The effects of monopoles on the electroweak phase transition

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Outline

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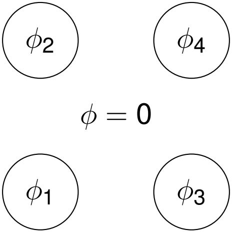
Motivation

- Sakharov's conditions for baryogenesis require out-of-equilibrium processes
- This is possible in a strongly first order phase transition
- The EWPT is second order in the standard model
- The energy density of monopoles can contribute to the energy of the broken phase, enhancing the phase transition

Outline

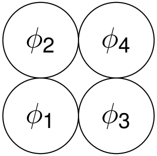
The Kibble Mechanism

- At $T = T_c$, domains of the broken phase will appear
- The higgs field in each domain takes independent directions on the vacuum manifold



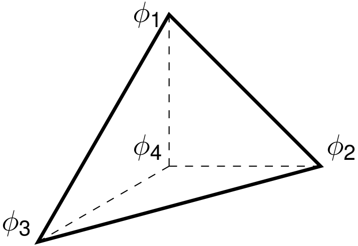
The Kibble mechanism

- As the Higgs field is continuous, it must be interpolated at the intersections.
- Consider an intersection of four of these domains:



The Kibble mechanism

- In field space, these points form the vertices of a tetrahedron.
- This tetrahedron should be shrunk to a point at the intersection.
- If these cannot be shrunk to a point continuously, a topological defect in the form of a monopole which continuously joins the two minima.
- The tetrahedron is homotopically equivalent to S^2.
- Therefore, $\pi_2(M_{\text{vac}}) \neq 0$ implies the existence of monopoles



The standard model

- For the standard model, $M_{\text{vac}} = (SU(2) \times U(1)_Y)/U(1)_Q$
- $\pi_2(M_{\text{vac}}) = \pi_2(S^3) = 0$
- No electroweak monopoles?

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The Ansatz

- Cho and Maison (1997) found electroweak monopoles through the ansatz:

$$
\begin{aligned}
\phi &= \frac{1}{\sqrt{2}}\rho\xi \\
\rho &= \rho(r) \\
\xi &= i\begin{pmatrix} \sin(\theta/2)e^{-i\varphi} \\ -\cos(\theta/2) \end{pmatrix} \\
A_\mu &= \frac{1}{g}A(r)\partial_\mu t\hat{\phi} + \frac{1}{g}(f(r)-1)\hat{\phi}\times\partial_\mu\hat{\phi} \\
B_\mu &= -\frac{1}{g'}B(r)\partial_\mu t - \frac{1}{g'}(1-\cos\theta)\partial_\mu\varphi
\end{aligned}
$$

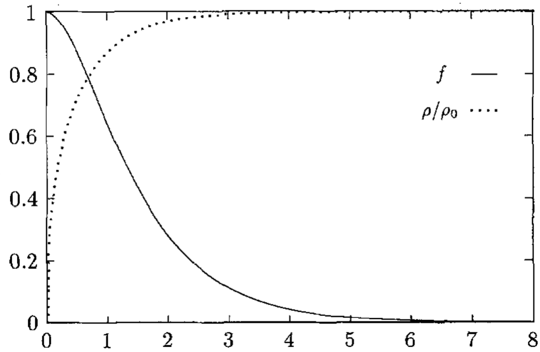
The Ansatz

$$\xi = i \begin{pmatrix} \sin(\theta/2)e^{-i\varphi} \\ -\cos(\theta/2) \end{pmatrix}$$

- Note that there is a missing phase
- The $U(1)_Y$ gauge freedom is used to remove this phase.
- $M_{\text{vac}} = SU(2)/U(1) \cong \mathbb{C}P^1$
- $\pi_2(M_{\text{vac}}) = \mathbb{Z}$

Solution

- Simple solution: $A = B = 0$ (Cho & Maison, 1997)
- $h = \frac{4\pi}{e}$
- Horizontal axis is $M_W r$.



