Test of the Einstein equivalence principle with CMB spectral distortions

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Einstein Equivalent Principle (EEP)

One of the fundamental principles in General Relativity

The structure of space-time felt by a test particle does not depend on its own energy

In GR, a metric does not have any energy-dependence

Test of the EEP with the PPN parameter γ

γ : how much space curvature is produced by a unit rest mass

$$g_{ij} = (1 + 2\gamma U)\delta_{ij} \qquad U = \int \frac{\rho'}{|\mathbf{x} - \mathbf{x}'|} d^3x'$$

In General Relativity $\gamma = 1$

The PPN parameter γ has energy-dependence

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The EEP is violated

Test of the EEP with the PPN parameter γ

The difference of the PPN parameters for different energies is measured by Relative Shapiro time delay

$$\Delta t = \frac{\gamma_1 - \gamma_2}{c^3} \int_{r_o}^{r_e} U(r) dr$$

γ_1 , γ_2 : the PPN parameters for photons with energies E_1, E_2

r_e, r_o : the locations of source and observer

$U(r)$: the gravitational field

Current constraints on the EEP

Cassini space craft $U(r)$: the solar system

$\Delta\gamma < 10^{-5}$ in radio frequencies (Bertotti et al 2003)

Gamma ray bursts
Fast radio bursts
$U(r)$: Milky way

$|\gamma_{\mathrm{GeV}} - \gamma_{\mathrm{MeV}}| < 2 \times 10^{-8}$ (Gao et al. 2015)

$\Delta\gamma < 10^{-9}$ in GHz (Tingay & Kaplan, 2016)

Test of the EEP with the PPN parameter γ

Test of the energy dependence
of a Schwarzschild-like metric

$$g_{ij} = (1 + 2\gamma U)\delta_{ij} \qquad U = \int \frac{\rho'}{|\mathbf{x} - \mathbf{x}'|} d^3x'$$

Question:
The energy dependence of the FLRW metric

- Cosmological distance
- Cosmological time scale

Cosmological redshift effect

The FLRW metric

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$$

The geodesic equation for a photon with energy E

$$\dot{E} = \frac{\dot{a}}{a}E$$

Cosmological Redshift effect

Cosmological redshift effect

The energy-dependent FLRW metric

$$ds^2 = -\frac{dt^2}{f^2(E)} + \frac{a^2(t)}{g^2(E)}\delta_{ij}dx^i dx^j$$

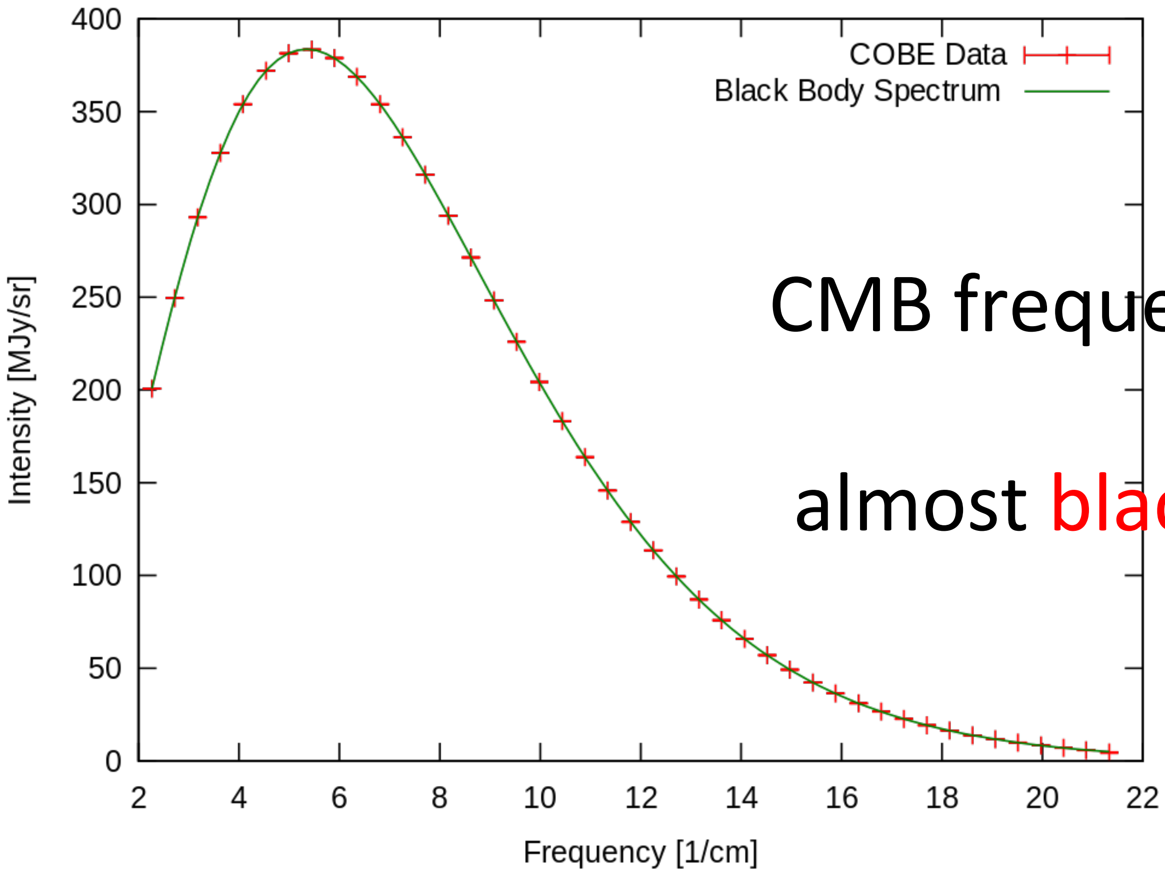
The geodesic equation for a photon with energy E

