

Flavour Physics and CP Violation

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Content

- Standard Model, Quark flavour mixing & the CKM Matrix
- Present status of the V_{CKM} matrix elements $|V_{ud}|, |V_{us}|, |V_{cd}|, |V_{cs}|$
- Determination of $|V_{cb}|$ and $|V_{ub}|$ from semileptonic B -decays
- Status of the third row of V_{CKM} : $|V_{td}|, |V_{ts}|, |V_{tb}|$
- CP violation in the charged and neutral hadron decays - Formalism
- CP violation in the Kaon sector
- $D^0 - \bar{D}^0$ mixing and CP violation in the Charm sector
- CP Violation in the B -meson sector & determination of the weak phases $\alpha, \beta, \gamma, \phi_s$
- Rare B -decays and their impact on the CKM phenomenology
- Summary

Recommended Reviews

- Particle Data Group C. Patrignani *et al.*, Chin. Phys. C, **40**, 100001 (2016)
 - The CKM quark-mixing matrix; A. Ceccucci, Z. Ligeti, Y. Sakai
 - CP violation in the quark sector; T. Gershon, Y. Nir
- Review of Lattice results concerning low energy particle physics
S. Aoki *et al.* (FLAG Collaboration); <http://itpwiki.unibe.ch/flag>
- Heavy Flavor Averaging Group (HFAG)
Y. Amhis *et al.*, arxiv:1412.7515; www.slac.stanford.edu/xorg/hfag/
- Three Lectures of Flavor and CP violation within and beyond the standard model; Stefania Gori, AEPSHEP 2015; arxiv:1610.02629
- Rare B -meson decays at the crossroads;
A. Ali, Int. J. Mod. Phys. A, Vol. 31, No. 23 (2016) 1630036
[arxiv:1607.04918 (hep-ph)]
- Weak decays beyond leading logarithms, G. Buchalla, A.J. Buras and M. Lautenbacher, Rev. Mod. Phys. 68 (1996) 1125 [hep-ph/9512380]
- References to the original papers cited later in these lectures

Standard Model Lagrangian

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{QCD}} + \mathcal{L}_{\text{GSW}}$$

QCD [SU(3)]: Strong Interacting Theory of Colored Quarks and Gluons

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4}F_{\mu\nu}^{(a)}F^{(a)\mu\nu} + i \sum_q \bar{\psi}_q^\alpha \gamma^\mu (D_\mu)_{\alpha\beta} \psi_q^\beta$$

with $F_{\mu\nu}^{(a)} = \partial_\mu A_\nu^{(a)} - \partial_\nu A_\mu^{(a)} - g_s f_{abc} A_\mu^{(b)} A_\nu^{(c)}$; $a, b, c = 1, \dots, 8$

and $(D_\mu)_{\alpha\beta} = \delta_{\alpha\beta} \partial_\mu + ig_s \sum_a \frac{1}{2} \lambda_{\alpha\beta}^{(a)} A_\mu^{(a)}$; $\alpha, \beta = 1, 2, 3$

- f_{abc} are SU(3) structure constants, $\lambda_{\alpha\beta}^{(a)}$ are Gell-Mann SU(3) matrices

Electroweak [$SU(2)_I \times U(1)_Y$]

$$\mathcal{L}_{\text{GSW}} = \mathcal{L}_{\text{gauge}}(W_i, B, \psi_j) + \mathcal{L}_{\text{Higgs}}(\phi_k, W_i, B, \psi_j)$$

$$\mathcal{L}_{\text{gauge}}(W_i, B, \psi_j) = -\frac{1}{4}F_{\mu\nu}^i F_i^{\mu\nu} - \frac{1}{4}B_{\mu\nu} B^{\mu\nu} + \sum_{\psi_L} \overline{\psi_L} i D_\mu \gamma^\mu \psi_L + \sum_{\psi_R} \overline{\psi_R} i D_\mu \gamma^\mu \psi_R$$

- $(W_i)_\mu$; $i = 1, 2, 3$ and B_μ electroweak gauge fields;
- ψ_L an $SU(2)_I$ doublet, ψ_R a $U(1)_Y$ singlet

Standard Model Lagrangian-Contd.

$$\mathcal{L}_{\text{Higgs}}(\phi_k, W_i, B, \psi_j) = \mathcal{L}_{\text{Higgs}}(\text{gauge}) + \mathcal{L}_{\text{Higgs}}(\text{fermions})$$

$$\mathcal{L}_{\text{Higgs}}(\text{gauge}) = (D_\mu \Phi)^* (D^\mu \Phi) - V(\Phi)$$

$$D_\mu \Phi = (I(\partial_\mu + i\frac{g_1}{2}B_\mu) + ig_2 \frac{\tau}{2} \cdot W^-)\Phi; V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

$$\mathcal{L}_{\text{Higgs}}(\text{fermions}) = Y_u^{ij} \bar{Q}_{L,i} \tilde{\Phi} u_{R,j} + Y_d^{ij} \bar{Q}_{L,i} \Phi d_{R,j} + \text{h.c.} + \dots$$

- 3 Quark families: $Q_{L,j} = (u_L, d_L); (c_L, s_L); (t_L, b_L); \bar{u}_R, \bar{d}_R, \dots$
- Flavour mixing resides in the Higgs-Yukawa sector of the theory
- Flavour symmetry broken by Yukawa interactions

$$Q_i Y_d^{ij} d_j \phi \longrightarrow Q_i M_d^{ij} d_j$$

$$Q_i Y_u^{ij} u_j \phi^c \longrightarrow Q_i M_u^{ij} u_j$$

- Quark mass matrices diagonalized by a biunitary transformation

$$M_d = \text{diag}(m_d, m_s, m_b); M_u^\dagger = \text{diag}(m_u, m_c, m_t) \times V_{\text{CKM}}$$

- $V_{\text{CKM}} = U_{uL} U_{dL}^\dagger$: a (3×3) unitary matrix \implies quark flavour mixing

The Cabibbo-Kobayashi-Maskawa Matrix

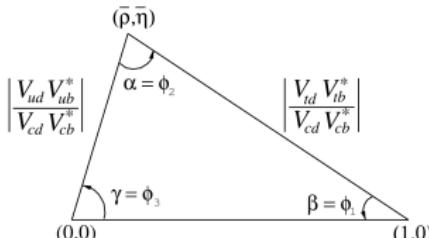
- Charged currents involving quarks: $J^\mu W_\mu^+ = -\frac{g}{\sqrt{2}} \bar{U}_L^i \gamma^\mu W_\mu^+ V_{\text{CKM}} D_L^i$

$$V_{\text{CKM}} \equiv \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- Requires four parameters, one of which is a phase inducing CP violation; customary to use the handy Wolfenstein parametrization

$$V_{\text{CKM}} \simeq \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda(1 + iA^2\lambda^4\eta) & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2(1 + i\lambda^2\eta) & 1 \end{pmatrix}$$

- Four parameters: A, λ, ρ, η ; $\bar{\rho} = \rho(1 - \lambda^2/2)$, $\bar{\eta} = \eta(1 - \lambda^2/2)$
- The CKM-Unitarity triangle [$\phi_1 = \beta$; $\phi_2 = \alpha$; $\phi_3 = \gamma$]



Phases and sides of the UT

$$\alpha \equiv \arg \left(-\frac{V_{tb}^* V_{td}}{V_{ub}^* V_{ud}} \right), \quad \beta \equiv \arg \left(-\frac{V_{cb}^* V_{cd}}{V_{tb}^* V_{td}} \right), \quad \gamma \equiv \arg \left(-\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}} \right)$$

- β and γ have simple interpretation

$$V_{td} = |V_{td}| e^{-i\beta}, \quad V_{ub} = |V_{ub}| e^{-i\gamma}$$

- α defined by the relation: $\alpha = \pi - \beta - \gamma$
- The Unitarity Triangle (UT) is defined by:

$$R_b e^{i\gamma} + R_t e^{-i\beta} = 1$$

$$R_b \equiv \frac{|V_{ub}^* V_{ud}|}{|V_{cb}^* V_{cd}|} = \sqrt{\bar{\rho}^2 + \bar{\eta}^2} = \left(1 - \frac{\lambda^2}{2}\right) \frac{1}{\lambda} \left| \frac{V_{ub}}{V_{cb}} \right|$$
$$R_t \equiv \frac{|V_{tb}^* V_{td}|}{|V_{cb}^* V_{cd}|} = \sqrt{(1 - \bar{\rho})^2 + \bar{\eta}^2} = \frac{1}{\lambda} \left| \frac{V_{td}}{V_{cb}} \right|$$

Present Status of the CKM Matrix Elements

$$|V_{ud}|$$

- From $O^+ \rightarrow O^+$ Nuclear Superallowed Fermi Transitions (charged vector current):

$$|V_{ud}| = 0.97425 \pm 0.00022 \quad (\text{Towner \& Hardy 2009})$$

- From Neutron β -decays $n \rightarrow pe^- \bar{\nu}_e$, (axial- and vector-currents)

- Great progress in precise measurements of τ_n and neutron polarization (e.g., at Grenoble), but g_A/g_V an issue at present

$$|V_{ud}| = 0.9773 \pm 0.0016; \quad g_A/g_V = -1.2701 \pm 0.0025$$

- From Pion β -decay: $\pi^+ \rightarrow \pi^0 e^+ \nu_e$ (involves only vector current)
 - Result from PIBETA Collaboration (Phys. Rev. Lett. 93, 181803 (2004))

$$BR(\pi^+ \rightarrow \pi^0 e^+ \nu_e) = (1.036 \pm 0.004(\text{stat}) \pm 0.004(\text{syst})) \times 10^{-8}$$

$$\Rightarrow |V_{ud}| = 0.9728 \pm 0.0030 \quad (\text{Blucher \& Marciano; PDG2014})$$

- Present World Average [PDG 2012]: $|V_{ud}| = 0.97425 \pm 0.00022$

$K_{\ell 3}$ Decays and $|V_{us}|$

- $K_{\ell 3}$ Decays: $K \rightarrow \pi \ell \nu_\ell$

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} V_{us}^* C_K \left[f_+^K(t)(p_K + p_\pi)_\mu + f_-^K(t)(p_K - p_\pi)_\mu \right] L^\mu$$

$$L^\mu = \bar{u}(p_\nu) \gamma^\mu (1 - \gamma_5) v(p_\ell); t = (p_K - p_\pi)^2; C_K = 1[\frac{1}{\sqrt{2}}] \text{ (for } K^0 [K^+])$$

- Partial Width: $\Gamma = C_K^2 \frac{G_F^2 |V_{us}|^2 M_K^5}{128\pi^3} \cdot |f_+^K(0)|^2 \cdot I_K(f_+, f_-)$
- Accurate determination of $|V_{us}|$ requires:
 - Evaluation of $f_+^K(0) - 1$ (enters QCD)
 - Momentum dependence of $f_\pm(t) \rightarrow I_K(f_+, f_-)$
 - Photonic radiative corrections [Ginsberg; Bytev et al.; Cirigliano et al.]
- Integrating out W and Z fields \implies Effective Low Energy Theory (LET)

$$\mathcal{L}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left(1 + \frac{\alpha}{\pi} \ln \frac{M_Z}{\mu} \right) \times H_\mu L^\mu$$

Theoretical Estimates of $f_+^K(0)$ in χ PT and Lattice QCD

- Matrix elements calculated in LET using Chiral perturbation theory;
Calculational tool: Chiral symmetry and expansion in p/Λ_χ , $\chi = 4\pi f_\pi$
- Low Energy Constants (LEC's) encode physics order by order in χ PT;
have to be determined from experiments

Theoretical Developments

- No linear corrections in $(m_s - m_u)$ [Ademollo-Gatto-Sirlin Theorem '64]
- In $\mathcal{O}(p^4)$: finite non-polynomial corrections induced by meson loops;
numerically small: $\delta^{(4)} = -2.2\%$ [Gasser-Leutwyler '85]
- In $\mathcal{O}(p^6)$: Appearance of $(m_s - m_u)^2/\Lambda_\chi^4$ terms; model-dependent
 $\delta^{(6)} = (-1.6 \pm 0.8)\%$ [Leutwyler-Roos, '84]
- Estimates (including $\mathcal{O}(e^2 p^2)$ terms) [Cirigliano et al. '01]
 $\delta = -(4.0 \pm 0.8)\% \implies f_+^K(0) = 0.961 \pm 0.008$
- Lattice QCD [FLAG 2016]: $f_+^K(0) = 0.9784(29)$
- Present World Average (including measurement of V_{us}/V_{ud}):
 $|V_{us}| = \lambda = 0.2253 \pm 0.0008$

Current Estimates of $|V_{cd}|$ and $|V_{cs}|$

- $|V_{cd}|$ can be determined from the old dimuon data in νN scattering:

$$\nu_\mu + d \rightarrow \mu^- c; \quad c \rightarrow s \mu^+ \nu_\mu \implies \nu_\mu \rightarrow \mu^+ \mu^- X$$

$$\bar{\nu}_\mu + \bar{d} \rightarrow \mu^+ \bar{c}; \quad \bar{c} \rightarrow \bar{s} \mu^- \bar{\nu}_\mu \implies \bar{\nu}_\mu \rightarrow \mu^+ \mu^- X$$

- Using the relation

$$\frac{\sigma(\nu_\mu \rightarrow \mu^+ \mu^- X) - \sigma(\bar{\nu}_\mu \rightarrow \mu^+ \mu^- X)}{\sigma(\nu_\mu \rightarrow \mu^- X) - \sigma(\bar{\nu}_\mu \rightarrow \mu^+ X)} = \mathcal{B}(c \rightarrow \mu^+ X) |V_{cd}|^2$$

- With L.H.S. = $(0.463 \pm 0.034) \times 10^{-2}$ and
 $\mathcal{B}(c \rightarrow \mu^+ X) = 0.087 \pm 0.005$ [PDG 2012] $\implies |V_{cd}| = 0.230 \pm 0.011$
- From $D \rightarrow \pi \ell \nu_\ell$ & Lattice QCD: $|V_{cd}| = 0.220 \pm 0.006 \pm 0.010$
- Averaging the two: $|V_{cd}| = 0.225 \pm 0.008$
- $|V_{cs}|$
 - From $D_s^+ \rightarrow \mu^+ \nu_\mu$, $D_s^+ \rightarrow \tau^+ \nu_\tau \implies |V_{cs}| = 1.008 \pm 0.021$;
 - From $D \rightarrow K \ell \nu_\ell \implies |V_{cs}| = 0.953 \pm 0.008 \pm 0.024$;
Averaging $\implies |V_{cs}| = 0.986 \pm 0.016$

$|V_{cb}|$ from Inclusive decays $B \rightarrow X_c \ell \nu_\ell$

- Theoretical Method

Heavy Quark Mass Expansion and Operator Product Expansion (OPE)

[Chay, Georgi, Grinstein; Voloshin, Shifman; Bigi et al.; Manohar, Wise; Blok et al.]

- Perform an OPE: m_b is the largest scale
- Decay rate for $B \rightarrow X_c \ell \bar{\nu}_\ell$

$$\Gamma = \Gamma_0 + \frac{1}{m_b} \Gamma_1 + \frac{1}{m_b^2} \Gamma_2 + \frac{1}{m_b^3} \Gamma_3 + \dots$$

- Γ_i are power series in $\alpha_s(m_b)$ → Perturbation theory
- Γ_0 is the decay of a free quark (“Parton Model”)
- Γ_1 vanishes due to the absence of dimension-5 operators
- Γ_2 is expressed in terms of two non-perturbative parameters

$$2M_B \lambda_1 = \langle B(v) | \bar{Q}_v (iD)^2 Q_v | B(v) \rangle$$

$$6M_B \lambda_2 = \langle B(v) | \bar{Q}_v \sigma_{\mu\nu} [iD^\mu, iD^\nu] Q_v | B(v) \rangle$$

λ_1 : Kinetic energy, λ_2 : Chromomagnetic moment (also called μ_π^2 and μ_G^2)

- Γ_3 is currently under investigation; involves several new parameters

Moment analysis of $B \rightarrow X_c \ell \nu_\ell$ with lepton energy cut

Lepton-energy and hadron mass moments [Gambino, Uraltsev; Benson et al.]

$$\langle E_\ell^{(n)} \rangle(E_{\text{cut}}) = \frac{\int_{E_{\text{cut}}} E_\ell^n \frac{d\Gamma}{dE_\ell} dE_\ell}{\int_{E_{\text{cut}}} \frac{d\Gamma}{dE_\ell} dE_\ell}, \langle M_X^\nu \rangle = \left(\langle M_X^2 \rangle \right)^{\frac{\nu}{2}} \left[1 + \sum_{k=2}^{\infty} C_{\frac{\nu}{2}}^k \frac{\langle (M_X^2 - \langle M_X^2 \rangle)^k \rangle}{\langle M_X^2 \rangle^k} \right]$$

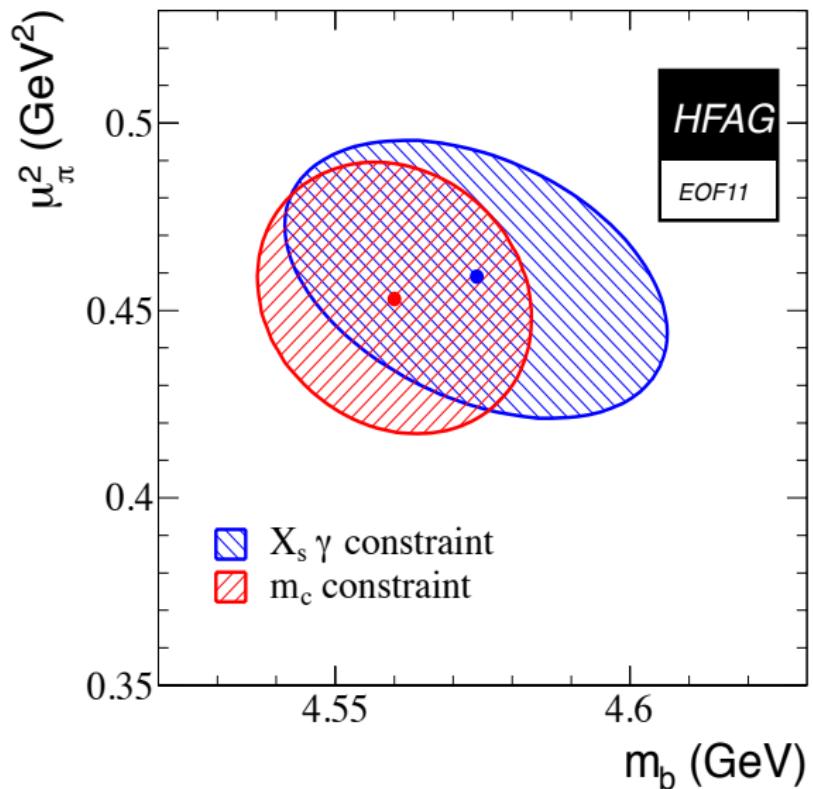
- Combined with the decay $B \rightarrow X_s \gamma$

$$\langle m_X^{2n} \rangle_{E_{\text{cut}}} = \frac{\int_{E_{\text{cut}}} (m_X^2)^n \frac{d\Gamma}{dm_X^2} dm_X^2}{\int_{E_{\text{cut}}} \frac{d\Gamma}{dm_X^2} dm_X^2}, \quad \langle E_\gamma^n \rangle_{E_{\text{cut}}} = \frac{\int_{E_{\text{cut}}} E_\gamma^n \frac{d\Gamma}{dE_\gamma} dE_\gamma}{\int_{E_{\text{cut}}} \frac{d\Gamma}{dE_\gamma} dE_\gamma}$$

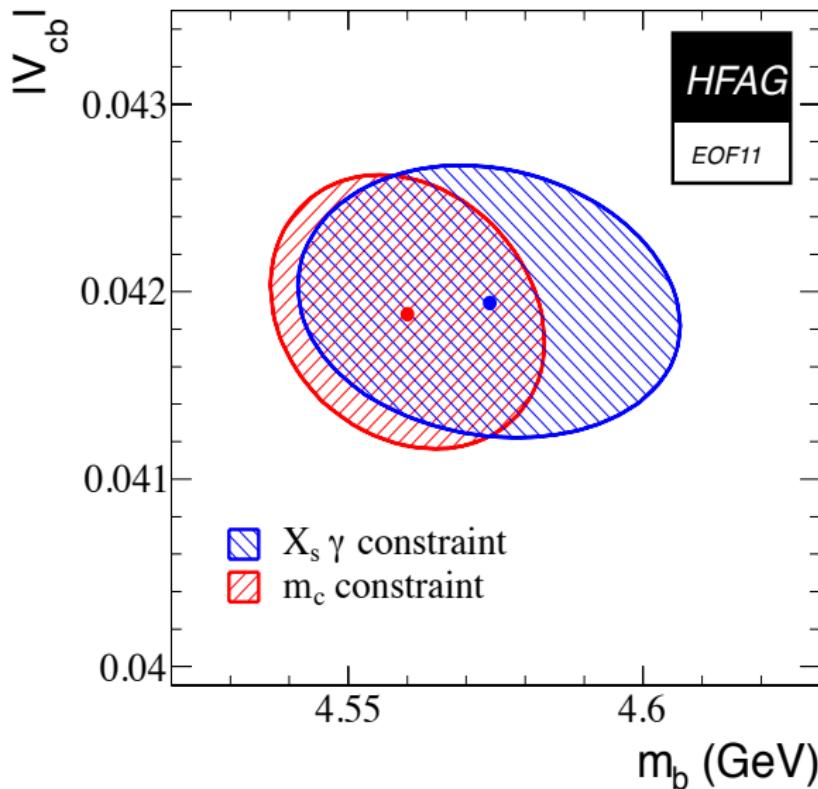
- Kinematic-mass scheme, $\mu \simeq 1 \text{ GeV}$

- Theory depends on $m_c(\mu), m_b(\mu)$, $\underbrace{\mu_\pi^2(\mu), \mu_G^2}_{\mathcal{O}(\Lambda_{\text{QCD}}^2/m_b^2)}$, $\underbrace{\rho_{\text{LS}}^3(\mu), \rho_{\text{D}}^3(\mu)}_{\mathcal{O}(\Lambda_{\text{QCD}}^3/m_b^3)}$

μ_π^2 vs. m_b (HFAG 2012)



$|V_{cb}|$ vs. m_b : $|V_{cb}| = (42.2 \pm 0.7) \times 10^{-3}$ (PDG 2014)



$|V_{cb}|$ from $B \rightarrow (D, D^*) \ell \nu_\ell$ decays

$\underline{B \rightarrow D^* \ell \nu_\ell \text{ decays}}$

$$\frac{d\Gamma}{d\omega} = \frac{G_F^2}{4\pi^3} (\omega^2 - 1)^{1/2} m_{D^*}^3 (m_B - m_{D^*})^2 \mathcal{G}(\omega) |V_{cb}|^2 |\mathcal{F}(\omega)|^2$$

- $\mathcal{F}(\omega)$ = Isgur–Wise function: $\mathcal{G}(\omega)$ phase space factor:
 $\mathcal{G}(1) = \mathcal{F}(1) = 1$,

- Leading Λ_{QCD}/m_b corrections absent (Luke's theorem)

- Need second order correction to $\mathcal{F}(\omega = 1)$, and slope ρ^2

$$\mathcal{F}(\omega) = \mathcal{F}(1) [1 - 8\rho^2 z + + (53\rho^2 - 15)z^2 - (231\rho^2 - 91)z^3]$$

where $z = (\sqrt{\omega+1} - \sqrt{2})/(\sqrt{\omega+1} + \sqrt{2})$, $\mathcal{F}(1) = \eta_A [1 + \delta_{1/m^2} + \dots]$

- strong correlation between $\mathcal{F}(1)$ and ρ^2 ,

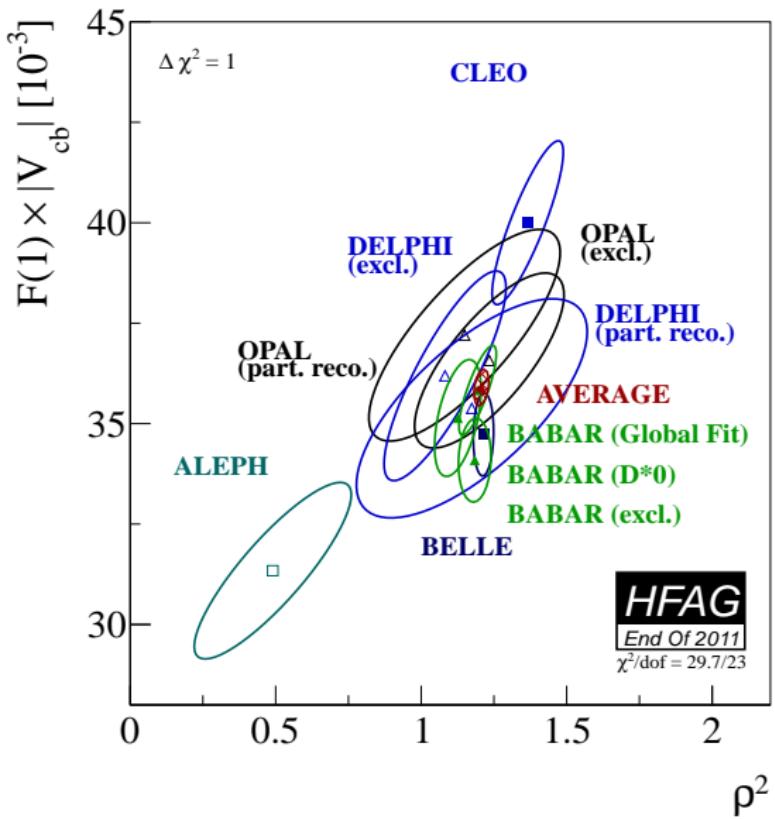
$$\mathcal{F}(1)|V_{cb}| = (36.0 \pm 0.5) \times 10^{-3}$$

Current values of $\mathcal{F}(1)$

$$\mathcal{F}(1) = 0.902 \pm 0.017 \quad [\text{Lattice QCD (Bernard et al., 2009)}]$$

- $|V_{cb}|_{B \rightarrow D^* \ell \nu_\ell} = (39.5 \pm 0.08) \times 10^{-3}$ (PDG 2014)

$\mathcal{F}(1)|V_{cb}|$ vs. ρ (HFAG 2012)



$|V_{ub}|$

From End-point spectra in $B \rightarrow X_u \ell \nu_\ell$ and $B \rightarrow X_s \gamma$

- Background from $B \rightarrow X_c \ell \nu_\ell$ removed by a large E_ℓ -cut
- Decay rate in the cut-region depends on the shape function $f(\omega)$

$$2M_B f(\omega) = \langle B | \bar{Q}_v \delta(\omega + \mathbf{n} \cdot (\mathbf{i}D)) Q_v | B \rangle; \quad (\mathbf{n} \cdot \mathbf{v} = 1, \mathbf{n}^2 = 0)$$

- Use of OPE to calculate inclusive spectra:

Example: Photon Spectrum in $B \rightarrow X_s \gamma$ [Neubert; Bigi et al.]

