Consistency Tests of LCDM

The 13th International Symposium on Cosmology and Particle Astrophysics (CosPA 2016)
Cosmological Observations

Cosmic Microwave Background (CMB)

Gravitational Lensing

Type Ia supernovae

Large-scale structure

Lyman Alpha Forest

Cosmology, from fiction to being science...
Era of Precision Cosmology
Combining theoretical works with new measurements and using statistical techniques to place sharp constraints on cosmological models and their parameters.

Initial Conditions:
- Form of the Primordial Spectrum and Model of Inflation and its Parameters
- Dark Energy: density, model and parameters
- Dark Matter: density and characteristics
- Neutrino species, mass and radiation density
- Baryon density, Neutrino species, mass and radiation density
- Curvature of the Universe
- FLRW?
- Hubble Parameter and the Rate of Expansion
- Epoch of reionization
Standard Model of Cosmology

Using measurements and statistical techniques to place sharp constraints on parameters of the standard cosmological models.

**Initial Conditions:**

- **Form of the Primordial Spectrum is** Power-law
- **Dark Energy is** Cosmological Constant: density
- **Dark Matter is** Cold and weakly Interacting: density
- Neutrino mass and radiation density: assumptions and CMB temperature

**Universe is** Flat

**FLRW**

**Epoch of reionization**

**Hubble Parameter and the Rate of Expansion**

**Power-law**
Standard Model of Cosmology

Using measurements and statistical techniques to place sharp constraints on parameters of the standard cosmological model.

Initial Conditions:

- Form of the Primordial Spectrum is Power-law
- Dark Energy is Cosmological Constant
- Dark Matter is Cold and weakly Interacting
- Neutrino mass and radiation density: fixed by assumptions and CMB temperature
- Baryon density
- Neutrino mass and radiation density: fixed by assumptions and CMB temperature
- Universe is Flat
- Hubble Parameter and the Rate of Expansion
- Epoch of reionization

Power-law

$$\Omega_\Lambda = 1 - \Omega_b - \Omega_{dm}$$
Combination of Assumptions

Dark Energy is **Cosmological Constant**: \( \Omega_\Lambda = 1 - \Omega_b - \Omega_{dm} \)

Universe is **Flat**

**FLRW**

**Universe is Flat**

** Epoch of reionization ** \( \tau \)

** Hubble Parameter and the Rate of Expansion ** \( H_0 \)
Standard Model of Cosmology

Using measurements and statistical techniques to place sharp constraints on parameters of the standard cosmological model.

Initial Conditions:
- Form of the Primordial Spectrum is Power-law
- Dark Energy is Cosmological Constant
- Dark Matter is Cold and weakly Interacting:
- Baryon density
- Neutrino mass and radiation density:
- Combination of reasonable assumptions, but.....

Universe is Flat

Power-law

Epoch of reionization

Hubble Parameter and the Rate of Expansion
Beyond the Standard Model of Cosmology

• The universe might be more complicated than its current standard model (Vanilla Model).
• There might be some extensions to the standard model in defining the cosmological quantities.
• This needs proper investigation, using advanced statistical methods, high performance computational facilities and high quality observational data.
Standard Model of Cosmology

- Universe is Flat
- Universe is Isotropic
- Universe is Homogeneous
- Dark Energy is Lambda (w=-1)
- Power-Law primordial spectrum (n_s=const)
- Dark Matter is cold

All within framework of FLRW (Present)
Planck 2015: $n_s$ vs $r$
Primordial Power Spectrum

\[ P(k) \]

Determined by background model and cosmological parameters

\[ C_l = \sum G(l,k)P(k) \]

Suggested by Model of Inflation

Detected by observation

Angular power Spectrum

Parameterization and Model Fitting

Cosmological Radiative Transport Kernel
We cannot anticipate the unexpected!!

