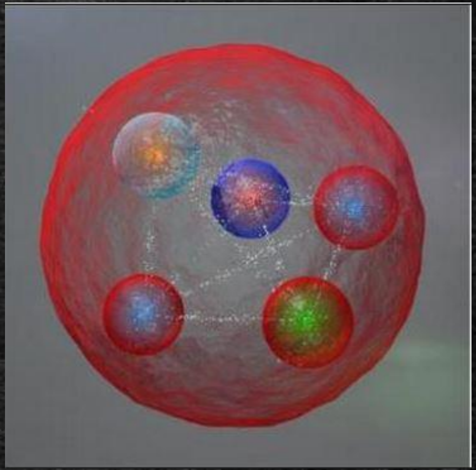
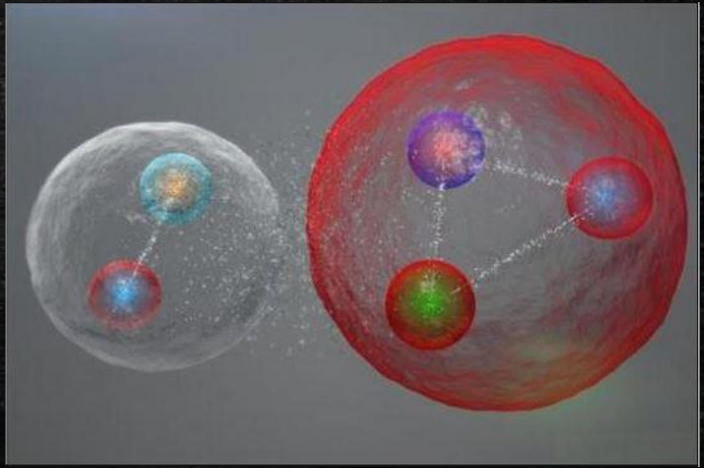
Observation of J/ψp Resonances Consistent with Pentaquark States

Giulio Mezzadri – INFN Ferrara
On behalf of Discussion Group 3 – Flavour Physics

Outline

- Brief history overview
- LHCb detector
- Event selection
- Results
  - An Helicity amplitude model to describe the data
  - Fit results
  - Systematic uncertainties





A Long History

Phys.Lett. 8 (1964) 214-215

Volume 8, number 3 PHYSICS LETTERS 1 February 1964

A SCHEMATIC MODEL OF BARYONS AND MESONS *

M. GELL-MANN
California Institute of Technology, Pasadena, California

Received 4 January 1964

. . .

A simpler and more elegant scheme can be constructed if we allow non-integral values for the charges. We can dispense entirely with the basic baryon b if we assign to the triplet t the following properties: spin $\frac{1}{2}$, $z = -\frac{1}{3}$, and baryon number $\frac{1}{3}$. We then refer to the members $u^{\frac{2}{3}}$, $d^{-\frac{1}{3}}$, and $s^{-\frac{1}{3}}$ of the triplet as "quarks" 6) q and the members of the anti-triplet as anti-quarks $\bar{q}$. Baryons can now be constructed from quarks by using the combinations $(q\,q\,q)$, $(q\,q\,q\,q\,\bar{q})$, etc., while mesons are made out of $(q\,\bar{q})$, $(q\,q\,\bar{q}\,\bar{q})$, etc. It is assuming that the lowest baryon configuration $(q\,q\,q)$ gives just the representations **1**, **8**, and **10** that have been observed, while

8419/TH.412
21 February 1964

AN SU_3 MODEL FOR STRONG INTERACTION SYMMETRY AND ITS BREAKING
II *)

G. Zweig
CERN---Geneva

*) Version I is CERN preprint 8182/TH.401, Jan. 17, 1964.

. . .

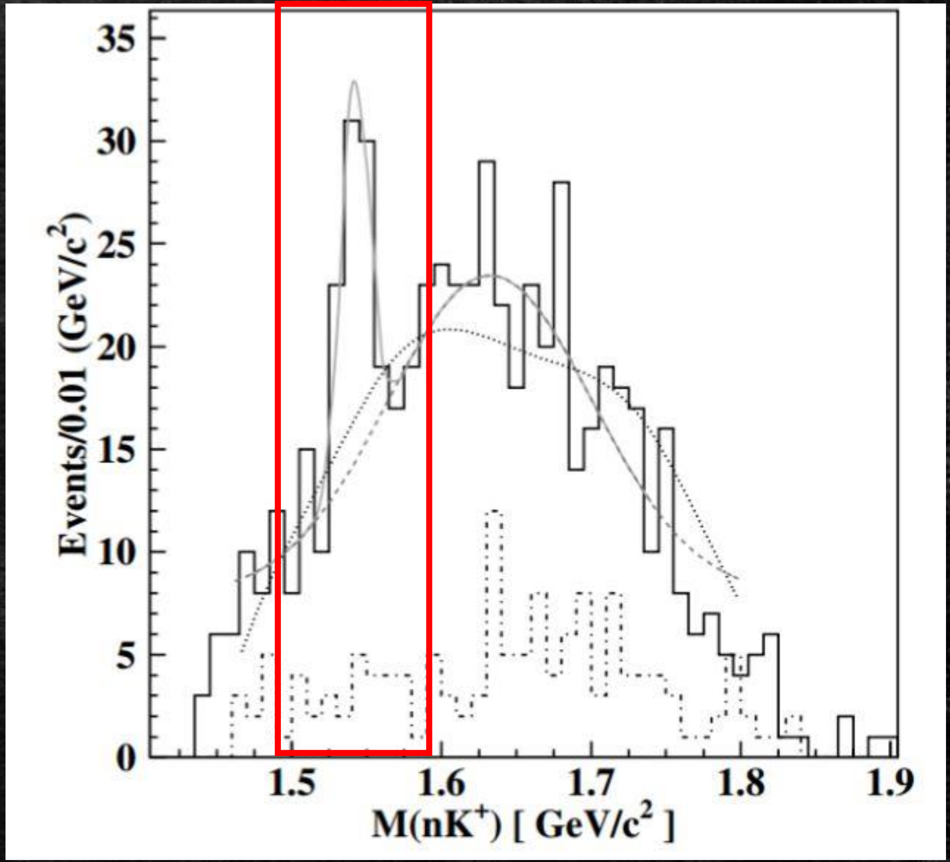
6) In general, we would expect that baryons are built not only from the product of three aces, AAA, but also from $\bar{A}AAAA$, $\bar{A}\bar{A}AAAAA$, etc., where $\bar{A}$ denotes an anti-ace. Similarly, mesons could be formed from $\bar{A}A$, $\bar{A}\bar{A}AA$ etc. For the low mass mesons and baryons we will assume the simplest possibilities, $\bar{A}A$ and AAA, that is, "deuces and treys".

The Θ(1540)⁺ Case

- Observed by a Japanese collaboration in 2002
  - In γd -> K+ K- p(n), in the invariant mass of Kp and Kn

- Several other claims

Experiment	Reaction	Mass (GeV)	Significance
LEPS [8]	$\gamma C \to K^+K^-X$	1.54±0.01	4.6σ
DIANA [9]	$K^+Xe \to K^0_s pX$	1.539±0.002	4.4σ
CLAS [10]	$\gamma d \to K^+K^-pn$	1.542±0.005	(5.2±0.6)σ
SAPHIR [11]	$\gamma p \to K^0_s K^+ n$	1.540±0.004	4.8σ



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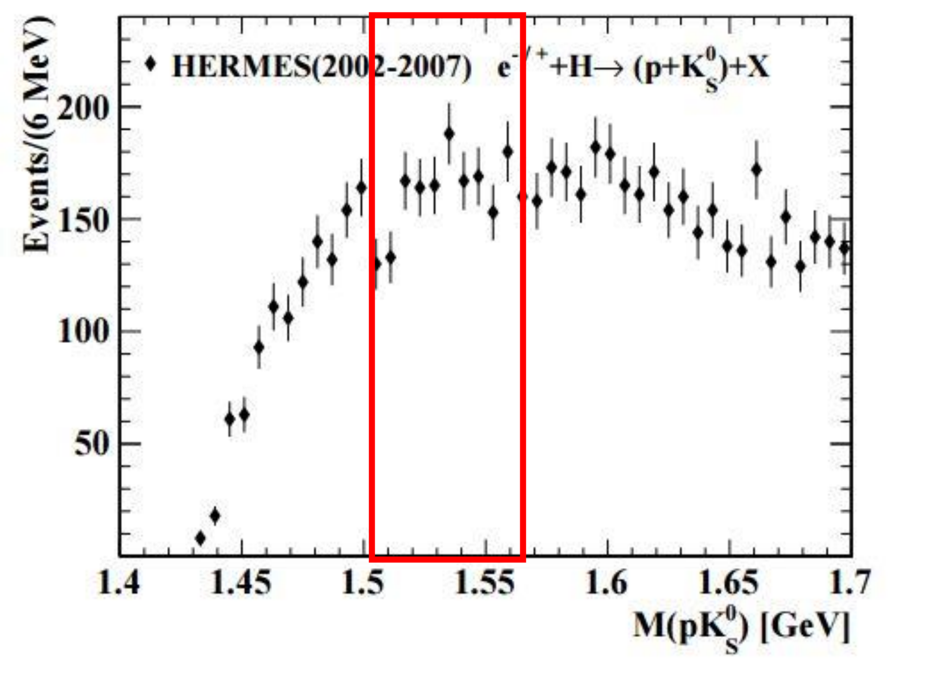
arXiv =1412.7317



With higher luminosities, signal disappeared

The $\Theta(1540)^+$ Case

From PDG review of 2006 (http://pdg.lbl.gov/2006/reviews/theta_b152.pdf)

for the Θ^+; and all attempts to confirm the two other claimed pentaquark states have led to negative results. The conclusion that pentaquarks in general, and the Θ^+, in particular, do not exist, appears compelling.

The LHC Era

"Take a look at this everyone - it just could be the signature we've been looking for!"

