Lectures on Particle Cosmology

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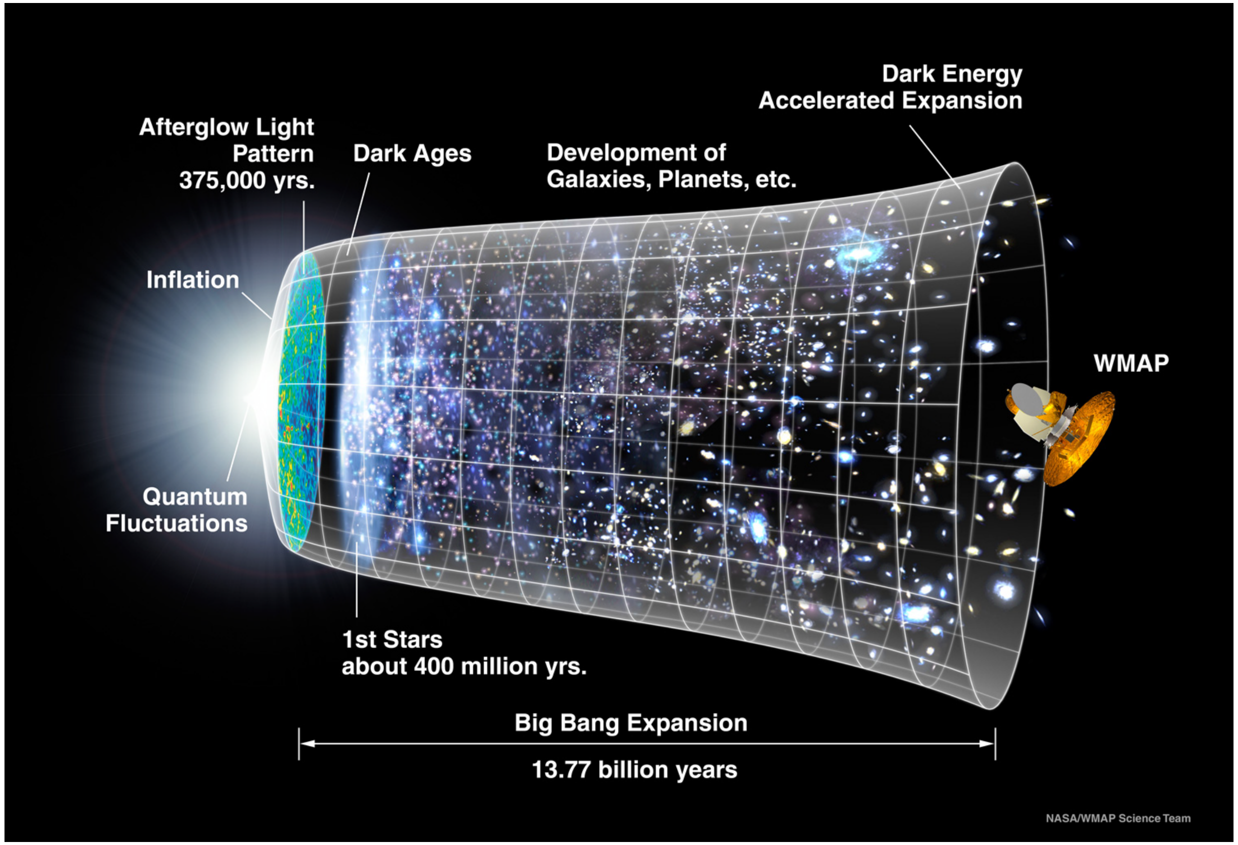
Outline

0. Introduction
1. Big-Bang Cosmology: Basic Issues
2. Big-Bang Nucleosynthesis
3. Inflation
4. Dark Matter

0. Introduction

Aim of particle-cosmology:

To understand the history of the universe



Cosmic history is determined once the followings are fixed:

- Equations for cosmic evolution
  - General relativity (or some other gravity theories)
  - Particle-physics models
- Initial conditions
  - Baryon number density
  - Dark matter density
  - Density perturbations

It is better to have natural explanations for initial conditions

$\Rightarrow$ Of course, we need an initial condition which gives thermal history consistent with observations

Observational fact 1: Universe is (almost) homogeneous

- "Small-scale" objects exist
  - Stars ($r \sim 10^6$ km)
  - Galaxies ($r \sim 10$ kpc)
  - Clusters ($r \sim 1 - 10$ Mpc)
  - C.f. Horizon scale: ~ 3000 Mpc
- For large scale, the universe is (almost) homogeneous
  - $\Delta T/T \sim O(10^{-5})$

Observational fact 2: Universe is expanding

- Red-shift of the absorption/emission lines are observed
- Einstein equation predicts expanding universe
- In the past, the universe had higher temperature and higher density

Observational fact 3: Universe is filled with radiation

- CMB is first observed by Penzias & Wilson in 1964
- $T_{\text{CMB}} = 2.735 \pm 0.06$ K
  [COBE]
- Anisotropy in the CMB is now confirmed by various experiments

Basic picture (“initial condition” of our universe):

- In the past, the universe was hotter and denser
- Universe was (almost) homogeneous from an early epoch
- There exist small density perturbations
  - $\delta\rho/\rho \sim O(10^{-5})$
- There exist various “components” in the universe
  - Radiations
  - Baryons (proton, D, ^{4}He, $\cdots$)
  - Dark matter
  - Dark energy (cosmological “constant”)

Questions

- How was the homogeneous universe formed?
- How were the stars and galaxies formed from the homogeneous universe?
  - What is the origin of the density perturbation?
- How were the matters generated in the early universe
  - Dark matter
  - Baryons
  - Dark energy
- Why is the dark energy so small (but finite)?
- $\cdots$

For cosmology, particle physics is very important

- High temperature $\Leftrightarrow$ High energy
- Exotic particles may significantly affect thermal history
  - Particle-physics candidate of the dark matter
  - Inflation
  - Baryogenesis
  - ...
- Inflation
- Birth of the universe: Quantum gravity?

1. Big-Bang Cosmology: Basic Issues

We need to solve Einstein equation:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G_{\mathsf{N}} T_{\mu\nu} = \frac{1}{M_{\mathsf{Pl}}^2} T_{\mu\nu}$$

$T_{\mu\nu}$: Energy-momentum tensor

$M_{\mathsf{Pl}} \simeq 2.4 \times 10^{18}$ GeV: Reduced Planck scale

Our universe is (almost) isotropic and homogeneous:

$\Rightarrow$ Usually, we adopt Robertson-Walker metric

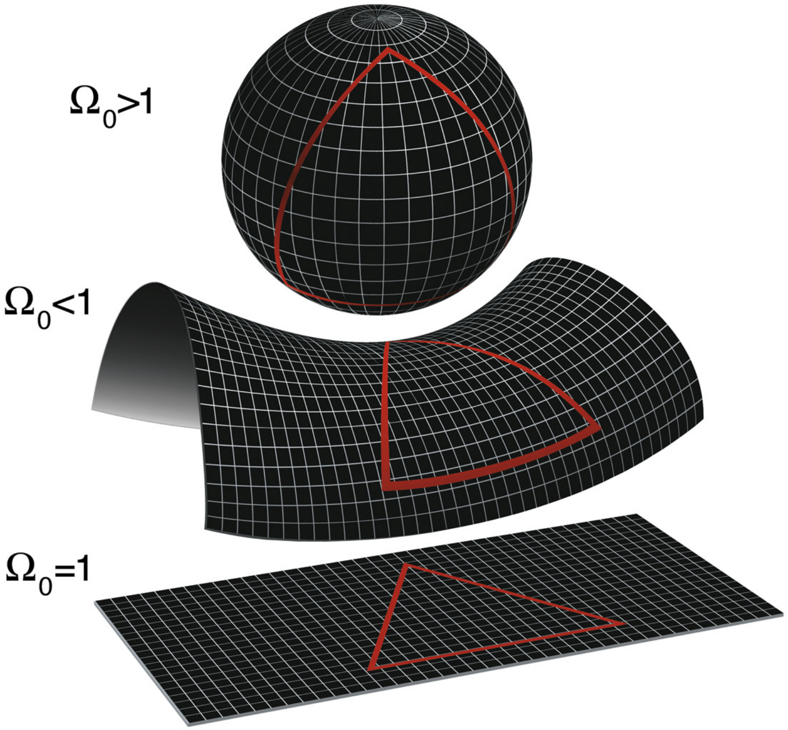
Robertson-Walker metric

$$ds^2 = dt^2 - a^2(t) \left[\frac{dr^2}{1 - Kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \right]$$

a: Scale factor

r: Comoving radius (dimensionless)

Geometry of the universe



- $K > 0$: Closed
- $K < 0$: Open
- $K = 0$: Flat

Observational fact: our universe is (almost) flat

$\Rightarrow$ Hereafter, I mostly discuss the case with $K = 0$

Comment on the spacial curvature:

$$ds^2 \simeq dt^2 - a^2(t)\left[\left(1 + Kr^2 + \cdots\right)dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right]$$

The spacial curvature becomes important for $r \sim |K|^{-1/2}$

$\Rightarrow$ Radius of the curvature of the space: $R_{\text{phys}} = |K|^{-1/2} a$

With rescaling r and a, we can take $K = -1$, 0, or $+1$

Observational fact: our universe is (almost) flat

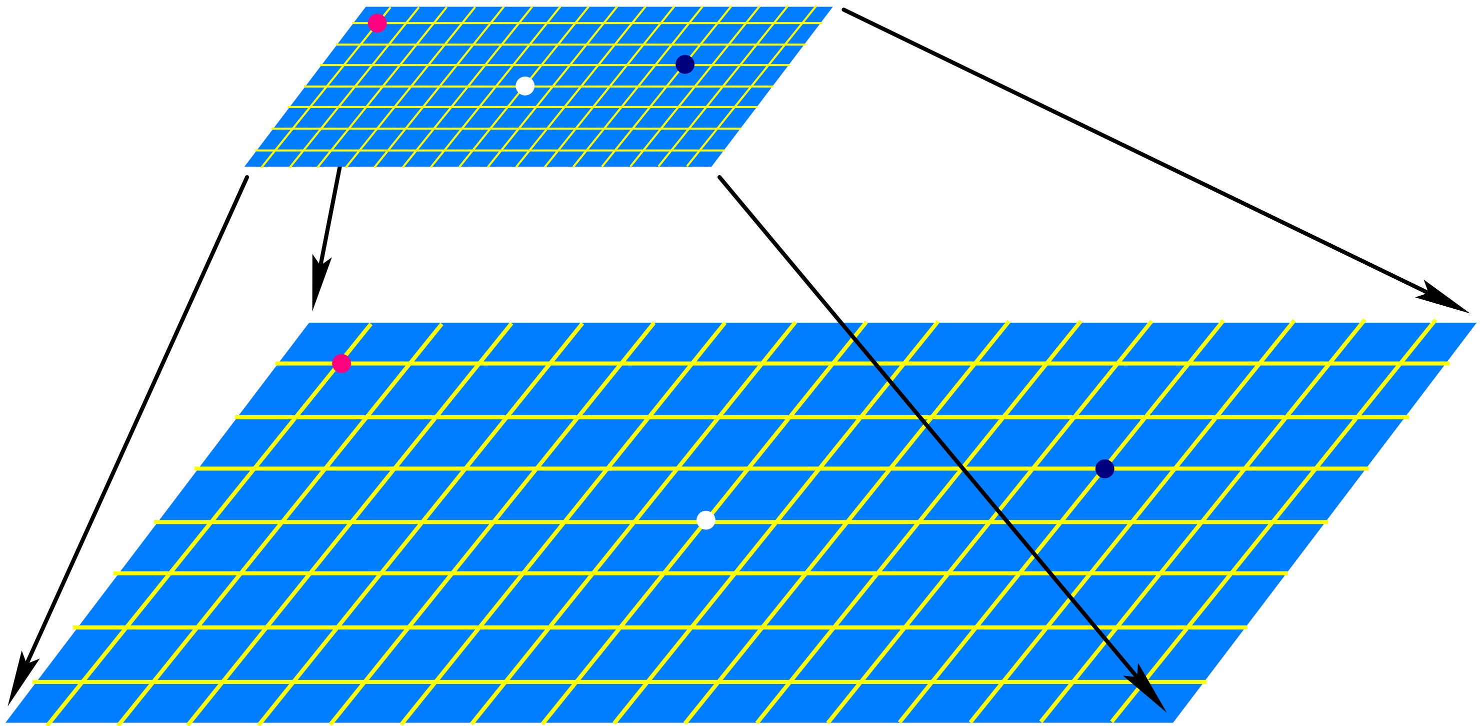
$\Rightarrow$ For the length scale of our interest, $Kr^2 \ll 1$

$\Rightarrow$ Taking $|K| = 1$, the scale factor a is much larger than the present horizon size ($a_{\text{now}} \gg H_{\text{now}}^{-1}$)

Spacial coordinate with RW metric: comoving coordinate

Physical distance: $dl_{\mathsf{phys}} \sim a(t)dr$

Comoving coordinate expands with the universe



Physical distance from the origin

$$l_{\text{phys}} = a(t) \int \frac{dr}{1 - Kr^2}$$

$\Rightarrow$ For two objects at "rest," physical distance changes with time

$\Rightarrow$ Three-momentum red-shifts as the universe expands

Comoving momentum (for the case of $K = 0$): $\vec{k}$

- Comoving momentum is constant of time for free particles

- Physical momentum: $\vec{k}_{\text{phys}} = a^{-1}\vec{k}$

Energy momentum tensor (for RW metric)

$$T_{\mu\nu} = \mathrm{diag}(\rho, p, p, p)$$

ρ: Energy density

p: Pressure

p is a function of ρ

$$p = w\rho$$

w: Equation-of-state parameter

$$w = \begin{cases} 1/3 & : \ \text{radiation} \\ 0 & : \ \text{matter} \\ -1 & : \ \text{cosmological constant} \end{cases}$$

Einstein equation for the RW metric

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{\rho}{3M_{\rm Pl}^2} - \frac{K}{a^2} \equiv \frac{\rho_{\rm crit}}{3M_{\rm Pl}^2}$$

$M_{\rm Pl} \simeq 2.4 \times 10^{18}$ GeV: Reduced Planck scale

$\rho_{\rm crit}$: Critical density

Energy conservation

$$\dot{\rho} = -3H(\rho + p) = -3H(1+w)\rho$$

$w = p/\rho$: Equation-of-state parameter

Density parameter:

$$\Omega_{\rm tot} \equiv \frac{\rho}{\rho_{\rm crit}} = 1 + \frac{K}{a^2 H^2}$$

Case with the flat universe ($K = 0$)

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{\rho}{3M_{\mathrm{Pl}}^2}$$

Equations can be analytically solved if w is constant

- $a \propto t^{2/3(1+w)}$ (for $w > -1$)
- $\rho \propto a^{-3(1+w)}$

Notice:

$$H = \frac{2}{3(1+w)} t^{-1}$$

In radiation or matter dominated universe:

- $t \sim O(H^{-1})$
- The causal connection is possible only for the physical distance shorter than $\sim H^{-1}$ (horizon)

At present:

- Age of the Universe: 13.8 billion years
- Present horizon scale: $\sim 10^{26}$ m $\sim 10^{3}$ Mpc

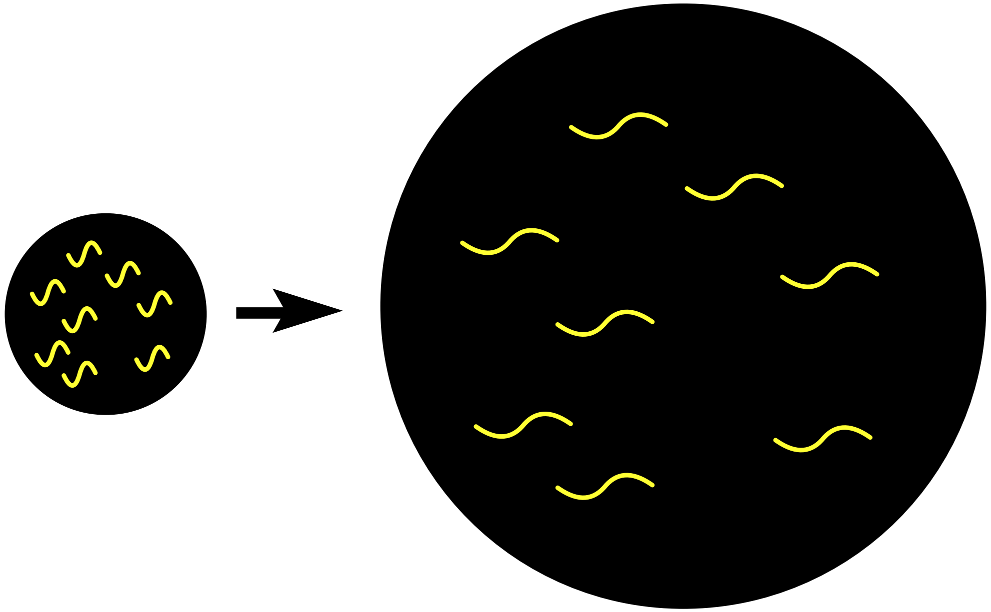
  1 Mpc $\simeq 3.26 \times 10^{6}$ light years $\simeq 3.09 \times 10^{22}$ m

Radiation-dominated (RD) universe

Radiation: relativistic object ($w = 1/3$)

Energy density is proportional to a^{-4}

Decrease of the number density & red-shift



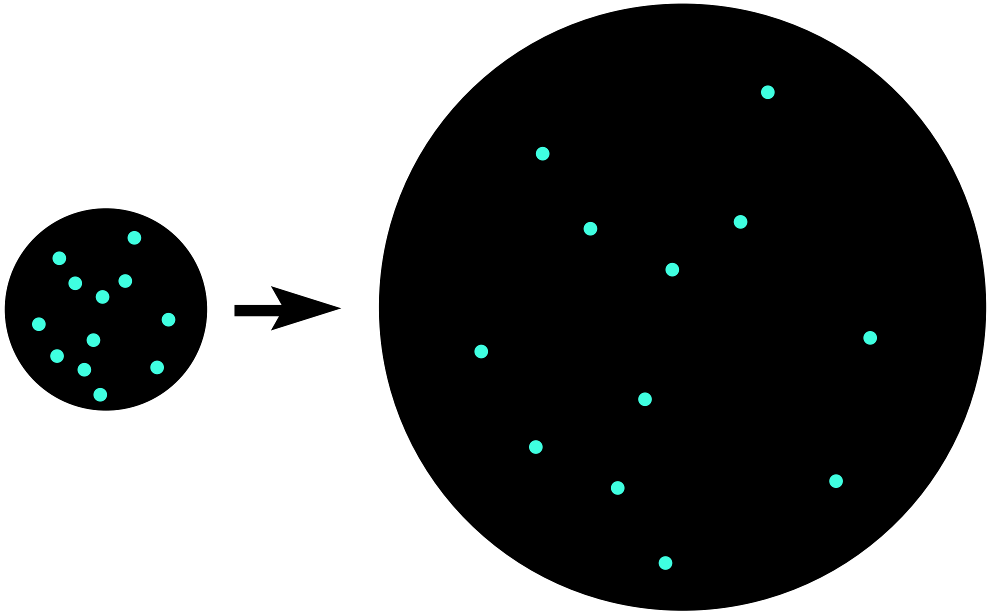
$\Rightarrow$ In RD, $t = \dfrac{1}{2} H^{-1}$

Matter-dominated (MD) universe

Matter: non-relativistic objects $(w = 0)$

Energy density is proportional to a^{-3}

Decrease of the number density, but no red-shift



$\Rightarrow$ In MD, $t = \frac{2}{3} H^{-1}$

We have see that:

- $t \sim O(H^{-1}) \sim O(\sqrt{\rho}/M_{\mathsf{Pl}})$

  $\Rightarrow$ In RD, $t \simeq 1$ sec $\times \left(\frac{T}{1\ \mathsf{MeV}}\right)^{-2}$

Notice: we may consider "cosmological constant"

$T_{\mu\nu}(\mathsf{C.C.}) = \Lambda g_{\mu\nu}$

$\Rightarrow \rho_\Lambda =$ const.

$\Rightarrow p_\Lambda/\rho_\Lambda = -1$

$\Rightarrow a \propto e^{Ht}$ with $H =$ const.

In the universe, there are several components

- Radiation (photon, neutrinos)

  $\Rightarrow \rho_{\mathsf{r}} \propto a^{-4}$

- Matter (baryon, CDM)

  $\Rightarrow \rho_{\mathsf{m}} \propto a^{-3}$

- Dark energy (which looks like cosmological constant)

  $\Rightarrow \rho_{\Lambda} \propto a^{0}$

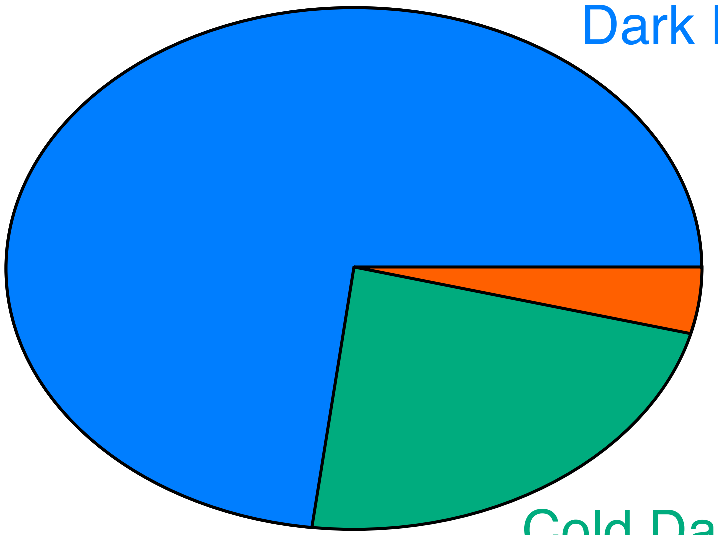
The dominant component changes with time

Total energy density of the present universe

[Planck Collaboration, 1502.01589]

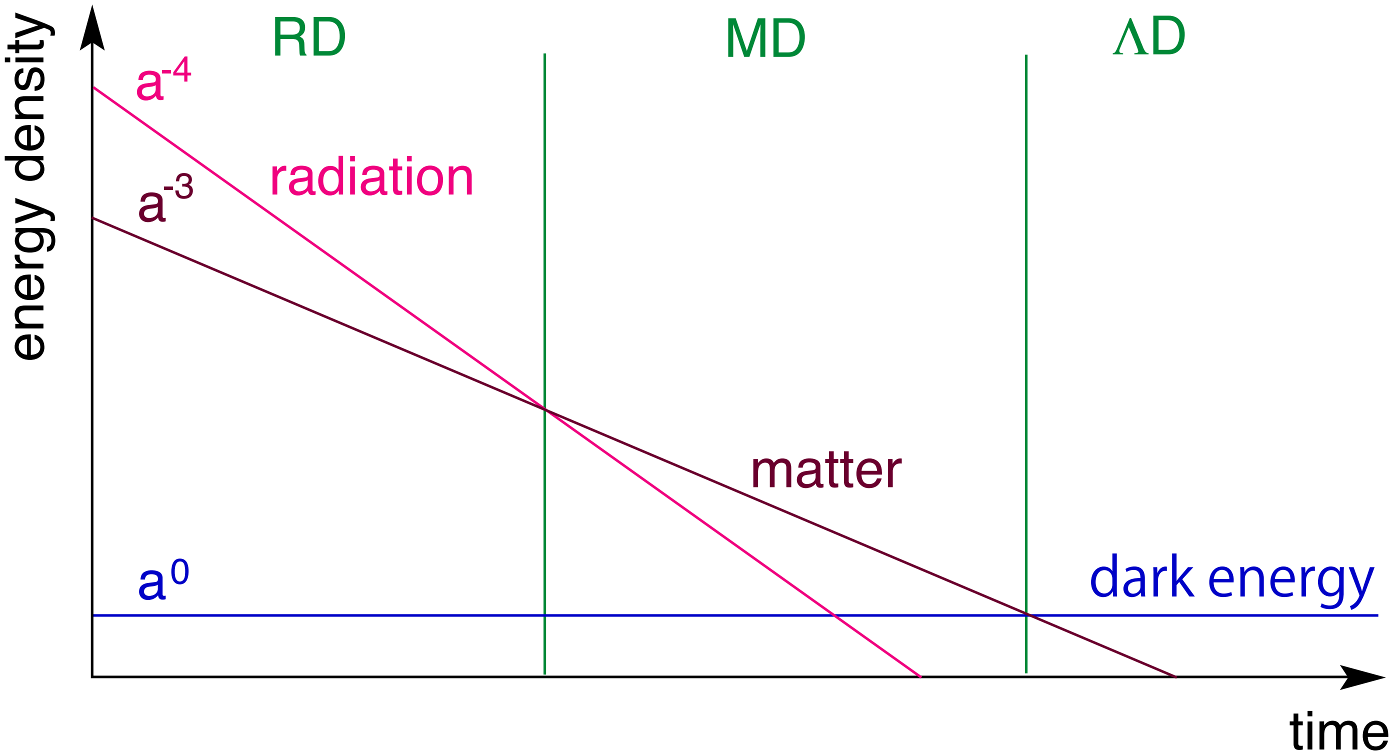
- $H_{\text{now}} = (67.8 \pm 0.9)$ km/sec/Mpc $\Rightarrow \rho_{\text{tot}}(t_{\text{now}}) \simeq 3.7 \times 10^{-47}$ GeV^4

The contents of the universe: $\Omega_X \equiv \dfrac{\rho_X(t_{\text{now}})}{\rho_{\text{tot}}(t_{\text{now}})}$



- $\Omega_c \simeq 0.26$ (CDM)
- $\Omega_b \simeq 0.05$ (baryons)
- $\Omega_\Lambda \simeq 0.69$ (dark energy)
- $\Omega_r \sim 10^{-4}$ (radiation)

Dominant component changes as the universe expands



- In the past, the universe was dominated by radiation
- Radiation-matter equality occurred when $z \sim 3000$ $(z + 1 \equiv a_{\mathrm{now}}/a)$

Thermal history of the universe (part I)

T	$1+z$	
2.7 K	1	Present
	$H = 67.8$ km/sec/Mpc	
	$\Omega_{\mathrm{m}} \simeq 0.31$, $\Omega_\Lambda \simeq 0.69$, $\Omega_{\mathrm{total}} \simeq 1$	
1 eV	1000	Recombination ($e^- p \to H$)
	Mean free path of γ becomes $\sim \infty$	
10 eV	3000	Radiation-matter equality
1 MeV	10^{10}	Big-bang nucleosynthesis (BBN)
	Baryogenesis should be before this epoch	
	No large entropy production after BBN	

Thermal history of the universe (part II)

T	$1+z$	
100 GeV	10^{15}	Electroweak phase transition
	Sphaleron process becomes insignificant	
???		Production of dark matter
???		Baryogenesis
		$\ldots$
???		Inflation
		$\ldots$
???		Birth of the universe

2. Big-Bang Nucleosynthesis (BBN)

In the present universe, there are various nuclei species

- Mostly H and ^{4}He

  ^{4}He mass fraction (now): $Y \equiv \dfrac{4n_{^4\mathrm{He}}}{n_\mathrm{H}} \sim \dfrac{1}{4}$

- Small amount of others (D, ^{3}He, $\cdots$)

Primary question:

How were the nuclei synthesized?

$\Rightarrow$ BBN

$\Rightarrow$ In the star

$\Rightarrow$ SN

BBN: prediction of the big-bang cosmology

- $T \gtrsim 1$ MeV: p and n do not form nuclei
- $T \lesssim 1$ MeV: It is energetically favorable to form nuclei

BBN provides interesting test of the SM and BSM

- In the SM, there is only one free parameter

  $\Rightarrow$ Baryon to photon ratio

- The light element abundance is sensitive to BSM physics

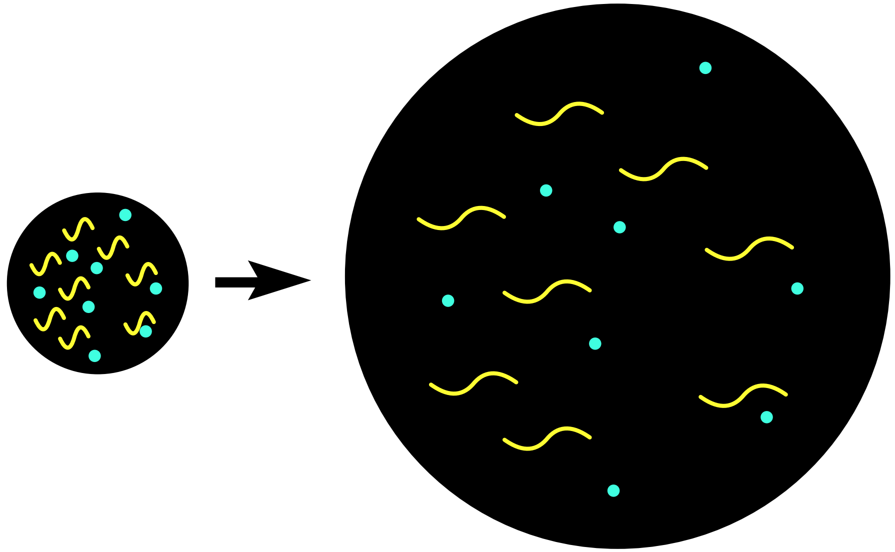
  $\Rightarrow$ Exotic long-lived (but unstable) particles

  $\Rightarrow$ Number of neutrino species (or dark radiation)

  $\Rightarrow \cdots$

Baryon-to-photon ratio: (almost) time-independent variable

$$\eta \equiv \left[\frac{n_B}{n_\gamma}\right]_{T \ll m_e}$$



$$\Rightarrow \eta \simeq 2.68 \times 10^{-8}(\Omega_b h^2)$$

$$\Rightarrow \eta \simeq 6.0 \times 10^{-10}$$

[with Planck 2015 result]

At $T \sim m_e$, photons are produced by the process $e^+e^- \to \gamma\gamma$

$\Rightarrow$ At such an epoch, $n_\gamma a^3 \neq$ const.