- Leading Shape Function ($x = \frac{2E_\gamma}{m_b}$):

$$\frac{d\Gamma}{dx} = \frac{G_F^2 \alpha m_b^5}{32\pi^4} |V_{ts} V_{tb^*}|^2 |C_7|^2 f(1-x)$$

- E_ℓ - and M_{X_u} -spectra in $B \rightarrow X_u \ell \nu_\ell$ governed also by $f(x)$
- $f(x)$ can be measured in $B \rightarrow X_s \gamma$

Theoretical Frameworks used to determine $|V_{ub}|$ by HFAG

→ Starting point: Triple differential rate in

$$P_\ell = M_B - 2E_\ell; \quad P_- = E_X + |\vec{P}_X|; \quad P_+ = E_X - |\vec{P}_X|$$

$$\begin{aligned} \frac{d^3\Gamma}{dP_+ dP_- dP_\ell} &= \frac{G_F^2 |V_{ub}|^2}{16\pi^2} \left[(P_- - P_\ell)(M_B - P_- + P_\ell - P_+) \mathcal{F}_1 \right] \\ &\quad + (M_B - P_-)(P_- - P_+) \mathcal{F}_2 + (P_- - P_\ell)(P_\ell - P_+) \mathcal{F}_3 \end{aligned}$$

• $\mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3$ "structure functions", calculated using Soft-Collinear-Effective Theory (SCET) – a technology developed to handle multi-scale problems

- The BLNP Method [Bosch, Lange, Neubert, Paz (2004 - 2006); updated to include $O(\alpha_s^2)$ contributions (Greub, Neubert, Pecjak (2010))]
 - Explained on the example of the E_γ -spectrum in $B \rightarrow X_s \gamma$

$$\frac{d\Gamma_s}{dE_\gamma} = \frac{G_F^2 \alpha}{2\pi^4} |V_{tb} V_{ts^*}|^2 \bar{m}_b^2(\mu_h) |C_{7\gamma}^{eff}(\mu_h)|^2 U(\mu_h, \mu_i) \mathcal{F}_\gamma(P_+)$$

- $C_{7\gamma}^{eff}(\mu_h)$: Wilson coefficient of the operator O_7 inducing $b \rightarrow s\gamma$ transition in the SM, calculated at the scale $\mu_h = m_b$;
- $U(\mu_h, \mu_i)$: Renormalization group running between the scale μ_h and $\mu_i = \sqrt{m_b \Lambda_{QCD}}$; $\mathcal{F}_\gamma(P_+)$: Structure function for $B \rightarrow X_s \gamma$

Theoretical Frameworks used to determine $|V_{ub}|$ by HFAG (Contd.)

- Factorization Theorem for the SF $\mathcal{F}_\gamma(P_+)$ (in leading power in $1/m_b$)

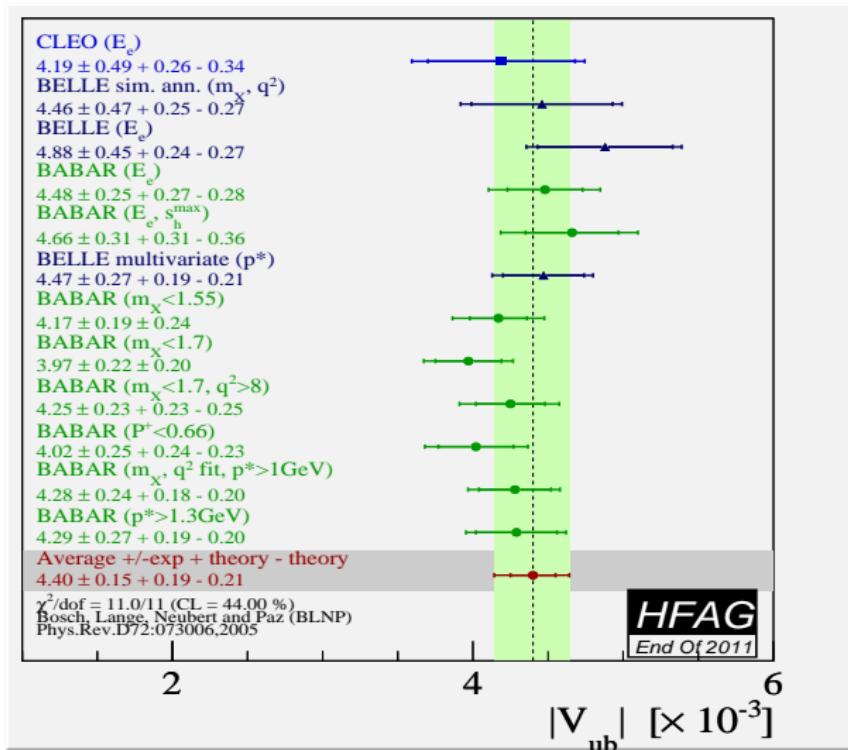
$$\mathcal{F}_\gamma^{(0)}(P_+) = |H_s(\mu_b)|^2 \int_0^{P_+} d\hat{w} m_b J(m_b(P_+ - \hat{w}), \mu_i) \hat{S}(\hat{\omega}, \mu_i)$$

- **H:** Perturbatively calculable hard coefficients;
- **J:** Perturbatively calculable jet functions;
- **S:** (soft) Light-cone dist. functions – the shape-functions
- This framework is developed by incorporating $O(\alpha_s^2)$ contributions in the various hard functions and in the RG running, and by including subleading power (in $1/m_b$) contributions
- Other variations on the BNLP Method:

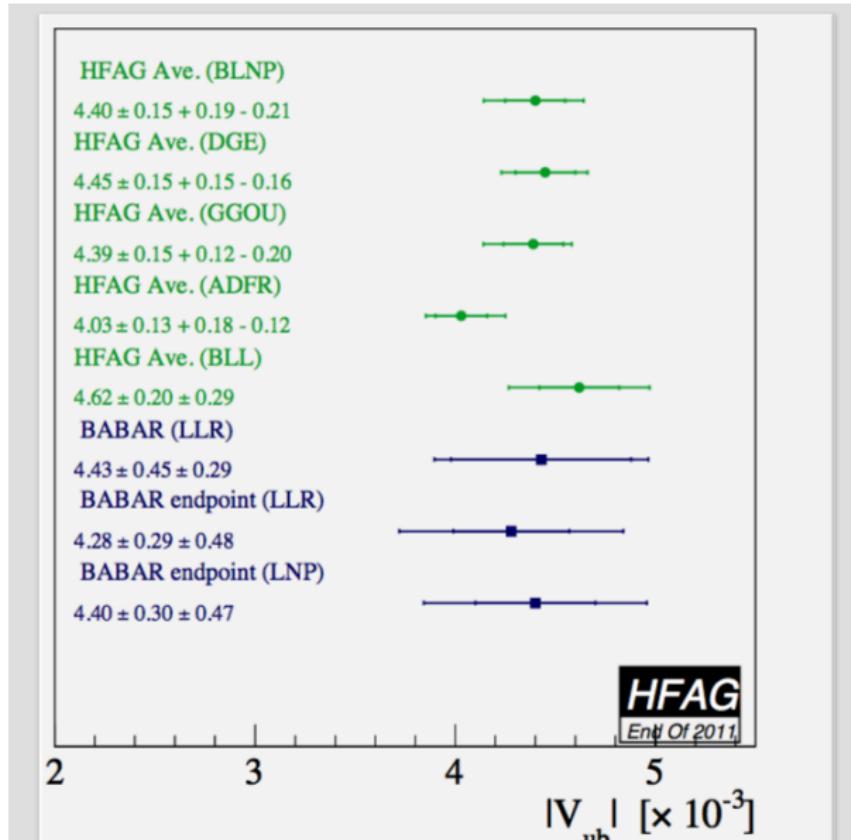
- The GGOU Method [Gambino, Giordano, Ossola, Uraltsev; (2007)]:
Uses the kinematic scheme for the definition of the quark masses and the shape functions are defined differently
- Dressed Gluon Exponentiation (DGE) Method [Anderson, Gardi (2006)]:

An attempt to calculate the SFs by extending the perturbative result into the infrared regime using renormalons

$|V_{ub}|$ from inclusive $B \rightarrow X_u \ell \nu_\ell$ decays (HFAG: ICHEP 2012 Update)



$|V_{ub}|$ from inclusive $B \rightarrow X_u \ell \nu_\ell$ using the SF Method (HFAG: ICHEP 2012)

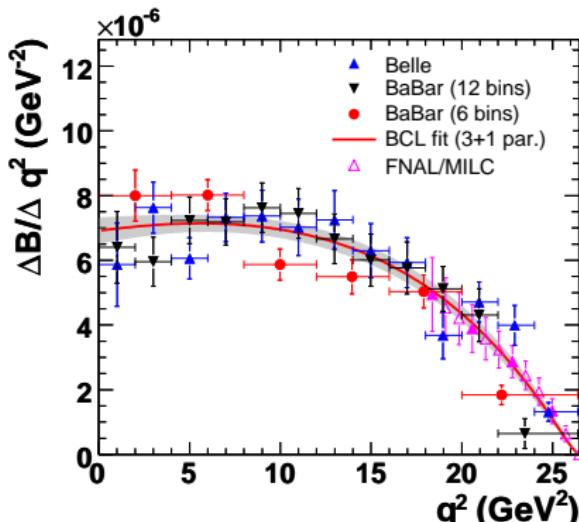


$|V_{ub}|$ from exclusive decays $B \rightarrow \pi \ell \nu_\ell$

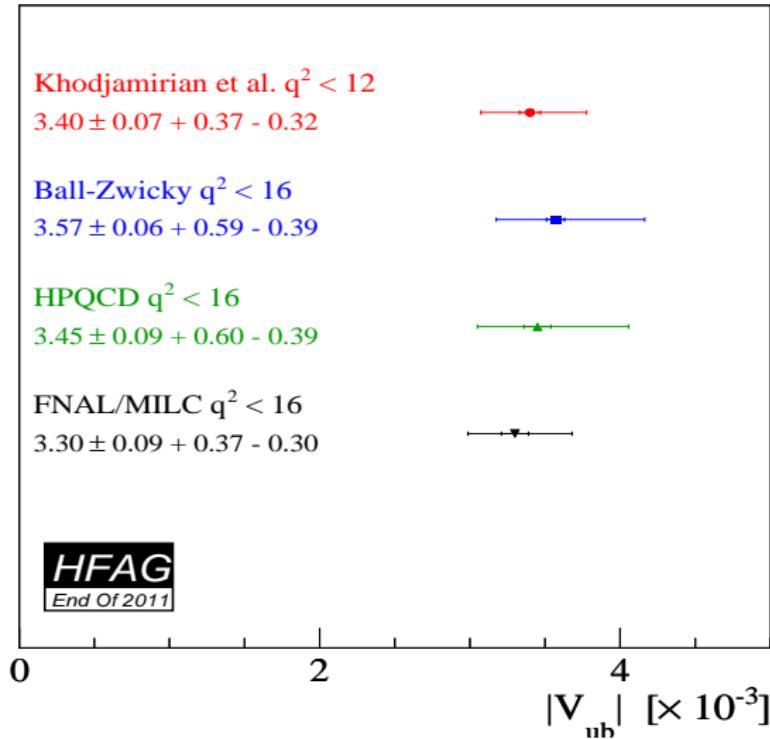
$$\langle \pi(p_\pi) | \bar{b} \gamma_\mu q | B(p_B) \rangle = \left((p_B + p_\pi)_\mu - \frac{m_B^2 - m_\pi^2}{q^2} q_\mu \right) F_+(q^2) + \frac{m_B^2 - m_\pi^2}{q^2} F_0(q^2) q_\mu$$

Techniques used to determine $F_+(q^2)$, $F_0(q^2)$

- Light-cone QCD sum rules [Colangelo, Khodjamirian; Ball, Zwicky]
- Lattice-QCD (Quenched) [APE, UKQCD, FNAL, JLQCD]
- Lattice-QCD (Unquenched) [HPQCD, FNAL/MILC]



$|V_{ub}|$ from exclusive decays $B \rightarrow \pi \ell \nu_\ell$



Summary of the First 2 Rows of V_{CKM}

- $|V_{ud}| = 0.97425(22)$
- $|V_{us}| = 0.2253(8)$
- $|V_{ub}| = (4.41 \pm 0.15^{+0.15}_{-0.17}) \times 10^{-3}$ [Inclusive; SF-based]
 $|V_{ub}| = (3.28 \pm 0.29) \times 10^{-3}$ [Exclusive; Lattice-QCD (updated)]

Unitarity of the 1st Row of V_{CKM}

$$|V_{ud}|^2 + |V_{us}|^2 + (|V_{ub}| = 0.0036)^2 = 0.9999 \pm 0.0006$$

- $|V_{cd}| = 0.225(8)$
- $|V_{cs}| = 0.986(16)$
- $|V_{cb}| = (41.1 \pm 1.3) \times 10^{-3}$

Unitarity of the 2nd Row of V_{CKM}

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1.024 \pm 0.032$$

- Room to improve the precision on $|V_{cd}|$ and $|V_{cs}|$

Status of the Third Row of V_{CKM}

$$\underline{|V_{tb}|}$$

- From direct production and decays of the top quark measured at the Tevatron ($p\bar{p}; \sqrt{s} = 1.96 \text{ TeV}$) and LHC ($pp; \sqrt{s} = 7 \& 8 \text{ TeV}$)

$$R \equiv \frac{\mathcal{B}(t \rightarrow W + b)}{\mathcal{B}(t \rightarrow W + \sum_q q)} = \frac{|V_{tb}|^2}{|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2} = |V_{tb}|^2$$

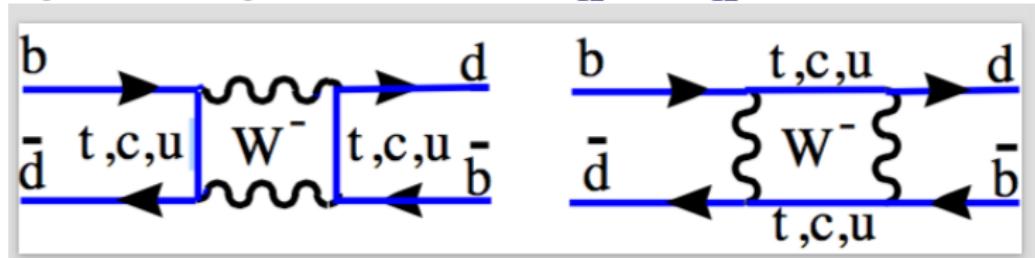
- Assuming CKM unitarity & Data \implies (95% C.L.) bounds on $|V_{tb}|$:
 - $|V_{tb}| > 0.78$ (CDF)
 - $0.99 > |V_{tb}| > 0.90$ (D0)
 - $|V_{tb}| > 0.92$ (CMS)
 - Single top-production X-section is a measure of $|V_{tb}|$. A typical process is: $g + b \rightarrow b^* \rightarrow W^- + t$
 - $\sigma(p\bar{p} \rightarrow tX; \sqrt{s} = 1.96 \text{ TeV}) = (3.51^{+0.40}_{-0.37}) \text{ pb} \implies |V_{tb}| = 1.03 \pm 0.06$
 - $\sigma(pp \rightarrow tX(\text{t-channel}); \sqrt{s} = 7 \text{ TeV}) = (68.5 \pm 5.8) \text{ pb} \implies |V_{tb}|_{7 \text{ TeV}} = 1.03 \pm 0.05; |V_{tb}|_{8 \text{ TeV}} = 0.99 \pm 0.07$
- PDG Average [2014]: $|V_{tb}| = 1.021 \pm 0.032$

Meson-Antimeson mixing formalism

- Time evolution of a decaying particle: $B(t) = \exp^{[im_B t - \Gamma_B t/2]}$
- In the case of two-state mixing, such as $B_q^0 = (\bar{b}q)$ and $\bar{B}_q^0 = (b\bar{q})$, the time evolution is governed by a (2×2) coupled-channel equation:

$$i \frac{d}{dt} \begin{pmatrix} |B_q(t)\rangle \\ |\bar{B}_q(t)\rangle \end{pmatrix} = \left(\hat{M}^q - \frac{i}{2} \hat{\Gamma}^q \right) \begin{pmatrix} |B_q(t)\rangle \\ |\bar{B}_q(t)\rangle \end{pmatrix}$$

- Weak interactions induce virtual transitions $B_{d,s} \rightarrow \bar{B}_{d,s}$ (*box diagrams*), generating the off-diagonal elements in M_{12}^q and Γ_{12}^q



- Diagonalization of \hat{M}_q and $\hat{\Gamma}_q$ yields mass eigenstates $B_{q,H}$ (H = heavy) and $B_{q,L}$ (L = light) with the masses M_H^q, M_L^q and decay rates Γ_H^q, Γ_L^q

Meson-Antimeson mixing formalism (Contd.)

- 3 Observables: M_{12}^q , Γ_{12}^q and the relative phase $\phi_q = \arg(-M_{12}^q/\Gamma_{12}^q)$
- The phase ϕ_q can be measured from the decays $B_q \rightarrow X\ell^+\nu_\ell$

$$A_{\text{SL}}^q = \frac{\Gamma(\bar{B}_q(t) \rightarrow f) - \Gamma(B_q(t) \rightarrow \bar{f})}{\Gamma(\bar{B}_q(t) \rightarrow f) + \Gamma(B_q(t) \rightarrow \bar{f})} = \frac{\Delta\Gamma_q}{\Delta M_q} \tan \phi_q$$

- Mass difference $\Delta M_q = M_H^q - M_L^q$ ($q = d, s$) is given by the modulus of the dispersive part of the Box-diagram

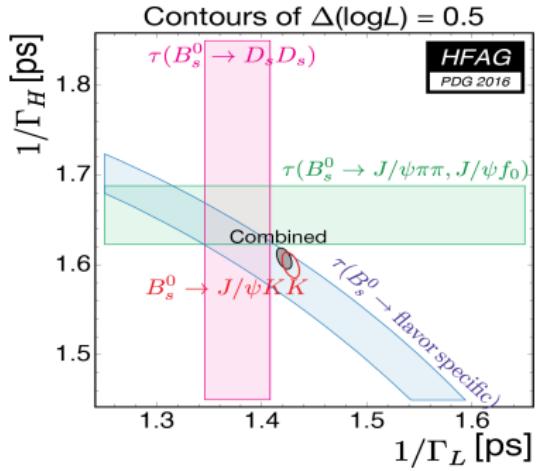
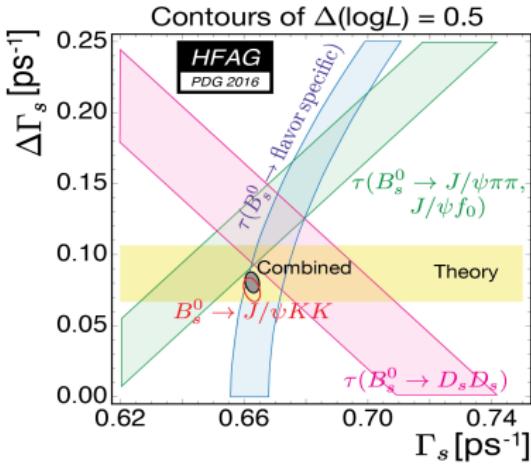
- Decay rate difference $\Delta\Gamma_q = \Delta\Gamma^q = \Gamma_L^q - \Gamma_H^q = 2|\Gamma_{12}^q| \cos \phi_q$ ($q = d, s$) is given by the absorptive part of the Box-diagram

- SM Estimates (ps^{-1}) [Lenz, Nierste]

$$\Delta\Gamma_d^{\text{th}} \ll \Delta\Gamma_s^{\text{th}}; \quad \Delta\Gamma_s^{\text{th}} = \Delta\Gamma_s^{(0)} (1 + \delta^{\text{Lattice}} + \delta^{\text{pQCD}} + \delta^{\text{HQE}}) = (0.087 \pm 0.021)$$

- $\Delta\Gamma_s^{\text{expt}} = (0.082 \pm 0.007) \text{ps}^{-1} \implies \Delta\Gamma_s^{\text{expt}} / \Delta\Gamma_s^{\text{th}} = 0.94 \pm 0.26$

Contours in $(1/\Gamma_s, \Delta\Gamma_s)$ & $(1/\Gamma_L, 1/\Gamma_H)$ plane [HFAG]



- $\Delta\Gamma_s = +0.082 \pm 0.007$ ps $^{-1}$
- $1/\Gamma_s = 1.510 \pm 0.005$ ps
- $1/\Gamma_L = 1.422 \pm 0.008$ ps
- $1/\Gamma_H = 1.610 \pm 0.012$ ps

Determination of $|V_{td}|$ and $|V_{ts}|$ from loop processes

$$\underline{|V_{td}|}$$

- From $B_d^0 - \bar{B}_d^0$ Mixing; $\Delta M_d = (0.5096 \pm 0.0034) \text{ ps}^{-1}$ [PDG 2016]
- SM (Box contribution with NLO QCD corrections) ($x_t = m_t^2/m_W^2$)

$$\Delta M_d = \frac{G_F^2}{6\pi^2} \hat{\eta}_B |V_{td} V_{tb}^*|^2 M_{B_d} (f_{B_d}^2 \hat{B}_{B_d}) M_W^2 S_0(x_t)$$

$$S_0(x) = x \cdot \left[\frac{1}{4} + \frac{9}{4} \frac{1}{(1-x)} - \frac{3}{2} \frac{1}{(1-x)^2} - \frac{3}{2} \frac{x^2 \ln x}{(1-x)^3} \right]$$

$$\langle \bar{B}_q^0 | (\bar{b} \gamma_\mu (1 - \gamma_5) q)^2 | B_q^0 \rangle \equiv \frac{8}{3} f_{B_q}^2 B_{B_q} M_{B_q}^2$$

- Unquenched Lattice-QCD [Gray et al., Wingate et al (HPQCD); Bernard et al. (Fermilab); Laiho et al., Updated in Aoki et al. (JLQCD) (2013)]:

$$\sqrt{\hat{B}_{B_d} f_{B_d}} = 216 \pm 15 \text{ MeV}; \quad S_0(x_t) = 2.29(5)$$

$$|V_{td}^* V_{tb}| = 8.4 \times 10^{-3} \left[\frac{216 \text{ MeV}}{\sqrt{\hat{B}_{B_d} f_{B_d}}} \right] \sqrt{\frac{2.29}{S_0(x_t)}}$$

- Lattice-QCD & SM $\implies |V_{td}^* V_{tb}| = (8.4 \pm 0.6) \times 10^{-3}$ [PDG 2014]

$|V_{ts}|$ and $|V_{td}|/|V_{ts}|$ from $\Delta M_{d,s}$ and Lattice-QCD

$$\underline{|V_{ts}|}$$

- $B_s^0 - \overline{B_s^0}$ Mixing: $\Delta M_s = (17.757 \pm 0.021) \text{ ps}^{-1}$ [CDF & LHCb]
- SM: $\Delta M_s = \frac{G_F^2}{6\pi^2} \hat{\eta}_B |V_{ts}^* V_{tb}|^2 M_{B_s} (f_{B_s}^2 \hat{B}_{B_s}) M_W^2 S_0(x_t)$
- Lattice-QCD: $\sqrt{\hat{B}_{B_s} f_{B_s}} = 266 \pm 18 \text{ MeV}$ [Aoki *et al.* (2013)]
 $\implies |V_{ts}^* V_{tb}| = (40.0 \pm 2.7) \times 10^{-3}$
- in quantitative agreement with $|V_{cb}| = (41.1 \pm 1.1) \times 10^{-3}$;

$$\underline{|V_{td}|/|V_{ts}|}$$

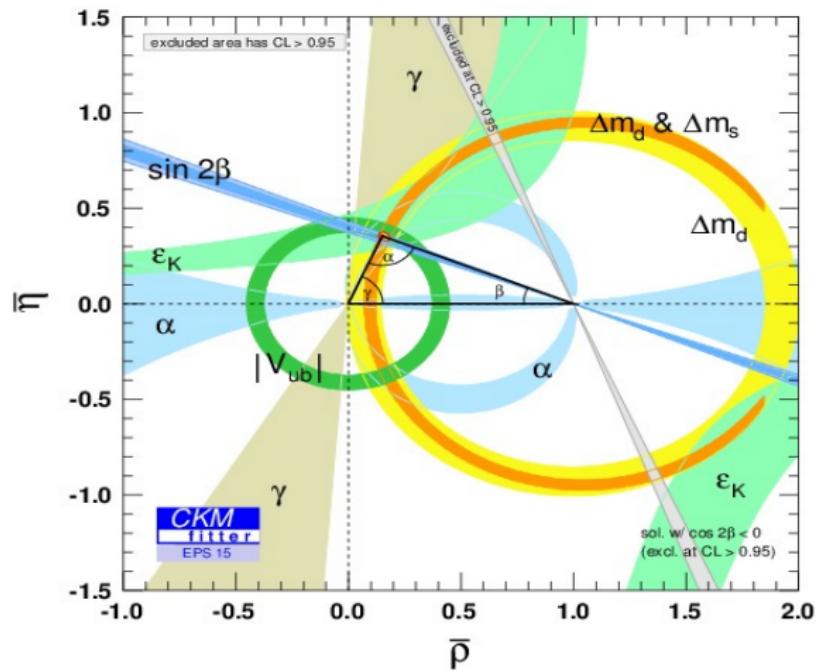
- Using the ratio $\Delta M_s/\Delta M_d$ and $\xi = 1.268 \pm 0.063$ [Lattice-QCD]

$$\frac{\Delta M_s}{\Delta M_d} = \xi \frac{M_{B_s}}{M_{B_d}} \frac{|V_{ts}|^2}{|V_{td}|^2}; \quad \xi = \sqrt{\frac{f_{B_s}^2 \hat{B}_{B_s}}{f_{B_d}^2 \hat{B}_{B_d}}}$$

$$\implies |V_{td}/V_{ts}| = 0.216 \pm 0.001(\text{exp}) \pm 0.011(\text{th})$$

- This is an important constraint on the CKM-Unitarity triangle

Current Status of the CKM-Unitarity Triangle [CKMfitter: 2015]



- Measurements of $|\mathcal{V}_{ub}|$, Δm_d and Δm_s do not determine the KM phase δ . Requires the measurement of a CP violating quantity.

