Natural Inflation with Hidden Scale Invariance

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Inflation

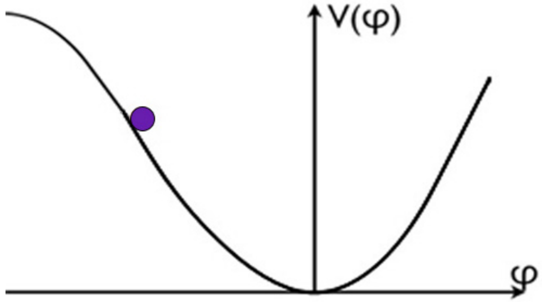
- It is a modification of standard Big Bang cosmology with a period of accelerated expansion:

$$w = -1, \quad H = \text{const.}, \quad a(t) = a_0 \exp(Ht)$$

- It was proposed by Guth (1981); Linde (1982); Albrecht and Steinhart (1982)
- It aims to solve:
  - The horizon problem
  - The flatness problem
  - Providing initial fluctuations
  - The magnetic monopole problem

Challenges of slow-roll Inflation

- The inflaton potential V(φ):
- It should satisfy the slow-roll conditions for successful inflation



- Obstacles in model building:
  - Fine-tuning problem:

$$\epsilon = \frac{M_p^2}{2}\left(\frac{V'_{\rm eff}}{V_{\rm eff}}\right)^2 \overset{?}{\ll} 1 \qquad \eta = M_p^2\left(\frac{V''_{\rm eff}}{V_{\rm eff}}\right) \overset{?}{\ll} 1$$

  - Higher-order operators

$$\left(\frac{\varphi}{\Lambda}\right)^n, \qquad \varphi \gg \Lambda$$

Scale Invariance

- Aim: to introduce scale invariance symmetry to V(φ)
- The scale transformation:

$$x^\mu \to t x^\mu$$

- The scale transformation on fields:

$$\phi \to t^{-1}\phi$$
$$A \to t^{-1}\phi$$
$$\psi \to t^{-3/2}\psi$$

- The transformed action: $S' = \int d^4x\, t^4 \mathcal{L}'$
- It is broken by massive scales

Scale Invariance

- Assume that the UV theory is scale-invariant
- Introduce the dilaton χ by rescaling massive scales:

$$M(\Lambda) \to M\left(\Lambda \frac{\chi}{f}\right)\frac{\chi}{f}$$

Λ: Wilsonian cut-off

- Breaking of scale invariance:

$$\chi \to \langle \chi \rangle$$

$$\lambda(\chi) = \lambda_0 + \beta_\lambda \ln\left(\frac{\chi}{\chi_0}\right) + \mathcal{O}\left(\beta_\lambda^2\right)$$

Scale Invariance and inflation

- Natural inflation (Freese et al. 1990)

$$\phi \to \phi + 2\pi f, \qquad V = \Lambda^4 \left(1 + \cos\left(\frac{\phi}{f}\right)\right)$$

- Scale-invariant inflationary models:
    - Garcia-Bellido et al. arXiv: 1107.2163
    - Kallosh et al. arXiv:1306.5220
    - Salvio et al. arXiv:1403.4226
    - Ellis et al. arXiv:1405.0271
    - Kannike et al. arXiv:1405.3987
    - Csaki et al. arXiv:1406.5192
    - Kannike et al. arXiv:1502.01334
    - Ozkan et al. arXiv:1507.03603
    - Kannike et al. EPS-HEP2015(2015)379
    - Farzinnia et al. arXiv:1512.05890
    - Rinaldi et al. arXiv:1512.07186

The model

- We propose a new class of inflation model with hidden scale invariance featuring the dilaton

> A SI model with generic scalar potential has a direction in field space that is flat in the classical limit, which is lifted by quantum corrections

- Inflation proceeds along the flat direction while the other fields remain in their sufficiently long-lived minima

The model

- The action of a Wilsonian EFT: an extended SM coupled to gravity at scale Λ

$$S_\Lambda = \int dx^4 \sqrt{-g} \left[\left(\frac{M_P^2}{2} + \sum_{i=1}^{N} \xi_i(\Lambda)\phi_i^2 \right) R - \frac{1}{2} \sum_{i=1}^{N} \partial_\mu \phi_i \partial^\mu \phi_i - V(\phi_i) + \ldots \right]$$

- The scalar potential: a generic polynomial in set of N scalar field (inc. the Higgs) respecting relevant symmetries

$$V(\phi_i) = \sum_{\{i_n\}} \lambda_{i_1,\ldots,i_n}(\Lambda)\phi_{i_1} \ldots \phi_{i_n}$$