Number densities in the kinetic equilibrium

$$n_A = g_A \left(\frac{m_A T}{2\pi}\right)^{3/2} \exp\left(\frac{\mu_A - m_A}{T}\right)$$

m_A: Mass of A

g_A: Inertial degree of freedom of A

μ_A: Chemical potential of A

Number densities of p and n

$$n_p = 2 \left(\frac{m_p T}{2\pi}\right)^{3/2} \exp\left(\frac{\mu_p - m_p}{T}\right)$$

$$n_n = 2 \left(\frac{m_n T}{2\pi}\right)^{3/2} \exp\left(\frac{\mu_n - m_n}{T}\right)$$

If the nuclear nuclear reaction rates are fast enough

$\mu_A = A\mu_B$: chemical equilibrium

A: Atomic number

μ_B: Chemical potential for the baryon number

Total baryon number density

$$n_B = n_p + n_n + n_{\mathrm{D}} + n_{^3\mathrm{He}} + n_{^4\mathrm{He}} + \cdots$$

Number density in nuclear statistical equilibrium (NSE)

$$n_A = 2^{-A} g_A A^{3/2} \left(\frac{2\pi T}{m_N}\right)^{3(A-1)/2} \left(\frac{n_p}{T^3}\right)^Z \left(\frac{n_n}{T^3}\right)^{A-Z} \exp(B_A/T)$$

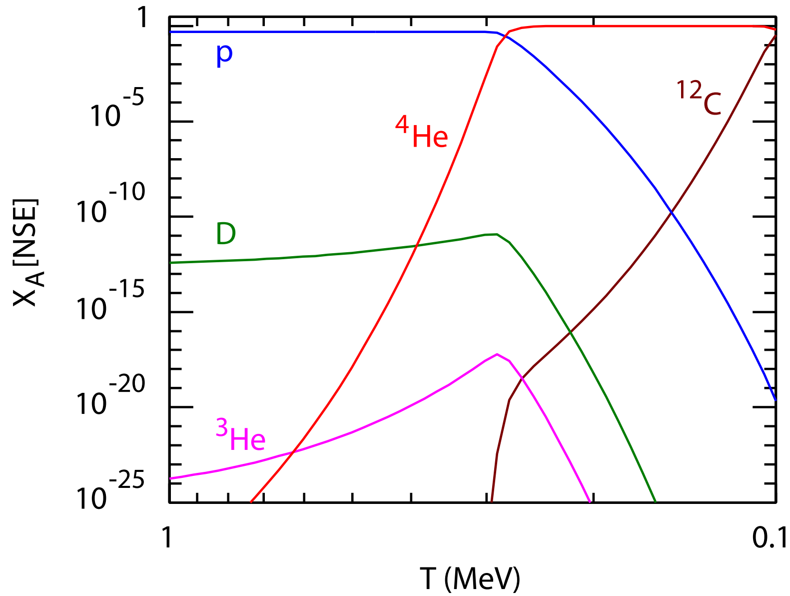
Z: Charge

$B_A \equiv Z m_p + (A - Z) m_n - m_A$: Binding energy

It is convenient to define the mass fraction

$$X_A \equiv \frac{A n_A}{n_B}$$

In the NSE, the nucleus A is formed when $T \lesssim B_A$



However, expansion of the universe cannot be neglected

$\Rightarrow$ At some point, expansion becomes so fast that the NSE cannot be maintained

Rough estimate of the freeze-out temperature of neutron

- $p \leftrightarrow n$ conversion occurs via weak interaction
  - $n + \nu_e \leftrightarrow p + e^-$
  - $n + e^+ \leftrightarrow p + \bar{\nu}_e$
  - $n \leftrightarrow p + e^- + \bar{\nu}_e$
- $p \leftrightarrow n$ conversion rate: $\Gamma_{p\leftrightarrow n} \sim G_{\mathsf{F}}^2 T^5$

  $G_{\mathsf{F}} \simeq 1.17 \times 10^{-5}$ GeV^{-2}: Fermi constant
- Expansion rate of the universe at BBN: $H \sim T^2/M_{\mathsf{Pl}}$ (RD)

Freeze-out: $H \sim \Gamma_{p\leftrightarrow n}$

$$T_{\mathsf{F}} \sim (1/G_{\mathsf{F}}^2 M_{\mathsf{Pl}})^{1/3} \sim O(1 \text{ MeV})$$

$T \gtrsim 1$ MeV

- NSE holds

$T \sim 1$ MeV

- $p \leftrightarrow n$ conversion freezes out

  $\Rightarrow p/n$ ratio deviates from the NSE value

- $p \leftrightarrow n$ conversion: weak interaction

$$\mathcal{L} \propto [\bar{n}(1 + g_A \gamma_5)\gamma_\mu p][\bar{\nu_e}(1 - \gamma_5)\gamma_\mu e] + \text{h.c.}$$

$$\Rightarrow \Gamma_{pe \to n\nu_e} = \tau_n^{-1}(T/m_e)^3 e^{-Q/T} \quad (T \ll Q, m_e)$$

$T \lesssim 1$ MeV

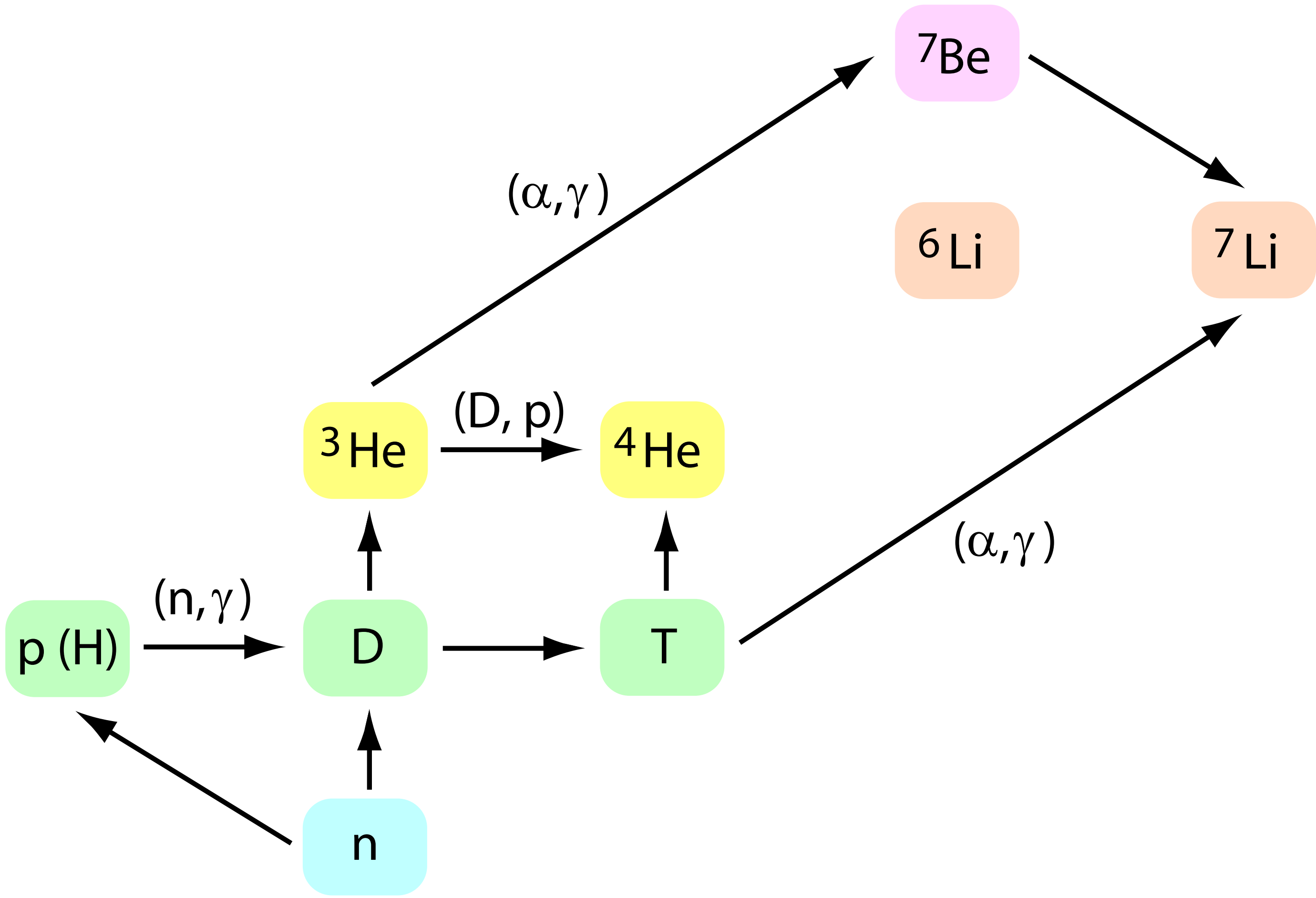
- The $p \leftrightarrow n$ conversion rate becomes smaller than the expansion rate of the universe

  $\Rightarrow$ Deviation from the NSE

$$
\begin{aligned}
X_n &\simeq 1/7 \\
X_p &\simeq 6/7 \\
X_{\mathrm{D}} &\simeq 10^{-12} \\
X_{^3\mathrm{He}} &\simeq 10^{-23} \\
X_{^4\mathrm{He}} &\simeq 10^{-28}
\end{aligned}
$$

- After this epoch, nucleosynthesis proceeds

Nuclear reaction network



$T \lesssim 0.3$ MeV

- For $T \gtrsim 0.1$ MeV, D is easily destroyed (c.f., $B_{\mathrm{D}} \simeq 2.22$ MeV)

  $\Rightarrow$ Nucleosynthesis does not proceed

- When $T \sim 0.1$ MeV, D is effectively produced

  $\Rightarrow$ D is transferred to ^{4}He

$$\mathrm{D}\ (\mathrm{D}, n)\ {}^3\mathrm{He}\ (\mathrm{D}, p)\ {}^4\mathrm{He}$$
$$\mathrm{D}\ (\mathrm{D}, p)\ {}^3\mathrm{H}\ (\mathrm{D}, n)\ {}^4\mathrm{He}$$
$$\mathrm{D}\ (\mathrm{D}, \gamma)\ {}^4\mathrm{He}$$

Nuclei heavier than ^{4}He are hardly produced

^{7}Li is synthesized using ^{4}He

$${}^4\mathrm{He}\ ({}^3\mathrm{H}, \gamma)\ {}^7\mathrm{Li}$$
$${}^4\mathrm{He}\ ({}^3\mathrm{He}, \gamma)\ {}^7\mathrm{Be}\ (e^-, \nu_e)\ {}^7\mathrm{Li}$$

$T \ll 0.1$ MeV

- Nuclear interaction rates becomes smaller than the expansion rate of the universe

  $\Rightarrow$ Abundances of the light elements are fixed

  $\Rightarrow$ The end of BBN

Rough estimate of the ^{4}He abundance:

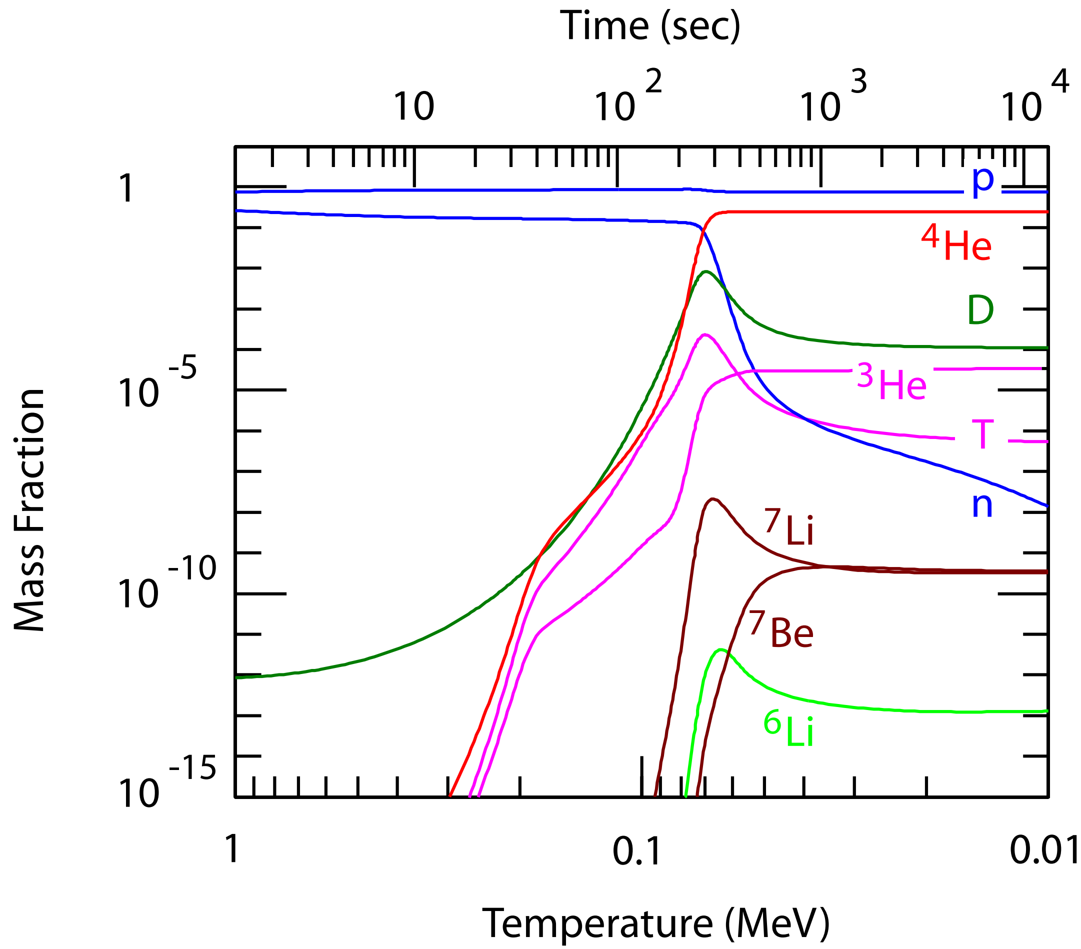
- Almost all the relic neutrons are synthesized into ^{4}He

$$Y_{\mathrm{p}} \equiv \frac{4n_{^4\mathrm{He}}}{n_B} \sim \left. \frac{2n_n}{n_B} \right|_{T=T_{\mathrm{F}}} \sim 0.25$$

- For a more accurate estimation, we need to solve Boltzmann equations

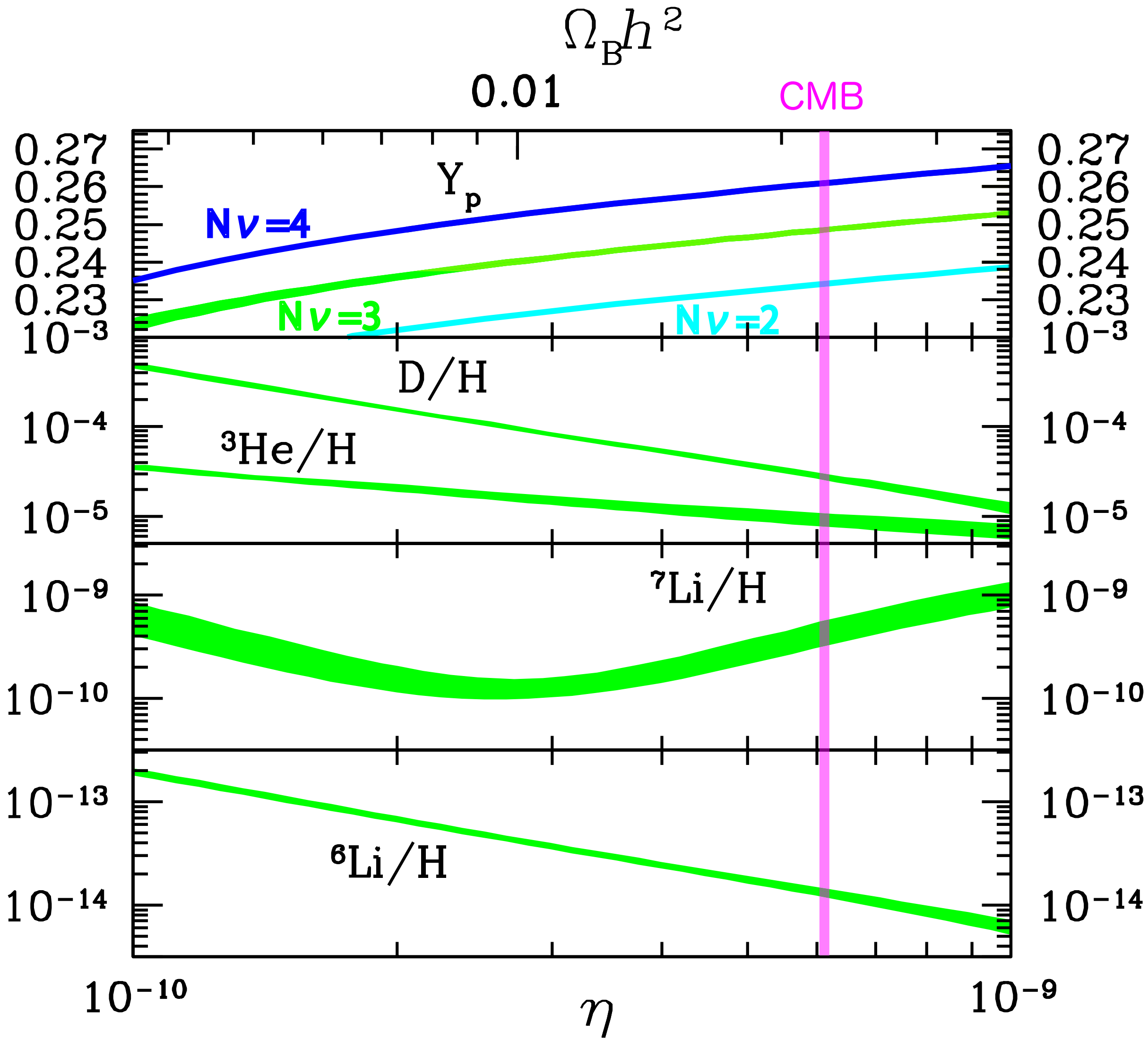
  [Kawano code; PArthENoPE code; $\cdots$]

Evolution of the light-element abundances



Abundances as functions of η

[Figure: Courtesy of K. Kohri]



$\eta_{\rm CMB} \simeq 6.0 \times 10^{-10}$

Light elements are produced by BBN

D, ^{3}He, ^{4}He, (^{6}Li, ^{7}Li)

Heavier elements are produced afterwords

- Nuclear reaction in stars
- Supernova (r-process)

In order to test the scenario of BBN, we should compare theoretical predictions with observations

- Light elements are also produced/destroyed in stars

We should find old environment

- High redshift
- Metal-poor

Deuterium

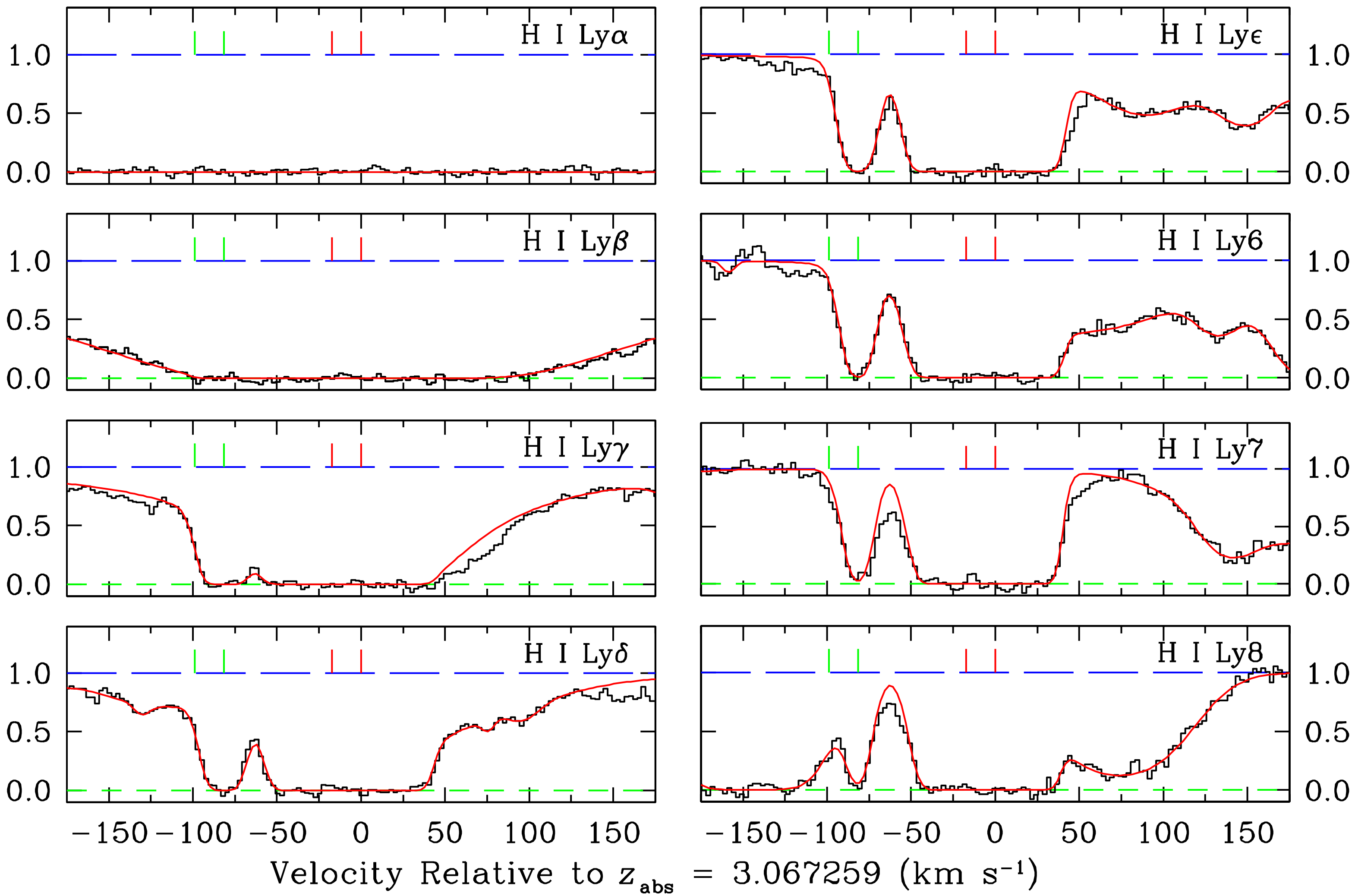
- D/H is inferred from D absorption in damped Lyα systems (DLAs)

$$E_{\mathsf{H}}^{(n)} \simeq -\frac{\alpha^2}{2n^2}\left(\frac{1}{m_e}+\frac{1}{m_{\mathrm{p}}}\right)^{-1} \quad \text{vs.} \quad E_{\mathsf{D}}^{(n)} \simeq -\frac{\alpha^2}{2n^2}\left(\frac{1}{m_e}+\frac{1}{m_{\mathrm{D}}}\right)^{-1}$$

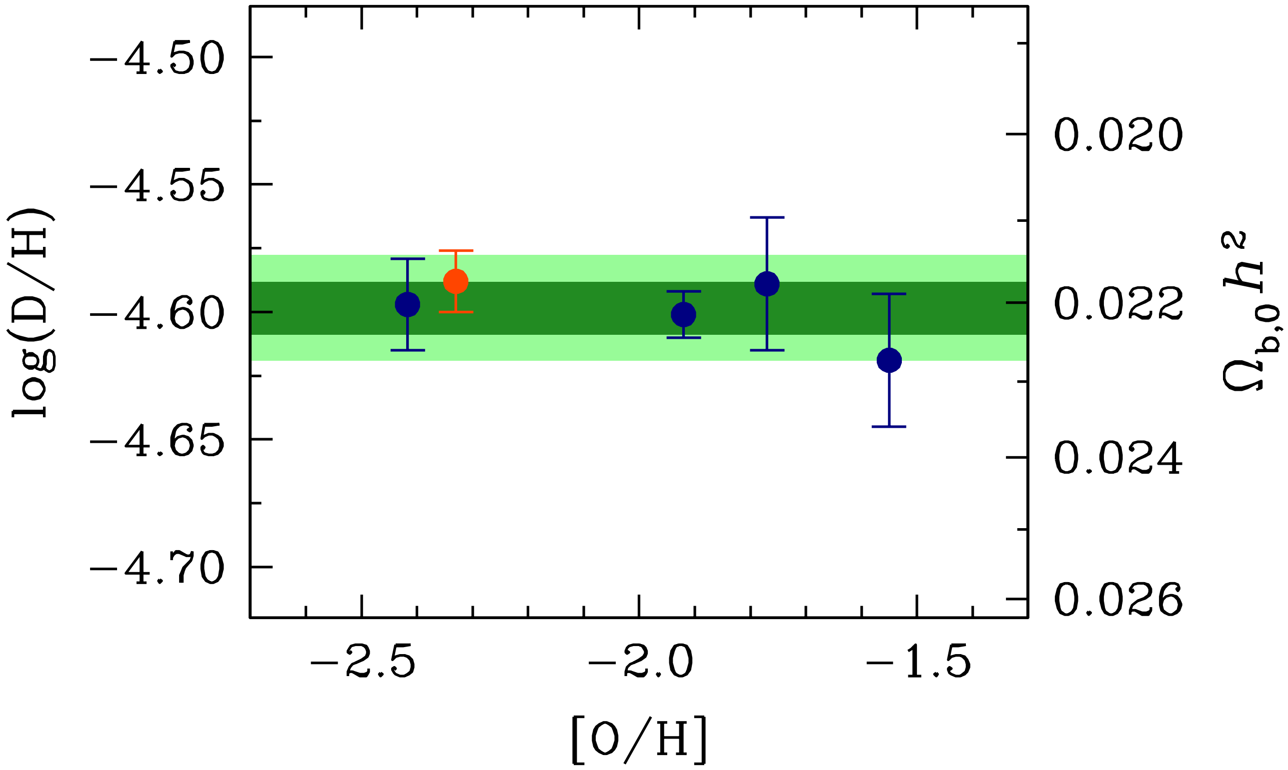
$$\Rightarrow \frac{E_{\mathsf{H}}^{(n)} - E_{\mathsf{D}}^{(n)}}{E_{\mathsf{H}}^{(n)}} \sim -2.7 \times 10^{-4} \quad \Rightarrow \quad \delta v \sim 80 \ \mathrm{km/sec}$$

Observation of DLA toward QSO SDSS J1358+6522

[Cooke et al., Astrophys.J. 781 (2014) 31]



Primordial abundance of D

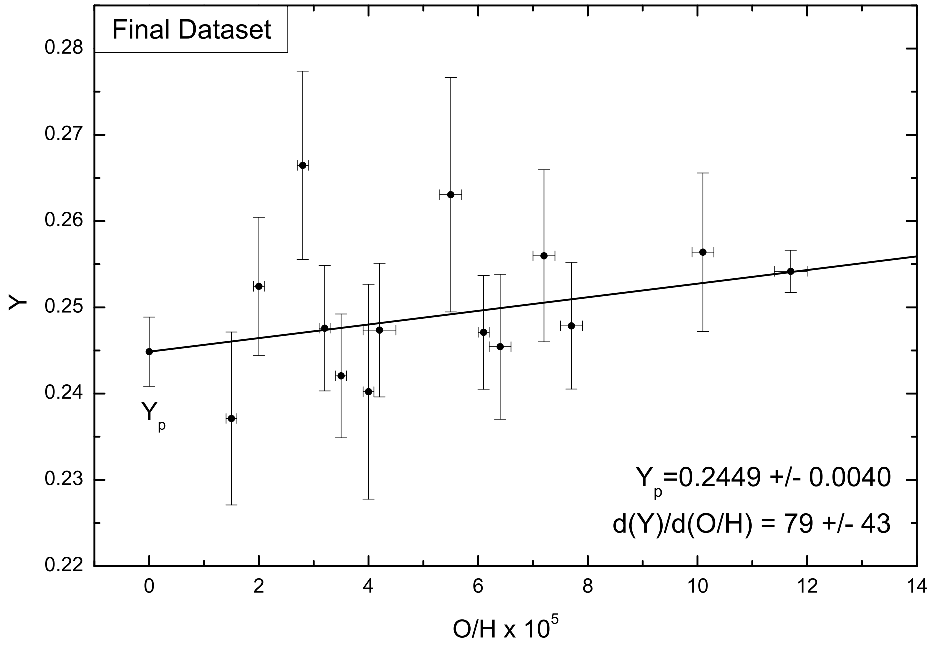


$\Rightarrow$ D/H $= (2.53 \pm 0.04) \times 10^{-5}$

[Cooke et al., Astrophys.J. 781 (2014) 31]

Helium-4

- ^{4}He abundance is inferred from observation of emission lines from extra galactic HII regions

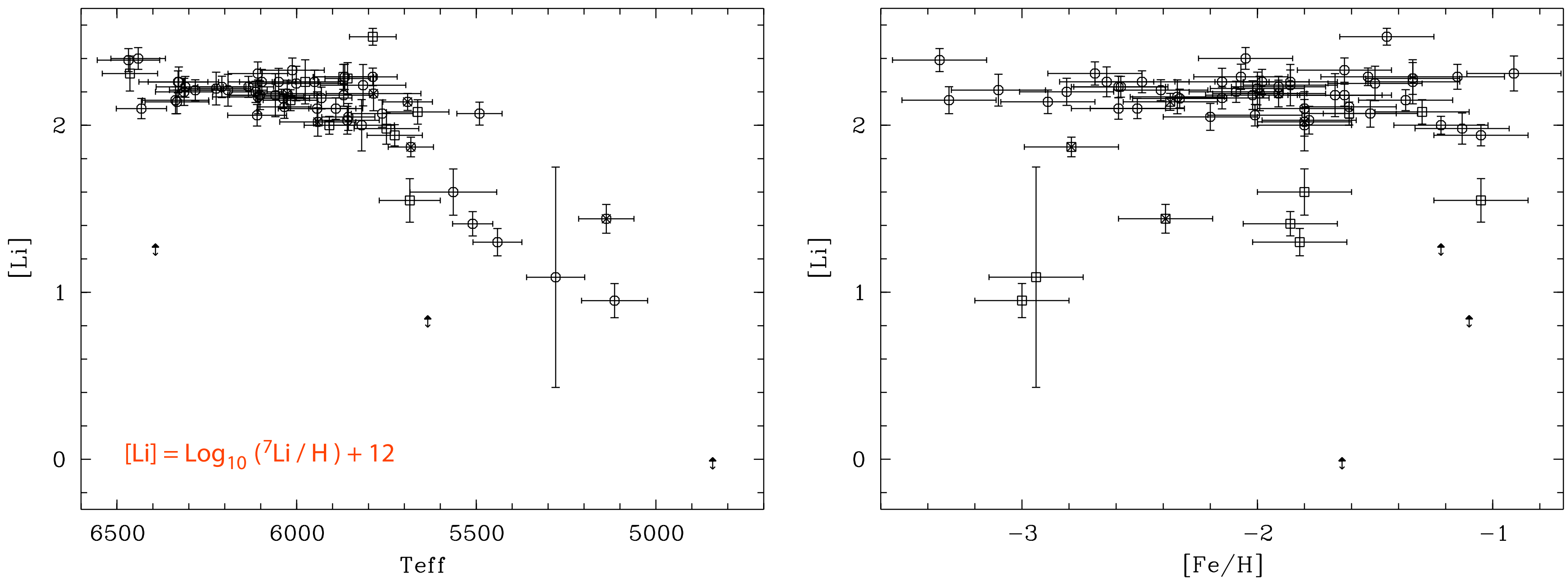


$$Y_{\text{BBN}} = 0.2449 \pm 0.0040$$

[Aver, Olive & Skillman, JCAP 1507 (2015) 011]

Lithium-7

- ^{7}Li has been observed in Pop-II old halo stars

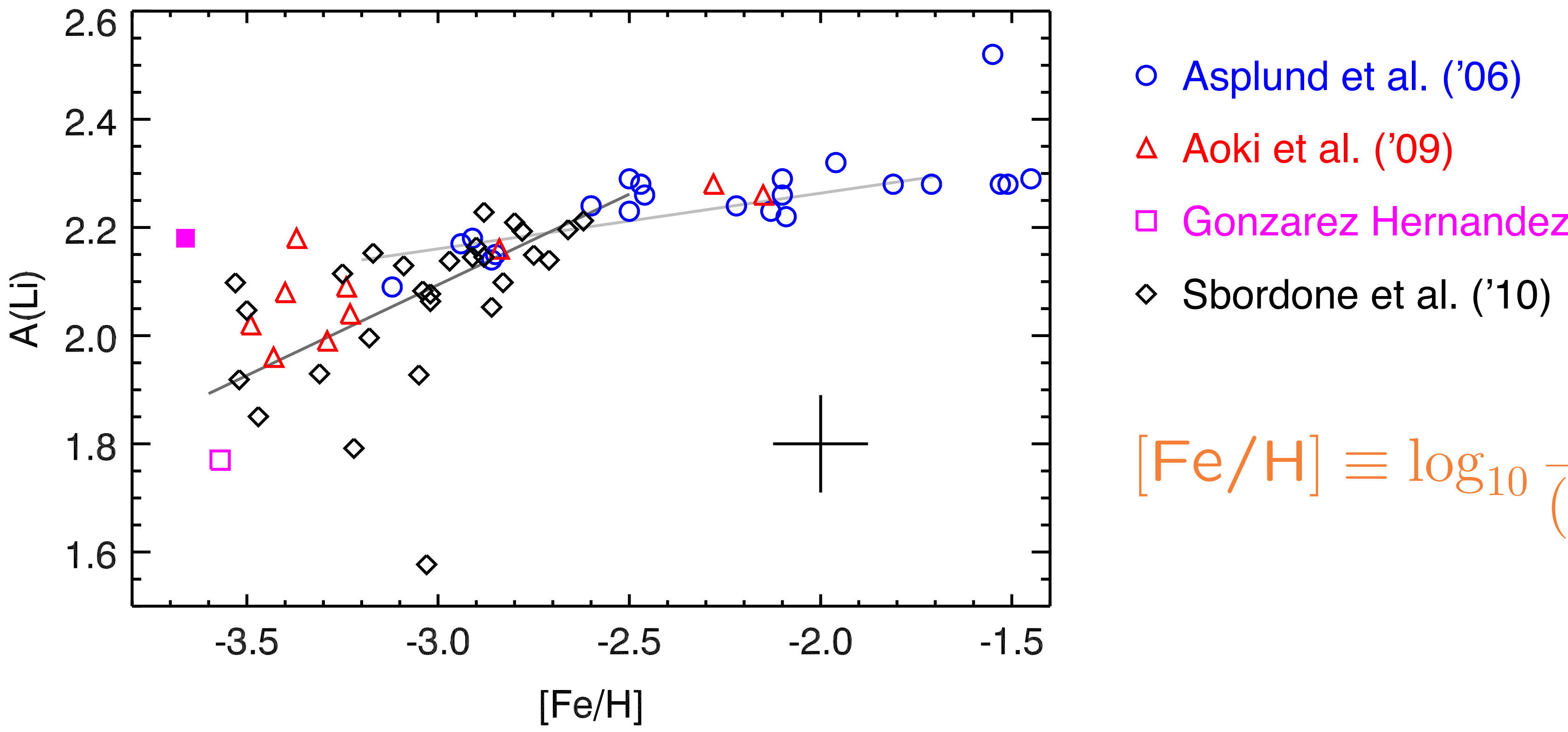


[Bonifacio and Malaro, MNRAS 285 (1997) 847]

