



Naturalness of the relaxion mechanism

A. Fowlie, C. Balazs, G. White, L. Marzola, and M. Raidal, JHEP 08, 100 (2016), arXiv:1602.03889 [hep-ph]

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Introduction to relaxation model

The relaxation mechanism: a clever new idea

Developed by Kaplan et al.¹ last year from related ideas by Abbott in the 1980s² and Dvali a decade ago.³

It's a very, very clever idea (Raman Sundrum)

It's definitely clever (Nima Arkani-Hamed)

Comparisons drawn to Dirac's large number hypothesis⁴ — large ratios explained by dynamics/age of Universe.

¹P. W. Graham, et al., Phys. Rev. Lett. 115, 221801 (2015), arXiv:1504.07551 [hep-ph].

²L. F. Abbott, Phys. Lett. B150, 427 (1985).

³G. Dvali, and A. Vilenkin, Phys. Rev. D70, 063501 (2004), arXiv:hep-th/0304043 [hep-th], G. Dvali, Phys. Rev. D74, 025018 (2006), arXiv:hep-th/0410286 [hep-th].

⁴P. A. M. Dirac, Nature 139, 323 (1937).

SM generic prediction: weak scale \sim Planck scale

- No symmetries protect scalar mass-squared parameter from quantum corrections
- In the SM as an EFT below the Planck scale, Higgs mass-squared parameter receives Planck scale quadratic corrections
- SM generic prediction is that $m_H^2 \approx m_0^2 + \beta M_P^2 \sim M_P^2$
- But, of course, we observe that $m_H^2 \lll M_P^2$

Hierarchy solutions

Traditional approaches:

- Protect weak scale with a symmetry/approximate symmetry — supersymmetry/pseudo-Goldstone
- No fundamental scalars near the weak scale — compositeness
- Close the gulf between the weak and Planck scales — large extra dimensions

Heretical approaches:

- Reinterpret quadratic divergences — physical naturalness/classical scale invariance, $\int \frac{d^4k}{k^2} \stackrel{!}{=} 0$
- Nothing — fine-tuning of $m_0^2 + \beta M_P^2 \sim (100 \text{ GeV})^2$

New, dynamical approach — the relaxion mechanism

Utilise peculiar interplay between axion-like field and Higgs.⁵

Relaxion-dependent Higgs mass

$$V = (\mu^2 - \kappa \langle a \rangle \phi) h^2 - m_b^3 \langle h \rangle \cos\left(\frac{\phi}{f}\right) - m^2 \langle a \rangle \phi + \lambda h^4$$

Dynamics imply that relaxion field halts once $m_H^2 \lesssim 0$ such that (generic?) prediction

$$|m_H^2| = |\mu^2 - \kappa \langle a \rangle \phi| \lll M_P^2$$

Whilst $m_H^2 = 0$ doesn't enhance symmetry, it is a critical point in dynamics as $m_H^2 < 0$ triggers EWSB and a *backreaction*.

⁵P. W. Graham, et al., Phys. Rev. Lett. 115, 221801 (2015), arXiv:1504.07551 [hep-ph].

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Backreaction to Higgs VEV

$$V = (\mu^2 - \kappa \langle a \rangle \phi) h^2 - m_b^3 \langle h \rangle \cos\left(\frac{\phi}{f}\right) - m^2 \langle a \rangle \phi + \lambda h^4$$

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New, *dynamical* approach — the relaxion mechanism

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Periodic barrier for axion field

$$V = (\mu^2 - \kappa \langle a \rangle \phi) h^2 - m_b^3 \langle h \rangle \cos\left(\frac{\phi}{f}\right) - m^2 \langle a \rangle \phi + \lambda h^4$$

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Linear slope for axion field

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Ordinary SM Higgs quartic

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How the relaxion dynamics insure $m_H \lll M_P$

