Entropy of Horava-Lifshitz Blackholes

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Nov. 28, 2016

COMSATS ,Lahore

-Introduction to the Black-hole Thermodynamics

- Hawking Spectrum

-Calculation of Hawking Spectrum Via Tunneling

- Horava's Model of gravity

- Quantum Corrections to the entropy of IR modified Blackholes solutions

Introduction to the Black-hole Thermodynamics

-Blackhole mechanics can be given the form of the laws of thermodynamics, with area of horizon playing the role of entropy, the surface gravity of temperature and the mass of the black hole as energy, the study of blackhole thermodynamics has been a very active area of research due to fundamental problems posed by such a connection.

- Hawking discovered that the connection between geometry and thermodynamical quantities is not incidental

-Nevertheless, this connection has also posed many theoretical challenges, such as information loss problem, solution of which is believed to reveal the nature of how quantum mechanics should be applied to gravity.

Some internal degrees of freedom ought to account for the entropy associated with the area of a blackhole.

Hawking Spectrum

-Therefore, it is of great interest to study how universal is the connection of thermodynamical quantities, in particular, entropy with the geometrical aspects of the black hole such as the horizon area.

-Hawking spectrum is the main tool available to us in this regard in order to identify the thermodynamical quantities related to a black hole.

-It basically works because one can directly derive the flux of radiated particles from blackholes using elementary field

theory ideas from which one can read off the temperature in terms of black hole parameters.

- Application of classical laws of blackhole mechanics then give the remaining thermodynamical quantities such as entropy.

Calculation of Hawking Spectrum Via Tunneling

ideas of tunneling which basically consists of solving the quantum equations of field propagation in semi-classical approximation in the background of the blackhole.

The advantage of this method is that one can invoke any order

in semi-classical approximation to compute quantum corrections to Hawking temperature and related thermodynamical quantities, the traditional temperature being related to the lowest order in the semi-classical approximation.

Horava's Model of gravity

On general grounds one can expect that the general relativity (GR) must be modified

at small scales due to quantum corrections as GR is not itself renormalizable.

These corrections, which become important at very small scale, may be at the heart of resolving the information and other paradoxes related to black hole thermodynamics.

A particular model of gravity which is renormalizable was presented by Horava , based on simple power counting

HORAVA'S ACTION

In order to write down the action for Horava's theory, one starts with the ADM decomposition of metric i.e.,

$$ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt),$$

For Horava gravity with IR modification, the action is

$$S = \int d^4x \sqrt{g} N \Big(\frac{2}{\kappa^2}(K_{ij}K^{ij} - \lambda K^2) - \frac{\kappa^2}{2w^4} C_{ij}C^{ij} + \frac{\kappa^2 \mu}{2w^2} \epsilon^{ijk} R^{(3)}_{il} \nabla_j R^{(3)l}{}_k$$
$$- \frac{\kappa^2 \mu^2}{8} R^{(3)}_{ij} R^{(3)ij} + \frac{\kappa^2 \mu^2}{8(1-3\lambda)} \left(\frac{1-4\lambda}{4} (R^{(3)})^2 + \Lambda_W R^{(3)} - 3\Lambda_W^2 \right) + \mu^4 R^{(3)} \Big)$$

Black Hole Solution

A spherically symmetric black hole solution is

$$ds^2 = -N^2 dt^2 + \frac{1}{f} dr^2 + r^2 d\Omega^2,$$

where

$$N^2 = f = 1 + \omega r^2 - \sqrt{\omega^2 r^4 + 4M\omega r}$$

There are two horizons at

$$R_0^{\pm} = M\left(1 \pm \sqrt{1 - \frac{1}{2\omega M^2}}\right).$$

Tunneling

The Klein Gordon Equation in this background is

$$\frac{1}{f(r)}\frac{\partial^2\Phi}{\partial t^2} - \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 f(r)\frac{\partial\Phi}{\partial r}\right) + (\text{terms involving angular derivatives}) = 0$$