CP violation in charged and neutral-hadron decays - Formalism

- Decay amplitude of M (and its CP conjugate \bar{M}) to decay into a final state f and its CP conjugate \bar{f} are defined as

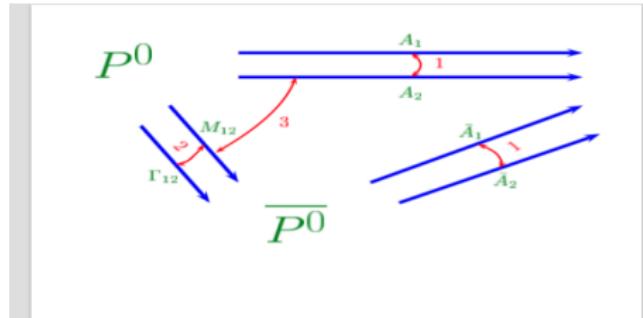
$$A_f \equiv \langle f | \mathcal{H} | M \rangle, \quad \bar{A}_f \equiv \langle f | \mathcal{H} | \bar{M} \rangle$$
$$A_{\bar{f}} \equiv \langle \bar{f} | \mathcal{H} | M \rangle, \quad \bar{A}_{\bar{f}} \equiv \langle \bar{f} | \mathcal{H} | \bar{M} \rangle$$

- \mathcal{H} is the weak interaction Hamiltonian. The operator CP introduces phases ξ_M and ξ_f , which depend on their flavour content

$$CP|M\rangle = \exp^{+i\xi_M} |\bar{M}\rangle, \quad CP|f\rangle = \exp^{+i\xi_f} |\bar{f}\rangle,$$
$$CP|\bar{M}\rangle = \exp^{-i\xi_M} |M\rangle, \quad CP|\bar{f}\rangle = \exp^{-i\xi_f} |f\rangle,$$

- The phases ξ_M and ξ_f are arbitrary and unobservable. If CP is conserved, i.e., $[CP, \mathcal{H}] = 0$, then A_f and $\bar{A}_{\bar{f}}$ have the same magnitude, and an arbitrary unphysical relative phase $\bar{A}_{\bar{f}} = \exp^{i(\xi_f - \xi_M)} A_f$
- CP violation in decay is defined by $|\bar{A}_{\bar{f}}/A_f| \neq 0$. This is the only possibility for charged meson (and baryon) decays.

CP violation in neutral meson decay into a CP eigenstate



- 1 In decay: $\bar{A}/A \neq 1$ $\left(\frac{\bar{A}}{A} = \frac{\bar{A}_1 + \bar{A}_2}{A_1 + A_2}\right)$
(For example, A_1 is a Tree amplitude & A_2 is Penguin)
- 2 In mixing: $|q/p| \neq 1$ $\left(\frac{q}{p} = \frac{2M_{12}^* - i\Gamma_{12}^*}{\Delta m - (i/2)\Delta\Gamma}\right)$
- 3 In interference: $\text{Im}\lambda \neq 1$ $\left(\lambda = \frac{q}{p} \frac{\bar{A}}{A}\right)$

■ The case theorists love!

- Decay dominated by a single CPV phase: $|\frac{\bar{A}}{A}| = 1$;
- CPV in mixing negligible $|\frac{q}{p}| = 1$;
- The remaining effect is: $S_f \sim \sin[\arg(M_{12}) - 2\arg(A)]$

CP Violation in the Kaon sector

- CP violation was discovered in $K \rightarrow \pi\pi$ decays in 1964
[J.H. Christenson et al., Phys. Rev. Lett. 13, 138 (1964)]
- The decays actually measured were the decays of the mass eigenstates, labelled as K_L (long-lived) and K_S (short-lived). In the CP conservation limit, $CP|K_S\rangle = +|K_S\rangle$ and $CP|K_L\rangle = -|K_L\rangle$
- As the final states $\pi^+\pi^-$ and $\pi^0\pi^0$ are CP even, in the CP conservation limit $K_L \rightarrow \pi\pi$ Not allowed.
- Defining: $\eta_{00} = |\eta_{00}| \exp^{i\phi_{00}} = \frac{\langle \pi^0\pi^0 | \mathcal{H} | K_L \rangle}{\langle \pi^0\pi^0 | \mathcal{H} | K_S \rangle}$,
 $\eta_{+-} = |\eta_{+-}| \exp^{i\phi_{+-}} = \frac{\langle \pi^+\pi^- | \mathcal{H} | K_L \rangle}{\langle \pi^+\pi^- | \mathcal{H} | K_S \rangle}$
- $|\eta_{00}| = (2.220 \pm 0.011) \times 10^{-3}$ and $|\eta_{+-}| = (2.232 \pm 0.011) \times 10^{-3}$
- $\phi_{00} = (43.7 \pm 0.6)^\circ$ and $\phi_{+-} = (43.4 \pm 0.5)^\circ$ (assuming CPT)
- Another important observable is the asymmetry of the semileptonic decays: $\delta_L = \frac{\Gamma(K_L \rightarrow \ell^+ \nu_\ell \pi^-) - \Gamma(K_L \rightarrow \ell^- \bar{\nu}_\ell \pi^+)}{\Gamma(K_L \rightarrow \ell^+ \nu_\ell \pi^-) + \Gamma(K_L \rightarrow \ell^- \bar{\nu}_\ell \pi^+)} = (3.32 \pm 0.06) \times 10^{-3}$

CP Violation in the Kaon sector (Contd.)

- CP violation in neutral K decays is described in terms of the (complex) parameters ϵ and ϵ' , and the observables η_{00} , η_{+-} , and δ_L are

$$\eta_{00} == \epsilon - 2\epsilon', \quad \eta_{+-} == \epsilon + \epsilon', \quad \delta_L = \frac{1 - |q/p|^2}{1 + |q/p|^2} = \frac{2\text{Re}(\epsilon)}{1 + |\epsilon|^2},$$

- Isospin decomposition of the decay amplitudes for $K^0 \rightarrow \pi^0 \pi^0, \pi^+ \pi^-$ having the weak phases ϕ_i and strong phases δ_i

$$A_{\pi^0 \pi^0} = \sqrt{\frac{1}{3}} |A_0| \exp^{i(\delta_0 + \phi_0)} - \sqrt{\frac{2}{3}} |A_2| \exp^{i(\delta_2 + \phi_2)}$$

$$A_{\pi^+ \pi^-} = \sqrt{\frac{2}{3}} |A_0| \exp^{i(\delta_0 + \phi_0)} + \sqrt{\frac{1}{3}} |A_2| \exp^{i(\delta_2 + \phi_2)}$$

- A_I parametrise the amplitudes with total isospin $I = 0, 2$

$$A_I \equiv \langle (\pi\pi)_I | \mathcal{H} | K^0 \rangle = A_I \exp^{i(\delta_I + \phi_I)},$$

$$\bar{A}_I \equiv \langle (\pi\pi)_I | \mathcal{H} | \bar{K}^0 \rangle = A_I \exp^{i(\delta_I - \phi_I)}$$

- $\epsilon \simeq \frac{\exp^{i\pi/4}}{\sqrt{2}} \frac{\text{Im}(M_{12})}{\Delta m}, \quad \epsilon' = \frac{i}{\sqrt{2}} \left| \frac{A_2}{A_0} \right| \exp^{i(\delta_2 - \delta_0)} \sin(\phi_2 - \phi_0)$

- A fit to the $K \rightarrow \pi\pi$ data yields

$$|\epsilon| = (2.228 \pm 0.011) \times 10^{-3}, \quad \text{Re}(\epsilon'/\epsilon) = (1.66 \pm 0.23) \times 10^{-3}$$

ϵ_K in the SM

- Indirect CP violation in $K \rightarrow \pi\pi$ is measured by ϵ_K

$$\epsilon_K = \frac{\exp^{i\pi/4}}{\sqrt{2}\Delta M_K} (\text{Im}M_{12} + 2\xi \text{Re}M_{12}); \quad \xi = \frac{\text{Im}A_0}{\text{Re}A_0}$$

- M_{12} calculated from the Box diagrams in which the charm and top quark loops dominate ($x_i = m_i^2/m_W^2$)

$$M_{12} = \frac{G_F^2}{12\pi^2} F_K^2 B_K m_K M_W^2 [\lambda_c^{*2} \eta_1 S_0(x_c) + \lambda_t^{*2} \eta_2 S_0(x_t) + 2\lambda_c^* \lambda_t^* \eta_3 S_0(x_c, x_t)]$$

- $S_0(x_c), S_0(x_t), S_0(x_c, x_t)$ are the Inami-Lim functions
 $S_0(x_c) = x_c$

$$S_0(x_t) = 4x_t - 11x_t^2 + x_t^3/4(1-x_t)^2 - 3x_t^2 \ln x_t/2(1-x_t)^3$$

$$S_0(x_c, x_t) = x_c [\ln x_t/x_c - 3x_t/4(1-x_t) - 3x_t^2 \ln x_t/4(1-x_t)^2]$$

- $\lambda_i = V_{id} V_{is}^*, (i = c, t)$, F_K is the K -meson decay constant, B_K is a bag constant, and η_i are QCD renormalization constants

$$\eta_1 = 1.38, \quad \eta_2 = 0.57, \quad \eta_3 = 0.47$$

ϵ_K in the SM (Contd.)

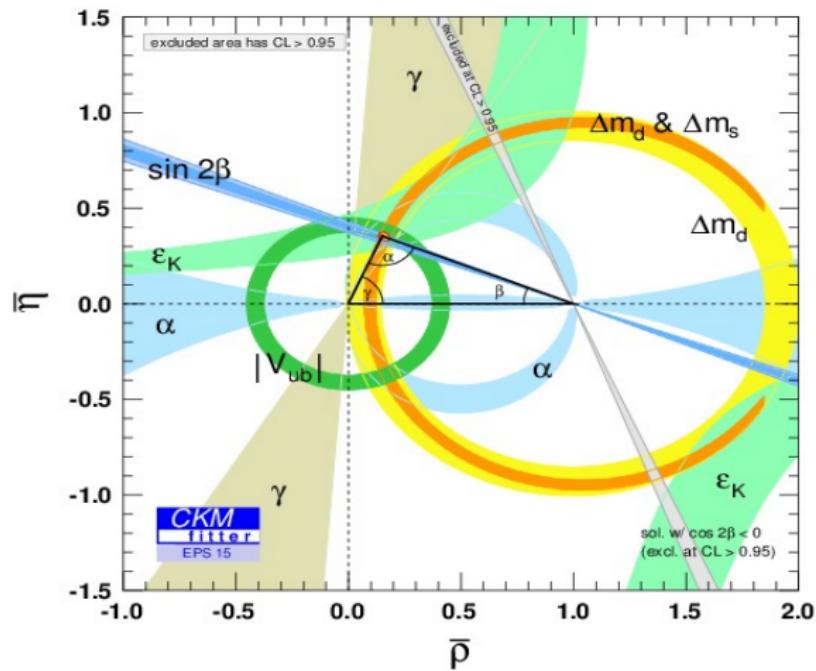
$$\epsilon_K = C_\epsilon B_K \operatorname{Im} \lambda_t \{ \operatorname{Re} \lambda_c [\eta_1 S_0(x_c) - \eta_3 S_0(x_c, x_t)] - \operatorname{Re} \lambda_t \eta_2 S_0(x_t) \} \frac{\exp^{i\pi}}{4}$$

- $C_\epsilon = G_F^2 F_K^2 m_K M_W^2 / 6\sqrt{2}\pi^2 \Delta M_K = 3.78 \times 10^{-4}$
- Using the Wolfenstein Parametrization in which
- $\operatorname{Im} \lambda_t = -\operatorname{Im} \lambda_c = \eta A^2 \lambda^5; \operatorname{Re} \lambda_c = -\lambda(1 - \lambda^2/2); \operatorname{Re} \lambda_t = -(1 - \lambda^2/2)A^2$
- This yields the following relation (a Hyperbola in the $(\bar{\rho}, \bar{\eta})$ plane)

$$\bar{\eta}[(1 - \bar{\rho})^2 A^2 \eta_2 S_0(x_t) + P_0(\epsilon)] A^2 B_K = 0.226$$

- where $P_0(\epsilon) = [\eta_3 S_0(x_c, x_t) - \eta_1 x_c]/\lambda^4$, intersecting the circle given by
- $$R_b = \frac{|V_{ud} V_{ub}^*|}{|V_{cd} V_{cb}^*|} = \sqrt{\bar{\rho}^2 + \bar{\eta}^2}$$
- \implies 2 solutions for the KM phase δ . However, the intersection of R_b, R_t and ϵ_K determines δ , to be tested by the CPV in the b -sector

Current Status of the CKM-Unitarity Triangle [CKMfitter: 2015]



- Do direct and indirect measurements of the angles agree well? Need to discuss the measurements of the angles of the Unitarity triangle

$D^0 - \bar{D}^0$ Mixing and CP violation in the charm sector

- The formalism of $D^0 - \bar{D}^0$ mixing and CP violation is the same as in the B -meson sector

$$\langle D^0 | \mathcal{H} | \bar{D}^0 \rangle = M_{12} - \frac{i}{2} \Gamma_{12}$$

- The three observables are defined likewise:

$$y_{12} = \frac{|\Gamma_{12}|}{\Gamma}; \quad x_{12} = 2 \frac{|M_{12}|}{\Gamma}; \quad \phi_{12} = \arg\left(\frac{M_{12}}{\Gamma_{12}}\right)$$

- HFAG 2014 Update: $x_{12} = (0.37 \pm 0.10) \times 10^{-2}$

$$y_{12} = (0.66^{+0.07}_{-0.10}) \times 10^{-2}$$

$$\phi_{12} = (-9^{+12}_{-10})^\circ$$

- Difficult to reliably estimate the SM contributions, as they are dominated by long-distance (non-perturbative) terms

- The most sensitive searches of CP violation involve the modes $D^0 \rightarrow K^+ K^-$, $D^0 \rightarrow \pi^+ \pi^-$, and $D^0 \rightarrow K^\pm \pi^\mp$, and their time-integrated asymmetries $\mathcal{A}_{\text{CP}}(D \rightarrow K^+ K^-)$, $\mathcal{A}_{\text{CP}}(D \rightarrow \pi^+ \pi^-)$ and $\Delta \mathcal{A}_{\text{CP}} = \mathcal{A}_{\text{CP}}(D \rightarrow K^+ K^-) - \mathcal{A}_{\text{CP}}(D \rightarrow \pi^+ \pi^-)$

$D^0 - \bar{D}^0$ Mixing and CP violation in the charm sector (contd.)

- The time-integrated CP asymmetry for a CP eigenstate f is defined as

$$a_f \equiv \frac{\Gamma(D^0 \rightarrow f) - \Gamma(\bar{D}^0 \rightarrow f)}{\Gamma(D^0 \rightarrow f) + \Gamma(\bar{D}^0 \rightarrow f)}.$$

- The decay amplitudes $A_f(\bar{A}_f)$ are:

$$\begin{aligned} A_f &= A_f^T e^{i\phi_f^T} [1 + r_f e^{i(\delta_f + \phi_f)}] \\ \bar{A}_f &= \eta_{CP} A_f^T e^{-i\phi_f^T} [1 + r_f e^{i(\delta_f - \phi_f)}] \end{aligned}$$

$\eta_{CP} = \pm 1$ is the CP eigenvalue of f ; $A_f^T e^{\pm i\phi_f^T}$ are the “tree” amplitudes; r_f denotes the relative magnitude of all the other (usually called the “penguin”) amplitudes, having different strong δ_f and weak ϕ_f phases

- $a_f = a_f^{\text{direct}} + a_f^{\text{mixing}} + a_f^{\text{interf.}}$
- Of these, direct CP asymmetry has the expression ($r_f \ll 1$)

$$a_f^{\text{dir}} = \frac{2r_f \sin \delta_f \sin \phi_f}{1 + 2r_f \cos \delta_f \cos \phi_f + r_f^2} \simeq 2r_f \sin \delta_f \sin \phi_f$$

$D^0 - \bar{D}^0$ Mixing and CP violation in the charm sector (contd.)

- $D^0 \rightarrow K^+ K^-$: Using the CKM unitarity relation $\lambda_d + \lambda_s + \lambda_b = 0$ (here $\lambda_i \equiv V_{ci} V_{ui}$), one can eliminate the λ_d term:

$$A_K = \lambda_s (A_K^s - A_K^d) + \lambda_b (A_K^b - A_K^d)$$

- $D^0 \rightarrow \pi^+ \pi^-$: Here, one can eliminate λ_s and write

$$A_\pi = \lambda_d (A_\pi^d - A_\pi^s) + \lambda_b (A_\pi^b - A_\pi^s)$$

- Define $\xi_s \equiv \lambda_b/\lambda_s = V_{cb} V_{ub} / V_{cs} V_{us}$ and $\xi_d \equiv \lambda_b/\lambda_d = V_{cb} V_{ub} / V_{cd} V_{ud}$, and the ratio of the hadronic amplitudes

$$R_K^{\text{SM}} \equiv \frac{A_K^b - A_K^d}{A_K^s - A_K^d}; \quad R_\pi^{\text{SM}} \equiv \frac{A_\pi^b - A_\pi^s}{A_\pi^d - A_\pi^s}$$

- We can express: $a_K^{\text{dir}} \simeq 2\xi_s \text{Im } (R_K^{\text{SM}})$; $a_\pi^{\text{dir}} \simeq 2\xi_d \text{Im } (R_\pi^{\text{SM}})$
- Current measurements: $\Delta a_{\text{CP}} = a_K^{\text{dir}} - a_\pi^{\text{dir}} = (2.6 \pm 1.0) \times 10^{-3}$
- In the SM: $\xi_s = -\xi_d = A^2 \lambda^4 \sqrt{(\rho^2 + \eta^2)}$
 $\Delta a_{\text{CP}} \simeq (1.3 \times 10^{-3}) \times \text{Im}(\Delta R^{\text{SM}} = R_K^{\text{SM}} + R_\pi^{\text{SM}}) \simeq O(10^{-3})$

$D^0 - \bar{D}^0$ Mixing and CP violation in the charm sector (contd.)

- a_f^{Mix} signals CP violation in Mixing; it and also $a_f^{\text{Int.}}$ are universal

$$a^{\text{Mix}} = -\frac{y}{2} \left(\left| \frac{q}{p} \right| - \left| \frac{p}{q} \right| \right) \cos \phi_D$$

- $a_f^{\text{Int.}}$ signals CP violation in the interference of mixing and decay

$$a^{\text{Int.}} = -\frac{x}{2} \left(\left| \frac{q}{p} \right| + \left| \frac{p}{q} \right| \right) \sin \phi_D$$

- One can also isolate the effects of indirect CP violation using the time-dependent decay rates involving $D^0 - \bar{D}^0$ mixing. For $(x, y) \ll 1$,

$$\Gamma(D^0 \rightarrow K^+ K^-) = \Gamma \left[1 + \left| \frac{q}{p} \right| (y \cos \phi_D - x \sin \phi_D) \right]$$

$$\Gamma(\bar{D}^0 \rightarrow K^+ K^-) = \Gamma \left[1 + \left| \frac{p}{q} \right| (y \cos \phi_D + x \sin \phi_D) \right]$$

- Define the CP-conserving and CP-violating combinations

$$y_{\text{CP}}(K^+ K^-) \equiv \frac{\Gamma(D^0 \rightarrow K^+ K^-) + \Gamma(\bar{D}^0 \rightarrow K^+ K^-)}{2\Gamma} - 1; A_\Gamma(K^+ K^-) \equiv \frac{\Gamma(D^0 \rightarrow K^+ K^-) - \Gamma(\bar{D}^0 \rightarrow K^+ K^-)}{2\Gamma}$$

- In the limit of CP conservation, $y_{\text{CP}} = (\Gamma_+ - \Gamma_-)/2\Gamma = y$ and $A_\Gamma = 0$. LHCb has shown that CP asymmetry in the charm sector is essentially 0

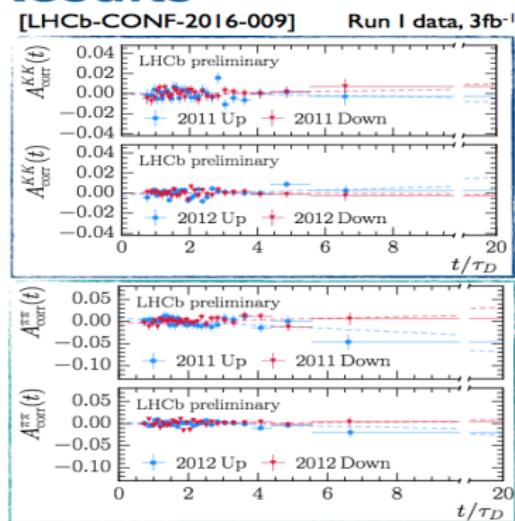
CP Asymmetry in the charm sector using effective lifetime measurements

$$A_{\Gamma}(KK) = (-0.14 \pm 0.37 \pm 0.10) \times 10^{-3};$$
$$A_{\Gamma}(\pi\pi) = (0.14 \pm 0.63 \pm 0.15) \times 10^{-3}$$

• Results

P. Marino (SNS & INFN-P)

07/09/2016

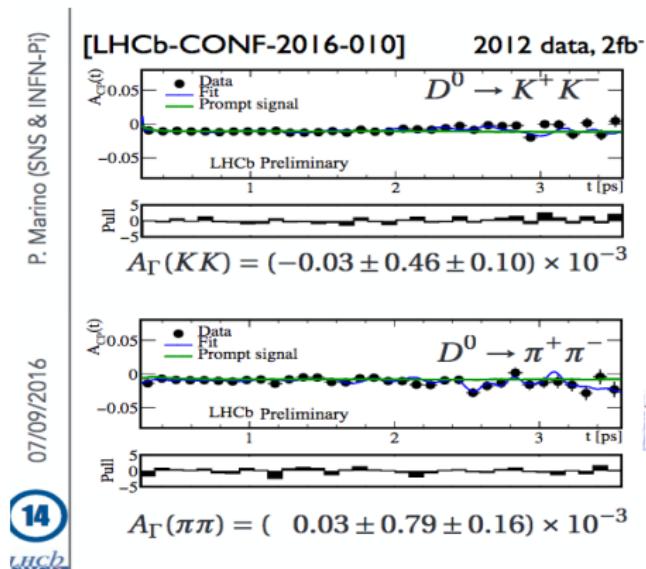


CP Asymmetry in the charm sector using bins of decay times

$$A_{\Gamma}(KK) = (-0.30 \pm 0.32 \pm 0.14) \times 10^{-3};$$

$$A_{\Gamma}(\pi\pi) = (0.46 \pm 0.58 \pm 0.16) \times 10^{-3}$$

- compatible with 0 at the level of 0.3 per mill



Introduction to CP Violation in B Decays & Mixings -1

- CKM Violation in the SM is due to the phase η in the CKM matrix
- CP violation in the SM is due to interference of two *different* amplitudes