The energy

$$E = E_0 + E_1$$

$$E_0 = 4\pi \int_0^\infty \frac{dr}{2r^2} \left\{ \frac{1}{g'^2} + \frac{1}{g^2}(f^2 - 1)^2 \right\}$$

$$\begin{aligned}
E_1 &= 4\pi \int_0^\infty dr \left\{ \frac{1}{2}(r\dot{\rho})^2 + \frac{1}{g^2}\left(\dot{f}^2 + \frac{1}{2}(r\dot{A})^2 + f^2 A^2 \right) \right. \\
&= \frac{1}{2g'^2}(r\dot{B})^2 + \frac{\lambda r^2}{8}(\rho^2 - \rho_0^2)^2 \\
&\left. + \frac{1}{4} f^2 \rho^2 + \frac{r^2}{8}(B - A)^2 \rho^2 \right\}
\end{aligned}$$

- The first term of E_0 is divergent at the origin.

Regularisation

- Cho, Kim and Yoon(2015) proposed a regularisation of the form:

$$g' \to \frac{g'}{\sqrt{\epsilon}}$$

$$\epsilon = \left(\frac{\phi}{\phi_0}\right)^n$$

- However, g' becomes non-peturbative as $\phi \to 0$.
- This is undesirable in an EFT framework.
- We instead propose a simple Wilsonian high energy cutoff.
- Mass is a free parameter.

Outline

The electroweak phase transition

- The Gibbs free energy:

$$G_u = V(0)$$
$$G_b = V(\phi_c(T)) + E_{\text{monopoloes}}$$

- At the critical temperature:

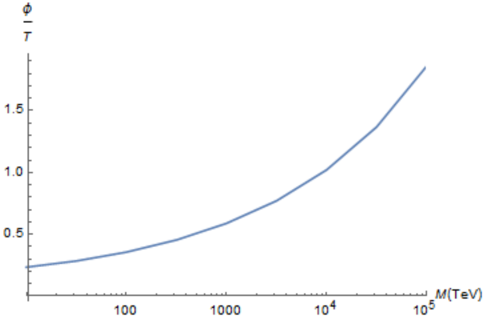
$$V(0) = V(\phi_c(T_c)) + E_{\text{monopoles}}$$

- Assuming $T << M$,the monopoles are decoupled and $E_{monopoles} = M \times n_M$

The initial density

- $n_M \approx \frac{1}{d^3}$ where d is the separation of two uncorrelated monopoles.
- This is chosen to be the Coulomb capture distance.
- Hence, $n_M \approx \left(\frac{4\pi}{h^2}\right)^3 T^3$

Results



- A strong electroweak phase transition for $M > 10^4$TeV.

Outline

The constraint

- The monopole density should not dominate the universe at the time of helium synthesis. This implies:
- $\frac{n}{T^3}\Big|_{T=1\text{MeV}} < \frac{1\text{MeV}}{M}$
- Hence, the evolution of the number density over time must be considered.

The number density at lower temperatures

- As the temperature cools,

$$\frac{dn}{dt} = -Dn^2 - 3\frac{\dot{R}}{R}n$$

- D is estimated by considering monopoles drifting in the plasma of charged particles.
- Preskill (1979) showed that

$$\frac{n}{T^3} = \frac{1}{Bh^2}\left(4\pi/h^2\right)^2 \frac{M}{CM_{pl}}$$

- $B = \frac{3}{4\pi^2}\zeta(3)\sum_i (hq_i/4\pi)^2$
- $C = (45/4\pi^3 N)^{1/2}$

Nucleosynthesis constraint

- $\frac{n}{T^3} = \frac{1}{Bh^2}\left(4\pi/h^2\right)^2 \frac{M}{CM_{pl}} < \frac{1\text{MeV}}{M}$
- This constrains the mass of the monopole to $M < O(10^8)$TeV

Summary and future work

- The phase transition can be enhanced by electroweak monopoles while remaining under the nucleosynthesis constraints.
- Other forms of regularisation can be analysed
- Estimating the monopole density in the context of bubble collisions could also prove interesting.