

GRAVITATIONAL WAVES FROM LOW TEMPERATURE PHASE TRANSITIONS^a

Adrian Manning

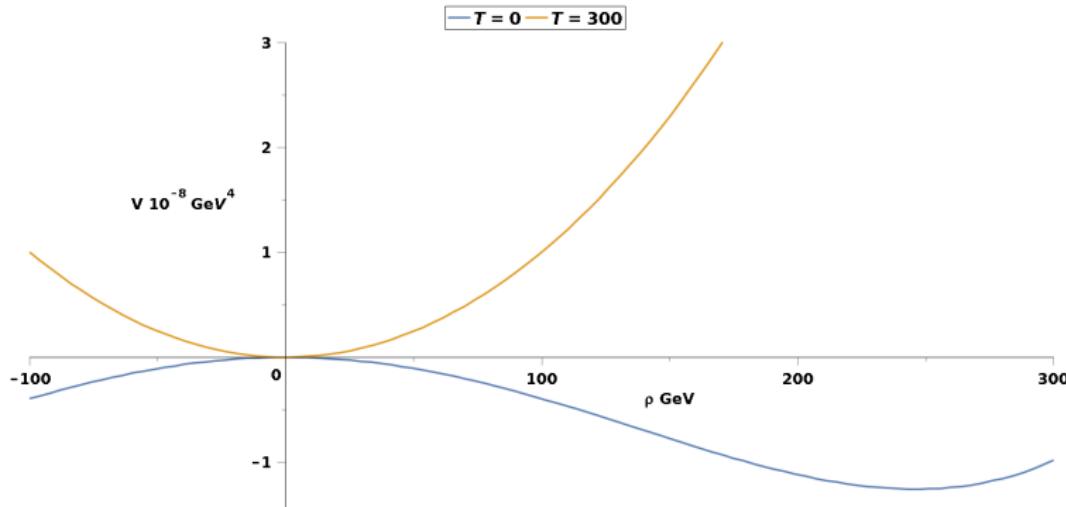
University of Sydney



^aA. Kobakhidze, A. Manning, J. Yue, arXiv:1607.00883 (2016)

FIRST-ORDER ELECTROWEAK PHASE TRANSITIONS

- We currently live in a universe with a broken Electroweak symmetry.
- We assume this occurs from the Higgs mechanism in the Standard Model.
- In the early universe this symmetry is restored.
- If the breaking of this symmetry deviates from the standard model, we can potentially measure the result through gravitational waves.



- The Higgs potential determines the type of phase transition.
- Generalise the Higgs mechanism allowing for the Higgs-like gauge singlet, ¹²

$$V(\rho) = -\frac{\mu^2}{2}\rho^2 + \frac{\kappa}{3}\rho^3 + \frac{\lambda}{4}\rho^4$$

- Minimum

$$V'(\rho) = -\mu^2\rho + \kappa\rho^2 + \lambda\rho^3 = 0$$

- Higgs mass

$$m_h^2(\rho) := V''(\rho) = -\mu^2 + 2\kappa\rho + 3\lambda\rho^2$$

- Tree Level Relations

$$\mu^2 = \frac{1}{2} (m_h^2 + v\kappa),$$

$$\lambda = \frac{1}{2v^2} (m_h^2 - v\kappa) > 0 \quad (\text{stability})$$

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- 1-Loop Potential

$$V^{(1)}(\rho, T) = V_0(\rho) + V_{CW}(\rho) + V_T(\rho, T) + V_{daisy}(\rho, T)$$

- Coleman-Weinberg Potential

$$V_{CW}(\rho) = \sum_{i=W,Z,t,h} n_i \frac{m_i^4(\rho)}{64\pi^2} \left(\ln \left(\frac{m_i^2(\rho)}{\mu_R^2} \right) - \frac{3}{2} \right)$$

- Finite Temperature Contribution

$$V_T(\rho, T) = \frac{T^4}{2\pi^2} \sum_{i=W,Z,t,h} n_i J \left[\frac{m_i^2(\rho)}{T^2} \right]$$

with

$$J[m^2 \beta^2] := \int_0^\infty dx x^2 \ln \left[1 - (-1)^{2s+1} e^{-\sqrt{x^2 + \beta^2 m^2}} \right]$$