$$\dot{E} = -\frac{\dot{a}}{a}\left(1 - \frac{d\log g}{d\log E}\right)^{-1} E$$

Energy-dependent Redshift effect

The EEP is violated
when this energy-dependence is confirmed

The measurement of the CMB frequency spectrum can test the energy-dependence of the redshift effect



CMB frequency spectrum:

almost blackbody spectrum

The evolution of the CMB spectrum:

Boltzmann equation

$$\frac{\partial n_E}{\partial t} - \frac{\dot{a}}{a} E \frac{\partial n_E}{\partial E} = CT$$

Redshift term

Collisional term

In the early epoch $(z > 10^6)$

the collision term (thermalization) is dominant

The spectrum is maintained in a blackbody spectrum

$$n_E = \frac{1}{\exp[E/T] - 1}$$

The evolution of the CMB spectrum

After decoupling $(z_{dc} \sim 10^3)$

collisionless Boltzmann equation

$$\frac{\partial n_E}{\partial t} - \frac{\dot{a}}{a} E \frac{\partial n_E}{\partial E} = 0$$

$$n_E = \frac{1}{\exp[E/T_z] - 1}, \quad T_z = T_{dc} \frac{(1+z)}{(1+z_{dc})}$$

assuming a blackbody spectrum at decoupling

The CMB can keep a blackbody shape until the present

In the energy-dependent FRW metric

$$ds^2 = -\frac{dt^2}{f^2(E)} + \frac{a^2(t)}{g^2(E)}\delta_{ij}dx^i dx^j$$

After decoupling $(z_{dc} \sim 10^3)$

$$\frac{\partial n_E}{\partial t} - \frac{\dot{a}}{a}E\left(1 - \frac{d\log g}{d\log E}\right)^{-1}\frac{\partial n_E}{\partial E} = 0$$

Energy-dependent redshift term

$$n_E = \frac{1}{\exp[\eta(E,z)E/T_z] - 1} \qquad \eta(E,z) = \frac{g(E_{dc}(E,z))}{g(E)}$$

The spectrum deviates from a blackbody spectrum

In the energy-dependent FRW metric

$$ds^2 = -\frac{dt^2}{f^2(E)} + \frac{a^2(t)}{g^2(E)}\delta_{ij}dx^i dx^j$$

The CMB spectrum after decoupling

$$n_E = \frac{1}{\exp[\eta(E,z)E/T_z] - 1} \qquad \eta(E,z) = \frac{g(E_{dc}(E,z))}{g(E)}$$

$$g^{-1}(E) \approx 1 + h(E)$$

assuming the deviation from GR is small

Evaluate the CMB shape in two simple cases of h(E)

