# Growth of perturbations in dark energy cosmologies

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# Outline:

- Background history in scalar-tensor gravity.
- Perturbations in ST gravity
- Spherical collapse model in ST gravity
- Growth factor
- SCM parameters
- Abundance of haloes
- Conclusion

# Background history in ST cosmology

Action of these models is given by

$$S = \frac{1}{16\pi G}\int d^4x\,\sqrt{-g}\,(F(\Phi)\,R - Z(\Phi)\,g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - 2U(\Phi)) + S_{\rm m}(g_{\mu\nu})\ , \qquad (1)$$

(Bergmann 1968; Nordtvedt 1970; Wagoner 1970)

Where G is coupling gravitational constant,
R is the Ricci scalar,
Sm is the action of matter field which does not evolve the scalar field,
$F(\Phi), Z(\Phi)$ are the arbitrary and dimensionless functions,
And $U(\Phi)$is the potential of the scalar field.
The term $F(\Phi)R$ represents the non-minimally coupling between the scalar field $\Phi$ and gravity.
In the limit of GR, it is obvious to have $F(\Phi) = 1$ meaning that there is no direct interaction between Scalar field and gravity.

# Background history in ST cosmology

By a re-definition of scalar field $\Phi$, we can recast $Z(\Phi) = 1$and in the context of FRW Metric

$$ds^2 = -dt^2 + a^2(t)\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right] , \qquad (2)$$

We can obtain the following modified Freidmann equation in ST cosmologies

$$\begin{aligned} 3F(\Phi)H^2 &= \rho_{\rm m} + \frac{1}{2}\dot{\Phi}^2 + U(\Phi) - 3H\dot{F}(\Phi) = \rho_{\rm tot} , \\ -2F(\Phi)\dot{H} &= \rho_{\rm m} + \dot{\Phi}^2 + \ddot{F}(\Phi) - H\dot{F}(\Phi) = \rho_{\rm tot} + p_{\rm tot} \end{aligned} \qquad (3)$$

The Klein-Gordon equation for the evolution of the scalar field
and the equation of motion for non-relativistic dust matter are, respectively, given by

$$\begin{aligned} \ddot{\Phi} + 3H\dot{\Phi} &= 3\frac{dF}{d\Phi}\left(\dot{H} + 2H^2\right) - \frac{dU}{d\Phi} . \\ \dot{\rho}_{\rm m} + 3H\rho_{\rm m} &= 0 , \end{aligned} \qquad (4)$$

# Background history in ST cosmology

Here we will assume the Ratra-Peebles form for the scalar field potential

$$U(\Phi) = \frac{M^{4+\alpha}}{\Phi^{\alpha}} \,, \qquad (5)$$

Where $M$ is an energy scale and the exponent $\alpha$ is a free positive constant.
We further assume a power-law for the dimensionless function

$$F(\Phi) = 1 + \xi(\Phi^2 - \Phi_0^2) \qquad (6)$$

where the constant ξ indicates the strength of the coupling between the scalar field and Ricci scalar and Φ_0 is the value of the scalar field at the present time.

From modified Freidmann equations we have:

$$\begin{aligned} \rho_\phi &= \frac{1}{2}\dot{\Phi}^2 + U(\Phi) - 3H\dot{F}(\Phi) \,, \\ p_\phi &= \frac{1}{2}\dot{\Phi}^2 - U(\Phi) + \ddot{F}(\Phi) + 2H\dot{F}(\Phi) \,. \end{aligned} \qquad (7)$$

# Background history in ST cosmology

Solving the coupled system of equations ( 3 & 4) from the early scale factor $a_i = 10^{-4}$ we calculate the background history of the cosmology in ST gravity.

We also need two initial conditions for Klein-Gordon equation in such a way that One get to $\Omega_\Phi = 0.7$ , $F(\Phi) = 1$ and $\Phi_0 = 1$ at the present time.