$C_l = \sum G(l,k)P(k)$

$G(l,k)$
Cosmological Radiative Transport Kernel

Detected by observation

Angular power Spectrum

Detected by observation

Determined by background model and cosmological parameters
$P(k)$
Primordial Power Spectrum

$G(l,k)$
Cosmological Radiative Transport Kernel

$C_l = \sum G(l,k) P(k)$

Detected by observation

Reconstructed by Observations

Determined by background model and cosmological parameters

Angular power Spectrum

DIRECT TOP DOWN Reconstruction
Primordial Power Spectrum from Planck
Hazra, Shafieloo & Souradeep, JCAP 2014

<table>
<thead>
<tr>
<th>Our symbol</th>
<th>Spectra</th>
<th>Multipoles(f)</th>
<th>Scales</th>
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</thead>
<tbody>
<tr>
<td>α</td>
<td>low-ř</td>
<td>2-49</td>
<td>Largest scales</td>
</tr>
<tr>
<td>a</td>
<td>100 GHz × 100 GHz</td>
<td>50-1200</td>
<td>Intermediate scales</td>
</tr>
<tr>
<td>b</td>
<td>143 GHz × 143 GHz</td>
<td>50-2000</td>
<td>Intermediate scales</td>
</tr>
<tr>
<td>l</td>
<td>217 GHz × 217 GHz</td>
<td>500-2500</td>
<td>Small scales</td>
</tr>
<tr>
<td>2</td>
<td>143 GHz × 217 GHz</td>
<td>500-2500</td>
<td>Small scales</td>
</tr>
</tbody>
</table>
Planck 2015: No feature

Power spectra reconstruction

2015
TT+lowP
+BAO+JLA
+Hlow

12-knots power spectra
(actually used 3 different methods, all with similar results)
**Cosmological Parameter Estimation with Power-Law Primordial Spectrum**

- Flat Lambda Cold Dark Matter Universe (LCDM) with power–law form of the primordial spectrum

- It has 6 main parameters.

\[
C_l = \sum G(l,k) P(k)
\]

- **Direct Reconstruction of PPS and Theoretical Implication**

\[
P(k) = A_s \left[ \frac{k}{k_*} \right]^{n_s - 1}
\]
Cosmological Parameter Estimation with Free form Primordial Spectrum

\[ C_l = \sum G(l,k)P(k) \]

Direct Reconstruction of PPS and Theoretical Implication
Cosmological Parameter Estimation with Free form Primordial Spectrum

Red Contours: Power Law PPS

Blue Contours: Free Form PPS

Hazra, Shafieloo & Souradeep, PRD 2013
there are some other models consistent to the data.
Beyond Power-Law: there are some other models consistent to the data.
Understanding the Early Universe:

- Form of the primordial spectrum (degenerate with other cosmological quantities).
- Tensor-to-scalar ratio of perturbation amplitudes (near future potential probe)
- Primordial non-Gaussianities (near future potential probe)
Plausible approach for the future:

Joint constraint on inflationary features using the two and three-point correlations of temperature and polarization anisotropies

Bispectrum in terms of the reconstructed power spectrum and its first two derivatives

Direct reconstruction of PPS from Planck

Appleby, Gong, Hazra, Shafieloo, Sypsas, PLB 2016
Figure 5. Wiggly Whipped Inflation: Matter power spectra (left) obtained from the best fit potential and background parameters (in Table 1) and the ratio (right) w.r.t. the matter power spectra obtained from power law best fit model. The DESI forecasted fractional errors are overlayed in the right panel as well. Note that from the future matter power spectrum data we shall be able to identify specific features in the primordial power spectrum.
From 2D to 3D (first step)

- Generating many N-body simulations (similar to stage IV dark energy measurements such as DESI) based on various inflationary scenarios with features in PPS (but still degenerate to be distinguished by CMB data).

- Try to distinguish them by implementing/designing appropriate statistics.
  (power spectrum, bi-spectrum etc may not work)
Standard Model of Cosmology
18 years after discovery of the acceleration of the universe:

From 60 Supernovae Ia at cosmic distances, we now have ~800 published distances, with better precision, better accuracy, out to $z=1.5$.

Accompanying figure:

- **Guess 0**: $(\Omega_m, \Omega_{\Lambda}, h) = (0.300, 0.700, 0.700); \chi^2 = 547.228$
- **\Lambda CDM best fit**: $(\Omega_m, \Omega_{\Lambda}, h) = (0.245, 0.712, 0.703); \chi^2 = 543.961$

L’Huillier & Shafieloo 2016
18 years after discovery of the acceleration of the universe:

CMB directly points to acceleration.

Didn't even have acoustic peak in 1998!
18 years after discovery of the acceleration of the universe:

BOSS collaboration (2016), arXiv: Alam et al, 1607.03155
Something seems to be there, but,

What is it?
Dark Energy Models

• Cosmological Constant
• Quintessence and k-essence (scalar fields)
• Exotic matter (Chaplygin gas, phantom, etc.)
• Braneworlds (higher-dimensional theories)
• Modified Gravity

But which one is really responsible for the acceleration of the expanding universe?!
To find cosmological quantities and parameters there are two general approaches:

1. Parametric methods
   - Easy to confront with cosmological observations to put constrains on the parameters, but the results are highly biased by the assumed models and parametric forms.

