A map of the non-thermal WIMP

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Based on arXiv:1611.02287
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At CosPA2016, Dec. 01, 2016
Effects of kinetic decoupling

- Elastic scattering between DM and nucleus
- Direct Detection of DM
- Supressing growth of the density perturbation of DM
- Dark Matter abundance
- Small scale structure
- DM annihilation cross section
Effects of kinetic decoupling

Elastic scattering between DM and nucleus

Direct Detection of DM

Suppressing growth of the density perturbation of DM

Small scale structure

Dark Matter abundance

DM annihilation cross section
Outline

• WIMP in standard thermal history
  - How to determine a DM relic abundance

• Early universe with low reheating temperatures
  - Thermal and Non-thermal production of WIMPs

• Non-thermal WIMP
  - Categorizing mechanisms for the final DM abundance

• Constraints
  - dark matter density, direct/indirect detections

• Summary and Outlook
WIMP in standard thermal history
Thermal freeze out

- At a high temperature, the WIMP DMs are in equilibrium. As the temperature drops below the DM mass, the number changing interactions are frozen so that the total number is preserved.

\[
\dot{n}_\chi + 3Hn_\chi = -\langle \sigma_{\text{ann}}v_{\text{rel}} \rangle \chi n_\chi^2 + \langle \sigma_{\text{ann}}v_{\text{rel}} \rangle T (n_{\chi}^{\text{eq}})^2 \\
= -\langle \sigma_{\text{ann}}v_{\text{rel}} \rangle T (n_\chi^2 - (n_{\chi}^{\text{eq}})^2)
\]

- Why is the annihilation cross-section averaged for a thermal distribution?

\[
\langle \sigma_{\text{ann}}v_{\text{rel}} \rangle_X \approx \frac{\alpha_{\text{ann}}^2}{m_X^2} \left( \frac{2\langle p_X^2 \rangle}{m_X^2} \right)^{k_{\text{ann}}} \\
= \frac{\alpha_{\text{ann}}^2}{m_X^2} \left( \frac{6T}{m_X} \right)^{k_{\text{ann}}}
\]

\[
\Gamma_{\text{ann}} = \langle \sigma_{\text{ann}}v_{\text{rel}} \rangle_X n_\chi \approx H \text{ at } T = T_{\text{fr}}
\]

\[
Y_\chi(t_0) = \frac{H_{\text{fr}}}{\langle \sigma_{\text{ann}}v_{\text{rel}} \rangle T_{\text{fr}} S_{\text{fr}}}
\]

\[
\langle \sigma_{\text{ann}}v_{\text{rel}} \rangle_X = \frac{\int d^3p_X d^3\tilde{p}_X f_X^{\text{eq}}(p_X, T) f_X^{\text{eq}}(\tilde{p}_X, T) (\sigma_{\text{ann}}v_{\text{rel}})}{\int d^3p_X d^3\tilde{p}_X f_X^{\text{eq}}(p_X, T) f_X^{\text{eq}}(\tilde{p}_X, T)}
\]

\[
f_X^{\text{eq}} = (e^{E/T} \pm 1)^{-1}
\]
Kinetic decoupling

- **Number conserving interactions** (elastic scatterings) are decoupled far later than the number changing interactions.

\[ \Gamma_{\text{ann}} = \langle \sigma_{\text{ann}} v_{\text{rel}} \rangle \chi n_{\chi} \]

\[ \Gamma_{\text{el}} = \langle \sigma_{\text{el}} v_{\text{rel}} \rangle \chi T n_{\gamma} \frac{T}{m_{\chi}} \quad n_{\chi} \ll n_{\gamma} \]

\[ x_{\text{fr}} \ll x_{\text{kd}} = m_{\chi} / T_{\text{kd}} \]

- Since the freeze-out happens during kinetic equilibrium, the relic abundance does not explicitly depend on the elastic scattering rate.

\[ x_{\text{fr}} \approx \ln m_{\chi} M_{\text{pl}} \langle \sigma_{\text{ann}} v_{\text{rel}} \rangle T_{\text{fr}} \]

\[ = 25 + \ln \frac{m_{\chi}}{100 \text{ GeV}} + \ln \frac{\langle \sigma_{\text{ann}} v_{\text{rel}} \rangle T_{\text{fr}}}{3 \times 10^{-26} \text{ cm}^3 / \text{sec}} \]

\[ x_{\text{kd}} \approx \sqrt{m_{\chi} M_{\text{pl}} \langle \sigma_{\text{el}} v_{\text{rel}} \rangle \chi T_{\text{kd}}} \]

\[ = 10^3 \left( \frac{m_{\chi}}{100 \text{ GeV}} \frac{\langle \sigma_{\text{el}} v_{\text{rel}} \rangle \chi T_{\text{kd}}}{5 \times 10^{-32} \text{ cm}^3 / \text{sec}} \right)^{1/2} \]
Early Universe with low reheating temperatures
Long lived heavy particles

