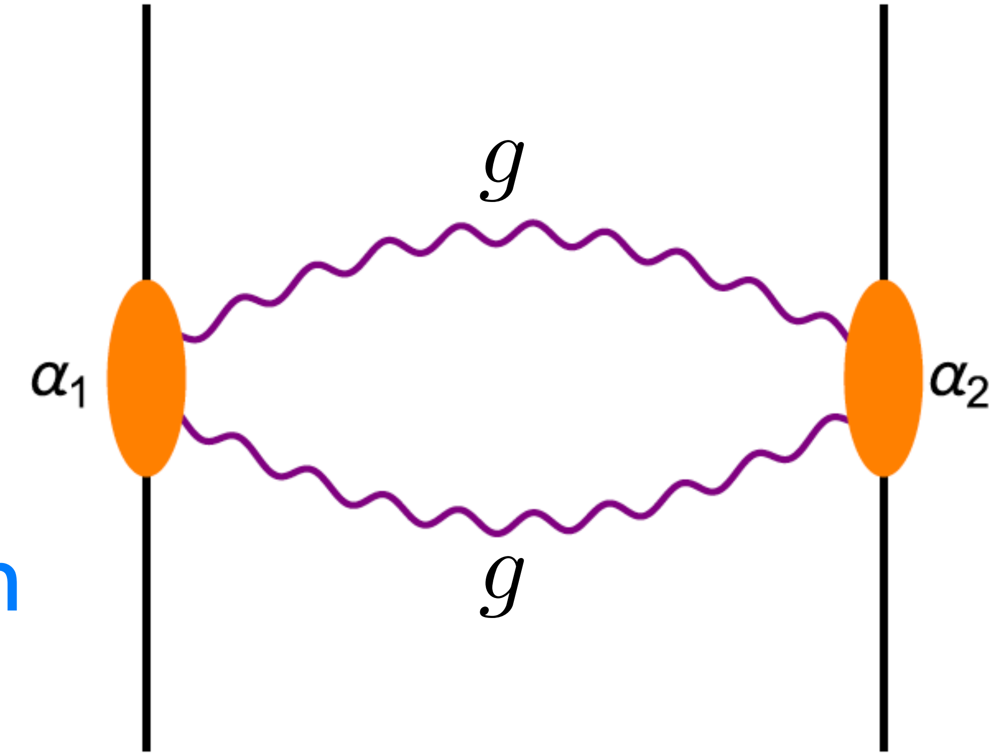
= Standard Theory (GR)

(Deser 1970)

Consistent Effective Theory GR A

- 2 distant polarizable objects

- 1-loop quantum gravity correction



$$V_{\mathrm{far}}(r) = -\frac{3987\,\hbar\,c\,G^2}{4\pi\,r^{11}}\alpha_{1S}\,\alpha_{2S}$$

(Ford, M.H, Karouby, PRL 2016)

Theory B

- Safe; immediately gauge invariant to this order

$$S_{int} = \int d^4x \, R^{(L)}_{\mu\nu\alpha\beta} \, \tilde{T}^{\mu\nu\alpha\beta}_M$$

- No higher order corrections needed

$$\boxed{S = \int d^4x \left[\text{“}\frac{1}{2}(\partial h)^2\text{”} + \mathcal{L}_M[\psi_i, \eta_{\mu\nu}] + R^{(L)}_{\mu\nu\alpha\beta} \tilde{T}^{\mu\nu\alpha\beta}_M \right]}$$

Theory B

- Safe; immediately gauge invariant to this order

$$S_{int} = \int d^4x \, R^{(L)}_{\mu\nu\alpha\beta} \, \tilde{T}^{\mu\nu\alpha\beta}_{M}$$

- No higher order corrections needed

$$\boxed{S = \sum_f \int d^4x \left[\text{``}\frac{1}{2}(\partial h_f)^2\text{''} + \mathcal{L}_M[\psi_i, \eta_{\mu\nu}] + R^{(L)f}_{\mu\nu\alpha\beta} \tilde{T}^{\mu\nu\alpha\beta}_{Mf} \right]}$$

- Many species allowed

Theory B

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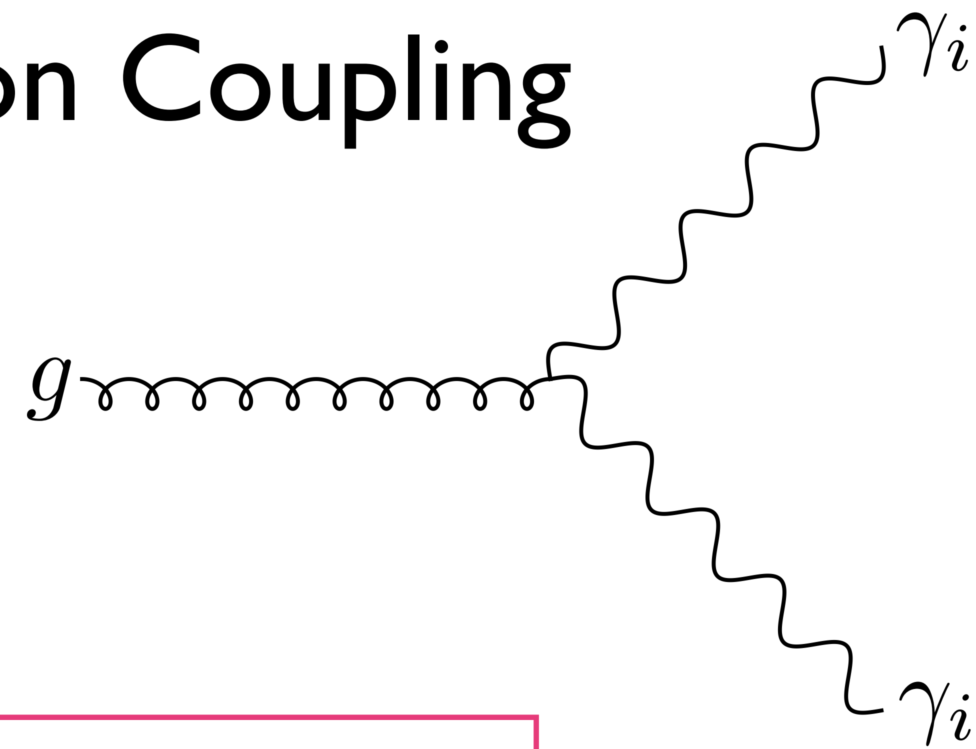
- Many species allowed