Three Classes of CP Violation in B Decays

- 1 Direct CP Violation: e.g., in $B^\pm \rightarrow K_s \pi^\pm$ decays

$$\mathcal{A}_{\text{CP}} = \frac{2|A_1||A_2|\sin(\delta_1 - \delta_2)\sin(\phi_1 - \phi_2)}{|A_1|^2 + 2|A_1||A_2|\cos(\delta_1 - \delta_2)\cos(\phi_1 - \phi_2) + |A_2|^2}$$

- Requires $(\delta_1 - \delta_2) \neq 0$ & $(\phi_1 - \phi_2) \neq 0$

- 2 Indirect CP Violation:

$$\mathcal{A}_{\text{SL}} = \frac{\Gamma(\overline{B^0} \rightarrow \ell^+ X) - \Gamma(B^0 \rightarrow \ell^- X)}{\Gamma(\overline{B^0} \rightarrow \ell^+ X) + \Gamma(B^0 \rightarrow \ell^- X)}$$

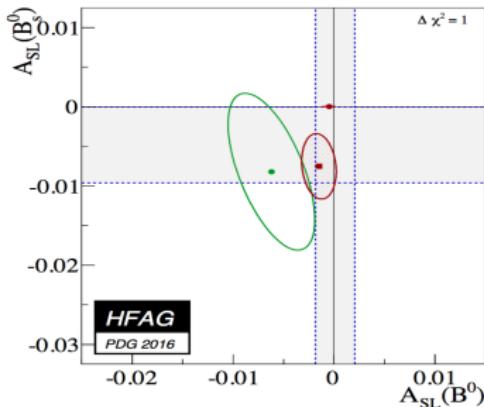
- Involves the relative phase in the absorptive & dispersive parts of the $B^0 - \overline{B^0}$ Mixing Amplitude: $\mathcal{A}_{\text{SL}} = \text{Im}(\frac{\Gamma_{12}}{M_{12}})$

- Writing $(\frac{\Gamma_{12}}{M_{12}})_q = r_q e^{i\zeta_q}$: $\mathcal{A}_{\text{SL}}(B_d) = r_d \sin \zeta_d$

- SM (NLO): $\mathcal{A}_{\text{SL}}(B_d) = -(4.1 \pm 0.6)(10^{-4})$ [Lenz & Nierste (2011)]

- SM (NLO): $\mathcal{A}_{\text{SL}}(B_s) = +(1.9 \pm 0.3)(10^{-5})$ [Lenz & Nierste (2011)]

Current Status of of $A_{\text{SL}}(B_s^0)$ vs. $A_{\text{SL}}(B_d^0)$ [HFAG 2016]



- $A_{\text{SL}}(B_d^0)$ and $A_{\text{SL}}(B_s^0)$ are defined as follows ($\phi_{12}^q = \arg(-M_{12}^q/\Gamma_{12}^q)$):

$$A_{\text{SL}}(B_q) = \frac{\Delta \Gamma_q}{\Delta M_q} \tan \phi_{12}^q$$

- $A_{\text{SL}}^{\text{SM}}(B_s) = (1.9 \pm 0.3) \times 10^{-5}$ vs. $A_{\text{SL}}^{\text{WA}}(B_s) = -0.007 \pm 0.0041$
- $A_{\text{SL}}^{\text{SM}}(B_d) = (-4.1 \pm 0.6) \times 10^{-4}$ vs. $A_{\text{SL}}^{\text{WA}}(B_d) = -0.0015 \pm 0.0017$
- Both $A_{\text{SL}}(B_d^0)$ and $A_{\text{SL}}(B_s^0)$ are consistent with SM within 1.5σ

Interplay of Mixing & Decays of B^0 and $\overline{B^0}$ to CP Eigenstate

- Tree-dominated B -decays ($b \rightarrow c\bar{c}s$): $(B^0/\overline{B^0} \rightarrow J/\psi K_s; J/\psi K_L)$

$$\begin{aligned}\mathcal{A}_f(t) &= \frac{\Gamma(\overline{B^0}(t) \rightarrow f) - \Gamma(B^0(t) \rightarrow f)}{\Gamma(\overline{B^0}(t) \rightarrow f) + \Gamma(B^0(t) \rightarrow f)} \\ &= C_f \cos(\Delta M_B t) + S_f \sin(\Delta M_B t)\end{aligned}$$

$$C_f = \frac{(|\lambda_f|^2)-1}{(|\lambda_f|^2+1)}; \quad S_f = \frac{2 \operatorname{Im} \lambda_f}{(|\lambda_f|^2+1)}$$

- Definitions: $\lambda_f \equiv (q/p) \rho(f); \quad \rho(f) = \frac{\bar{A}(f)}{A(f)}$

$$A(f) = \langle f | H | B^0 \rangle; \quad \bar{A}(f) = \langle f | H | \overline{B^0} \rangle$$

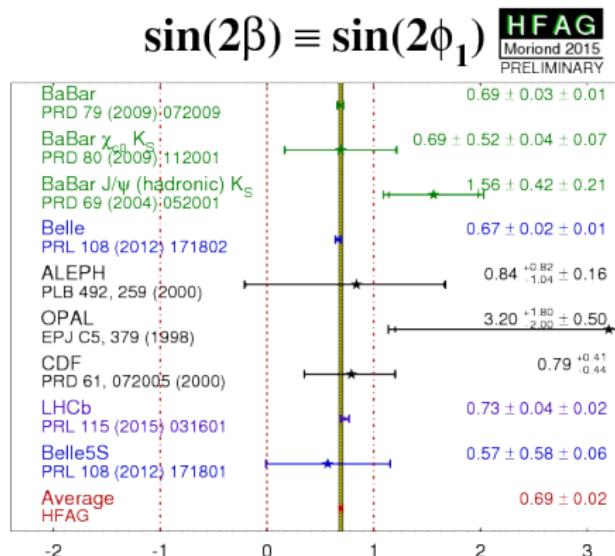
$$q/p = \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} = e^{-2i\phi_{\text{mixing}}} = e^{-2i\beta}$$

- If only a Single Amplitude dominant: $\rho(f) = \eta_f e^{-2i\phi_{\text{decay}}}$

$\eta_f = \pm 1$ is the intrinsic CP-Parity of the state $f \Rightarrow |\rho(f)| = 1$

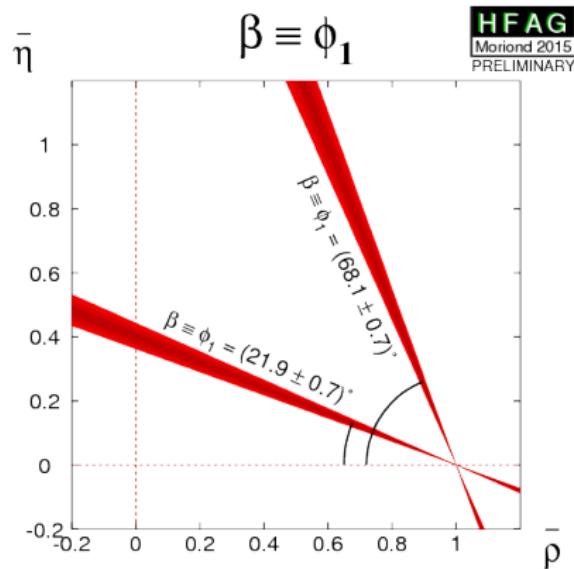
$$\mathcal{A}_f(t) = S_f \sin(\Delta M_B t); \quad S_f = -\eta_f \sin 2(\phi_{\text{mixing}} + \phi_{\text{decay}}); \quad C_f = 0$$

S_{CP} from the $b \rightarrow c\bar{c}s$ decays [HFAG 2016]



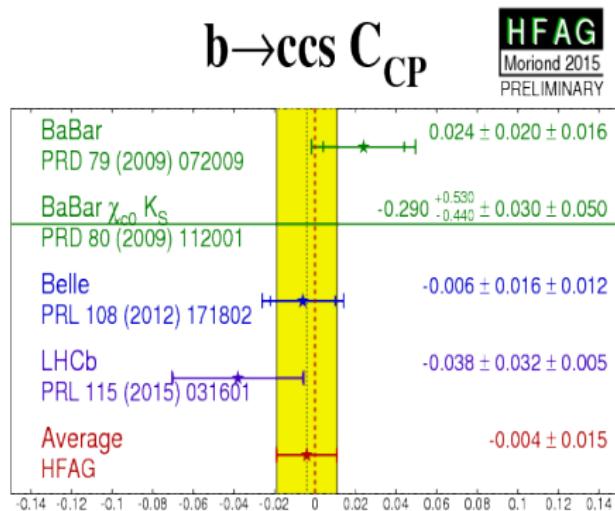
- $\sin(2\beta) = \sin(2\phi_1) = 0.691 \pm 0.017$ (HFAG: Summer 2016)

Constraints on $\bar{\rho} - \bar{\eta}$ from $\sin(2\phi_1)$ decays [HFAG 2016]

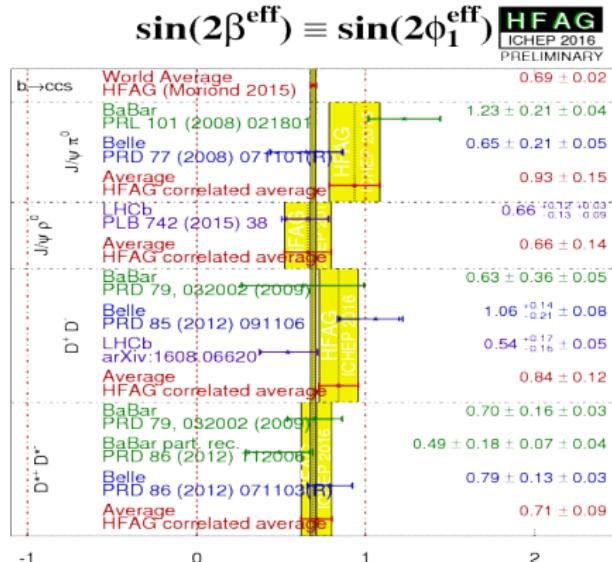


- $\beta = \phi_1 = (21.9 \pm 0.7)^\circ$ or $\beta = \phi_1 = (68.1 \pm 0.7)^\circ$

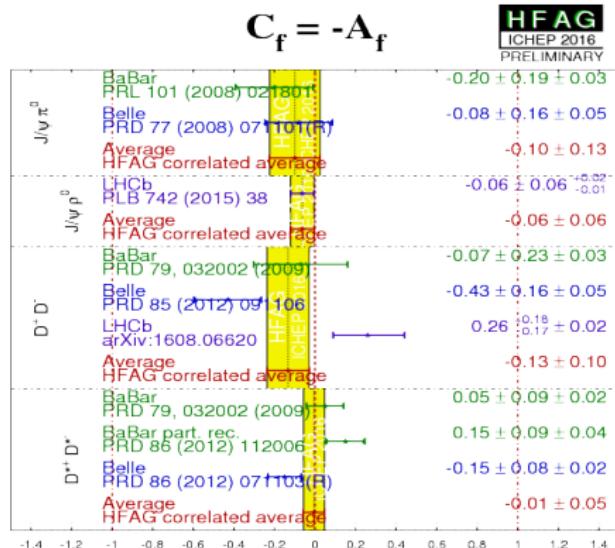
C_{CP} from the $b \rightarrow c\bar{c}s$ decays [HFAG 2016]



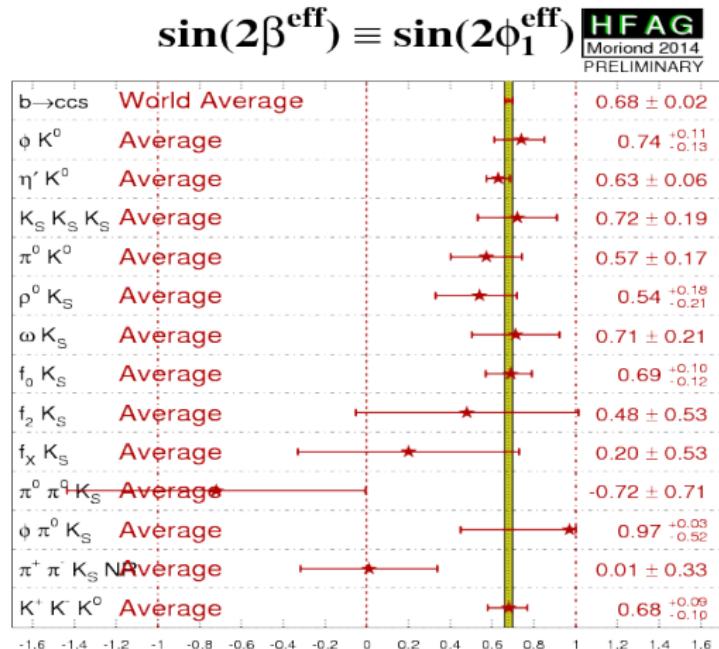
S_{CP} from the $b \rightarrow c\bar{c}d$ decays [HFAG 2016]



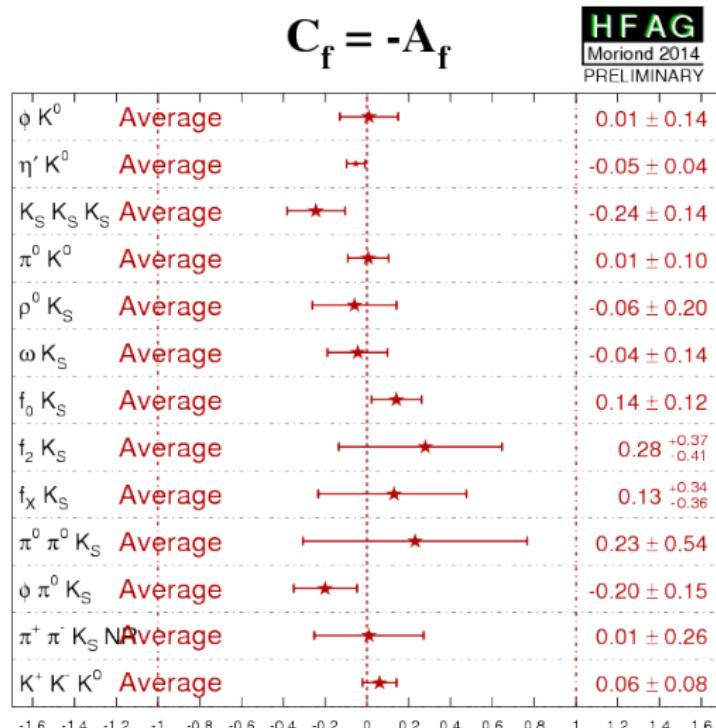
C_{CP} from the $b \rightarrow c\bar{c}d$ decays [HFAG 2016]



S_{CP} from the $b \rightarrow s\bar{s}(s,d)$ decays [HFAG 2016]

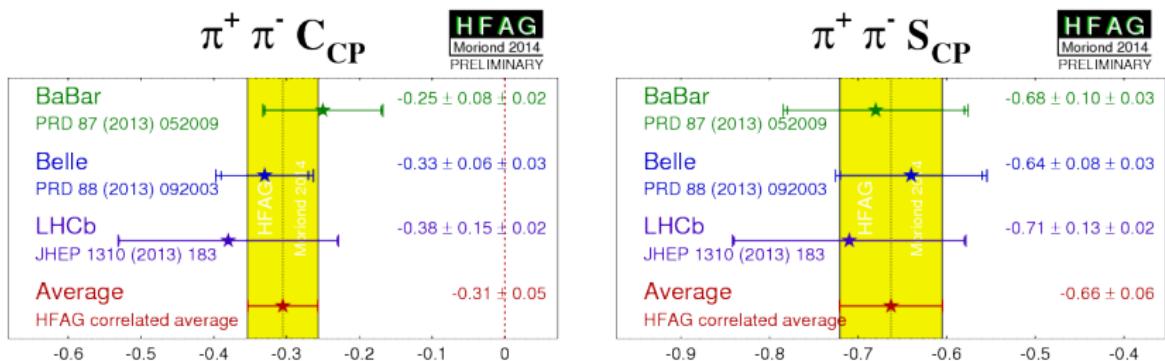


C_{CP} from the $b \rightarrow s\bar{s}(s,d)$ decays [HFAG 2016]



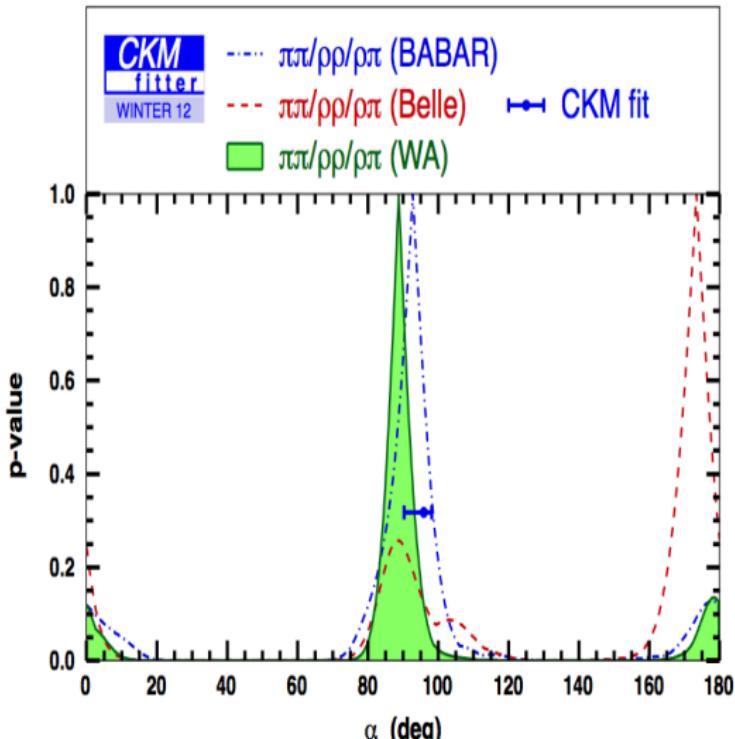
Time-dependent CP-asymmetry in $B^0/\bar{B}^0 \rightarrow \pi^+ \pi^-$

$$\begin{aligned}
 a_{\pi\pi}^{+-}(t) &\equiv \frac{\Gamma[\bar{B}^0(t) \rightarrow \pi^+ \pi^-] - \Gamma[B^0(t) \rightarrow \pi^+ \pi^-]}{\Gamma[\bar{B}^0(t) \rightarrow \pi^+ \pi^-] + \Gamma[B^0(t) \rightarrow \pi^+ \pi^-]} \\
 &= S_{\pi\pi}^{+-}(t) \sin(\Delta M_B t) - C_{\pi\pi}^{+-}(t) \cos(\Delta M_B t)
 \end{aligned}$$



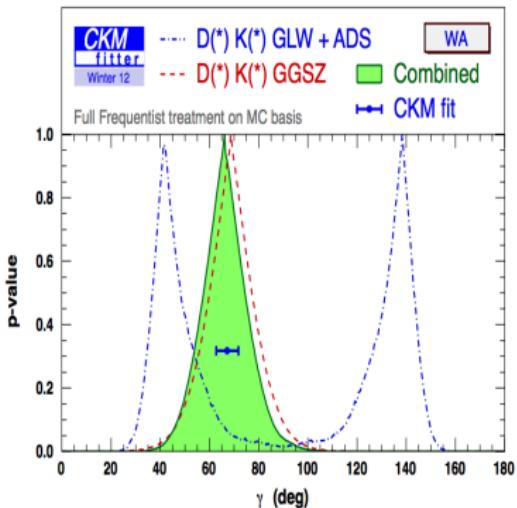
- Individual measurements imprecise, but HFAG average now significant numbers

Current World Average of α [CKMfitter 2012] & Update



2016 Update: $\alpha = [88.0 \pm 5]^\circ$ (Belle & BaBar measurements)

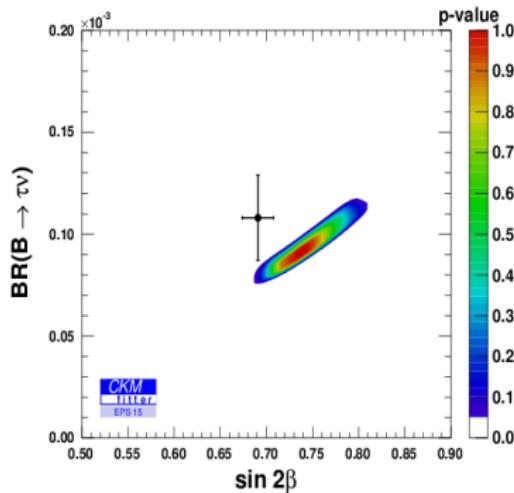
Current World Average of γ [CKMfitter 2012] & Updates



- $\gamma = (69^{+17}_{-16})^\circ$ (BaBar)
- $\gamma = (70.9^{+7.1}_{-8.5})^\circ$ (LHCb)
- $\gamma = (68.0^{+8.0}_{-8.5})^\circ$ (PDG 2014)

Some open issues (tensions) in the CKM-UT fits

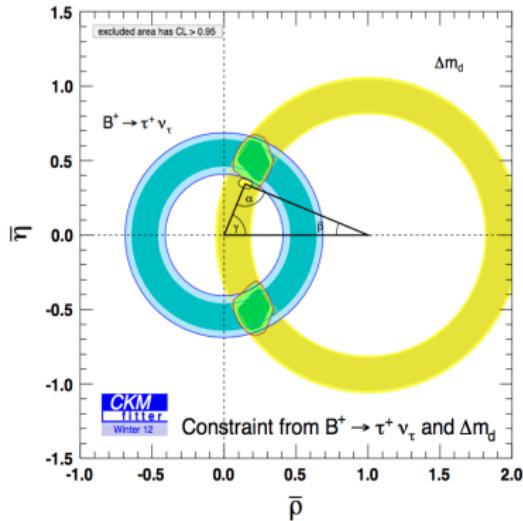
- The correlation between $BR(B^+ \rightarrow \tau^+\nu_\tau)$ and $\sin 2\beta$ is at odds with their direct experimental determinations
- $BR(B^+ \rightarrow \tau^+\nu_\tau) = (10.8 \pm 2.1) \times 10^{-5}$ is too large compared to the indirect SM-based estimates: $(7.58^{+0.80}_{-0.59}) \times 10^{-5}$ [Pull: 1.6σ]



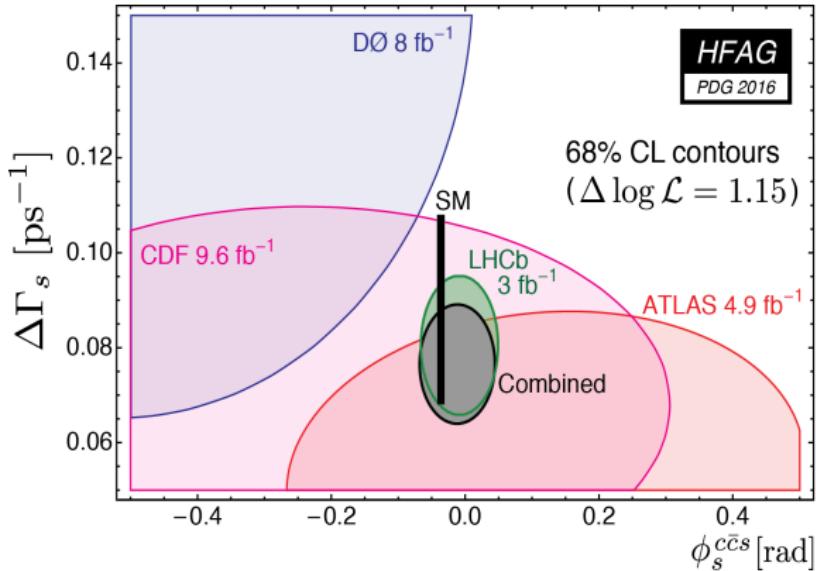
Some open issues (tensions) in the CKM-UT fits (contd.)