2. Non Parametric methods
   - Difficult to apply properly on the raw data, but the results will be less biased and more reliable and independent of theoretical models or parametric forms.

Reconstructing Dark Energy
Problems of Dark Energy Parameterizations

Holsclaw et al, PRD 2011

Chevallier-Polarski-Linder ansatz (CPL).

Brane Model
Quintessence DE?!

Kink Model
Phantom DE?!

\[ w(z) = w_0 + w_a \frac{z}{1+z}. \]
Dealing with observational uncertainties in matter density (and curvature)

- Small uncertainties in the value of matter density affects the reconstruction exercise dramatically.
- Uncertainties in matter density is in particular bound to affect the reconstructed $w(z)$.

\[
H(z) = \left[ \frac{d}{dz} \left( \frac{d_L(z)}{1+z} \right) \right]^{-1}
\]

\[
\omega_{DE} = \frac{\left( \frac{2(1+z) H'}{H} \right)^{-1}}{1 - \left( \frac{H_0}{H} \right)^2 \Omega_{0M} (1+z)^3}
\]
Cosmographic Degeneracy

\[ d_l(z) = \frac{1 + z}{\sqrt{1 - \Omega_m - \Omega_{de}}} \sinh \left( \sqrt{1 - \Omega_m - \Omega_{de}} \int_0^z \frac{dz'}{h(z')} \right) \]

\[ h(z)^2 = \left[ \frac{H(z)}{H_0} \right]^2 = \left( \frac{\dot{a}}{a} \right)^2 \]

\[ = \Omega_m (1 + z)^3 + (1 - \Omega_m - \Omega_{de})(1 + z)^2 \]

\[ + \Omega_{de} \exp \left[ 3 \int_0^z \frac{dz'}{1 + z'} \left[ 1 + w(z') \right] \right] \]
Cosmographic Degeneracies would make it so hard to pin down the actual model of dark energy even in the near future.

Shafieloo & Linder, PRD 2011

Cosmographic Degeneracy

No Curvature Constraint
Flat Universe

\[ \omega_{DE} = \frac{2(1+z)H'}{3H} - 1 \]

\[ \frac{1}{1 - \left(\frac{H_0}{H}\right)^2 \Omega_M (1+z)^3} \]

Indistinguishable from each other!
Reconstruction  Falsification

Considering (low) quality of the data and cosmographic degeneracies we should consider a new strategy sidewise to reconstruction: Falsification.

Yes-No to a hypothesis is easier than characterizing a phenomena.

But, How? We should look for special characteristics of the standard model and relate them to observables.
Instead of looking for $w(z)$ and exact properties of dark energy at the current status of data, we can concentrate on a more reasonable problem: **Falsification of Cosmological Constant**

Yes-No to a hypothesis is easier than characterizing a phenomena
$w(z) = -0.7$

$w(z) = -1.3$

$$H^2(z) = H_0^2 \left[ \Omega_{0m} (1+z)^3 + \Omega_{DE} \right]$$

$$\Omega_{DE} = (1 - \Omega_{0m}) \exp \left\{ 3 \int_0^z \frac{1 + w(z')}{1 + z'} \, dz' \right\}$$
Falsification: Null Test of Lambda

\[ Om(z) = \frac{h^2(z) - 1}{(1 + z)^3} - 1 \]

We Only Need \( h(z) \)

\[ h(z) = \frac{H(z)}{H_0} \]

\( Om(z) \) is constant only for FLAT LCDM model

\[ w = -1 \rightarrow Om(z) = \Omega_{0m} \]
\[ w < -1 \rightarrow Om(z) < \Omega_{0m} \]
\[ w > -1 \rightarrow Om(z) > \Omega_{0m} \]

Quintessence
\[ w = -0.9 \]

Phantom
\[ w = -1.1 \]
Om diagnostic is very well established
Model Independent Evidence for Dark Energy Evolution from Baryon Acoustic Oscillation

\[
Omh^2(z_1, z_2) = \frac{H^2(z_2) - H^2(z_1)}{(1 + z_2)^3 - (1 + z_1)^3} = \Omega_{0m} H_0^2
\]