Using the ansatz $\Phi = e^{(i/\hbar)S(r,t)}$ for the semi-classical solution, the equation becomes

$$\left(\frac{\partial S}{\partial t}\right)^2 - f^2\left(\frac{\partial S}{\partial r}\right)^2 + \frac{\hbar}{i}\left(\frac{\partial^2 S}{\partial t^2} - f^2\frac{\partial^2 S}{\partial r^2} - f\frac{df}{dr}\frac{\partial S}{\partial r} - \frac{2}{r}f\frac{\partial S}{\partial r}\right) = 0$$

Tunneling Solution

Expanding in powers of (hbar)

$$S = S_0 + \left(\frac{\hbar}{i}\right) S_1 + \left(\frac{\hbar}{i}\right)^2 S_2 + \cdots$$

The first order equation becomes

$$\left(\frac{\partial S_0}{\partial t}\right)^2 - f^2 \left(\frac{\partial S_0}{\partial r}\right)^2 = 0.$$

With the solution

$$S_0 = -E\left(t \pm \int^r \frac{dr'}{f(r')}\right).$$

The propagator is the simply

$$K(r_2, t_2; r_1, t_1) \sim e^{\frac{i}{\hbar} S_0(r_2, t_2; r_1, t_1)}$$

$$S_0(r_2, t_2; r_1, t_1) = -E(t_2 - t_1) \pm E \int_{r_1}^{r^2} \frac{dr}{f(r)}$$

Performing the integral by deforming the path slightly above the singularity we get tunneling probability

$$P \sim e^{-\frac{4\pi E}{\hbar F_0}}.$$

For our solution

$$F_0 = \frac{\partial f}{\partial r}|_{r=R_0^+} = \frac{2\omega {R_0^+}^2 - 1}{\omega {R_0^+}^3 + R_0^+}$$

Which gives on comparing with boltzman distribution

$$T = \frac{\hbar}{4\pi k} \frac{2\omega {R_0^+}^2 - 1}{\omega {R_0^+}^3 + R_0^+}$$

Using the second law dS=dM/T

$$S = \frac{k}{\hbar}\left(\pi R_0^{+2} + \frac{2\pi}{\omega}\log(R_0^+)\right)$$

To consider quantum corrections, we go to higher order in expansion.

The next order equation is

$$\frac{\partial S_1}{\partial t} + f\frac{\partial S_1}{\partial r} = 2\frac{f(r)}{r}$$

Solution to higher order eqtns

- Since tunneling only involves considerations near Horizon we have $\frac{f(r)}{r} \sim 0$
- The solutions are of the form

$$S_1 \sim -E\left(1 - \int^r \frac{dr'}{r'}\right) = S_0$$

- But S_1 comes with a factor of hbar. hbar has dimensions of length^2 in our units. Only relevant scale is outer horizon. Therefore, the solution is

$$S = S_0 + a\frac{\hbar}{R_0^{+2}} S_0$$

Quantum Corrected Temperature

Now the tunneling probability becomes

$$P(E) = e^{-\frac{4\pi E\left(1+a\frac{\hbar}{R_0^{+2}}\right)}{\hbar F_0}}$$

Giving a temperature

$$T = \frac{\hbar}{4\pi k}\left(\frac{2\omega R_0^{+2} - 1}{\omega R_0^{+3} + R_0^{+}}\right)\left(1 + a\frac{\hbar}{R_0^{+2}}\right)^{-1}$$

And entropy becomes

$$S = \frac{k}{\hbar}(1 + 2a\hbar)\left(\pi R_0^{+2} + \frac{2\pi}{\omega}\log(R_0^{+})\right) - \frac{2\pi a k}{w}\frac{1}{R_0^{+2}}$$

Conclusion

We have studied quantum corrections to the entropy of a class of solutions to Horava's
Gravity.
 It turns out that the entropy is drastically changed from classical expression for small (Planck size) black holes. Since, Horava's correction regime is supposed to take over for only small black hole, controlling the ultra-violet behavior of the theory, where hcutR0+2 is of order 1, this term can not be ignored in this regime. This shows that th Entropy and thermodynamic behavior of a Horava black hole at Planck regime is fundamentally different from expectations from a simple quantum correction expectation of Einstein's GR.