- There are many candidates for long lived particles in beyond SM such as string moduli, saxion/axino, Q-ball, etc. ➞ Early Matter domination

\[ \Gamma_\phi = \frac{1}{16\pi} \frac{m_\phi^3}{M_{\text{GUT}}^2} \approx 3.4 \times 10^{-20} \left( \frac{m_\phi}{100 \text{ TeV}} \right)^3 \]

\[ \Gamma_\phi = H \Rightarrow T_{\text{reheating}} \approx \sqrt{\Gamma_\phi M_{\text{pl}}} = 150 \text{ MeV} \sqrt{\frac{\Gamma_\phi}{10^{-20} \text{ GeV}}} \]

- The “reheating temperature” could be lower than the DM freeze-out temperature, and also lower than the kinetic decoupling temperature.

![Graphs showing the evolution of density with scale factor](image)
Probing the effect of kinetic decoupling

- Depending on the branching fractions, histories of the dark matter abundances are different.
  - Non-thermal WIMP ($\text{Br}(\phi \to \chi + \cdots) > 0$)

✓ The abundance generated at the end of reheating is important.

[ M. Fujii, K. Hamaguchi 2002]

\[
\Gamma_{\text{ann}}^{\text{ini}} > H_{\text{reh}} \Rightarrow \Gamma_{\text{ann}} \to H_{\text{reh}}
\]

\[
Y_{\chi}(t_0) \simeq \frac{H_{\text{reh}}}{\langle \sigma_{\text{ann}} v_{\text{rel}} \rangle T_{\text{reh}} s_{\text{reh}}}
\]

- However...
  - complete thermalization could not happen for a low reheating temp.
  - cross-sections depend on the momentum evolution
  - relative size between $\Gamma_{\text{ann}}$ and $\Gamma_{\text{el}}$ could be important for $p$-wave ann. DM

In this plot, we assume instantaneous (kinetic) thermalization of DM:

\[
p_\chi \to p_\chi^{\text{eq}} = \sqrt{3m_\chi T} \ll m_\chi
\]

\[
\Gamma_{\text{el}} > H_{\text{reh}}
\]
Non-thermal WIMP

[H. Kim, J-P. Hong, CSS 1611.02287]
Evolution of the DM momentum

\[ \frac{dp_X}{dt} + H p_X = -\langle \sigma_{el} v_{rel} \Delta p_X \rangle_{\chi,T} \ n_\gamma \]

- After the DMs are produced by decay of heavy particles at the end of the reheating (at \( T = T_{reh} \)), Elastic scatterings between DM and radiations determine the evolution of the DM momentum.
Categorizing mechanisms

- **(N.A.) No Annihilation**:
  \[ \Gamma_{\text{ann}}(T, E_\chi) < H(T) \quad \text{for } T \leq T_{\text{reh}} \]

- **(I.A.) Instantaneous Annihilation**:
  \[ \Gamma_{\text{el}}(T_{\text{reh}}, E_\chi) > H_{\text{reh}} : E_\chi \rightarrow E_{\chi}^{\text{eq}} \]
  \[ \Gamma_{\text{ann}}(T_{\text{reh}}, E_\chi) \rightarrow H_{\text{reh}} \quad \text{at } T \simeq T_{\text{reh}} \]

- **(C.A.) Continuous Annihilation**:
  \[ \Gamma_{\text{el}}(T_{\text{reh}}, E_{\chi}^{\text{dec}}) = H_{\text{reh}} \quad \text{for } m_\chi \ll E_{\chi}^{\text{dec}} \leq E_{\chi}^{\text{ini}} \]
  \[ \Gamma_{\text{ann}}(T_*, E_\chi \sim m_\chi) \rightarrow H_* \quad \text{until } T_* < T_{\text{reh}} \]
  \[ \langle \sigma_{\text{ann}} v_{\text{rel}} \rangle_\chi \propto 1/E_\chi^2 \propto a^2 \quad \text{for } T < T_{\text{reh}} \]

\[ \Gamma_{\text{ann}}(T, E_\chi) = n_\chi \langle \sigma_{\text{ann}} v_{\text{rel}} \rangle_\chi, \]
\[ \Gamma_{\text{el}}(T, E_\chi) = n_{\gamma} \left\langle \sigma_{\text{el}} v_{\text{rel}} \frac{\Delta p_\chi}{p_\chi} \right\rangle_{\chi,T} \]
Evolution of the annihilation cross-section

\[
\dot{n}_\chi + 3Hn_\chi = -\langle \sigma_{\text{ann}}v_{\text{rel}} \rangle \chi n_\chi^2
\]

\[
Y_\chi(t_0) = Y_\chi(t_{\text{reh}}) \left(1 + \frac{n_\chi}{H_{\text{reh}}} \int_0^1 du \langle \sigma_{\text{ann}}v_{\text{rel}} \rangle p_\chi(u) \right)^{-1}
\]

