DYNAMICAL BRANE BACKGROUNDS

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[arXiv:1510.01496 [hep-th]]

[arXiv:1603.01948 [hep-th]]

[1] Introduction

❖ **String theory :**

♠ **This is the only viable unified fundamental theories at present.**
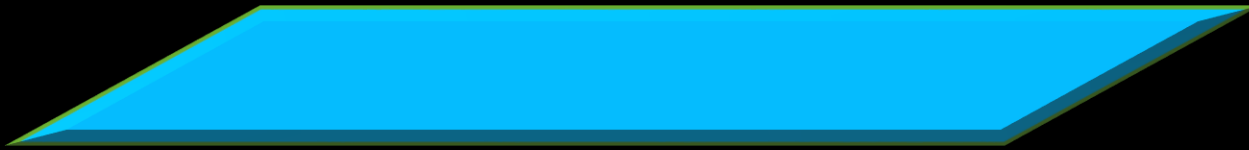
♣ **String theory contains p-branes as well as strings.**

✎ What is "p-brane" ?
(Gary T. Horowitz, Andrew Strominger, Nucl.Phys. B360 (1991) 197-209)

★ Classical mem*brane* solution of Einstein equation

$$X^1, X^2, \cdots, X^p$$

- **This is extended in *p* direction.**

- **p-brane has p spacelike translational Killing vectors.**

❀ **An innumerable number of static brane solutions have been discovered so far.**

But …

☺ **Cosmological brane solutions may also exist !**

☞ **Dynamical brane background**

▶ "Dynamical" means time-dependent.

◆ Dynamical brane may be related to

• brane collision
  (Gibbons & Lu & Pope, Phys.Rev.Lett. 94 (2005) 131602)

• cosmic Big-Bang of our universe
  (Chen, et al., Nucl.Phys. B732 (2006) 118-135)

• black hole in expanding universe
  (Maeda & Ohta & Uzawa, JHEP 0906 (2009) 051)
  (Maeda & Nozawa, Phys.Rev. D81 (2010) 044017)

★Outline my talk

∗ The property of dynamical brane

∗ Singularities and cosmic censorship

∗ Summary and comments

[2] **Property of dynamical brane solutions:**
★ **Cosmology:**
  (Binetruy, Sasaki, Uzawa, Phys.Rev.D80:026001,2009)
  (K. Maeda, K. Uzawa, [arXiv:1603.01948 [hep-th]])

We assume an isotropic and homogeneous three space in the four-dimensional spacetime.

Note that the time dependence in the metric comes from only one brane even if we consider several branes.

Solutions in the original higher-dimensional theory (10D or 11D).

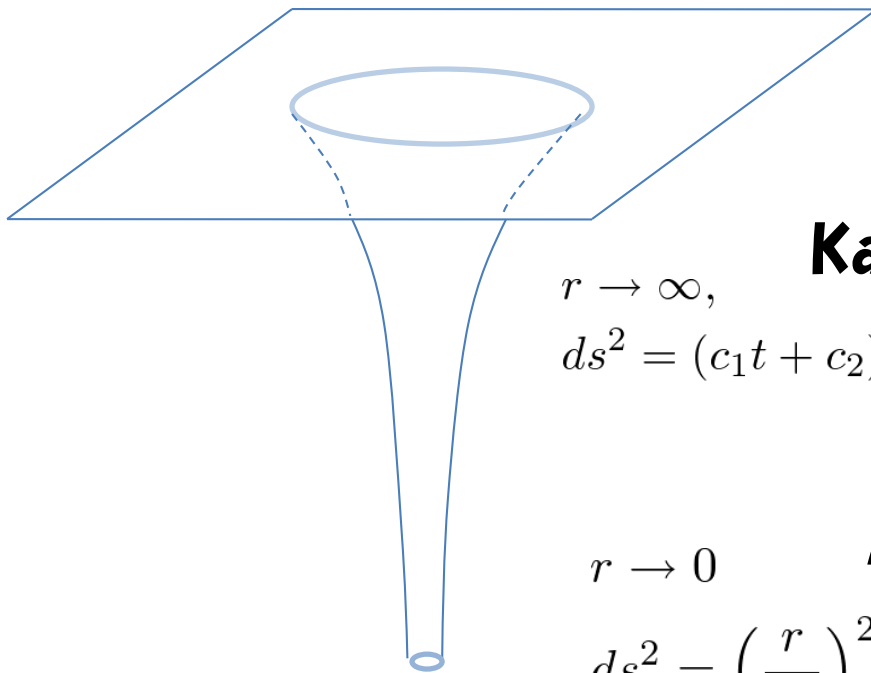# ■ Dynamical solution of D- and M-brane system