- In stars with high surface temperature, ^{7}Li abundance was though to be almost constant (Spike plateau)

Currently, the situation is more controversial about ^{7}Li

- (^{7}Li/H) is not constant in stars with very low metallicity



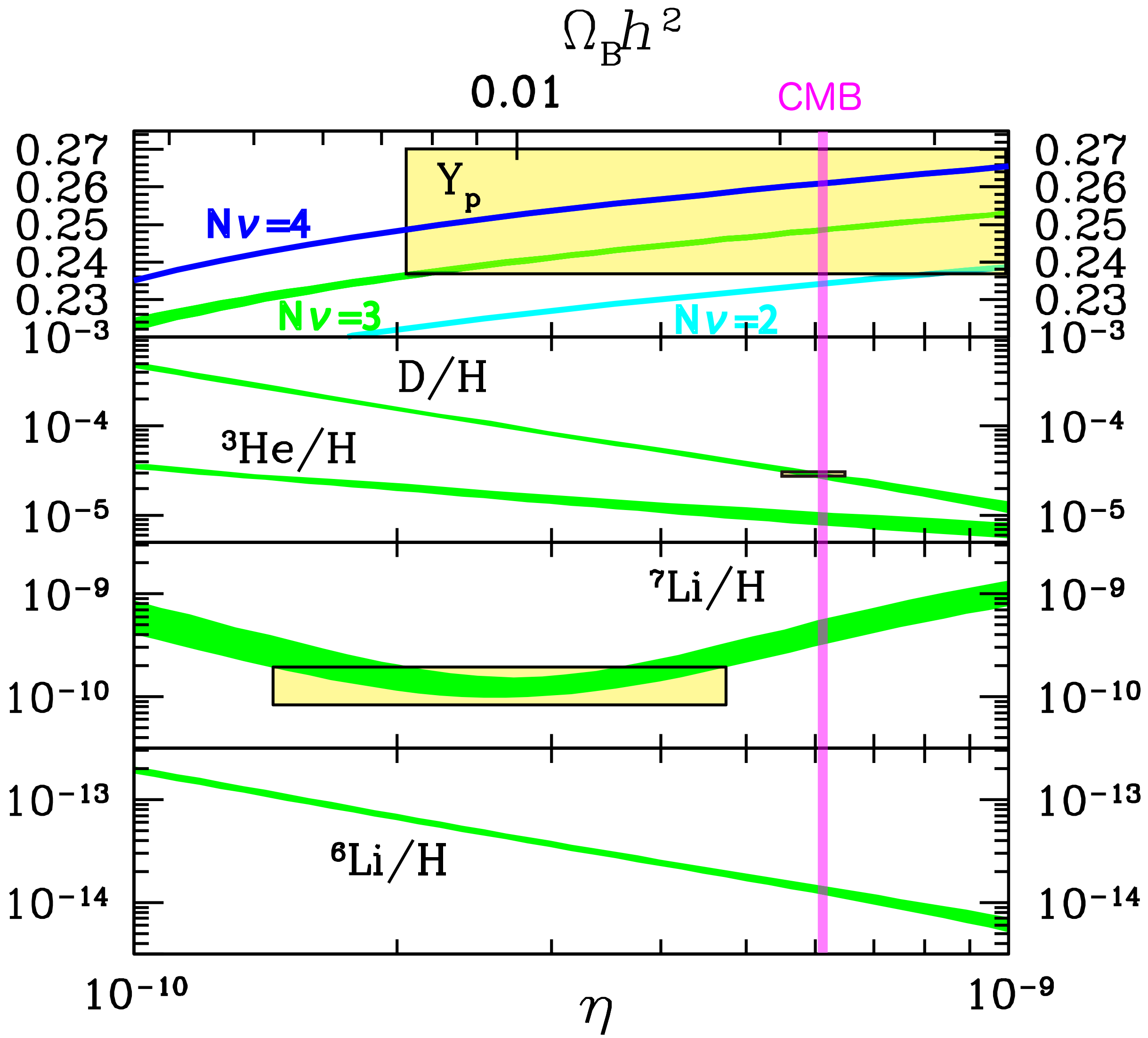
$$[\mathrm{Fe/H}] \equiv \log_{10} \frac{(\mathrm{Fe/H})}{(\mathrm{Fe/H})_\odot}$$

[Sbordone et al., Astron.Astrophys. 522 (2010) A26]

- We do not use ^{7}Li to test BBN

Theory vs. Observations: Reasonable agreements

[Figure: Courtesy of K. Kohri]



Predictions of the standard BBN (more or less) agree with observations

$\Rightarrow$ Any new physics which significantly affects the BBN is disfavored / excluded

- Extra energy density at the neutron freeze-out
- Late-time emissions of high energy particles
- $\cdots$

BBN constrains physics beyond the standard model

- Weakly interacting light particles
- Supersymmetry
- $\cdots$

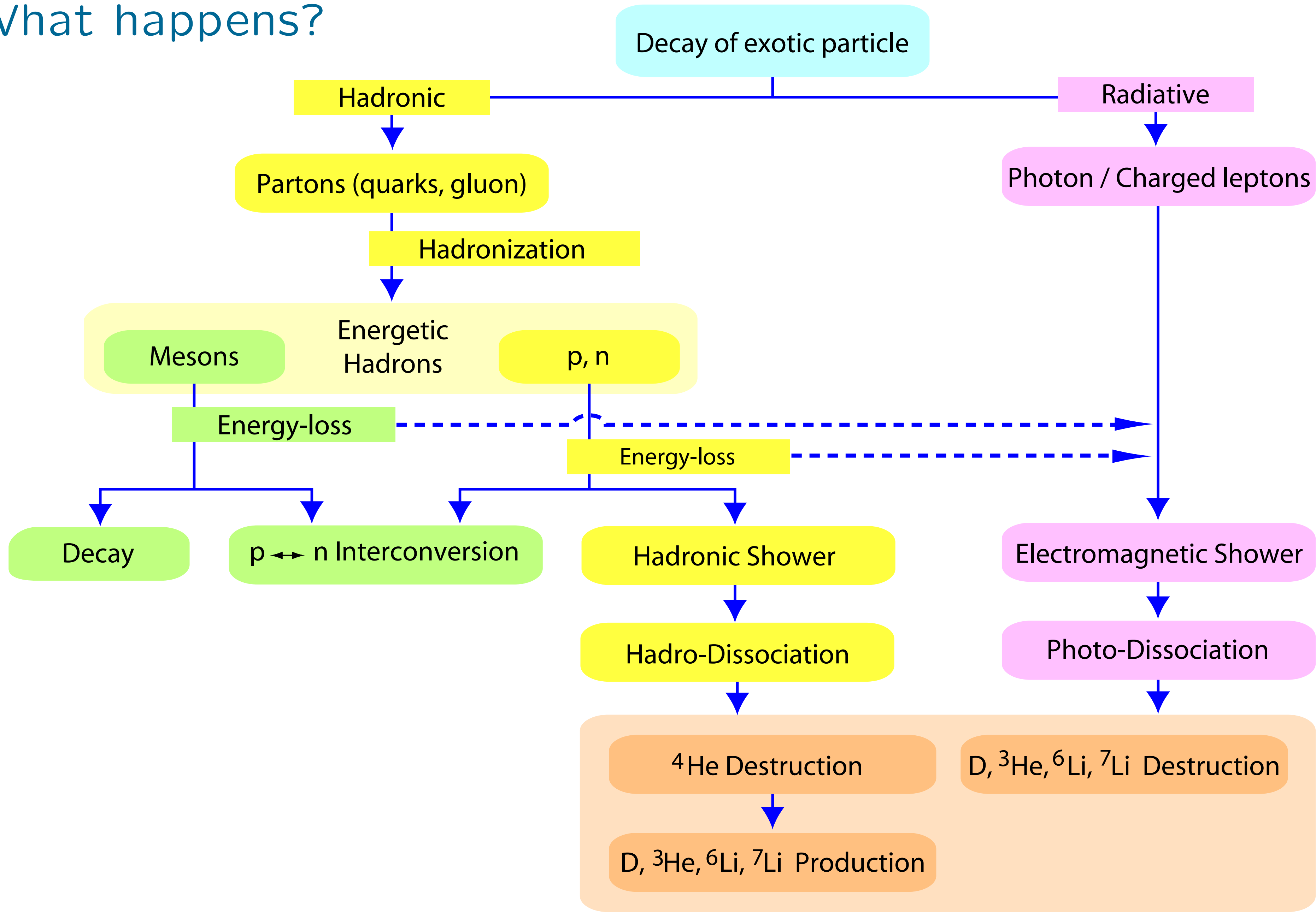
If high energy photons / hadrons are emitted after BBN, they dissociate light elements

$\Rightarrow$ They may change the prediction of the standard BBN

If we consider physics beyond the standard model, it may contain particle with very long lifetime

- Gravitino (superpartner of the graviton)
- Moduli fields in string theory
- $\cdots$

What happens?

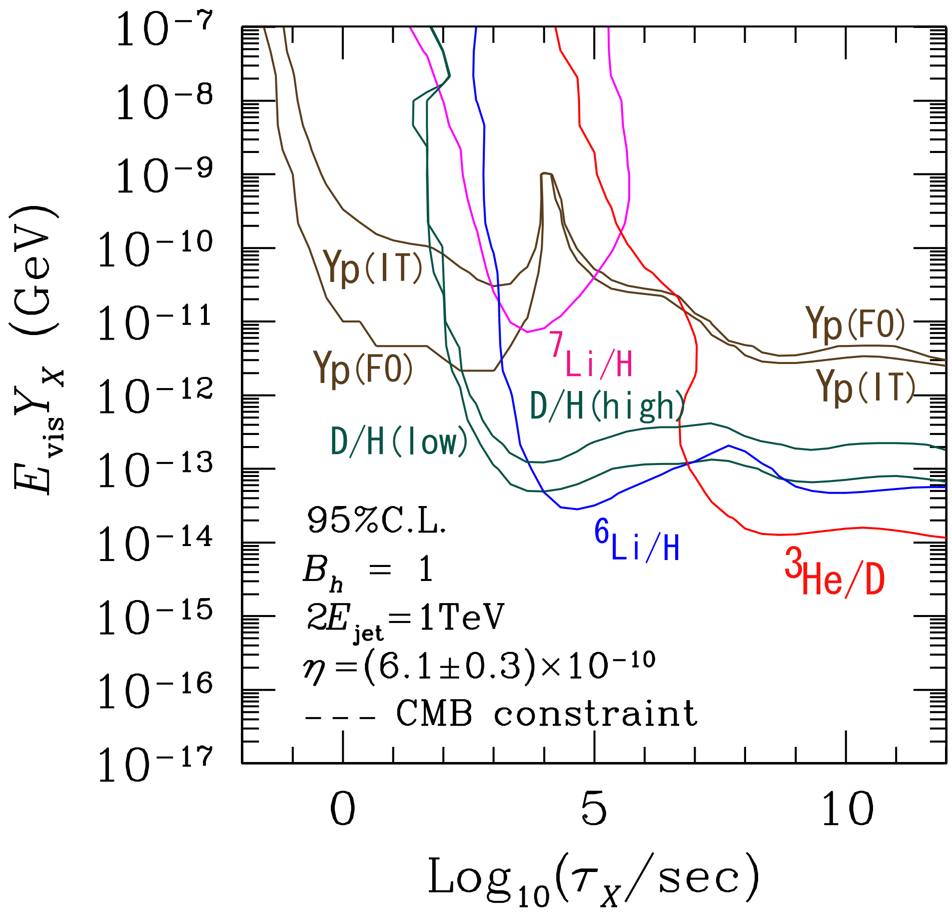


As an application, we may obtain upper bound on the reheating temperature in supergravity models

- In supergravity, gravitino exists
  - Its interaction is suppressed by $1/M_{\mathsf{Pl}}$
  - Long lifetime:
    $\tau \simeq 10^8$ sec $\times \left(m_{3/2}/100 \text{ GeV}\right)^{-3}$
  - Its relic abundance is proportional to T_{R}:
    $Y_{3/2} \simeq 2 \times 10^{-11} \times \left(T_{\mathsf{R}}/10^{10} \text{ GeV}\right)$
  - Possible decay mode:
    $\psi_\mu \to \gamma + \chi^0$, hadrons, $\cdots$

$\Rightarrow$ Too much gravitino spoils the success of BBN

Upper bound on the abundance of a long-lived particle X



$$Y_X \equiv \frac{n_X}{s}$$

$$s = \frac{2\pi^2}{45} g_* T^3$$

[Kawasaki, Kohri & Moroi ('05)]

3. Inflation

Unsolvable problems in standard cosmology:

- Why is the Universe so homogeneous and flat at large-scale?
- What is the origin of the small-scale inhomogeneity?
- Why is the curvature term so small?
- Where did unwanted relics go?

Inflation solves these problems by one fine-tuning

Inflation does not explain the smallness of the cosmological constant

Fine-tuning of the cosmological constant is needed for viable scenarios of inflation

Here, I discuss

- Motivations of inflation
  - Horizon problem
  - Flatness problem
  - Unwanted relics
- Slow roll inflation
  - Basic features
  - Several Models (chaotic inflation, new inflation)
- Density perturbation

Horizon problem

Why is our universe so homogeneous and smooth?

In RD / MD epoch, particle travels at most $\sim H^{-1}$

- For $r \gg H^{-1}$, no causal contact is expected

  $\Rightarrow$ Inhomogeneity is possible within the horizon

In the RD / MD epoch, the Hubble horizon expands faster than the the comoving distance

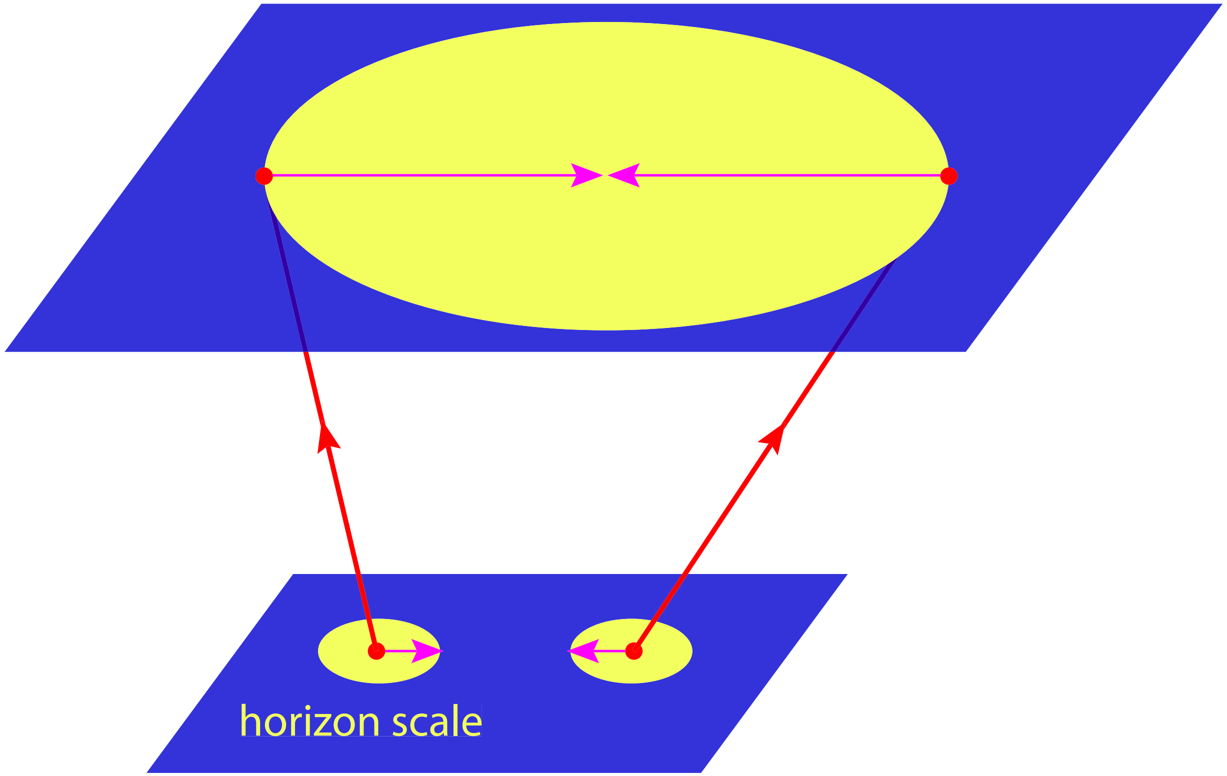
- Physical distance $\propto a$

- Hubble horizon $\propto H^{-1}$

  $H^{-1} \propto a^{3(1+w)/2} \quad \Rightarrow \quad \frac{d}{dt}(aH) < 0$ for RD/MD

Present horizon consists of many regions which were causally disconnected

$\Leftrightarrow$ In the past, horizon size was smaller than the present one



Homogeneity of the CMB ($\Delta T/T \sim O(10^{-5})$)

- The CMB is from the last-scattering surface
- Entropy within the horizon scale
  - $S_{\mathsf{H}}(z=0) \sim 10^{88}$
  - $S_{\mathsf{H}}(z \sim 1000) \sim 10^{83}$

$\Rightarrow$ Present horizon contains about 10^5 causally disconnected regions at the recombination

$\Rightarrow$ This scale corresponds to the angular scale of about 0.8 degree on the present sky

What is the reason for the homogeneity of the present universe?

Flatness problem

Our universe is (almost) flat, but why?

Robertson-Walker metric

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right]$$

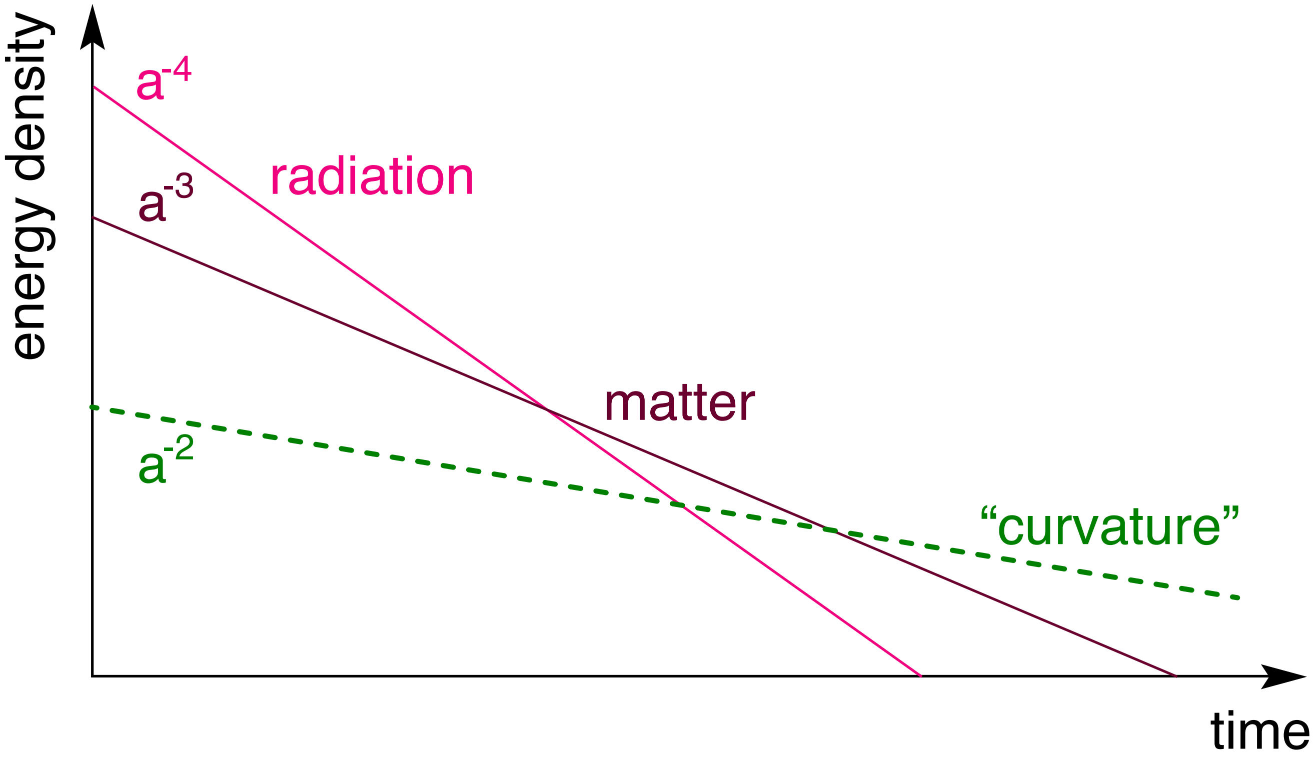
FRW equation:

$$H^2 = \frac{\dot{a}^2}{a^2} = \frac{\rho}{3M_{\rm Pl}^2} - \frac{K}{a^2} \quad \text{where } \rho \propto \begin{cases} a^{-3}\text{: MD} \\ a^{-4}\text{: RD} \end{cases}$$

The curvature term becomes important when $K^{-1/2}a \sim H^{-1}$

$\Leftrightarrow$ Radius of the curvature of the space: $R_{\rm phys} = K^{-1/2}a$

Evolution of each components



$\Rightarrow$ As time goes, the curvature term dominates over the energy density of radiation and matter

Tuning required for the curvature term:

- $\Omega_K = 0.0008^{+0.0040}_{-0.0039}$

  $\Rightarrow K^{-1/2}a(t_{\rm now}) \gtrsim O(10) \times H^{-1}(t_{\rm now})$

- At the time of recombination (for example)

  $\Rightarrow K^{-1/2}a(t_{\rm rec}) \gtrsim O(10^3) \times H^{-1}(t_{\rm rec})$

- In the very early epoch (birth of the universe)

  $\Rightarrow K^{-1/2}a(t_{\rm early}) \gg H^{-1}(t_{\rm early})$

Why is the curvature term so suppressed compared to the expansion rate?

Unwanted relics

In many models, there exist various unwanted relics

- Monopole from GUT
- Gravitino in supergravity models
- Topological defects from PQ (and other) symmetry
- $\cdots$

These objects are produced when universe was very hot

- If they are stable, they may overclose the universe
- If they decay after BBN, they dissociate light elements and spoil the success of BBN

$\Rightarrow$ Somehow, we need to dilute them

If the universe once experienced a de Sitter phase, these problems may be solved

$$T_{\mu\nu} \simeq \rho_{\mathsf{v}} g_{\mu\nu} \text{ with } \rho_{\mathsf{v}} \simeq \text{const.} \quad \Rightarrow \quad \frac{\dot{a}}{a} = \frac{\sqrt{\rho_{\mathsf{v}}}}{\sqrt{3} M_{\mathsf{Pl}}} \equiv H_{\mathsf{inf}}$$

The universe expands rapidly

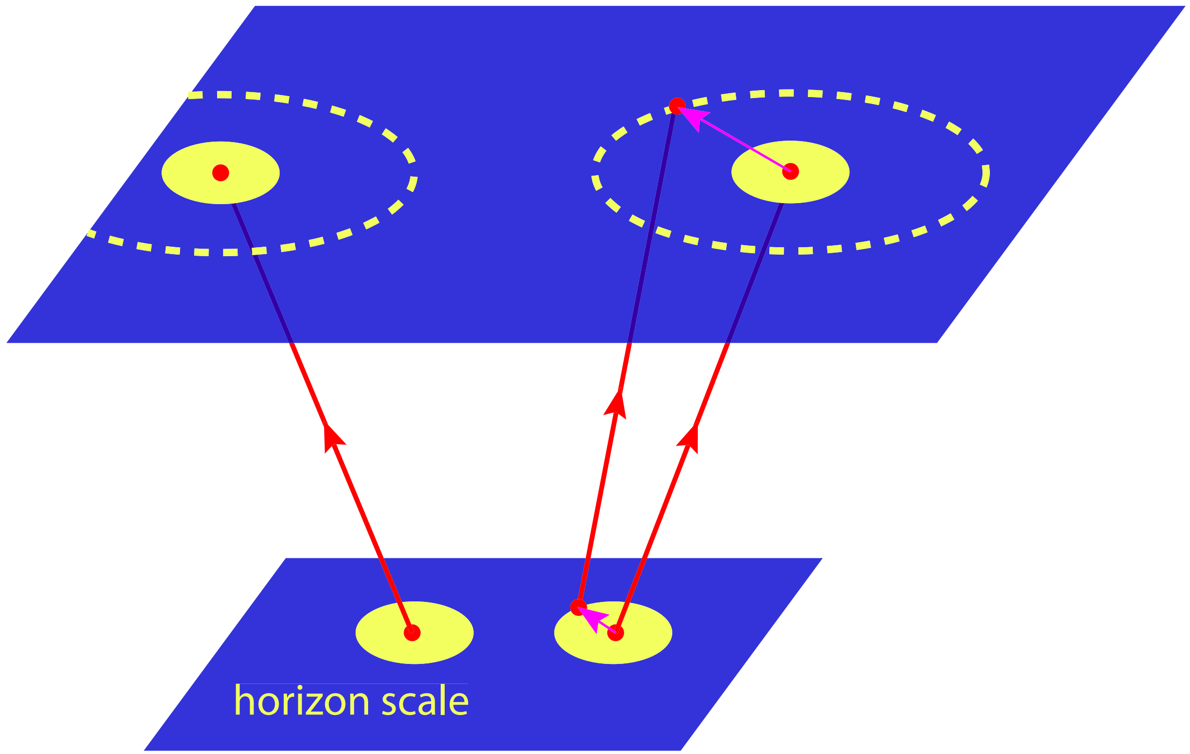
$$\dot{a}/a = H_{\mathsf{inf}} \ \Rightarrow a = a_0 e^{H_{\mathsf{inf}} t} \Rightarrow \frac{d}{dt}(aH) > 0$$

With the exponential growth of the scale factor:

$\Rightarrow$ Horizon problem may be solved

$\Rightarrow$ Spacial curvature as well as energy densities of radiation and matter become (almost) zero

Causally connected region becomes larger than the horizon scale



As time goes, causally connected region goes out of the horizon during inflation

Meaning of the Hubble horizon during inflation

- At $t = 0$, a photon is emitted from A to B

  $\Rightarrow ds^2 = dt^2 - a(t)^2 dx^2 = 0$

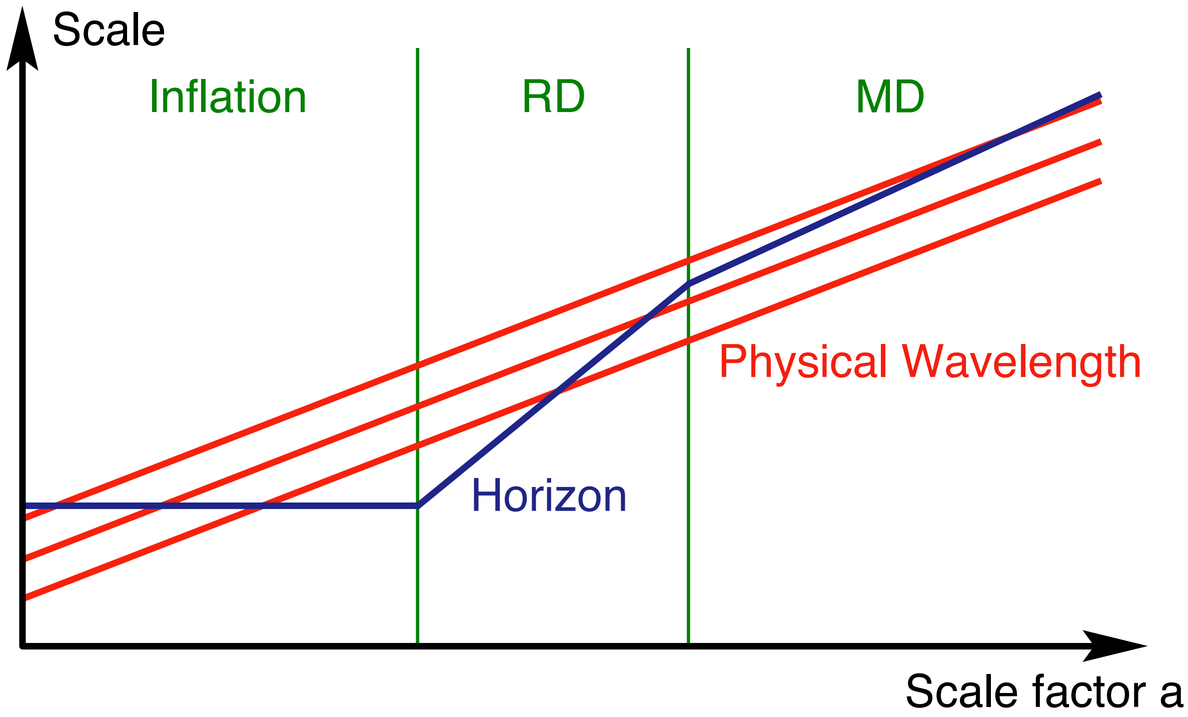
  $\Rightarrow$ Comoving coordinate at time t:

$$x_\gamma(t) = \int dx = \int_0^t \frac{dt}{a(t)} = \frac{1}{H_{\rm inf}}(1 - e^{-H_{\rm inf}t})$$