= Alternate class of theories of spin 2

Example: Graviton-Photon Coupling



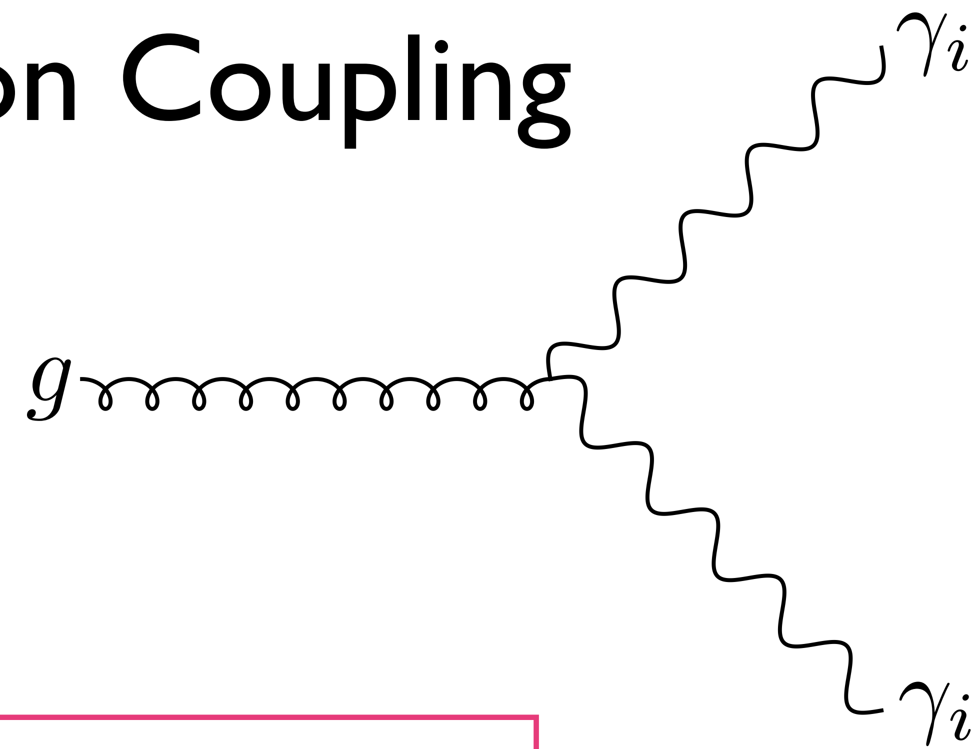
A

$$S_{int} = \sum_i \int d^4x \sqrt{G_N}\, h_{\mu\nu} \left[F_i^{\mu\alpha} F_{i\ \alpha}^{\nu} - \frac{\eta^{\mu\nu}}{4} F_i^2 \right] + \ldots$$

B

$$S_{int} = \sum_i \int d^4x\, c_i\, R^{(L)}_{\mu\nu\alpha\beta} \left[F_i^{\mu\nu} F_i^{\alpha\beta} - 4\eta^{\mu\alpha} F^{\nu\sigma} F^{\beta}_{i\ \sigma} + \eta^{\mu\alpha}\eta^{\nu\beta} F_i^2 \right]$$

Example: Graviton-Photon Coupling



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Arbitrary coefficients (no equivalence principle)

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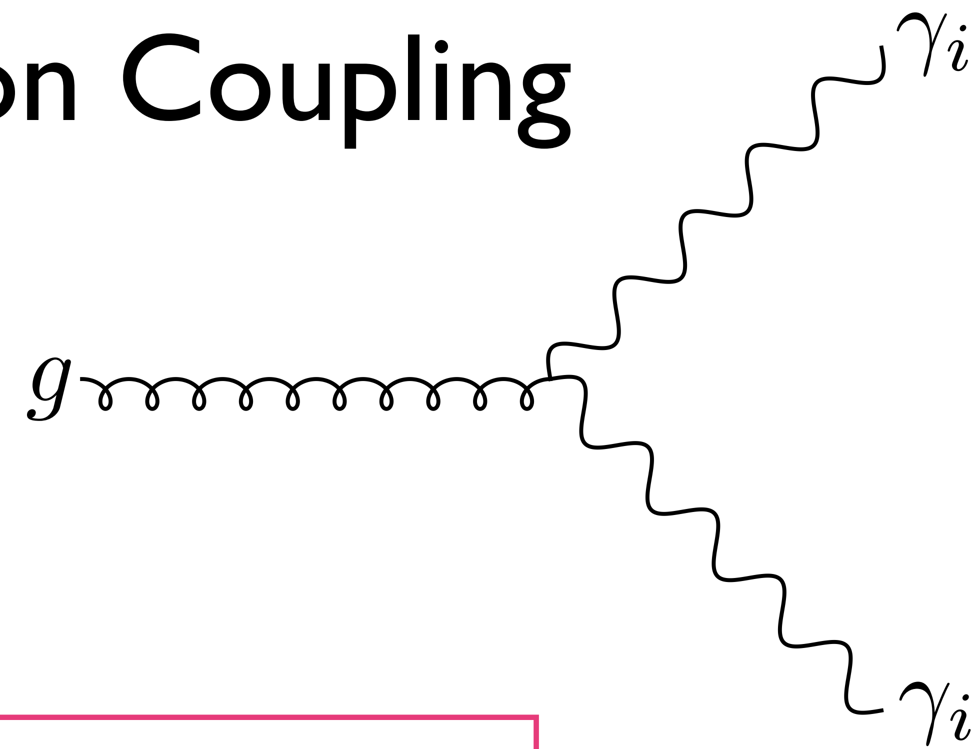

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B

$$S_{int} = \sum_{if} \int d^4x\, c_{if}\, R^{(L)f}_{\mu\nu\alpha\beta} \left[F_i^{\mu\nu} F_i^{\alpha\beta} - 4\eta^{\mu\alpha} F^{\nu\sigma} F^{\beta}_{i\ \sigma} + \eta^{\mu\alpha}\eta^{\nu\beta} F_i^2 \right]$$