$$ds^2 = h^{-(D-p-3)/(D-2)}\eta_{\mu\nu}dx^\mu dx^\nu + h^{(p+1)/(D-2)}(dr^2 + r^2 d\Omega_{D-p-2}^2),$$

$$h(t,r) = c_1 t + c_2 + Mr^{-D+p+3}, \quad F_{p+2} = d(h^{-1}) \wedge dt \wedge dx^1 \wedge \cdots \wedge dx^p,$$

$$e^\phi = h^{c/2}, \quad c^2 = 4 - 2(p+1)(D-p-3)(D-2)^{-1}$$

· **10-dim D3-brane solution**

Kasner solution

$$r \to \infty,$$

$$ds^2 = (c_1 t + c_2)^{-1/2}\eta_{\mu\nu}dx^\mu dx^\nu + (c_1 t + c_2)^{1/2}(dr^2 + r^2 d\Omega_5^2)$$

AdS$_5 \times$ S^5

$$r \to 0$$

$$ds^2 = \left(\frac{r}{M}\right)^2 \eta_{\mu\nu}dx^\mu dx^\nu + \left(\frac{M}{r}\right)^2 dr^2 + d\Omega_5^2$$

- For each case, the scale factor of 4-dimensional universe is given by $a(\tau) \propto \tau^{\lambda}$, where τ denotes the cosmic time.

- Since the three-dimensional spatial space of our universe stays in the transverse space to the brane, D-dimensional theory gives the fastest expansion of our universe.

- The power of the scale factor becomes
  $\lambda = (p+1)/(D+p-1) < 1$ for $D > 2$.
  It is impossible to find the cosmological model that our universe is accelerating expansion.

☆ Dynamical D3-brane backgournd

In the case of no 3-form, the 10-dim metric becomes

$$ds^2 = h^{-1/2} ds^2(\mathrm{E}^{3,1}) + h^{1/2} \left[dr^2 + r^2 ds^2(\mathrm{Z}_5) \right],$$

$$h(x,r) = \left[h_0(x) + \frac{L^4}{r^4} \right]$$

For static background, $AdS_5 \times S^5$, the background has the full supersymmetry.

- **Solution for dynamical background**
  (H. Kodama & K. Uzawa, JHEP 0507:061,2005)

$$\partial_\mu h_0\, \gamma^\mu\, \varepsilon = 0\,, \quad -i\, \gamma^0 \gamma^1 \gamma^2 \gamma^3\, \varepsilon = \varepsilon$$

(i) Induced effective mass for the spinor field

$$\sim (Dh_0)^2 / h^2$$

(ii) This mass scale diverges at the naked singularity where the function h vanishes.

(iii) the degree of supersymmetry breaking increases as the universe approaches the singularity.

(iv) In the region with a large warp factor, the SUSY breaking becomes negligible.

[3] Singularity in dynamical brane

☀ **It is of great significance to understand the cosmological backgrounds profoundly.**

☠ **There is a naked singularity in the dynamical brane background due to ⋯**

(i) the divergence of non-trivial dilaton
(This also appears in the static brane).