- $x_\gamma \to H_{\mathsf{inf}}^{-1}$ as $t \to \infty$

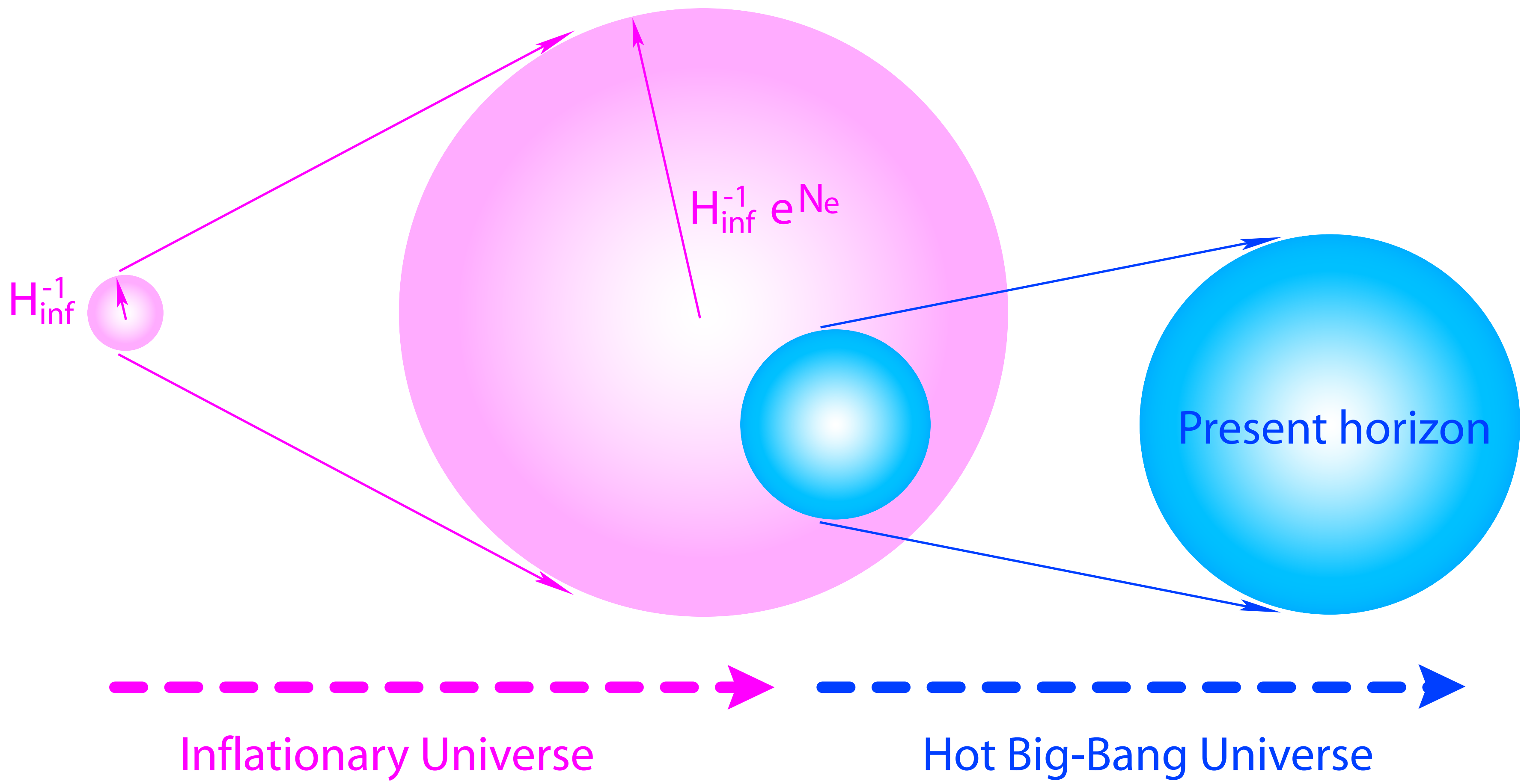
- If the distance between A and B is longer than H_{inf}^{-1} at $t = 0$, the photon cannot arrive the point B

Evolution of the physical scale and horizon scale



$\Rightarrow$ Mode with longer wavelength exits the horizon at earlier epoch during inflation

Schematic picture of the evolution of horizon



For inflation, (effective) cosmological constant should be provided in some form

- Energy density of the scalar field (inflaton)
- Energy density of inflaton should be finally converted to that of radiation (reheating)

There are various models of inflation

- The “vacuum energy” is provided as a potential energy of a scalar field
- The inflaton decays after inflation, and reheat the universe

Energy momentum tensor of scalar field χ

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \chi)^2 - V(\chi) \quad \Rightarrow \quad T_{\mu\nu} = \partial_\mu \chi \partial_\nu \chi - g_{\mu\nu} \mathcal{L}$$

Assumption: $\chi = \chi(t)$

$$\rho_\chi = T_{00} = \frac{1}{2}\dot{\chi}^2 + V$$

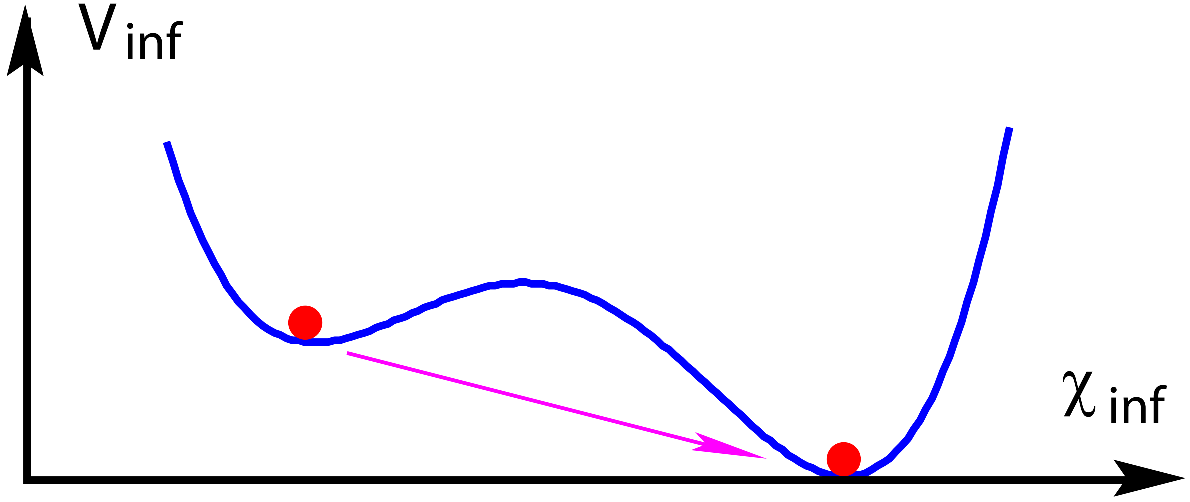
$$p_\chi = T_{ii} = \frac{1}{2}\dot{\chi}^2 - V$$

Vacuum energy: $p/\rho = -1$

$\Rightarrow$ Condition for successful inflation: $\dot{\chi}^2 \ll V$

Original idea of inflation: Inflation with false vacuum

[Guth; Sato]



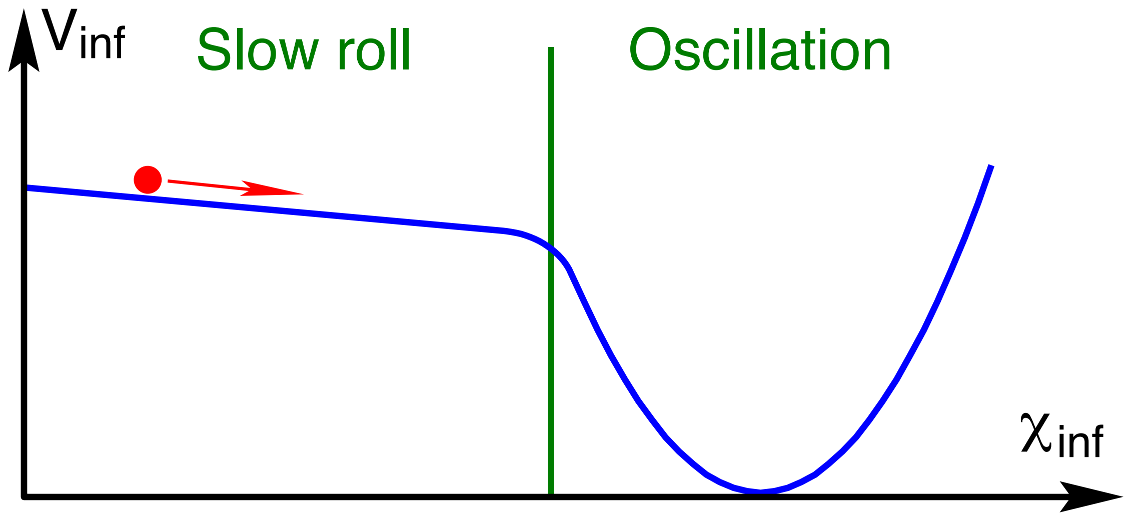
Guth and Weinberg pointed out that, in this case, the rate of the 1st-order phase transition is too slow

$\Rightarrow$ Inflation does not end

Slow-roll inflation

- Inflation occurs when inflaton field is “slow-rolling”
- Inflaton potential has a very flat “slow-roll” region

$$\ddot{\chi} + 3H\dot{\chi} + V' = 0$$



$\dot{\chi}$ should not change rapidly

$\Rightarrow$ Slow-roll condition: $H^{-1}\ddot{\chi} \ll \dot{\chi}$

Conditions for slow-roll inflation

$\dot{\chi}_{\mathsf{inf}}^2 \ll V_{\mathsf{inf}}$ (potential energy dominance)

$\ddot{\chi}_{\mathsf{inf}} \ll 3H\dot{\chi}_{\mathsf{inf}}$ (slow-roll condition)

Equation of motion of inflaton: $\ddot{\chi}_{\mathsf{inf}} + 3H\dot{\chi}_{\mathsf{inf}} + V'_{\mathsf{inf}} = 0$

$$\Rightarrow \quad \dot{\chi}_{\mathsf{inf}} \simeq -\frac{V'_{\mathsf{inf}}}{3H_{\rm inf}} \quad \text{with} \quad H_{\rm inf}^2 \simeq \frac{V_{\mathsf{inf}}}{3M_{\mathsf{Pl}}^2}$$

Condition for the potential energy dominance

With the slow-roll condition: $3H\dot{\chi}_{\mathsf{inf}} + V'_{\mathsf{inf}} \simeq 0$

$$\epsilon \equiv \frac{1}{2}M_{\mathsf{Pl}}^2 \left[\frac{V'_{\mathsf{inf}}}{V_{\mathsf{inf}}}\right]^2 \simeq \frac{3}{2}\frac{\dot{\chi}_{\mathsf{inf}}^2}{V_{\mathsf{inf}}} \ll 1$$

Slow-roll condition: $\ddot{\chi}_{\text{inf}} \ll H\dot{\chi}_{\text{inf}}$

Derivative of $3H\dot{\chi}_{\text{inf}} + V'_{\text{inf}} \simeq 0$ with respect to time

$$3H\ddot{\chi}_{\text{inf}} + 3\dot{H}\dot{\chi}_{\text{inf}} + V''_{\text{inf}}\dot{\chi}_{\text{inf}} \simeq 0$$

Then, divide by $3H^2\dot{\chi}_{\text{inf}}$

$$\frac{\ddot{\chi}_{\text{inf}}}{H\dot{\chi}_{\text{inf}}} + \frac{\dot{H}}{H^2} + \frac{V''_{\text{inf}}}{3H^2} \simeq 0$$

Using the relations $\dot{H} \simeq -\epsilon H^2$ and $3H^2 M_{\text{Pl}}^2 \simeq V_{\text{inf}}$

$$\eta \equiv M_{\text{Pl}}^2 \frac{V''_{\text{inf}}}{V_{\text{inf}}} \simeq \epsilon - \frac{\ddot{\chi}^2_{\text{inf}}}{H_{\text{inf}}\dot{\chi}_{\text{inf}}} \ll 1$$

The scenario of slow-roll inflation

1. Slow-roll epoch (inflation)

   - Energy density of the universe is dominated by potential energy of inflaton

   - Background is almost de Sitter

2. Oscillating epoch

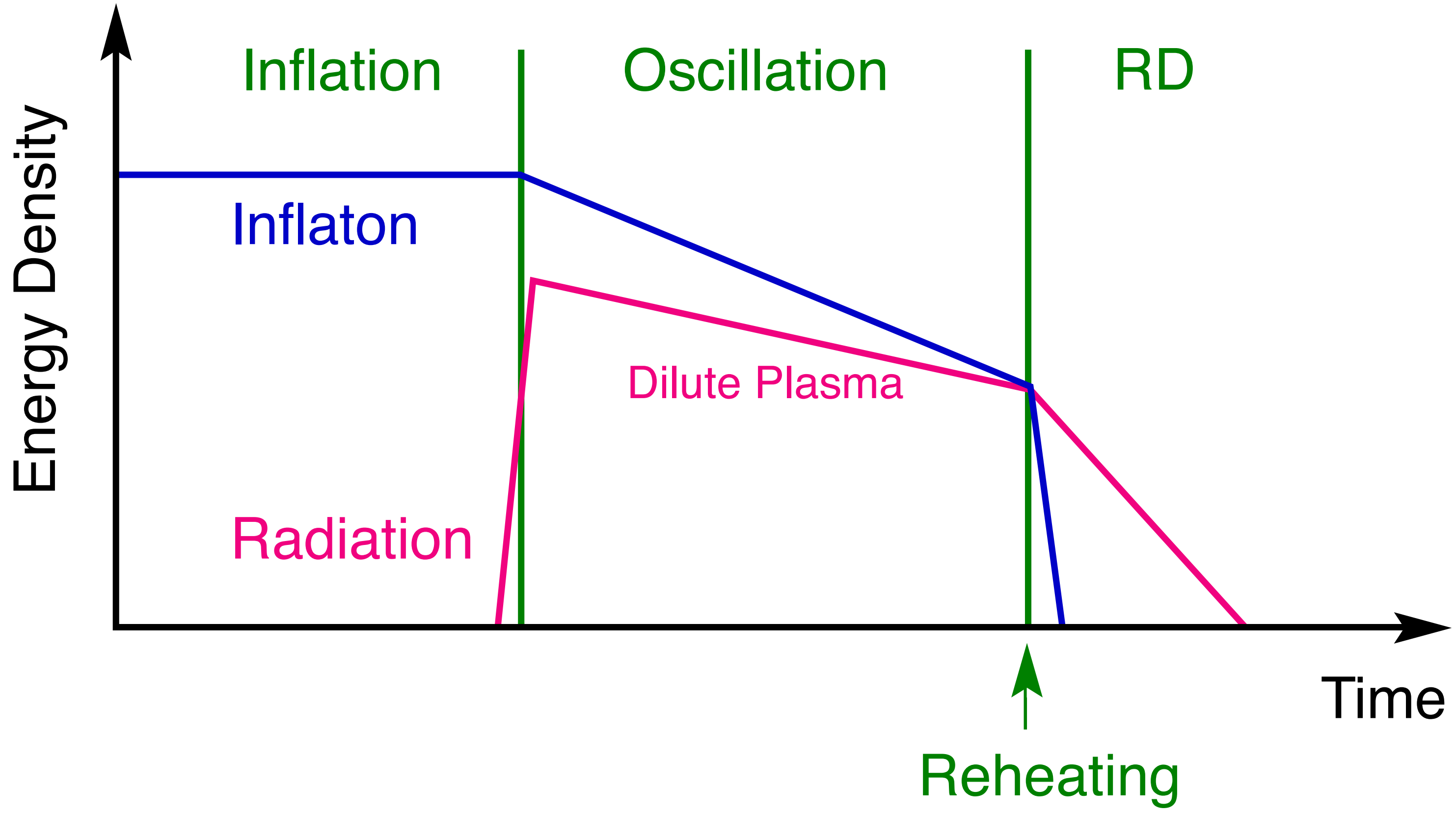
   - Inflaton field oscillates around the vacuum

3. Reheating

   - Inflaton field decays

   - Energy density of inflaton is converted to that of radiation, and radiation-dominated universe is realized

Evolution of energy densities



In the de Sitter phase, the universe expands rapidly

$\Rightarrow$ Unwanted relics are diluted at the reheating

The de Sitter phase should continue long enough to solve the horizon problem

$\Rightarrow$ How long should inflation continue?

We usually use "e-folding number"

$$N_e(t) = \ln \frac{a_{\rm end}}{a(t)} = \int_t^{t_{\rm end}} H dt$$

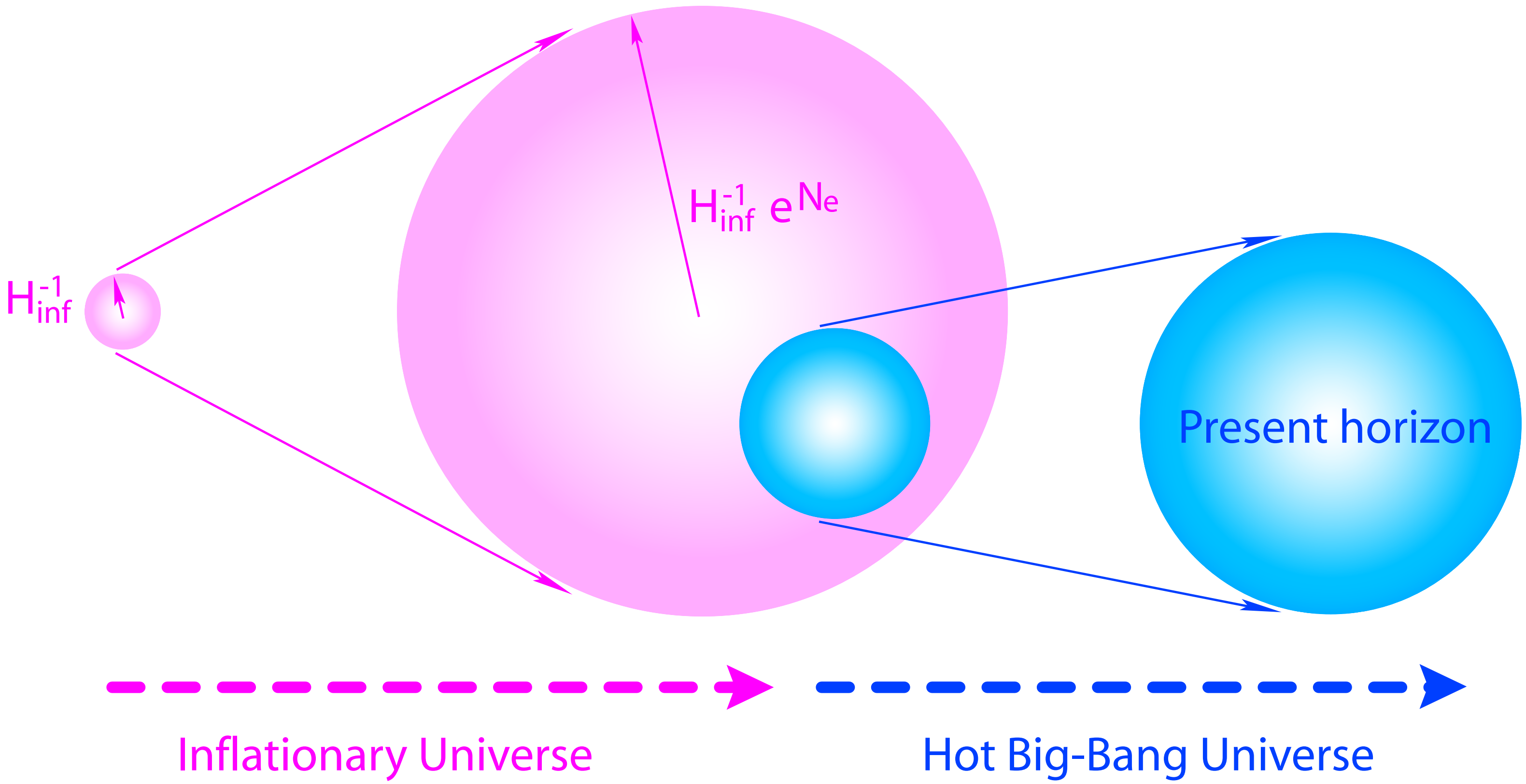
$a_{\rm end}$: scale factor at the end of inflation

Total e-foldings:

$$N_e^{(\rm tot)} = \ln \frac{a_{\rm end}}{a_{\rm start}}$$

Size of the causally connected region after inflation: $H_{\text{inf}}^{-1} e^{N_e}$

$\Rightarrow$ Physical length corresponding to the present horizon scale should be smaller than $H_{\text{inf}}^{-1} e^{N_e}$



Entropy in the causally connected region at $T \sim T_R$

- Assume instantaneous reheating (for simplicity)

$$T_{\mathsf{R}}^4 \sim \rho_{\mathsf{inf}} \sim M_{\mathsf{Pl}}^2 H_{\mathsf{inf}}^2$$

- Total entropy in the causally connected region

$$S_{\mathsf{C}} \sim T_{\mathsf{R}}^3 (H_{\mathsf{inf}}^{-1} e^{N_e})^3 \sim \left(\frac{M_{\mathsf{Pl}}}{H_{\mathsf{inf}}} \right)^3 e^{3N_e}$$

In order to solve the horizon problem

$$S_{\mathsf{C}} \gtrsim S_{\mathsf{now}}$$

- $S_{\mathsf{now}} \sim 10^{88}$: Entropy within the current horizon
- Adiabatic expansion of the universe is assumed

Lower bound on the e-foldings:

$$N_e \gtrsim 60 + \frac{1}{2} \ln \frac{H_{\mathsf{inf}}}{10^{13}\ \mathsf{GeV}}$$

If we take account of the oscillation epoch after inflation

$$N_e \gtrsim 60 + \frac{1}{3} \ln \frac{H_{\mathsf{inf}}}{10^{13}\ \mathsf{GeV}} + \frac{1}{3} \ln \frac{T_{\mathsf{R}}}{10^{16}\ \mathsf{GeV}}$$

Our horizon scale exits the horizon when $N_e \sim 25 - 60$

$\Rightarrow$ Universe expands significantly during inflation

The reheating temperature is related to the decay rate of inflaton field

- Before the reheating, the energy density of the universe is dominated by the (oscillating) inflaton

  $\Rightarrow \rho_{\chi_{\rm inf}} \propto a^{-3}$

- At $H \sim \Gamma_{\rm inf}$, inflaton decays (reheating)

Reheating temperature

$$\Gamma_{\rm inf} \sim H\big|_{T=T_{\rm R}} \sim \sqrt{\frac{\rho_\chi}{M_{\rm Pl}^2}} \sim \frac{T_{\rm R}^2}{M_{\rm Pl}} \quad \Rightarrow \quad T_{\rm R} \sim \sqrt{M_{\rm Pl}\Gamma_{\rm inf}}$$

Notice that $T_{\rm R}$ is model-dependent

The reheating temperature should be low enough to dilute unwanted relics

$\Rightarrow$ Constraints on inflation model

Monopole

- $T_{\mathsf{R}} \lesssim M_{\mathsf{GUT}}$
- In SUSY GUT, $T_{\mathsf{R}} \lesssim 10^{16}$ GeV

Gravitino (superpartner of graviton)

- Severe upper bound on T_{R}

These problems can be solved by assuming late-time entropy production

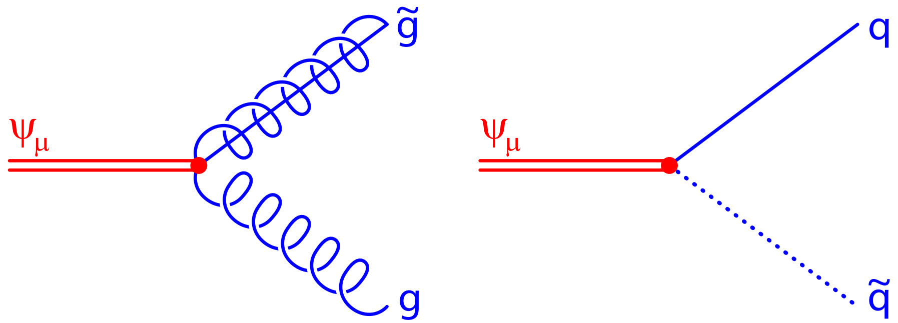
Gravitino problem

[Weinberg (1982)]

- In supersymmetric model, there exists superpartner of the graviton, called gravitino
- Interaction of gravitino is very weak
- Gravitinos are produced in the early universe
- Gravitinos decay during the BBN epoch because its lifetime is extremely long

Interaction of gravitino

- Gravitino couples to supercurrent
- Interaction of graivitino is Planck suppressed



Gravitino is produced by the scatterings of MSSM particles

- $g + g \to \psi_\mu + \tilde{g}, \ \cdots$

Gravitino has very long lifetime (if unstable)

$$\tau_{3/2}(\psi_\mu \to g + \tilde{g}) \simeq 50 \text{ sec} \times \left(\frac{m_{3/2}}{10 \text{ TeV}} \right)^{-3}$$

Gravitino production after the inflation

- Gravitino production rate

$$\langle \sigma_{\text{prod}} v_{\text{rel}} \rangle \sim \frac{\alpha_{\text{gauge}}}{M_{\text{Pl}}^2} \quad \Rightarrow \quad \Gamma_{\text{prod}} \sim \langle \sigma_{\text{prod}} v_{\text{rel}} \rangle T^3$$

- Yield variable (s: entropy density)

$$Y_{3/2} \equiv \frac{n_{3/2}}{s} \sim \Gamma_{\text{prod}} H^{-1} \to \frac{\alpha_{\text{gauge}} T_{\text{R}}}{M_{\text{Pl}}}$$

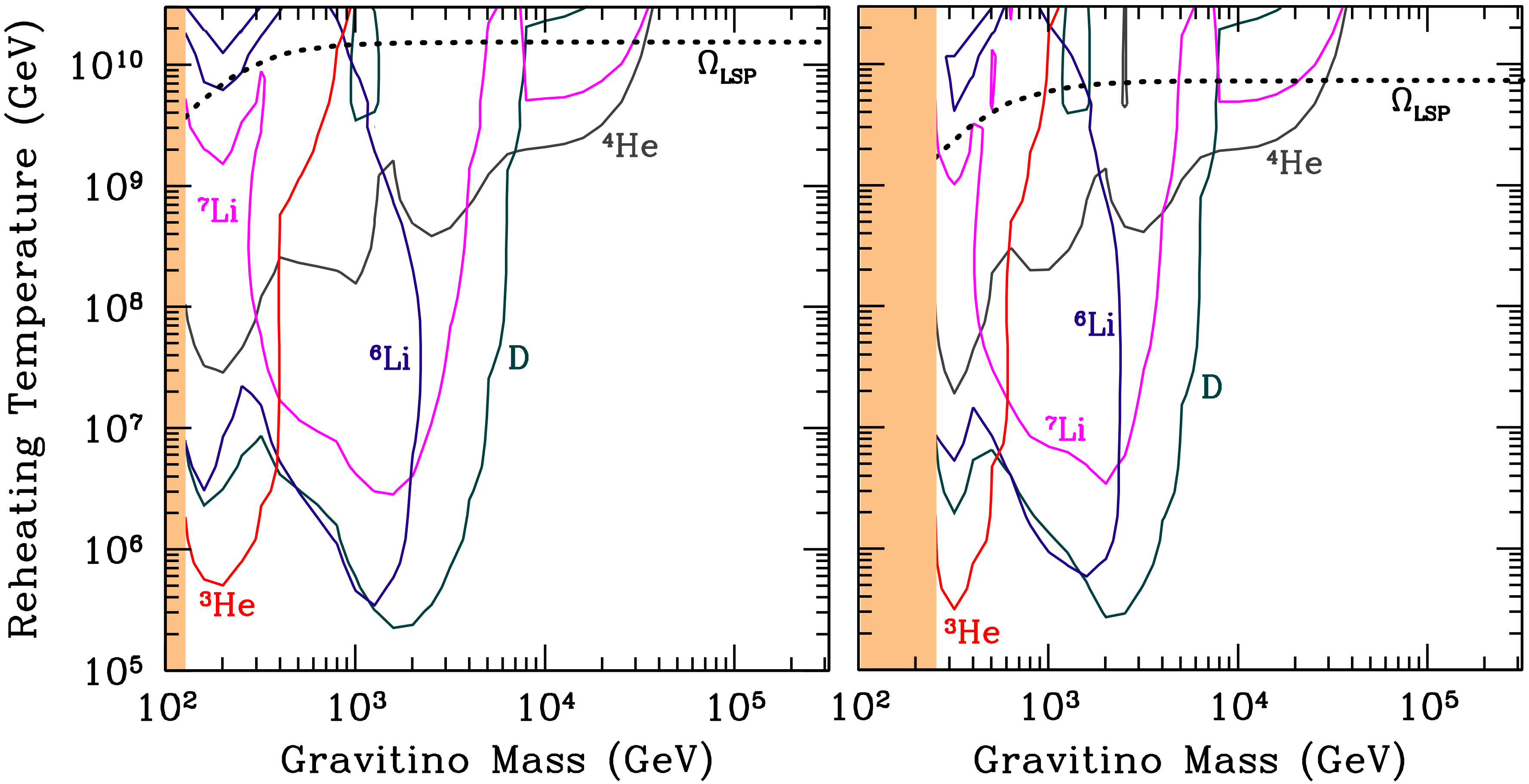
$\Rightarrow$ (Gravitino abundance) $\propto$ (reheating temperature T_{R})

Result of the detailed calculation

[with $\langle \sigma_{\text{prod}} v_{\text{rel}} \rangle$ by Bolz, Brandenburg & Buchmuller]

$$\frac{n_{3/2}}{s} \simeq 1.9 \times 10^{-12} \times \left(\frac{T_{\text{R}}}{10^{10}\ \text{GeV}} \right) \quad \text{with} \quad T_{\text{R}} \equiv \left(\frac{10}{g_* \pi^2} M_{\text{Pl}}^2 \Gamma_\Phi^2 \right)^{1/4}$$

In order not to spoil the success of the BBN, T_{R} is bounded from above



[Kawasaki, Kohri, TM, Yotsuyanagi (2008)]

Chaotic inflation

[Linde]

Inflation may occur even with parabolic (or polynomial) potential

$$V_{\text{inf}} = \frac{1}{2} m_\chi^2 \chi_{\text{inf}}^2$$

V_{inf} becomes larger than kinetic energy if $\chi_{\text{inf}} \gg M_{\text{Pl}}$

$\Rightarrow$ Chaotic inflation (more to be discussed later)

As time goes on, χ_{inf} becomes smaller than M_{Pl}

$\Rightarrow$ Slow-roll condition is not satisfied any longer

$\Rightarrow$ Inflation ends

Chaotic inflation with parabolic potential

$$V_{\mathsf{inf}} = \frac{1}{2} m_{\mathsf{inf}}^2 \chi_{\mathsf{inf}}^2 \quad \Rightarrow \quad \epsilon = \eta = \frac{2M_{\mathsf{Pl}}^2}{\chi_{\mathsf{inf}}^2}$$

Slow-roll condition is satisfied when $\chi_{\mathsf{inf}} \gg M_{\mathsf{Pl}}$

$\Rightarrow$ Can we use the field theory beyond the Planck scale?

From the observations of the cosmic density fluctuations, m_{inf} is determined

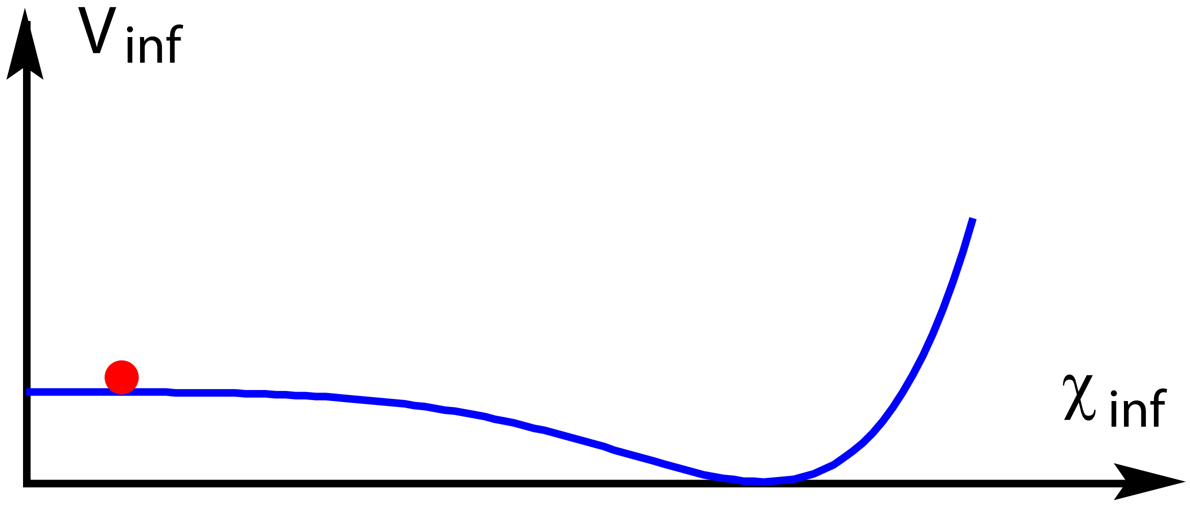
$$m_{\mathsf{inf}} \sim 10^{13}\ \mathsf{GeV}$$

Reheating with Planck-suppressed interaction

$$\Gamma_{\mathsf{inf}} \sim \frac{m_{\mathsf{inf}}^3}{M_{\mathsf{Pl}}^2} \quad \Rightarrow \quad T_{\mathsf{R}} \sim 10^{10}\ \mathsf{GeV} \times \left[\frac{m_{\mathsf{inf}}}{10^{13}\ \mathsf{GeV}}\right]^{3/2}$$

New inflation

Inflaton starts from a "top of a hill"



One might ask: Why such an initial condition?