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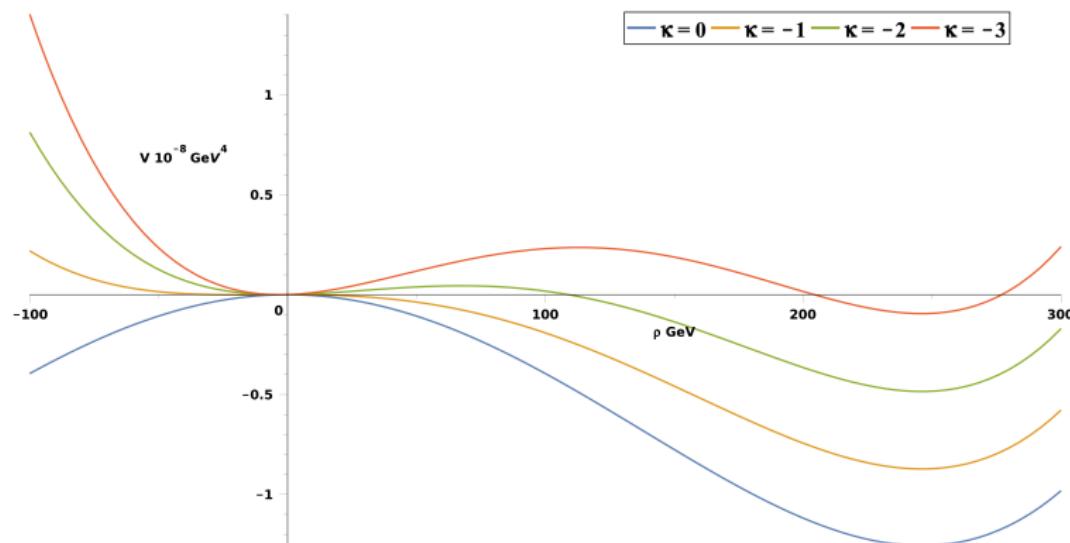
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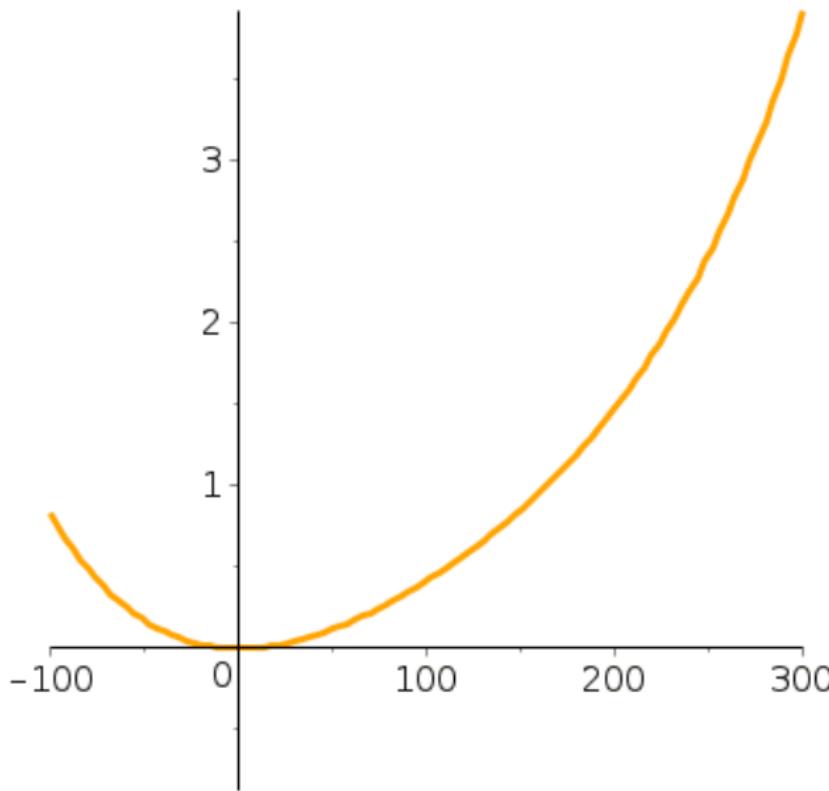


κ in units of $\frac{m_h^2}{|v|}$

The Potential

$$\kappa = -\frac{m_h^2}{v}$$

$T = 200.$



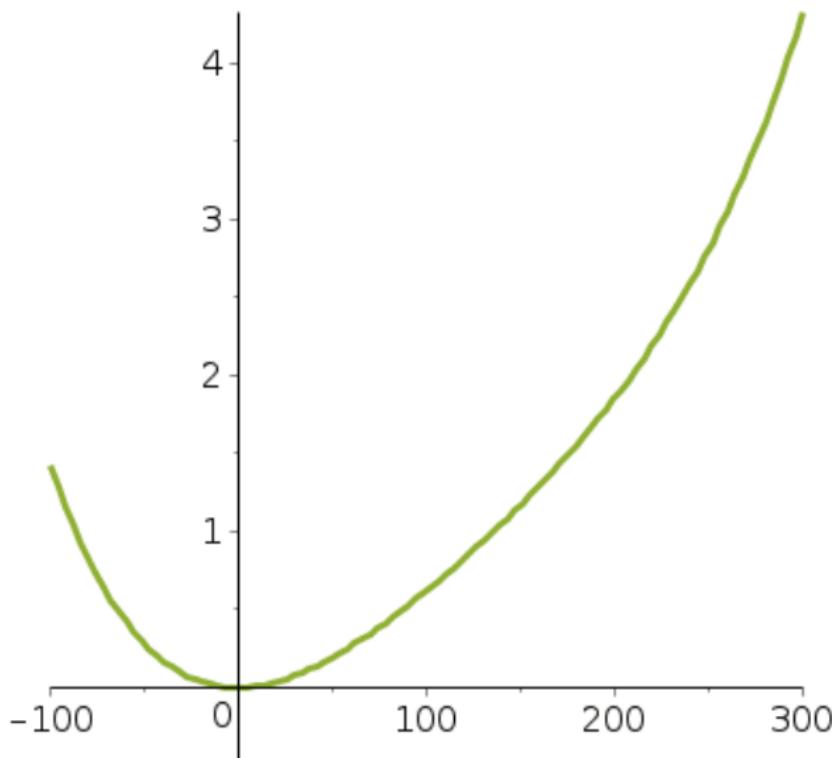
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$$\kappa = -2 \frac{m_h^2}{v}$$