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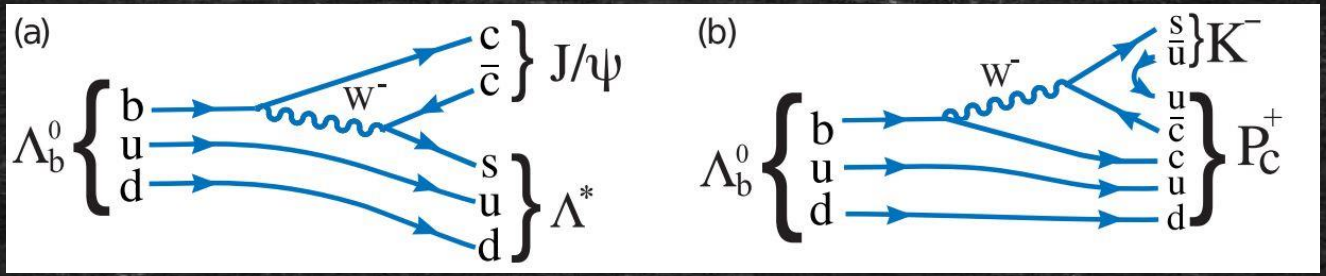
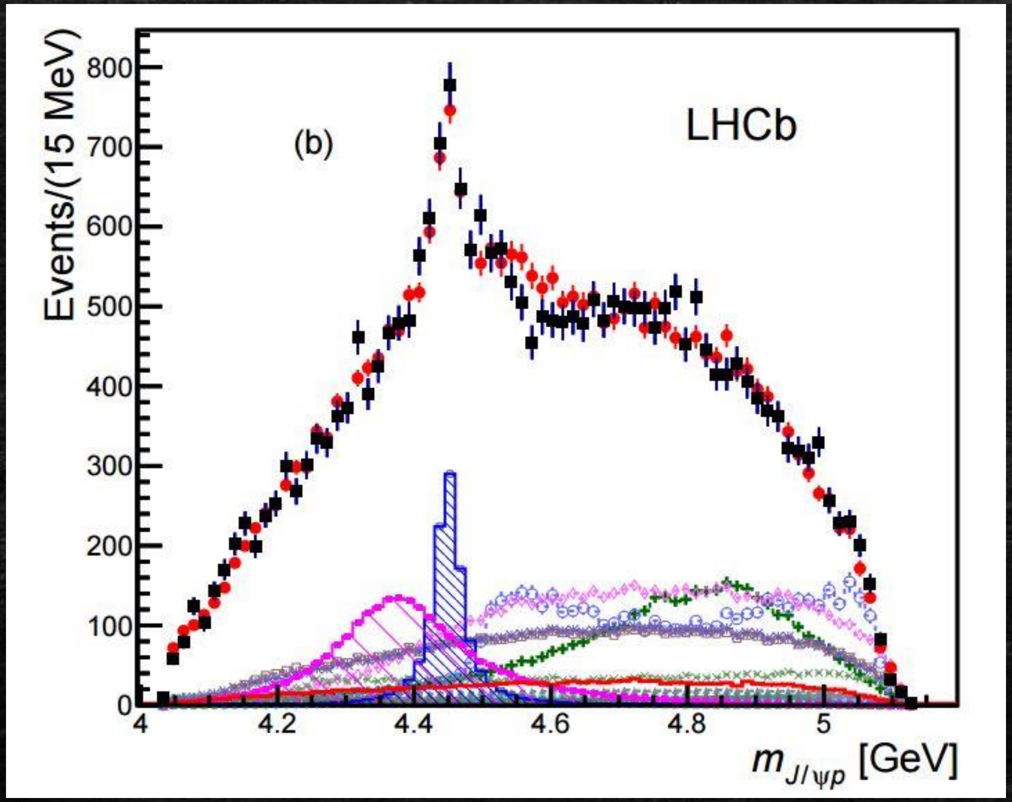
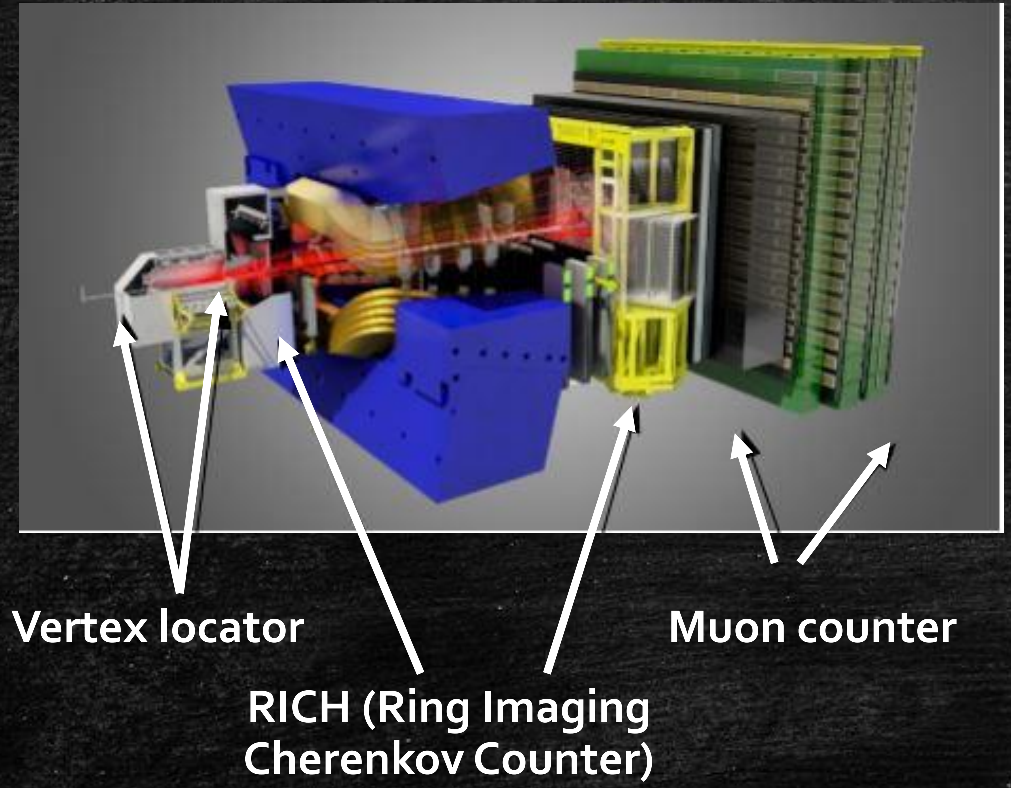
Observation in Λ_b -> $J/\psi pK^-$



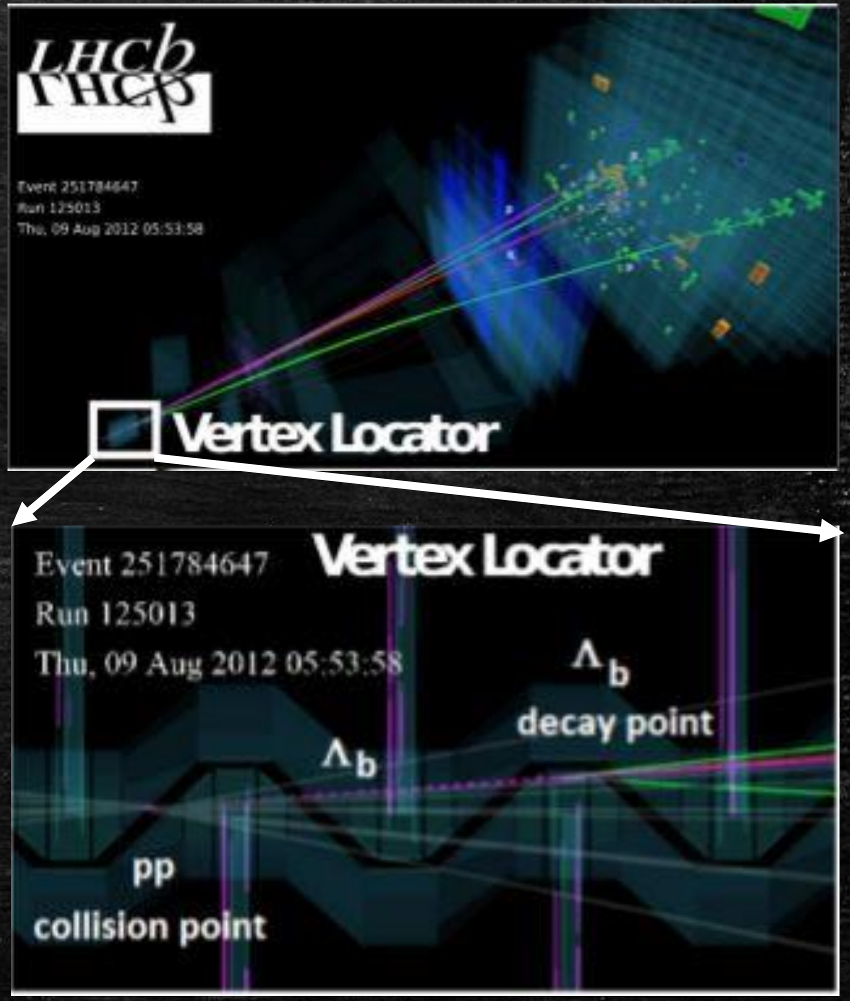


Based on 3 fb^{-1} data

Detector

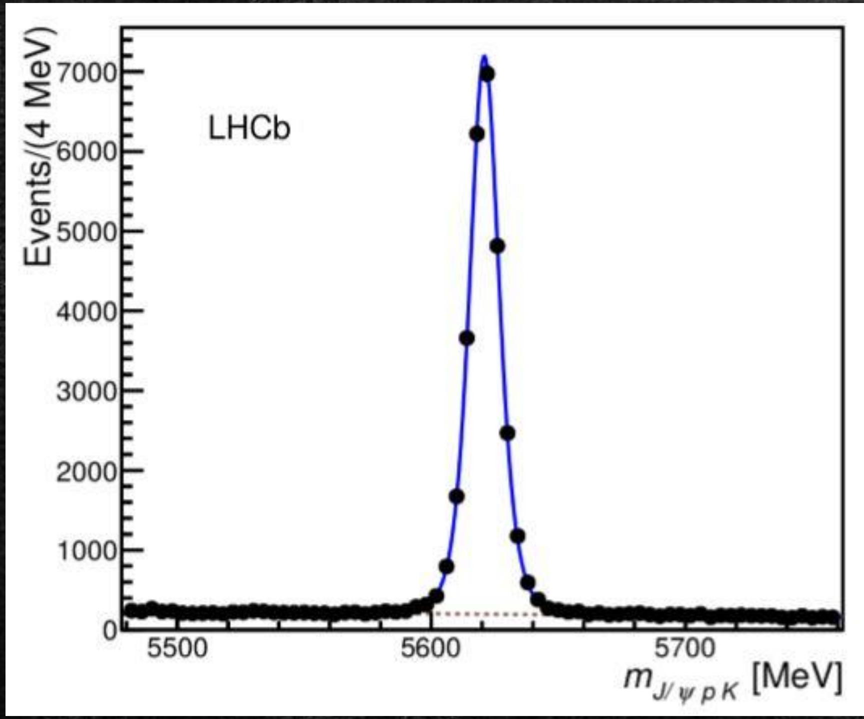


Visualization of a pentaquark event



Pseudorapidity acceptance $-2 < \eta < 5$

Event Selection



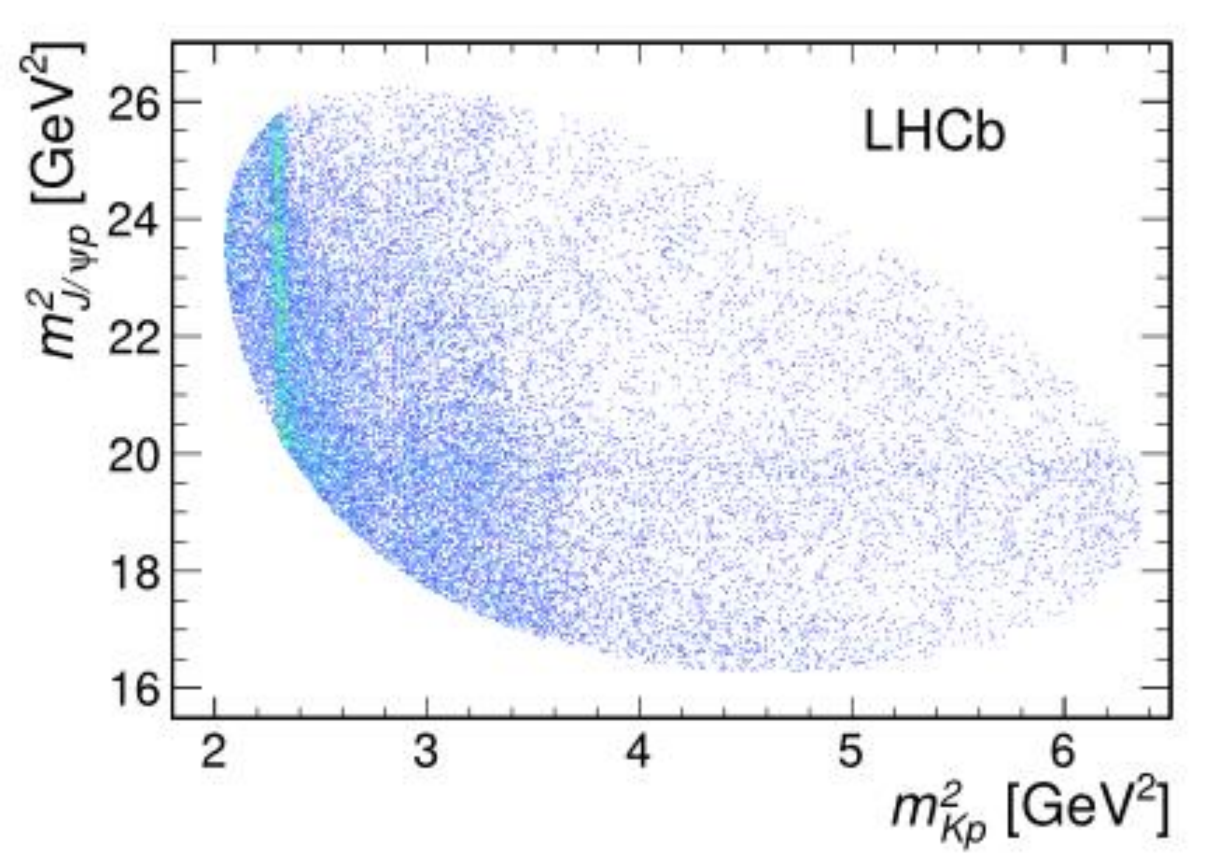
Ref: Nathan Jurik PhD dissertation (U-Syracuse)