(ii) the time-dependence in the theory.

✍ **The naked singularity in the 4-dim Einstein-Maxwell-dilaton theory with cosmological constant gives the violation of cosmic censorship.**

(Horne & Horowitz, Phys.Rev. D48 (1993) 5457-5462)

☆ **Question**

Does the smooth initial data in the dynamical brane background evolve into the naked singularity?

☀ Cosmic censorship conjecture

- ## Weak :

  "Singularities have to be hidden by the event horizon of a black hole."

- ## Strong :

  "For smooth initial data with suitable matter systems, the maximal Cauchy development is not extendible."

❏ **Our results:**

❀ **The cosmic censorship is violated in dynamical M-brane background.**

☞ **This is similar to the result which has been obtained in Einstein-Maxwell-dilaton theory (with cosmological constant).**

(Horne & Horowitz, Phys.Rev. D48 (1993) 5457-5462)

[3] Cosmic censorship in dynamical M5-brane

☆ Logic :

- We can set a regular and smooth initial data for the M5-brane.

- These initial data in the far past evolve into the curvature singularity.

- The cosmic censorship is violated.

❀ M5-brane
(Duff & Stelle, Phys.Lett. B253 (1991) 113-118)
(Güven, Phys.Lett. B276 (1992) 49-55)
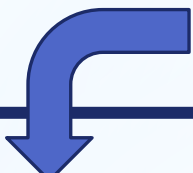
✎ matter (bosonic) :
gravity, 4-form field strength

M5 $t, x^1, \cdots, x^5$

	0	1	2	3	4	5	6	7	8	9	10
M5	○	○	○	○	○	○					
x^N	t	x^1	x^2	x^3	x^4	x^5	r	y^1	y^2	y^3	y^4

◆ Dynamical M5-brane background

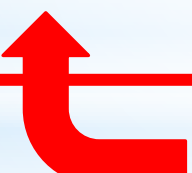(Binetruy & Sasaki & Uzawa, Phys.Rev. D80 (2009) 026001)
(Maeda & Ohta & Uzawa, JHEP 0906 (2009) 051)

(1+5)-dim worldvolume spacetime

$$ds^2 = \left(a\,t + b + \frac{M}{r^3} \right)^{-1/3} \eta_{\mu\nu} dx^\mu dx^\nu$$