$T = 200.$

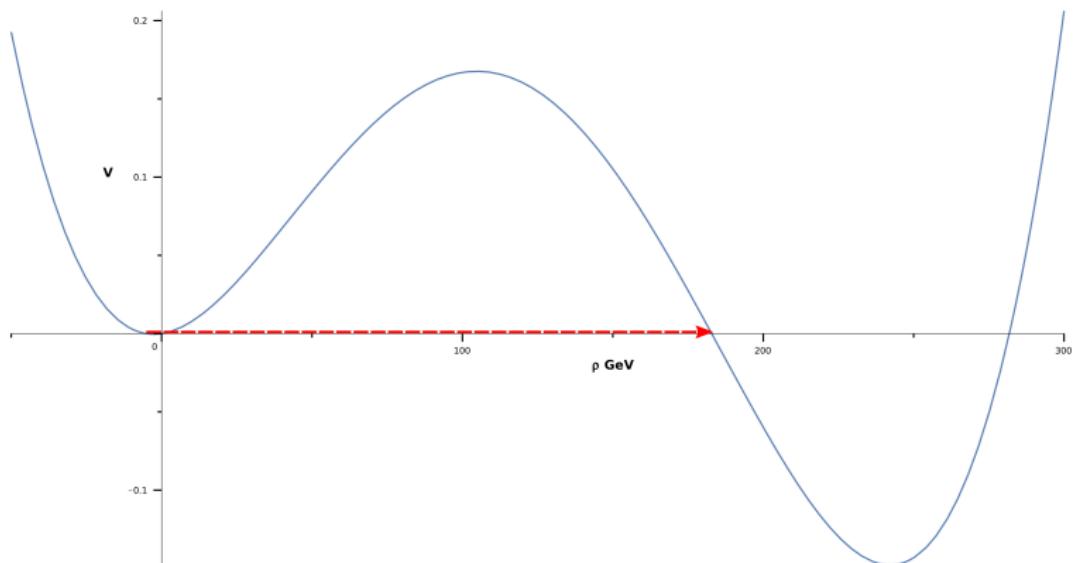


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BUBBLE FORMATION

At a given temperature bubbles can form from a phase transition when the field tunnels from one vacuum state to another



- Nucleation rate is proportional to the tunnelling action:

$$\Gamma \propto T^4 e^{-S}$$

- General Action³

$$S = 4\pi \int_0^{T^{-1}} dt_E \int_0^\infty dr r^2 \left[\frac{1}{2} \left(\frac{\partial \rho}{\partial t_E} \right)^2 + \frac{1}{2} \left(\frac{\partial \rho}{\partial r} \right)^2 + V(\rho, T) \right]$$

Common Approximations

$O(3)$ symmetry - $\frac{S_3}{T}$

$O(4)$ symmetry - $S_4 = S_3(T=0)$

$$S_3[\rho] = \int_0^\infty d^3x \left[\frac{1}{2} (\nabla \rho)^2 + V(\rho, T) \right]$$

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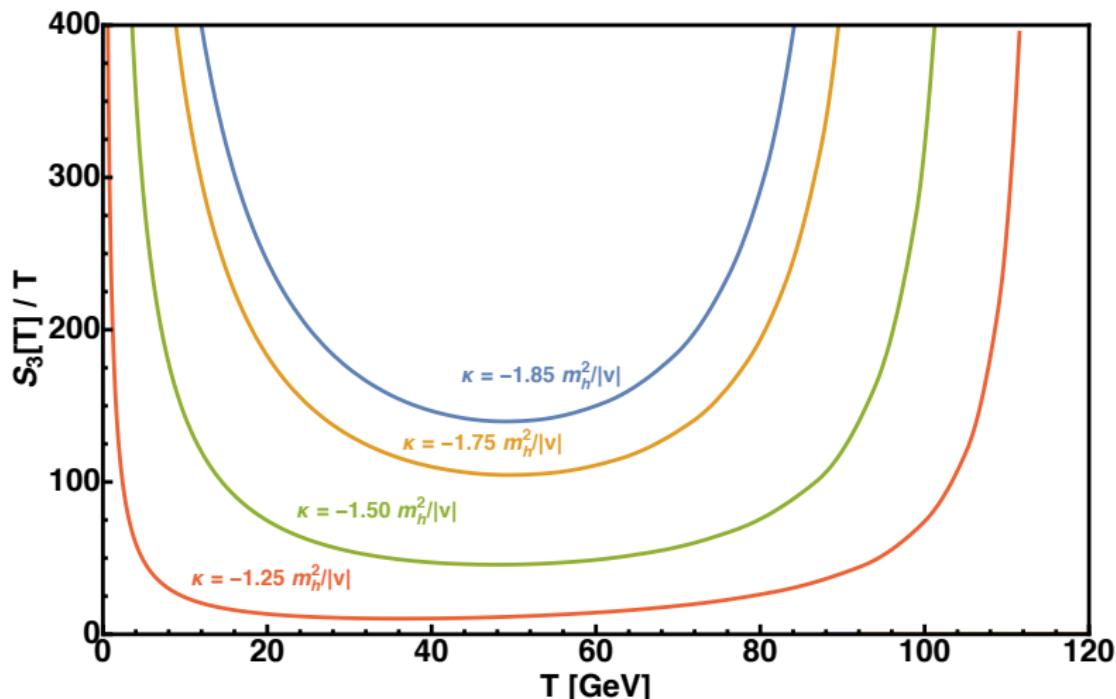
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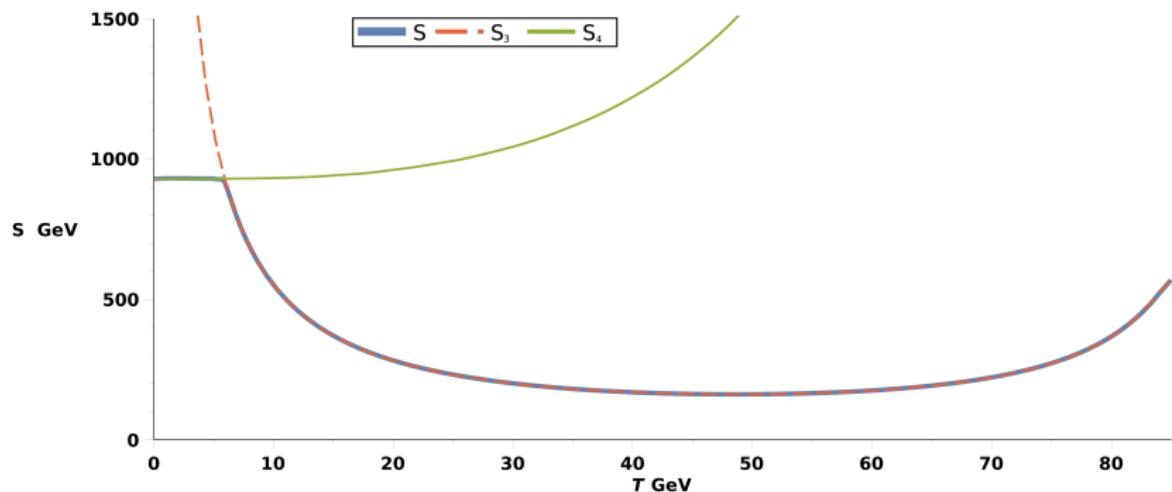
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Low Temperature Action Approximations