$\Rightarrow$ Maybe, thermal effect (or ???)

$\Rightarrow$ Otherwise, one may go to the "chaotic" scenario

During inflation, scalar fields (and also metric) fluctuate

$\Rightarrow$ Origin of the density fluctuation of the universe

Equations of motion of a scalar field ϕ during inflation

$$\ddot{\phi} + 3H_{\mathsf{inf}}\dot{\phi} + \frac{k^2}{a^2}\phi + m_\phi^2\phi = 0$$

With $\nu^2 = 9/4 - m_\phi^2/H_{\mathsf{inf}}^2$ and $d\tau = adt$

$$\phi(\tau, \vec{x}) = \frac{\sqrt{-\pi\tau}}{2a}\int\frac{d^3k}{(2\pi)^{3/2}} \times \left[a_k H_\nu^{(1)}(-k\tau)e^{i\vec{k}\vec{x}} + a_k^\dagger H_\nu^{(2)}(-k\tau)e^{-i\vec{k}\vec{x}}\right]$$

$H_\nu^{(1)}(-k\tau) \propto e^{-ik\tau}$ for $k \to \infty$

Quantization

We identify a_k as annihilation operator: $a_k|0\rangle_{\text{in}} = 0$

Two point function

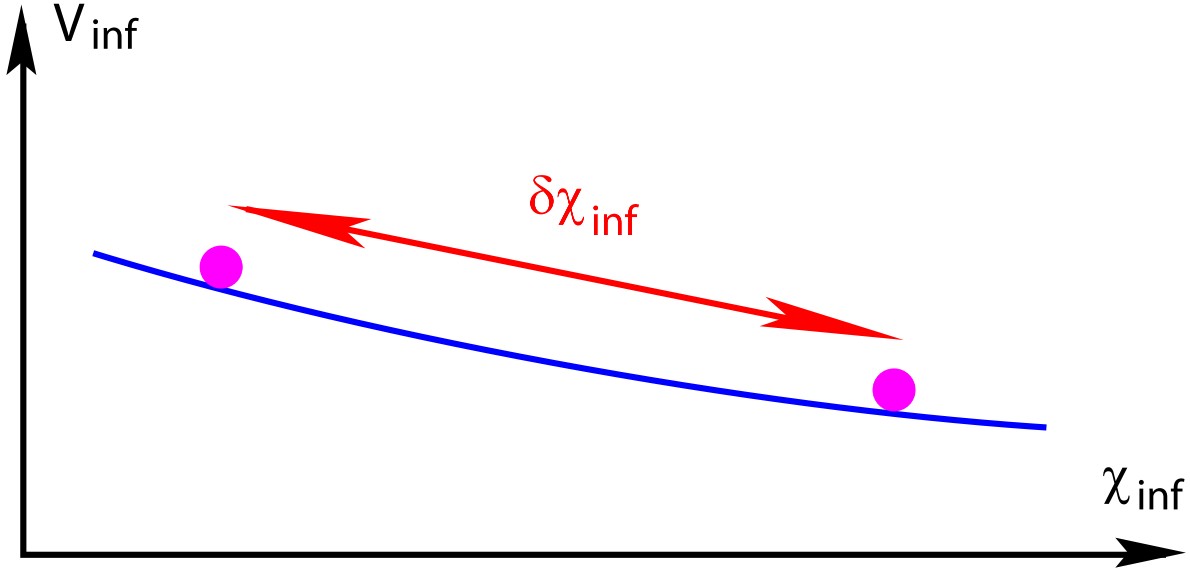
$$\langle 0|\delta\phi(t,\vec{x})\delta\phi(t,\vec{y})|0\rangle = \frac{1}{4\pi}\int d\log k d\Omega_k |\delta\phi(k)|^2 e^{i\vec{k}(\vec{x}-\vec{y})}$$

$\Rightarrow$ For the long-wavelength limit

$$\delta\phi(k) = \left(\frac{k_{\text{phys}}}{2H_{\text{inf}}}\right)^{2m_\phi^2/3H_{\text{inf}}^2} \frac{H_{\text{inf}}}{2\pi}$$

- When $m_\phi \ll H_{\text{inf}}$, $\delta\phi(k) \simeq H_{\text{inf}}/2\pi$
- When $m_\phi \gtrsim H_{\text{inf}}$, $\delta\phi(k) \simeq 0$

Different region has different e-folding at the end of inflation (if we go back to the coordinate space)



$$\delta N_e(k) = H_{\text{inf}} \delta t(k) = H_{\text{inf}} \frac{\delta \chi_{\text{inf}}}{\dot{\chi}_{\text{inf}}}$$

$\delta N_e(k)$ becomes the origin of cosmic density fluctuations

$\Rightarrow$ Fluctuation of CMB

$\Rightarrow$ Density fluctuations (galaxies, clusters)

Fluctuation is often parametrized by curvature perturbation

$$\mathcal{R}(k) = H_{\mathrm{inf}} \frac{\delta \chi_{\mathrm{inf}}}{\dot{\chi}_{\mathrm{inf}}} = \left[\frac{H_{\mathrm{inf}}}{2\pi} \frac{3H_{\mathrm{inf}}^2}{V'} \right]_{k=aH}$$

- Curvature perturbation for the wavelength k^{-1} is evaluated when such scale exits the horizon during inflation
- $\mathcal{R}(k)$ determines the size and the scale-dependence of density fluctuations

During the slow-roll inflation, H and V' changes

$\Rightarrow$ Scale dependence of $\mathcal{R}$

Scale dependence is often approximated as

$$\langle \mathcal{R}^2(k) \rangle \simeq \langle \mathcal{R}^2(k_0) \rangle \left(\frac{k}{k_0} \right)^{n_S - 1}$$

Notice: observables are second (or higher) order in $\mathcal{R}$

n_S is called (scalar) spectral index

- n_S is usually approximated by a constant
- n_S is close to 1 in most of inflation models
  (Scale invariant density fluctuation)

Estimation of the spectral index: when $n_S \sim 1$

$$\langle \mathcal{R}^2(k) \rangle \simeq \langle \mathcal{R}^2(k_0) \rangle \left[1 + (n_S - 1) \ln \frac{k}{k_0} + \cdots \right]$$

Expansion of $\langle \mathcal{R}^2(k) \rangle$

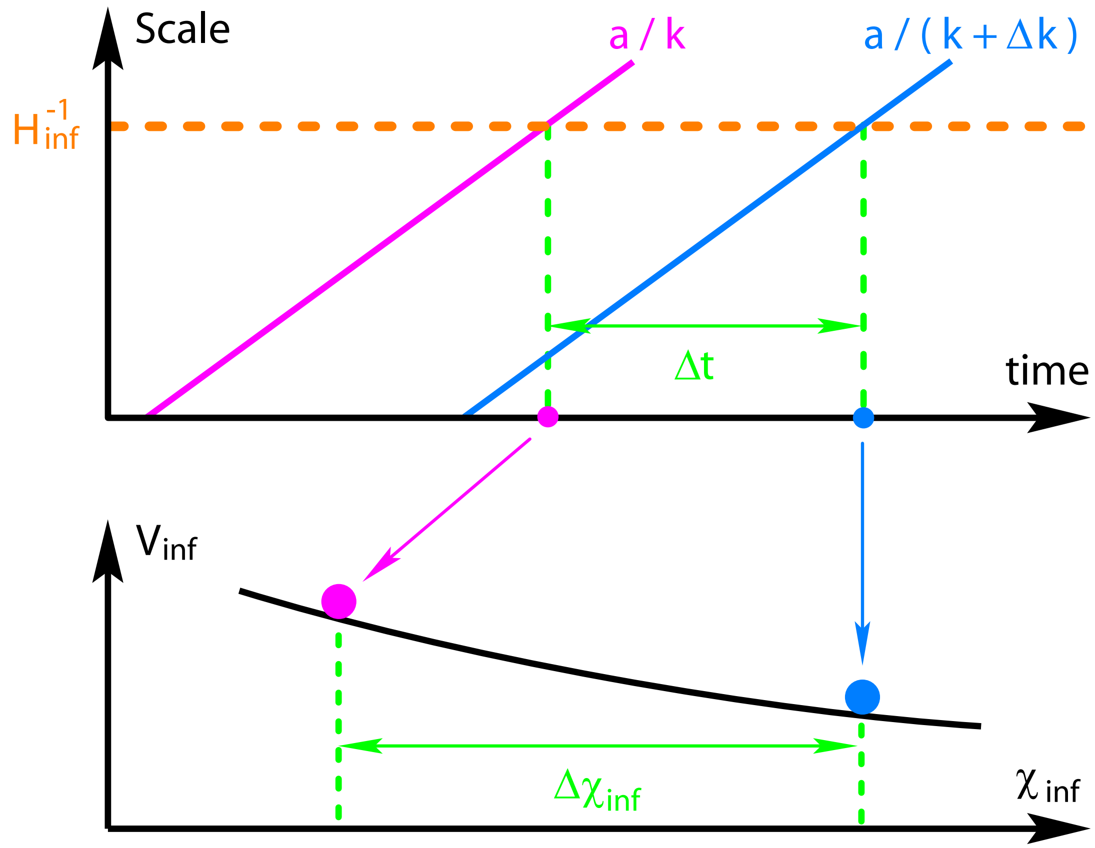
$$\begin{aligned} \langle \mathcal{R}^2(k) \rangle &= \langle \mathcal{R}^2(k_0) \rangle \left[1 + \frac{1}{\mathcal{R}^2(k_0)} \frac{d\langle \mathcal{R}^2(k) \rangle}{d \ln k} \ln \frac{k}{k_0} + \cdots \right] \\ &= \langle \mathcal{R}^2(k_0) \rangle \left[1 + \frac{d\chi_{\mathsf{inf}}(k)}{d \ln k} \frac{d \ln \langle \mathcal{R}^2(k) \rangle}{d\chi_{\mathsf{inf}}(k)} \ln \frac{k}{k_0} + \cdots \right] \end{aligned}$$

$\chi_{\mathsf{inf}}(k)$: amplitude when the mode k exits the horizon

Spectral index is obtained by comparing above two relations

Derivative of $\chi_{\rm inf}(k)$ with respect to k?

$\Rightarrow$ We have to know the inflaton amplitude when the mode k exits the horizon



From the figure, we can see

$$H_{\mathsf{inf}}^{-1} = \frac{a(t)}{k} = \frac{a(t+\Delta t)}{k+\Delta k} \quad \Rightarrow \quad \frac{\Delta k}{k} = \frac{\dot{a}}{a}\Delta t \quad \Rightarrow \quad \Delta \ln k = H \Delta t$$

In addition

$$\Delta t = \frac{\Delta \chi_{\mathsf{inf}}}{\dot{\chi}_{\mathsf{inf}}} \quad \Rightarrow \quad \frac{d\chi_{\mathsf{inf}}}{d\ln k} = \frac{\dot{\chi}_{\mathsf{inf}}}{H_{\mathsf{inf}}} \simeq -M_{\mathsf{Pl}}^2 \frac{V'_{\mathsf{inf}}}{V_{\mathsf{inf}}}$$

Derivative of $\langle \mathcal{R}^2 \rangle$ with respect to χ_{inf}

$$\frac{d\ln\langle \mathcal{R}^2 \rangle}{d\chi_{\mathsf{inf}}} = \frac{d\ln(V_{\mathsf{inf}}^3/V_{\mathsf{inf}}'^2)}{d\chi_{\mathsf{inf}}} = \frac{3V'_{\mathsf{inf}}}{V_{\mathsf{inf}}} - \frac{2V''_{\mathsf{inf}}}{V'_{\mathsf{inf}}}$$

Spectral index

$$n_S - 1 = \frac{d\chi_{\mathsf{inf}}}{d\ln k}\frac{d\ln\langle\mathcal{R}^2\rangle}{d\chi_{\mathsf{inf}}} = -M_{\mathsf{Pl}}^2\frac{V'_{\mathsf{inf}}}{V_{\mathsf{inf}}}\left[\frac{3V'_{\mathsf{inf}}}{V_{\mathsf{inf}}} - \frac{2V''_{\mathsf{inf}}}{V'_{\mathsf{inf}}}\right]$$

n_S can be expressed with the slow-roll parameters

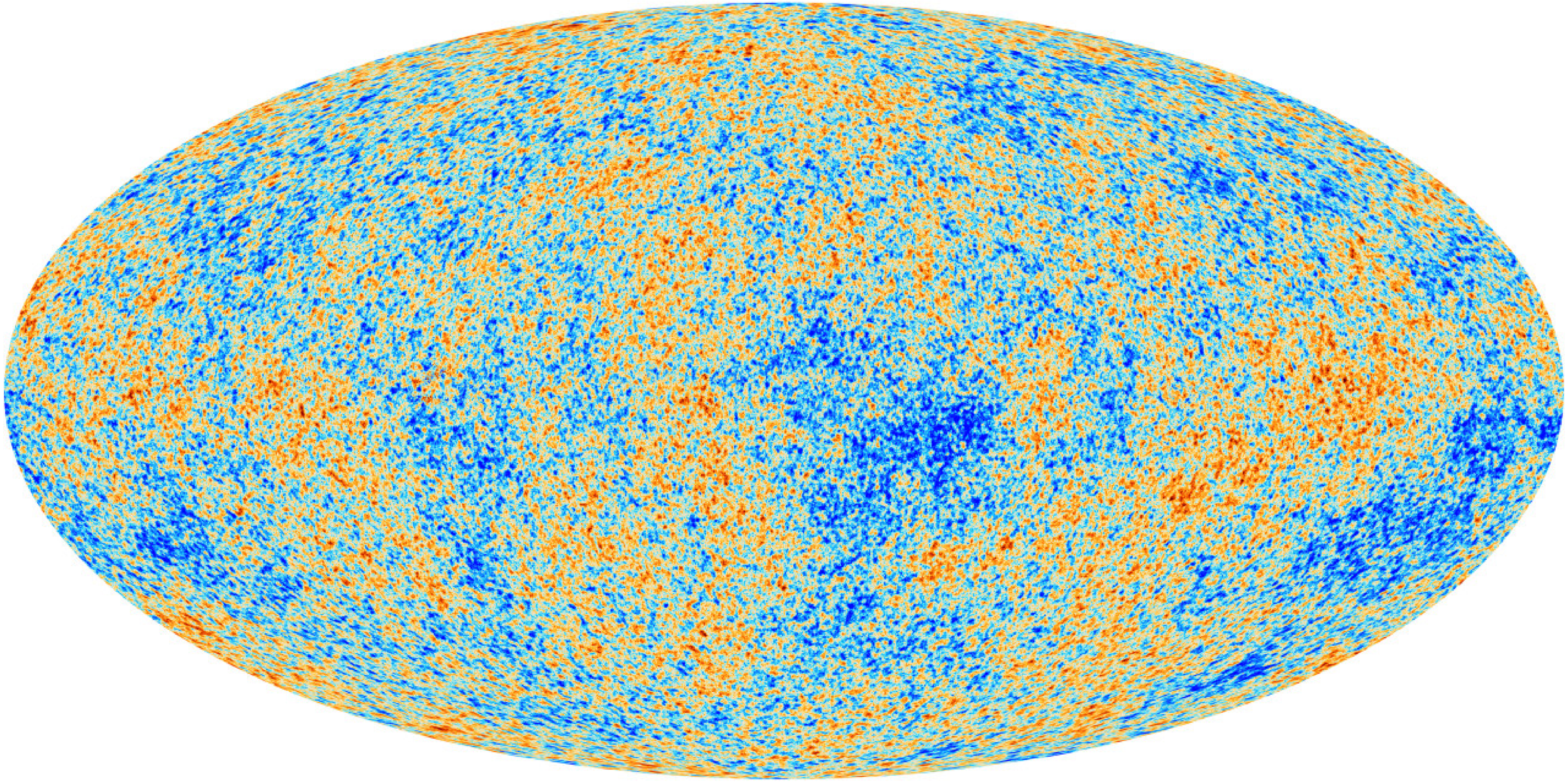
$$n_S - 1 = -6\epsilon + 2\eta$$

During inflation, ϵ and η are small

$\Rightarrow$ Density fluctuation from the slow-roll inflation has weak scale-dependence

- ϵ: change of H_{inf}
- η: change of V'_{inf}

We learn a lot about $\mathcal{R}(k)$ from CMB fluctuations



Planck Collaboration [http://www.cosmos.esa.int/web/planck]

CMB angular power spectrum

$$\Delta T(\vec{x},\vec{\gamma}) = \sum_{l,m} a_{l,m}(\vec{x}) Y_{l,m}(\vec{\gamma}) \quad \Rightarrow \quad C_l = \frac{1}{2l+1}\sum_m |a_{l,m}|^2$$

Notice:

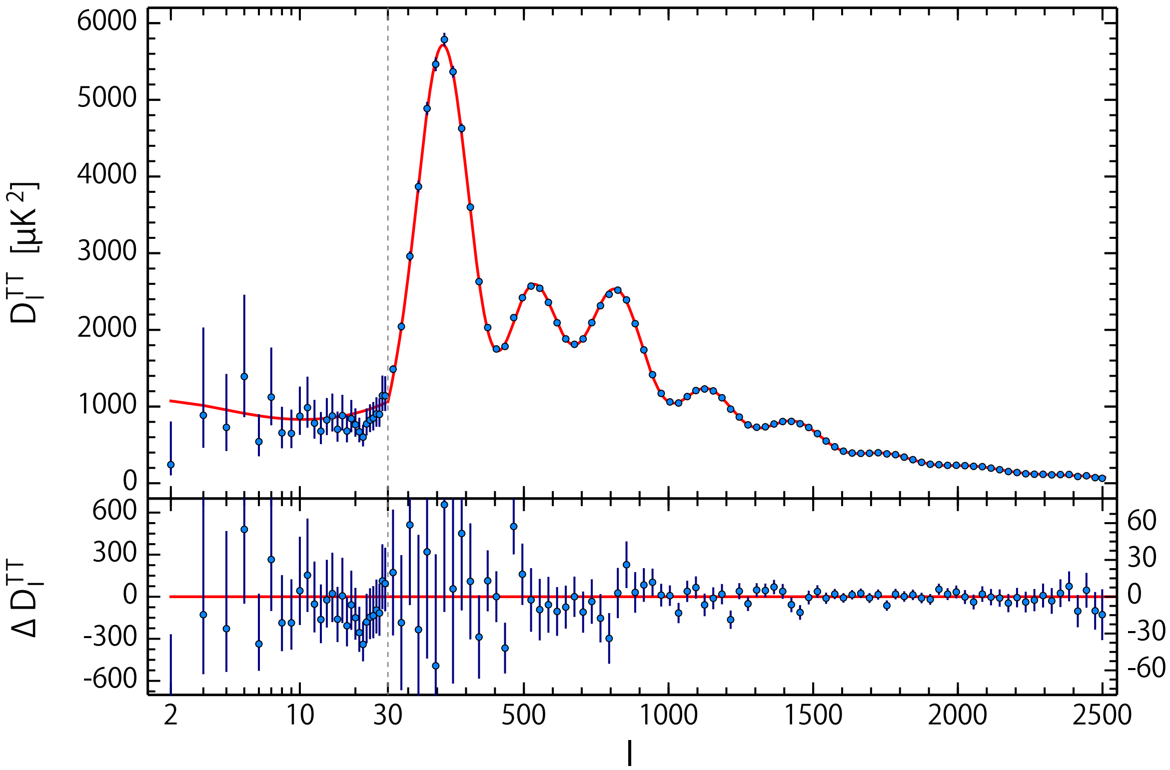
- C_l is second order in the fluctuation
- $C_l \sim (\Delta T)^2$ for the angular scale $\theta \sim \pi/l$

Sometimes, we also use

$$\mathcal{D}_l \equiv \frac{l(l+1)}{2\pi} C_l$$

Result of Planck 2015

[Planck Collaboration, 1502.01589]



$$\mathcal{D}_l \equiv \frac{l(l+1)}{2\pi} C_l$$

$\Rightarrow$ Very good agreement with the prediction of inflation (with a proper choice of cosmological parameters)

CMB angular power spectrum contains rich information

- Density perturbations
- Dark radiation, dark matter, baryon, and CDM densities
- Other cosmological parameters

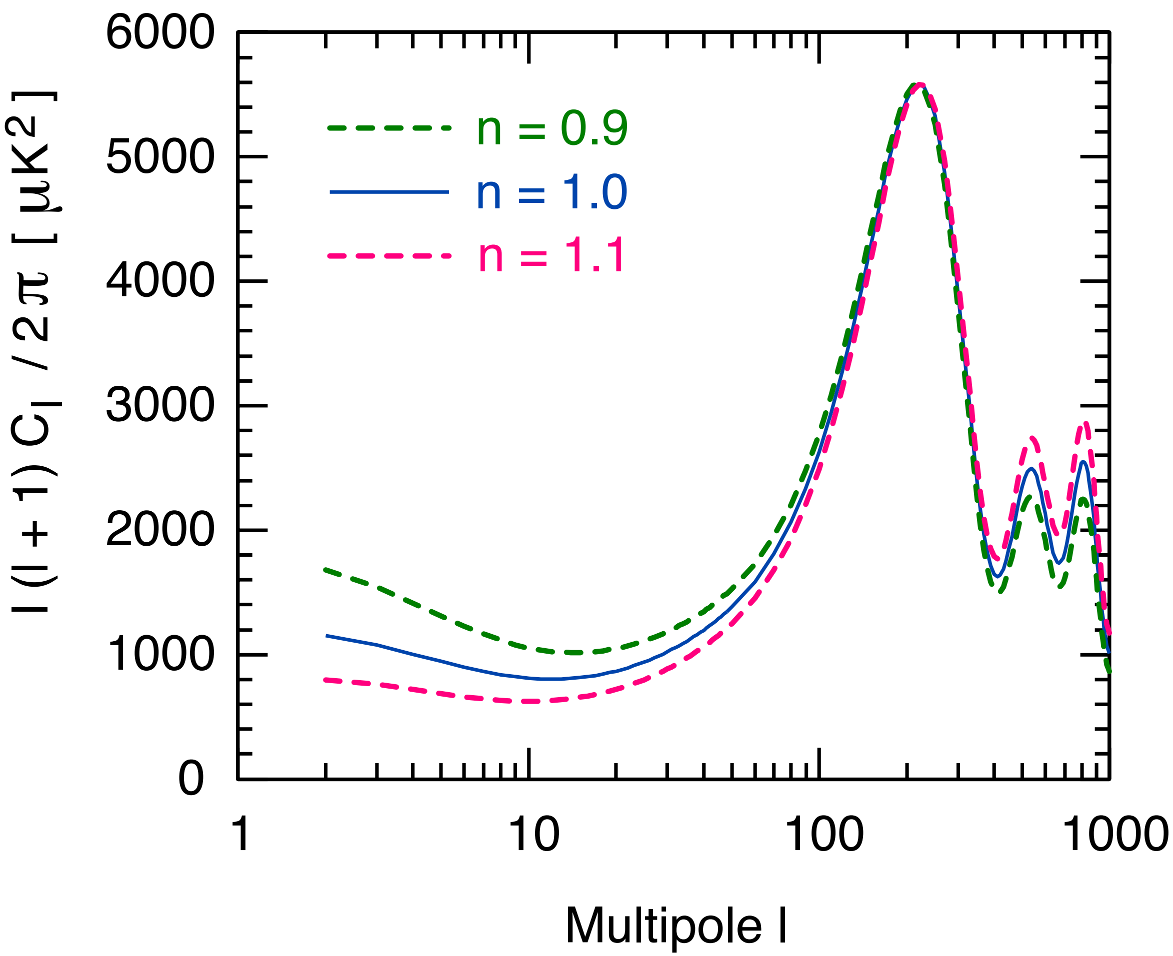
Amplitude of the scalar perturbation

[Planck Collaboration, 1502.01589]

$$A_{\mathsf{S}} = \langle \mathcal{R}^2(k = 0.05\ \mathsf{Mpc}^{-1}) \rangle = (2.142 \pm 0.049) \times 10^{-9}$$

Prediction of inflation: scale-invariant density fluctuation

$\Leftrightarrow C_l$ is sensitive to n_S



Observed value of the scalar spectral index:

$$n_S = 0.9645 \pm 0.0049$$

[Planck (2015)]

$$\Leftrightarrow n_S = 0.961 \pm 0.017$$

[WMAP (2006)]

There are strong (but circumstantial) evidences of inflation

- Flat geometry
- Almost scale-invariant (and adiabatic) density fluctuation

Tensor perturbation

During inflation, the metric (i.e., graviton) also fluctuates

Physical d.o.f.: transverse & traceless mode

$$g_{ij} = -a^2(\delta_{ij} + h_{ij}) \text{ with } \begin{cases} \partial_i h_{ij} = 0 \\ h_{ii} = 0 \end{cases}$$

For $h_{ij} \propto e^{ikz}$, for example:

$$h_{ij} = -a^2 \begin{pmatrix} 1 + h_+ & h_\times & 0 \\ h_\times & 1 - h_+ & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Transverse & traceless mode in de Sitter space:

$$
\begin{aligned}
S &= \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{\mathsf{Pl}}^2 R-\Lambda\right] \\
&= \int d^4x a^3\sum_{\alpha}\left[\frac{1}{2}(\partial_t\tilde{h}_\alpha)^2-\frac{1}{2}a^{-2}(\partial_i\tilde{h}_\alpha)^2\right]+\cdots
\end{aligned}
$$

$$\tilde{h}_\alpha=\frac{1}{\sqrt{2}}M_{\mathsf{Pl}}h_\alpha$$

$\tilde{h}_\alpha$ behaves like canonically normalized massless scalar field

$$\langle\tilde{h}_\alpha^2\rangle=\frac{M_{\mathsf{Pl}}^2}{2}\langle h_\alpha^2\rangle=\left(\frac{H}{2\pi}\right)^2$$

Tensor-to-scalar ratio:

$$r \equiv \frac{4\langle \tilde{h}_\alpha^2 \rangle}{\langle \mathcal{R}^2 \rangle}$$

In slow-roll inflation, r is related to the slow-roll parameter:

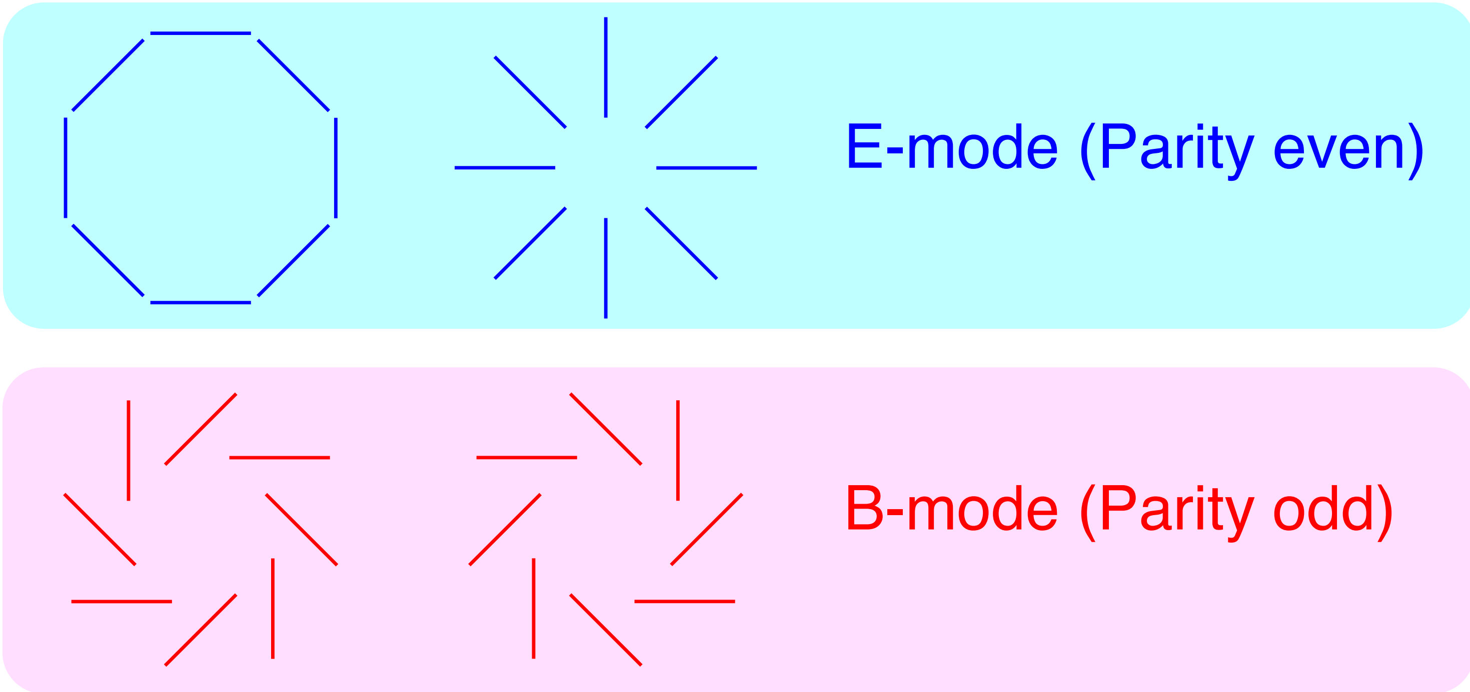
$$r = 16\epsilon$$

$$\epsilon \equiv \frac{1}{2} M_{\mathsf{Pl}}^2 \left(\frac{V'}{V} \right)^2$$

The tensor fluctuation affects

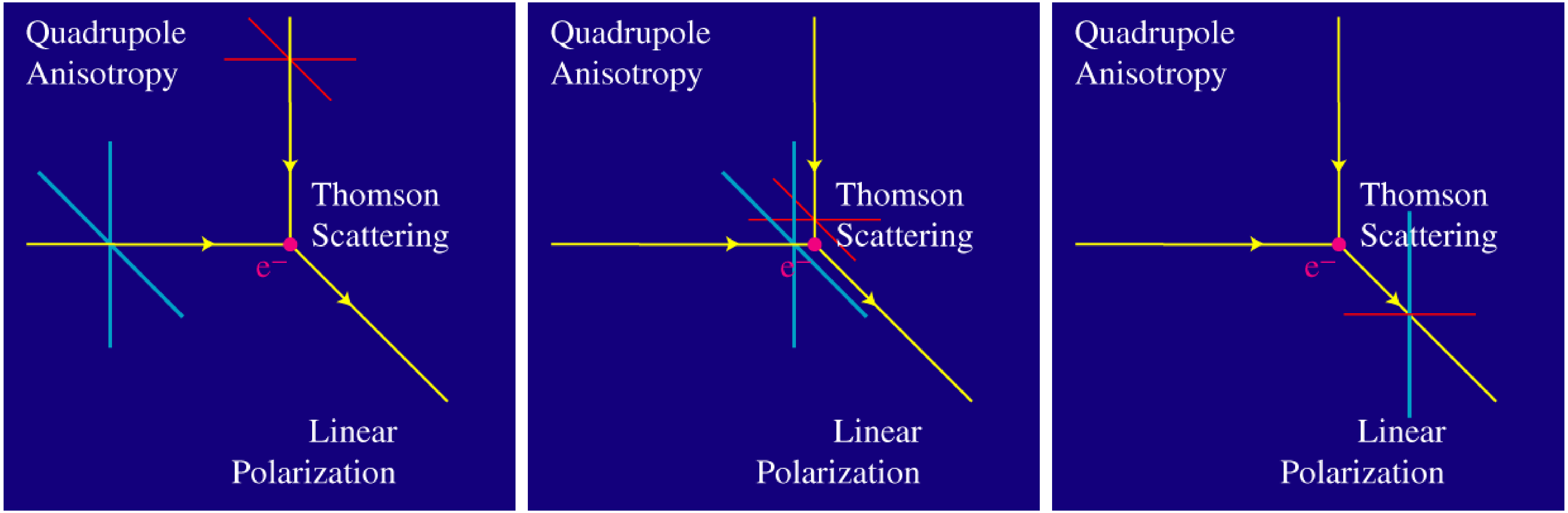
- CMB temperature fluctuations
- CMB polarization (in particular, B-mode)