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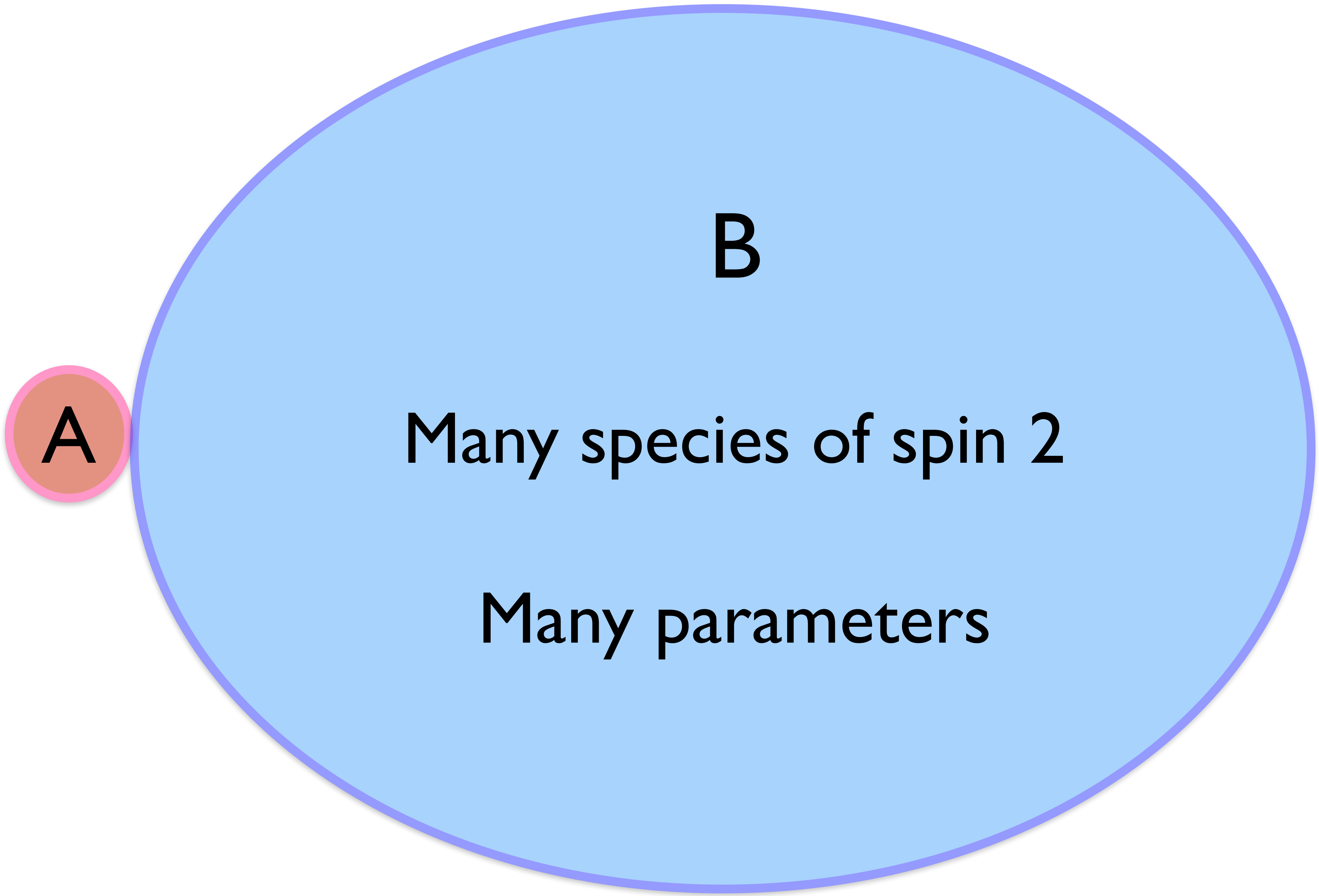
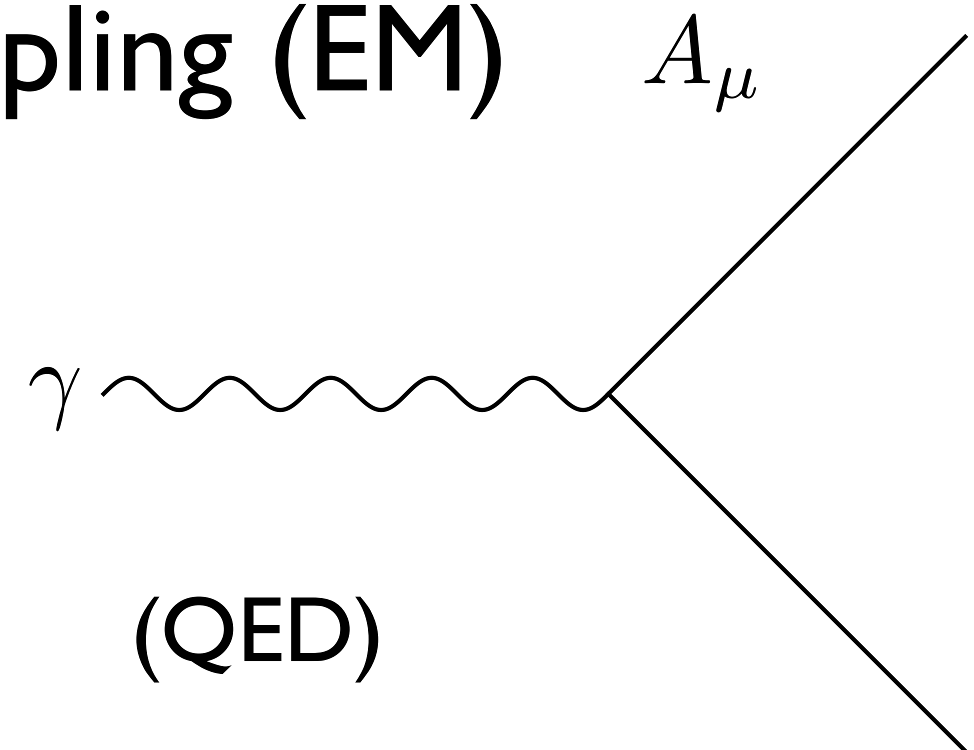
Space of Possible Theories



Photon-Matter Coupling (EM)

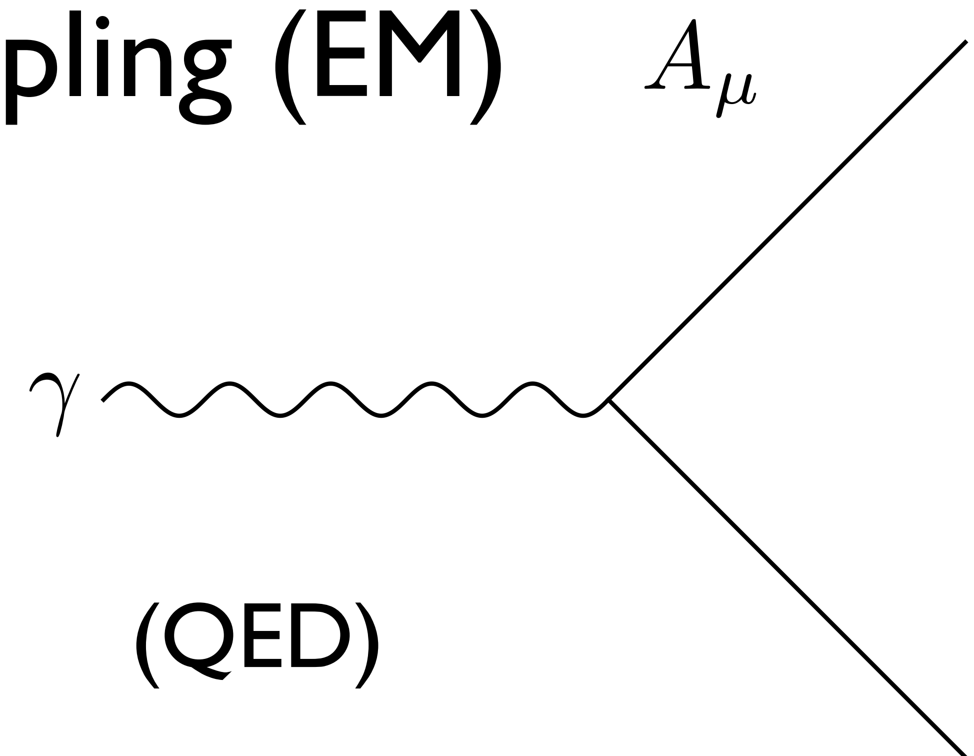


A $\quad \boxed{S_{int} = \int d^4x \, A_\mu \, J^\mu}$ (QED)

B $\quad \boxed{S_{int} = \int d^4x \, F_{\mu\nu} \, \tilde{J}^{\mu\nu}}$ $\quad$ (e.g., $\quad \tilde{J}^{\mu\nu} \propto \bar{\psi} \, \sigma^{\mu\nu} \, \psi$)

Question: How did nature choose A (QED)?