A very recent result.
Important discovery if no systematic in the SDSS Quasar BAO data

\[
Omh^2 = 0.1426 \pm 0.0025
\]

\[
Omh^2(z_1; z_2) = 0.124 \pm 0.045
\]

\[
Omh^2(z_1; z_3) = 0.122 \pm 0.010
\]

\[
Omh^2(z_2; z_3) = 0.122 \pm 0.012
\]
A null diagnostic customized for reconstructing the properties of dark energy directly from BAO data

\[ \text{Observables} \]

Shafieloo, Sahni, Starobinsky, PRD 2013
Om is constant only for Flat LCDM model
Om3 is equal to one for Flat LCDM model

\[ Om3(z_1; z_2; z_3) = \frac{H(z_2; z_1)^2 - 1}{x_2^3 - x_1^3} \left/ \frac{H(z_3; z_1)^2 - 1}{x_3^3 - x_1^3} \right., \quad \text{where} \quad x = 1 + z, \]

\[ H(z_i; z_j) = \left( \frac{z_j}{z_i} \right)^2 \left[ \frac{D(z_i)}{D(z_j)} \right]^2 \left[ \frac{A(z_j)}{A(z_i)} \right] \left[ \frac{d(z_i)}{d(z_j)} \right]^3 = \frac{z_i}{z_j} \left[ \frac{D(z_i)}{D(z_j)} \right]^2 \left[ \frac{d(z_i)}{d(z_j)} \right]^3, \]

Om3 is independent of H0 and the early universe models and can be derived directly using BAO observables.

Shafieloo, Sahni, Starobinsky, PRD 2013
Future perspective
P. Bull et al,
1501.04088
• Om3 will show its power as it can be measured very precisely and used as a powerful litmus test of Lambda.

\[
\begin{align*}
\sigma_{om3} &\approx 1.0 \times 10^0 \text{[WiggleZ]} \\
\sigma_{om3} &\approx 2.0 \times 10^{-1} \text{[DESI]} \\
\sigma_{om3} &\approx 5.7 \times 10^{-1} \text{[SKA1 – SUR(Gal)]} \\
\sigma_{om3} &\approx 5.6 \times 10^{-1} \text{[SKA1 – MID(Gal)]} \\
\sigma_{om3} &\approx 4.0 \times 10^{-2} \text{[SKA1 – MID(IM)]} \\
\sigma_{om3} &\approx 2.5 \times 10^{-2} \text{[SKA1 – SUR(IM)]} \\
\sigma_{om3} &\approx 1.4 \times 10^{-2} \text{[Euclid]} \\
\sigma_{om3} &\approx 9.3 \times 10^{-3} \text{[SKA2(Gal)]}
\end{align*}
\]
Standard Model of Cosmology

- Universe is Flat
- Universe is Isotropic
- Universe is Homogeneous (large scales)
- Dark Energy is Lambda (w=-1)
- Power-Law primordial spectrum (n_s=const)
- Dark Matter is cold

All within framework of FLRW
Falsification: Is Universe Isotropic?

Method of Smoothed Residuals
- Residual Analysis,
- Tomographic Analysis,
- 2D Gaussian Smoothing,
- Frequentist Approach
- Insensitive to non-uniform distribution of the data
Method of Smoothed Residuals is well received and was used recently by Supernovae Factory collaboration.
### Bias in the Sky