GRAVITATION WAVE PRODUCTION

Gravitational Wave Parameters

Typical parameters to determine characteristics of gravitational wave production

- β - Inverse of the average size of a produced bubble

$$\tilde{\beta} = \frac{\beta}{H_n} = \frac{1}{H_n d} \approx T \frac{d(S_3/T)}{dT} \Big|_{T=T_n}$$

- α - Ratio of latent heat density to radiation density

$$\alpha = \frac{\epsilon_*}{\rho_{rad}} = \frac{1}{\frac{\pi^2}{30} g_* T_*^4} \left(-V_{true}(T_*) + T \frac{d}{dT} V_{true}(T) \Big|_{T=T_*} \right)$$

$\kappa(m_h^2/ v)$	T _n GeV	α	$\tilde{\beta}$	$\frac{\rho_c}{T_c}$
-1.25	106.06	0.037	1771.15	1.64
-1.5	91.85	0.057	989.13	1.87
-1.75	72.03	0.11	307.57	2.11
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Three main sources

- Bubble Collisions ⁴
- Sound Waves in the Plasma ⁵
 - Acoustic production of gravitational waves after bubble collisions.
- Turbulence in the Plasma ⁶
 - Ionized plasma can create mhd turbulence which source Gravitational waves

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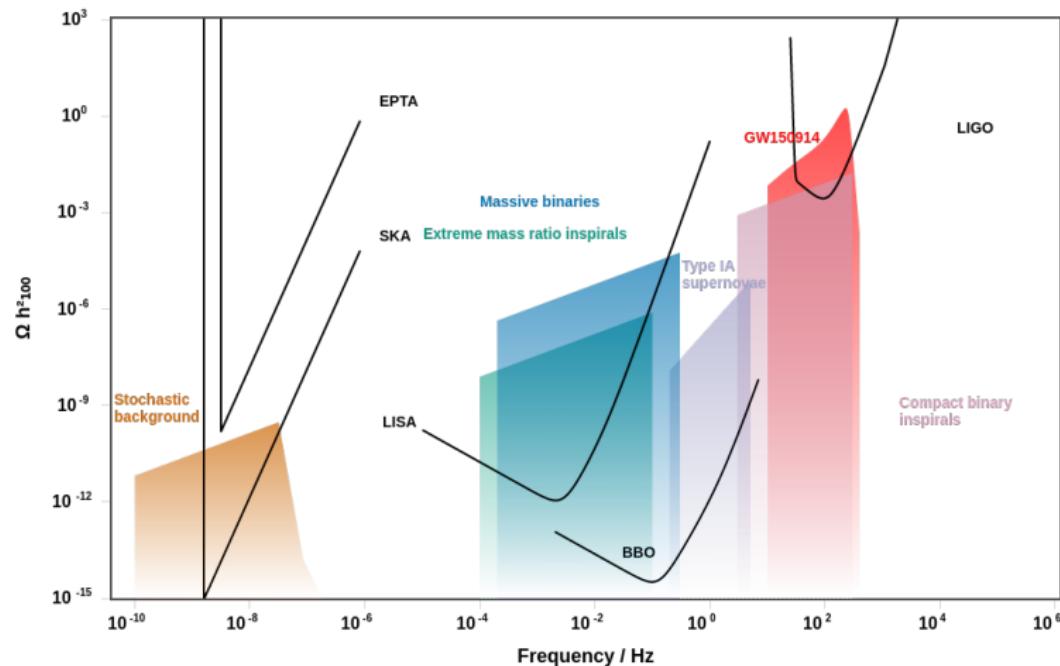
Measuring Gravitational Waves