Photon-Matter Coupling (EM)



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Question: How did nature choose A (QED)?

Answer: Nature chose A and B (neutrinos)

Did Nature have a Choice for Gravity?

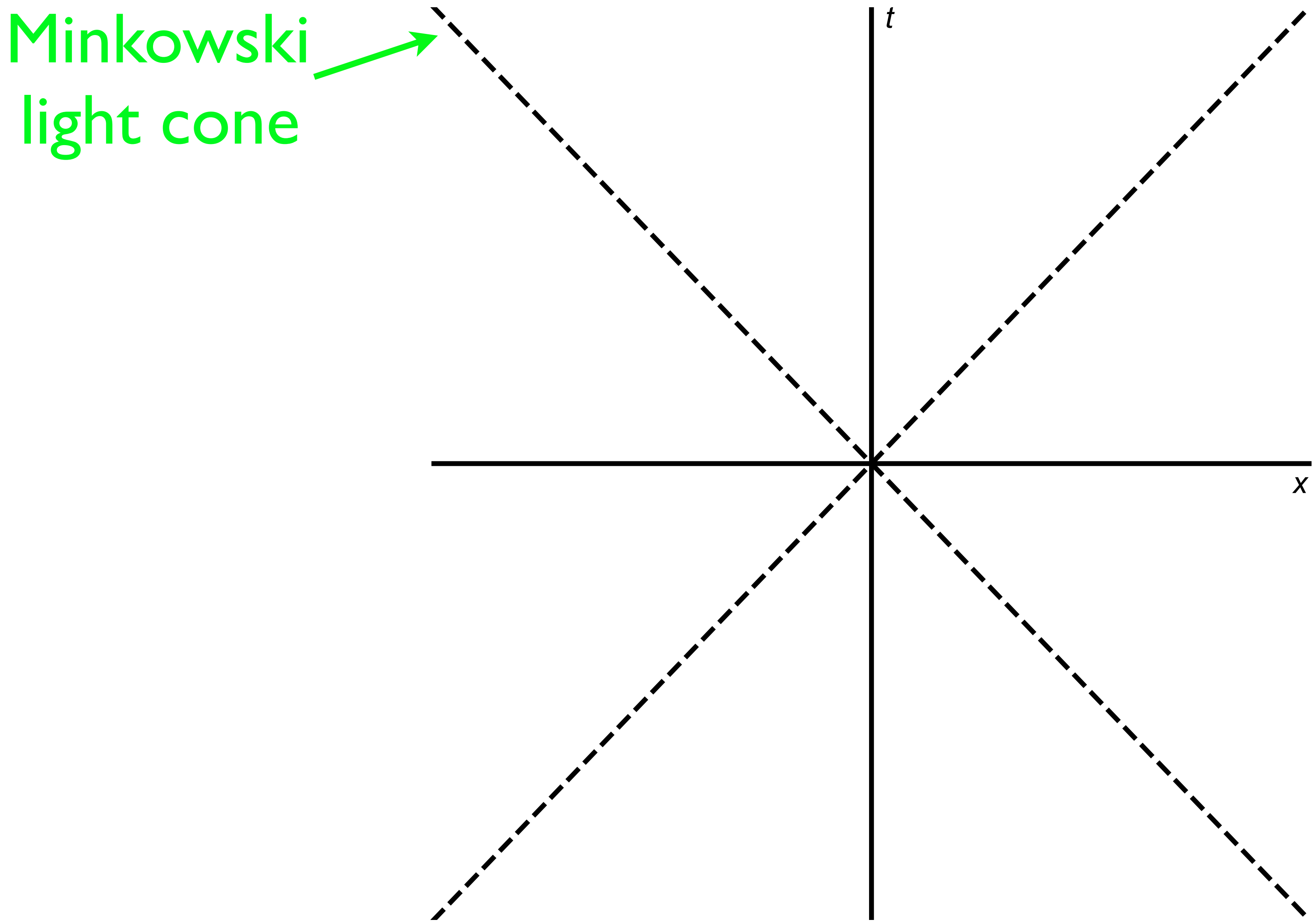
A $\sim h\, F^2 + \ldots$

- Single species
- Infinite tower of terms
- Equivalence Principle
- $1/r^2$ Force Law
- Space-time Curvature
- C.C. Problem
- Black Holes; Paradoxes