Information about r is imprinted in CMB polarizations



- E-mode can be generated from temperature (scalar) perturbations
- B-mode provides important test of inflation

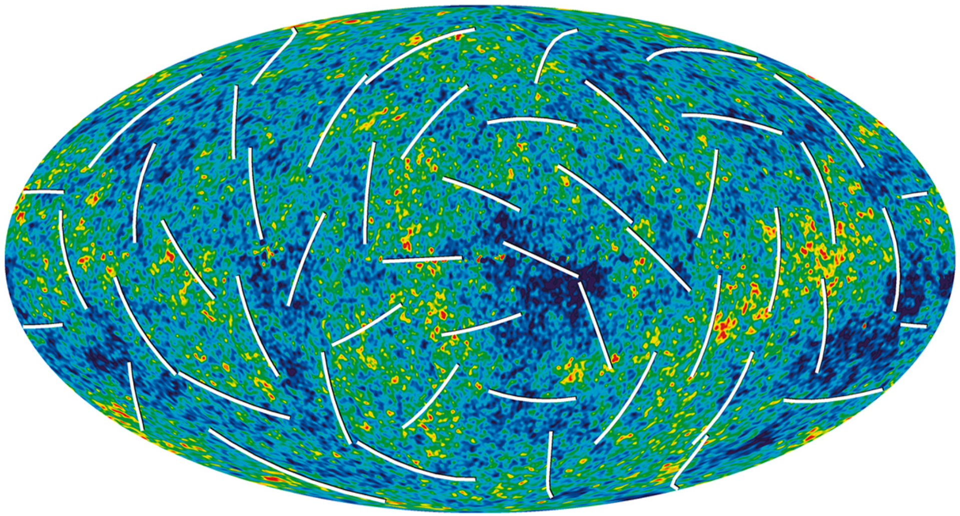
CMB polarization from temperature fluctuations



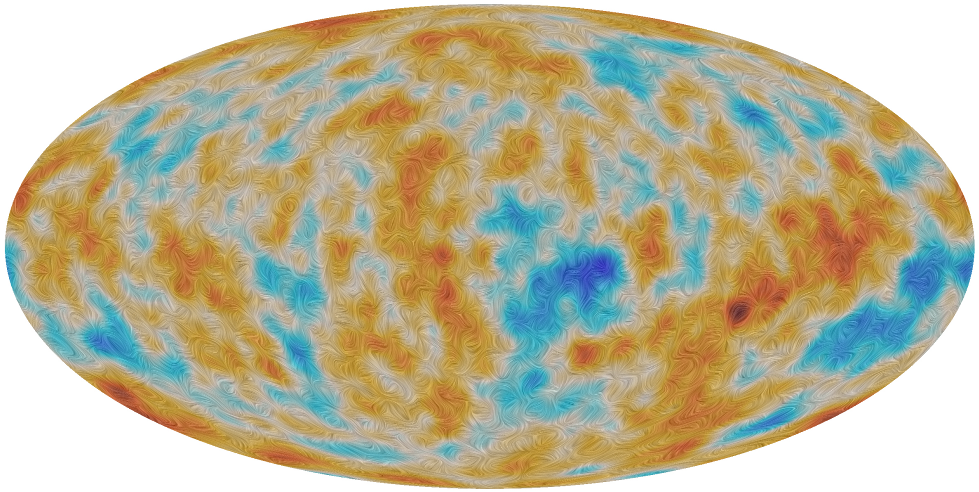
[W. Hu]

$\Rightarrow$ Temperature quadrupole anisotropy causes polarization of CMB

WMAP and Planck results



[WMAP 3 years]



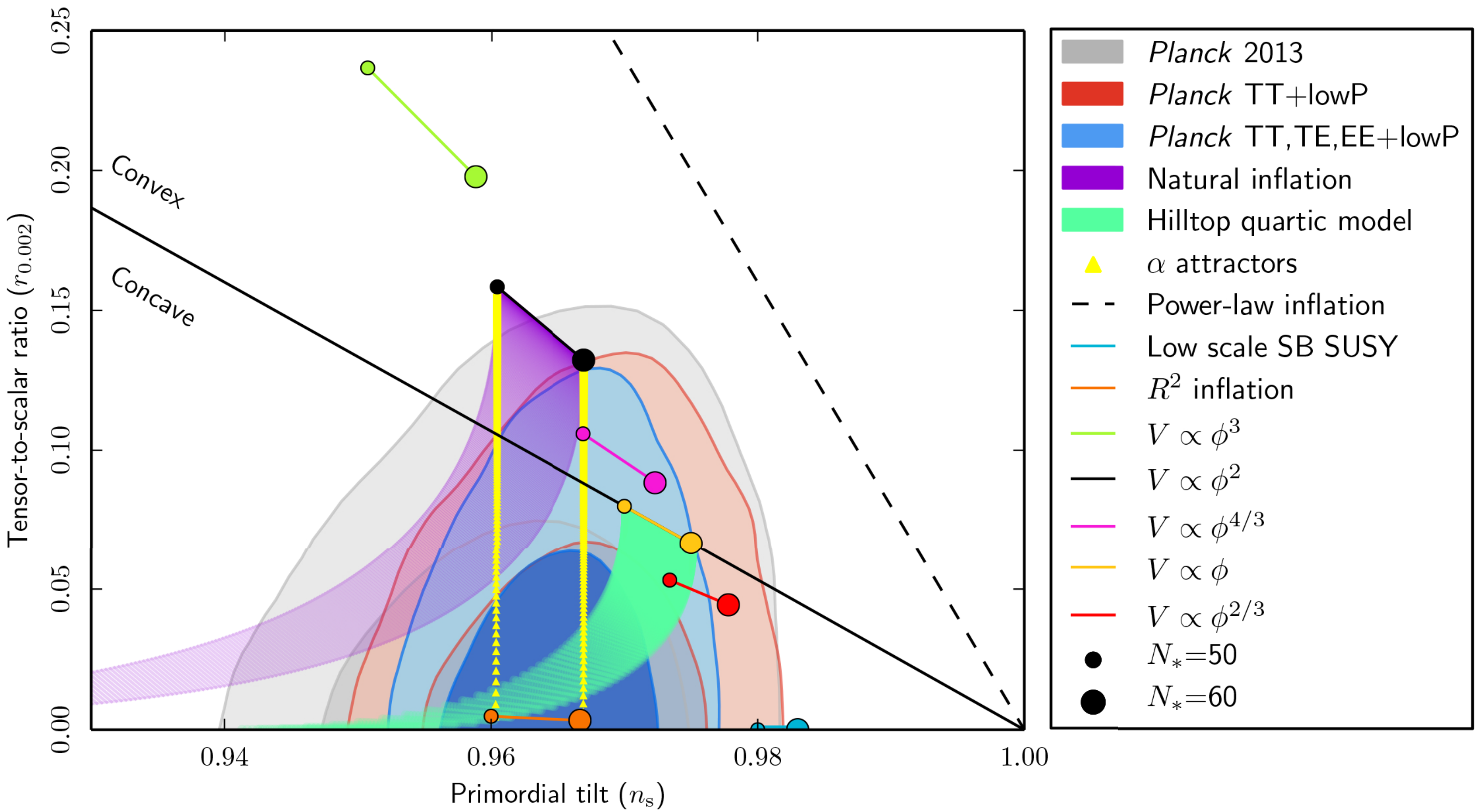
[Planck 2015]

Currently:

- E-mode signal has been already observed
- B-mode signal has not been observed yet

Current constraints on the inflation models

[Planck Collaboration, 1502.02114]



4. Dark Matter

Our universe is mostly "dark"

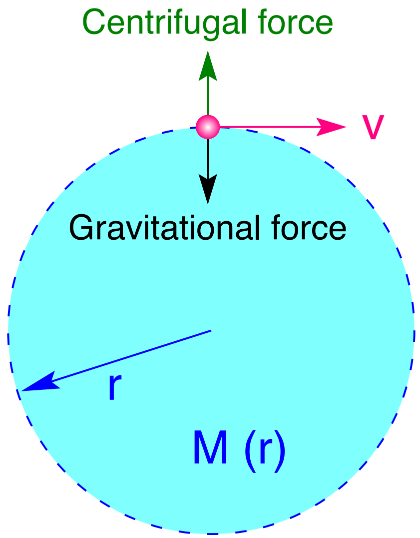
- Dark energy (cosmological constant?)
- Dark matter

Properties of dark matter

- Very weakly interacting
- Pressure is (almost) zero (like non-relativistic particles)

Nobody knows what is playing the role of dark matter in the present universe

Evidence 1: Rotation curve

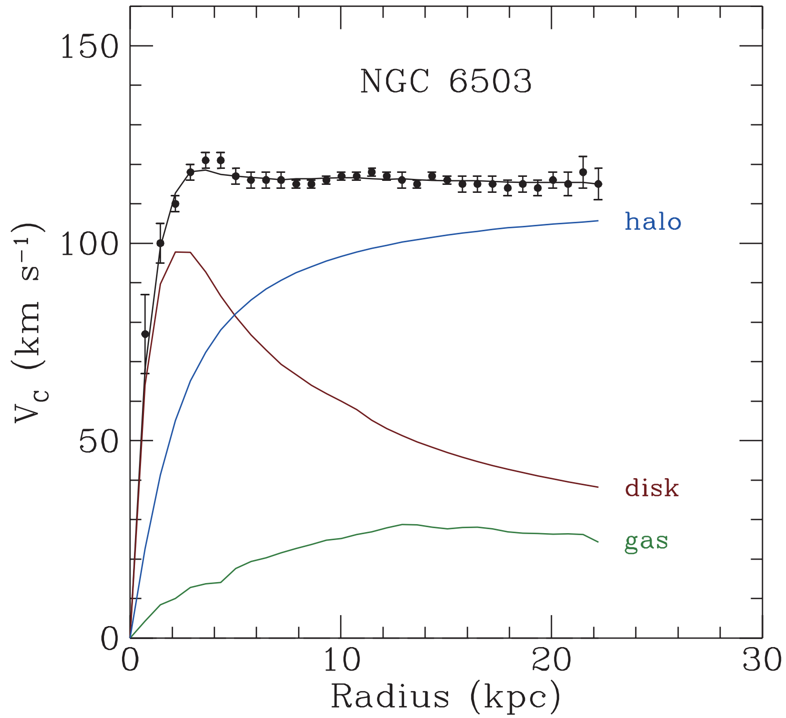


Stars (and gases) are rotating around the center of galaxy

$$v(r) \simeq \sqrt{\frac{GM(r)}{r}}$$

$M(r)$: Total mass within the radius r

We can estimate $v(r)$: usually 21cm spectral line is used

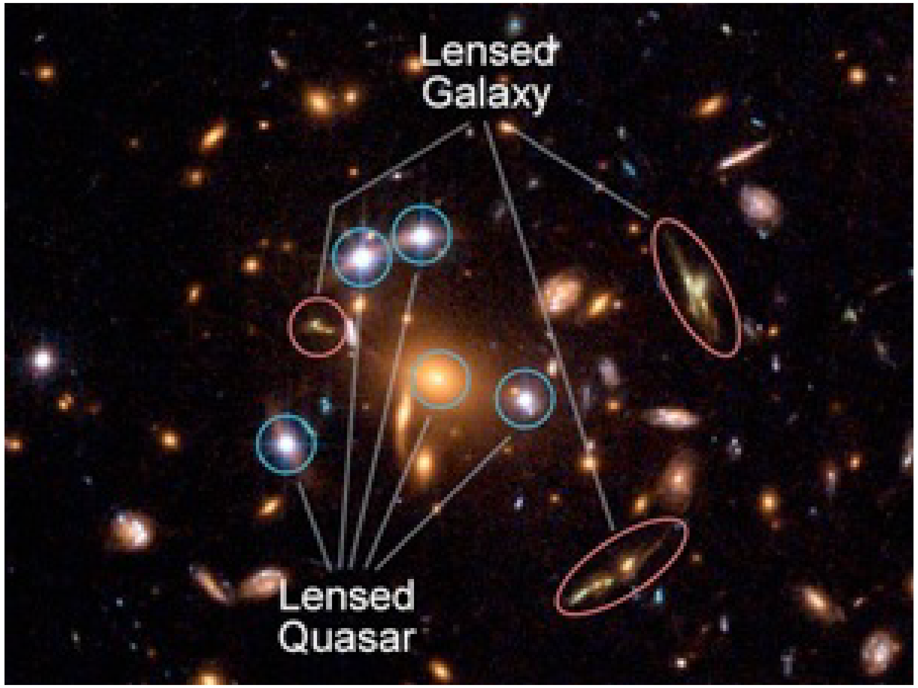


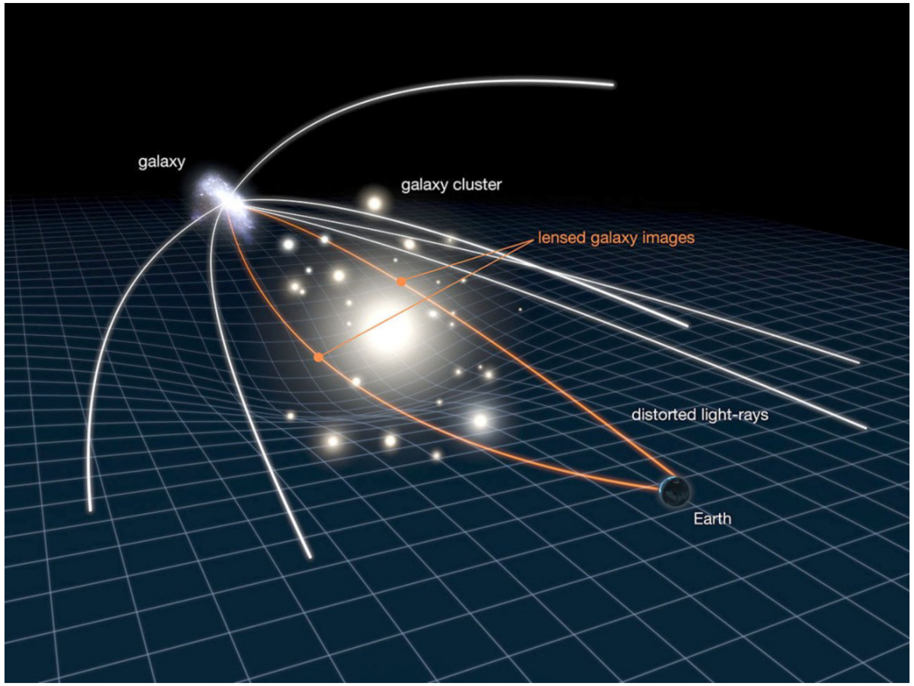
[Begeman, Broeils & Sanders, MNRAS 249 (1991) 523]

Visible objects are not enough to explain the rotation curve

$\Rightarrow$ Dark matter

Evidence 2: Gravitational lensing





[NASA / ESA]

- Visible objects are not enough to explain the lensing

  ⇒ Dark matter

Evidence 3: Bullet clusters

[HST homepage]

⇒ See also the movie

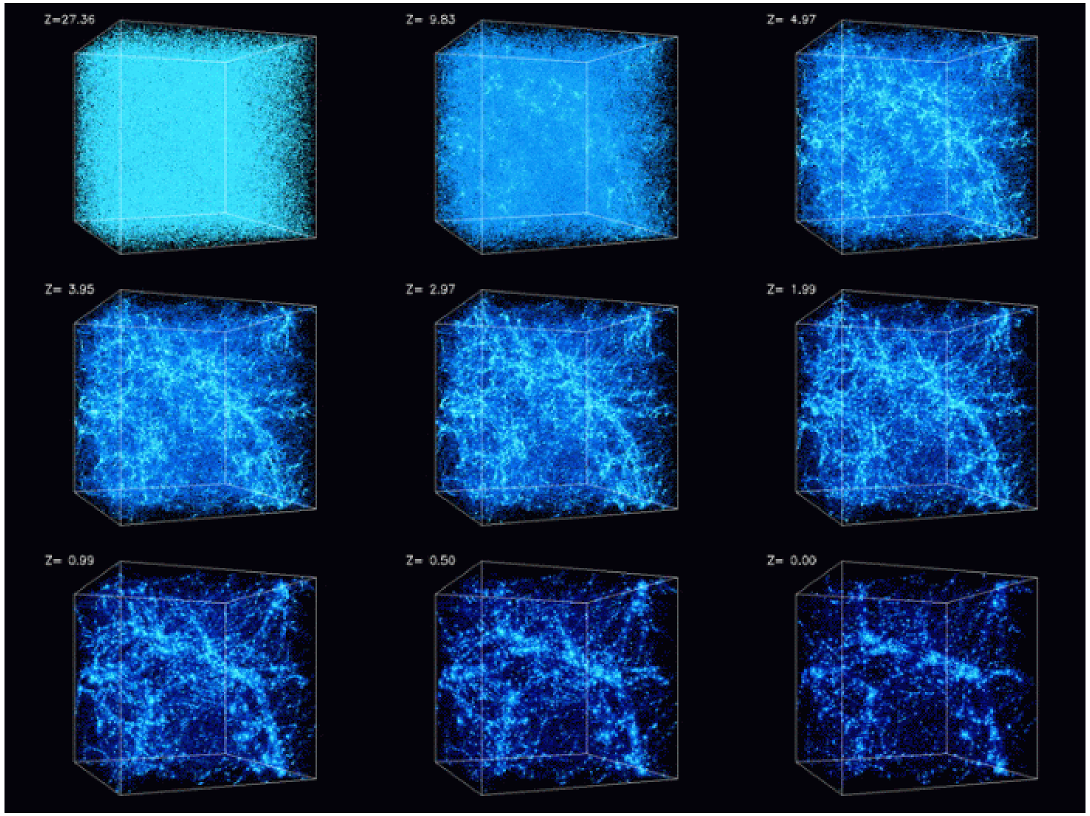
- Pink: Intense X-ray emission observed

  ⇒ Baryon

- Blue: Significant effect of gravitational lensing observed

  ⇒ Dark matter

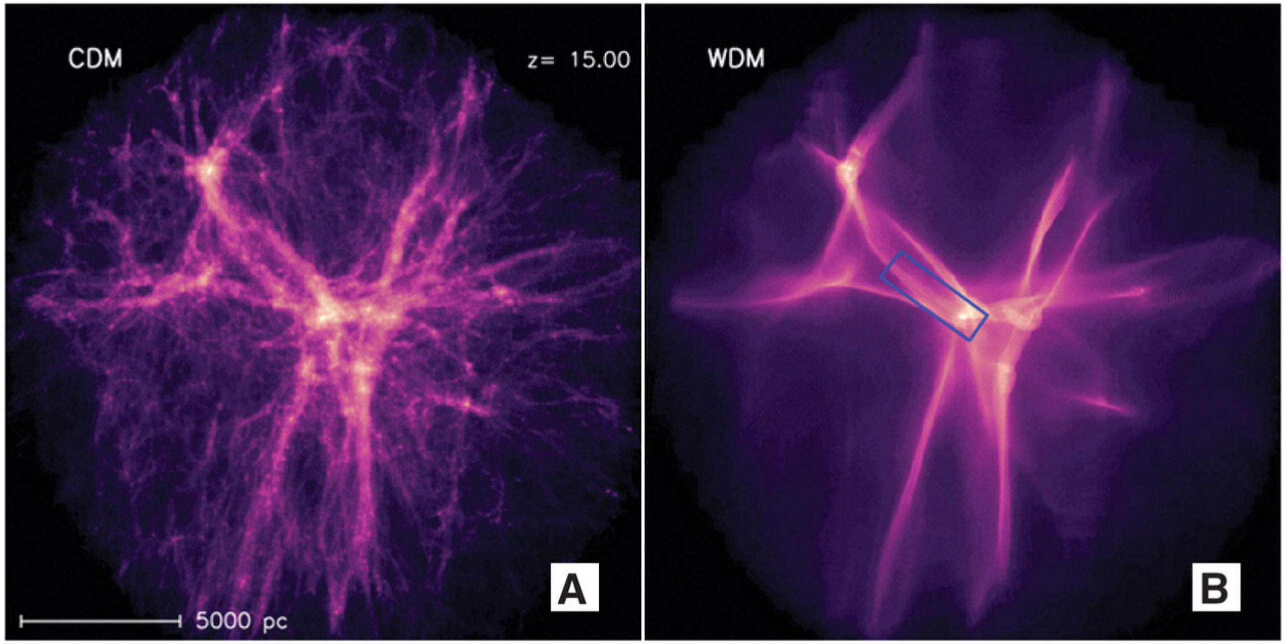
Evidence 4: Structure formation



[The Center for Cosmological Physics (U. Chicago)]

$\Rightarrow$ Without dark matter, current structure of the universe could not be formed

Dark matter: (almost) no pressure (i.e., $w = 0$)



[Gao & Theuns, Science 317 (2007) 1527]

Left: CDM

Right: WDM (mass = 3 keV)

$\Rightarrow$ Dark-matter pressure should be negligible when $T \sim 1$ keV

Requirements for dark matter

- Weakly interacting

  $\Rightarrow$ Charge-neutral, color singlet, $\cdots$

- Stable (or long-lived)

  $\Rightarrow \tau_{\text{DM}} \gg 10^{10}$ years

- $p/\rho \simeq 0$ (when $T \sim 1$ keV)

  $\Rightarrow$ For "particle" dark matter, $m_{\text{DM}} \gtrsim 1$ keV

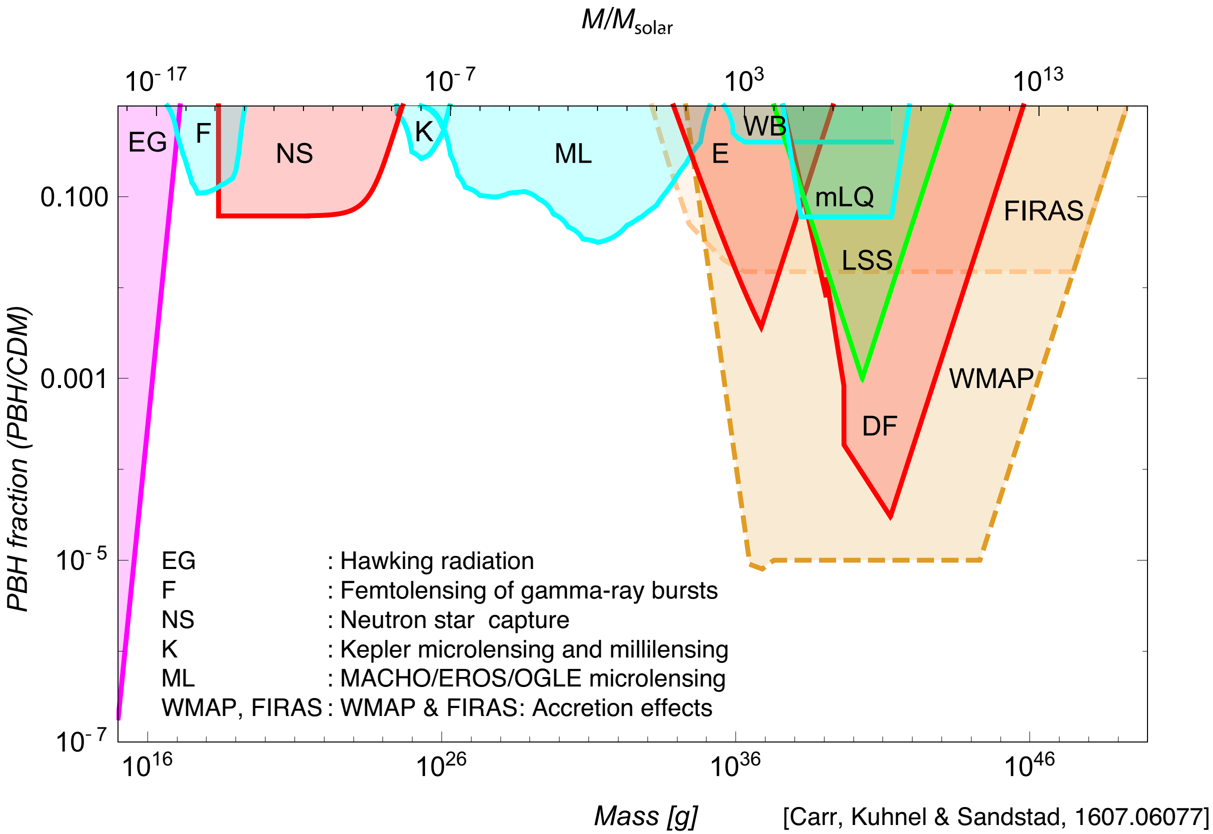
- $\Omega_{\text{DM}} \simeq 0.26$

  [Planck15]

Open question:

What is dark matter (from particle-physics point of view)?

Primordial black holes (PBHs) as DM



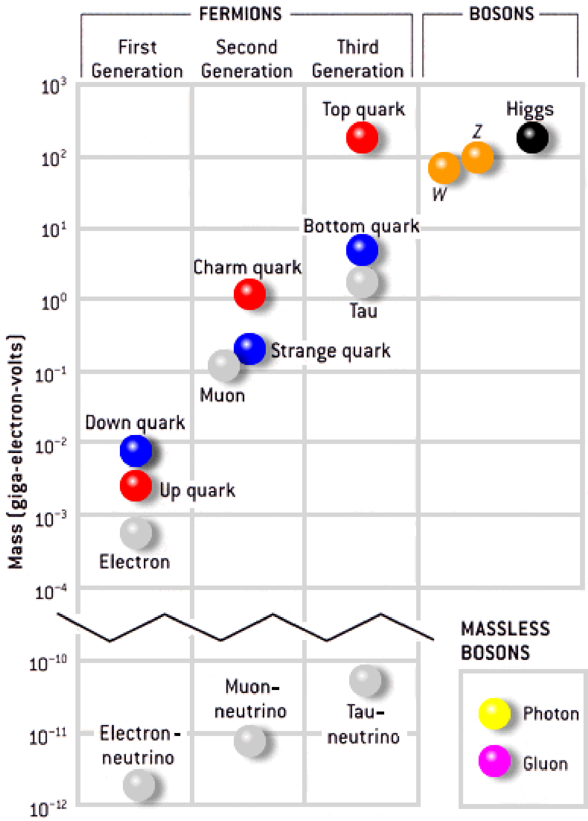
M(Galaxy) $\sim 10^{12} M_\odot$

- PBH DM looks difficult, if the spectrum is monochromatic
- Origin of PBHs is unknown

An elementary particle as DM

$\Rightarrow$ No candidate of CDM in the particle content of the SM

$\Rightarrow$ Need physics beyond the SM



Physics beyond the SM is necessary for DM

$\Rightarrow$ There are many candidates

Particle dark matter

- Lightest syperparticle (LSP) in SUSY
- Lightest Kaluza-Klein particle in UED model
- $\cdots$

Coherent oscillation of scalar field

- Axion in models with Peccei-Quinn symmetry
- $\cdots$

WIMP dark matter

WIMP = Weakly Interacting Massive Particle

Standard approach

1. Introduce dark-matter candidate X
2. Assume some symmetry to stabilize X
   - R-parity in SUSY models
   - KK-parity in UED models
   - $\cdots$

Production mechanism of X depends on scenario

- Thermal, if the interaction of X is non-negligible
- Non-thermal, if the interaction of X is super-weak

An example: Minimal SUSY standard model with R parity

	$R=+$	$R=-$
Gauge Multiplets	$g,\ W,\ B$	$\tilde{g},\ \tilde{W},\ \tilde{B}$
Quark Multiplets	Q	$\tilde{Q}$
Lepton Multiplets	L	$\tilde{L}$
Higgs Multiplets	$H_u,\ H_d$	$\tilde{H}_u,\ \tilde{H}_d$

$$\begin{aligned}\mathcal{L} \;&=\; \mathcal{L}_{\mathsf{kin}} - \frac{1}{2}\tilde{B}\tilde{B} - \frac{1}{2}\tilde{W}\tilde{W} - \frac{1}{2}\tilde{g}\tilde{g} - \mu\tilde{H}_u\tilde{H}_d - m_{\tilde{Q}}^2\tilde{Q}^*\tilde{Q} \\ &\quad -i\sqrt{2}g_1 Y_Q \tilde{B}Q\tilde{Q}^* + \cdots\end{aligned}$$

$$\mathcal{L} \;\not\ni\; \lambda\tilde{\ell}_L\ell_L e_R^c + \lambda'\tilde{\ell}_L q_L d_R^c + \lambda''\tilde{u}_R^c d_R^c d_R^c + \cdots$$

$\Rightarrow$ The lightest superparticle (LSP) is stable

Popular Scenario: Thermal relic WIMP as dark matter

1. X is in chemical equilibrium when $T \gg m_X$
2. X freezes out from thermal bath when $T \ll m_X$

   $\Rightarrow n_X \propto a^{-3}$

X should interact with SM particles for thermalization

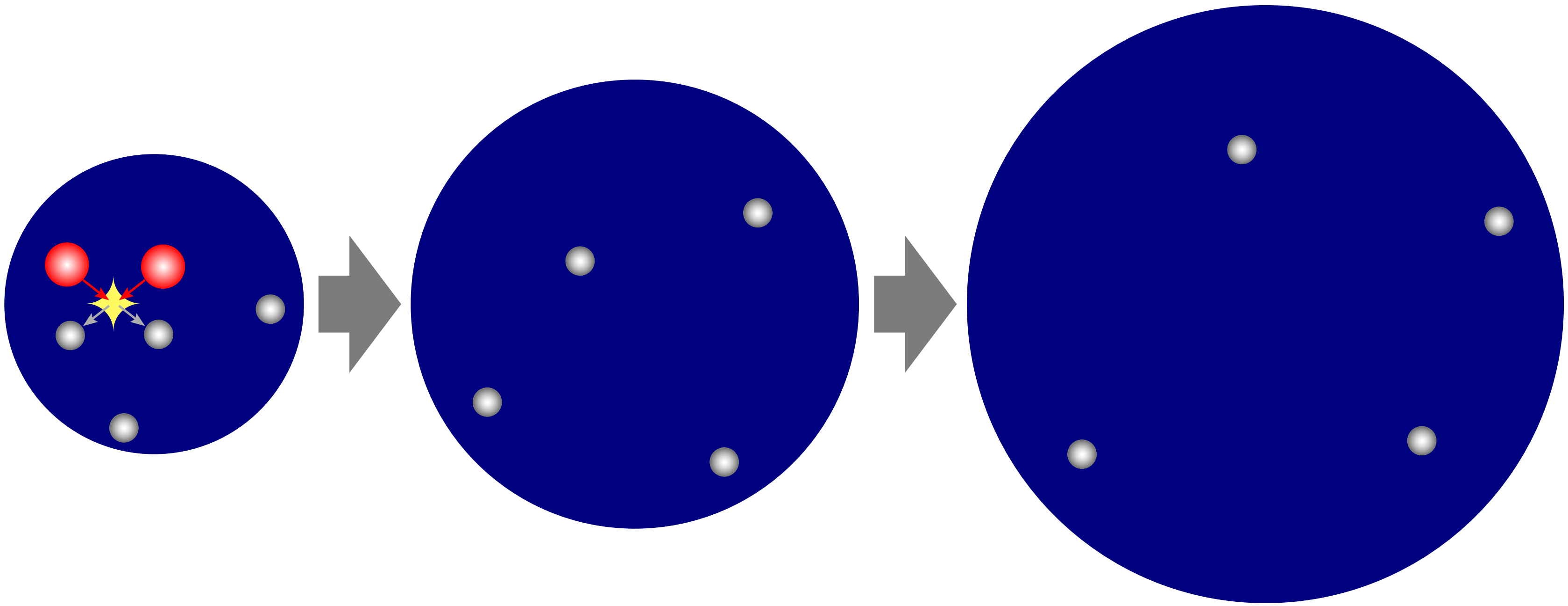
- $X + X \leftrightarrow$ SM particles
- Important quantity: pair-annihilation cross section $\langle \sigma v_{\mathrm{rel}} \rangle$

  $\Rightarrow$ (Scattering rate) $=$ (Mean free time)$^{-1} = n_X \langle \sigma v_{\mathrm{rel}} \rangle$

  $\Rightarrow \langle \sigma v_{\mathrm{rel}} \rangle$ is model-dependent

Dark matter in the early universe

- When $T \gtrsim T_{\mathsf{F}}$, DM is in thermal bath
- When $T \sim T_{\mathsf{F}}$, DM freezes out
- When $T \lesssim T_{\mathsf{F}}$, $n_{\mathsf{DM}} a^3 \sim$ const.