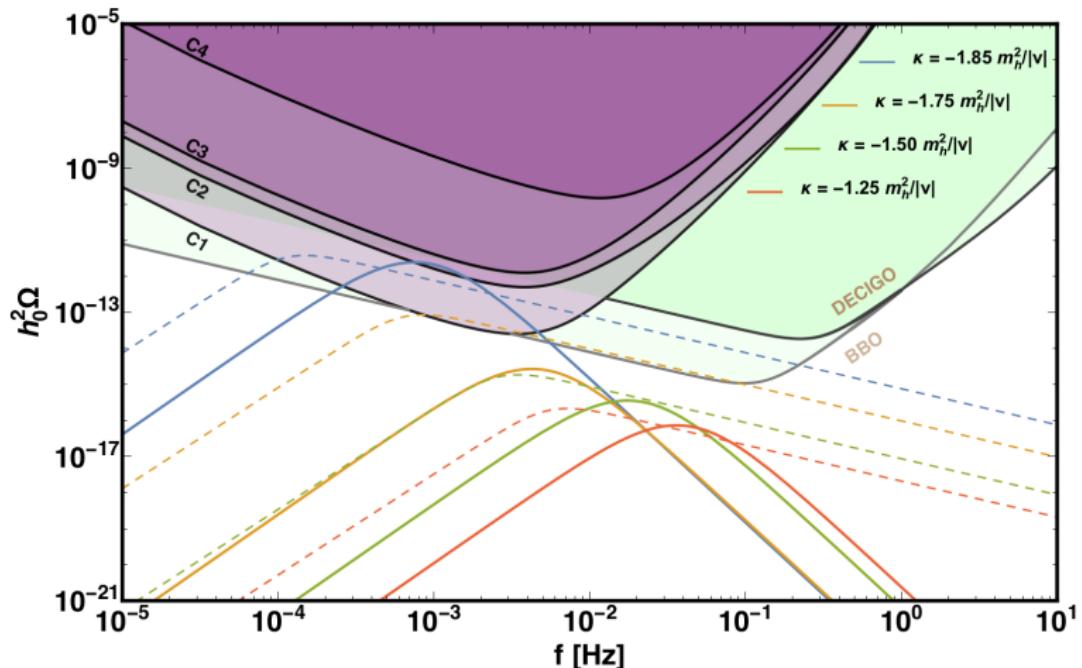
SKA

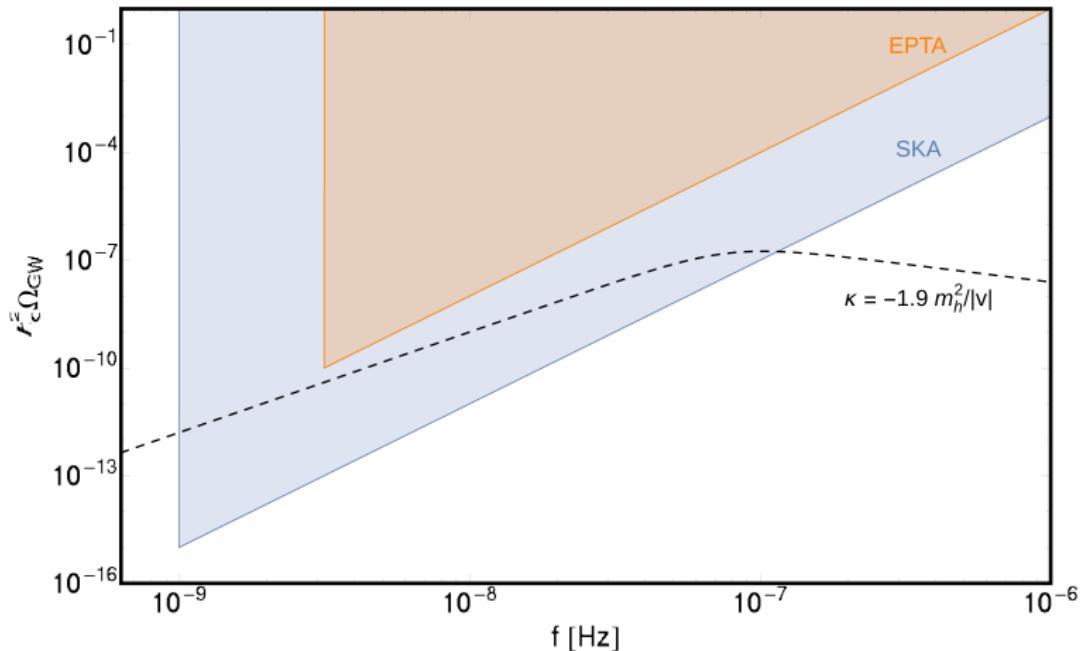
eLISA

BBO

LIGO







- First-order electroweak phase transitions can occur in the early universe.
- Bubbles nucleated from these can give rise to detectable gravitational waves.
- Can have a super-cooling effect, creating low-temperature phase transitions.
- Low temperature phase transition could potentially be detected in future PTA.
- Gravity wave measurements can therefore also test the Standard Model.