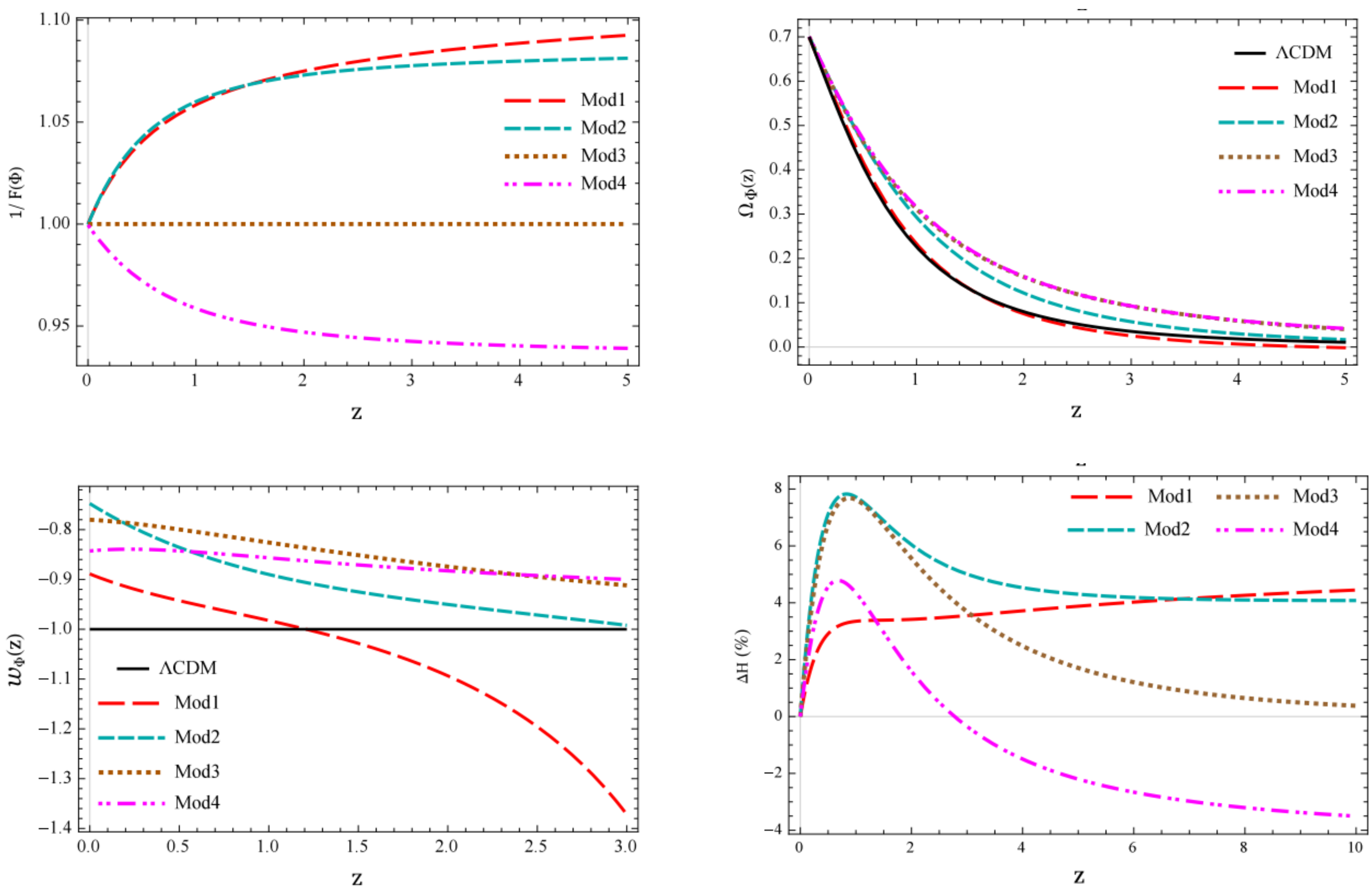
To do this we select the following cosmological models:

| Model | $\xi$ | $\alpha$ |
|---|---|---|
| Model (1) | 0.123 | 0.261 |
| Model (2) | 0.088 | 0.679 |
| Model (3) | 0.000 | 0.877 |
| Model (4) | -0.087 | 0.877 |
| $\Lambda$CDM | 0.000 | 0.000 |

**Models (1,2,4): non-minimally coupled Quintessence models.**
**Models (3): minimally coupled quintessence Model.**

Model (3) represents a minimally coupled quintessence model in which there is no direct coupling between scalar field and gravity.

# Background history in ST cosmology



# Perturbations in ST cosmologies

## We consider two different cases:

A) The homogenous non-minimally coupled quintessence models, where the perturbations in scalar field is ignored. In this case only the dust matter is perturbed.