Appleby, Shafieloo, JCAP 2014


**Method of Smoothed Residuals**

<table>
<thead>
<tr>
<th>Catalog</th>
<th>0.015 ≤ z &lt; 0.025</th>
<th>0.025 ≤ z &lt; 0.035</th>
<th>0.035 ≤ z &lt; 0.045</th>
<th>0.045 ≤ z &lt; 0.06</th>
<th>0.06 ≤ z &lt; 0.1</th>
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<tbody>
<tr>
<td>Union 2.1</td>
<td>61</td>
<td>51</td>
<td>15</td>
<td>17</td>
<td>19</td>
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<tr>
<td>Constitution</td>
<td>53</td>
<td>40</td>
<td>11</td>
<td>12</td>
<td>8</td>
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<tr>
<td>LOSS</td>
<td>76</td>
<td>64</td>
<td>23</td>
<td>17</td>
<td>19</td>
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<tr>
<td>Combined</td>
<td>98</td>
<td>67</td>
<td>22</td>
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<table>
<thead>
<tr>
<th>Δz</th>
<th>Catalog</th>
<th>( b_{\text{max}} )</th>
<th>( \ell_{\text{max}} )</th>
<th>( p )</th>
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<td>0.015 ≤ z &lt; 0.025</td>
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<td>Const (SALT II)</td>
<td>20°</td>
<td>284°</td>
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<td>Const (MLCS 17)</td>
<td>67°</td>
<td>241°</td>
<td>0.692</td>
<td></td>
</tr>
<tr>
<td>LOSS</td>
<td>4°</td>
<td>247°</td>
<td>0.412</td>
<td></td>
</tr>
<tr>
<td>Combined</td>
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<td>241°</td>
<td>0.179</td>
<td></td>
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<td>0.202</td>
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<td>LOSS</td>
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<td>32°</td>
<td>0.574</td>
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<td>Const (MLCS 17)</td>
<td>58°</td>
<td>65°</td>
<td>0.156</td>
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<td>LOSS</td>
<td>52°</td>
<td>349°</td>
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<td>Combined</td>
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<td>65°</td>
<td>0.788</td>
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<table>
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<th>Δz</th>
<th>( p_A )</th>
<th>( p_B )</th>
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<td>0.015 ≤ z &lt; 0.025</td>
<td>0.179</td>
<td>0.371</td>
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<td>0.015 ≤ z &lt; 0.035</td>
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<td>0.070</td>
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<td>0.015 ≤ z &lt; 0.060</td>
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<td>0.015 ≤ z &lt; 0.100</td>
<td>0.270</td>
<td>0.531</td>
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<table>
<thead>
<tr>
<th>( V_{\text{bary}} ) (kms(^{-1}))</th>
<th>North (( b_v &gt; 20° ))</th>
<th>South (( b_v &lt; -20° ))</th>
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<td>(( \Delta b, \Delta \ell ))</td>
<td>(( \Delta b, \Delta \ell ))</td>
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<tr>
<td>400</td>
<td>(13°, -3°)</td>
<td>(14°, 28°)</td>
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<tr>
<td>800</td>
<td>(15°, -4°)</td>
<td>(9°, 22°)</td>
</tr>
</tbody>
</table>
Falsification: Testing Isotropy of the Universe in Matter Dominated Era through Lyman Alpha forest

- Comparing statistical properties of the PDF of the Lyman-alpha transmitted flux in different patches
- Different redshift bins and different signal to noise
- Results for BOSS DR9 quasar sample

Results consistent to Isotropy
Using Local Variance to Test Statistical Isotropy in CMB maps

- Based on Crossing Statistic
- Residual Analysis,
- Real Space Analysis
- Low Sensitivity to Systematics
- 2D Adaptive Gaussian Smoothing
- Frequentist Approach

**TABLE 1**

<table>
<thead>
<tr>
<th>Map</th>
<th>(l, b) [°]</th>
<th>Significance or p-value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Planck-VA</td>
<td>(212, -13)</td>
<td>0/1000</td>
<td>present work</td>
</tr>
<tr>
<td>WMAP5-VA</td>
<td>(219, -24)</td>
<td>10/1000</td>
<td>present work</td>
</tr>
<tr>
<td>Planck-DP</td>
<td>(227, -15)</td>
<td>3.5σ</td>
<td>Ade et al. (2013a)</td>
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<tr>
<td>WMAP5-DP</td>
<td>(224, -22)</td>
<td>3.5σ</td>
<td>Hofvath et al. (2009)</td>
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<td>Planck-PA</td>
<td>(224, 6)</td>
<td>0/500</td>
<td>Ade et al. (2013a)</td>
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<tr>
<td>WMAP5-PA</td>
<td>(227, -27)</td>
<td>7/10000</td>
<td>Axelsson et al. (2013)</td>
</tr>
</tbody>
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**Curvature and Metric Test**

\[
\Theta(z) = \frac{1 + z}{c} \left( H(z) r_d \frac{d_A(z)}{r_d} \right) \left( \frac{D'(z)}{D(z)} \right),
\]

\[
\Omega_k(z) = \frac{\Theta^2(z) - 1}{D^2(z)},
\]

\[
\Theta(z) \equiv h(z) D'(z) = \frac{H(z)}{H_0} D'(z) = 1.
\]
Gaussian Processes:

Modeling of the data around a mean function searching for likely features by looking at the likelihood space of the hyperparameters.

Bayesian Interpretation of Crossing Statistic:

Comparing a model with its own possible variations.