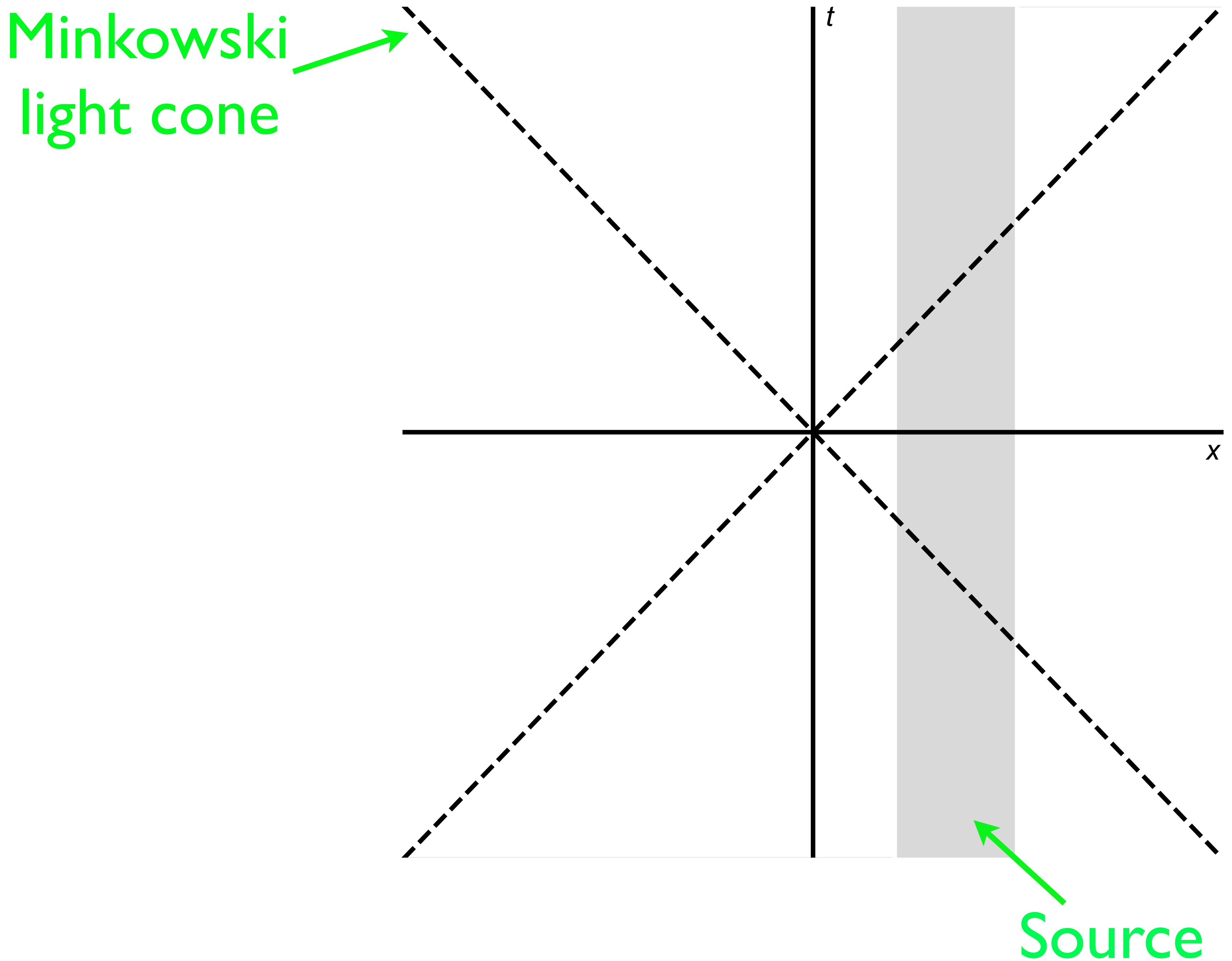
B $\sim R_f^{(L)} F^2$

- Many species
- Finite number of terms
- No Equivalence Principle
- $1/r^6$ Force Law
- No Space-time Curvature
- No C.C. Problem
- No Black Holes

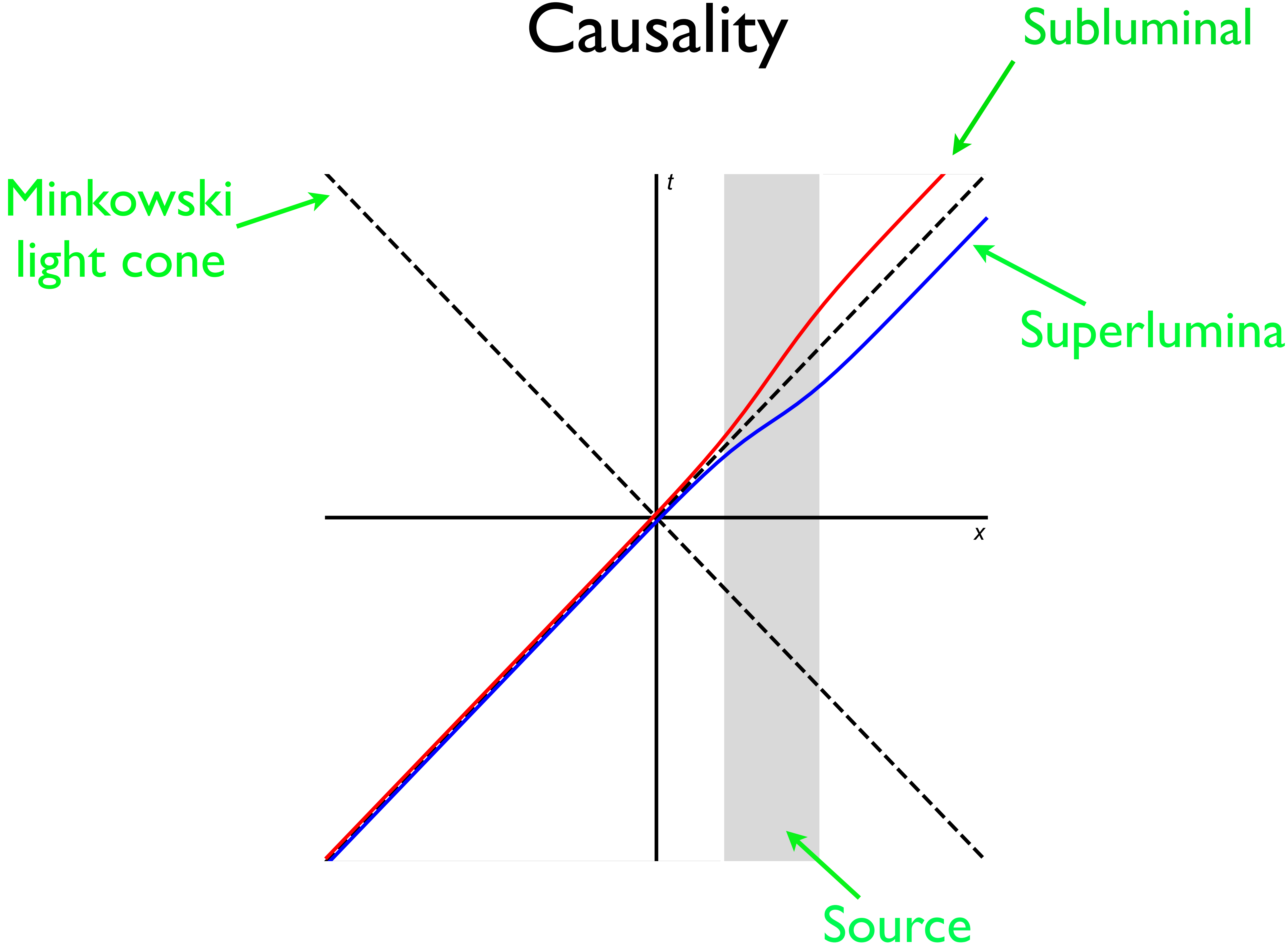
Causality



Causality



Causality



Propagation of Light in GR A

- Geometrics optics limit $k_\mu k_\nu g^{\mu\nu} = 0$

- Weak field regime $g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu}$

$$\boxed{k_\mu k_\nu \eta^{\mu\nu} = k_\mu^{(0)} k_\nu^{(0)} h^{\mu\nu}}$$

(leading order deflection)

(Visser et al 2000, Adams/Nicolis et al 2006)