B) The clustering non-minimally coupled quintessence models, where the perturbations in scalar field is included.

## Linear regimes:

$$\ddot{\delta}_{\rm m} + 2H\dot{\delta}_{\rm m} - 4\pi G_{\rm eff}\rho_{\rm m}\delta_{\rm m} = 0 \qquad (8)$$

Where $G_{eff}$ in the case of clustering quintessence model is given by

$$G_{\rm eff}^{(p)} = \frac{G_{\rm N}}{F}\left(\frac{2F + 4F_{,\Phi}^2}{2F + 3F_{,\Phi}^2}\right) \qquad (9)$$

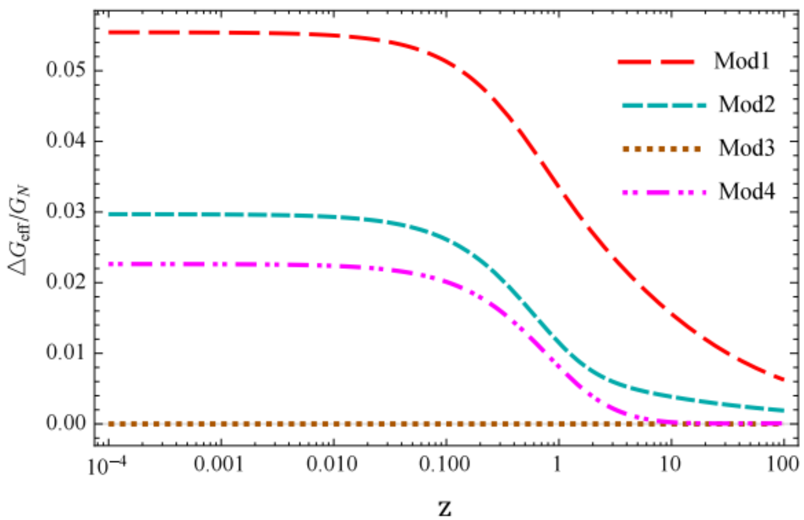
Where $F_{,\Phi} = dF/d\Phi$ ( N. Nazari-Pooya, **M. Malekjani**, F. Pace 2016 MNRAS)

# Perturbations in ST cosmologies

In the case of homogeneous non-minimally quintessence models where we ignore the perturbations of scalar field, $\delta\Phi = 0$, it is easy to show the effective gravitational constant as (Pace et al 2014 MNRAS)

$$G_{\rm eff}^{(h)} = \frac{G_{\rm N}}{F}\left(\frac{2F + 2F_{,\Phi}^2}{2F + 3F_{,\Phi}^2}\right) \quad (10)$$

From figure, we see that at low redshift the Gravitational coupling $G_{eff}$ in clustering Non-minimally quintessence models is bigger than the corresponding quantity defined in homogenous Quintessence Models.




$$\Delta G_{\rm eff}/G_{\rm N} = (G_{\rm eff}^{(p)} - G_{\rm eff}^{(h)})/G_{\rm N}$$