$$+ \left(a\,t + b + \frac{M}{r^3} \right)^{2/3} \left(dr^2 + r^2 d\Omega_{(4)} \right)$$
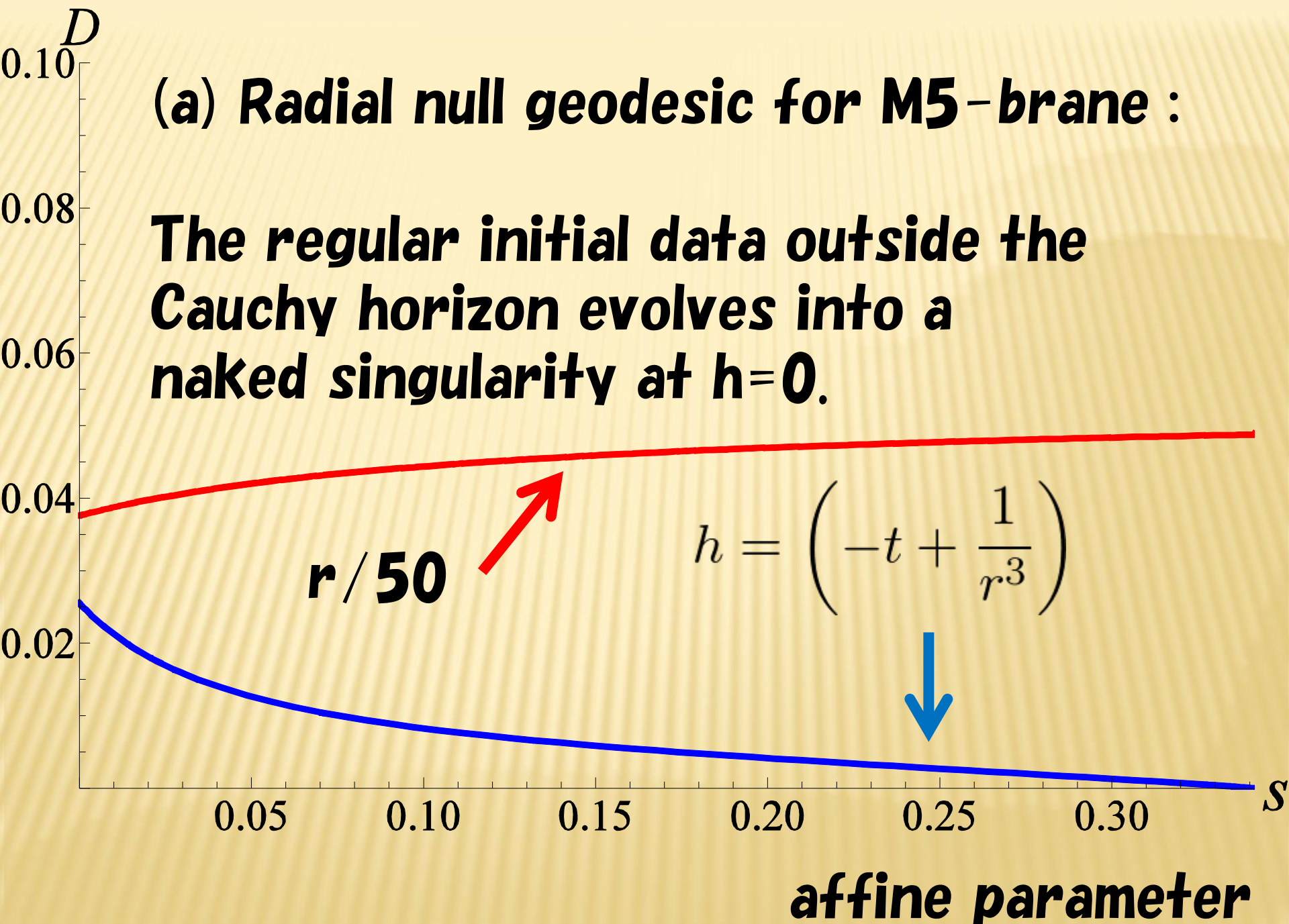
5-dim transverse space to brane