Boltzmann equation:

$$\frac{dn_X}{dt} = -3Hn_X - \langle \sigma v_{\mathsf{rel}} \rangle (n_X^2 - \langle n_X \rangle_T^2)$$

$\langle n_X \rangle_T$: number density in chemical equilibrium

$$\Rightarrow n_X = g_X \left(\frac{m_X T}{2\pi} \right)^{3/2} e^{-m_X/T} \text{ (for } T \ll m_X)$$

- First term: cosmic expansion
- Second term: pair annihilation and its inverse process

High temperature: $H \ll n_X \langle \sigma v_{\mathsf{rel}} \rangle$

$$\dot{n}_X \simeq -\langle \sigma v_{\mathsf{rel}} \rangle (n_X^2 - \langle n_X \rangle_T^2) \quad \Rightarrow \quad n_X \simeq \langle n_X \rangle_T$$

$\Rightarrow$ X is in chemical equilibrium

$\Rightarrow$ $n_X \propto e^{-m_X/T}$ for $T \lesssim m_X$

Low temperature: $H \gg n_X \langle \sigma v_{\mathsf{rel}} \rangle$

$$\dot{n}_X \simeq -3Hn_X \quad \Rightarrow \quad n_X \propto a^{-3}$$

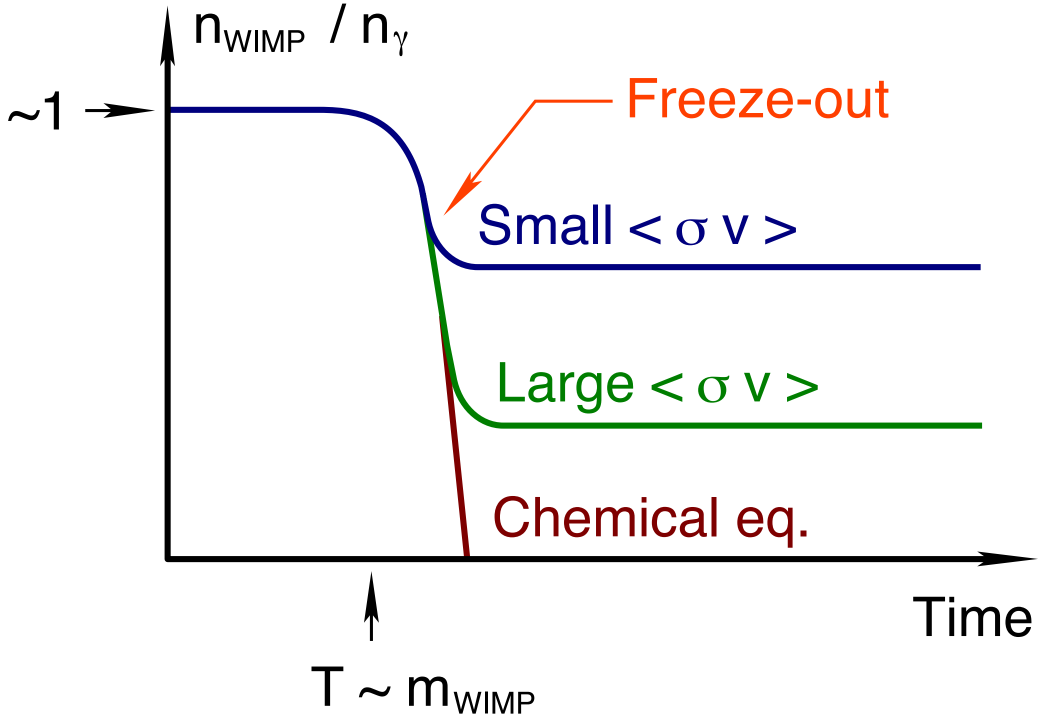
$\Rightarrow$ X freezes out from thermal bath

$\Rightarrow$ The number of X in comoving volume is conserved

Freeze-out temperature: $H(T_F) \sim \langle n_X \rangle_{T_F} \langle \sigma v_{\mathrm{rel}} \rangle$

$$n_X(T_F) \sim \left. \frac{H(T_F)}{\langle \sigma v_{\mathrm{rel}} \rangle} \right|_{T=T_F}$$

Dark matter density is inversely proportional to $\langle \sigma v_{\mathrm{rel}} \rangle$



It is convenient to use:

$sa^3 =$ const. (if there is no entropy production)

$s = \frac{2\pi^2}{45} g_{*S}(T) T^3$: entropy density

Thermal relic density of dark-matter particle

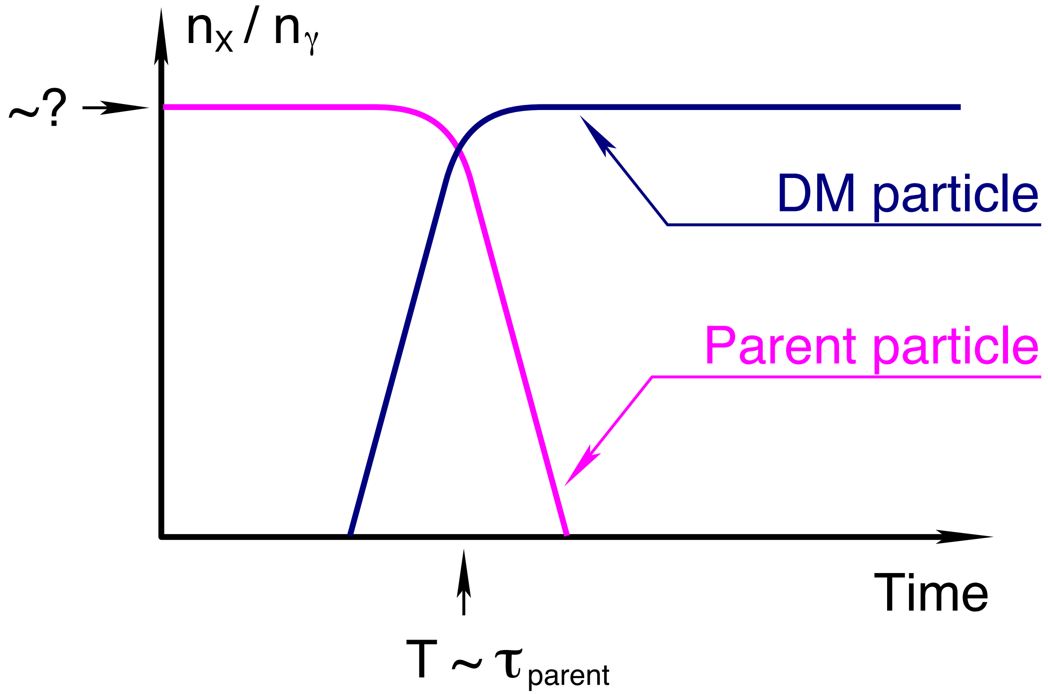
$$\left.\frac{\rho_X}{s}\right|_{\text{now}} \sim \left.\frac{m_X n_X}{s}\right|_{T=T_F} \sim \left.\frac{m_X}{\langle \sigma v_{\text{rel}} \rangle T M_{\text{Pl}}}\right|_{T=T_F}$$

$$\Rightarrow \quad \Omega_{\text{WIMP}}^{(\text{thermal})} \simeq 0.2 \times \left(\frac{\langle \sigma v_{\text{rel}} \rangle}{1 \text{ pb}} \right)^{-1}$$

Notice: $1 \text{ pb} \sim \frac{4\pi\alpha^2}{(500 \text{ GeV})^2}$

Other classes of scenarios are also possible

- DM may be somehow produced in the early universe (like gravitino in SUSY model)
- DM may originate from the decay of "parent" particle

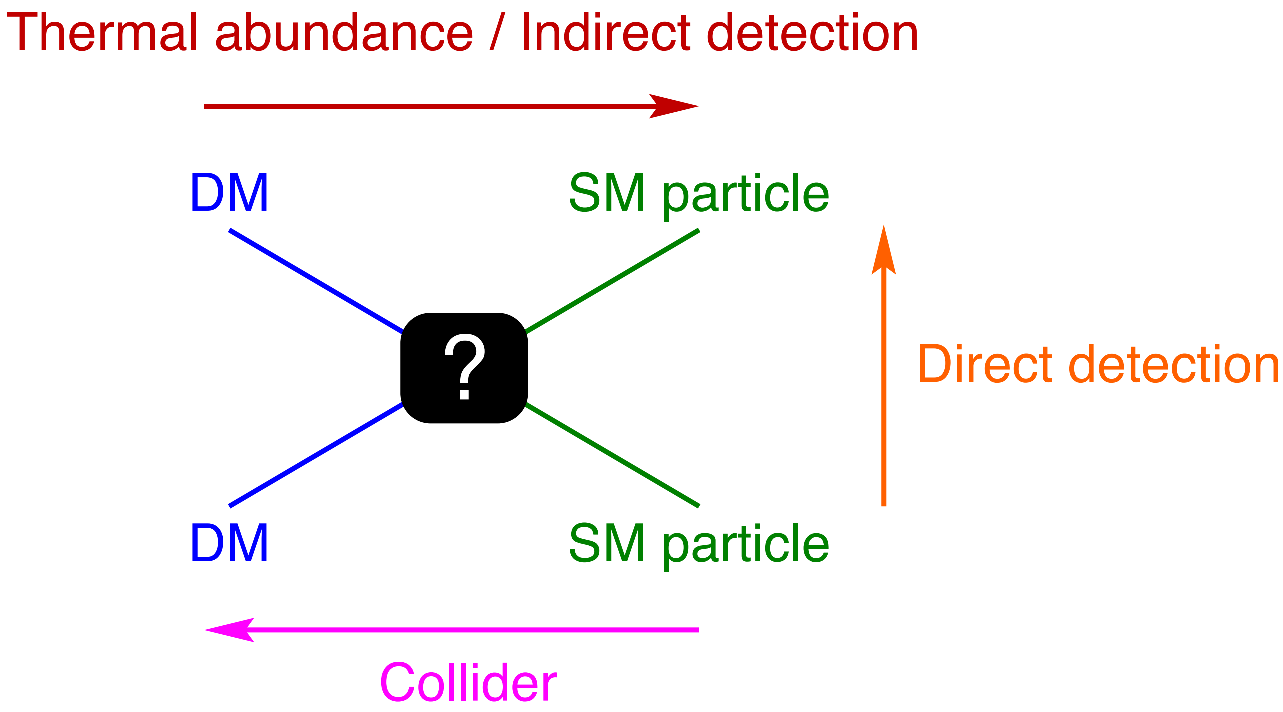


$$\Omega_{DM} = \frac{m_{DM}}{m_{parent}} \Omega^{(would\text{-}be)}_{parent}$$

In non-thermal scenarios, $\langle \sigma v_{rel} \rangle \simeq 1$ pb is not necessary

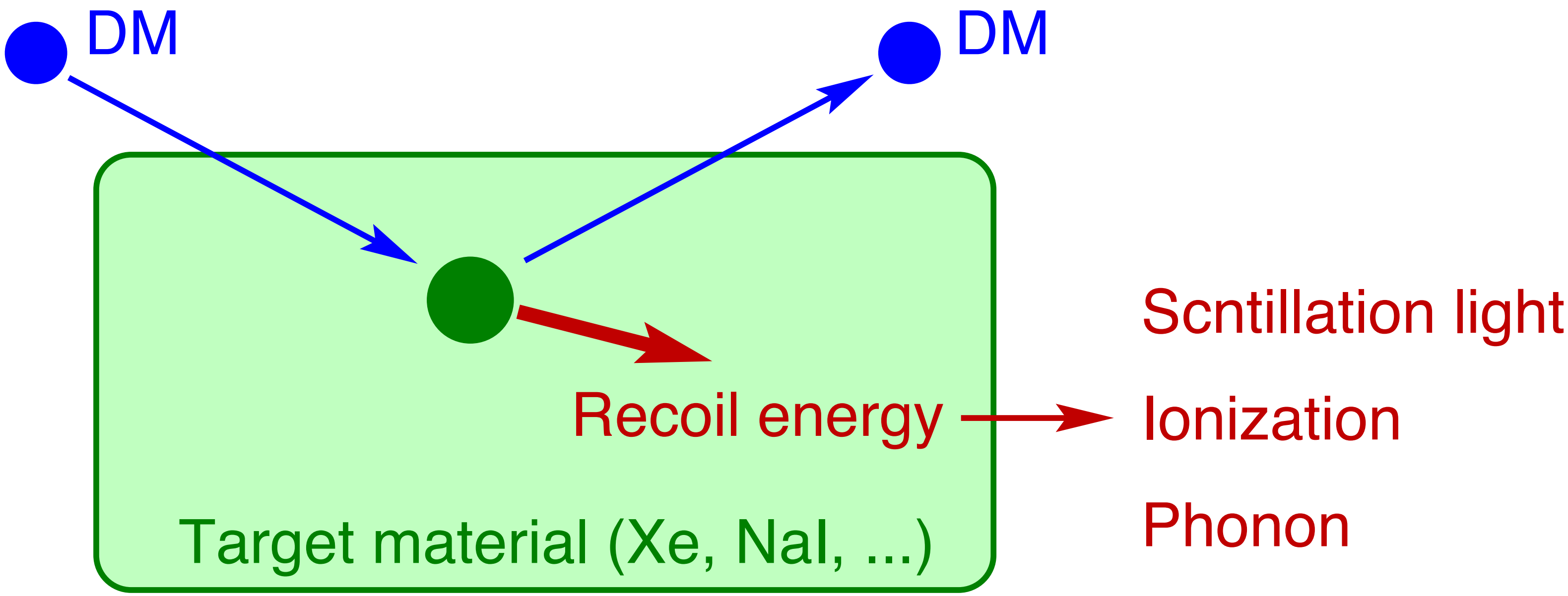
How to detect dark matter particle?

- Direct detection
- Indirect detection (with cosmic rays)
- Collider experiments



Direct detection

- There are DMs around us
- We look for scattering of DM and target materials



DM in our galaxy

- DM velocity

$$v \sim 10^{-3} c$$

- Typical recoil energy

$$E_{\text{recoil}} \sim \frac{m_{\text{DM}}^2 m_N}{(m_{\text{DM}} + m_N)^2} v^2 \sim O(10\ \text{keV})$$

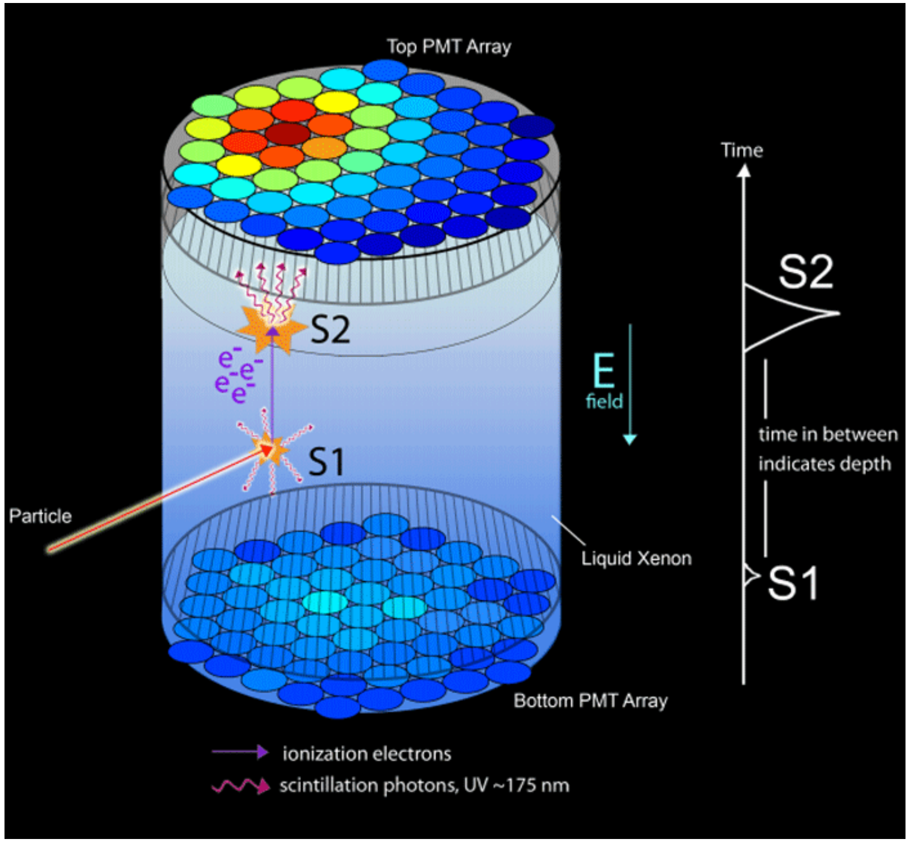
Event rate

$$R \sim 10^{-4}\ \text{kg}^{-1}\text{day}^{-1} \times \left(\frac{\sigma_{\chi N}}{10^{-40}\ \text{cm}^2}\right) \left(\frac{m_{\text{DM}}}{100\ \text{GeV}}\right)^{-1} \left(\frac{m_N}{1\ \text{GeV}}\right)^{-1}$$

$\sigma_{\chi N}$: WIMP scattering cross section

Example: LUX (@ deep underground in SD, U.S.)

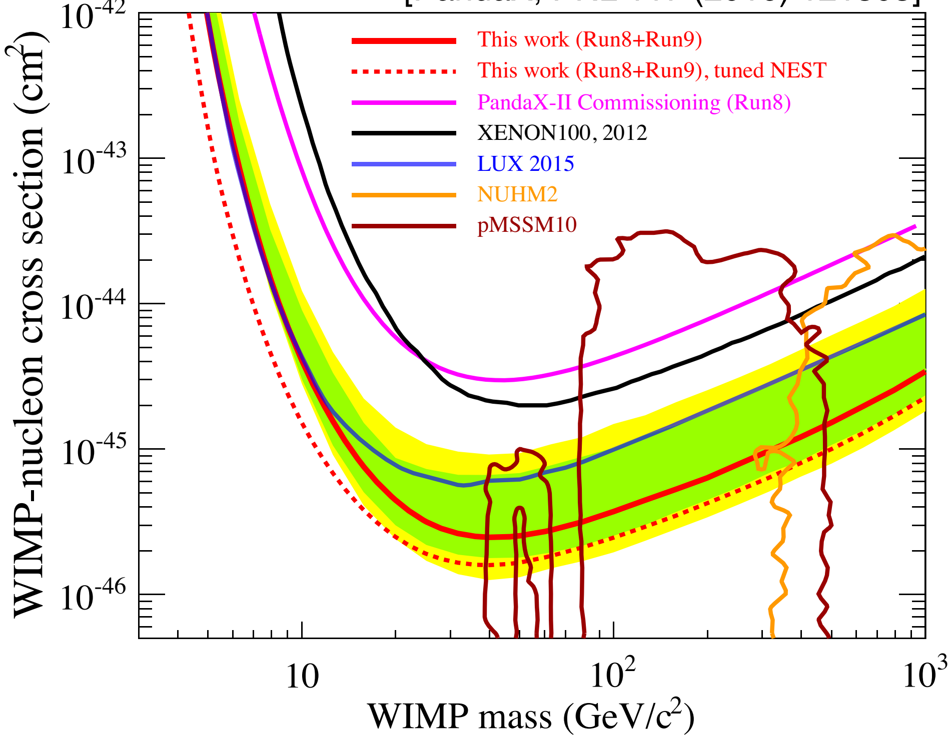
- Scintillation photons (S1)
- Ionization electrons (S2)



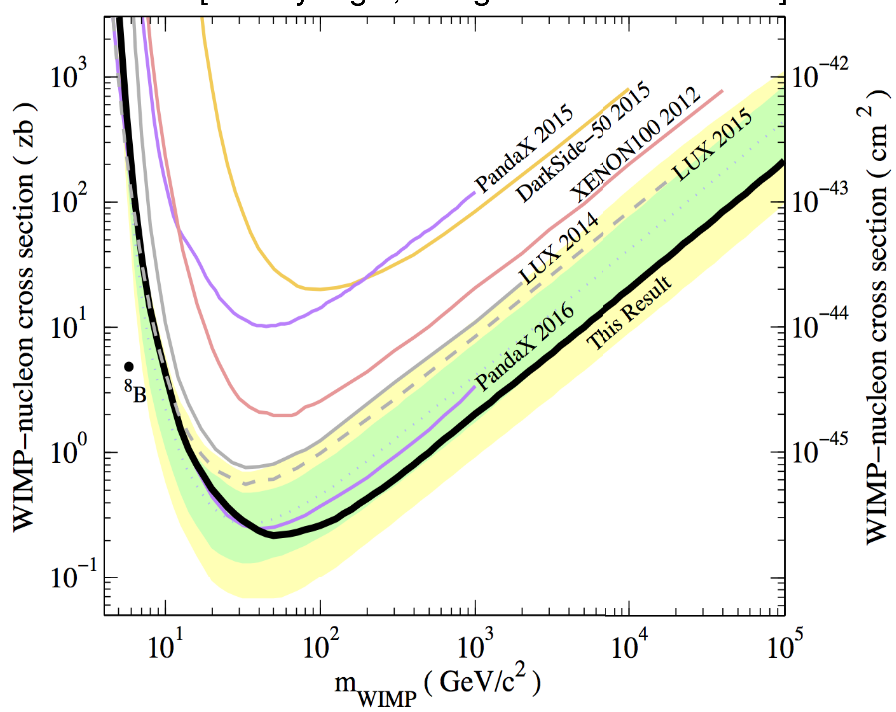
Recent results from liquid Xe experiments

- PandaX: 3.3×10^4 kg-day
- LUX: 370 kg $\times$ 332 live day $\simeq 1.2 \times 10^5$ kg-day

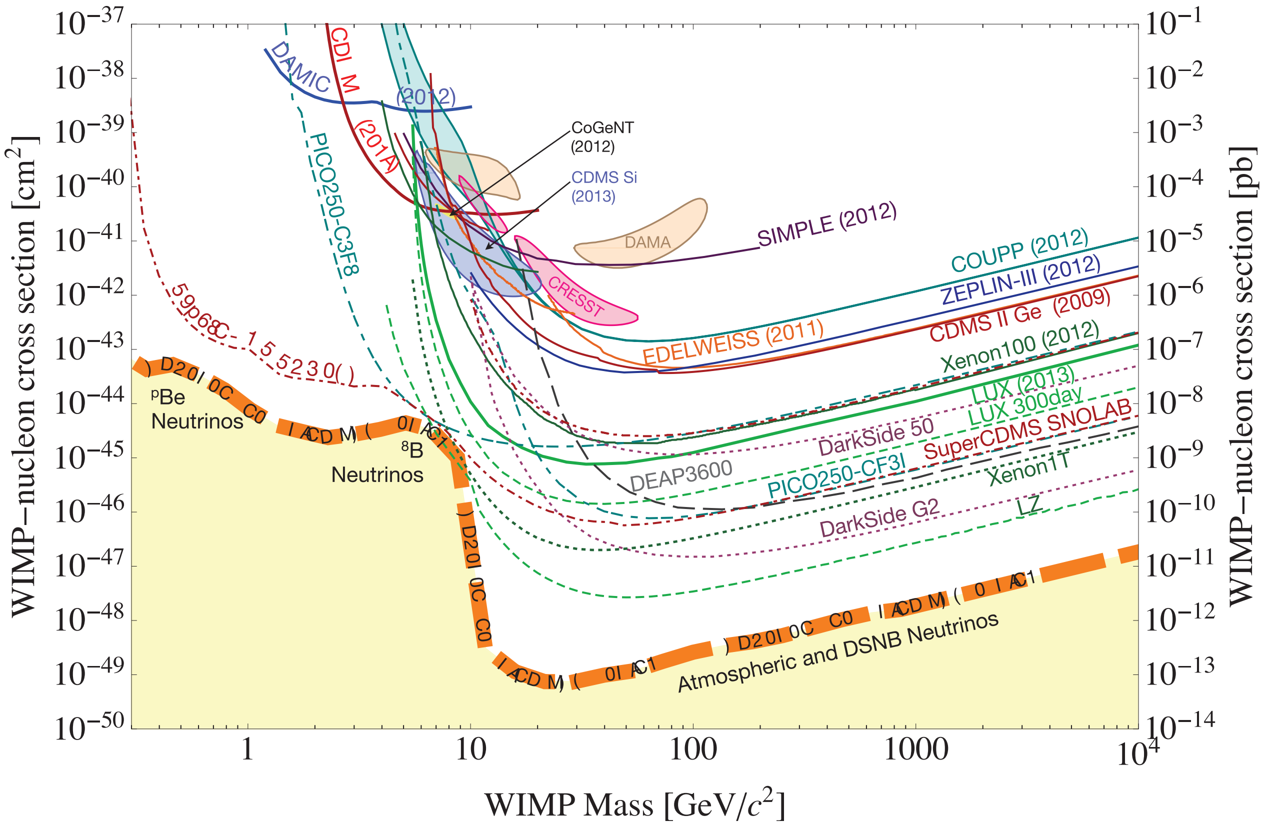
[PandaX, PRL 117 (2016) 121303]



[M. Szydagis, talk given at ICHEP2016]



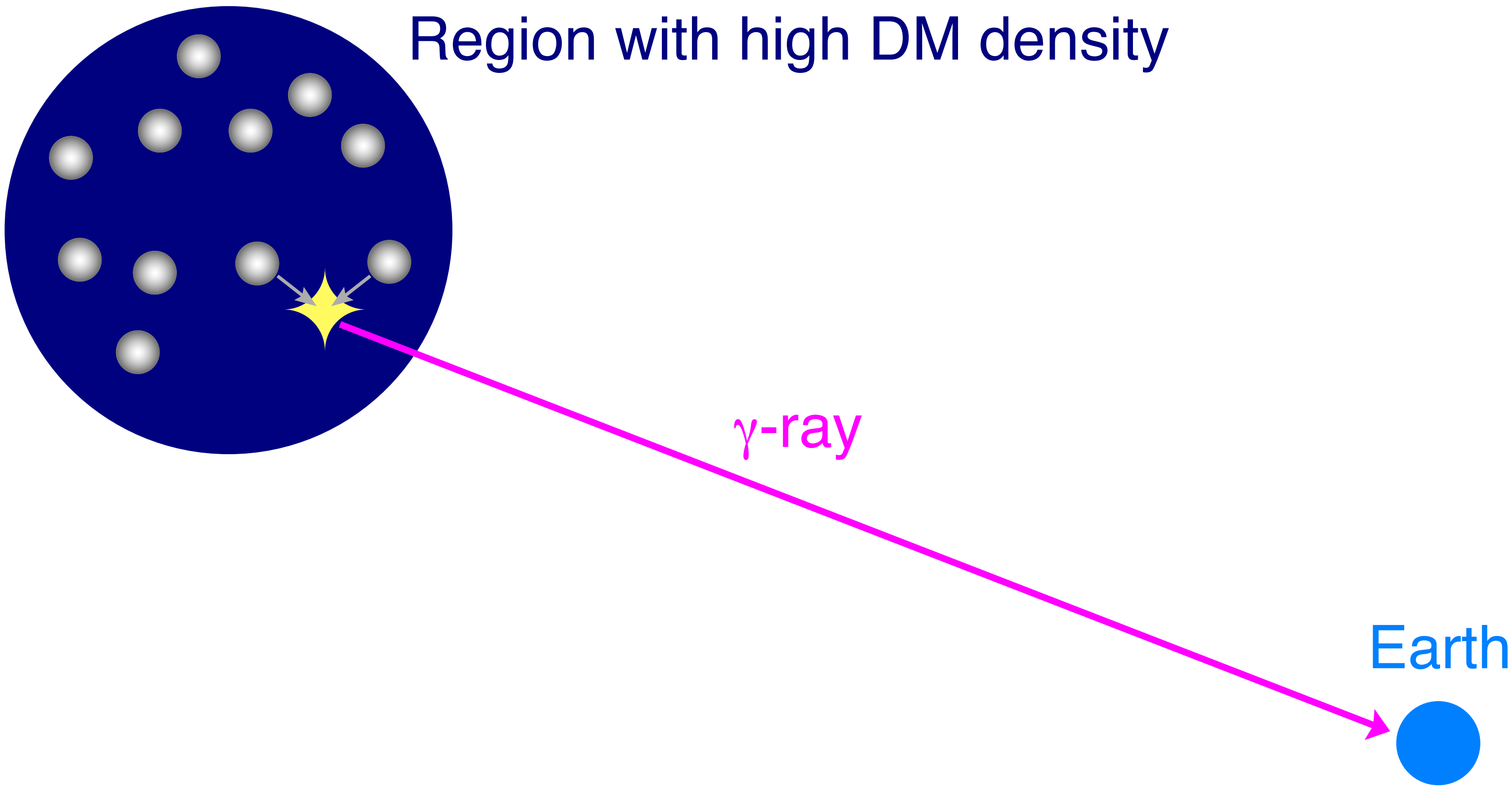
Future prospect



⇒ Eventually, we should worry about neutrino backgrounds

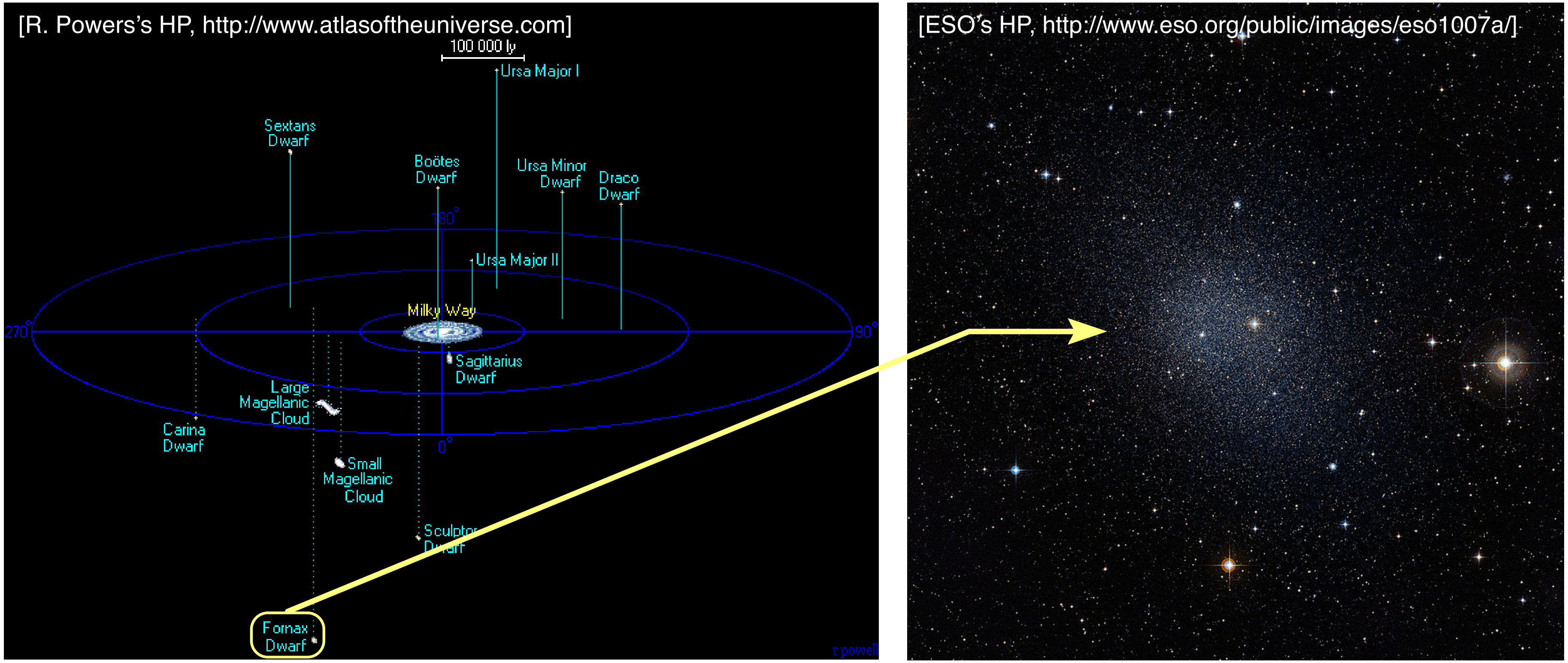
Indirect detection

- We can look for a signal of DM annihilation in the present universe



Interesting environment: dwarf spheroidal galaxies (dSph)