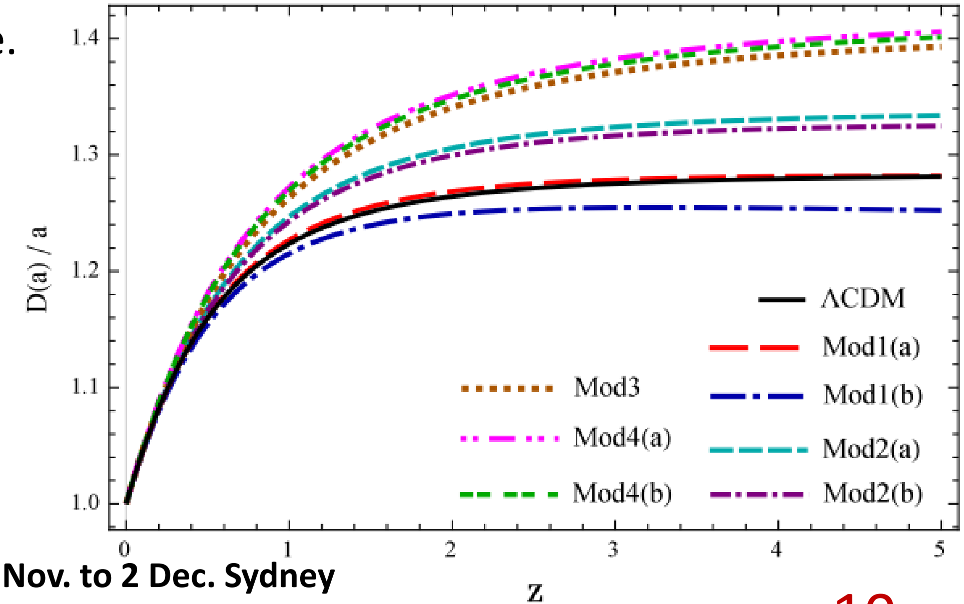
## Growth of sub-horizon perturbations

- **Linear growth factor:**

Solving equation (8) and using the relations (9) and (10), respectively, for clustering And homogenous non-minimally coupled quintessence models, we obtain the linear growth factor quantity

$$D(a) = [\delta_m(a)/\delta_m(a=1)]/\mathrm{a} \quad (11)$$

In all models (a) represent homogenous Case and (b) indicates the clustering case.



# Growth of sub-horizon perturbations

- **SCM parameters:**

We now follow the growth of matter perturbations in non-linear regime to get the linear overdensity parameter $\delta_c$ and the virial overdensity $\Delta_{vir}$, the two main quantities characterizing the SCM.
(Press & Schechter 1974; Bond et al. 1991; Sheth & Tormen 2002 )

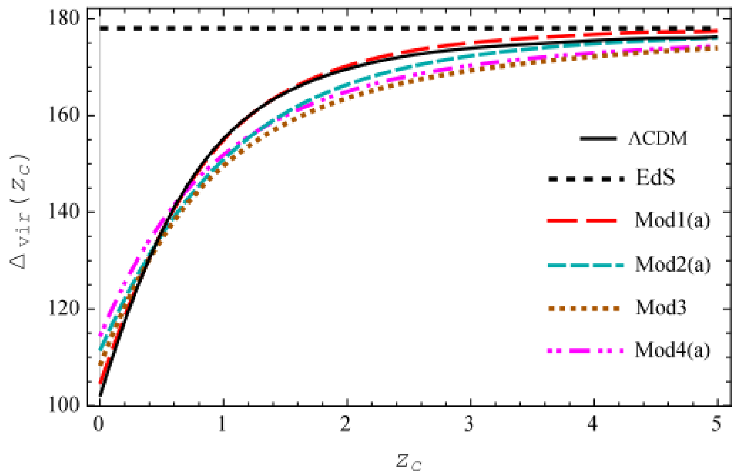
$$\ddot{\delta}_{\rm m} + 2H\dot{\delta}_{\rm m} - \frac{4}{3}\frac{\dot{\delta}_{\rm m}^2}{1+\delta_{\rm m}} - 4\pi G_{\rm eff}\rho_{\rm m}\delta_{\rm m} = 0 \qquad (12)$$

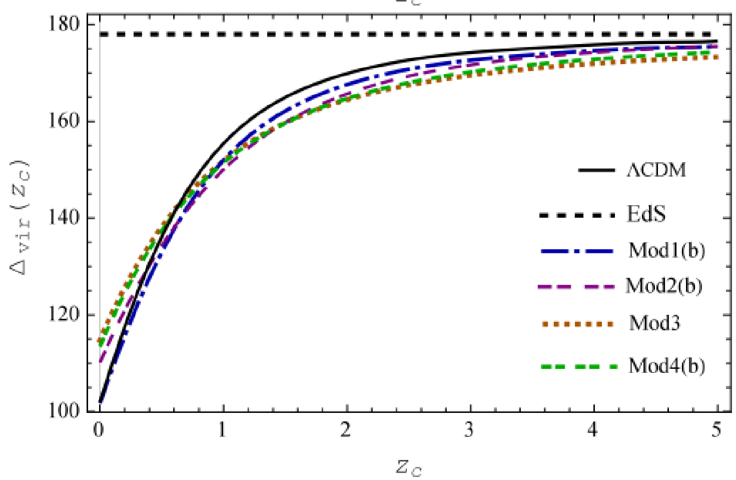
Notice that for homogenous non-minimally coupled quintessence model we use Eq.(10) and in the case of clustering non-minimally coupled quintessence model we adopt Eq.(9).