- $J/\psi \to \mu^+\mu^-$
  - $p_t > 550$ MeV/c
  - -48 MeV/c2 $< M_{\mu+\mu-} - M_{J/\psi} < 43$ MeV/c2
  - Separated vertex from primary vertex (PV)
- K/p selection
  - Vertex fit compatible with J/ψ
  - $p_t > 250$ MeV/c
  - Proton does not point to PV
- Λ_b
  - Flight distance > 1.5 mm
  - Direction between PV and decay vertex matches the Λ_b momentum

$\underline{B}^0/\underline{B}_S^0$ veto and Boosted Decision Tree to reject the background

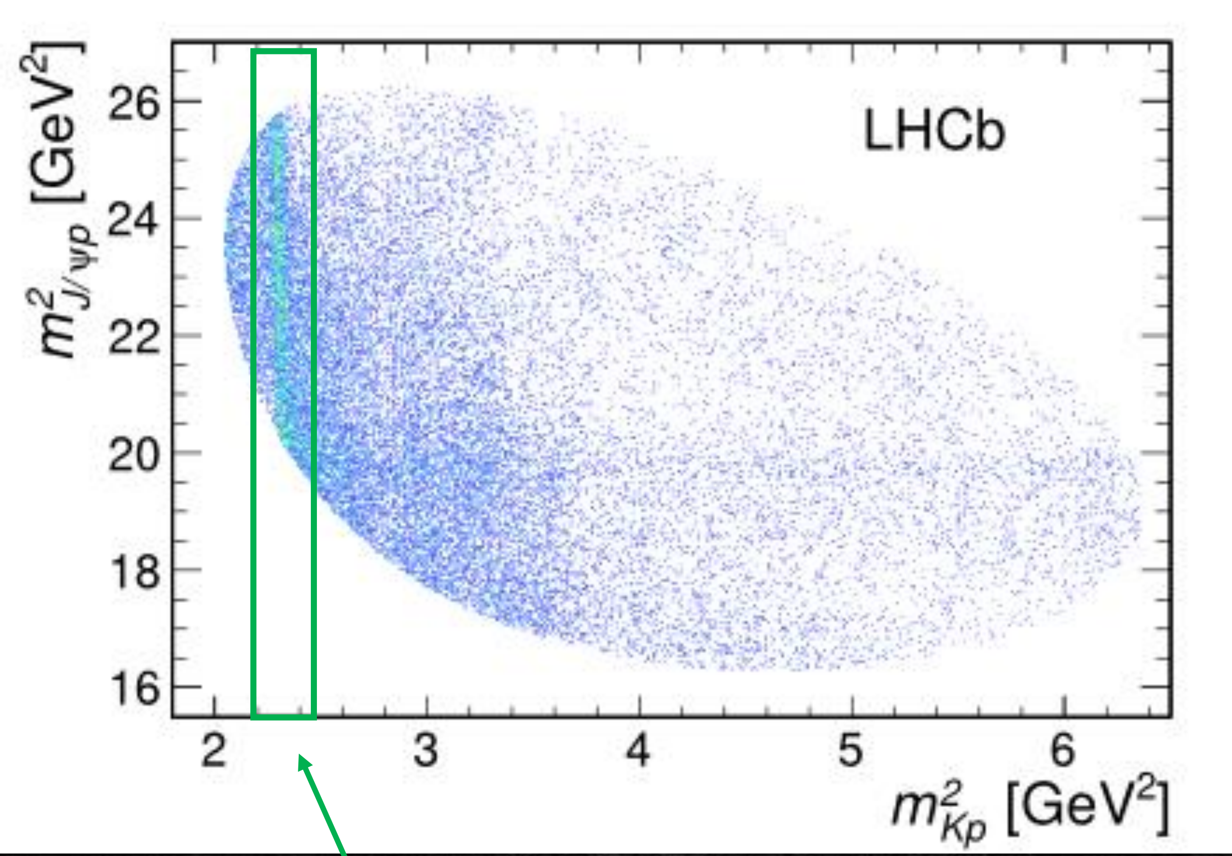
Results

Dalitz plot



Results

Dalitz plot



Expected signal from Λ(1520)

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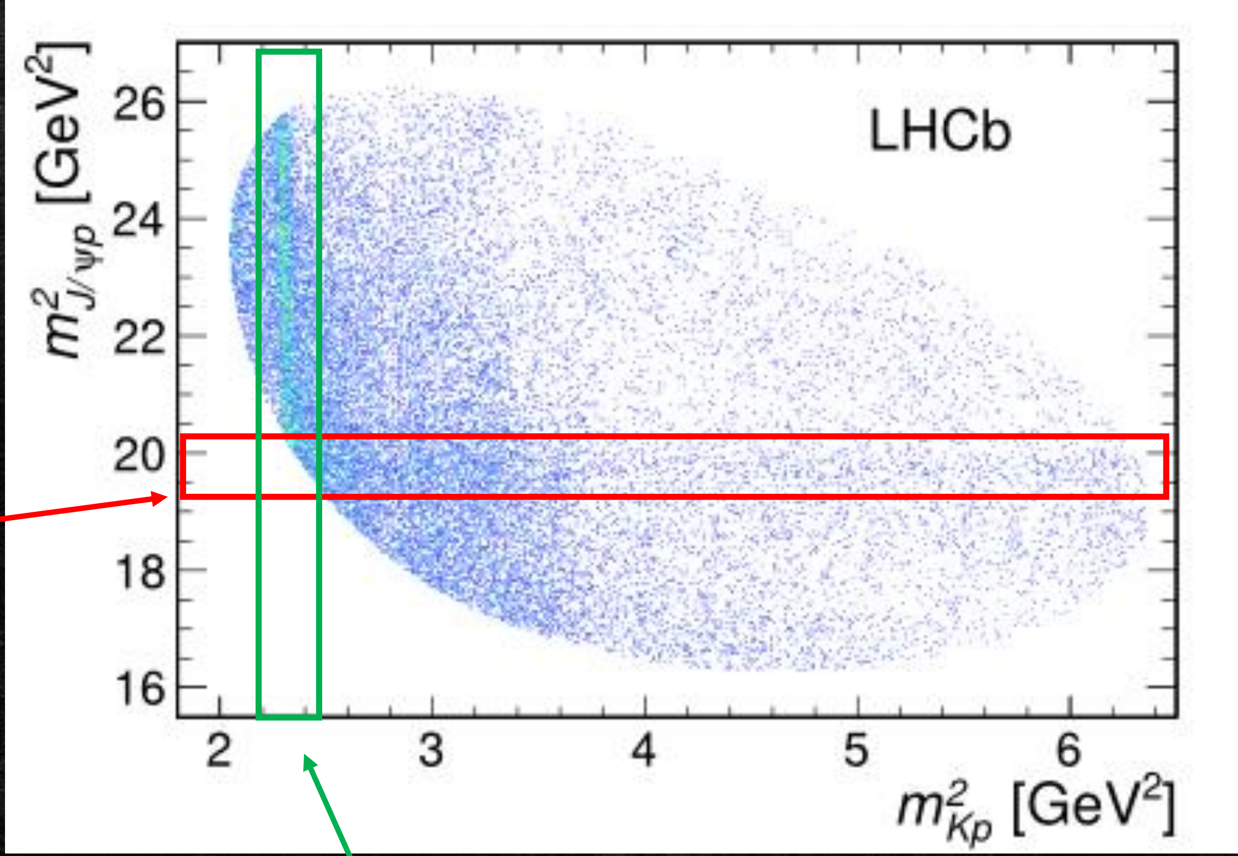
Results