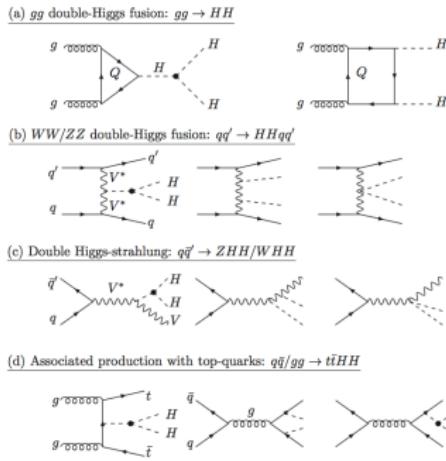
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- Double Higgs production⁷
- Sensitive to other anomalous couplings ($t\bar{t}h$, VVh , $VVhh$)



[JHEP 04 (2013) 151]

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Table 1-22. Signal significance for $pp \rightarrow HH \rightarrow bb\gamma\gamma$ and percentage uncertainty on the Higgs self-coupling at future hadron colliders, from [102].

	HL-LHC	HE-LHC	VLHC
\sqrt{s} (TeV)	14	33	100
$\int \mathcal{L} dt$ (fb $^{-1}$)	3000	3000	3000
$\sigma \cdot \text{BR}(pp \rightarrow HH \rightarrow bb\gamma\gamma)$ (fb)	0.089	0.545	3.73
S/\sqrt{B}	2.3	6.2	15.0
λ (stat)	50%	20%	8%

[Snowmass Higgs Report 1310.8361]

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- Are we sure the Higgs mechanism is correct?

$$\Sigma = \exp \left[-\frac{i}{2} (\sigma^a \pi^a - \mathbb{1} \pi^3) \right] \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Σ parameterizes the coset group $SU(2)_L \times U(1)_Y / U(1)_{EM}$ and transforms non-linearly.⁸

- We can then have $SU(2)_L \times U(1)_Y$ terms such as

$$D_\mu \Sigma D^\mu \Sigma + m_t \bar{Q}_L \Sigma^\dagger t_R$$

- We want unitarity in the gauge sector. Introduce a singlet scalar, ρ ⁹

$$\rho^2 D_\mu \Sigma D^\mu \Sigma + m_t \bar{Q}_L \Sigma^\dagger t_R + y_t \bar{Q}_L \rho \Sigma^\dagger t_R$$

- Identify

$$H = \rho \Sigma$$

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- Are we sure the Higgs mechanism is correct?

$$\Sigma = \exp \left[-\frac{i}{2} (\sigma^a \pi^a - \mathbb{1} \pi^3) \right] \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Σ parameterizes the coset group $SU(2)_L \times U(1)_Y / U(1)_{EM}$ and transforms non-linearly.⁸

- We can then have $SU(2)_L \times U(1)_Y$ terms such as

$$D_\mu \Sigma D^\mu \Sigma + m_t \bar{Q}_L \Sigma^\dagger t_R$$

- We want unitarity in the gauge sector. Introduce a singlet scalar, ρ ⁹

$$\rho^2 D_\mu \Sigma D^\mu \Sigma + m_t \bar{Q}_L \Sigma^\dagger t_R + y_t \bar{Q}_L \rho \Sigma^\dagger t_R$$

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$$H = \rho \Sigma$$

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$$V(\rho) = -\frac{\mu^2}{2}\rho^2 + \frac{\kappa}{3}\rho^3 + \frac{\lambda}{4}\rho^4$$

- Minimum

$$V'(\rho) = -\mu^2\rho + \kappa\rho^2 + \lambda\rho^3 = 0$$

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$$m_h^2(\rho) := V''(\rho) = -\mu^2 + 2\kappa\rho + 3\lambda\rho^2$$

- Tree Level Relations

$$\mu^2 = \frac{1}{2} (m_h^2 + v\kappa),$$

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$$\Sigma(x) \rightarrow g\Sigma(x)h^{-1}$$

$$\Sigma'(x) = e^{i\alpha^a \frac{\sigma^a}{2}} \Sigma(x) e^{-\frac{i}{2}\beta\sigma^3}$$

Determining the Waveform

- Consider general 0 PN waveform

$$h_{+,\times} = A_{+,\times}(t) \cos(\phi_{+,\times}(t))$$

- Use a stationary phase approximation to take the Fourier transform

$$\tilde{h}_{+,\times} = \frac{1}{2} A_{+,\times}(t_*) \sqrt{\frac{2\pi}{\dot{\phi}_{+,\times}(t_*)}} e^{-i\psi_{+,\times}}$$

- The phase, $\Psi_{+,\times}$ has been calculated to 3.5 PN¹⁰

$$\psi_{+,\times}(f) = 2\pi f t_c - \phi_c - \frac{\pi}{4} + \sum_{j=0}^7 \psi_j f^{(j-5)/3}$$

- PN order coefficients defining the waveform

$$\psi_j = \frac{3}{128\nu} (\pi M)^{(j-5)/3} \alpha_j$$

$$\alpha_0 = 1$$

$$\alpha_3 = -16\pi$$

$$\alpha_1 = 0$$

$$\alpha_4 = \frac{15293365}{508032} + \frac{27145}{504}\nu + \frac{3085}{72}\nu^2$$

$$\alpha_2 = \frac{3715}{756} + \frac{55}{9}\nu$$

$$\dots$$

¹⁰T. Damour, B. Iyer and B. Sathyaprakash, Phys. Rev. D 63.4 (2001)