- The shape of this correlation can be understood by the ratio $BR(B^+ \rightarrow \tau^+ \nu_\tau)/\Delta M_d$, which depends on $\sin \beta / \sin \gamma$ and B_{B_d}

$$\frac{BR(B^+ \rightarrow \tau^+ \nu_\tau)}{\Delta M_d} = \frac{3\pi}{4} \frac{m_\tau^2}{m_W^2 S(x_t)} \left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 \tau_{B^+} \frac{1}{B_{B_d}} \frac{1}{|V_{ud}|^2} \left(\frac{\sin \beta}{\sin \gamma}\right)^2$$

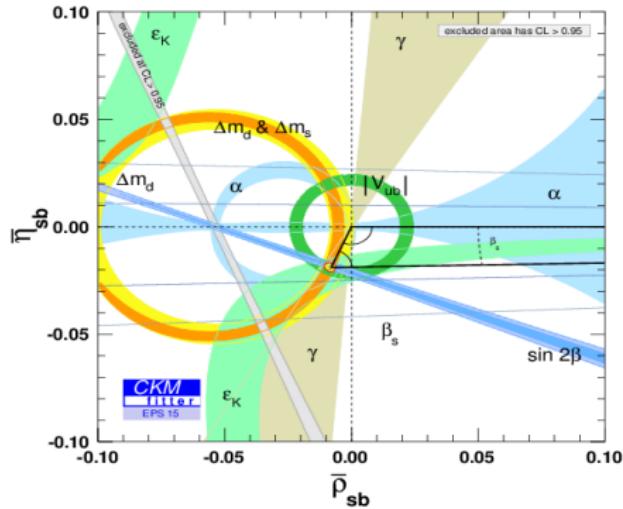


Contours in the $(1/\Gamma_s, \phi_s)$ plane [HFAG]



- $\Delta\Gamma_s = +0.082 \pm 0.007 \text{ ps}^{-1}$
- $\phi_s(c\bar{c}s) = -0.013 \pm 0.37$

Current Status of the Squashed UT_s Triangle [CKMfitter]

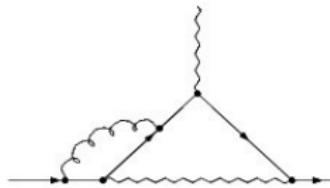
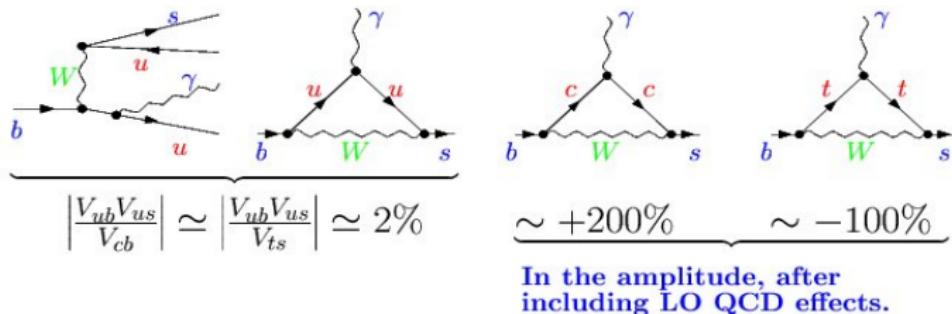


- $\bar{\rho}_{B_s} = -0.0078 \pm 0.0015$ [Fit-value]
- $\bar{\eta}_{B_s} = -0.01837^{+0.00080}_{-0.00082}$ [Fit-value]
- $\sin 2\beta_s = 0.0364 \pm 0.0016$ [Fit-value]
where $\beta_s = -\arg(-V_{cs}V_{cb}^*/V_{ts}V_{tb}^*)$

Rare B -decays on the Crossroads

- The Standard Candle in Rare B -Decays: $\mathbf{B} \rightarrow X_s \gamma$
- Electroweak Penguins: $\mathbf{B} \rightarrow X_s \ell^+ \ell^-$
- Exclusive Radiative Decays $\mathbf{B} \rightarrow K^* \gamma$ & $\mathbf{B}_s \rightarrow \phi \gamma$
- Test of Lepton Universality in $B \rightarrow D^{(*)} \tau \nu_\tau$ Decays
- Exclusive Decays $\mathbf{B} \rightarrow (K, K^*, \pi) \ell^+ \ell^-$ & Tension on the SM
- Current Frontier of Rare B Decays: $\mathbf{B}_s \rightarrow \mu^+ \mu^-$ & $\mathbf{B}_d \rightarrow \mu^+ \mu^-$
- Current Pulls on the SM Parameters

Examples of leading electroweak diagrams for $B \rightarrow X_s \gamma$



- QCD logarithms $\alpha_s \ln \frac{M_W^2}{m_b^2}$ enhance $\text{BR}(B \rightarrow X_s \gamma)$ more than twice
- Effective field theory (obtained by integrating out heavy fields) used for resummation of such large logarithms

The effective Lagrangian for $B \rightarrow X_s \gamma$ and $B \rightarrow X_s \ell^+ \ell^-$

$$\mathcal{L} = \mathcal{L}_{QCD \times QED}(q, l) + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{10} C_i(\mu) O_i$$

$$(q = u, d, s, c, b, l = e, \mu, \tau)$$

$$O_i = \begin{cases} (\bar{s}\Gamma_i c)(\bar{c}\Gamma'_i b), & i = 1, 2, \quad |C_i(m_b)| \sim 1 \\ (\bar{s}\Gamma_i b)\Sigma_q(\bar{q}\Gamma'_i q), & i = 3, 4, 5, 6, \quad |C_i(m_b)| < 0.07 \\ \frac{em_b}{16\pi^2} \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}, & i = 7, \quad C_7(m_b) \sim -0.3 \\ \frac{gm_b}{16\pi^2} \bar{s}_L \sigma^{\mu\nu} T^a b_R G_{\mu\nu}^a, & i = 8, \quad C_8(m_b) \sim -0.15 \\ \frac{e^2}{16\pi^2} (\bar{s}_L \gamma_\mu b_L)(\bar{l} \gamma^\mu (\gamma_5) l), & i = 9, (10) \quad |C_i(m_b)| \sim 4 \end{cases}$$

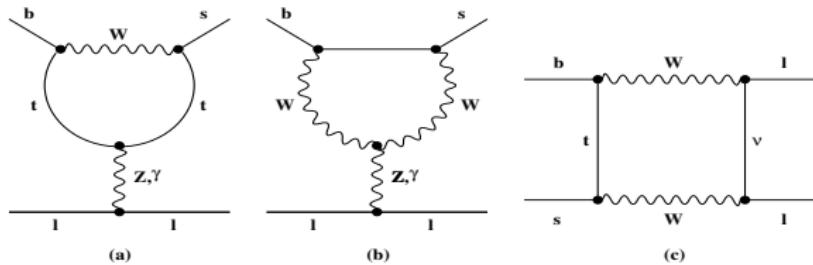
Three steps of the calculation:

Matching: Evaluating $C_i(\mu_0)$ at $\mu_0 \sim M_W$ by requiring equality of the SM and the effective theory Green functions

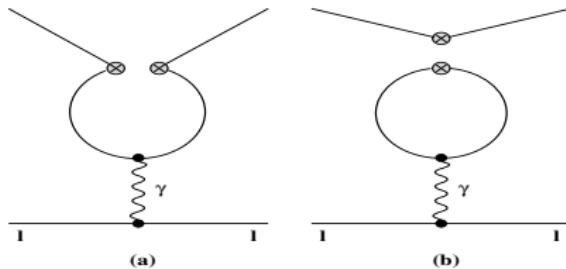
Mixing: Deriving the effective theory RGE and evolving $C_i(\mu)$ from μ_0 to $\mu_b \sim m_b$

Matrix elements: Evaluating the on-shell amplitudes at $\mu_b \sim m_b$

The decay $b \rightarrow s\ell^+\ell^-$: Leading Feynman diagram



Diagrams in the full theory



Diagrams in the effective theory

Structure of the SM calculations for $\bar{B} \rightarrow X_s \gamma$ & $\bar{B} \rightarrow X_s \ell^+ \ell^-$

$$\mathcal{H}_{\text{eff}} \sim \sum_{i=1}^{10} C_i(\mu) O_i$$

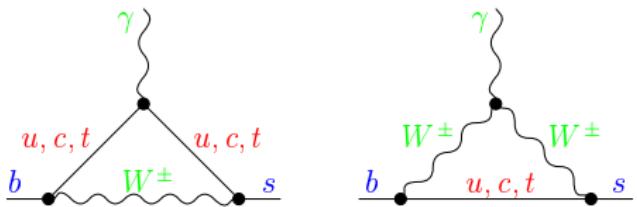
- \mathcal{H}_{eff} independent of the scale μ , while $C_i(\mu)$ and $O_i(\mu)$ depend on μ
Renormalization Group Equation (RGE) for $C_i(\mu)$:

$$\mu \frac{d}{d\mu} C_i(\mu) = \gamma_{ij}^T C_j(\mu)$$

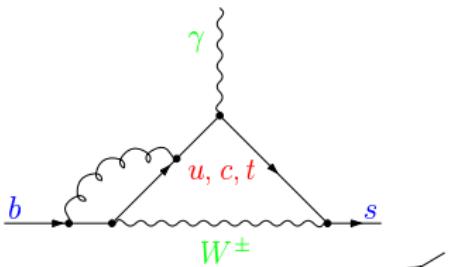
- γ_{ij} : anomalous dimension matrix
- Matching usually done at high scale ($\mu_0 \sim M_W, m_t$)
- SM and the matrix elements of the operators have the same large logs
 $\mu_0 \sim O(M_W)$
 \downarrow RGE
 $\mu_b \sim O(m_b)$: matrix elements of the operators at this scale don't have large logs; they are contained in the $C_i(\mu_b)$
- Evaluation of the on-shell amplitudes at $\mu_b \sim m_b$

Examples of SM diagrams for the matching of $C_7(\mu_0)$

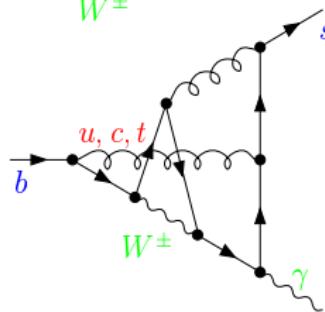
LO:
[Inami, Lim, 1981]



NLO:
[Adel, Yao, 1993]



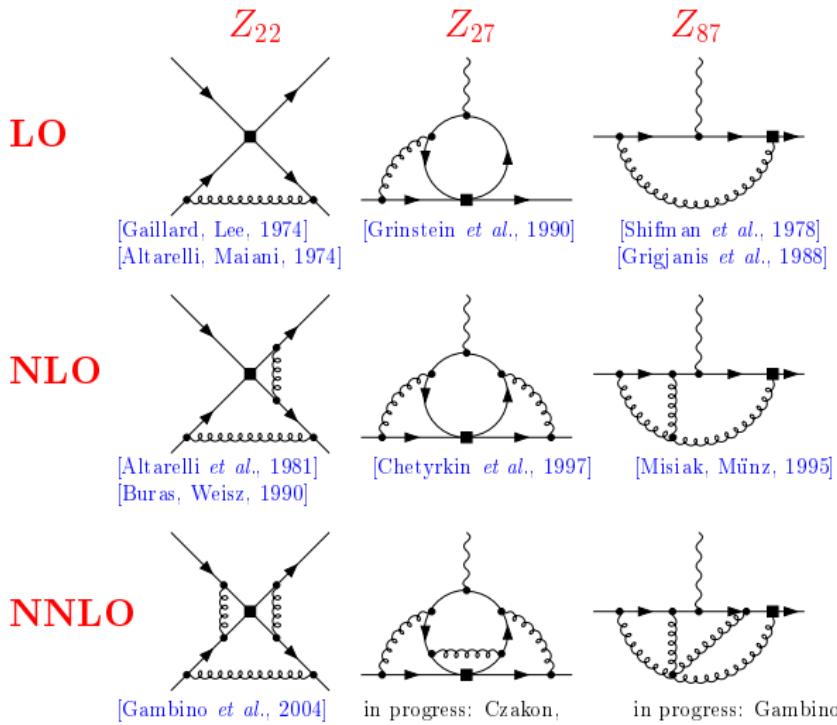
NNLO:
[Steinhauser, Misiak, 2004]



Resummation of large logarithms $\left(\alpha_s \ln \frac{M_W^2}{m_b^2}\right)^n$ in $b \rightarrow s\gamma$ amplitude

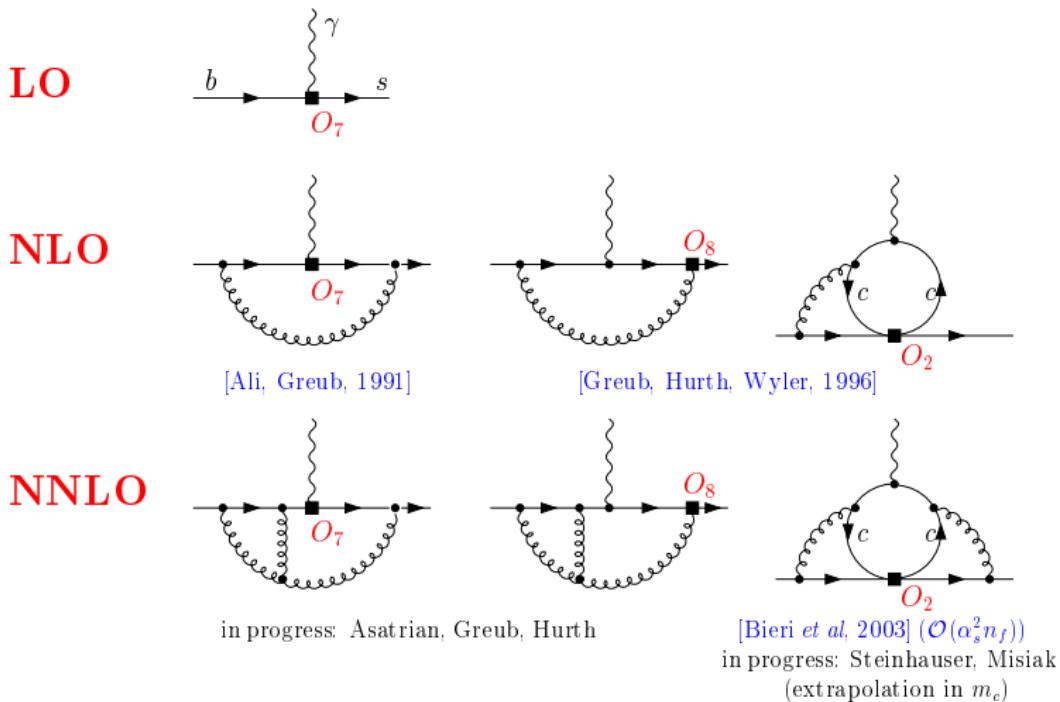
$$\text{RGE for the Wilson coefficients } \mu \frac{d}{d\mu} C_j(\mu) = C_i(\mu) \gamma_{ij}(\mu)$$

- Renormalization constants $\Rightarrow \gamma_{ij}$: $C_j(\mu)$ known to NLL accuracy



The $b \rightarrow s\gamma$ matrix elements

Perturbative on-shell amplitudes



Wilson Coefficients in the SM

Wilson Coefficients of Four-Quark Operators

	$C_1(\mu_b)$	$C_2(\mu_b)$	$C_3(\mu_b)$	$C_4(\mu_b)$	$C_5(\mu_b)$	$C_6(\mu_b)$
LL	-0.257	1.112	0.012	-0.026	0.008	-0.033
NLL	-0.151	1.059	0.012	-0.034	0.010	-0.040

Wilson Coefficients of the dipole and semileptonic Operators

	$C_7^{\text{eff}}(\mu_b)$	$C_8^{\text{eff}}(\mu_b)$	$C_9(\mu_b)$	$C_{10}(\mu_b)$
LL	-0.314	-0.149	2.007	0
NLL	-0.308	-0.169	4.154	-4.261
NNLL	-0.290		4.214	-4.312

- Obtained for the following input:

$$\mu_b = 4.6 \text{ GeV} \quad \bar{m}_t(\bar{m}_t) = 167 \text{ GeV}$$

$$M_W = 80.4 \text{ GeV} \quad \sin^2 \theta_W = 0.23$$

$\mathcal{B}(B \rightarrow X_s \gamma)$: Experiment vs. SM & BSM Effects

[Misiak et al., PRL 114 (2015) 22, 221801]

- Expt.: CLEO, Belle, BaBar [HFAG 2014]: ($E_\gamma > 1.6$ GeV):

$$\mathcal{B}(B \rightarrow X_s \gamma) = (3.43 \pm 0.21 \pm 0.07) \times 10^{-4}$$

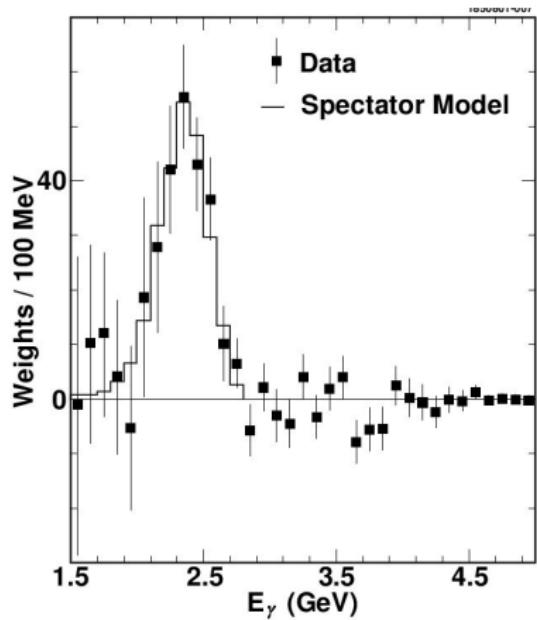
- SM [NNLO]: $\mathcal{B}(B \rightarrow X_s \gamma) = (3.36 \pm 0.23) \times 10^{-4}$
- Expt./SM = 1.02 ± 0.08
- Excellent agreement; restricts most NP models
- BSM effects can be parametrized as additive contributions to the Wilson Coeffs. of the dipole operators C_7 and C_8

$$\mathcal{B}(B \rightarrow X_s \gamma) \times 10^4 = (3.36 \pm 0.23) - 8.22\Delta C_7 - 1.99\Delta C_8$$

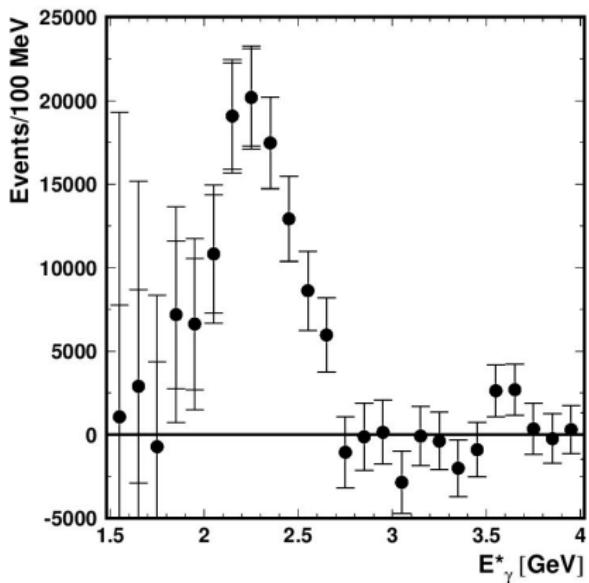
- In 2HDM, $\mathcal{B}(B \rightarrow X_s \gamma)$ puts strict bounds on M_{H^+}

Photon Energy Spectrum in $B \rightarrow X_s \gamma$

Spectator Model: Greub, AA; PLB 259, 182 (1991)



CLEO
PRL 87 (2001) 251807



BELLE
PRL 93 (2004) 061803

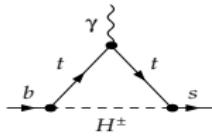
$B \rightarrow X_s \gamma$ in 2HDM

- NNLO in 2HDM [Hermann, Misiak, Steinhauser; JHEP 1211 (2012) 036]; Updated [Misiak et al., Phys. Rev. Lett. 114 (2015) 22, 221801]

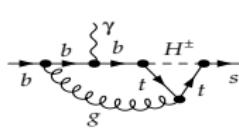
$$\mathcal{L}_{H+} = (2\sqrt{2}G_F)^{1/2} \sum_{i,j=1}^3 \bar{u}_i (A_u m_{u_i} V_{ij} P_L - A_d m_{d_j} V_{ij} P_R) d_j H^* + h.c.$$

- 2HDM contributions to the Wilson coefficients are proportional to $A_i A_j^*$
 - 2HDM of type-I: $A_u = A_d = \frac{1}{\tan \beta}$
 - 2HDM of type-II: $A_u = -1/A_d = \frac{1}{\tan \beta}$

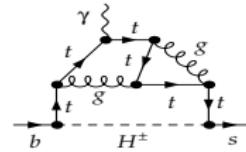
(a)



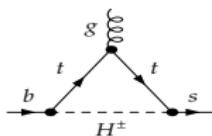
(b)



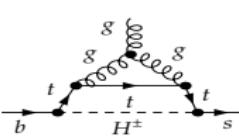
(c)



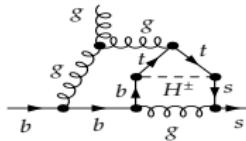
(d)



(e)

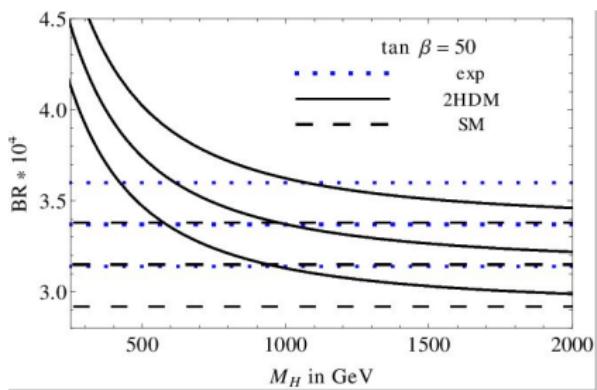
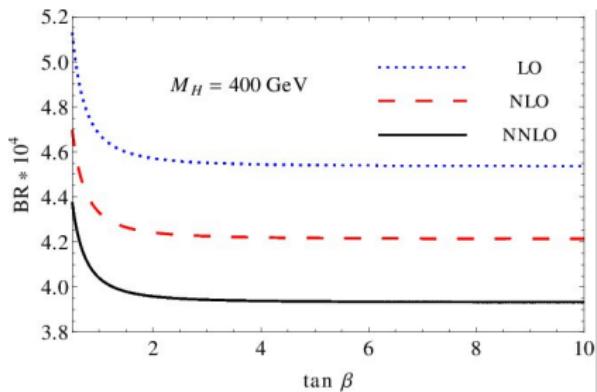


(f)



$B \rightarrow X_s \gamma$ in Type-II 2HDM

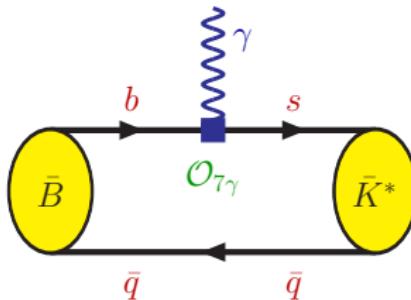
[Hermann, Misiak, Steinhauser JHEP 1211 (2012) 036]



- Updated NNLO [Misiak et al., Phys. Rev. Lett. 114 (2015) 22, 221801]
- $M_{H^+} > 480$ GeV (at 95% C.L.)
- $M_{H^+} > 358$ GeV (at 99% C.L.)
- Limits on 2HDM competitive to direct H^\pm searches at the LHC

The decay $B \rightarrow K^* \gamma$

- In LO, only the electromagnetic penguin operator $\mathcal{O}_{7\gamma}$ contributes to the $B \rightarrow K^* \gamma$ amplitude; involves the form factor $T_1^{(K^*)}(0)$



$$\mathcal{M}^{\text{LO}} \propto V_{tb} V_{ts}^* C_7^{(0)\text{eff}} \frac{e \bar{m}_b}{4\pi^2} T_1^{(K^*)}(0) [(Pq)(e^* \varepsilon^*) - (e^* P)(\varepsilon^* q) + i \text{eps}(e^*, \varepsilon^*, P, q)]$$

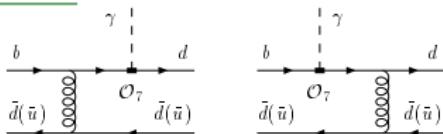
Here, $P^\mu = p_B^\mu + p_K^\mu$; $q^\mu = p_B^\mu - p_K^\mu$ is the photon four-momentum; e^μ is its polarization vector; ε^μ is the K^* -meson polarization vector

- Branching ratio:

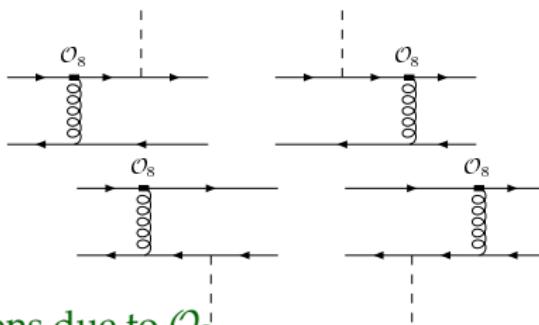
$$\mathcal{B}^{\text{LO}}(B \rightarrow K^* \gamma) = \tau_B \frac{G_F^2 |V_{tb} V_{ts}^*|^2 \alpha M^3}{32\pi^4} \bar{m}_b^2(\mu_b) |C_7^{(0)\text{eff}}(\mu_b)|^2 |T_1^{(K^*)}(0)|^2$$

Hard spectator contributions in $B \rightarrow (K^*, \rho) \gamma$

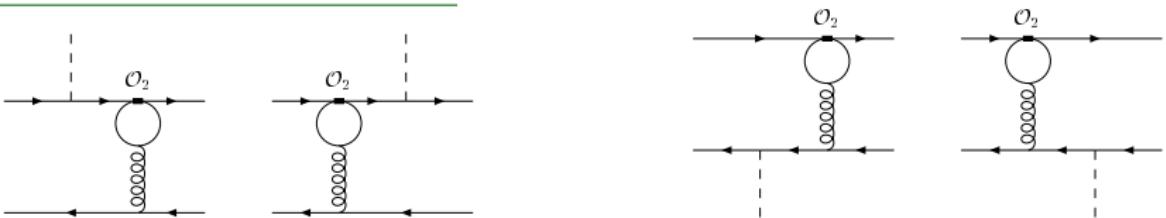
Spectator corrections due to \mathcal{O}_7



Spectator corrections due to \mathcal{O}_8



Spectator corrections due to \mathcal{O}_2



$B \rightarrow K^* \gamma$ decay rates in NLO

- Perturbative approaches: QCD-F; PQCD; SCET

Factorization Ansatz (QCDF):

[Beneke, Buchalla, Neubert, Sachrajda; Beneke & Feldmann]

$$\langle V\gamma | Q_i | \bar{B} \rangle = t_i^I \zeta_{V_\perp} + t_i^{II} \otimes \phi_+^B \otimes \phi_\perp^V + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m_b}\right)$$