Dalitz plot

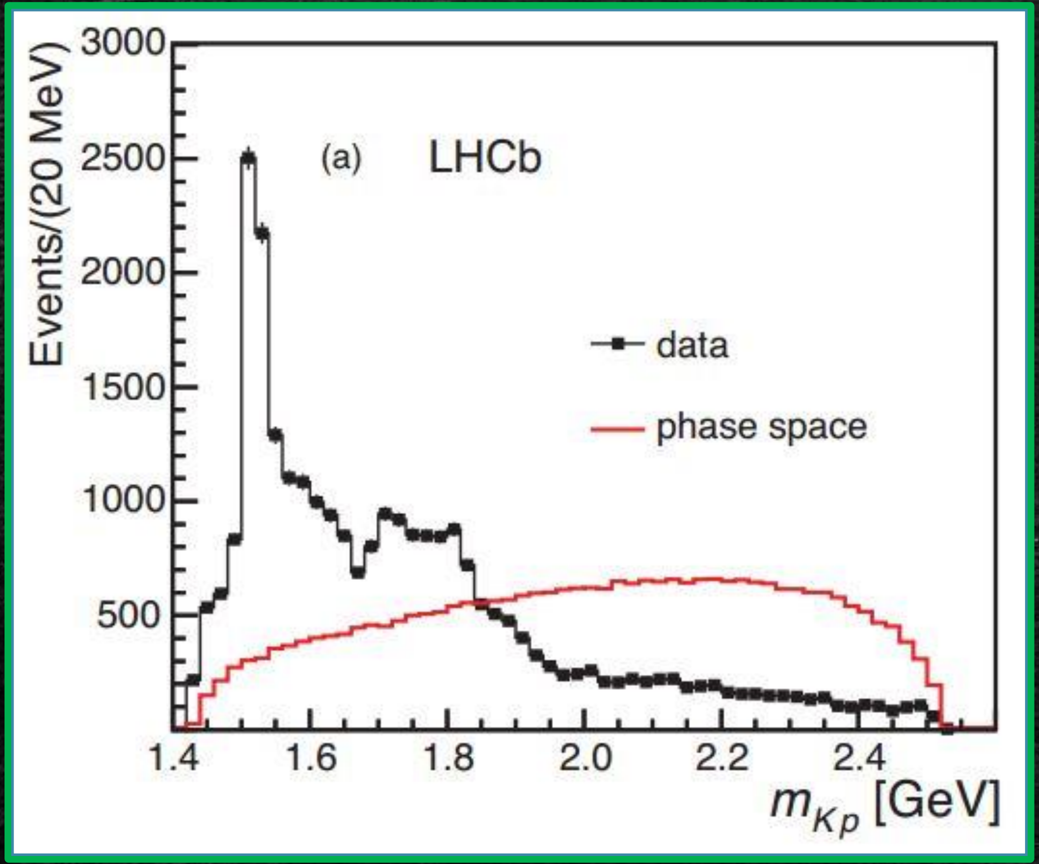


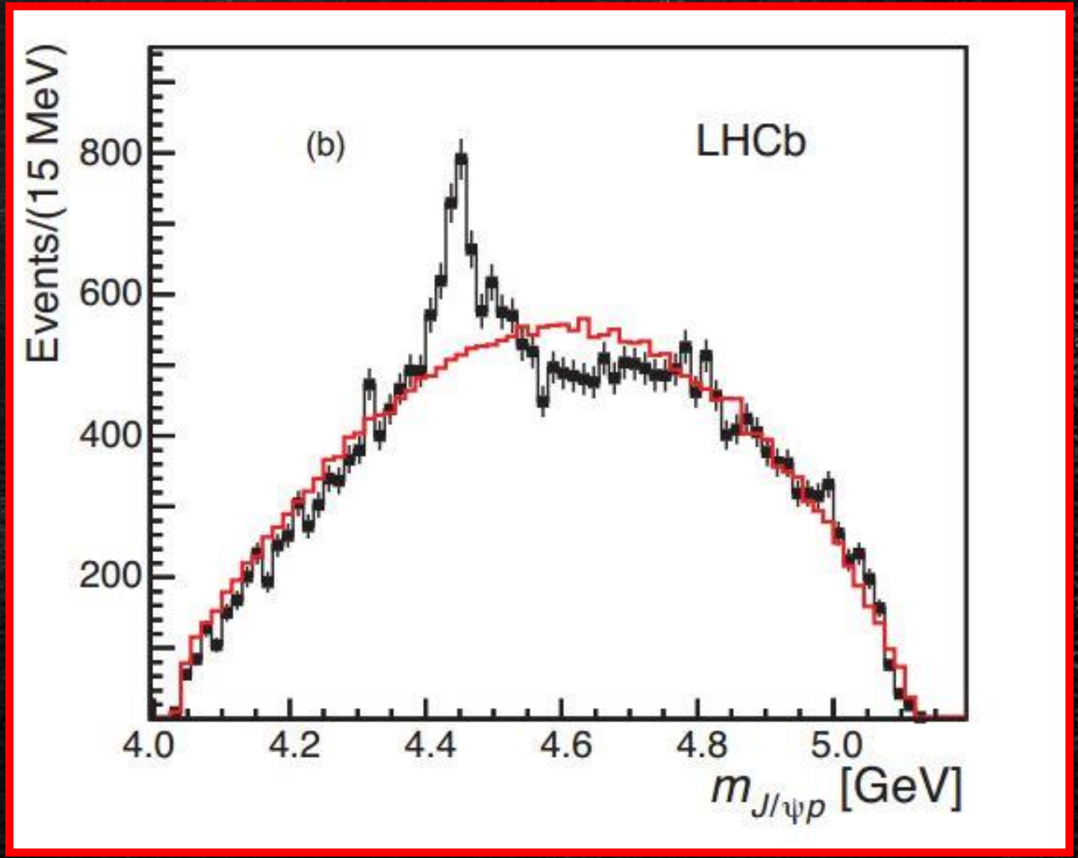
Unexpected signal in J/ψp invariant mass

Expected signal from Λ(1520)

Results

Dalitz plot: projections





Can they both be described as Λ_b -> $\Lambda^* J/\psi$?

Helicity Amplitude Model

Ref: J.D. Richman, CALT-68-1148 (for experimentalist)

- Multi-body final states have complex decay dynamics
- Partial wave analysis is a powerful tool to describe the data

$$|J, m >= \sum_{m'} D^{J}_{m,m'}(\alpha, \beta, \gamma)|J, m' >$$

General description of a rotation from state J,m to J',m'

$$\mathcal{H}^{A\to BC}_{\lambda_B,\lambda_C} * D^{J_A}_{\lambda_A,\lambda_B-\lambda_C}(\phi_B, \theta_A, 0) * R_A(m_{BC})$$

For A -> BC (helicity formalism)

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Helicity Amplitude Model

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For A -> BC (helicity formalism)

Complex Helicity amplitude matrix

Term of helicity axis rotation

Proportional to natural width of state A

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Helicity Amplitude Model

$\Lambda_b \to \Lambda^* J/\psi$ amplitude

$$\mathcal{M}^{\Lambda^*}_{\lambda_{\Lambda_b^0},\lambda_p,\Delta\lambda_\mu} = \sum_n \sum_{\lambda_{\Lambda^*}} \sum_{\lambda_\Psi} \boxed{\mathcal{H}^{\Lambda_b^0\to\Lambda_n^*\Psi}_{\lambda_{\Lambda^*},\lambda_\Psi} D^{1/2}_{\lambda_{\Lambda^*},\lambda_{\Lambda^*}-\lambda_\Psi}(0,\theta_{\Lambda_b^0},0)^*}$$

$$\times \boxed{\mathcal{H}^{\Lambda_n^*\to Kp}_{\lambda_p,0} D^{J_{\Lambda_n^*}}_{\lambda_{\Lambda^*},\lambda_p}(\phi_K,\theta_{\Lambda^*},0)^* R_{\Lambda_n^*}(m_{Kp})}$$

$$\times \boxed{D^1_{\lambda_\Psi,\Delta\lambda_\mu}(\phi_\mu,\theta_\Psi,0)^*}$$

$\Lambda_b \to \Lambda^* J/\psi$

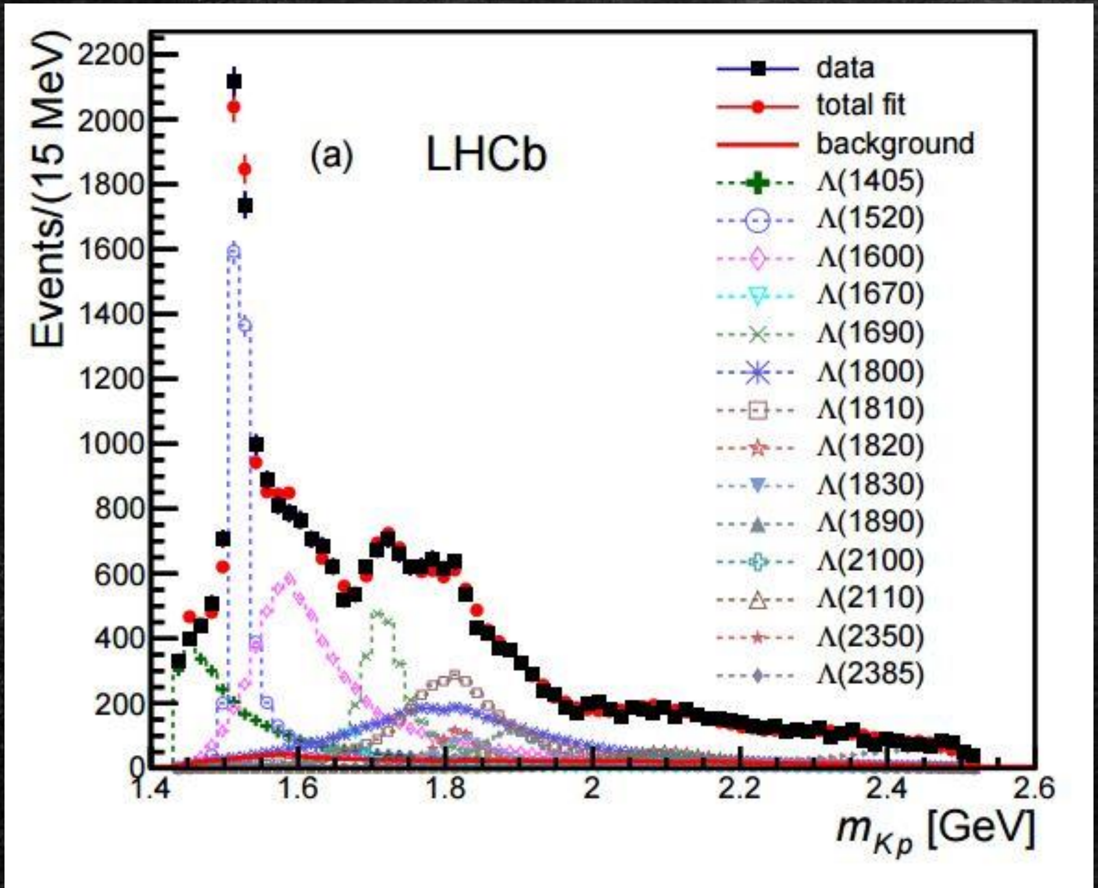
$\Lambda^* \to K^- p$

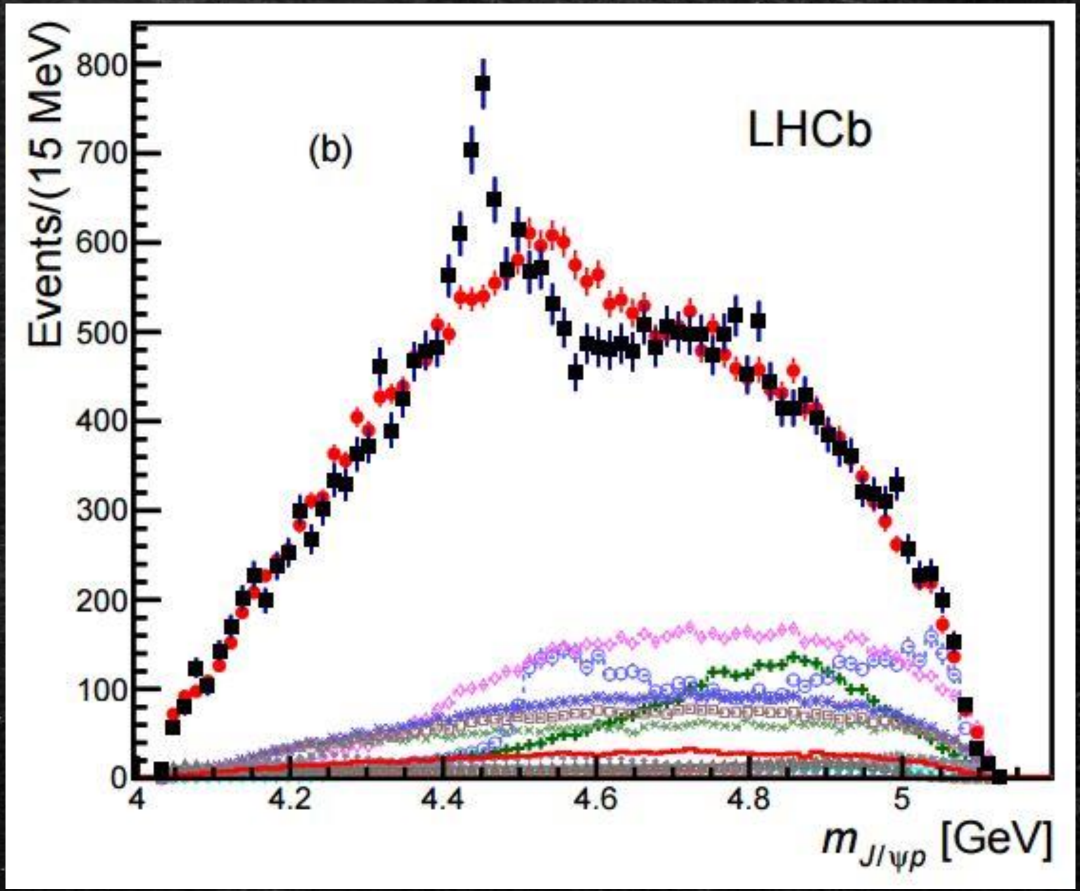
$J/\psi \to \mu^+ \mu^-$

146 free-parameter fit considering all the known Λ^* states

Results of the Extended Fit

Considering 14 possible Λ^* resonances





Fit does not match the data -> Add new components to the fit

P_c Helicity Decay Amplitude

$\Lambda_b \to P_c K^-$ amplitude

$$\mathcal{M}^{P_c}_{\lambda_{\Lambda_b^0},\lambda_p^{P_c},\Delta\lambda_\mu^{P_c}} = \sum_j \sum_{\lambda_{P_c}} \sum_{\lambda_\Psi^{P_c}} \boxed{\mathcal{H}^{\Lambda_b^0\to P_{c,j}K}_{\lambda_{P_c},0}\, D^{1/2}_{\lambda_{\Lambda_b^0},\lambda_{P_c}}(\phi_{P_c},\theta^{P_c}_{\Lambda_b^0},0)^*}$$

$$\times \boxed{\mathcal{H}^{P_{c,j}\to\Psi p}_{\lambda_\Psi^{P_c},\lambda_p^{P_c}}\, D^{J_{P_{c,j}}}_{\lambda_{P_c},\lambda_\Psi^{P_c}-\lambda_p^{P_c}}(\phi_\Psi,\theta_{p_c},0)^*\, R_{P_{c,j}}(m_{\Psi p})}$$

$$\times \boxed{D^1_{\lambda_\Psi^{P_c},\Delta\lambda_\mu^{P_c}}(\phi_\mu^{P_c},\theta_\Psi^{P_c},0)^*}$$