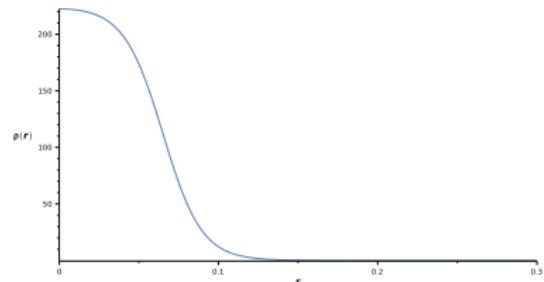
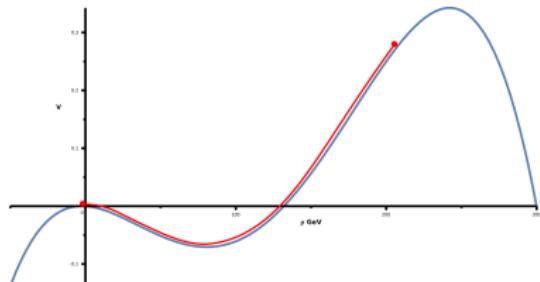
We seek $O(3)$ symmetric solutions to S_3 , i.e

$$\frac{d^2\rho}{dr^2} + \frac{2}{r} \frac{d\rho}{dr} - \frac{\partial V(\rho, T)}{\partial \rho} = 0$$

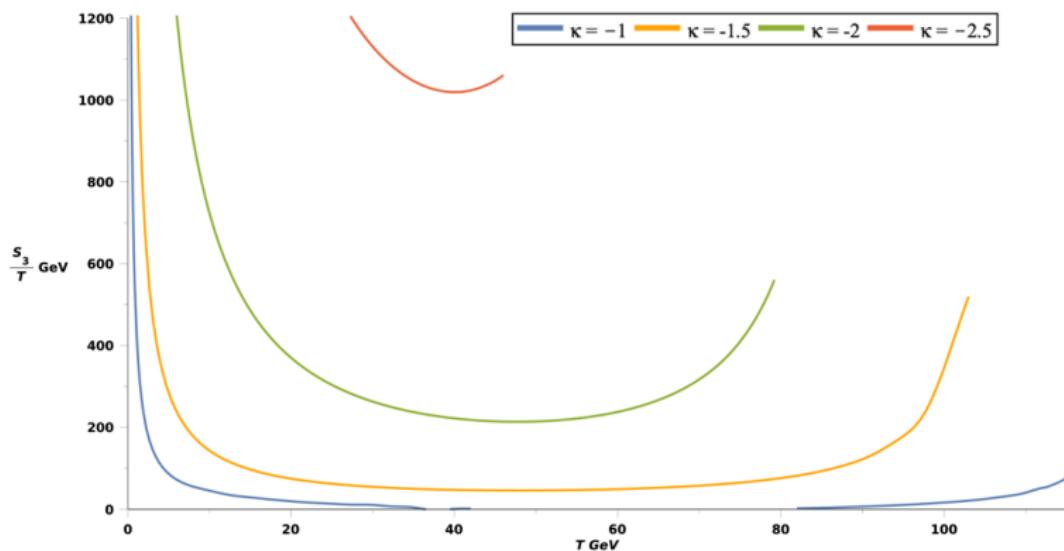
where

$$\left. \frac{d\rho}{dr} \right|_{r=0} = 0, \quad \rho|_{r=\infty} = v_{false}$$

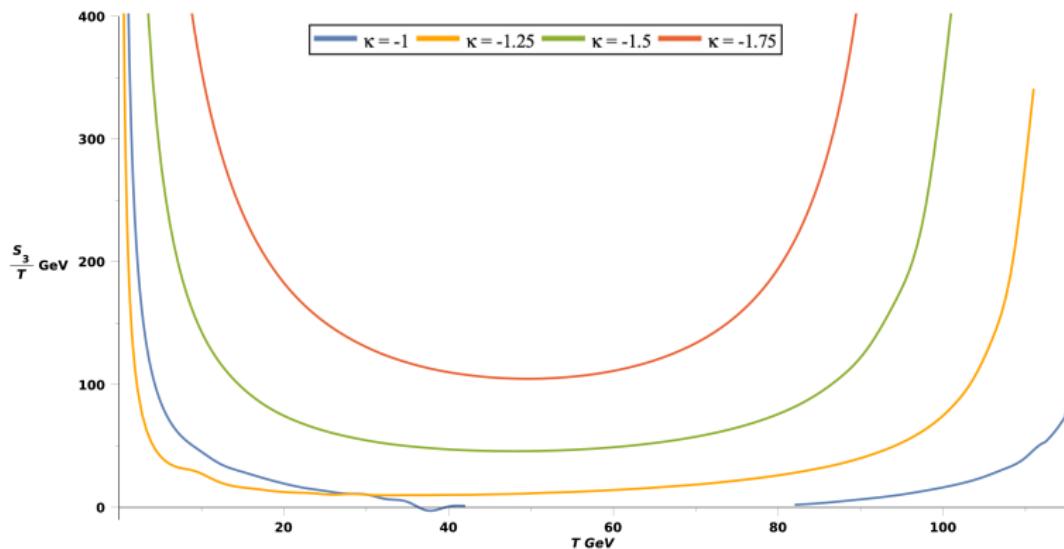
Solve by “rolling” down the inverted potential

