DYNAMICAL BRANE BACKGROUNDS

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[arXiv:1510.01496 [hep-th]]
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1 Introduction

❖ String theory:

♦ This is the only viable unified fundamental theories at present.

♣ String theory contains $p$-branes as well as strings.
What is “p-brane”? 

★ Classical membrane solution of Einstein equation

\[ X^1, X^2, \ldots, X^p \]

- This is extended in \( p \) direction.
- \( p \)-brane has \( p \) spacelike translational Killing vectors.
An innumerable number of static brane solutions have been discovered so far.

But ...

Cosmological brane solutions may also exist!

Dynamical brane background
“Dynamical” means time-dependent.

Dynamical brane may be related to

- brane collision
  (Gibbons & Lu & Pope, Phys.Rev.Lett. 94 (2005) 131602)

- cosmic Big-Bang of our universe

- black hole in expanding universe
  (Maeda & Ohta & Uzawa, JHEP 0906 (2009) 051)
Outline my talk

* The property of dynamical brane

* Singularities and cosmic censorship

* Summary and comments
[2] Property of dynamical brane solutions:
★ Cosmology:
(Binetruy, Sasaki, Uzawa, Phys.Rev.D80:026001,2009)

We assume an isotropic and homogeneous three space in the four-dimensional spacetime.

Note that the time dependence in the metric comes from only one brane even if we consider several branes.

Solutions in the original higher-dimensional theory (10D or 11D).
**Dynamical solution of D- and M-brane system**


\[
\begin{align*}
    ds^2 &= h^{-(D-p-3)/(D-2)} \eta_{\mu\nu} dx^\mu dx^\nu + h^{(p+1)/(D-2)} (dr^2 + r^2 d\Omega_{D-p-2}^2), \\
    h(t, r) &= c_1 t + c_2 + M r^{-D+p+3}, \quad F_{p+2} = d(h^{-1}) \wedge dt \wedge dx^1 \wedge \cdots \wedge dx^p, \\
    e^{\phi} &= h^{c/2}, \quad c^2 = 4 - 2(p+1)(D-p-3)(D-2)^{-1}
\end{align*}
\]

- 10-dim D3-brane solution

Kasner solution

\[
\begin{align*}
    r \to \infty,
    ds^2 &= (c_1 t + c_2)^{-1/2} \eta_{\mu\nu} dx^\mu dx^\nu + (c_1 t + c_2)^{1/2} (dr^2 + r^2 d\Omega_5^2)
\end{align*}
\]

- AdS$_5 \times $S$^5$

\[
\begin{align*}
    r \to 0,
    ds^2 &= \left(\frac{r}{M}\right)^2 \eta_{\mu\nu} dx^\mu dx^\nu + \left(\frac{M}{r}\right)^2 dr^2 + d\Omega_5^2
\end{align*}
\]
For each case, the scale factor of 4-dimensional universe is given by \( a(\tau) \propto \tau^\lambda \), where \( \tau \) denotes the cosmic time.

Since the three-dimensional spatial space of our universe stays in the transverse space to the brane, \( D \)-dimensional theory gives the fastest expansion of our universe.

The power of the scale factor becomes
\[
\lambda = \frac{p+1}{D+p-1} < 1 \quad \text{for} \quad D \geq 2.
\]
It is impossible to find the cosmological model that our universe is accelerating expansion.
Dynamical D3-brane background

In the case of no 3-form, the 10-dim metric becomes

\[ ds^2 = h^{-1/2} ds^2(\mathbb{E}^3, 1) + h^{1/2} \left[ dr^2 + r^2 ds^2(Z_5) \right], \]

\[ h(x, r) = \left[ h_0(x) + \frac{L^4}{r^4} \right] \]

For static background, AdS$_5 \times$S$_5$, the background has the full supersymmetry.
- Solution for dynamical background
  (H. Kodama & K. Uzawa, JHEP 0507:061,2005)

\[ \partial_\mu h_0 \gamma^\mu \varepsilon = 0, \quad -i \gamma^0 \gamma^1 \gamma^2 \gamma^3 \varepsilon = \varepsilon \]

(i) Induced effective mass for the spinor field

\[ \sim (Dh_0)^2 / h^2 \]

(ii) This mass scale diverges at the naked singularity where the function h vanishes.
(iii) the degree of supersymmetry breaking increases as the universe approaches the singularity.

(iv) In the region with a large warp factor, the SUSY breaking becomes negligible.
Singularity in dynamical brane

It is of great significance to understand the cosmological backgrounds profoundly.

There is a naked singularity in the dynamical brane background due to ...

(i) the divergence of non-trivial dilaton
   (This also appears in the static brane).

(ii) the time-dependence in the theory.

Question

Does the smooth initial data in the dynamical brane background evolve into the naked singularity?
**Cosmic censorship conjecture**
(Penrose, "Singularities and time-asymmetry", (1979) 617–629)

- **Weak:**
  "Singularities have to be hidden by the event horizon of a black hole."

- **Strong:**
  "For smooth initial data with suitable matter systems, the maximal Cauchy development is not extendible."
Our results:

The cosmic censorship is violated in dynamical M-brane background.

This is similar to the result which has been obtained in Einstein–Maxwell–dilaton theory (with cosmological constant).

Logic:

- We can set a regular and smooth initial data for the M5-brane.
- These initial data in the far past evolve into the curvature singularity.
- The cosmic censorship is violated.
**M5-brane**


⇒ matter (bosonic): gravity, 4-form field strength

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<th>M5 ( x^N )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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**Dynamical M5-brane background**

(Maeda & Ohta & Uzawa, JHEP 0906 (2009) 051)

\[ ds^2 = \left( a t + b + \frac{M}{r^3} \right)^{-1/3} \eta_{\mu\nu} dx^\mu dx^\nu \]

\[ + \left( a t + b + \frac{M}{r^3} \right)^{2/3} \left( dr^2 + r^2 d\Omega_{(4)} \right) \]

5-dim transverse space to brane

\[ \left( a t + b + \frac{M}{r^3} \right) = 0 : \text{ curvature singularity} \]
Geodesic equation:

\[ \frac{d^2 r}{ds^2} + \Gamma^r_{MN} \frac{dx^M}{ds} \frac{dx^N}{ds} = 0 \]

We can set a regular and smooth initial data for the M5-brane.

The asymptotic behavior of the null curves depends crucially on whether \( r \) is inside or outside the Cauchy horizon.
(a) Radial null geodesic for $M5$-brane:

The regular initial data outside the Cauchy horizon evolves into a naked singularity at $h=0$. 

\[ h = \left( -t + \frac{1}{r^3} \right) \]
(b) Radial null geodesic for M5-brane:

The null geodesic inside the Cauchy horizon never hits the timelike singularity.

\[ h = \left( -t + \frac{1}{r^3} \right) \]
$\dot{r} = 0$

Cauchy horizon: $H^+(S)$

Null geodesic: Case (a)

$S$: Initial Surface

Null geodesic: Case (b)

$$h = -t + r^{-3} = 0$$

$t = \text{constant}$

$r = \text{constant}$

$r = 0$

$\left( \begin{array}{c} t = -\infty \\ r = \infty \end{array} \right)$
Summary and comments

(1) The dynamical brane background describes the new SUSY solution.

(2) The solutions of field equations cannot give a realistic expansion law. This means that we have to consider additional matter in order to get a realistic expanding universe.

(3) For dynamical M5-brane, we can set smooth initial data evolving into a timelike curvature singularity.