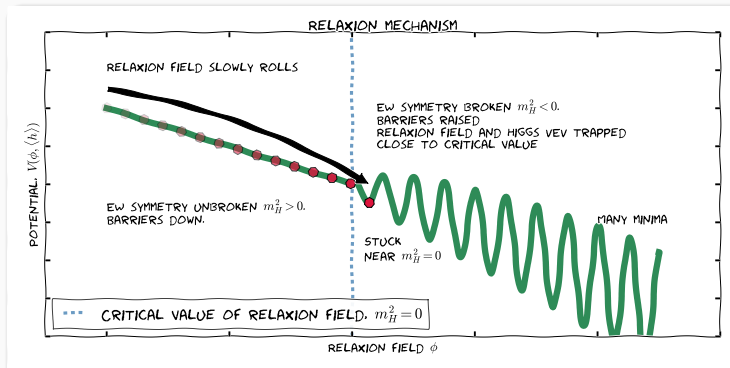


Figure 1: Higgs mass is field dependent. Relaxion field dynamics halt at $m_H \lll M_P$.

Require extra ingredient: Hubble friction, so that relaxion field dissipates energy and cannot surmount barriers.

Approximate solution for dynamics

The potential involves a cosine: the tadpoles are transcendental and have no closed-form solutions.

- OK: just write squiggles \sim
- **Better: find intervals bounding solutions to tadpoles and solve numerically**
- Better still: solve the dynamics by evolving initial conditions

We chose **middle** approach. Sidesteps issues about prior for initial conditions (e.g. constructing a Liouville measure⁶).

Assume that, because of Hubble friction, ultimately field always stuck in first minima. This may not be true.⁷

⁶G. W. Gibbons, et al., Nucl. Phys. B281, 736 (1987).

⁷J. Jaeckel, et al., Phys. Rev. D93, 063522 (2016), arXiv:1508.03321 [hep-ph].

Find intervals by graphing

Solving tree-level tadpoles, we find

$$\sin(\phi/f) = \frac{f\kappa\langle a \rangle}{m_b^3} \left(\frac{m^2/\kappa + \langle h \rangle^2}{\langle h \rangle} \right)$$

For phenomenologically viable points, confirm literature expressions for weak scale

$$\langle h \rangle \approx f \frac{m^2 \langle a \rangle}{m_b^3}$$

and that $\theta_{\text{QCD}} \approx \pi/2$.

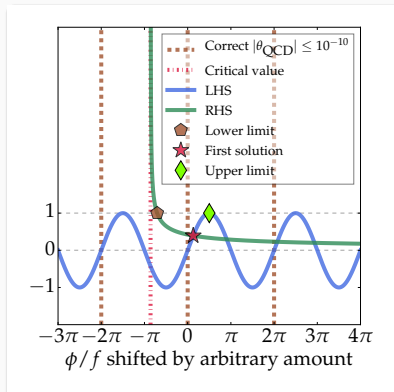


Figure 2: Tadpole equations. Frequency actually much higher, such that $\theta_{\text{QCD}} \gg 0$.

First epoch of inflation

Slowly-rolling fields generate Hubble friction during relaxation mechanism. Assume that it satisfies a constraint on the vacuum energy,

$$H^2 M_P^2 < \frac{\mu^2 m^2}{\kappa}$$

that classical beats quantum behaviour,

$$H^3 < m^3 \langle a \rangle$$

and that

$$H \lesssim m_b$$

This results in a constraint upon the relaxation parameters,

$$\sqrt{\frac{\mu^2 m^2}{\kappa}} < M_P \min(m_b, m^{2/3} \langle a \rangle^{1/3})$$

We impose this latter constraint, but assume that an acceptable Hubble constant is generated at no cost.

Second epoch of inflation

The first epoch of inflation does not satisfy constraints on density fluctuations $\delta\rho/\rho$. We require a second epoch of inflation that generates our cosmological observables.

The second epoch of inflation must satisfy

$$H \lesssim m_b$$

to prevent destruction of the periodic barriers. Fine-tuned as typically $H \gg M_W$.

Calculate inflationary observables measured by Planck/BICEP, n_S , A_S , and r , with η and ϵ slow-roll parameters.