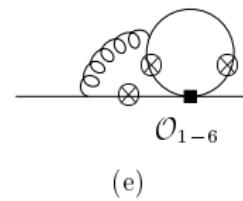
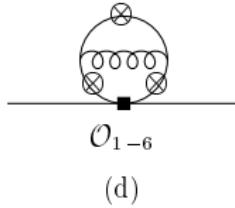
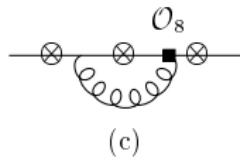
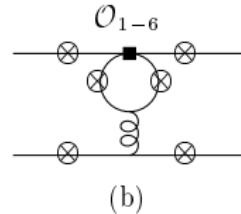
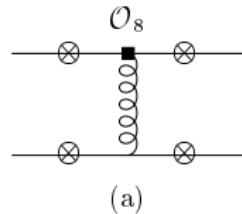
- ζ_{V_\perp} (form factor) and $\phi^{B,V}$ (LCDAs) are non-perturbative functions
- t^I and t^{II} are perturbative hard-scattering kernels

$$t^I = \mathcal{O}(1) + \mathcal{O}(\alpha_s) + \dots, \quad t^{II} = \mathcal{O}(\alpha_s) + \dots$$

- The kernels t^I and t^{II} are known at $\mathcal{O}(\alpha_s)$;
include Hard-scattering and Vertex corrections
[Parkhomenko, AA; Bosch, Buchalla; Beneke, Feldmann, Seidel 2001]

$B \rightarrow K^* \gamma$ Decays

Nonfactorizable α_s Corrections



- First line: hard-spectator corrections
- Second line: $b \rightarrow s\gamma$ vertex corrections

SCET factorization formula for $B \rightarrow K^* \gamma$

[Chay, Kim '03; Grinstein, Grossman, Ligeti '04; Becher, Hill, Neubert '05]

$$\langle V\gamma | Q_i | \bar{B} \rangle = \Delta_i C^A \zeta_{V\perp} + (\Delta_i C^{B1} \otimes j_\perp) \otimes \phi_\perp^V \otimes \phi_+^B$$

- $\zeta_{V\perp}, \phi_\perp^V, \phi_+^B$ are matrix elements of SCET operators
- Hard-scattering kernels t^I, t^{II} = SCET matching coefficients
- $t_i^I = \Delta_i C^A(m_b); \quad t_i^{II} = \Delta_i C^{B1}(m_b) \otimes j_\perp(\sqrt{m_b \Lambda})$ (**subfactorization**)
- Derivation of factorization in SCET
 - 1) QCD \rightarrow SCET_I: Integrate out m_b ; defines vertex corrections
 $\Delta_i C^A = t_i^I$

$$Q_i \rightarrow \Delta_i C^A(m_b) J^A + \Delta_i C^{B1}(m_b) \otimes J^{B1} + \dots$$

- 2) SCET_I \rightarrow SCET_{II}: Integrate out $\sqrt{m_b \Lambda_{\text{QCD}}}$; defines spectator corr.

$$J^{B1} \rightarrow j_\perp(\sqrt{m_b \Lambda_{\text{QCD}}}) \otimes O^{B1,\text{SCET}_{II}}(\Lambda_{\text{QCD}})$$

- 3) Large logs in t_i^{II} resummed by solving RG equations

Vertex Corrections

$$\Delta_i C^A = \Delta_7 C^{A(0)} \left[\Delta_{i7} + \frac{\alpha_s(\mu)}{4\pi} \Delta_i C^{A(1)} + \frac{\alpha_s^2(\mu)}{(4\pi)^2} \Delta_i C^{A(2)} \right]$$

- Contr. from O_7 and O_8 exact to NNLO $O(\alpha_s^2)$
- Contr. from O_2 exact at NLO $O(\alpha_s)$ but only large- β_0 limit at $O(\alpha_s^2)$

Spectator Corrections at $O(\alpha_s^2)$

$$t_i^{II(1)}(u, \omega) = \Delta_i C^{B1(1)} \otimes j_\perp^{(0)} + \Delta_i C^{B1(0)} \otimes j_\perp^{(1)}$$

- The one-loop jet-function $j_\perp^{(1)}$ known; [Becher and Hill '04; Beneke and Yang '05]
- The one-loop hard coefficient $\Delta_7 C^{B1(1)}$ known; [Beneke, Kiyo, Yang '04; Becher and Hill '04]
- The one-loop hard coefficient $\Delta_8 C^{B1(1)}$ known; [Pecjak, Greub, AA '07]
- $\Delta_i C^{B1(1)} (i = 1, \dots, 6)$ remain unknown (require two loops)

Estimates of $\text{BR}(B \rightarrow K^* \gamma)$ in SCET at NNLO

[Pecjak, Greub, AA; EPJ C55: 577 (2008)]

Estimates at NNLO in units of 10^{-5}

$$\mathcal{B}(B^+ \rightarrow K^{*+} \gamma) = 4.6 \pm 1.2[\zeta_{K^*}] \pm 0.4[m_c] \pm 0.2[\lambda_B] \pm 0.1[\mu]$$

[Expt. **4.21 ± 0.18** (HFAG 2014)];

$$\mathcal{B}(B^0 \rightarrow K^{*0} \gamma) = 4.3 \pm 1.1[\zeta_{K^*}] \pm 0.4[m_c] \pm 0.2[\lambda_B] \pm 0.1[\mu]$$

[Expt.: **4.33 ± 0.15** (HFAG 2014)];

$$\mathcal{B}(B_s \rightarrow \phi \gamma) = 4.3 \pm 1.1[\zeta_\phi] \pm 0.3[m_c] \pm 0.3[\lambda_B] \pm 0.1[\mu]$$

[Expt.: **3.59 ± 0.36** (HFAG 2014)])

Comparison with current experiments

- $\frac{\mathcal{B}(B^+ \rightarrow K^{*+} \gamma)_{\text{NNLO}}}{\mathcal{B}(B^+ \rightarrow K^{*+} \gamma)_{\text{exp}}} = 1.10 \pm 0.35[\text{theory}] \pm 0.04[\text{exp}]$
- $\frac{\mathcal{B}(B^0 \rightarrow K^{*0} \gamma)_{\text{NNLO}}}{\mathcal{B}(B^0 \rightarrow K^{*0} \gamma)_{\text{exp}}} = 1.00 \pm 0.32[\text{theory}] \pm 0.04[\text{exp}]$
- $\frac{\mathcal{B}(B_s \rightarrow \phi \gamma)}{\mathcal{B}(B^0 \rightarrow K^{*0} \gamma)} = 1.0 \pm 0.2[\text{theory}]; \quad 0.81 \pm 0.08[\text{exp}]$
- Theory error is about 30%; dominantly from ζ_{V_\perp} , m_c and λ_B

$$B \rightarrow X_s l^+ l^-$$

- There are two $b \rightarrow s$ semileptonic operators in SM:

$$O_i = \frac{e^2}{16\pi^2} (\bar{s}_L \gamma_\mu b_L) (\bar{l} \gamma^\mu (\gamma_5) l), \quad i = 9, (10)$$

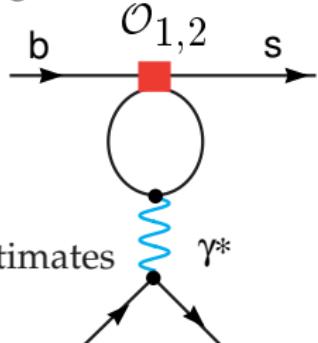
- Their Wilson Coefficients have the following perturbative expansion:

$$\begin{aligned} C_9(\mu) &= \frac{4\pi}{\alpha_s(\mu)} C_9^{(-1)}(\mu) + C_9^{(0)}(\mu) + \frac{\alpha_s(\mu)}{4\pi} C_9^{(1)}(\mu) + \dots \\ C_{10} &= C_{10}^{(0)} + \frac{\alpha_s(M_W)}{4\pi} C_{10}^{(1)} + \dots \end{aligned}$$

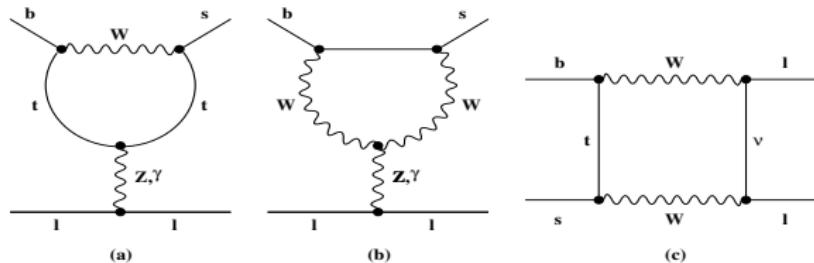
- $C_9^{(-1)}(\mu)$ reproduces the electroweak logarithm that originates from the photonic penguins with charm quark loops:

$$\frac{4\pi}{\alpha_s(m_b)} C_9^{(-1)}(m_b) = \frac{4}{9} \ln \frac{M_W^2}{m_b^2} + \mathcal{O}(\alpha_s) \simeq 2$$

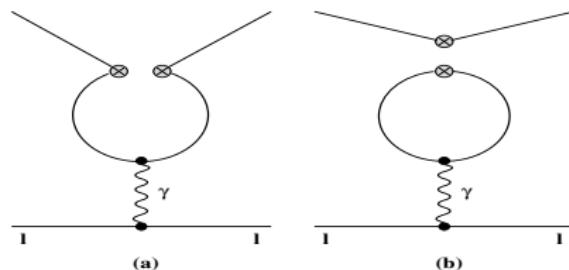
- $C_9^{(0)}(m_b) \simeq 2.2$; need to calculate NNLO for reliable estimates



The decay $b \rightarrow s\ell^+\ell^-$: Leading Feynman diagram



Diagrams in the full theory



Diagrams in the effective theory

NNLO Calculations of $\mathcal{B}(B \rightarrow X_s \ell^+ \ell^-)$

- 2-loop matching, 3-loop mixing and 2-loop matrix elements are available

- Matching: [Bobeth, Misiak, Urban]

- Mixing: [Gambino, Gorbahn, Haisch]

- Matrix elements:

[Asatryan, Asatrian, Greub, Walker; Asatrian, Bieri, Greub, Hovhannissyan;
Ghinculov, Hurth, Isidori, Yao; Bobeth, Gambino, Gorbahn, Haisch]

- Power corrections in $B \rightarrow X_s \ell^+ \ell^-$ decays

- $1/m_b$ corrections [A. Falk et al.; AA, Handoko, Morozumi, Hiller; Buchalla, Isidori]

- $1/m_c$ corrections [Buchalla, Isidori, Rey]

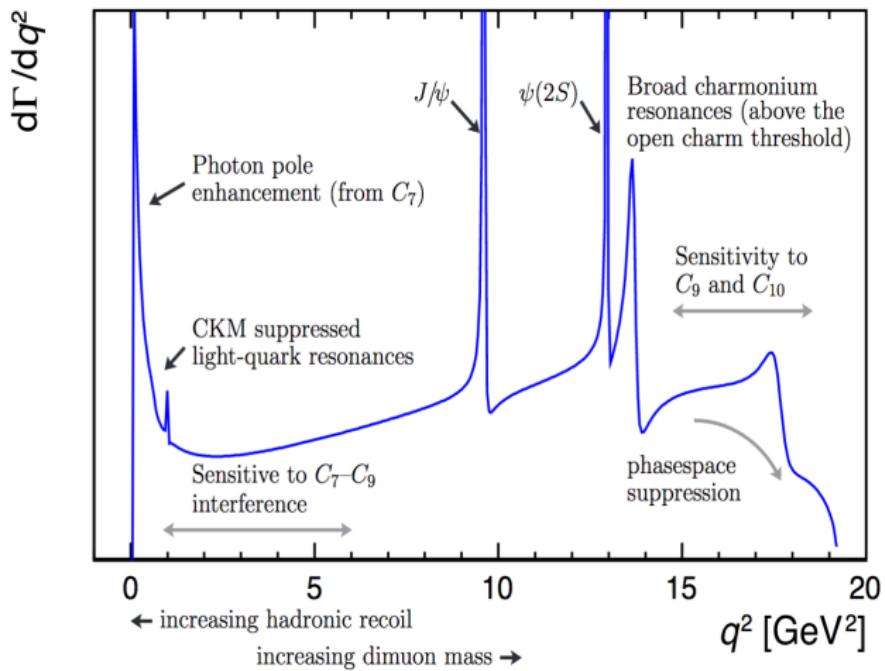
- NNLO Phenomenological analysis of $B \rightarrow X_s \ell^+ \ell^-$ decays

[AA, Greub, Hiller, Lunghi, Phys. Rev. D66, 034002 (2002)]

- $\mathbf{BR}(B \rightarrow X_s \mu^+ \mu^-); \quad q^2 > 4m_\mu^2 = (4.2 \pm 1.0) \times 10^{-6}$

- $\mathbf{BR}(B \rightarrow X_s e^+ e^-) = (6.9 \pm 0.7) \times 10^{-6}$

Sensitivity of the different q^2 regions to SD- & LD-pieces



Forward-Backward Asymmetry in $B \rightarrow X_s \ell^+ \ell^-$

[Proposed in AA, Mannel, Morozumi, PLB 273, 505 (1991)]

[NNLL: Asatrian, Bieri, Greub, Hovhannisyan; Ghinculov, Hurth, Isidori, Yao]

Normalized FB Asymmetry

$$\bar{A}_{\text{FB}}(\hat{s}) = \frac{\int_{-1}^1 \frac{d^2\Gamma(B \rightarrow X_s \ell^+ \ell^-)}{d\hat{s} dz} \text{sgn}(z) dz}{\int_{-1}^1 \frac{d^2\Gamma(B \rightarrow X_s \ell^+ \ell^-)}{d\hat{s} dz} dz}$$

$$\begin{aligned} \int_{-1}^1 \frac{d^2\Gamma(b \rightarrow X_s \ell^+ \ell^-)}{d\hat{s} dz} \text{sgn}(z) dz &= \left(\frac{\alpha_{\text{em}}}{4\pi}\right)^2 \frac{G_F^2 m_{b,\text{pole}}^5 |V_{ts}^* V_{tb}|^2}{48\pi^3} (1 - \hat{s})^2 \\ &\times \left[-3\hat{s} \text{Re}(\tilde{C}_9^{\text{eff}} \tilde{C}_{10}^{\text{eff}*}) \left(1 + \frac{2\alpha_s}{\pi} f_{910}(\hat{s})\right) - 6 \text{Re}(\tilde{C}_7^{\text{eff}} \tilde{C}_{10}^{\text{eff}*}) \left(1 + \frac{2\alpha_s}{\pi} f_{710}\right) \right] \end{aligned}$$

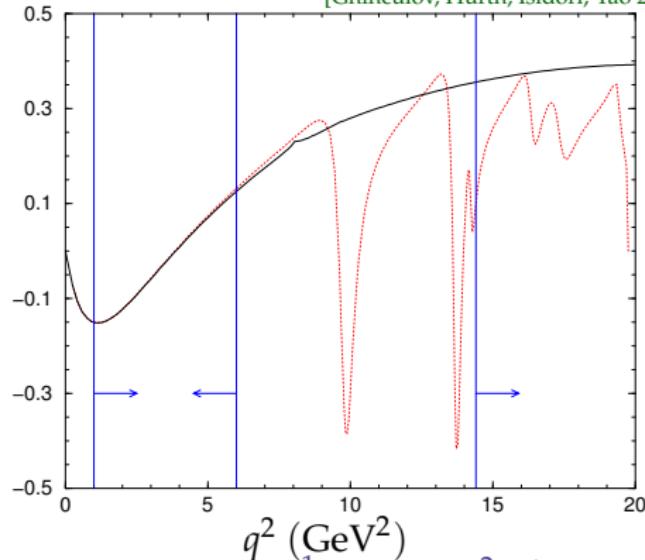
- NNLL stabilize the scale ($= \mu$) dependence of the FB Asymmetry

$$A_{\text{FB}}^{\text{NLL}}(0) = -(2.51 \pm 0.28) \times 10^{-6};$$
$$A_{\text{FB}}^{\text{NNLL}}(0) = -(2.30 \pm 0.10) \times 10^{-6}$$

- Zero of the FB Asymmetry is a precise test of the SM, correlating \tilde{C}_7^{eff} and \tilde{C}_9^{eff}

Normalized FB-Asymmetry in $\bar{B} \rightarrow X_s \ell^+ \ell^-$

[Ghinculov, Hurth, Isidori, Yao 2004]



$$\bar{\mathcal{A}}_{\text{FB}}(q^2) = \frac{1}{d\mathcal{B}(B \rightarrow X_s \ell^+ \ell^-)/dq^2} \int_{-1}^1 d\cos \theta_\ell \frac{d^2\mathcal{B}(B \rightarrow X_s \ell^+ \ell^-)}{dq^2 d\cos \theta_\ell} \text{sgn}(\cos \theta_\ell)$$

- Zero of the FB-Asymmetry is a precision test of the SM

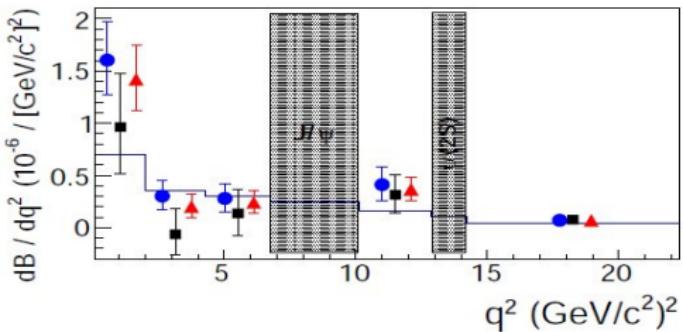
$$q_0^2 = (3.90 \pm 0.25) \text{ GeV}^2$$

[Ghinculov, Hurth, Isidori, Yao]

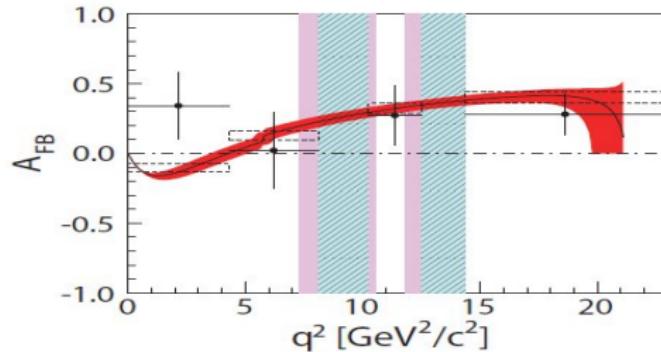
$$q_0^2 = (3.76 \pm 0.22_{\text{theory}} \pm 0.24_{m_b}) \text{ GeV}^2$$

[Bobeth, Gambino, Gorbahn, Haisch]

Dilepton invariant mass spectrum in $B \rightarrow X_s \ell^+ \ell^-$ [BaBar 2013]



Forward-Backward Asymmetry in $B \rightarrow X_s \ell^+ \ell^-$ [Belle 2014]



Exclusive Decays $B \rightarrow (K, K^*)\ell^+\ell^-$

- $B \rightarrow K$ & $B \rightarrow K^*$ transitions involve the currents:

$$\Gamma_\mu^1 = \bar{s}\gamma_\mu(1 - \gamma_5)b, \quad \Gamma_\mu^2 = \bar{s}\sigma_{\mu\nu}q^\nu(1 + \gamma_5)b$$

- \implies 10 non-perturbative q^2 -dependent functions (Form factors)

$$\langle K | \Gamma_\mu^1 | B \rangle \supset f_+(q^2), f_-(q^2)$$

$$\langle K | \Gamma_\mu^2 | B \rangle \supset f_T(q^2)$$

$$\langle K^* | \Gamma_\mu^1 | B \rangle \supset V(q^2), A_1(q^2), A_2(q^2), A_3(q^2)$$

$$\langle K^* | \Gamma_\mu^2 | B \rangle \supset T_1(q^2), T_2(q^2), T_3(q^2)$$

- Data on $B \rightarrow K^*\gamma$ provides normalization of $T_1(0) = T_2(0) \simeq 0.28$
- HQET/SCET-approach allows to reduce the number of independent form factors from 10 to 3 in low- q^2 domain ($q^2/m_b^2 \ll 1$)

Experimental data vs. SM in $B \rightarrow (X_s, K, K^*)\ell^+\ell^-$ Decays

Branching ratios (in units of 10^{-6}) [PDG: 2014]

SM: [A.A., Greub, Hiller, Lunghi PR D66 (2002) 034002]

Decay Mode	Expt. (BELLE & BABAR)	Theory (SM)
$B \rightarrow K\ell^+\ell^-$	0.48 ± 0.04	0.35 ± 0.12
$B \rightarrow K^*e^+e^-$	1.19 ± 0.20	1.58 ± 0.49
$B \rightarrow K^*\mu^+\mu^-$	1.06 ± 0.09	1.19 ± 0.39
$B \rightarrow X_s\mu^+\mu^-$	4.2 ± 1.3	4.2 ± 0.7
$B \rightarrow X_se^+e^-$	4.7 ± 1.3	4.2 ± 0.7
$B \rightarrow X_s\ell^+\ell^-$	4.5 ± 1.3	4.2 ± 0.7

Test of Lepton Universality using $B^\pm \rightarrow K^\pm \ell^+ \ell^-$ decays

[R.Aaij *et al.* (LHCb) PRL 113, 151601 (2014)]

- Precise measurements of the differential branching ratios in $B^\pm \rightarrow K^\pm e^+ e^-$ & $B^\pm \rightarrow K^\pm \mu^+ \mu^-$

$$R_K \equiv \frac{\int_{1 \text{ GeV}^2}^{6 \text{ GeV}^2} d\Gamma/dq^2 [B^\pm \rightarrow K^\pm \mu^+ \mu^-] dq^2}{\int_{1 \text{ GeV}^2}^{6 \text{ GeV}^2} d\Gamma/dq^2 [B^\pm \rightarrow K^\pm e^+ e^-] dq^2} = 0.745^{+0.090}_{-0.074} \pm 0.036$$

- SM Predictions [Bobeth, Hiller, Piranishvili, JHEP 12 (2007) 040]

$$R_K = 1.0003 \pm 0.0001 \implies 2.6\sigma \text{ deviation}$$

- Radiative corrections for the experimental setup is an issue
- BRs(expt.) smaller than the SM for both $K^\pm \mu^+ \mu^-$ and $K^\pm e^+ e^-$

$$\mathcal{B}(B \rightarrow K e^+ e^-) = (1.56^{+0.19-0.06}_{-0.15-0.04}) \times 10^{-7}$$

$$\mathcal{B}(B \rightarrow K \mu^+ \mu^-) = (1.20 \pm 0.09 \pm 0.07) \times 10^{-7}$$

$$\mathcal{B}^{\text{SM}}(B \rightarrow K \mu^+ \mu^-) = \mathcal{B}^{\text{SM}}(B \rightarrow K e^+ e^-) = (1.75^{+0.60}_{-0.29}) \times 10^{-7}$$

Test of Lepton Universality from the ratio $B \rightarrow D^{(*)} \tau \nu_\tau / B \rightarrow D^{(*)} \ell \nu_\ell$

[J.P. Lees *et al.* (BaBar), Phys. Rev. D88, 072012 (2013); M. Husche *et al.* (Belle) Phys. Rev. D92, 072014 (2015); R. Aaij *et al.* (LHCb) PRL 115, 159901 (2015)]

- A 3.9σ deviation from τ/ℓ ; ($\ell = e, \mu$) universality in charged current semileptonic $B \rightarrow D^{(*)}$ decays is reported by BaBar, Belle and LHCb

$$R_{D^{(*)}}^{\tau/\ell} \equiv \frac{\mathcal{B}(B \rightarrow D^{(*)} \tau \nu_\tau) / \mathcal{B}(B \rightarrow D^{(*)} \tau \nu_\tau)_{\text{SM}}}{\mathcal{B}(B \rightarrow D^{(*)} \ell \nu_\ell) / \mathcal{B}(B \rightarrow D^{(*)} \ell \nu_\ell)_{\text{SM}}}$$

$$R_D^{\tau\ell} = 1.37 \pm 0.17; \quad R_{D^*}^{\tau\ell} = 1.28 \pm 0.08$$

- A 30% deviation from the SM in a tree-level charged current interaction calls for a drastic contribution to an effective 4-fermi interaction proportional to the ***LL*** operator $(\bar{c} \gamma_\mu b_L)(\tau_L \gamma_\mu \nu_L)$
- Lepton non-universality in loop-induced R_K can be due to an ***LL*** operator $(\bar{s}_L \gamma_\mu b_L)(\bar{\mu}_L \gamma_\mu \mu_L)$, or an ***RL*** operator $(\bar{s}_L \gamma_\mu b_R)(\bar{\mu}_L \gamma_\mu \mu_L)$

Leptoquark models for R_K and $B \rightarrow D^{(*)}\tau\nu_\tau/B \rightarrow D^{(*)}\ell\nu_\ell$ anomalies

- Several suggestions along these lines have been made involving a leptoquark mediator
- A leptoquark model, with the leptoquark ϕ transforming as $(3, 3, -1/3)$ under the SM gauge groups, yielding an LL operator for muons:

$$\mathcal{L} = -\lambda_{b\mu}\phi^* q_3 \ell_2 - \lambda_{s\mu}\phi^* q_2 \ell_2$$