- Evolution of the annihilation cross-section is important to determine the relic density.

\[
\langle \sigma_{\text{ann}}v_{\text{rel}} \rangle \chi = \frac{\alpha_{\text{ann}}^2}{E_\chi^2} \left(\frac{2p_\chi^2}{E_\chi^2}\right)^{k_{\text{ann}}}, \quad \langle \sigma_{\text{el}}v_{\text{rel}} \rangle \chi,T = \frac{\alpha_{\text{el}}^2}{m_\chi^2} \left(\frac{E_\chi^2T^2}{m_\chi^4}\right)^{k_{\text{el}}}
\]

\[
k_{\text{ann}} = 0 \ (s\text{-wave}) \quad k_{\text{el}} = 0 \ (e.g.\ Thomson\ scattering)\\n= 1 \ (p\text{-wave}) \quad = 1 \ (e.g.\ t\text{-channel\ scalar\ mediator})
\]

- Introducing momentum independent variables,

\[
\langle \sigma_{\text{ann}}v_{\text{rel}} \rangle_0 \equiv \frac{\alpha_{\text{ann}}^2}{m_\chi^2}, \quad \langle \Gamma_{\text{ann}} \rangle_0 \equiv \langle \sigma_{\text{ann}}v_{\text{rel}} \rangle_0 n_{\text{reh}}
\]

\[
\langle \sigma_{\text{el}}v_{\text{rel}} \rangle_0 \equiv \frac{\alpha_{\text{el}}^2T_{\text{reh}}^2}{m_\chi^4}, \quad \langle \Gamma_{\text{el}} \rangle_0 \equiv \langle \sigma_{\text{el}}v_{\text{rel}} \rangle_0 \frac{T_{\text{reh}}n_{\text{reh}}}{m_\chi}
\]
Evolution of the annihilation cross-section

\[ \dot{n}_\chi + 3H n_\chi = -\langle \sigma_{\text{ann}} v_{\text{rel}} \rangle \chi n_\chi^2 \]

\[ Y_\chi(t_0) = Y_\chi(t_{\text{reh}}) \left( 1 + \frac{n_\chi}{H_{\text{reh}}} \int_0^1 du \langle \sigma_{\text{ann}} v_{\text{rel}} \rangle_{p_\chi(u)} \right)^{-1} \]

- **Evolution of the annihilation cross-section is important to determine the relic density.**

\[ \langle \sigma_{\text{ann}} v_{\text{rel}} \rangle_\chi = \frac{\alpha_{\text{ann}}^2}{E_{\chi}^2} \left( \frac{2p_{\chi}^2}{E_{\chi}^2} \right)^{k_{\text{ann}}} \]

\[ \langle \sigma_{\text{el}} v_{\text{rel}} \rangle_\chi, T = \frac{\alpha_{\text{el}}^2}{m_{\chi}^2} \left( \frac{E_{\chi}^2 T^2}{m_{\chi}^4} \right)^{k_{\text{el}}} \]

- \( k_{\text{ann}} = 0 \) (s-wave)
- \( k_{\text{el}} = 0 \) (e.g. Thomson scattering)
- \( = 1 \) (p-wave)
- \( = 1 \) (e.g. t-channel scalar mediator)

- **Introducing momentum independent variables,**

\[ \langle \sigma_{\text{ann}} v_{\text{rel}} \rangle_0 \equiv \frac{\alpha_{\text{ann}}^2}{m_{\chi}^2}, \quad \langle \Gamma_{\text{ann}} \rangle_0 \equiv \langle \sigma_{\text{ann}} v_{\text{rel}} \rangle_0 n_{\chi}^{\text{reh}}, \]

\[ \langle \sigma_{\text{el}} v_{\text{rel}} \rangle_0 \equiv \frac{\alpha_{\text{el}}^2 T_{\text{reh}}^2}{m_{\chi}^4}, \quad \langle \Gamma_{\text{el}} \rangle_0 \equiv \langle \sigma_{\text{el}} v_{\text{rel}} \rangle_0 \frac{T_{\text{reh}} n_{\chi}^{\text{reh}}}{m_{\chi}}. \]
Analytic approximation