Bayesian approach to fine-tuning/naturalness

Built/solved relaxation model. Does it *actually* solve fine-tuning problem? Is it less fine-tuned than SM?

What is fine-tuning?

Something to do with QFT? EFT? Quadratic divergences? Barbieri-Giudice? Aesthetic? A lot of confusion.

Bayesian statistics is a *unique* logical framework for quantifying the **plausibility** of a model.⁸

Includes an automatic Occam's razor/penalty for fine-tuning.

Anything that was correct/logical about old-fashioned fine-tuning arguments is automatically included in Bayesian statistics. Everything that wasn't, isn't.

⁸H. Jeffreys, (Oxford University Press, 1939), E. T. Jaynes, (Cambridge University Press, 2003).

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What do you calculate?

Calculate the Bayesian evidence for each model under consideration

$$p(D | M) = \int p(D | M, \boldsymbol{p}) \cdot p(\boldsymbol{p} | M) \prod d\boldsymbol{p}$$

Probability of data given point in model (likelihood).

Probability of point given model (prior). *Somewhat* subjective, though should reflect knowledge or ignorance about parameters.

Compare the evidences in a so-called Bayes-factor:

$$p(D | M_b) / p(D | M_a) \propto p(M_b | D) / p(M_a | D)$$

which is proportional to the posterior odds. May not agree with frequentist methods, even with *informative* data.¹⁰

¹⁰D. V. Lindley, *Biometrika* 44, 187–192 (1957), M. S. Bartlett, *Biometrika* 44, 533–534 (1957).

Why Bayesian evidence captures fine-tuning in one slide¹¹

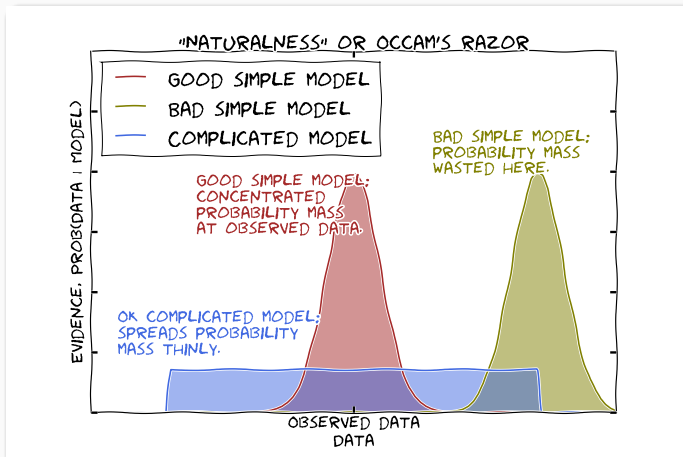


Figure 3: Bayesian evidence captures old-fashioned ideas about FT. SM is a bad simple model: concentrates probability at $M_Z \sim M_P$.

¹¹D. Mackay, (1992).

Bayes-factors (i.e. naturalness) of SM
vs. relaxion model

What we calculated: models/priors

We looked at the **SM + mixed inflation**

$$V = \frac{1}{2}m_\sigma^2\sigma^2 + \lambda_\sigma\sigma^4.$$

A canonical model of the weak scale and inflation.

Two relaxation models: **QCD relaxation and non-QCD relaxation** augmented with a **renormalizable inflationary potential**,

$$V = m_3^3\sigma + \frac{1}{2}m_2^2\sigma^2 + \frac{1}{3}m_1\sigma^3 + \frac{1}{4}\lambda_\sigma\sigma^4.$$

A general model of inflation that could satisfy $H \lesssim m_b$.

We picked non-informative priors for the Lagrangian parameters (typically logarithmic, since we are ignorant of their scale).

What we calculated: data/likelihood functions

We considered $M_Z \approx 90$ GeV, constraints on the axion decay constant and θ_{QCD} , and the inflationary observables r , n_S , and A_S measured by Planck/BICEP.