$\Lambda_b \to P_c K^-$

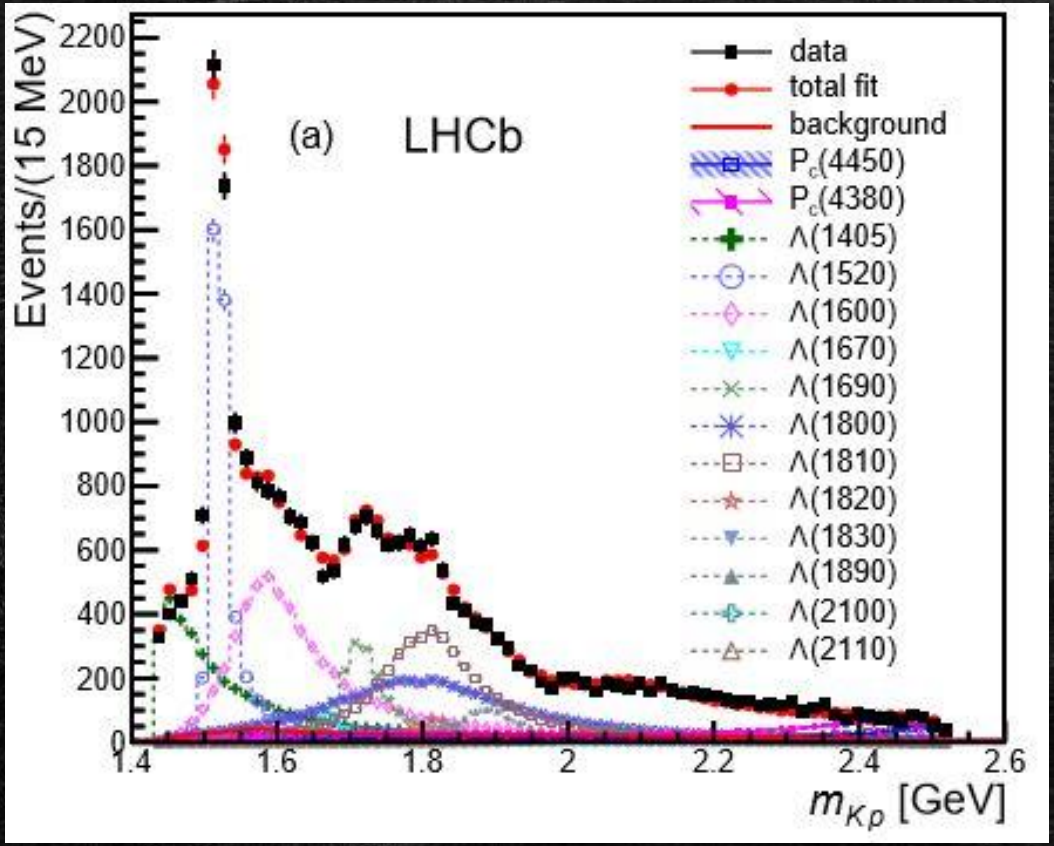
$P_c \to J/\psi p$

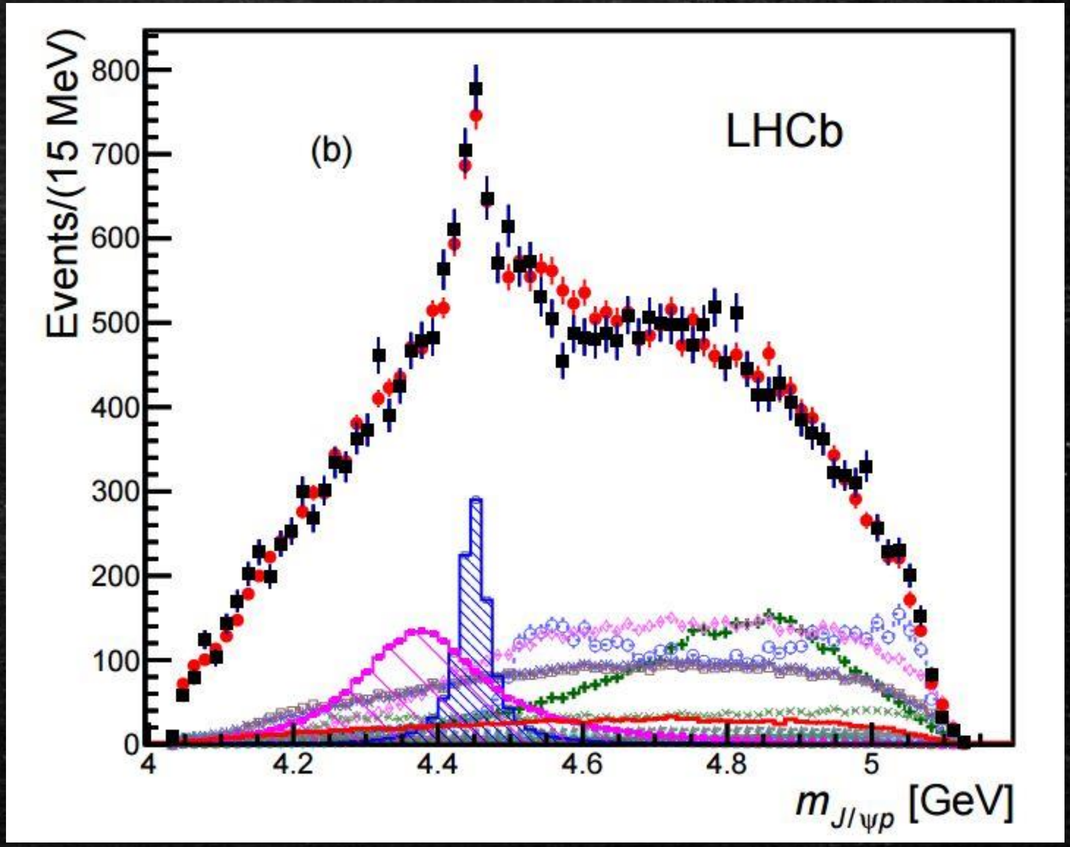
$J/\psi \to \mu^+\mu^-$

Reduced fit for Λ^* (64 free parameters) + P_c components

Results of the Reduced Fit

Higher mass Λ^* states excluded from the fit





Two more new states are necessary to fit the data!

Best Fit Results

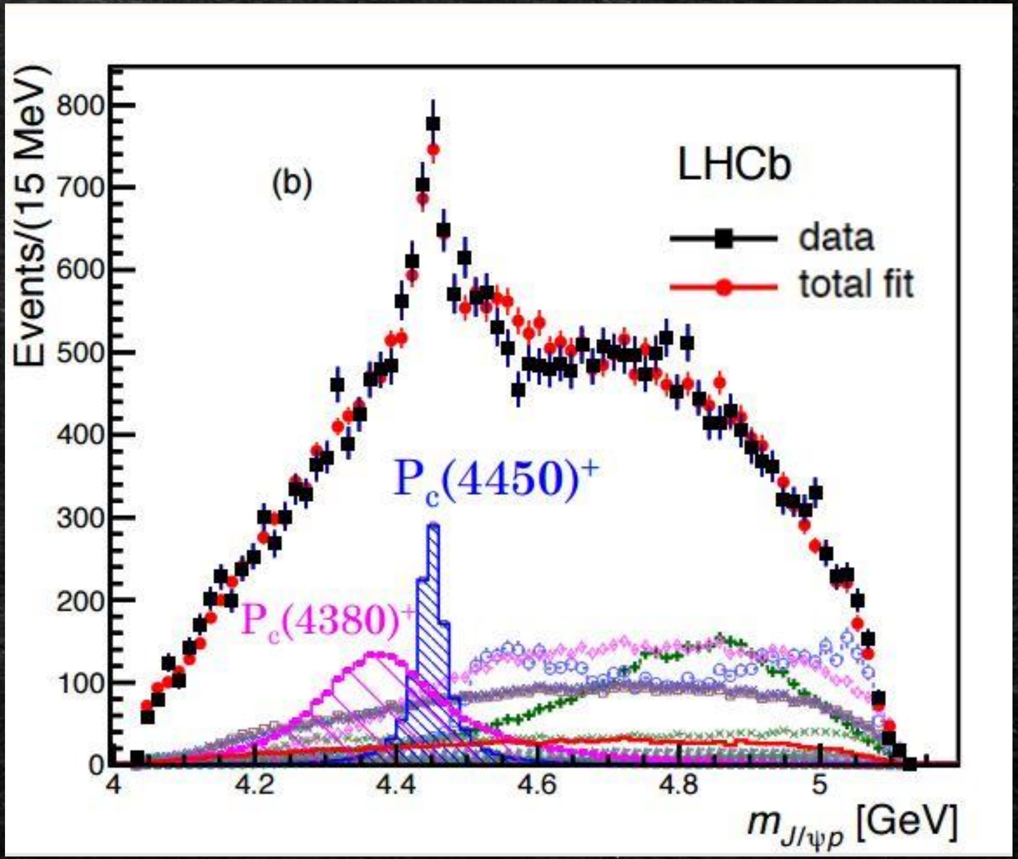
- Two states:

$P_c(4380)^+$:

Mass = 4380 ± 8 MeV
Width = 205 ± 18 MeV
Fit fraction = (4.1 ± 0.5)%
$J^p = 3/2^-$

$P_c(4450)^+$:

Mass = 4449.8 ± 1.7 MeV
Width = 205 ± 18 MeV
Fit fraction = (8.4 ± 0.7)%
$J^p = 5/2^+$



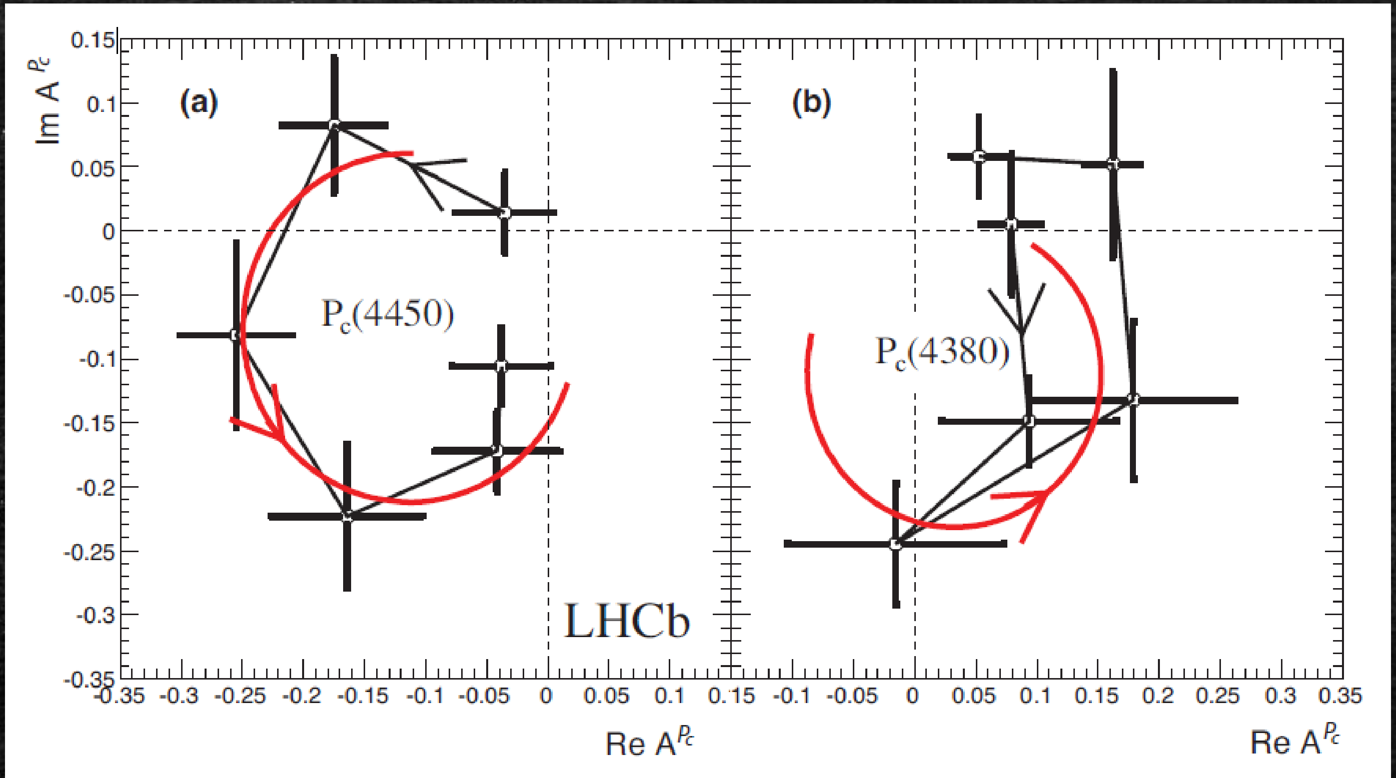
But also $J^p = (3/2^+, 5/2^-)$ and $J^p = (5/2^+, 3/2^-)$ are possible