Propagation of Light in GR A

- Linearized Einstein
  (Lorenz gauge)

$$\Box \bar{h}^{\mu\nu} = -16\pi G_N \, T_M^{\mu\nu}$$

$$\bar{h}^{\mu\nu} = 4G_N \int d^3x' \, \frac{T_M^{\mu\nu}(\mathbf{x}', t_R)}{|\mathbf{x} - \mathbf{x}'|}$$

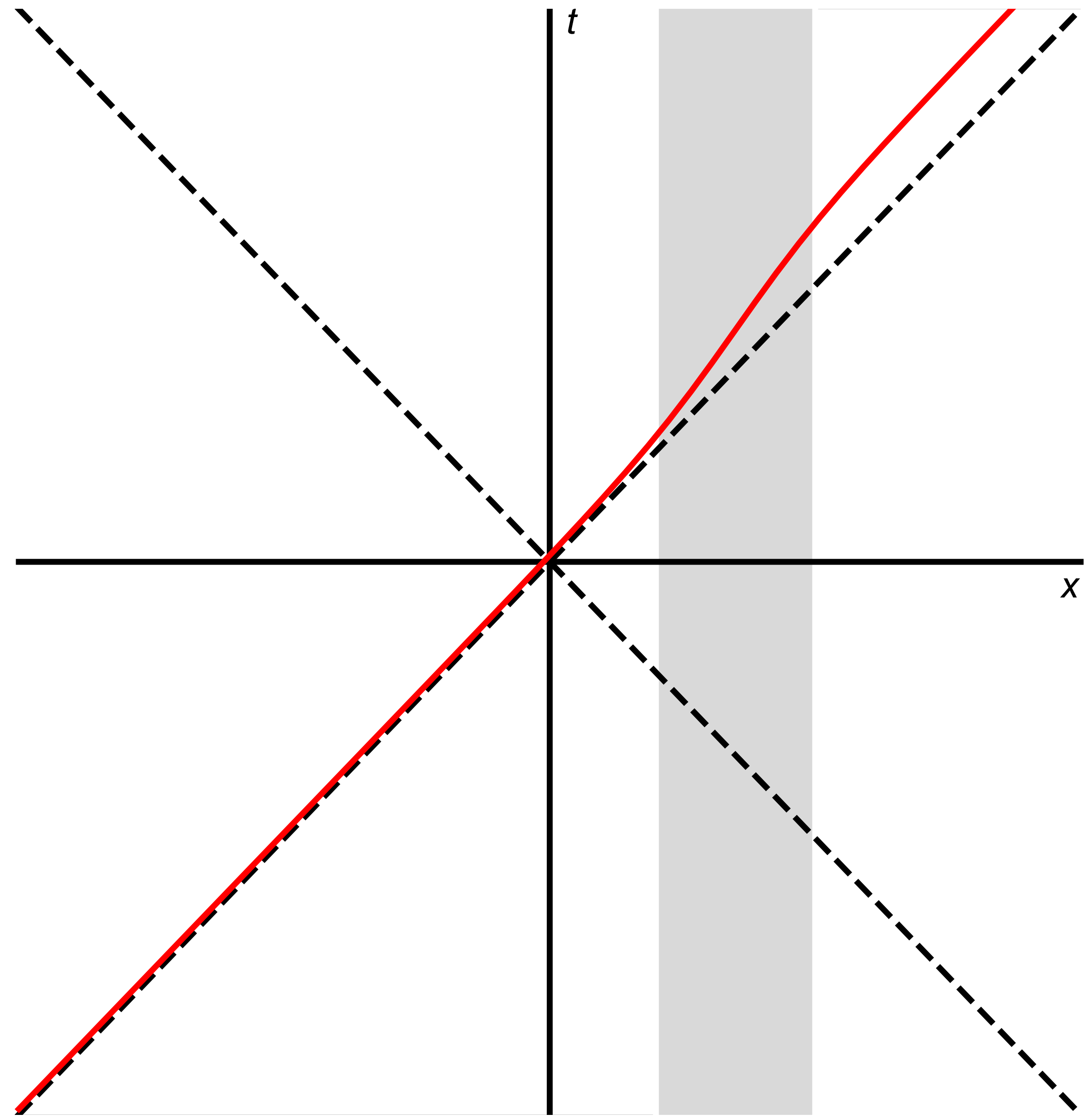
$$\boxed{k_\mu k_\nu \eta^{\mu\nu} = 4G_N \int d^3x' \, \frac{k_\mu^{(0)} k_\nu^{(0)} T_M^{\mu\nu}(\mathbf{x}', t_R)}{|\mathbf{x} - \mathbf{x}'|}}$$

≥ 0 Null energy condition

(Visser et al 2000, Adams/Nicolis et al 2006)

Causality in GR A

(Shapiro)
time delay



Light moves “inside light cone” - Causal!

Propagation of Light in Alternate Theory B

- Modified Maxwell equation (Ricci flat)

$$\partial_\mu F^\mu_{\ \nu} - 4c\, R^{(L)}_{\mu\nu\alpha\beta}\partial^\mu F^{\alpha\beta} = 0$$

- Geometrics optics limit, leading order

$$\boxed{k_\mu k_\nu \eta^{\mu\nu} = -8c\, R^{(L)}_{\mu\nu\alpha\beta} k^\mu k^\alpha a_i^\nu a_i^\beta \Big|_{(0)}}$$

$$(A_i^\mu = A_i\, a_i^\mu, \quad a_{i\mu} a_i^\mu = -1, \quad a_i^\mu k_\mu = 0)$$

2 Linear Polarization vectors i=1,2

Propagation of Light in Alternate Theory (B)