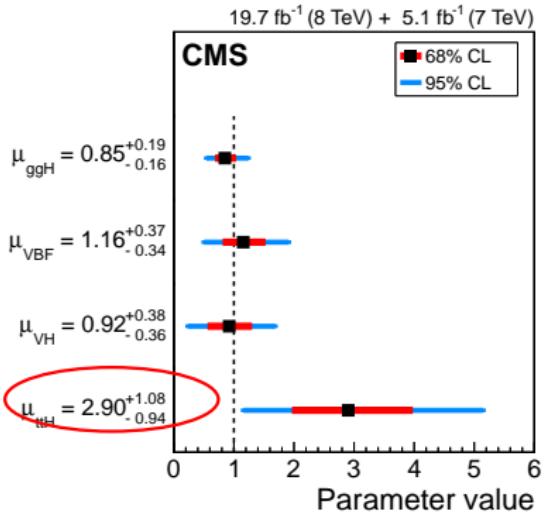


$$P \sim \int_{T_n}^{T_c} \left(\sqrt{\frac{45}{4\pi^3 g_*}} \frac{M_P}{T} \right)^4 e^{-S_3/T} \frac{dT}{T} \sim \mathcal{O}(1)$$

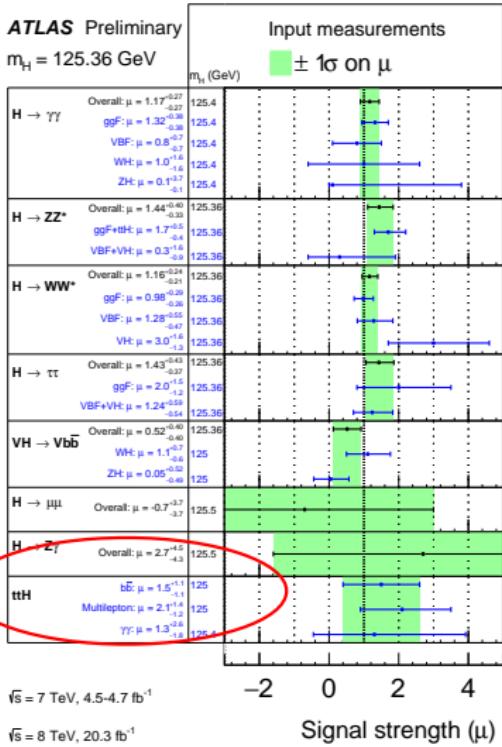


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[EPJC 75 (2015) 212]



[ATLAS-CONF-2015-007]

Gravitational Wave Parameters

Typical parameters to determine characteristics of gravitational wave production

- β - Inverse duration of phase transition

$$\tilde{\beta} = \frac{\beta}{H_n} = T \frac{d(S_3/T)}{dT} \Big|_{T=T_n}$$

- α - Ratio of latent heat density to radiation density

$$\alpha = \frac{\epsilon_*}{\rho_{rad}} = \frac{1}{\frac{\pi^2}{30} g_* T_*^4} \left(-V_{true}(T_*) + T \frac{d}{dT} V_{true}(T) \Big|_{T=T_*} \right)$$

$\kappa(m_h^2/ v)$	T_n GeV	α	$\tilde{\beta}$	$\frac{\rho_c}{T_c}$
-1.25	106.06	0.037	1771.15	1.64
-1.5	91.85	0.057	989.13	1.87
-1.75	72.03	0.11	307.57	2.11
-1.85	56.10	0.24	69.50	2.21
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$$\begin{aligned}
V(\rho, T) = & -\frac{\mu^2}{2}\rho^2 + \frac{\kappa}{3}\rho^3 + \frac{\lambda}{4}\rho^4 + \frac{T^4}{2\pi^2} \left\{ \frac{\pi^2}{12T^2} \left(6\frac{g_2^2}{4}\rho^2 + 3\frac{g_2^2 + g_1^2}{4}\rho^2 + \color{blue}{3\lambda\rho^2 + 2\kappa\rho^3} \right. \right. \\
& - \frac{\pi^2}{24T^2} \left(-12\frac{y_t^2}{2}\rho^2 \right) \\
& - \frac{\pi}{6T^3} \left[3 \left(\frac{g_2^2 + g_1^2}{4}\rho^2 \right)^{3/2} + 6 \left(\frac{g_2^2}{4}\rho^2 \right)^{3/2} + \color{red}{(3\lambda\rho^2 + 2\kappa\rho - \mu^2)^{3/2}} \right] \\
& - \frac{1}{32T^4} \left[3 \left(\frac{g_2^2 + g_1^2}{4}\rho^2 \right)^2 \ln \left(\frac{\mu_R^2}{c_B T^2} \right) + 6 \left(\frac{g_2^2}{4} \right)^2 \ln \left(\frac{\mu_R^2}{c_B T^2} \right) \right. \\
& \left. \left. + \color{red}{(3\lambda\rho^2 + 2\kappa\rho - \mu^2)^2} \ln \left(\frac{\mu_R^2}{c_B T^2} \right) - 12 \left(\frac{y_t^2}{2}\phi_c^2 \right)^2 \ln \left(\frac{\mu_R^2}{c_F T^2} \right) \right] \right\} \tag{1}
\end{aligned}$$