- Evolution of the (p-wave annihilating DM) annihilation cross-section

\[ H_{\text{reh}} \gg \langle \Gamma_{\text{el}} \rangle_0 \quad Y_\chi(t_0) \simeq \min \left[ Y_\chi(t_{\text{reh}}), \frac{H_{\text{reh}}}{\langle \sigma_{\text{ann}} v_{\text{rel}} \rangle_0 s_{\text{reh}}} \left( \frac{c_0 H_{\text{reh}}}{\Gamma_{\text{el}}_0} \right)^{1/3} \right] \quad \text{(N.A.)} \]

\[ H_{\text{reh}} \ll \langle \Gamma_{\text{el}} \rangle_0 \quad Y_\chi(t_0) \simeq \min \left[ Y_\chi(t_{\text{reh}}), \frac{H_{\text{reh}}}{\langle \sigma_{\text{ann}} v_{\text{rel}} \rangle_0 s_{\text{reh}}} \left[ \frac{c_0 H_{\text{reh}}}{\Gamma_{\text{el}}_0} + \left( 3 - \frac{T_{\text{kd}}^2}{T_{\text{reh}}^2} \right) \frac{T_{\text{reh}}}{m_\chi} \right]^{-1} \right] \quad \text{(I.A.)} \]

\[ \Delta u \simeq \frac{H_{\text{reh}}}{\Gamma_{\text{el}}_0} \ll 1 \]

Peak contribution becomes important for \( H_{\text{reh}} \ll \langle \Gamma_{\text{el}} \rangle_0 < \langle \Gamma_{\text{ann}} \rangle_0 \)

Continuous annihilation can happen

\[ T_{\text{reh}} = 0.2 \text{ GeV} \]
\[ T_{\text{reh}} = 30 \text{ MeV} \]
\[ T_{\text{reh}} = 5 \text{ MeV} \]
Constraints
Dark matter density

- **Assumption**: $\alpha_{\text{ann}} = \alpha_{\text{el}} = \alpha \leq 1$

$$\langle \sigma_{\text{ann}} v_{\text{rel}} \rangle_\chi = \frac{\alpha_{\text{ann}}^2}{E_X^2} \left( \frac{2p_X^2}{E_X^2} \right)^{k_{\text{ann}}}$$, $$\langle \sigma_{\text{el}} v_{\text{rel}} \rangle_X T = \frac{\alpha_{\text{el}}^2}{m_X^2} \left( \frac{E_X^2 T^2}{m_X^4} \right)^{k_{\text{el}}}$$

- **Present dark matter density**: $\Omega_X h^2 \simeq 0.11$

$$\Omega_X h^2 = 0.11 \left( \frac{m_X}{100 \text{ GeV}} \right) \left( \frac{Y_X(t_0)}{4 \times 10^{-12}} \right)$$

\(\Omega_X h^2 \propto \langle \sigma_{\text{ann}} v_{\text{rel}} \rangle_0^{-4/3} T_{\text{reh}}^{-7/3} m_X^2\)  

\(\Omega_X h^2 \propto T_{\text{reh}}^3 m_X^{-2}\)

\(\Omega_X h^2 \propto \langle \sigma_{\text{ann}} v_{\text{rel}} \rangle_0^{-1} T_{\text{reh}}^{-2} m_X^2\)

\(\Omega_X h^2 \propto \langle \sigma_{\text{el}} v_{\text{rel}} \rangle_0 T_{\text{reh}}^{-2} m_X^2\)

- (C.A.) : not thermalized, annihilation happens continuously
- (I.A.) : most of annihilation happens before complete thermalization
- (I.A.) : most of annihilation happens after complete thermalization
Direct/indirect detections

- The DM should be *lepto-philic* to be thermalized at a low temperature and *quark-phobic* by direct detection constraints.

- The effective operator as a benchmark example:

$$\mathcal{L}_{\text{eff}} = \frac{\langle \bar{\chi} \chi (\bar{l} l) \rangle}{\Lambda^2}$$

- Two loop induced elastic scattering between the DM and nucleus.
Summary and Outlook

- We have studied the effect of kinetic decoupling/elastic scattering on non-thermally produced WIMP dark matter phenomenology.

- We categorize each mechanisms and present the approximated analytic formulae.

- With low reheating temperatures, a p-wave annihilating DM is still viable and the corresponding relic abundance explicitly depends on the elastic scattering rate.

- It is also probable to think “dark wimp” in which dark matters are thermalized by dark radiations. Interesting connections between dark matter property and early history of the universe and late time cosmology can be made and predicted.