Blue Tensor Spectra
(with slightly parity-violated)
from axion-gauge couplings

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(in preparation)
Primordial GWs from the inflation

Tensor power spectrum (vacuum fluctuations):

\[ \Delta_{h}^{\pm} = \left. \frac{H^2}{\pi^2 M_p^2} \right|_{k=\alpha H} = A_t \left( \frac{k}{k_0} \right)^{n_t<0} \]

- Energy scale of early Universe
- Red-tilted.
- Parity-symmetric.
- Tiny amplitude.

\[ r < 0.07 \]

BICEP2/Keck Array (2016)
Is it detectable directly?

Sensitivity curves of DECIGO: Ando (2013, DECIGO workshop)

challenging for $r < \mathcal{O}(0.01)$!

if the source of GWs is vacuum fluctuations
Additional sources of GWs

(inflationary axion-gauge coupling)

\[ \mathcal{L} = -\frac{1}{2} (\partial \varphi)^2 - V(\varphi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} f F_{\mu\nu} \tilde{F}^{\mu\nu} \]

EOM for gauge field:

\[ \frac{d^2 A_k^\pm}{d\tau^2} + \left( k^2 \pm \frac{2k\xi}{\tau} \right) A_k^\pm = 0 \]

\[ \xi \equiv \frac{\dot{\varphi}}{2fH} > 0, \quad \tau = -(Ha)^{-1} < 0 \]

\[ A_k^+ \rightarrow e^{\pi \xi} \quad \text{(One helicity mode enhancement!)} \]

\[ \rightarrow \text{Additional sources for scalar and tensor perturbations!} \]
Testable **chiral** blue tensor spectrum

**Tensor spectrum:**

\[
\Delta^V_h \simeq \Delta^V_{h_-} + \Delta^S_{h_+} \\
= \Delta^V_{h_0} \left[ 1 + 10^{-6} \frac{H^2}{M_p^2 \xi^6} e^{4\pi \xi} \right] \\
\xi(t) \propto \dot{\varphi}(t) : \text{increasing in time}
\]

Preferable parameter region

\[
\xi \sim \frac{\sqrt{\epsilon \varphi}}{f} \gtrsim \mathcal{O}(1) \leftrightarrow f \lesssim 10^{-2} M_p
\]

Valerie et al. 2016

Valerie et al. 2016: Chaotic type

Valerie et al. 2016: Hilltop type

Valerie et al. 2016: Starobinsky type

\( r = \mathcal{O}(0.01) \)
Testable **chiral** blue tensor spectrum

Tensor spectrum:

\[ \sim \Delta^V_h + \Delta^S_h \]

\[ = \Delta^V_h \left[ 1 + 10^{-6} \frac{H^2}{M_p^2 \xi^6} e^{4\pi \xi} \right] \]

\[ \xi(t) \propto \dot{\varphi}(t) : \text{increasing in time} \]

Preferable parameter region

\[ \xi \sim \frac{\sqrt{\epsilon \varphi}}{f} \gtrsim \mathcal{O}(1) \leftrightarrow f \lesssim 10^{-2} M_p \]

Natural scale predicted by the heterotic superstring!

\[ (f \sim \frac{\alpha_G}{2\pi} M_p, \ \alpha_G : \text{GUT gauge coupling}) \]

Valerie *et al.* 2016
Our question and study

Question...

? : Testable blue tensor spectrum $\leftrightarrow$ EFT of the heterotic string theory

In this work...

✓ We explore the concrete model building based on the heterotic string theory and reconsider the form of axion-gauge couplings.

✓ We found that rich varieties of blue tensor spectra could be provided sourced by gauge fields coupled to two types of axions.
Axions from the heterotic superstring

\[ E_8 \times E_8 \]

Choi & Kim (1985), ...

Axions arise from the B field:

\[ B = B_{MN} dx^M \wedge dx^N \quad (M, N = 0 \sim 9) \]

\[ H = dB - \omega_Y + \omega_L \quad : \text{its field strength} \]

\[ (d\omega_Y = \text{Tr} F^2, d\omega_L = \text{Tr} R^2) \]

Two types of axion

“Model-independent” type

\[ H_{\mu \nu \rho} = M_1 \epsilon_{\mu \nu \rho \alpha} \partial^\alpha \varphi_1 \quad (\mu, \nu, ... = 0 \sim 3) \]

“Model-dependent” type

\[ H_{\mu mn} = M_2 \epsilon_{mn} \partial_\mu \varphi_2 \quad (m, n, ... = 4 \sim 9) \]
Axions from the heterotic superstring

\[ \mathbb{E}_8 \times \mathbb{E}_8 \]

Choi & Kim (1985), ...