The likelihood functions were Gaussians or step-functions. We added data one by one to assess their impact.

All scalar masses received Planck-scale quadratic corrections $\Delta m^2 \sim M_P^2$.

We calculated the evidence, $p(D | M)$, for each model with MultiNest.¹²

¹²F. Feroz, et al., Mon. Not. Roy. Astron. Soc. 398, 1601–1614 (2009), arXiv:0809.3437 [astro-ph], F. Feroz, et al., (2013), arXiv:1306.2144 [astro-ph.IM].

Parameter	Measurement
M_Z	$91.1876 \pm 0.0021 \text{ GeV}$
f_a	$f_a \gtrsim 10^9 \text{ GeV}$
$ \theta_{\text{QCD}} $	$ \theta_{\text{QCD}} \lesssim 10^{-10}$
r	$r < 0.12 \text{ at } 95\%$
n_s	0.9645 ± 0.0049
$\ln(10^{10} A_s)$	3.094 ± 0.034

Table 1: Data included in our evidences, $p(D | M)$.

For table of priors, see full paper.¹³

¹³A. Fowlie, et al., JHEP 08, 100 (2016), arXiv:1602.03889 [hep-ph].

SM vs. relaxion: Applying no constraints (not even inequalities)

SM vs. QCD relaxion

- SM wastes probability near Planck scale
- Relaxion model makes a broad prediction, much greater at correct weak scale
- Z -mass vastly favours relaxion models in Bayes-factor
- Compare with Fig. 3

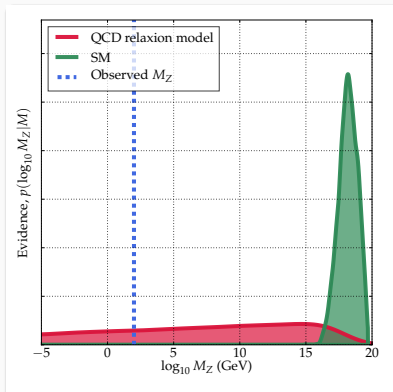


Figure 4: Evidence of model as function of $\log_{10} M_Z$

Weak scale + physicality conditions + inflation

- QCD and non-QCD relaxion *much worse* than SM
- Bayes-factors favour SM by many orders of magnitude
- Relaxion model destroyed by fine-tuned inflation and constraints on Hubble parameter during relaxation
- SM + scalar-field inflation 10^{24} times more plausible than relaxion model (after seeing all data)
- Hierarchies introduce enormous factors in Bayes-factors

SM vs. relaxion: in numbers

Data-set	M_Z only	All data
Evidence of SM + inflation · GeV	10^{-34}	10^{-53}
Evidence of non-QCD relaxion · GeV	10^{-4}	10^{-77}
Evidence of QCD relaxion · GeV	10^{-4}	$\lll 10^{-81}$

Table 2: Bayesian evidences for SM and relaxion models.

Many more numbers in paper.

Considering only M_Z , relaxion models favoured, but with all data, relaxion models much worse than SM.

The evidence for the QCD relaxion model is effectively zero, as it makes a bad prediction for $|\theta_{\text{QCD}}|$.

Bayes-factors *valid only for models under consideration* and may not apply to your favourite relaxation + inflation model

These Bayes-factors are *not* the final word for relaxation models in general




But any claims that more complicated relaxation models improve naturalness should be accompanied by calculations of Bayes-factors (or follow similar qualitative reasoning)

Naturalness of the relaxion mechanism = unnatural





- First statistical analysis of relaxion model.
- Bayesian statistics includes *automatic* penalties for fine-tuning/naturalness. *No cheating. Nothing by hand. No Barbieri-Giudice fine-tuning measures.*
- Found that, all told, relaxion models were much less plausible than SM + single-field inflation.
- Problems with unusual cosmology.
- Arguably overlooked issues with relaxion model that would further damage its plausibility.
- Strictly speaking, conclusions applicable only to simple relaxion models under consideration, though results are not promising for relaxion models in general.

Questions?






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