- Form null vectors from each polarization

$$a_\mu^\pm \equiv \frac{1}{\sqrt{2}}\left(a_1^\mu \pm i a_2^\mu\right)$$

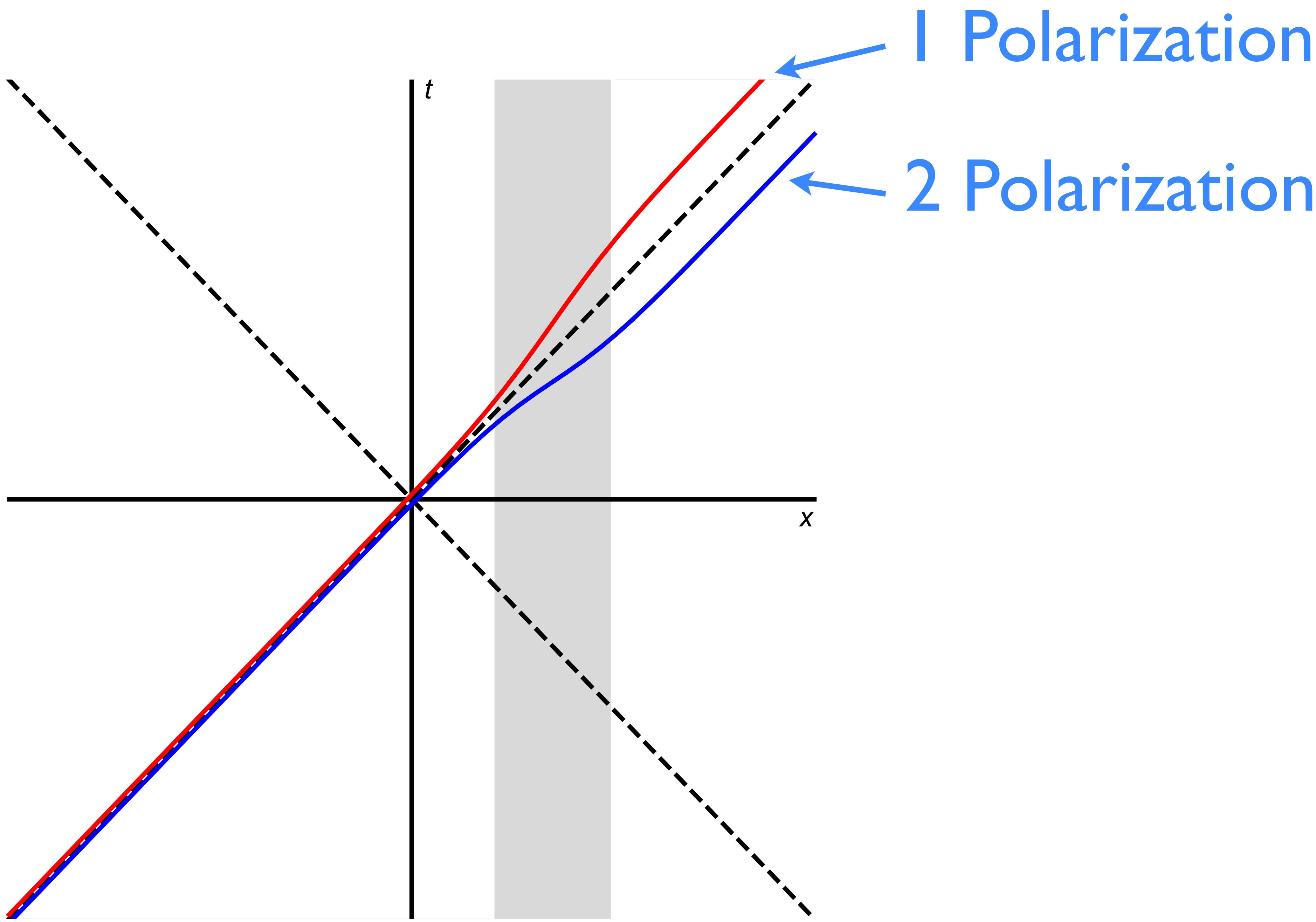
- Obtain

$$\boxed{k_\mu k_\nu \eta^{\mu\nu} = \pm 4c\, R^{(L)}_{\mu\nu\alpha\beta} k^\mu k^\alpha \left(a_+^\nu a_+^\beta + a_-^\nu a_-^\beta\right)\Big|_{(0)}}$$

Sign flip for different polarizations i=1,2

(Related: Drummond 1980, Shore 2003, Camanho et al 2014, Goon/Hinterbichler 2016)

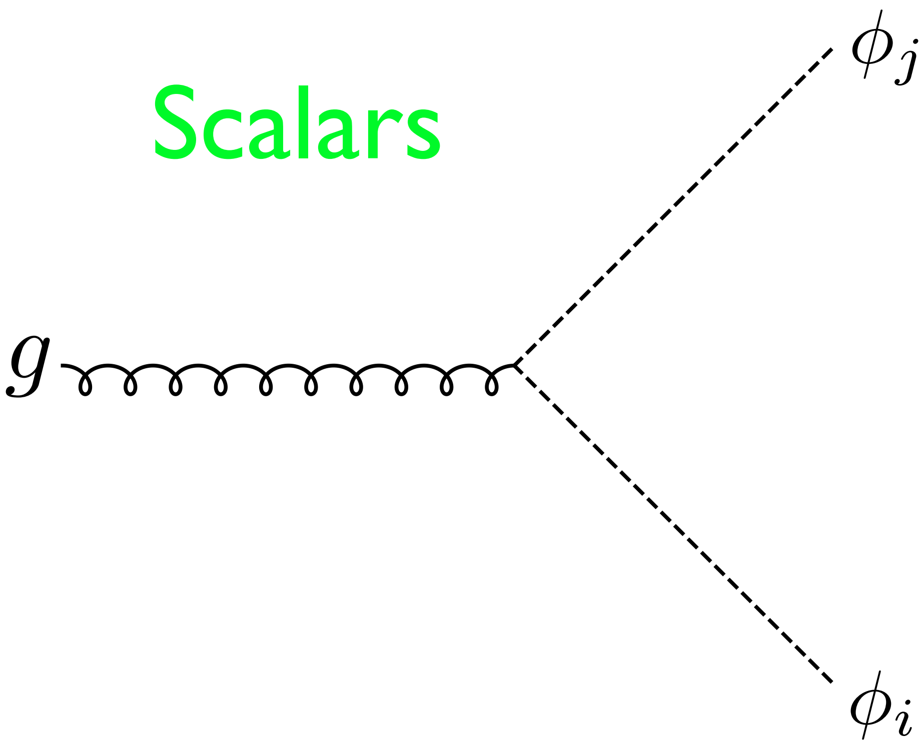
Causality in Alternate Theory (B)



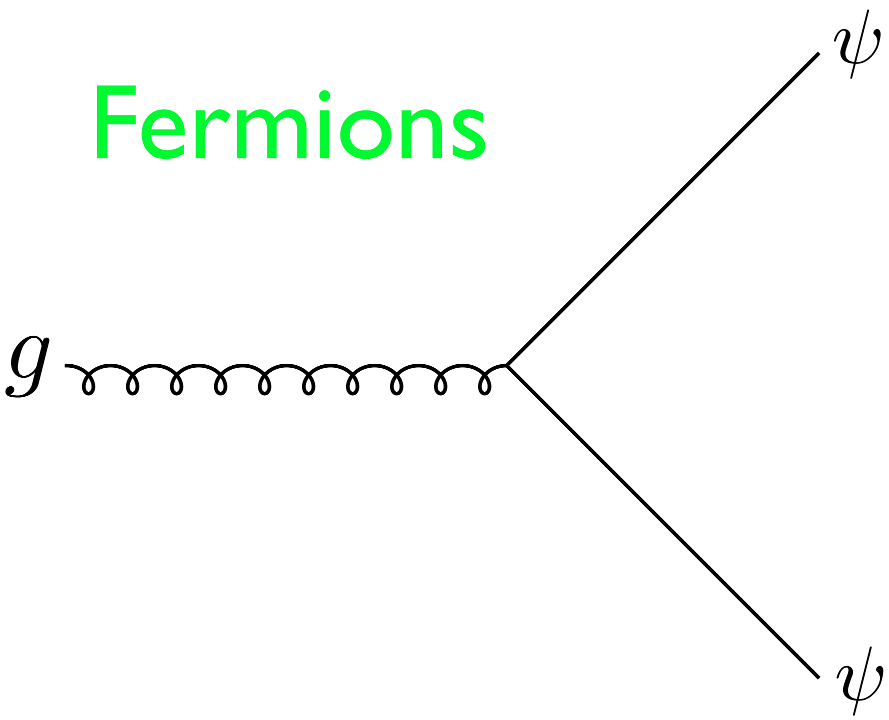
Superluminal and Lorentz invariant- Acausal!