Argand Plot

- Test of the resonant nature of the P_c candidates

Relativistic Breit-Wigner divided into 6 complex amplitudes

12 parameters



/ Expected Breit-Wigner
+ data

Argand diagrams of two pentaquark states.
$P_c(4450)$ (5/2+) is good, $P_c(4380)$ (3/2-) has 2 sigma difference for one bin.

Systematic Uncertainties

Source	M_0 (MeV) low	M_0 (MeV) high	Γ_0 (MeV) low	Γ_0 (MeV) high	Fit fractions (%) low	Fit fractions (%) high	$\Lambda^*(1405)$	$\Lambda^*(1520)$
Extended vs. reduced	21	0.2	54	10	3.14	0.32	1.37	0.15
Λ^* masses & widths	7	0.7	20	4	0.58	0.37	2.49	2.45
Proton ID	2	0.3	1	2	0.27	0.14	0.20	0.05
$10 < p_p < 100$ GeV	0	1.2	1	1	0.09	0.03	0.31	0.01
Non-resonant	3	0.3	34	2	2.35	0.13	3.28	0.39
Separate sidebands	0	0	5	0	0.24	0.14	0.02	0.03
J^P $(3/2^+, 5/2^-)$ or $(5/2^+, 3/2^-)$	10	1.2	34	10	0.76	0.44		
$d = 1.5 - 4.5$ GeV^{-1}	9	0.6	19	3	0.29	0.42	0.36	1.91
$\ell^{P_c}_{\Lambda_b}$ $\Lambda_b \to P_c^+$ (low/high)K^-	6	0.7	4	8	0.37	0.16		
ℓ_{P_c} P_c^+ (low/high) $\to J/\psi p$	4	0.4	31	7	0.63	0.37		
$\ell^{\Lambda^*_n}_{\Lambda_b}$ $\Lambda_b^* \to J/\psi \Lambda^*$	11	0.3	20	2	0.81	0.53	3.34	2.31
Efficiencies	1	0.4	4	0	0.13	0.02	0.26	0.23
Change $\Lambda^*(1405)$ coupling	0	0	0	0	0	0	1.90	0
Overall	29	2.5	86	19	4.21	1.05	5.82	3.89
sFit/cFit cross check	5	1.0	11	3	0.46	0.01	0.45	0.13

Resonance Parameterization

Λ^* Modeling

Quantum Number Assignment

Final Remarks

- LHCb found two resonance states decaying into $J/\psi p$ compatible with pentaquark structures
  - Higher mass state: $P_c(4450)^+$ with mass = (4449.8 ± 1.7 ± 2.5) MeV and width = (39 ± 5 ± 19) MeV with a significance of 12 standard deviations
  - Lower mass state: $P_c(4380)^-$ with mass = (4380 ± 8 ± 29) MeV and width = (205 ± 18 ± 86) MeV with a significance of 9 standard deviations
- Helicity amplitude analysis points towards a (3/2, 5/2) spin assignment, with opposite parity
- Several models are proposed to explain the structure (hadro-charmonium, molecular state, meson-baryon bound state)
  - Data are not yet conclusive and more studies are needed

THANK YOU!

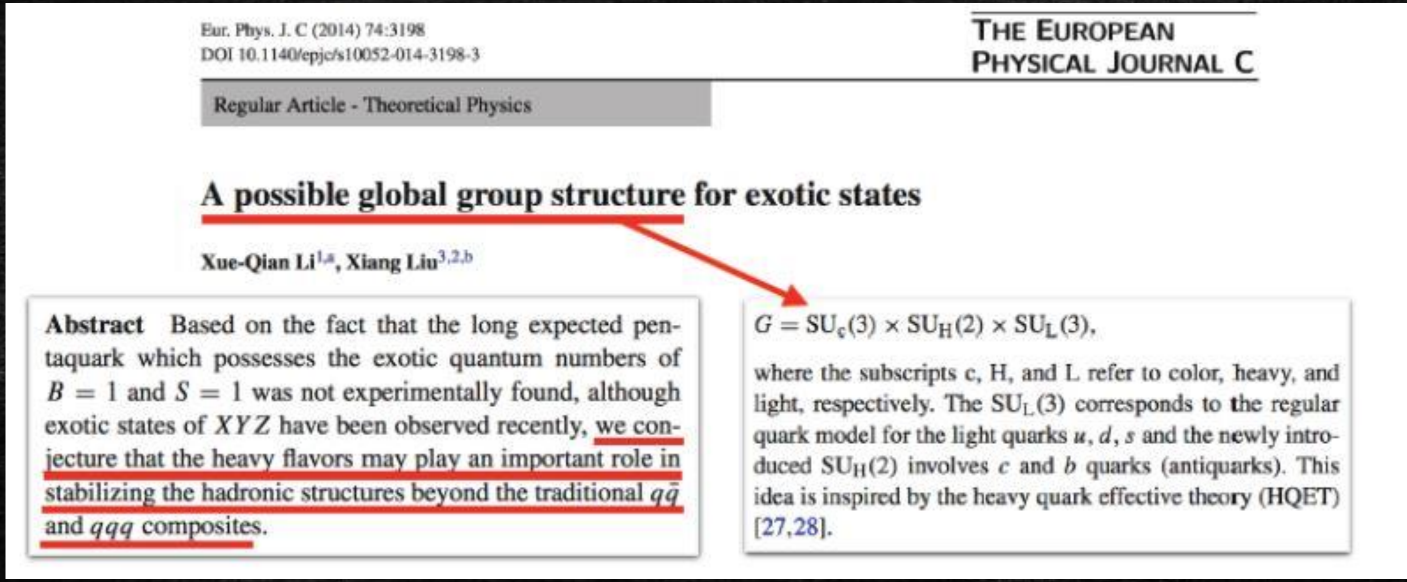
谢谢

You want to vote for this presentation! ;)

Additional contents

Why no light multiquarks state? A possible explanation

- In heavy quarkonium region, several multiquark, i.e. XYZ states, were observed
  - No counterpart with low quark masses (u,d,s)
- Some theorist argue that heavy quark plays important role in binding the structure together
- For a reference:


Eur. Phys. J. C (2014) 74:3198
DOI 10.1140/epjc/s10052-014-3198-3

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

A possible global group structure for exotic states

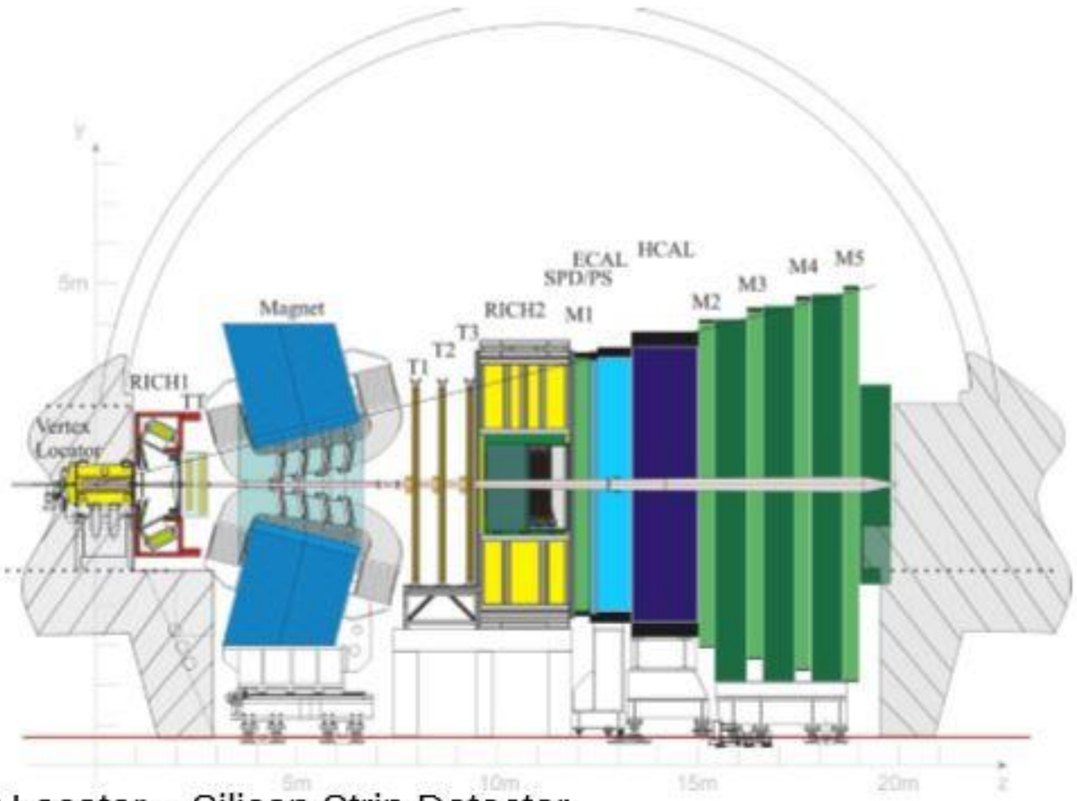
Xue-Qian Li[1,a], Xiang Liu[3,2,b]

Abstract Based on the fact that the long expected pentaquark which possesses the exotic quantum numbers of $B = 1$ and $S = 1$ was not experimentally found, although exotic states of XYZ have been observed recently, we conjecture that the heavy flavors may play an important role in stabilizing the hadronic structures beyond the traditional $q\bar{q}$ and qqq composites.

$G = SU_c(3) \times SU_H(2) \times SU_L(3)$,

where the subscripts c, H, and L refer to color, heavy, and light, respectively. The $SU_L(3)$ corresponds to the regular quark model for the light quarks u, d, s and the newly introduced $SU_H(2)$ involves c and b quarks (antiquarks). This idea is inspired by the heavy quark effective theory (HQET) [27,28].

Detector details



- Vertex Locator – Silicon Strip Detector.
- RICH1/RICH2 – Ring imaging cherenkov detectors.
- TT/Magnet/T1/T2/T3 – Tracking.
- Ecal/HCAL – Calorimeters
- M1-M5 – Muon detection.