In the case with $h \propto E^{-1}$

When $h(E) = \delta_{T_0} T_0 / E \qquad (\delta_{T_0} \ll 1)$

The CMB spectrum

$$n_E \approx \left(\exp\left[\frac{E}{T_0} \left(1 + \frac{z_{\rm dc}}{1+z_{\rm dc}} \frac{T_0}{E} \delta_{T_0} \right) \right] - 1 \right)^{-1}$$

The Bose-Einstein distribution with $\mu = z_{\rm dc} \delta_{T_0} / (1 + z_{\rm dc})$

$$n_E = \left(\exp(E/T_0 + \mu) - 1 \right)^{-1}$$

COBE/FIRAS constraint $\quad |\mu| \lesssim 9 \times 10^{-5}$

$$|\delta_{T_0}| \lesssim 9 \times 10^{-5}$$

In the case with $h \propto E$

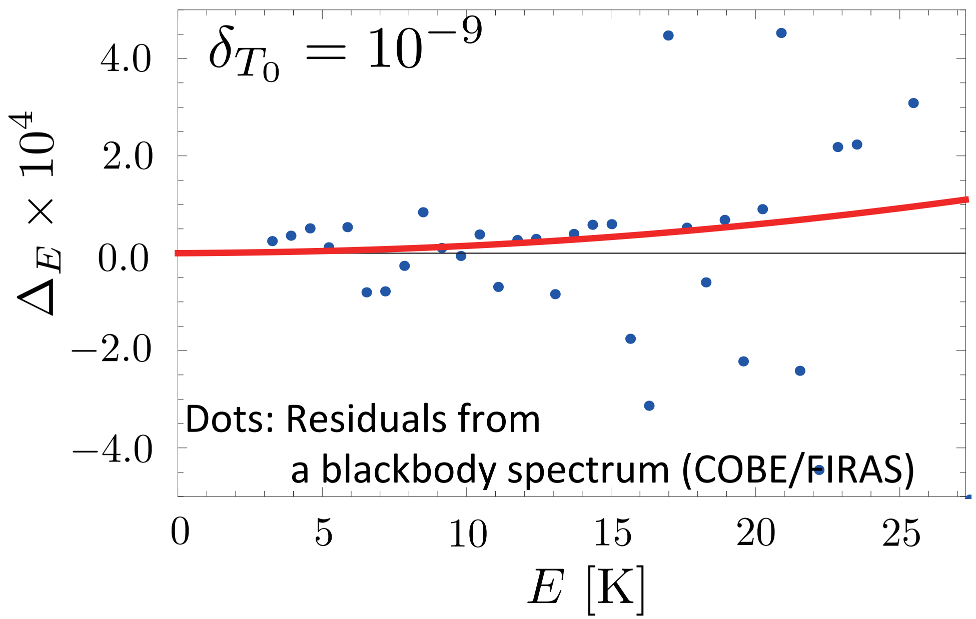
When $h(E) = \delta_{T_0} E/T_0 \qquad (\delta_{T_0} \ll 1)$

The CMB spectrum

$$n_E \approx \frac{1}{\exp(E/T_0) - 1} + \underline{\frac{\exp(E/T_0)}{[\exp(E/T_0) - 1]^2} \left(\frac{E}{T_0}\right)^2 z_{\rm dc}\delta_{T_0}}.$$

Deviation from a blackbody shape

$$\Delta_E = n_E/n_{\rm BB} - 1$$



COBE/FIRAS constraint

$$\delta_{T_0} \lesssim 10^{-9}$$

Summary

Measurement of CMB distortions can provide a constraint on the EEP
(energy-dependence of cosmological redshift effect)

- Cosmological distance (z < 1000)
- CMB energy scales ($10^{-4}\mathrm{eV} < E < 1\mathrm{eV}$)

Distortion depends on the energy-dependence of the FLRW metric

In the case with $h \propto E^{-1}$

no energy-dependence with a precision of 10^-5

In the case with $h \propto E$

no energy-dependence with a precision of 10^-9

SDSS provides the constraint on the energy-dependence of the redshift effect

no energy-dependence of redshift effect with a precision of 10^-6 (Ferreras & Trujillo 2016)

$$(0.1 < z < 0.25 \text{ with } 3700 - 6800\text{Å})$$