REACT:

Risk Estimation and Adaptation after Coordinate Transformation
Efficient in statistical modeling of stochastic variables
Derivatives of Gaussian Processes are Gaussian Processes
Provides us with all covariance matrices

\[
\begin{bmatrix}
y \\
f \\
f' \\
f''
\end{bmatrix} \sim \mathcal{N}
\begin{bmatrix}
m(Z) \\
m(Z_1) \\
m'(Z_1) \\
m''(Z_1)
\end{bmatrix},
\begin{bmatrix}
\Sigma_{00}(Z, Z) & \Sigma_{00}(Z, Z_1) & \Sigma_{01}(Z, Z_1) & \Sigma_{02}(Z, Z_1) \\
\Sigma_{00}(Z_1, Z) & \Sigma_{00}(Z_1, Z_1) & \Sigma_{01}(Z_1, Z_1) & \Sigma_{02}(Z_1, Z_1) \\
\Sigma_{10}(Z_1, Z) & \Sigma_{10}(Z_1, Z_1) & \Sigma_{11}(Z_1, Z_1) & \Sigma_{12}(Z_1, Z_1) \\
\Sigma_{20}(Z_1, Z) & \Sigma_{20}(Z_1, Z_1) & \Sigma_{21}(Z_1, Z_1) & \Sigma_{22}(Z_1, Z_1)
\end{bmatrix}
\]

\[
\frac{\partial K}{\partial z_i} = \frac{d^{(\alpha+\beta)\alpha+\beta}}{dz_i^{\alpha+\beta}},
\]

\[
k(z, z') = \sigma_f^2 \exp \left( -\frac{|z - z'|^2}{2l^2} \right),
\]

\[
2 \ln p(y|f) = -y^T \Sigma_{00}(Z, Z)^{-1} y - \ln \det \Sigma_{00}(Z, Z) - n \ln(2\pi),
\]

Data
Mean Function
Kernel
GP Hyper-parameters
GP Likelihood
Detection of the features in the residuals

Simulations

Signal Detectable

Signal Undetectable

GP to test GR
Shafieloo, Kim, Linder, PRD 2013
\[ C^{TT}_{\ell} \big|_{\text{modified}} = C^{TT}_{\ell} \big|_{\Omega_b, \Omega_{CDM}, H_0, \tau, A_S, n_S, \ell} \times T_i(C_0, C_1, C_2, \ldots, C_N, \ell). \]

Consistent only at 2~3 sigma CL
\[ C_{\ell}^{\text{TT}} |_{\text{modified}}^N = C_{\ell}^{\text{TT}} |_{\Omega_b, \Omega_{\text{CDM}}, H_0, \tau, A_S, n_S, \ell} \times T_i(C_0, C_1, C_2, \ldots, C_N, \ell). \]

Completely Consistent
$C_{\ell}^{TT}|_{\text{modified}} = C_{\ell}^{TT}|_{\Omega_b, \Omega_{\text{CDM}}, H_0, \tau, A_S, n_S, \ell} \times T_i(C_0, C_1, C_2, \ldots, C_N, \ell).$

**Completely Consistent**
\[ C_{\ell}^{TT} \bigg|_{N_{\text{modified}}} = C_{\ell}^{TT} \bigg|_{\Omega_b, \Omega_{CDM}, H_0, \tau, A_S, n_S, \ell} \times T_i(C_0, C_1, C_2, \ldots, C_N, \ell). \]

Amplitude discrepancy!
(issue was later on resolved)
Where is ISW?!
• The current standard model of cosmology seems to work fine but this does not mean all the other models are wrong. Data is not yet good enough to distinguish between various models.

• Using parametric methods and model fitting is tricky and we may miss features in the data. Non-parameteric methods of reconstruction can guide theorist to model special features.

• First target can be testing different aspects of the standard ‘Vanilla’ model. If it is not ‘Lambda’ dark energy or power-law primordial spectrum then we can look further. It is possible to focus the power of the data for the purpose of the falsification. Next generation of astronomical/cosmological observations, (DESI, Euclid, SKA, LSST, WFIRST etc) will make it clear about the status of the concordance model.
• We can (will) describe the constituents and pattern of the universe (soon). But still we do not understand it. Next challenge is to move from inventory to understanding, by the help of new generation of experiments.
Power spectra reconstruction

\[ \ell \equiv kD_{\text{rec}} \]