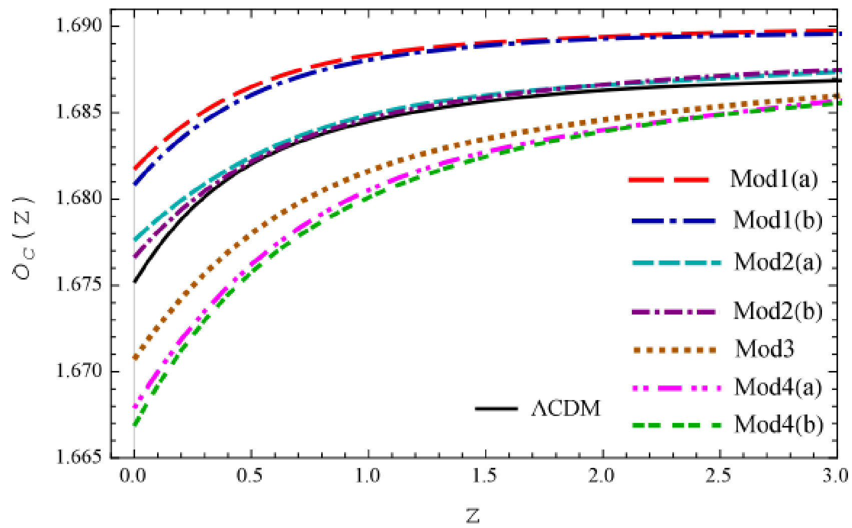
$\delta_c$ is an important quantity in Press-Schechter formalism to count the number of virialized haloes.

$\Delta_{vir}$ is used to determine the size of virialized haloes.

# Growth of sub-horizon perturbations




In early times, $\delta_c and\ \Delta_{vir}$, respectively, tend to fiducial values in EdS Universe 1.686 and 178.

# SCM parameters

- The rapidly decrements of $\delta_c$ and $\Delta_{vir}$ at low-redshifts is due to domination of quintessence field at these redshifts.
- From the Press-Schechter point of view $\delta > \delta_c$ means that a perturbed region with overdensity $\delta$ was collapsed.
- In EdS universe $\delta_c = 1.686$ independently of redshift z for any cosmic time. Hence the overdense regions collapse to form virialized haloes in a uniform rate at any cosmic redshift after matter-radiation equality epoch.
- In ST cosmologies, decreasing of $\delta_c$at low redshifts indicates that an overdense regions with lower values of overdensity $\delta$ can collapse to form bound structures. At high redshifts this effect is vanished and all models mimic the EdS Universe.

## SCM parameters

- The size of spherically symmetric haloes can be well defined by the virial overdensity parameter $\Delta_{vir}$.
- In EdS Universe $\Delta_{vir} = 178$ at any cosmic redshifts which means that the consendation of collapsed haloes virializing at different epochs are equal to each others. This is not the case for ST cosmologies, where $\Delta_{vir}$ is much lower at low redshifts compare to high redshifts.
- A decrease in $\Delta_{vir}$ with redshift z in ST cosmologies indicates that the quintessence sector prevents the more collapse and condensation of overdense regions. Hence we can conclude that in ST cosmologies the virialized haloes can be formed with less condensations of dust matters.