Event selection – further details

Selection variables	Requirements
All tracks χ^2/ndf	< 4
Muon PID	$\mathrm{DLL}(\mu - \pi) > 0$
p_T of muon	> 550 MeV
J/ψ vertex χ^2	< 16
J/ψ χ^2/ndf DLS	> 3
J/ψ mass window	$-48 < m(\mu^+\mu^-) - m(J/\psi) < 43$ MeV
p_T of hadron	> 250 MeV
Hadron χ^2_{IP}	> 9
K^- ID	$\mathrm{DLL}(K - \pi) > 0$ and $\mathrm{DLL}(p - K) < 3$
p ID	$\mathrm{DLL}(p - \pi) > 10$ and $\mathrm{DLL}(p - K) > 3$
Clone track rejection on hadron	Ghost probability < 0.2
pK^- vertex	DOCA $\chi^2 < 16$
Λ_b^0 χ^2_{IP}	< 25
Λ_b^0 vertex χ^2/ndf	< 10
Λ_b^0 fight distance	> 1.5 mm
Λ_b^0 pointing	DIRA> 0.999

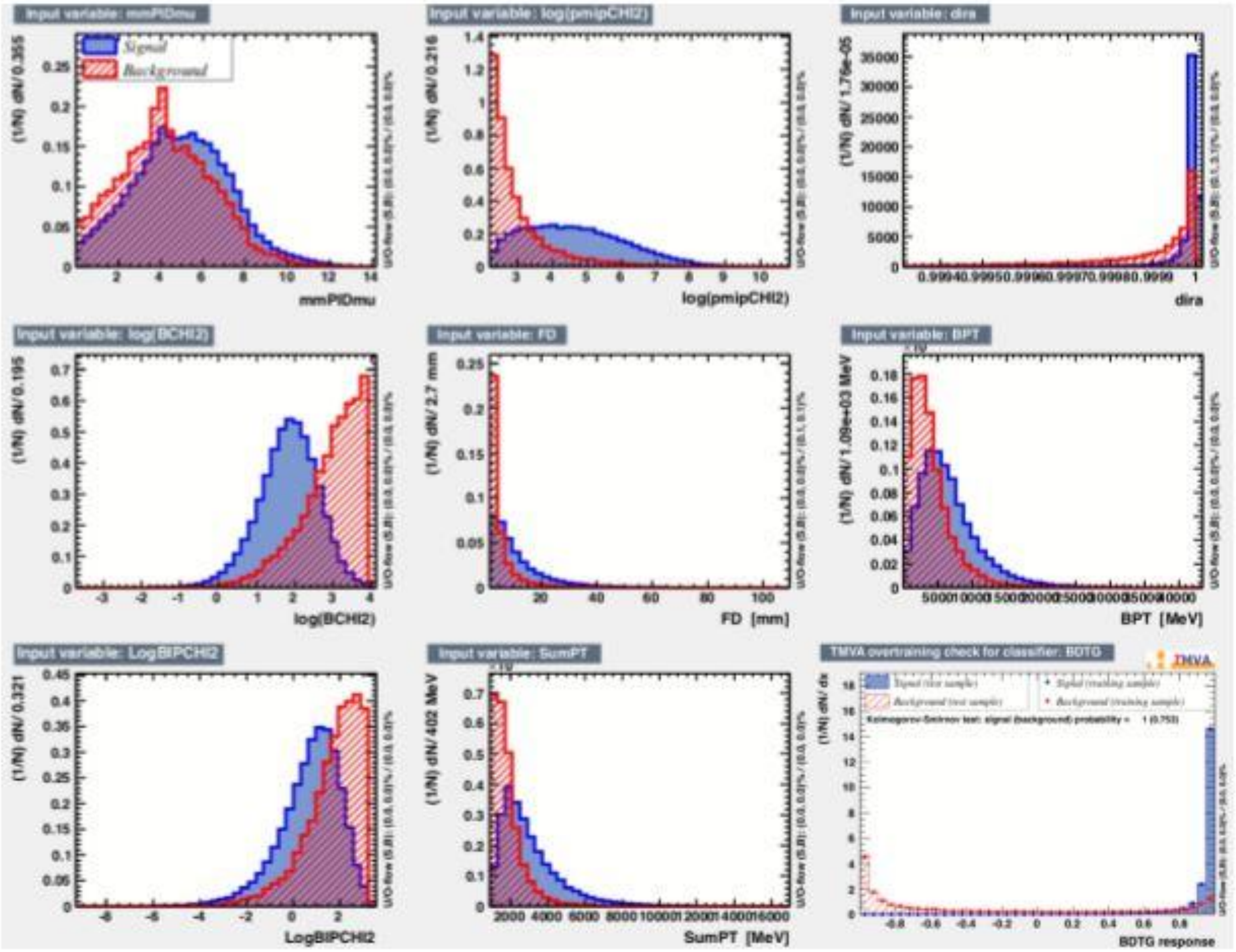
Reference: Nathan Jurik pentaquark dissertation.

BTDG (Gradient Boosted decision tree)

- mmPIDmu – is the log of the likelihood ratio of the pion and the muon hypotheses
- pmipCHI2 – The minimum of χ_{ip}^2
- Cos of the angle between the direction of the PV to decay vertex and the Λ^0_b momentum
- BCHI2 - $\chi_{ip}^2(\Lambda^0_b)$
- FD - Flight distance of the Λ^0_b
- BPT – p_t of the Λ^0_b
- BIPCHI2 - $\chi_{vtx}^2(\Lambda^0_b)$
- SumPT – Sum of the K^- and proton p_t

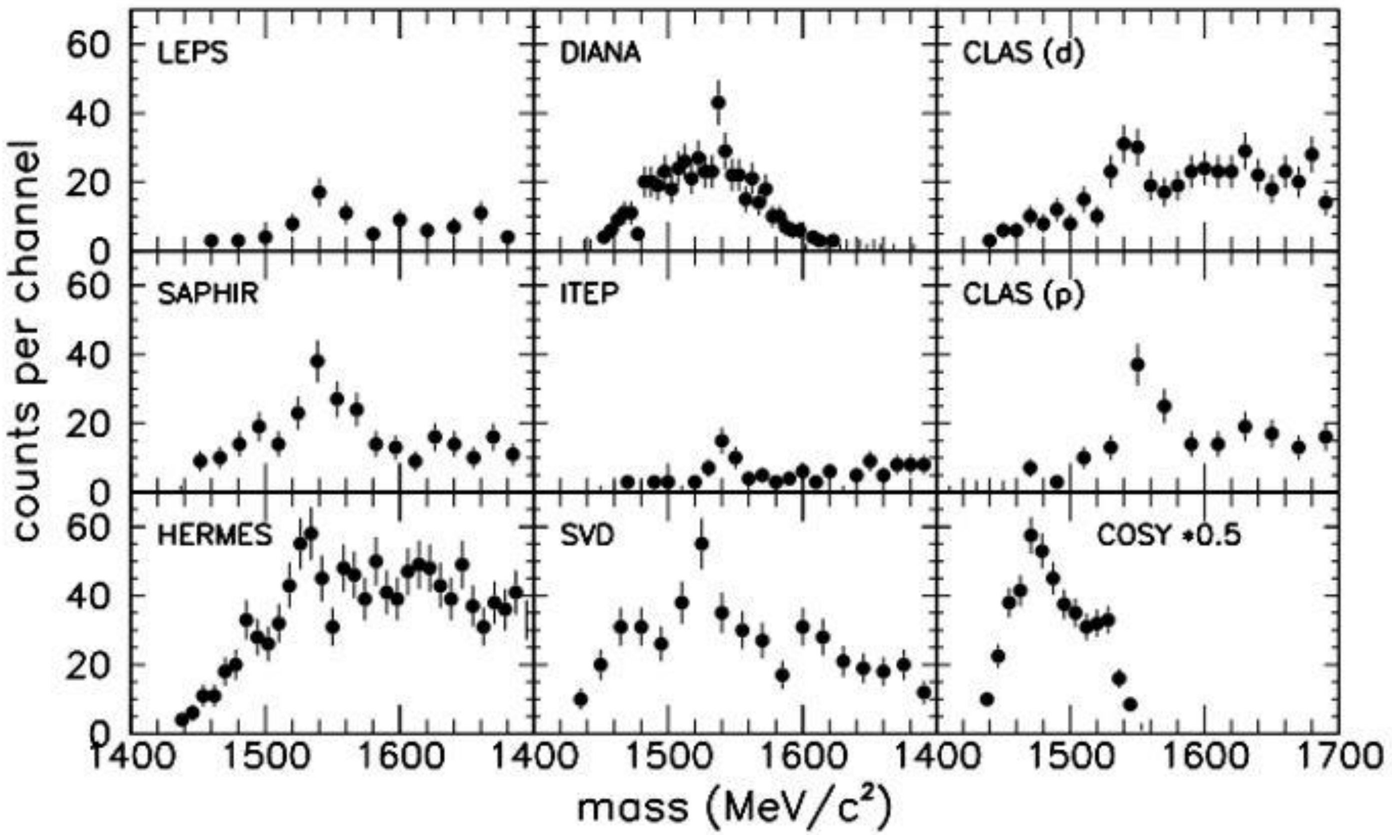
- BTDG Response– The output variable from the BDST – Cut at 0.9

BTDG (Gradient Boosted decision tree)



From 1 to 8 The BDST Input Variables. 9 The BSDST output variable.
Reference: Nathan Jurik pentaquark dissertation.

Θ(1540)+ searches



A summary of early experiments searching for the θ^+ presented without fitted curves.

Helicity amplitudes – more info

$$|J, m >= \sum_{m'} D^J_{m,m'}(\alpha, \beta, \gamma)|J, m' >$$

$$D^J_{m,m'}(\alpha, \beta, \gamma)^* =< J, m|\hat{\mathcal{D}}(\mathcal{R}(\alpha, \beta, \gamma))|J, m' >^*= e^{im\alpha} d^J_{m,m'}(\beta) e^{im'\gamma}$$

Helicity amplitudes – more info

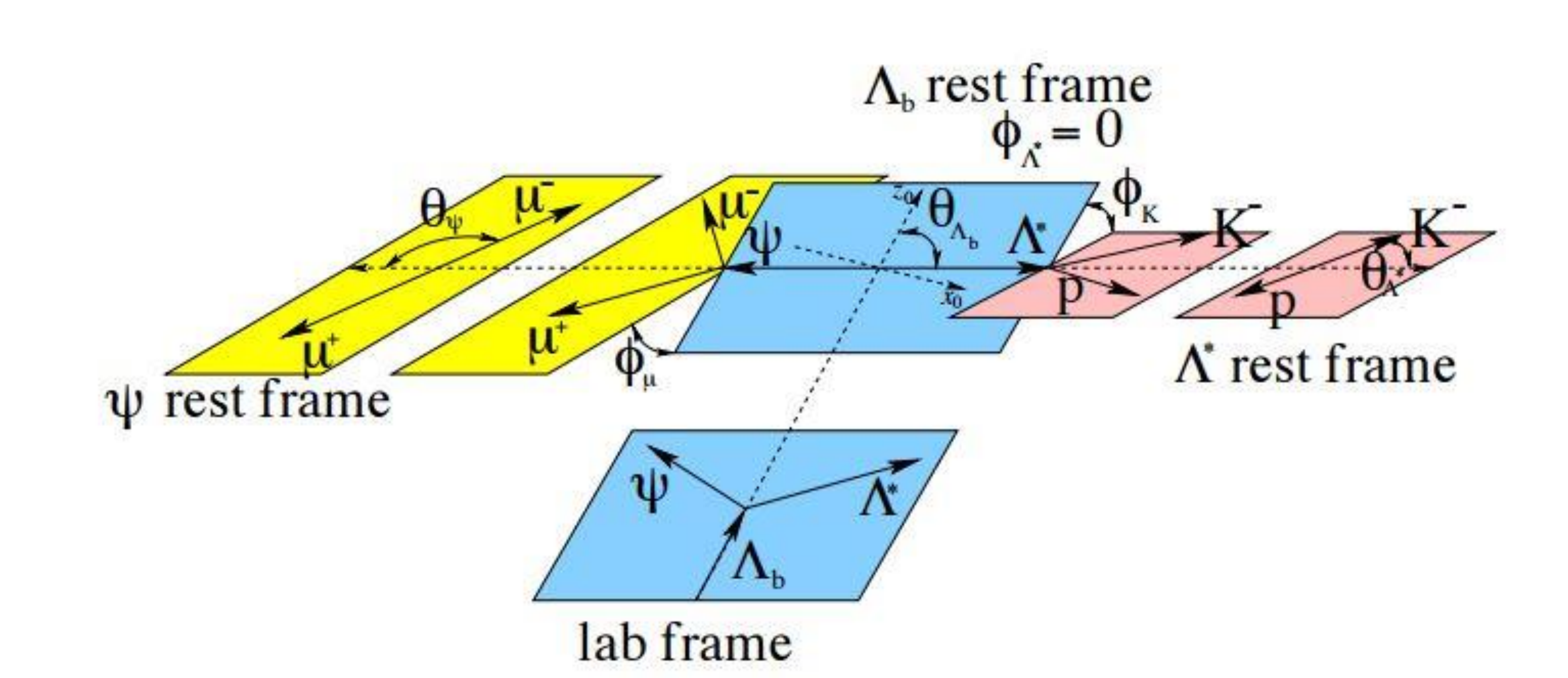
$$\mathcal{H}^{A\to BC}_{\lambda_B,\lambda_C} * D^{J_A}_{\lambda_A,\lambda_B-\lambda_C}(\phi_B,\theta_A,0) * R_A(m_{BC})$$

$$R_X(m) = B'_{L^X_{\Lambda_b^0}}(p,p_0,d)\left(\frac{p}{M_{\Lambda_b^0}}\right)^{L^X_{\Lambda_b^0}} \mathrm{BW}(m|M_{0X},\Gamma_{0X})\, B'_{L_X}(q,q_0,d)\left(\frac{q}{M_{0X}}\right)^{L_X}.$$