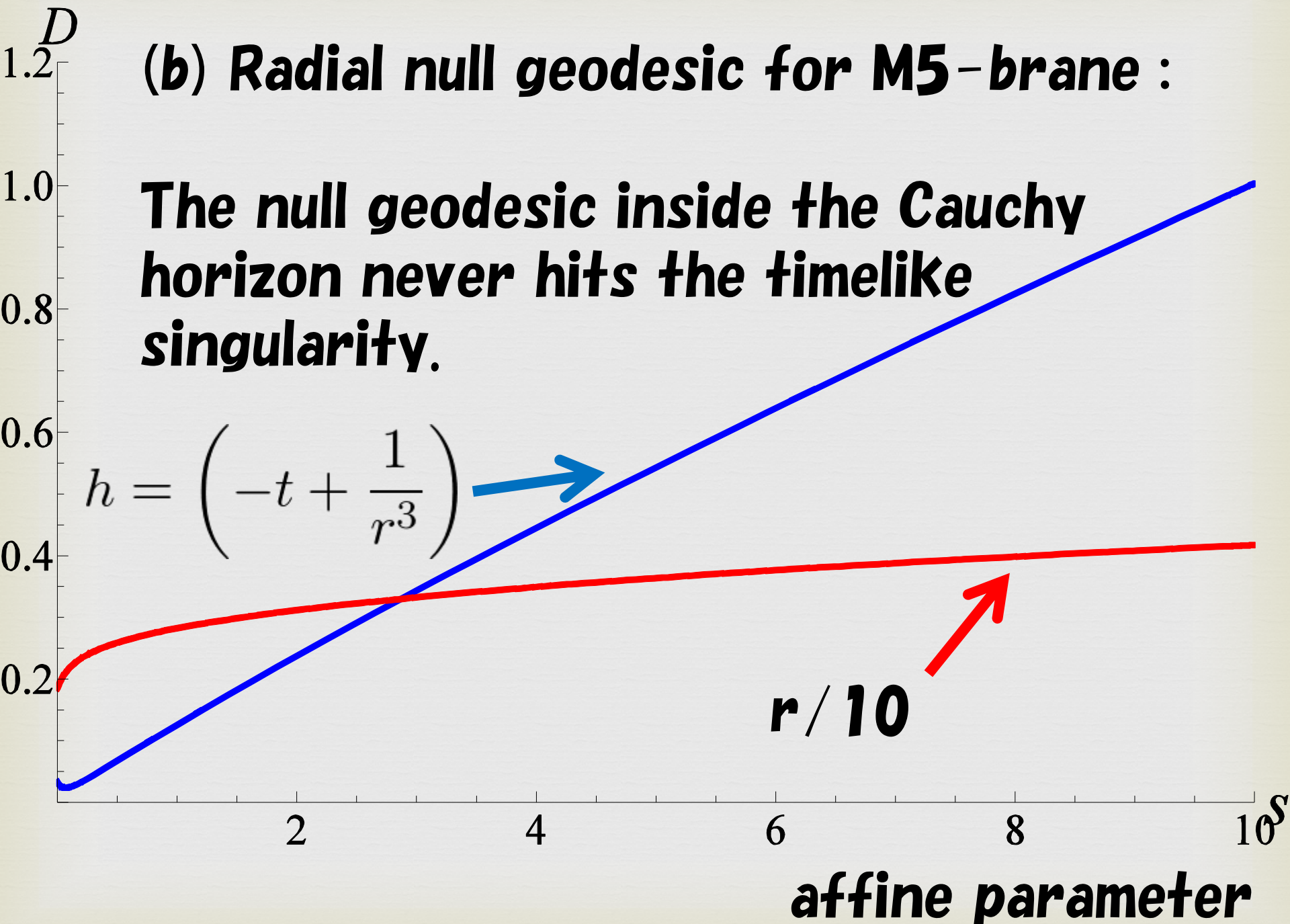$$\left(a\,t + b + \frac{M}{r^3} \right) = 0 \; : \; \text{curvature singularity}$$
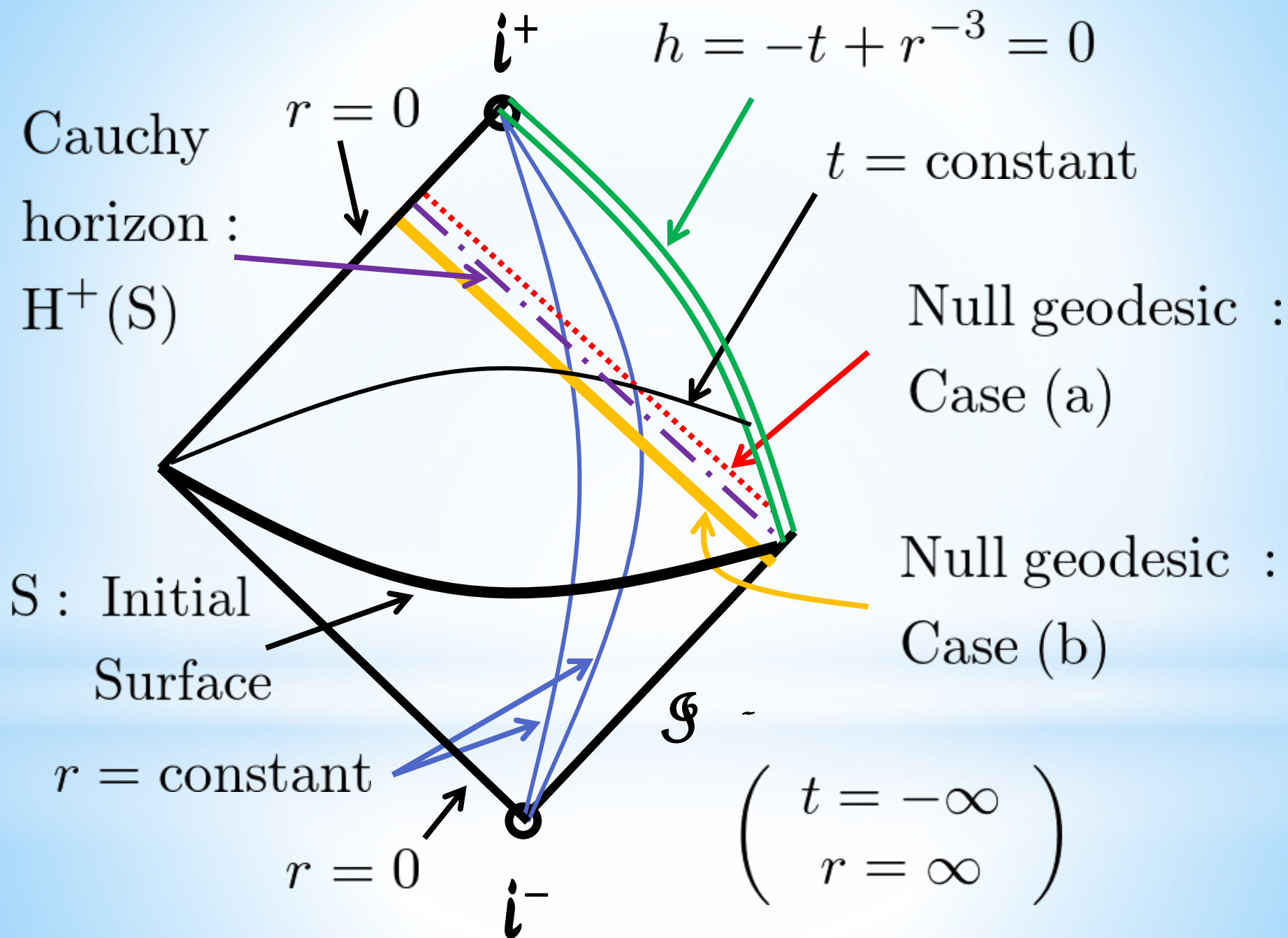
- **Geodesic equation :**

$$\frac{d^2 r}{ds^2} + \Gamma^r_{MN} \frac{dx^M}{ds} \frac{dx^N}{ds} = 0$$

✠ **We can set a regular and smooth initial data for the M5-brane.**

✪ **The asymptotic behavior of the null curves depends crucially on whether r is inside or outside the Cauchy horizon.**

(a) Radial null geodesic for M5-brane:

The regular initial data outside the Cauchy horizon evolves into a naked singularity at h=0.

r/50

$$h = \left(-t + \frac{1}{r^3} \right)$$

affine parameter

(b) Radial null geodesic for M5-brane :

The null geodesic inside the Cauchy horizon never hits the timelike singularity.

$$h = \left(-t + \frac{1}{r^3} \right)$$

r/10

affine parameter

[4] Summary and comments

(1) The dynamical brane background describes the new SUSY solution.

(2) The solutions of field equations cannot give a realistic expansion law. This means that we have to consider additional matter in order to get a realistic expanding universe.

(3) For dynamical M5-brane, we can set smooth initial data evolving into a timelike curvature singularity.