- A leptoquark model with an RL operator for electrons, with ϕ transforming as $(3, 2, 1/6)$

$$\mathcal{L} = -\lambda_{be}\phi(\bar{b}P_L\ell_e) - \lambda_{se}\phi(\bar{s}P_L\ell_e)$$

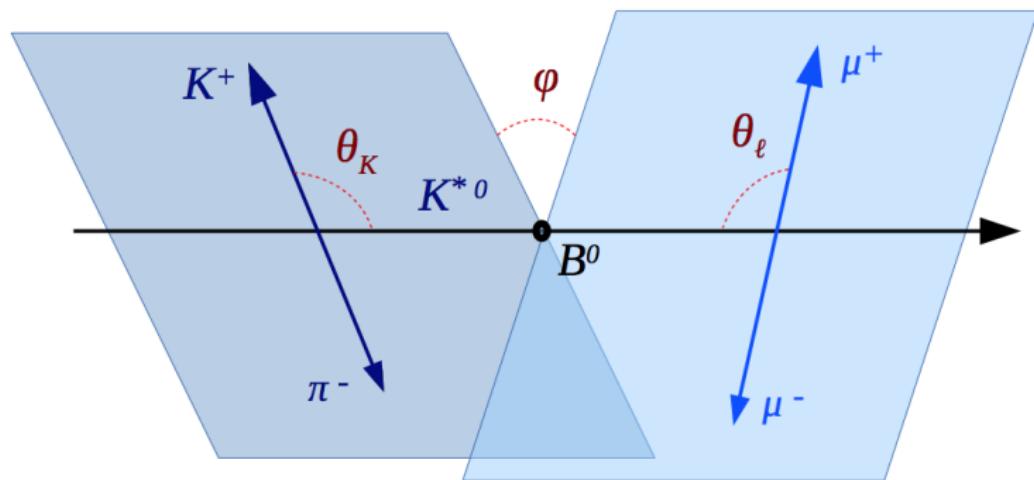
[G. Hiller, M. Schmaltz, Phys.Rev. D90, 054014 (2014)]

- A scalar leptoquark ϕ transforming as $(3, 1, -1/3)$ under the SM gauge groups, with $m_\phi = O(1)$ TeV and $O(1)$ couplings
[M. Bauer, M. Neubert, arxiv 1511.01900 (2015)]
- Anomalies in B decays and $U(2)$ flavor symmetry
[R. Barbieri *et al.*, Eur.Phys. J. C (2016) 76]

Angular analysis in $B \rightarrow K^* \mu^+ \mu^-$

$$B^0 \rightarrow K^{*0} (\rightarrow K^+ \pi^-) \mu^+ \mu^-$$

- Decay is $P \rightarrow VV'$ (since $K^*(892)^0$ is $J^P = 1^-$).
- System fully described by q^2 and three angles $\vec{\Omega} = (\cos \theta_l, \cos \theta_K, \phi)$



Observables in $B \rightarrow K^* \mu^+ \mu^-$

$$\begin{aligned} \frac{1}{d(\Gamma + \bar{\Gamma})/dq^2} \frac{d^4(\Gamma + \bar{\Gamma})}{dq^2 d\vec{\Omega}} = & \frac{9}{32\pi} \left[\frac{3}{4}(1 - F_L) \sin^2 \theta_K + F_L \cos^2 \theta_K \right. \\ & + \frac{1}{4}(1 - F_L) \sin^2 \theta_K \cos 2\theta_l \\ & - F_L \cos^2 \theta_K \cos 2\theta_l + S_3 \sin^2 \theta_K \sin^2 \theta_l \cos 2\phi \\ & + S_4 \sin 2\theta_K \sin 2\theta_l \cos \phi + S_5 \sin 2\theta_K \sin \theta_l \cos \phi \\ & + \frac{4}{3}A_{FB} \sin^2 \theta_K \cos \theta_l + S_7 \sin 2\theta_K \sin \theta_l \sin \phi \\ & \left. + S_8 \sin 2\theta_K \sin 2\theta_l \sin \phi + S_9 \sin^2 \theta_K \sin^2 \theta_l \sin 2\phi \right]. \end{aligned}$$

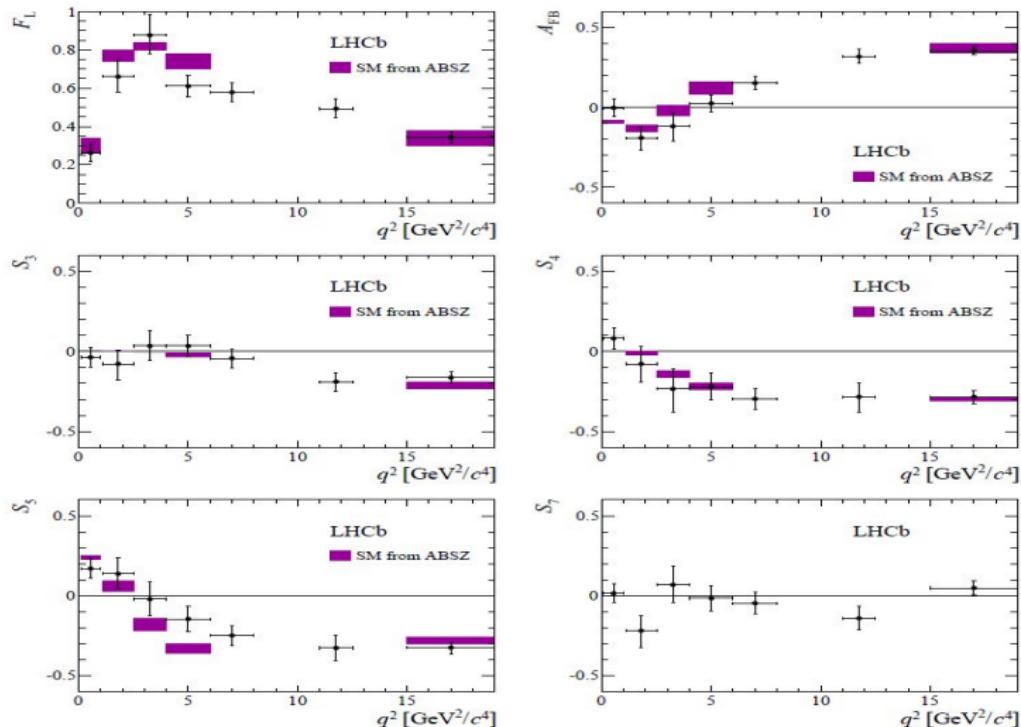
Optimized variables with reduced FF uncertainties

$$P_1 = 2S_3/(1 - F_L); \quad P_2 = 2A_{FB}/3(1 - F_L); \quad P_3 = -S_9/(1 - F_L)$$

$$P_{4,5,6,8} = S_{4,5,6,8}/\sqrt{F_L(1 - F_L)}$$

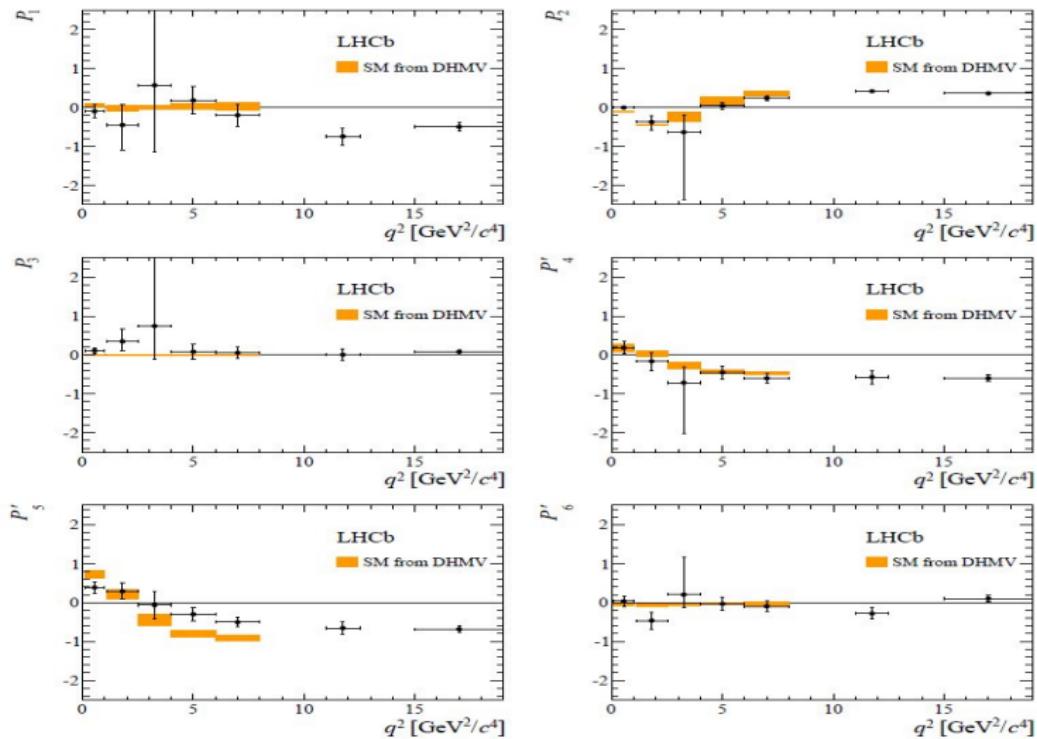
Latest Update from the LHCb: LHCb-Paper-2015-051

SM Estimates: Altmannshofer & Straub, EPJC 75 (2015) 382



Analysis of the optimised angular variables: LHCb-Paper-2015-051

SM Estimates: Descotes-Genon, Hofer, Matias, Virto; JHEP 12 (2014) 125



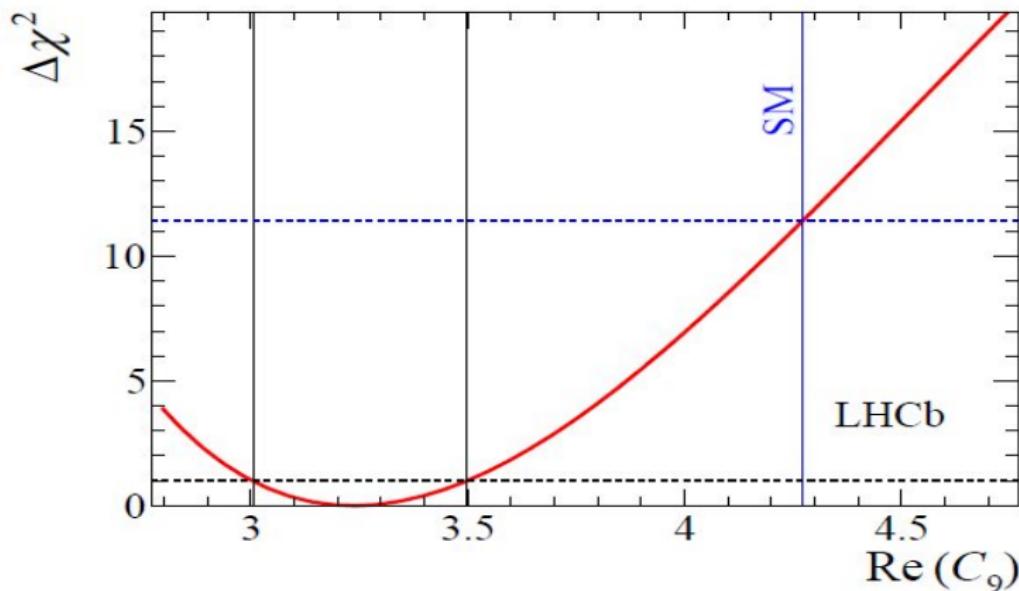
Recent Updates: Pull on the SM [Altmannshofer, Straub (2015)]

W. Altmannshofer & D.M. Straub, EPJ C75 (2015) 8, 382

Decay	obs.	q^2 bin	SM pred.	measurement	pull
$\bar{B}^0 \rightarrow \bar{K}^{*0} \mu^+ \mu^-$	$10^7 \frac{d\text{BR}}{dq^2}$	[2, 4.3]	0.44 ± 0.07	0.29 ± 0.05	LHCb +1.8
$\bar{B}^0 \rightarrow \bar{K}^{*0} \mu^+ \mu^-$	$10^7 \frac{d\text{BR}}{dq^2}$	[16, 19.25]	0.47 ± 0.06	0.31 ± 0.07	CDF +1.8
$\bar{B}^0 \rightarrow \bar{K}^{*0} \mu^+ \mu^-$	F_L	[2, 4.3]	0.81 ± 0.02	0.26 ± 0.19	ATLAS +2.9
$\bar{B}^0 \rightarrow \bar{K}^{*0} \mu^+ \mu^-$	F_L	[4, 6]	0.74 ± 0.04	0.61 ± 0.06	LHCb +1.9
$\bar{B}^0 \rightarrow \bar{K}^{*0} \mu^+ \mu^-$	S_5	[4, 6]	-0.33 ± 0.03	-0.15 ± 0.08	LHCb -2.2
$B^- \rightarrow K^{*-} \mu^+ \mu^-$	$10^7 \frac{d\text{BR}}{dq^2}$	[4, 6]	0.54 ± 0.08	0.26 ± 0.10	LHCb +2.1
$\bar{B}^0 \rightarrow \bar{K}^0 \mu^+ \mu^-$	$10^8 \frac{d\text{BR}}{dq^2}$	[0.1, 2]	2.71 ± 0.50	1.26 ± 0.56	LHCb +1.9
$\bar{B}^0 \rightarrow \bar{K}^0 \mu^+ \mu^-$	$10^8 \frac{d\text{BR}}{dq^2}$	[16, 23]	0.93 ± 0.12	0.37 ± 0.22	CDF +2.2
$B_s \rightarrow \phi \mu^+ \mu^-$	$10^7 \frac{d\text{BR}}{dq^2}$	[1, 6]	0.48 ± 0.06	0.23 ± 0.05	LHCb +3.1
$B \rightarrow X_s e^+ e^-$	10^6 BR	[14.2, 25]	0.21 ± 0.07	0.57 ± 0.19	BaBar -1.8

Tension on the SM from $B \rightarrow K^* \mu^+ \mu^-$ measurements

- Perform χ^2 fit of the measured observables $F_L, A_{FB}, S_3, \dots, S_9$
- Float the generic vector coupling, i.e., $\text{Re}(C_9)$
- Best fit: $\Delta\text{Re}(C_9) = \text{Re}(C_9)^{\text{LHCb}} - \text{Re}(C_9)^{\text{SM}} = -1.04 \pm 0.25$



Effective Weak $b \rightarrow d$ Hamiltonian

$$H_{\text{eff}}^{(b \rightarrow d)} = -\frac{4G_F}{\sqrt{2}} \left[V_{tb}^* V_{td} \sum_{i=1}^{10} C_i(\mu) \mathcal{O}_i(\mu) + V_{ub}^* V_{ud} \sum_{i=1}^2 C_i(\mu) \left(\mathcal{O}_i(\mu) - \mathcal{O}_i^{(u)}(\mu) \right) \right] + \text{h.c.}$$

- G_F (Fermi constant), $C_i(\mu)$ (Wilson coefficients), and $\mathcal{O}_i(\mu)$ (dimension-six operators) are the same (modulo $s \rightarrow d$) as in $H_{\text{eff}}^{(b \rightarrow s)}$
- CKM structure of the matrix elements more interesting in $H_{\text{eff}}^{(b \rightarrow d)}$, as $V_{tb}^* V_{td} \sim V_{ub}^* V_{ud} \sim \lambda^3$ are of the same order in $\lambda = \sin \theta_{12}$
- Anticipate sizable CP-violating asymmetries in $b \rightarrow d$ transitions compared to $b \rightarrow s$

Operator Basis

- Tree operators

$$\mathcal{O}_1 = \left(\bar{d}_L \gamma_\mu T^A c_L \right) \left(\bar{c}_L \gamma^\mu T^A b_L \right), \quad \mathcal{O}_2 = \left(\bar{d}_L \gamma_\mu c_L \right) \left(\bar{c}_L \gamma^\mu b_L \right)$$

$$\mathcal{O}_1^{(u)} = \left(\bar{d}_L \gamma_\mu T^A u_L \right) \left(\bar{u}_L \gamma^\mu T^A b_L \right), \quad \mathcal{O}_2^{(u)} = \left(\bar{d}_L \gamma_\mu u_L \right) \left(\bar{u}_L \gamma^\mu b_L \right)$$

- Dipole operators

$$\mathcal{O}_7 = \frac{e m_b}{g_{\text{st}}^2} \left(\bar{d}_L \sigma^{\mu\nu} b_R \right) F_{\mu\nu}, \quad \mathcal{O}_8 = \frac{m_b}{g_{\text{st}}} \left(\bar{d}_L \sigma^{\mu\nu} T^A b_R \right) G_{\mu\nu}^A$$

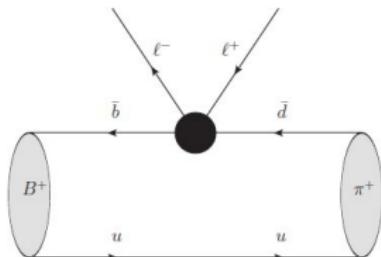
- Semileptonic operators

$$\mathcal{O}_9 = \frac{e^2}{g_{\text{st}}^2} \left(\bar{d}_L \gamma^\mu b_L \right) \sum_\ell (\bar{\ell} \gamma_\mu \ell), \quad \mathcal{O}_{10} = \frac{e^2}{g_{\text{st}}^2} \left(\bar{d}_L \gamma^\mu b_L \right) \sum_\ell (\bar{\ell} \gamma_\mu \gamma_5 \ell)$$

$B \rightarrow \pi$ transition matrix elements

Momentum transfer:

$$q = p_B - p_\pi = p_{\ell^+} + p_{\ell^-}$$



The Feynman diagram for the $B^+ \rightarrow \pi^+ \ell^+ \ell^-$ decay.

$$\langle \pi(p_\pi) | \bar{b} \gamma^\mu d | B(p_B) \rangle = f_+(q^2) \left(p_B^\mu + p_\pi^\mu \right) + \left[f_0(q^2) - f_+(q^2) \right] \frac{m_B^2 - m_\pi^2}{q^2} q^\mu$$

$$\langle \pi(p_\pi) | \bar{b} \sigma^{\mu\nu} q_\nu d | B(p_B) \rangle = \frac{i f_T(q^2)}{m_B + m_\pi} \left[\left(p_B^\mu + p_\pi^\mu \right) q^2 - q^\mu \left(m_B^2 - m_\pi^2 \right) \right]$$

- Dominant theoretical uncertainty is in the form factors $f_+(q^2), f_0(q^2), f_T(q^2)$; require non-perturbative techniques, such as Lattice QCD
- Their determination is the main focus of the theory

$B \rightarrow \pi \ell^+ \nu_\ell$ decay

$$\langle \pi | \bar{u} \gamma^\mu b | B \rangle = f_+(q^2) \left(p_B^\mu + p_\pi^\mu - \frac{m_B^2 - m_\pi^2}{q^2} q^\mu \right) + f_0(q^2) \frac{m_B^2 - m_\pi^2}{q^2} q^\mu$$

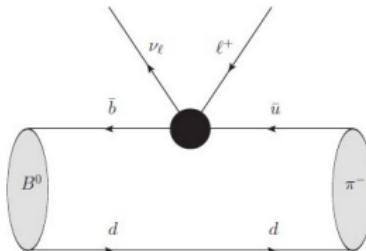
- $f_0(q^2)$ contribution is suppressed by m_ℓ^2/m_B^2 for $\ell = e, \mu$
- Differential decay width

$$\frac{d\Gamma}{dq^2}(B^0 \rightarrow \pi^- \ell^+ \nu_\ell) = \frac{G_F^2 |V_{ub}|^2}{192\pi^3 m_B^3} \lambda^{3/2}(q^2) |f_+(q^2)|^2$$

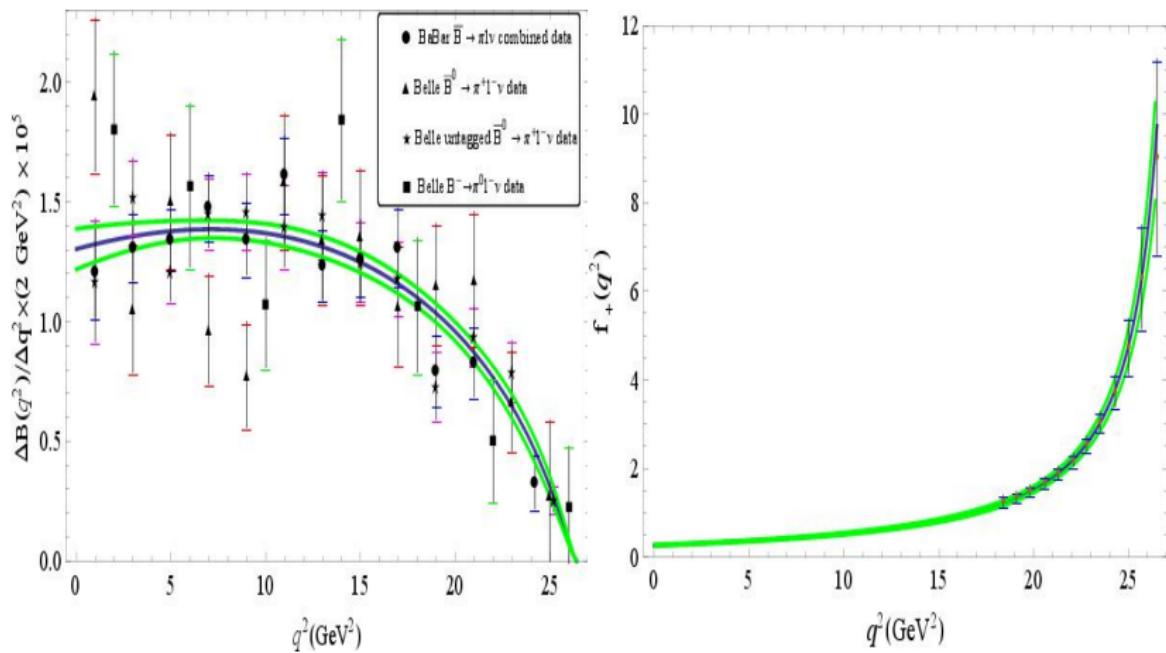
$$\text{with } \lambda(q^2) = (m_B^2 + m_\pi^2 - q^2)^2 - 4m_B^2 m_\pi^2$$

- Assuming Isospin symmetry: $f_+(q^2)$ and $f_0(q^2)$ in charged current $B \rightarrow \pi \ell \nu_\ell$ and neutral current $B \rightarrow \pi \ell^+ \ell^-$ decays are equal
- Global fit of the CKM matrix element
[PDG, 2012]

$$|V_{ub}| = (3.51^{+0.15}_{-0.14}) \times 10^{-3}$$



Fits of the data on $B \rightarrow \pi^+ \ell^- \nu_\ell$ yield $f_+(q^2)$



Heavy-Quark Symmetry (HQS) relations

- Including symmetry-breaking corrections, Heavy Quark Symmetry relates $f_+(q^2)$, $f_0(q^2)$ and $f_T(q^2)$ (for $q^2/m_b^2 \ll 1$) [Beneke, Feldmann (2000)]

$$f_0(q^2) = \left(\frac{m_B^2 + m_\pi^2 - q^2}{m_B^2} \right) \left[\left(1 + \frac{\alpha_s(\mu) C_F}{4\pi} (2 - 2L(q^2)) \right) f_+(q^2) \right.$$

$$\left. + \frac{\alpha_s(\mu) C_F}{4\pi} \frac{m_B^2(q^2 - m_\pi^2)}{(m_B^2 + m_\pi^2 - q^2)^2} \Delta F_\pi \right],$$

$$f_T(q^2) = \left(\frac{m_B + m_\pi}{m_B} \right) \left[\left(1 + \frac{\alpha_s(\mu) C_F}{4\pi} \left(\ln \frac{m_b^2}{\mu^2} + 2L(q^2) \right) \right) f_+(q^2) \right.$$

$$\left. - \frac{\alpha_s(\mu) C_F}{4\pi} \frac{m_B^2}{m_B^2 + m_\pi^2 - q^2} \Delta F_\pi \right],$$

$$L(q^2) = \left(1 + \frac{m_B^2}{m_\pi^2 - q^2} \right) \ln \left(1 + \frac{m_\pi^2 - q^2}{m_B^2} \right), \quad \Delta F_\pi = \frac{8\pi^2 f_B f_\pi}{N_c m_B} \left\langle l_+^{-1} \right\rangle_+ \left\langle \bar{u}^{-1} \right\rangle_\pi$$