2015 TT+lowP +BAO+JLA +Hlow

12-knots power spectra

(fixed \( r = 0.1 \))
Planck likelihood codes are released but not the data in a usable form in practice. Struggle is going on.....
The one dimensional marginalized likelihood of dark energy density $\Omega_\Lambda$ obtained using free form of primordial spectrum (in solid blue line) and using power law (in dashed red line). $\Omega_\Lambda = 0$ is clearly not favored by the data even if we allow a power spectra free of forms. Quantitatively, in $4\sigma$ the data rules out $\Omega_\Lambda < 0.25$. This is probably the first indication towards presence of dark energy with a very high confidence using CMB data alone.
Importance of the Features

Kink in the potential

Pre-inflationary radiation dominated era

Pre-inflationary kinetic dominated era

Exponential cut off

Direct Reconstruction

TABLE II: Best fit values of parameters specifying the initial power spectrum \((k_*, \alpha, R_*, n_s)\) and other relevant cosmological parameters for a class of model power spectra with an infrared cutoff (dataset used: WMAP TT data).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Expo-cutoff</th>
<th>Starobinsky</th>
<th>Kin. Dom.</th>
<th>VF</th>
<th>Expo-staro(a)</th>
<th>Expo-staro(b)</th>
<th>Power Law</th>
</tr>
</thead>
<tbody>
<tr>
<td>(k_* \times 10^{-4}) Mpc(^{-1})</td>
<td>3.6(^{+4.8}_{-2.9})</td>
<td>3.5(^{+2.8}_{-2.3})</td>
<td>3.5(^{+3.0}_{-2.3})</td>
<td>0.4(^{+0.7}_{-0.5})</td>
<td>3.0(^{+0.5}_{-2.0})</td>
<td>3.1(^{+5.5}_{-2.1})</td>
<td>–</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>9.6(^{+0.3}_{-0.6})</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0.58(^{+4.6}_{-0.43})</td>
<td>0.72(^{+9.1}_{-0.55})</td>
<td>–</td>
</tr>
<tr>
<td>(R_*)</td>
<td>–</td>
<td>0.73(^{+0.25}_{-0.14})</td>
<td>–</td>
<td>–</td>
<td>0.17(^{+0.80}_{-0.15})</td>
<td>0.35(^{+0.43}_{-0.20})</td>
<td>–</td>
</tr>
<tr>
<td>(n_s)</td>
<td>0.98(^{+0.16}_{-0.03})</td>
<td>0.96(^{+0.14}_{-0.07})</td>
<td>1.4(^{+0.09}_{-0.90})</td>
<td>1.6(^{+0.04}_{-0.15})</td>
<td>0.96(^{+0.15}_{-0.08})</td>
<td>0.99(^{+0.08}_{-0.12})</td>
<td>0.96(^{+0.30}_{-0.10})</td>
</tr>
<tr>
<td>(\tau)</td>
<td>0.014(^{+0.037}_{-0.004})</td>
<td>0.01(^{+0.025}_{-0.014})</td>
<td>0.1(^{+0.09}_{-0.15})</td>
<td>0.01(^{+0.035}_{-0.001})</td>
<td>0.26(^{+0.15}_{-0.08})</td>
<td>0.28(^{+0.12}_{-0.27})</td>
<td>0.014(^{+0.50}_{-0.04})</td>
</tr>
<tr>
<td>(\sigma_8)</td>
<td>3.2(^{+2.17}_{-0.67})</td>
<td>16.3(^{+11.5}_{-12.9})</td>
<td>17.8(^{+4.9}_{-15.2})</td>
<td>2.7(^{+2.55}_{-0.22})</td>
<td>23.8(^{+5.9}_{-6.0})</td>
<td>23.5(^{+3.9}_{-21.0})</td>
<td>3.2(^{+26.6}_{-0.63})</td>
</tr>
<tr>
<td>(\Omega_m)</td>
<td>0.70(^{+0.16}_{-0.18})</td>
<td>0.71(^{+0.37}_{-0.24})</td>
<td>0.70(^{+0.13}_{-0.21})</td>
<td>0.71(^{+0.12}_{-0.26})</td>
<td>0.74(^{+0.13}_{-0.10})</td>
<td>0.75(^{+0.12}_{-0.25})</td>
<td>0.67(^{+0.24}_{-0.25})</td>
</tr>
<tr>
<td>(\Omega_b h^2)</td>
<td>0.023(^{+0.006}_{-0.001})</td>
<td>0.022(^{+0.005}_{-0.004})</td>
<td>0.024(^{+0.004}_{-0.002})</td>
<td>0.024(^{+0.005}_{-0.002})</td>
<td>0.024(^{+0.007}_{-0.003})</td>
<td>0.025(^{+0.007}_{-0.005})</td>
<td>0.024(^{+0.006}_{-0.002})</td>
</tr>
<tr>
<td>(-\ln \mathcal{L})</td>
<td>481.89</td>
<td>481.89</td>
<td>485.18</td>
<td>486.46</td>
<td>483.44</td>
<td>484.45</td>
<td>486.28</td>
</tr>
<tr>
<td>(\chi^2_{\text{eff}} \equiv -2 \ln \mathcal{L})</td>
<td>969.78</td>
<td>969.78</td>
<td>970.36</td>
<td>972.92</td>
<td>966.88</td>
<td>968.90</td>
<td>972.56</td>
</tr>
<tr>
<td>d.o.f.</td>
<td>891</td>
<td>891</td>
<td>892</td>
<td>892</td>
<td>890</td>
<td>890</td>
<td>893</td>
</tr>
</tbody>
</table>
Inflationary scenarios