- SI is broken by Λ, the Einstein-Hilbert term, and any dimensionful couplings

The model

- Suppose the underlying theory exhibits SI, broken at some high scale f
- The breaking lead to the pGB dilaton X appearing at low scales
- We incorporate X by rescaling the dimensionful parameters with powers of X/f equal to their mass dimensions

$$\Lambda \to \Lambda \frac{\chi}{f} \equiv \lambda\chi \ , \quad M_P^2 \to M_P^2 \left(\frac{\chi}{f}\right)^2 \equiv \xi\chi^2 \ ,$$

$$\lambda_{i_1,\ldots,i_n}(\Lambda) \to \lambda_{i_1,\ldots,i_n}(\Lambda\chi/f) \left(\frac{\chi}{f}\right)^{4-n} \equiv \sigma_{i_1,\ldots,i_n}(\lambda\chi)\chi^{4-n}$$

The model

- We introduce the hyper-spherical representation for the scalar fields

$$\{\phi_i,\ \chi\} \to \{\theta_i,\ \rho\} \qquad\qquad \begin{aligned} \phi_i &= \rho \cos(\theta_i) \prod_{k=1}^{i-1} \sin(\theta_k) \ , \quad (i = 1, 2, ..., N) \\ \chi &= \rho \prod_{k=1}^{N} \sin(\theta_k) \end{aligned}$$

- Assume that the θ fields are relaxed in their stable (or sufficiently long-lived) minimum $\{\theta_i^c\}$ at the early stage of the universe, ζ and σ are just functions of those

The model

- Effective single-field model with a quartic potential and non-minimal coupling, but without the standard Einstein-Hilbert term:

$$\bar{S}_\rho = \int dx^4 \sqrt{-g}\left[\zeta(\rho)\rho^2 R - \frac{1}{2}\partial_\mu\rho\partial^\mu\rho - V(\rho)\right]$$

$$V(\rho) = \sigma(\rho)\rho^4$$

ζ, σ: evaluated at $\{\theta_i^c\}$

- If V(ρ) describes our present vacuum:

$$\rho_0 = \frac{M_P}{\sqrt{2\zeta(\rho_0)}} \qquad V(\rho_0) = \frac{\sigma(\rho_0)M_P^4}{4\zeta^2(\rho_0)}$$

- In the Einstein frame V(ρ) is independent of ρ classically

Quantum corrections

- The couplings have ρ dependence of quantum origin, at one loop:

$$\begin{aligned}\zeta(\rho) &= \zeta_0 + \frac{(12\zeta_0+1)\sigma_0}{8\pi^2}\ln\left(\frac{\rho}{\rho_0}\right) \\ \sigma(\rho) &= \sigma_0 + \frac{9\sigma_0^2}{2\pi^2}\ln\left(\frac{\rho}{\rho_0}\right)\end{aligned}$$

Odintsov, arXiv:gr-qc/9302004

- In the Einstein frame, V(ρ) is constant in ρ near the conformal fixed points:

$$\sigma_0 \to 0 \qquad\qquad \zeta_0 \to -1/12$$

- Near these points SI is recovered and the theory is natural in the technical sense

The Einstein frame

- We take the action to the Einstein frame via the Weyl rescaling

$$g_{\mu\nu} \to \Omega^2 g_{\mu\nu} \qquad \Omega^2 = \frac{2\zeta\rho^2}{M_P^2}$$

- The kinetic term for ρ can be put into the canonical form with the field redefinition

$$\rho = \rho_0 \exp\left(\frac{\sqrt{\tilde{\zeta}}}{M_P}\varphi\right) \qquad \tilde{\zeta} = \frac{2\zeta}{1+12\zeta}$$

- The action in the Einstein frame

$$\bar{S}_\varphi = \int dx^4 \sqrt{-g}\left[\frac{M_P^2}{2}R - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V(\varphi)\right]$$

$$V(\rho(\varphi)) = \frac{M_P^4}{4}\frac{\sigma(\rho(\varphi))}{\zeta^2(\rho(\varphi))}$$

Observables

- The potential evaluated in the Einstein frame in the conformal limit

$$V(\varphi) \approx \frac{162C}{\pi^2} M_P^3 \varphi \qquad \sigma_0^2 \sqrt{\frac{2\zeta_0}{1+12\zeta_0}} \xrightarrow{\sigma_0 \to 0,\, \zeta_0 \to -1/12} C$$