# Abundance of virialized Haloes in homogeneous quintessence models

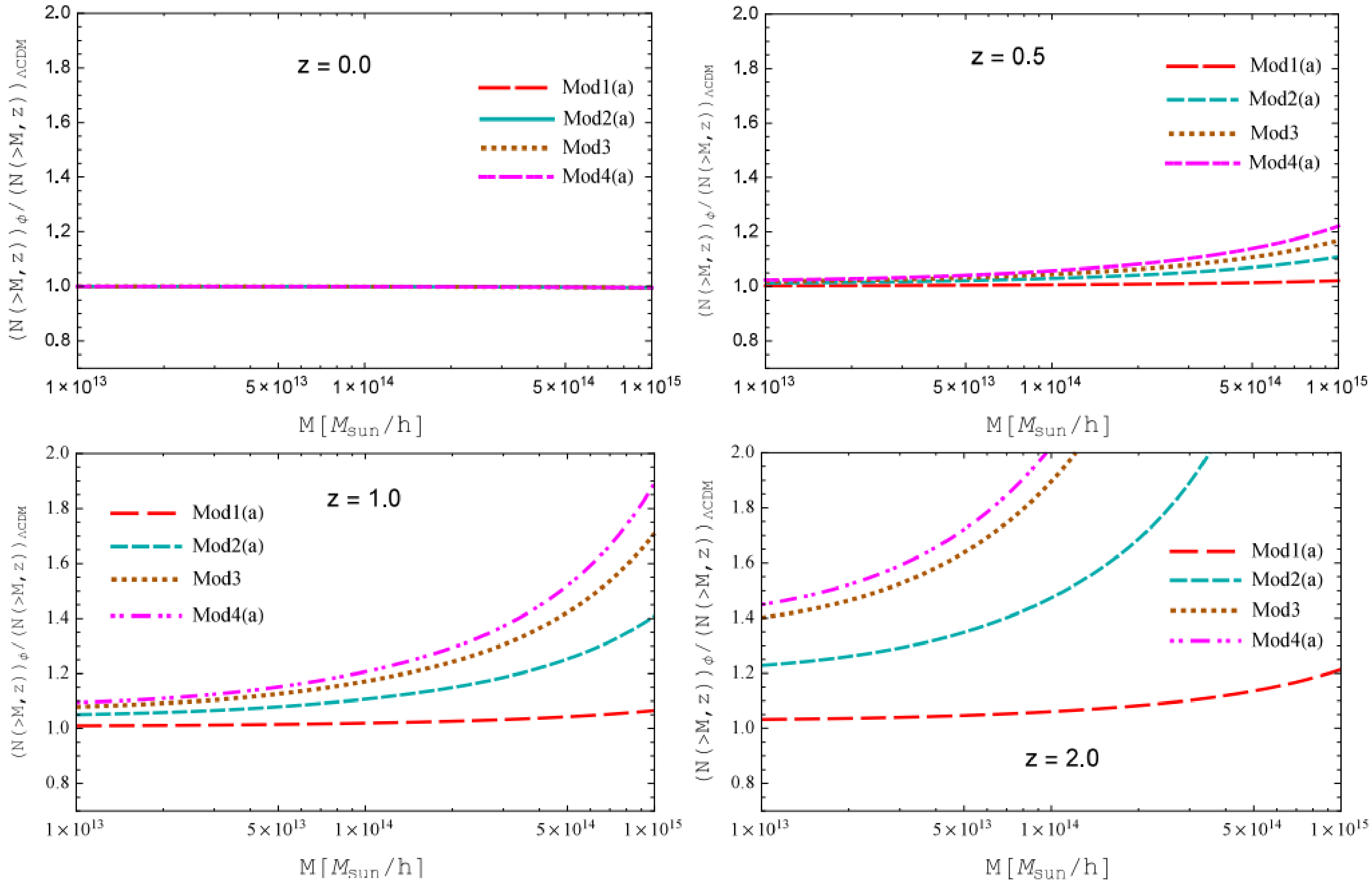
- The comoving number density of virialized haloes in a certain mass range using the Press-Schechter formalism (Press & Schechter 1974; Bond et al. 1991) reads

$$\frac{dn(M,w,z)}{dM} = \frac{\bar{\rho}_0}{M}\frac{d\nu(M,w,z)}{dM}f(\nu) \qquad \nu(M,w,z) = \frac{\delta_c}{\sigma}$$

$$\sigma^2 = \frac{1}{2\pi^2}\int_0^\infty k^2 P(k) W^2(kR)dk \qquad n(>M,z) = \int_M^\infty \frac{dn(z)}{dM'}dM'$$

$$f(\nu) = 0.2709\sqrt{\frac{2}{\pi}}\left(1+1.1096\nu^{0.3}\right)\exp\left(-\frac{0.707\nu^2}{2}\right)$$

Mass function presented By Sheth & Tormen 2002


z = 0.0
z = 0.5
z = 1.0
z = 2.0
Mod1(a)
Mod2(a)
Mod3
Mod4(a)
$(N(>M,z))_\phi/(N(>M,z))_{\Lambda CDM}$
$M\,[M_{sun}/h]$

# Abundance of virialized Haloes in clustering quintessence models