Generalizations: Graviton-XY Coupling

Scalars



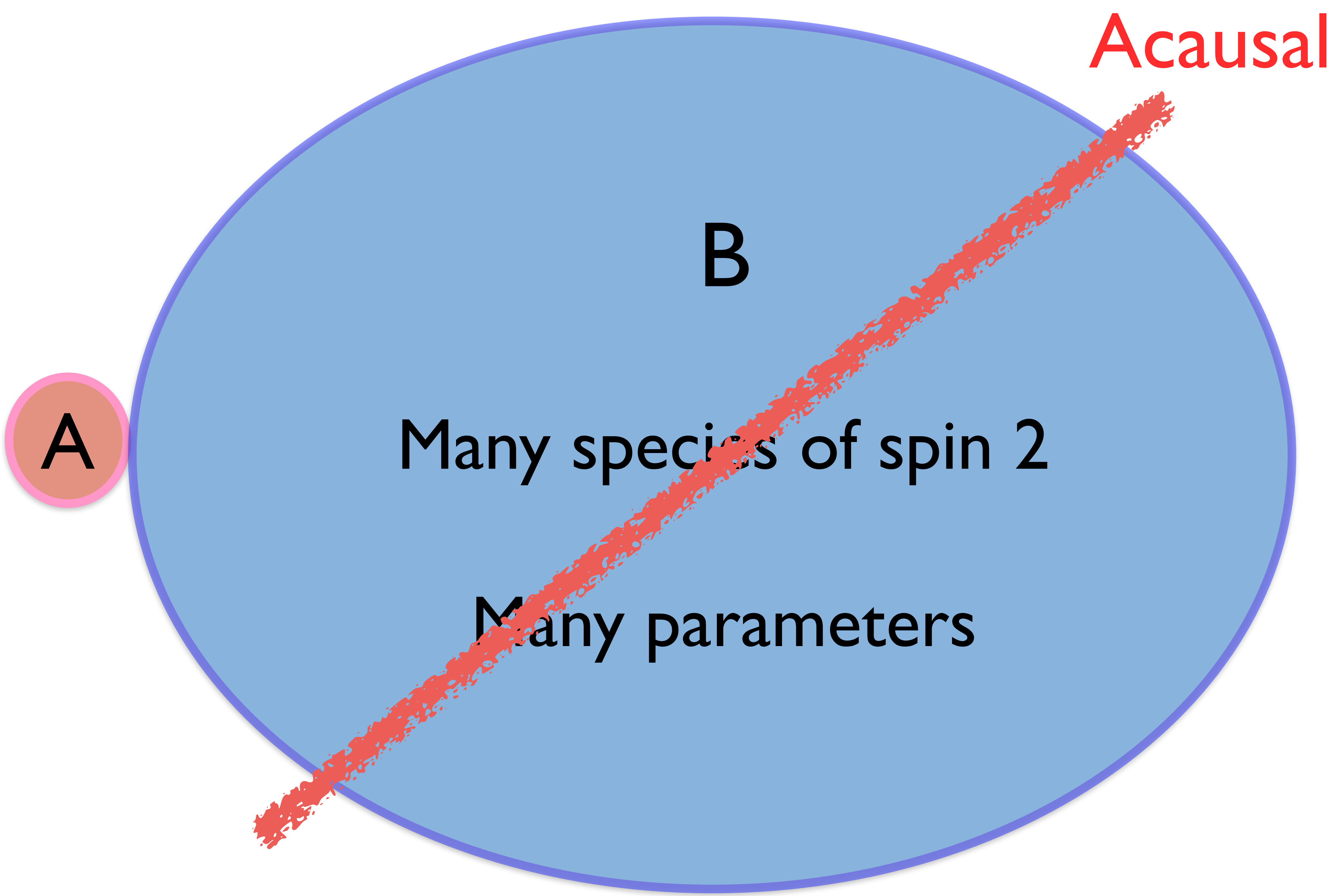
Fermions



All versions of Theory B are acausal!

(Work with M. Sandora; to appear soon)

General Relativity from Causality



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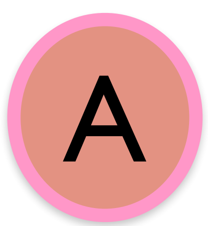

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Causal Modification of GR - F(R)

- Applications to inflation/dark energy

$$R \to F(R) \qquad \boxed{S = \int d^4x\sqrt{-g}\left[\frac{F(R)}{16\pi G_N} + \mathcal{L}_M[\psi_i, g_{\mu\nu}]\right]}$$

- Can be recast as scalar-tensor theory

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G_N} + \frac{1}{2}(\partial\phi)^2 - V(\phi) + \mathcal{L}_M[\psi_i, g_{\mu\nu}e^{2\phi/M_{Pl}}]\right]$$

- Additional, on-shell, d.o.f with its own dynamics

General Relativity from Quantum Consistency

- Tower of counter-terms required for quantum finiteness

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{16\pi G_N} + \frac{1}{2}(\partial\phi)^2 - V(\phi) + \mathcal{L}_M[\psi_i, g_{\mu\nu} e^{2\phi/M_{Pl}}] \right]$$
$$+ \frac{c_1}{M_{Pl}^4}(\partial\phi)^4 + \frac{c_2}{M_{pl}^2} R(\partial\phi)^2 + \frac{c_3}{M_{pl}^6}\phi^2(\partial\phi)^4 + \ldots$$

Not connected to F(R) theory

F(R) contains infinities without counter-terms- inconsistent!

Historical Developments in Understanding of GR

1915 - Einstein amazingly constructs GR

1956 - Feynman suggests GR arises from consistent classical Lagrangian of a spin-2 field

1964 - Weinberg derives equivalence principle from requiring Lorentz invariance of S-matrix

1970 - Deser completes derivation of GR

All use single flavor and minimal coupling assumptions

2016 - Causality, quantum, arguments appear to remove the need for any additional assumptions, leaving GR as unique theory of spin 2

Thank you