Origin of axion-gauge couplings:

Bianchi identity: 
\[ dH = -\text{Tr} F^2 + \text{Tr} R^2 \quad (F^2 \equiv F \wedge F, R^2 \equiv R \wedge R) \]
\[ (= f_1 \Box \varphi_1 ) \]

Anomaly cancellation term: 
\[ S_{GS} = \frac{1}{48(2\pi)^5} \int_{M_4} \text{Tr} F^2 \int_{M_6} B \left[ \text{Tr} F^2 - \frac{1}{2} \text{Tr} R^2 + \ldots \right] \]

4-dim. reduced axion-gauge coupling

Model-independent axion: 
\[ -\frac{1}{4} \frac{\varphi_1}{f_1} \left( F_1 \tilde{F}_1 + F_2 \tilde{F}_2 \right) \]

Model-dependent axion: 
\[ -\frac{1}{4} \frac{\varphi_2}{f_2} \left( F_1 \tilde{F}_1 - F_2 \tilde{F}_2 \right) \]

(omitting Lorentz indices)
Source of GWs from superstring

Model example (1):

\[ \mathcal{L} \supset -\frac{1}{4} \frac{\varphi_2}{f_2} \left( F_1 \tilde{F}_1 - F_2 \tilde{F}_2 \right) - V(\varphi_2) \]

\( F_{1,2} \): Abelian gauge field (for simplicity)

\[
\frac{d^2 A_{1k}^\pm}{d\tau^2} + \left( k^2 \mp \frac{2k\xi}{\tau} \right) A_{1k}^\pm = 0 \quad \rightarrow \quad A_{1k}^+ e^{\pi\xi} \quad \rightarrow \quad h^+
\]

\[
\frac{d^2 A_{2k}^\pm}{d\tau^2} + \left( k^2 \mp \frac{2k\xi}{\tau} \right) A_{2k}^\pm = 0 \quad \rightarrow \quad A_{2k}^- e^{\pi\xi} \quad \rightarrow \quad h^-
\]

→ parity-symmetric blue tensor spectra!
Source of GWs from superstring

Model example (2):

\[\mathcal{L} \supset -\frac{1}{4} \left( \frac{\varphi_1}{f_1} + \frac{\varphi_2}{f_2} \right) F_1 \tilde{F}_1 - \frac{1}{4} \left( \frac{\varphi_1}{f_1} - \frac{\varphi_2}{f_2} \right) F_2 \tilde{F}_2 - V(\varphi_1, \varphi_2)\]

\[F_{1,2} : \text{Abelian gauge field (for simplicity)}\]

diagonalising

\[\varphi \equiv \frac{f_2}{\sqrt{f_1^2 + f_2^2}} \varphi_1 - \frac{f_1}{\sqrt{f_1^2 + f_2^2}} \varphi_2, \quad \tilde{\varphi} \equiv \frac{f_1}{\sqrt{f_1^2 + f_2^2}} \varphi_1 + \frac{f_2}{\sqrt{f_1^2 + f_2^2}} \varphi_2\]

\[\rightarrow -\frac{1}{4} \varphi \left( F_2 \tilde{F}_2 + \frac{f_2^2 - f_1^2}{f_1^2 + f_2^2} F_1 \tilde{F}_1 \right) - \frac{1}{4} \tilde{\varphi} F_1 \tilde{F}_1 - V(\varphi, \tilde{\varphi})\]

if \(\tilde{\varphi}\) has been stabilized and \(f_1 \gg f_2\)...

\[\rightarrow -\frac{1}{4} \varphi \left( F_2 \tilde{F}_2 - (1 - 2\epsilon)F_1 \tilde{F}_1 \right)\]

\[\epsilon \equiv \frac{f_2^2}{f_1^2} \ll 1\]

\[\rightarrow \text{slightly parity-violated blue tensor spectra!}\]
Main Results

ex: \[ V(\varphi) = V_0 \left[ 1 - \cos \left( \frac{\varphi}{h} \right) \right] \]

✓ Bule-tilted and slightly parity violated spectra.

✓ Testable by future gravitational wave interferometers (DECIGO, BBO).

✓ Avoiding the overproduction of scalar modes.
Summary & Outlook

✓ We explore the concrete model building based on the heterotic string theory and reconsider the form of axion-gauge couplings.

✓ We found that rich varieties of blue tensor spectra sourced by gauge fields coupled to the model-(in)dependent axion could be provided.

? Possible potential forms of axion, the non-Abelian gauge interaction, ...

Working in progress!