- The SR parameters:

$$\epsilon_\star \equiv \frac{M_P^2}{2}\left(\frac{V_\varphi}{V}\right)^2\Bigg|_{\varphi=\varphi_\star} \qquad \eta_\star \equiv M_P^2 \frac{V_{\varphi\varphi}}{V}\Bigg|_{\varphi=\varphi_\star} \qquad N_\star \simeq \frac{1}{M_P}\int_0^{\varphi_\star} \frac{d\varphi}{\sqrt{2\epsilon}}$$

- The power spectrum of scalar perturbations, P_s, the tensor-to-scalar ratio, r, and the spectral index n_s

$$P_s = \frac{1}{24\pi^2 M_P^4}\frac{V_\star}{\epsilon_\star} \qquad r = 16\epsilon_\star \qquad n_s = 1 - 6\epsilon_\star + 2\eta_\star$$

Results

- The predictions derived from our theory, with the observable number of e-folds N* =35-60

$$n_s - 1 \approx -0.025 \left(\frac{N_\star}{60}\right)^{-1}$$

$$r = 0.0667 \left(\frac{N_\star}{60}\right)^{-1}$$

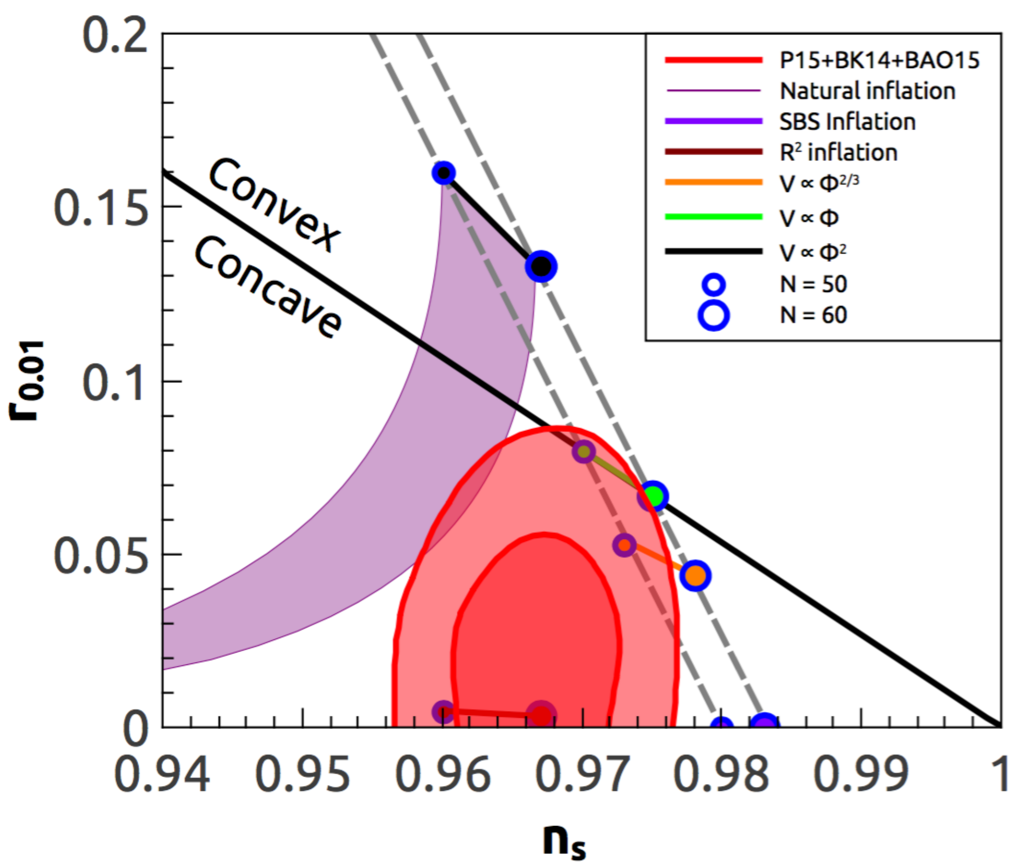
- The predictions are in reasonable agreement with the above estimate from the most recent analysis of cosmological data

$$n_s = 0.9669 \pm 0040 \quad (68\%\text{C.L.})$$

$$r_{0.01} < 0.0685 \quad (95\%\text{C.L.})$$

Results

- Analysis by Huang et al. arXiv:1512.07769:



Conclusions

- We proposed a new class of natural inflation models with hidden scale invariance realised via the dilaton field
- A very generic Wilsonian potential with an arbitrary number of scalar fields contains a flat direction in the classical limit, which is lifted by quantum corrections. Thus inflation can naturally proceed when the inflaton field evolves along this direction without fine-tuning
- In the conformal limit, the inflaton potential is linear
- While the results are in agreement with current observations, more accurate cosmological measurements may become critical in falsifying our scenario