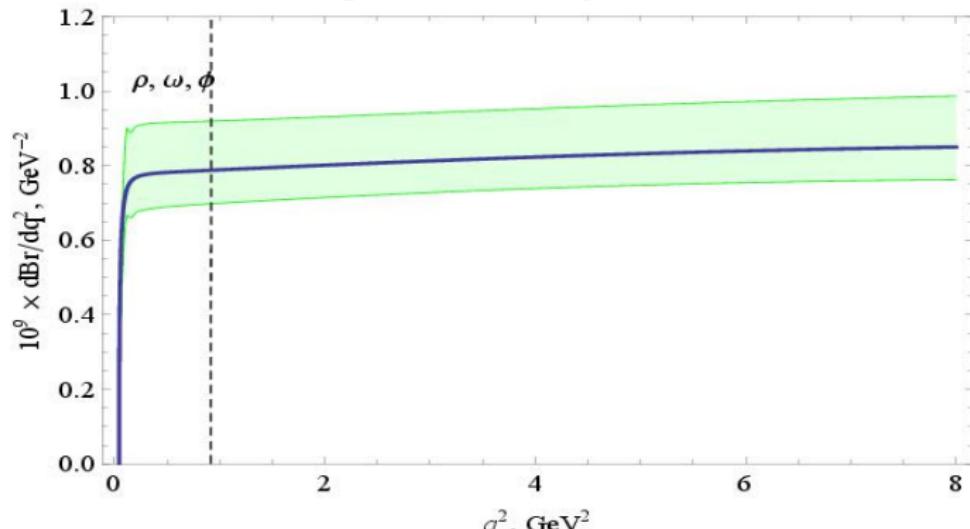
$B^\pm \rightarrow \pi^\pm \ell^+ \ell^-$ at large hadronic recoil ($q^2/m_b^2 \ll 1$)

[AA, A. Parkhomenko, A. Rusov; Phys. Rev. D89 (2014) 094021]

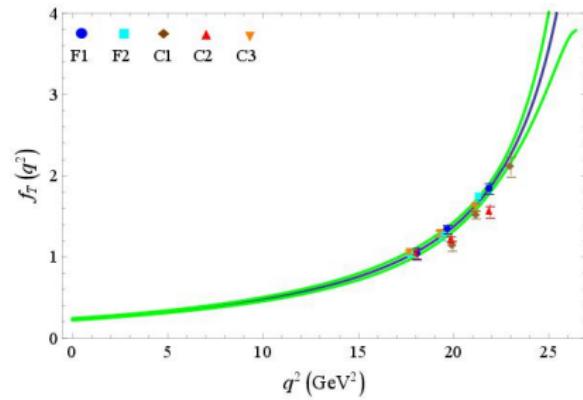
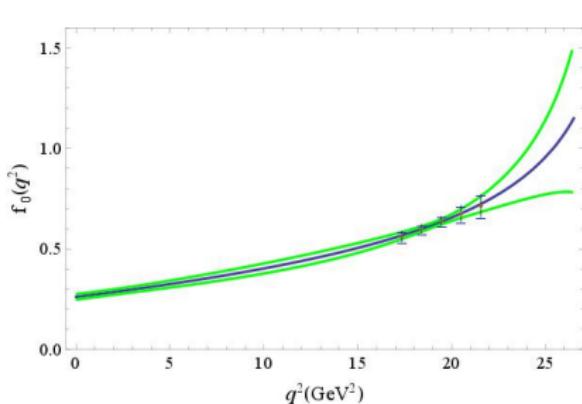
- Partially integrated branching fractions for $B^\pm \rightarrow \pi^\pm \ell^+ \ell^-$

$$\text{BR}_{\text{SM}}(B^+ \rightarrow \pi^+ \mu^+ \mu^-; 1 \text{ GeV}^2 \leq q^2 \leq 8 \text{ GeV}^2) = (0.57^{+0.07}_{-0.05}) \times 10^{-8}$$

- Dimuon invariant mass spectrum at large hadronic recoil



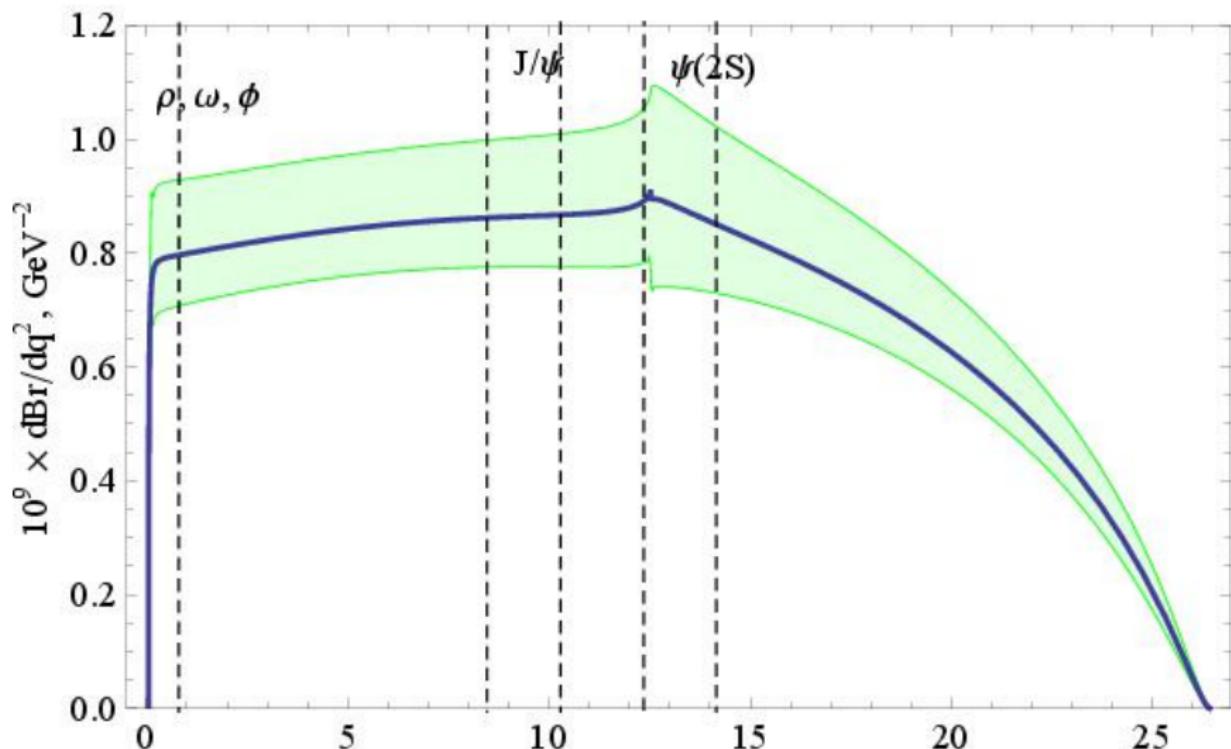
Determination of $f_0^{B\pi}(q^2)$ and $f_T^{B\pi}(q^2)$ and comparison with Lattice QCD



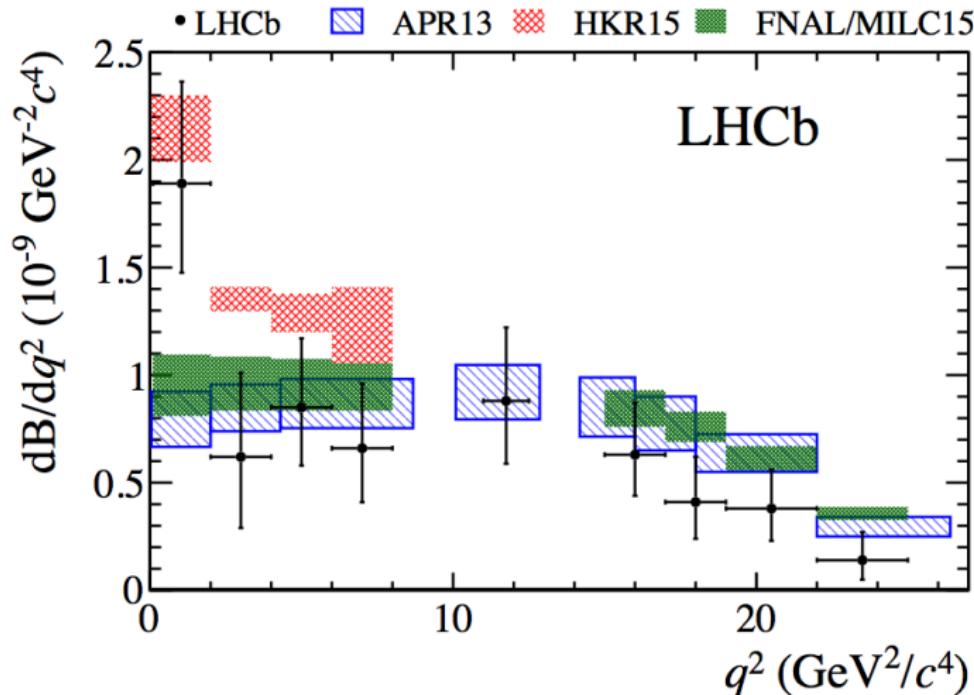
- FFs are obtained by the z -expansion [Boyd, Grinstein, Lebed] and constraints from data in low- q^2
- Lattice data (in high- q^2) are obtained by the HPQCD Collab.
 - for $f_0^{B\pi}(q^2)$ from [arXiv:hep-lat/0601021]
 - for $f_T^{B\pi}(q^2)$ from [arXiv:1310.3207]
- In almost the entire q^2 -domain, the form factors are now determined accurately. Recent Fermilab/MILC lattice results are in agreement

$B^+ \rightarrow \pi^+ \mu^+ \mu^-$ in the entire range of q^2

[AA, A. Parkhomenko, A. Rusov; Phys. Rev. D89 (2014) 094021]



Dimuon invariant mass spectrum in $B \rightarrow \pi \ell^+ \ell^-$



- In excellent agreement with the APR2013 predictions, as well as with the Lattice results

SM vs. experimental data

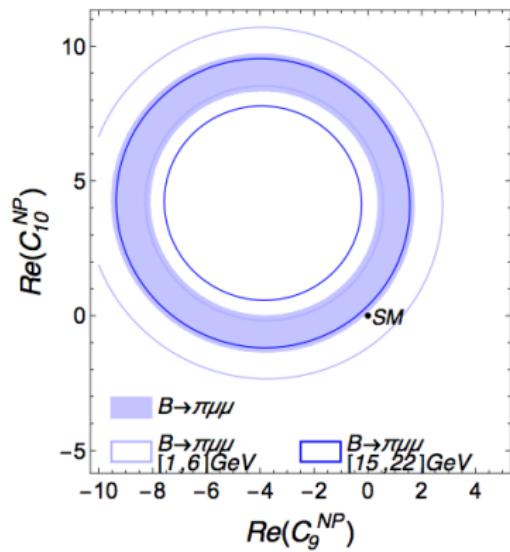
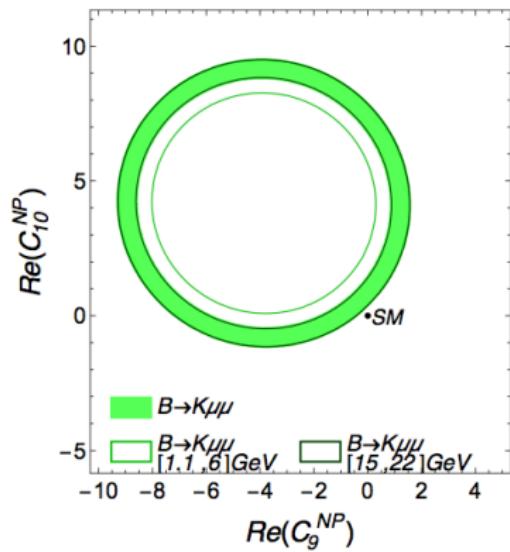
- SM theoretical estimate of the total branching fraction
[AA, A. Parkhomeno, A. Rusov; Phys. Rev. D89 (2014) 094021] :

$$\text{BR}_{\text{SM}}(B^\pm \rightarrow \pi^\pm \mu^+ \mu^-) = (1.88^{+0.32}_{-0.21}) \times 10^{-8}$$

- Uncertainty from the form factors is now reduced greatly.
Residual theoretical uncertainty is mainly from the scale dependence and the CKM matrix elements
- LHCb has measured the BR and dimuon invariant mass distribution in $B^\pm \rightarrow \pi^\pm \mu^+ \mu^-$ based on 3 fb^{-1} integrated luminosity
[LHCb-PAPER-2015-035; arXiv:1509.00414] :
$$\text{BR}_{\text{exp}}(B^\pm \rightarrow \pi^\pm \mu^+ \mu^-) = (1.83 \pm 0.24(\text{stat}) \pm 0.05(\text{syst})) \times 10^{-8}$$
- Excellent agreement with SM-based APR2013-theory within errors, but significant improvement expected from the future analysis

Determination of Wilson Coeffs. from $B \rightarrow (\pi/K)\mu^+\mu^-$

[Fermilab/MILC, arxiv:1510.02349]



$B_s \rightarrow \mu^+ \mu^-$ in the SM & BSM

- Effective Hamiltonian

$$\mathcal{H}_{\text{eff}} = -\frac{G_F \alpha}{\sqrt{2}\pi} V_{ts}^* V_{tb} \sum_i [C_i(\mu) \mathcal{O}_i(\mu) + C'_i(\mu) \mathcal{O}'_i(\mu)]$$

- Operators: \mathcal{O}_i (SM) & \mathcal{O}'_i (BSM)

$$\mathcal{O}_{10} = (\bar{s}_\alpha \gamma^\mu P_L b_\alpha) (\bar{l} \gamma_\mu \gamma_5 l), \quad \mathcal{O}'_{10} = (\bar{s}_\alpha \gamma^\mu P_R b_\alpha) (\bar{l} \gamma_\mu \gamma_5 l)$$

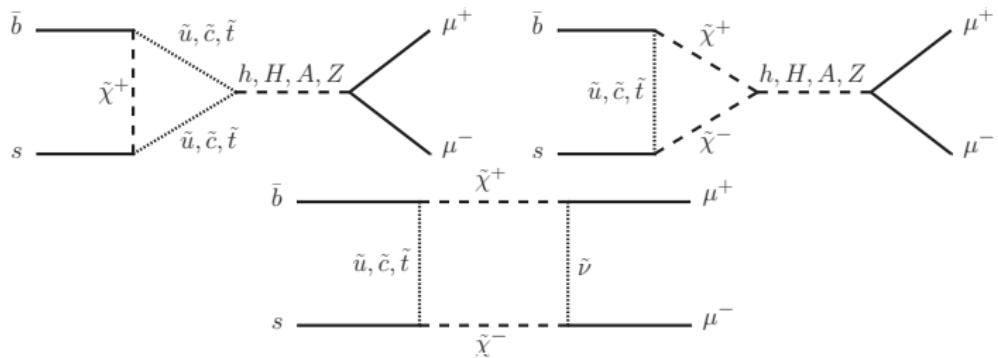
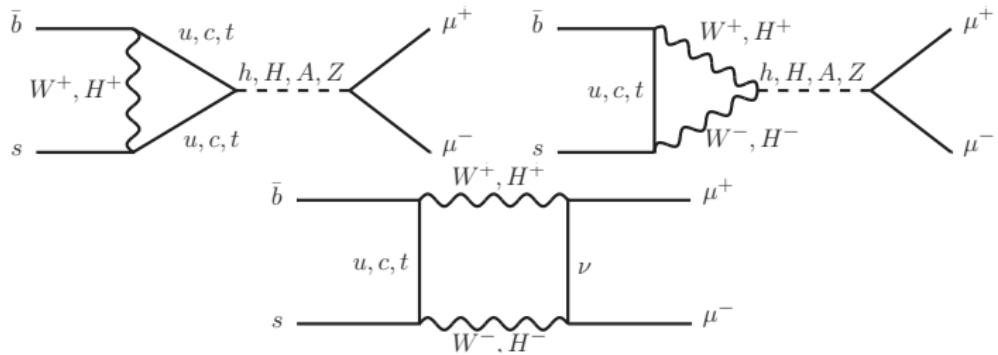
$$\mathcal{O}_S = m_b (\bar{s}_\alpha P_R b_\alpha) (\bar{l} l), \quad \mathcal{O}'_S = m_s (\bar{s}_\alpha P_L b_\alpha) (\bar{l} l)$$

$$\mathcal{O}_P = m_b (\bar{s}_\alpha P_R b_\alpha) (\bar{l} \gamma_5 l), \quad \mathcal{O}'_P = m_s (\bar{s}_\alpha P_L b_\alpha) (\bar{l} \gamma_5 l)$$

$$\begin{aligned} \text{BR} (\bar{B}_s \rightarrow \mu^+ \mu^-) &= \frac{G_F^2 \alpha^2 m_{B_s}^2 f_{B_s}^2 \tau_{B_s}}{64\pi^3} |V_{ts}^* V_{tb}|^2 \sqrt{1 - 4\hat{m}_\mu^2} \\ &\times \left[(1 - 4\hat{m}_\mu^2) |F_S|^2 + |F_P + 2\hat{m}_\mu^2 F_{10}|^2 \right] \end{aligned}$$

$$F_{S,P} = m_{B_s} \left[\frac{C_{S,P} m_b - C'_{S,P} m_s}{m_b + m_s} \right], \quad F_{10} = C_{10} - C'_{10}, \quad \hat{m}_\mu = m_\mu / m_{B_s}$$

Leading diagrams for $B_s \rightarrow \mu^+ \mu^-$ in SM, 2HDM & MSSM



$B_s \rightarrow \mu^+ \mu^-$ in the SM

- SM predictions depend somewhat on the input parameters [Blanke & Buras, arxiv: 1602.04021; Bobeth et al., Phys. Rev. Lett. 112, 101801 (2014)]

$$\mathcal{B}(\bar{B}_s \rightarrow \mu^+ \mu^-) = (3.65 \pm 0.06) \times 10^{-9} \left(\frac{m_t(m_t)}{163.5 \text{ GeV}} \right)^{3.02} \left(\frac{\alpha_s(M_Z)}{0.1184} \right)^{0.032} R_s$$

$$R_s = \left(\frac{f_{B_s}}{227.7 \text{ MeV}} \right)^2 \left(\frac{\tau_{B_s}}{1.516 \text{ ps}} \right) \left(\frac{0.938}{r(y_s)} \right) \left(\frac{|V_{ts}|}{41.5 \times 10^{-3}} \right)^2$$

- $\Delta\Gamma_s$ effects are taken into account through $r(y_s) = 1 - y_s$, with $y_s = \tau_{B_s} \Delta\Gamma_s / 2 = 0.062 \pm 0.005$

$$\mathcal{B}(\bar{B}_s \rightarrow \mu^+ \mu^-) = (3.78 \pm 0.23) [3.65 \pm 0.23] \times 10^{-9}$$

$$\mathcal{B}(B_d \rightarrow \mu^+ \mu^-) = (1.06 \pm 0.02) \times 10^{-10} \left(\frac{m_t(m_t)}{163.5 \text{ GeV}} \right)^{3.02} \left(\frac{\alpha_s(M_Z)}{0.1184} \right)^{0.032} R_d$$

$$R_d = \left(\frac{f_{B_d}}{190.5 \text{ MeV}} \right)^2 \left(\frac{\tau_{B_d}}{1.519 \text{ ps}} \right) \left(\frac{|V_{td}|}{8.8 \times 10^{-3}} \right)^2$$

$$\mathcal{B}(B_d \rightarrow \mu^+ \mu^-) = (1.02 \pm 0.08) [1.06 \pm 0.09] \times 10^{-10}$$

Compatibility of the SM with $B_{(s)}^0 \rightarrow \mu^+ \mu^-$ measurements

$B_s^0 \rightarrow \mu^+ \mu^-$

Combined analysis with CMS

[Nature 522(2015)]

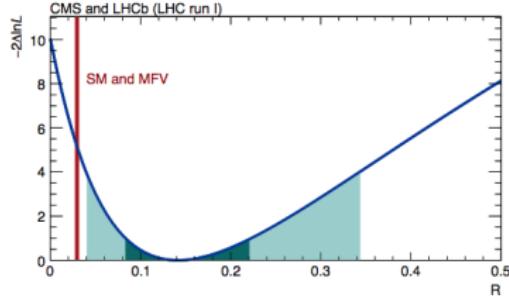
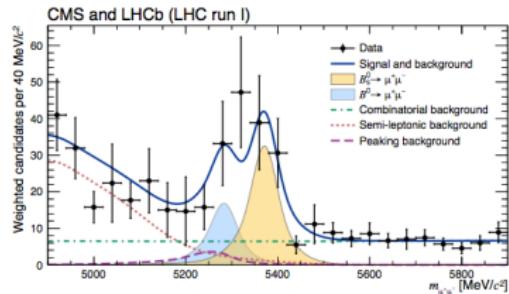
- ▶ First observation of $B_s^0 \rightarrow \mu^+ \mu^-$ and evidence for $B^0 \rightarrow \mu^+ \mu^-$.
 - ▶ 6.2σ and 3.2σ respectively.

- ▶ Measurement of branching fractions and ratio of branching fractions.

$$\mathcal{B} [B_s^0 \rightarrow \mu^+ \mu^-] = 2.8^{+0.7}_{-0.6} \times 10^{-9}$$

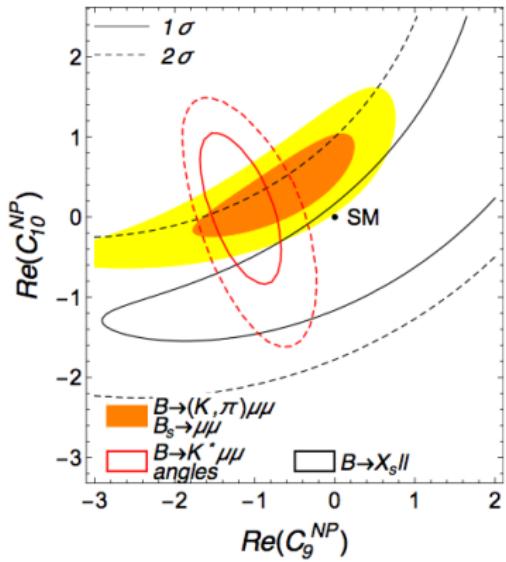
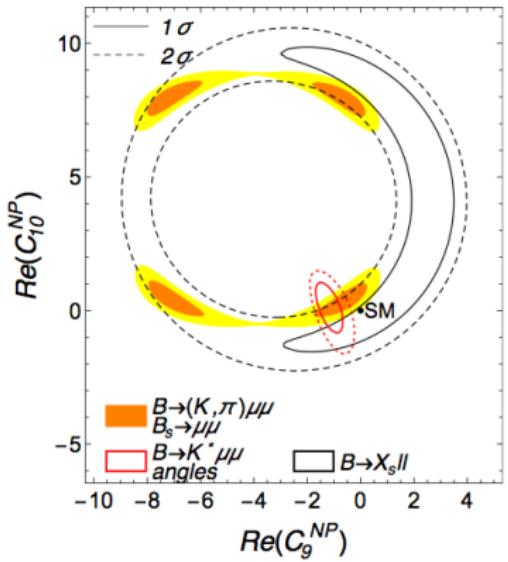
$$\mathcal{B} [B^0 \rightarrow \mu^+ \mu^-] = 3.9^{+1.6}_{-1.4} \times 10^{-10}$$

- ▶ Ratio found to be compatible with SM to 2.3σ .

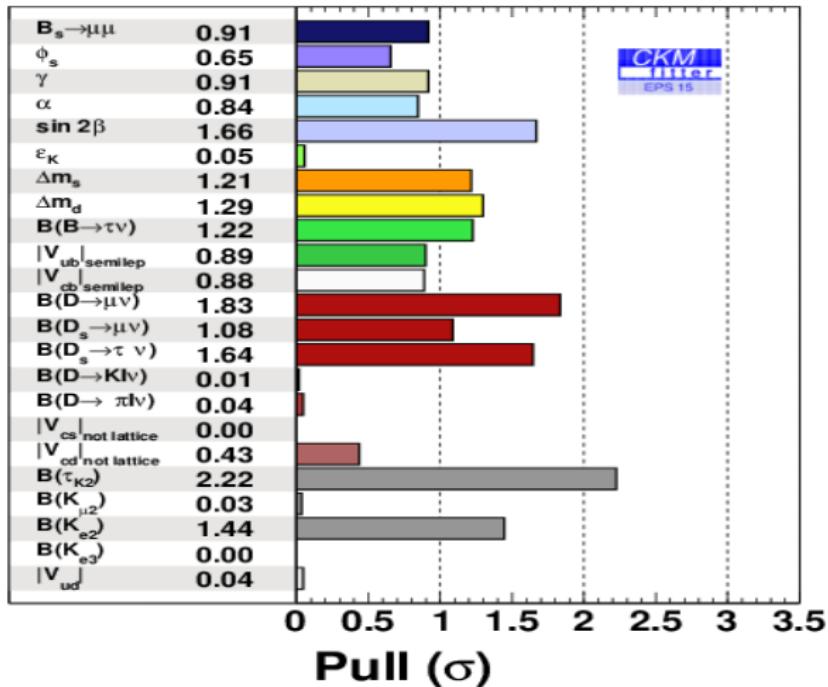


Test of the SM in Semileptonic B -decays and $B_s \rightarrow \mu^+ \mu^-$

[Fermilab/MILC, arxiv:1510.02349]



Current Pulls on the SM Parameters from Unitarity Fits [CKMfitter 2015]



Summary and outlook

- Experiments carried out during the last 50 years in flavour physics have led to a consistent theoretical framework - the CKM theory
- In particular, all observed CP violations in the quark sector are described by the single Kobayashi-Maskawa phase
- FCNC processes in the SM are governed by the GIM mechanism and have been measured in a number of K , D , D_s , B and B_s decays
- In particular, rare B -decays have been measured over 6 orders of magnitude and are found to be compatible with the SM, in general
- There is some tension on the SM in a number of rare B decays, typically $2 - 4 \sigma$; whether this is due to New Physics or QCD remains to be seen
- FCNC processes remain potentially very promising to search for physics beyond the SM, and they complement direct searches of BSM physics
- We look forward to improved theory and even more precise measurements at the LHC and the Super-B factory at KEK