- Low luminosity galaxies accompanying the Milky Way
- Stars in dSph are expected to be trapped by the gravitational field by dark matter



FERMI γ-ray space telescope:

- Satellite experiment to observe high energy cosmic-ray γ
- Fermi satellite observed 15 dSphs from 2008 to 2014

Non-observation of high energy γ from dSphs

$\Rightarrow$ Upper bound on DM annihilation cross section

γ-ray flux from the DM annihilation

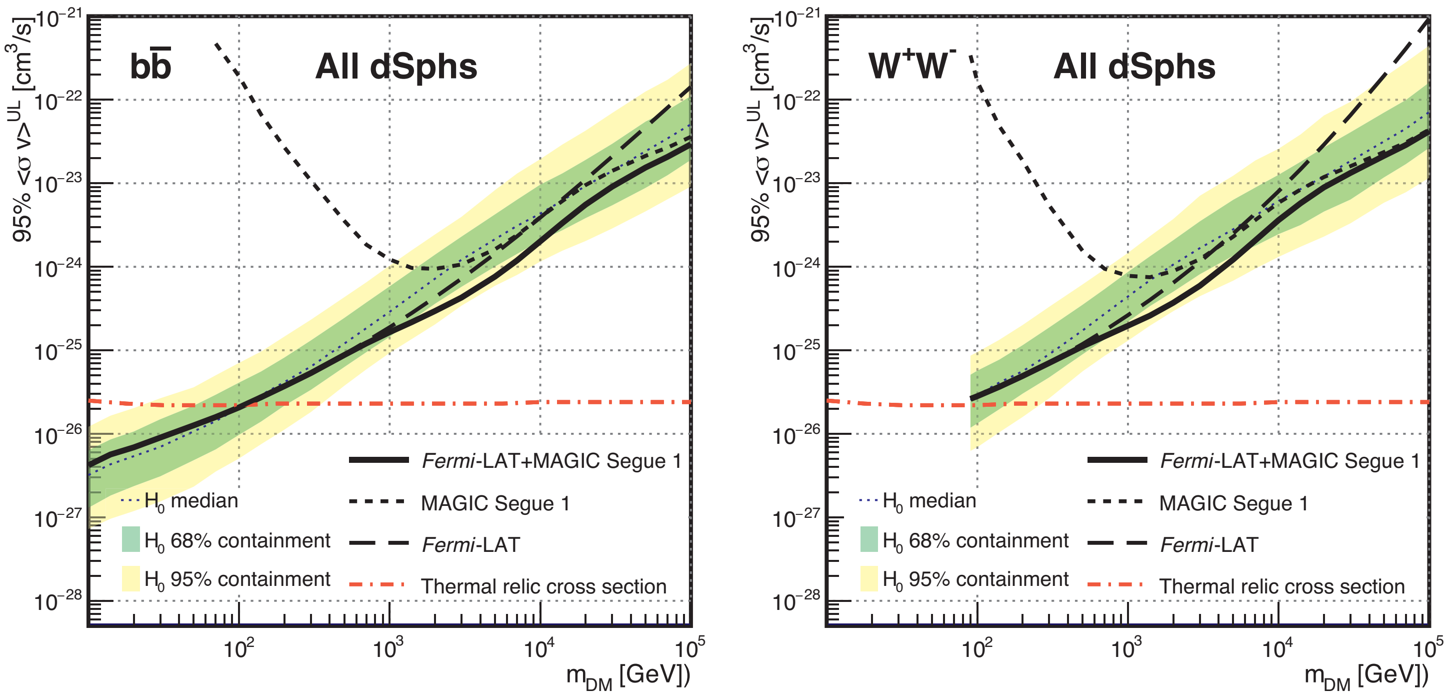
$$\frac{d\Phi}{dE_\gamma} = \frac{\langle \sigma v \rangle}{8\pi m_{\mathrm{DM}}^2} \sum_f b_f \frac{dN_\gamma}{dE_\gamma} \times J$$

J-factor (astrophysical factor)

$$J = \int_{\Delta\Omega} d\Omega \int_{\mathrm{l.o.s.}} dl \rho_{\mathrm{DM}}^2(l)$$

Bound on the annihilation cross sections

[MAGIC + Fermi-LAT, 1601.06590]



[Magic and Fermi-LAT, 1601.06590]

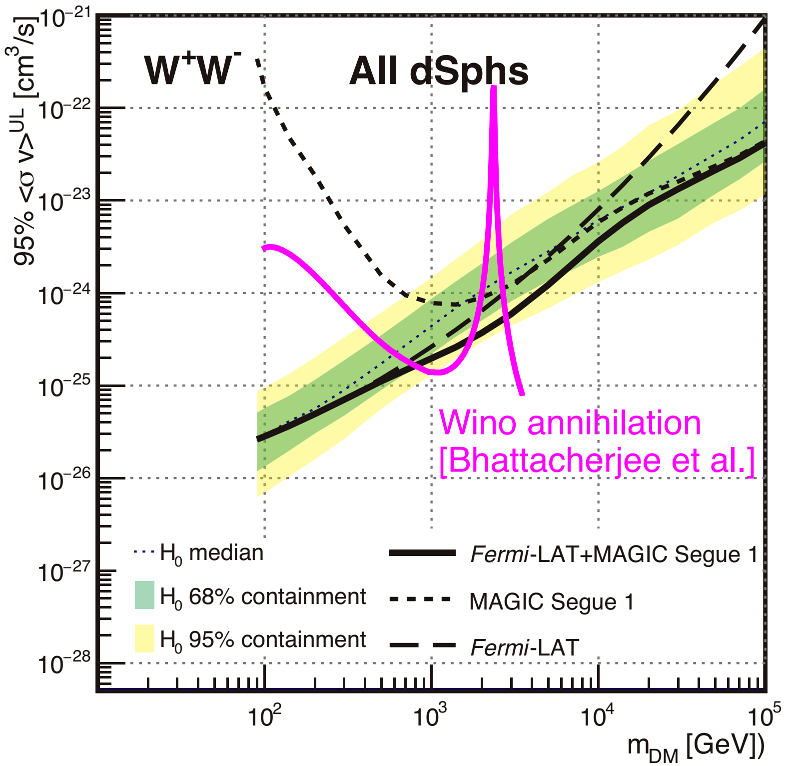
Notice:

- Uncertainties in the J-factor may be underestimated

  [Bhattacherjee et al., 1405.4914; Ichikawa et al., 1608.01749]

Case of neutral-Wino DM (in SUSY model): $\tilde{W}^0\tilde{W}^0 \to W^+W^-$

- Wino is a well-motivated candidate of DM in some class of scenarios, like anomaly mediation
- Annihilation cross section of Wino is relatively large

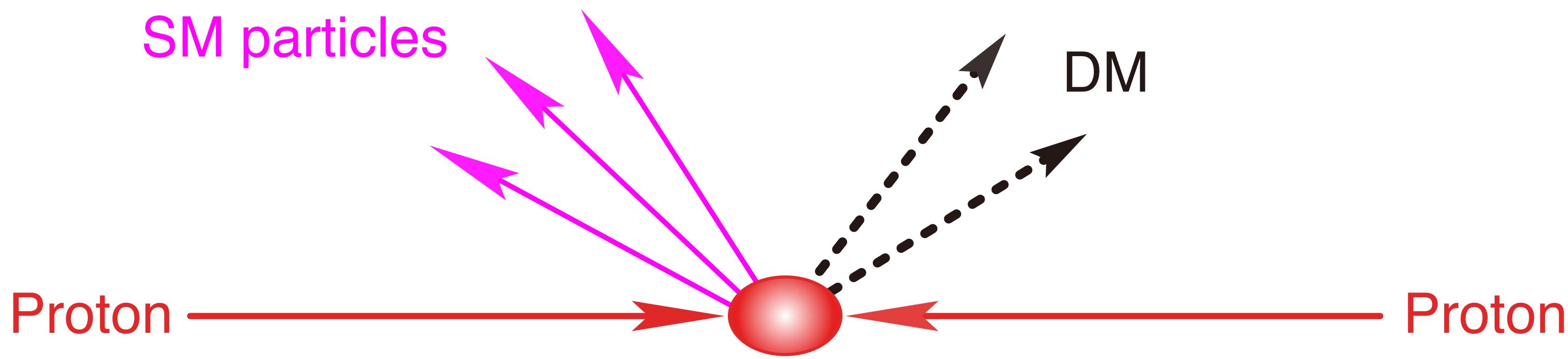


Based on MAGIC + Fermi-LAT:

- 780 GeV $\lesssim m_{\tilde{W}} \lesssim 1.7$ TeV, or
- $m_{\tilde{W}} \gtrsim 2.7$ TeV

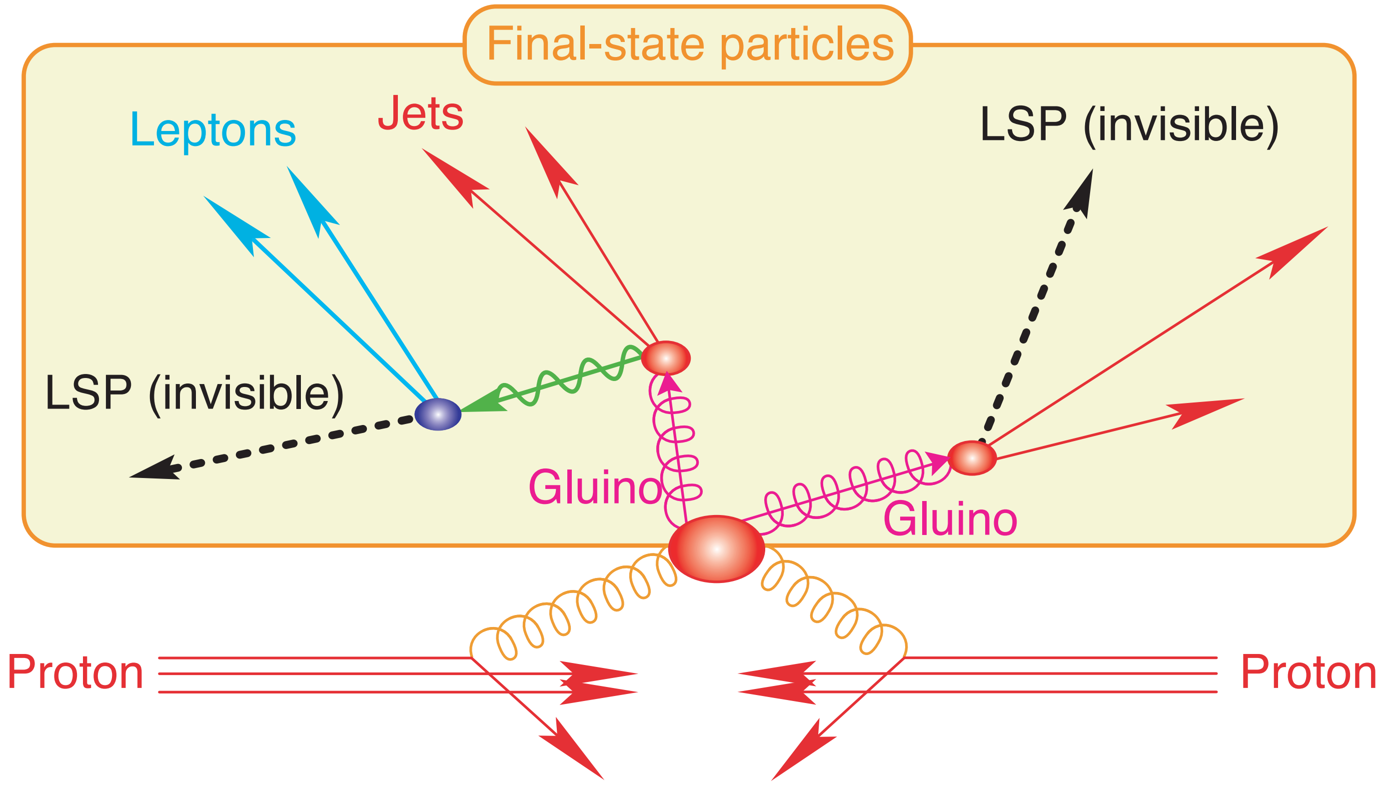
DM at Colliders

- DM can be produced at pp, e^+e^-, and other colliders, if the DM particles interacts with SM particles
- Because DM is "invisible," extra hadronic or leptonic activities are necessary to see the signal of DM production
- Events are (usually) characterized by sizable missing p_T (at the LHC) or missing momentum (at e^+e^- colliders)

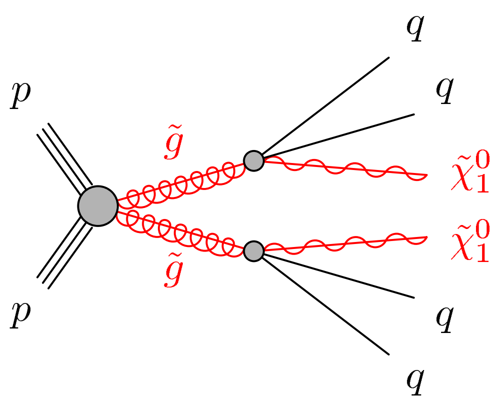


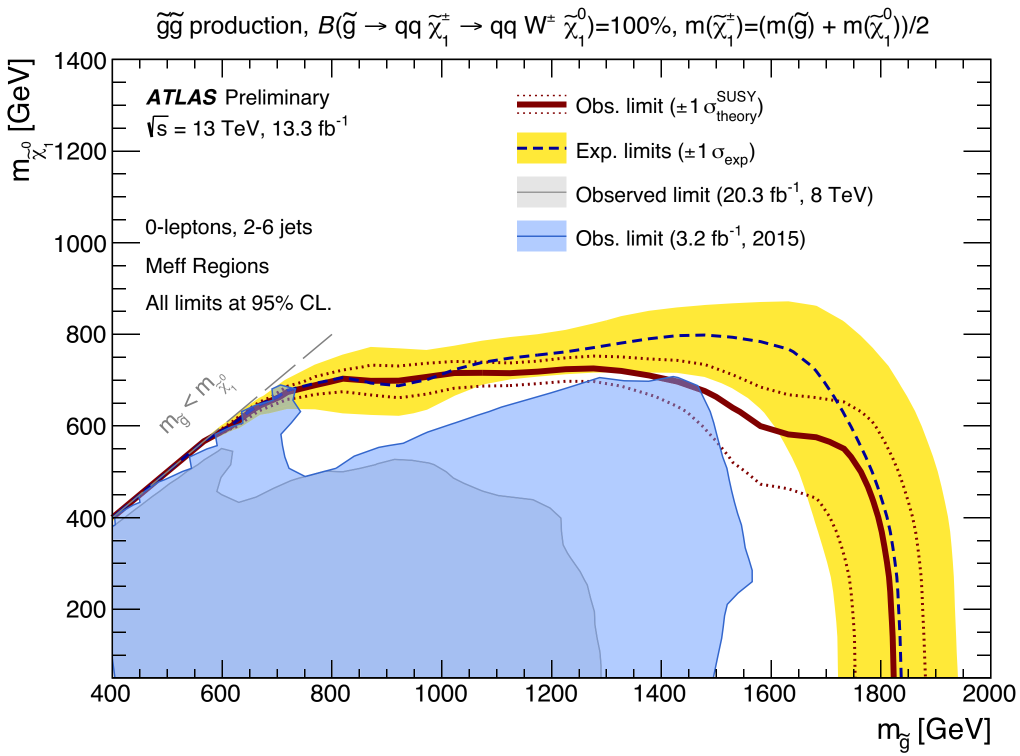
Example: SUSY search at the LHC

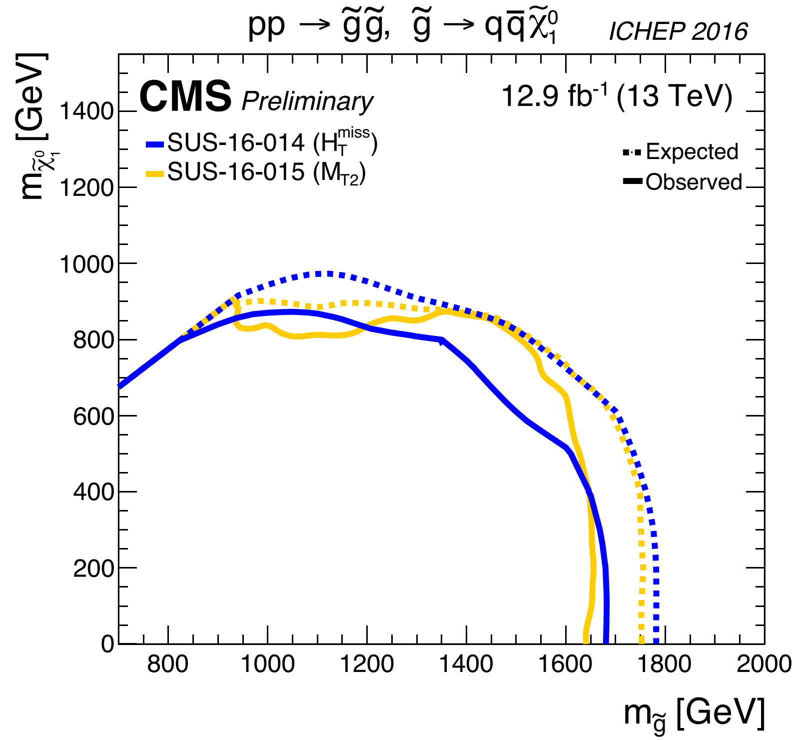
- Primary process: pair productions of gluino or squarks
- ATLAS and CMS look for signals with high p_T jets and sizable missing momentum



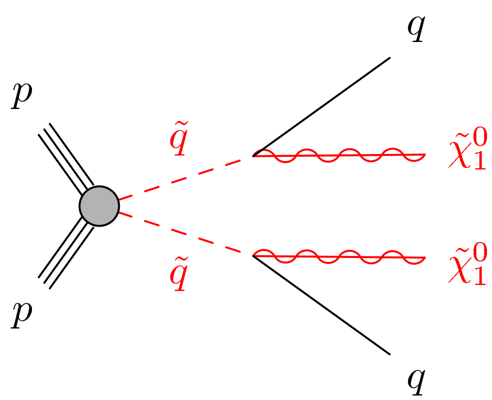
Recent result of Gluino search (ICHEP 2016)

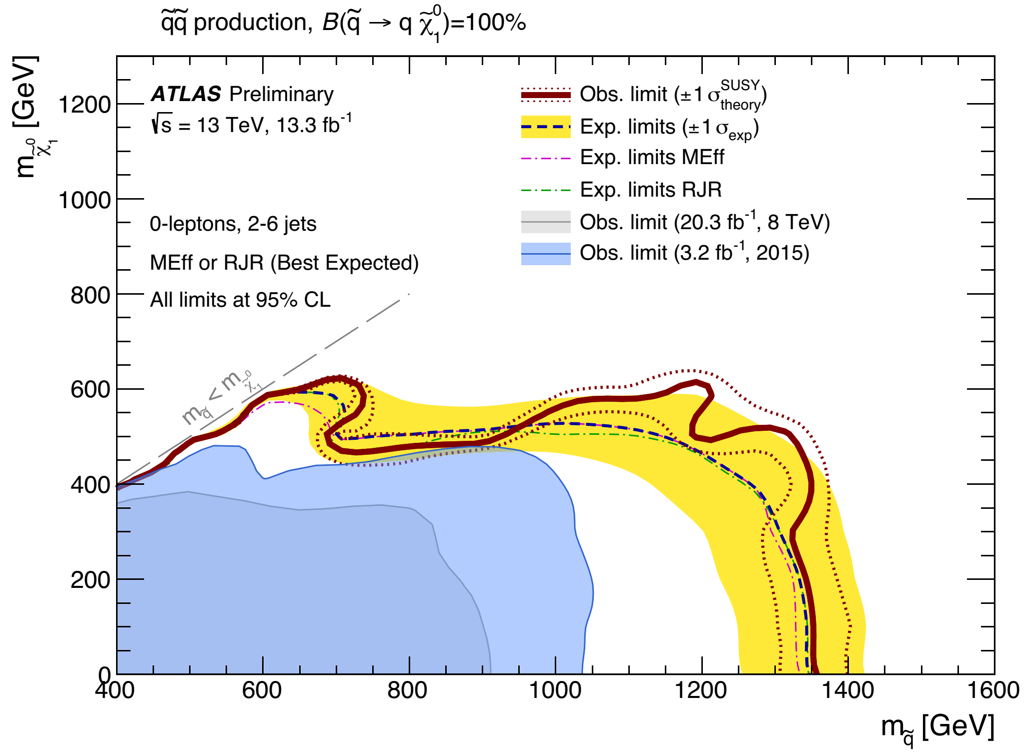


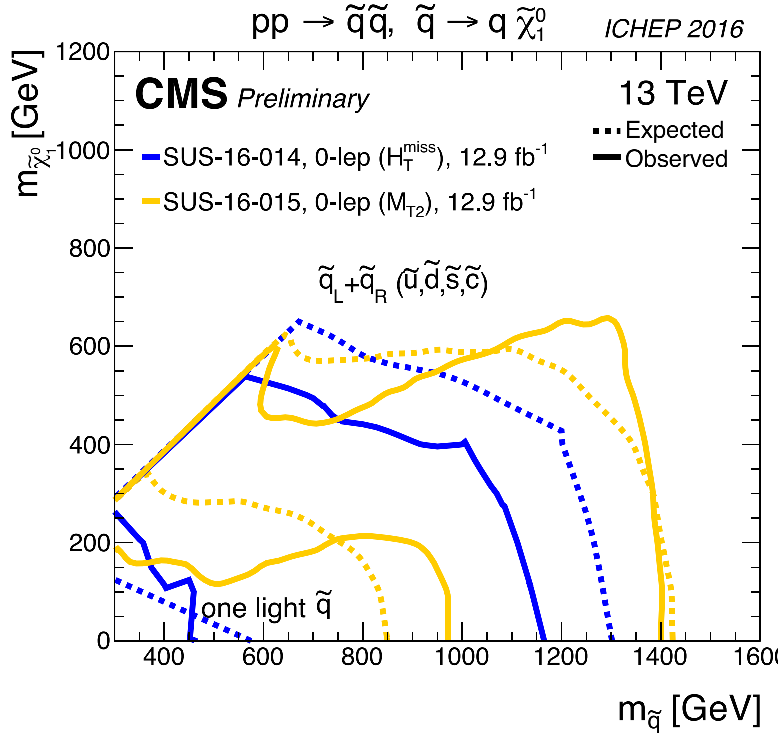




Recent result of squark search (ICHEP 2016)







LHC constraints on other “models” are also available

$\Rightarrow$ Please check ATLAS and CMS publications

You can find simplified models in 1507.00966

- List of simple Lagrangians for DM candidates are given, assuming Z_2 symmetry
- The model may contain new mediators
  - Scalar
  - Vector
  - Fermion

Another DM candidate: oscillating scalar field ϕ

- Let us assume there exists a massive scalar field

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m_\phi^2 \phi^2 \right]$$

- We assume that the interaction of ϕ is very weak

$\Rightarrow$ We neglect the interaction terms

- Energy-momentum tensor

$$T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu} \mathcal{L}$$

How does the scalar field play the role of DM?

- Let's consider the homogeneous mode: $\phi = \phi(t)$
- Equation of motion:

$$\ddot{\phi} + 3H\dot{\phi} + m_\phi^2\phi = 0 \qquad \text{where } H \equiv \frac{\dot{a}}{a}$$

- Energy density and pressure:

$$\rho_\phi = T_{00} = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}m_\phi^2\phi^2, \qquad p_\phi = T_{ii} = \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}m_\phi^2\phi^2$$

- When $H \ll m_\phi$, the scalar field rapidly oscillates

$$\langle\dot{\phi}^2\rangle \simeq m_\phi^2\langle\phi^2\rangle \quad \Rightarrow \quad \langle p_\phi\rangle \simeq 0 \quad \Rightarrow \quad w_\phi \simeq 0$$

Evolution of the energy density (for $H \ll m_\phi$)

- Equation of motion: $\ddot{\phi} + 3H\dot{\phi} + m_\phi^2 \phi = 0$

$$\Rightarrow \dot{\phi}\ddot{\phi} + m_\phi^2 \dot{\phi}\phi = -\frac{3\dot{a}}{a}\dot{\phi}^2$$

$$\Rightarrow \frac{d}{dt}\left[\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}m_\phi^2\phi^2\right] = -\frac{3\dot{a}}{a}\dot{\phi}^2$$

- Then, take the oscillation average

$$\Rightarrow \frac{d}{dt}\langle\rho_\phi\rangle \simeq -\frac{3\dot{a}}{a}\langle\dot{\phi}^2\rangle \simeq -\frac{3\dot{a}}{a}\langle\rho_\phi\rangle$$

- ρ_ϕ behaves as the energy density of NR matter

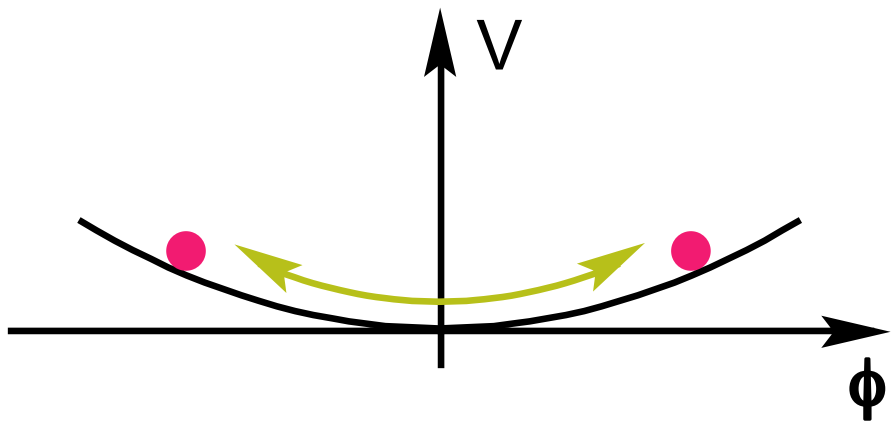
$\Rightarrow \rho_\phi \propto a^{-3}$ (consistent with $w = 0$)

Stable scalar field with $m_\phi \gg H$ can be DM

$$\Omega_\phi h^2 \simeq \Omega_c h^2 \times \left(\frac{m_\phi}{1\ \mathrm{eV}}\right)^2 \left(\frac{\phi_{\mathrm{now}}}{4\times 10^{-15}\ \mathrm{GeV}}\right)^2$$

Candidates:

- Axion
- $\cdots$



Axion

- Pseudo Nambu-Goldstone boson, associated with Peccei-Quinn symmetry

$$\mathcal{L} = \frac{\alpha_s}{8\pi f_a} a G^{(A)}_{\mu\nu} \tilde{G}^{(A)}_{\mu\nu}$$

f_a : axion decay constant

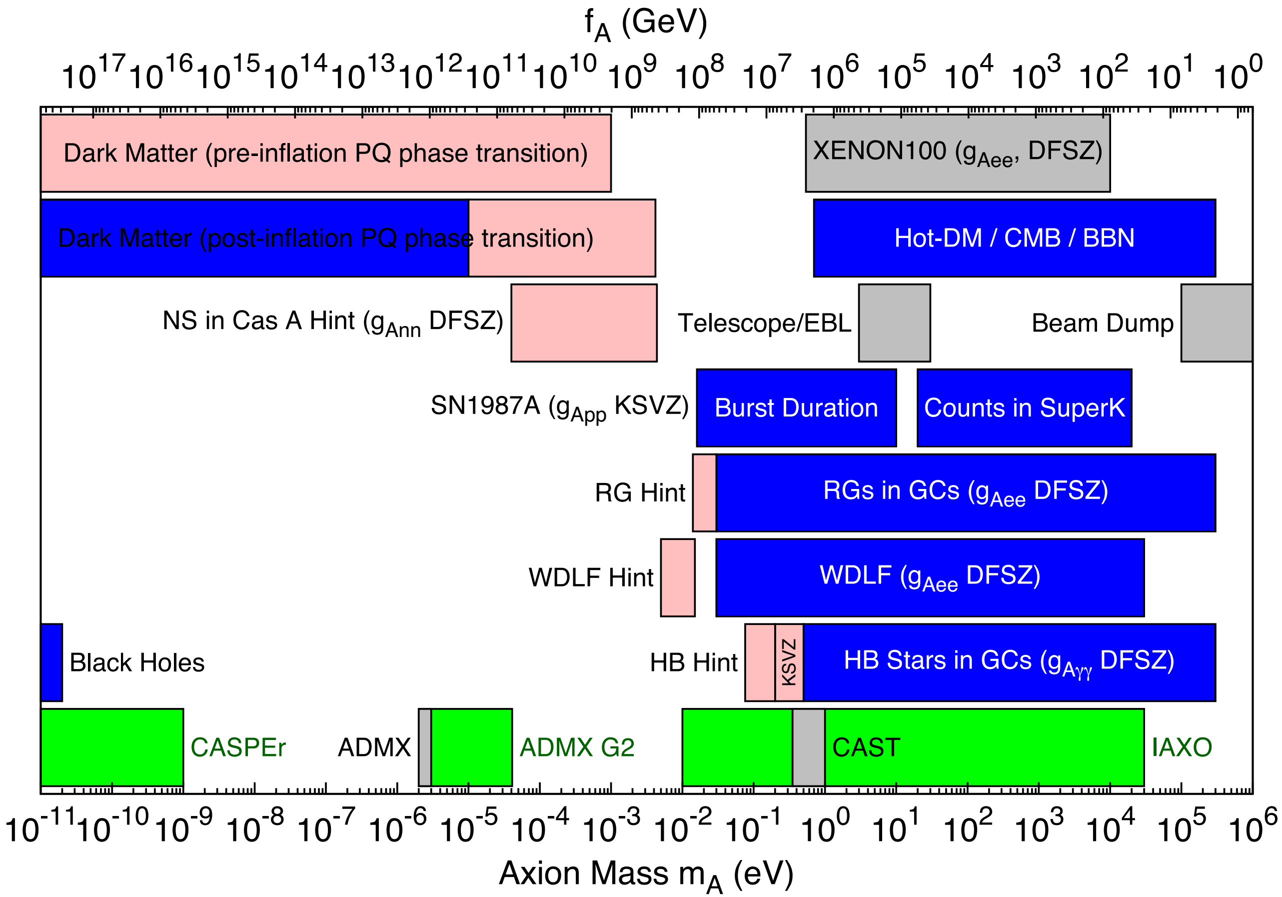
- Axion acquires mass after QCD condensation

$$m_a \simeq \frac{z^{1/2}}{1+z} \frac{f_\pi m_\pi}{f_a} \simeq 0.6 \text{ meV} \times \left(\frac{f_a}{10^{10} \text{ GeV}} \right)^{-1}$$

$z = m_u/m_d \simeq 0.38 - 0.58$

Bound on the axion decay constant

[Figure from PDG 2015]



Blue: Excluded

Green: Projected reach

Pink: Suggested

$$\Rightarrow 10^8 \text{ GeV} \lesssim f_a \lesssim 10^{10} \text{ GeV}$$