Is the recovered spectrum unusual for inflationary scenarios?

• Starobinsky (1992): sharp changes in the slope in the inflation potential.

\[
P(k) = P_0(k)D(k, k_c, r) = A_s k^{1-n_s} \left[ 1 - 3(r - 1) \frac{1}{y} \left( \frac{1 - 1}{y^2} \sin 2y + \frac{2}{y} \cos 2y + \frac{9}{2} (r - 1)^2 \frac{1}{y^2} \left( \frac{1 + 1}{y^2} \cos 2y - \frac{2}{y} \sin 2y \right) \right) \right]
\]

Starobinsky

\[
y = \frac{k}{k_c}
\]

Vilenkin and Ford

\[
P(k) = A_s k^{1-n_s} \frac{1}{4y^4} \left| e^{-2iy} (1 + 2iy) - 1 - 2y^2 \right|^2
\]
The recovered spectrum is NOT unusual!!
Motivating Inflationary Scenarios

Punctuated Inflation
Step Model

Hazra, Shafieloo, Souradeep, PRD 2013
Beyond Power-Law: there are some other models consistent to the data.

Phenomenological Models

Hazra, Shafieloo, Smoot, JCAP 2013
Beyond Power-Law: there are some other models consistent to the data.

Phenomenological Models

Hazra, Shafieloo, Smoot, JCAP 2013
there are some other models consistent to the data.
18 years after discovery of the acceleration of the universe:

WiggleZ collaboration,
Planck 2015: Testing Concordance Model using GP and its hyper-parameters

Aghamousa, Shafieloo, Hamann 2016 (in prep)
Calibrated REACT

Consistent only at 2~3 sigma CL

Excluding 217 Ghz, consistent at 1~2 sigma CL
• Target:

• Tools:

• Aim:
From 2D to 3D

Using LSS data to test early universe scenarios

• Targets:
  Features in PPS, primordial non-Gaussianity, spherical asymmetry

• Tools:
  Simulations, developing statistics, cross correlation with other data.

• Aim:
  To be well prepared for the future data (DESI).
Om is constant only for Flat LCDM model
Om3 is equal to one for Flat LCDM model

\[ \frac{H(z_2; z_1)^2 - 1}{x_2^3 - x_1^3} \Bigg/ \frac{H(z_3; z_1)^2 - 1}{x_3^3 - x_1^3}, \quad \text{where} \quad x = 1 + z, \]
18 years after discovery of the acceleration of the universe:

D. Sherwin et. al, PRL 2011

ACT CMB Survey
18 years after discovery of the acceleration of the universe:

Cosmological Parameter Estimation with Free form Primordial Spectrum

Red Contours:
Power Law PPS

Blue Contours:
Free Form PPS

Hazra, Shafieloo & Souradeep PRD 2013
Direct Reconstruction of angular power spectrum from Planck 2015 using Gaussian Processes

Aghamousa & Shafieloo 2016 (in prep)

Aghamousa, Shafieloo, Hamann 2016 (in prep)
Chebishev Polynomials as Crossing Functions

\[ T_I(C_1, z) = 1 + C_1 \left( \frac{z}{z_{\text{max}}} \right) \]

\[ T_{II}(C_1, C_2, z) = 1 + C_1 \left( \frac{z}{z_{\text{max}}} \right) + C_2 \left[ 2 \left( \frac{z}{z_{\text{max}}} \right)^2 - 1 \right], \]

Comparing a model with its own variations

Shafieloo, JCAP 2012 (a)
Shafieloo, JCAP 2012 (b)
18 years after discovery of the acceleration of the universe:

CMB