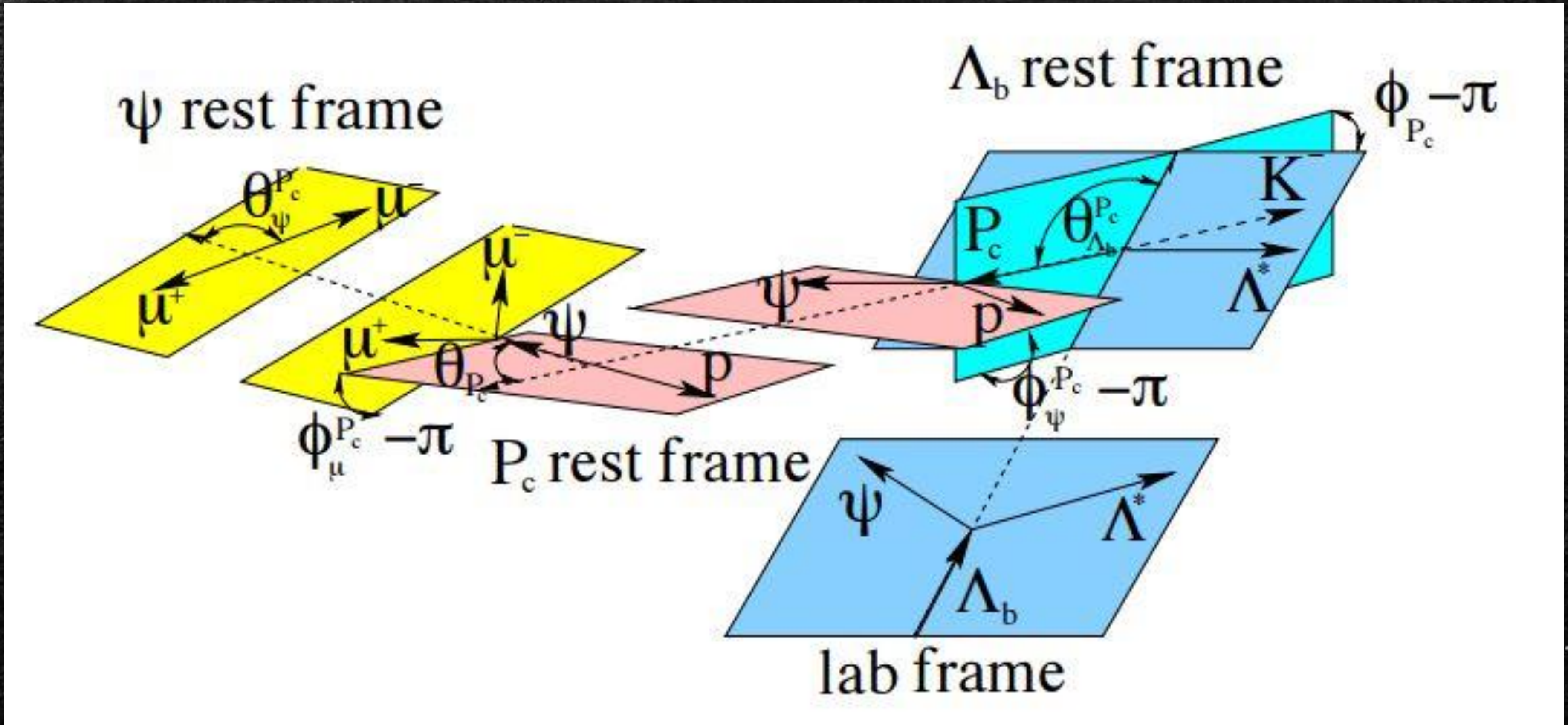
$$\begin{aligned}\phi_B &= \mathrm{atan2}\left(p_B{}^{\{A\}}_y, p_B{}^{\{A\}}_x\right)\\ &= \mathrm{atan2}\left(\hat{y}_0^{\{A\}}\cdot\vec{p}_B^{\{A\}}, \hat{x}_0^{\{A\}}\cdot\vec{p}_B^{\{A\}}\right)\\ &= \mathrm{atan2}\left((\hat{z}_0^{\{A\}}\times\hat{x}_0^{\{A\}})\cdot\vec{p}_B^{\{A\}}, \hat{x}_0^{\{A\}}\cdot\vec{p}_B^{\{A\}}\right),\\ \cos\theta_A &= \hat{z}_0^{\{A\}}\cdot\hat{p}_B^{\{A\}}.\end{aligned}$$

$$\mathcal{H}^{A\to BC}_{\lambda_B,\lambda_C} = \sum_L\sum_S\sqrt{\frac{2L+1}{2J_A+1}}B_{L,S}\times\begin{pmatrix} J_B & J_C & S\\ \lambda_B & -\lambda_C & \lambda_B-\lambda_C\end{pmatrix}\times\begin{pmatrix} L & S & J_A\\ 0 & \lambda_B-\lambda_C & \lambda_B-\lambda_C\end{pmatrix}$$

Helicity amplitude – more info – Λ* angles



Helicity amplitude – more info – Pc angles



Λ* parametrization

Masses, widths were taken from PDG and fixed in the fit

Each Λ*: J = 1/2 (> 1/2) has 4(6) coupling

State	J^P	M_0 (MeV)	Γ_0 (MeV)	# Reduced	# Extended
Λ(1405)	$1/2^-$	$1405.1^{+1.3}_{-1.0}$	50.5 ± 2.0	3	4
Λ(1520)	$3/2^-$	1519.5 ± 1.0	15.6 ± 1.0	5	6
Λ(1600)	$1/2^+$	1600	150	3	4
Λ(1670)	$1/2^-$	1670	35	3	4
Λ(1690)	$3/2^-$	1690	60	5	6
Λ(1800)	$1/2^-$	1800	300	4	4
Λ(1810)	$1/2^+$	1810	150	3	4
Λ(1820)	$5/2^+$	1820	80	1	6
Λ(1830)	$5/2^-$	1830	95	1	6
Λ(1890)	$3/2^+$	1890	100	3	6
Λ(2100)	$7/2^-$	2100	200	1	6
Λ(2110)	$5/2^+$	2110	200	1	6
Λ(2350)	$9/2^+$	2350	150	0	6
Λ(2585)	?	≈2585	200	0	6

Λ* parametrization

In order to reduce the number of free parameter, it was considered that Higher mass state with high L are most likely not in the data

State	J^P	M_0 (MeV)	Γ_0 (MeV)	# Reduced	# Extended
Λ(1405)	$1/2^-$	$1405.1^{+1.3}_{-1.0}$	50.5 ± 2.0	3	4
Λ(1520)	$3/2^-$	1519.5 ± 1.0	15.6 ± 1.0	5	6
Λ(1600)	$1/2^+$	1600	150	3	4
Λ(1670)	$1/2^-$	1670	35	3	4
Λ(1690)	$3/2^-$	1690	60	5	6
Λ(1800)	$1/2^-$	1800	300	4	4
Λ(1810)	$1/2^+$	1810	150	3	4
Λ(1820)	$5/2^+$	1820	80	1	6
Λ(1830)	$5/2^-$	1830	95	1	6
Λ(1890)	$3/2^+$	1890	100	3	6
Λ(2100)	$7/2^-$	2100	200	1	6
Λ(2110)	$5/2^+$	2110	200	1	6
Λ(2350)	$9/2^+$	2350	150	0	6
Λ(2585)	?	≈2585	200	0	6

In light blue the parameters for the reduced fit

1 P_c state fit result

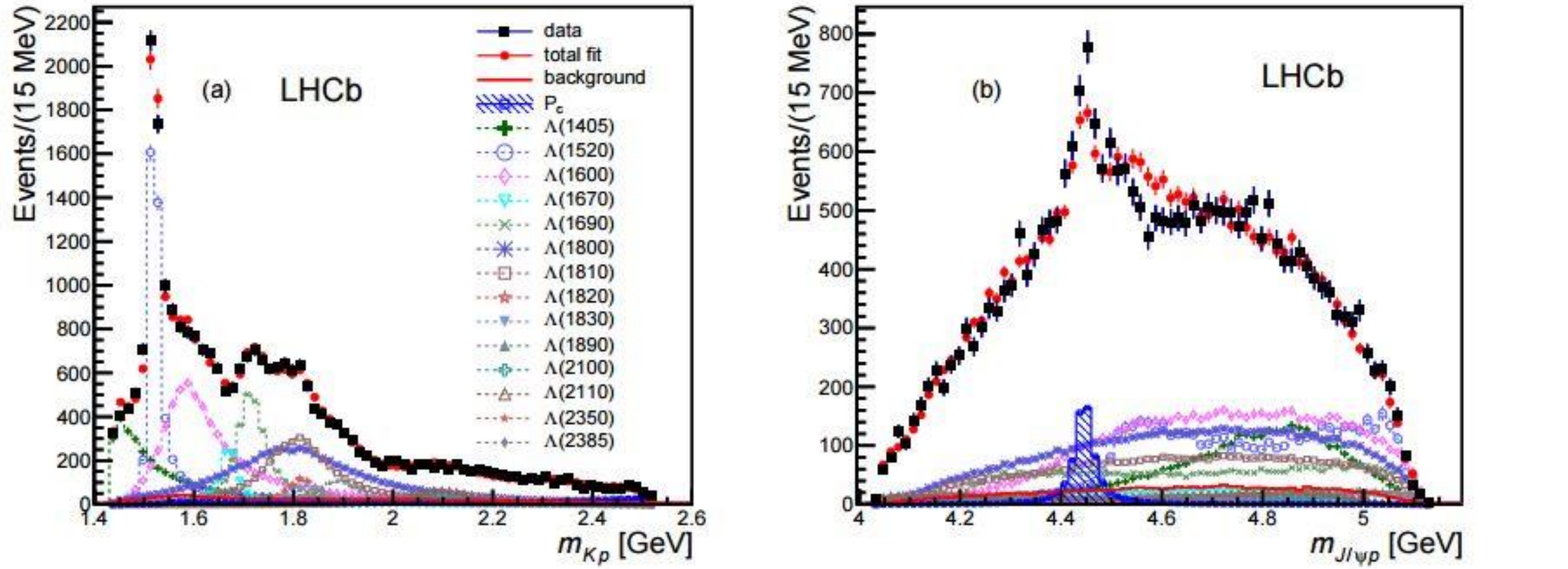
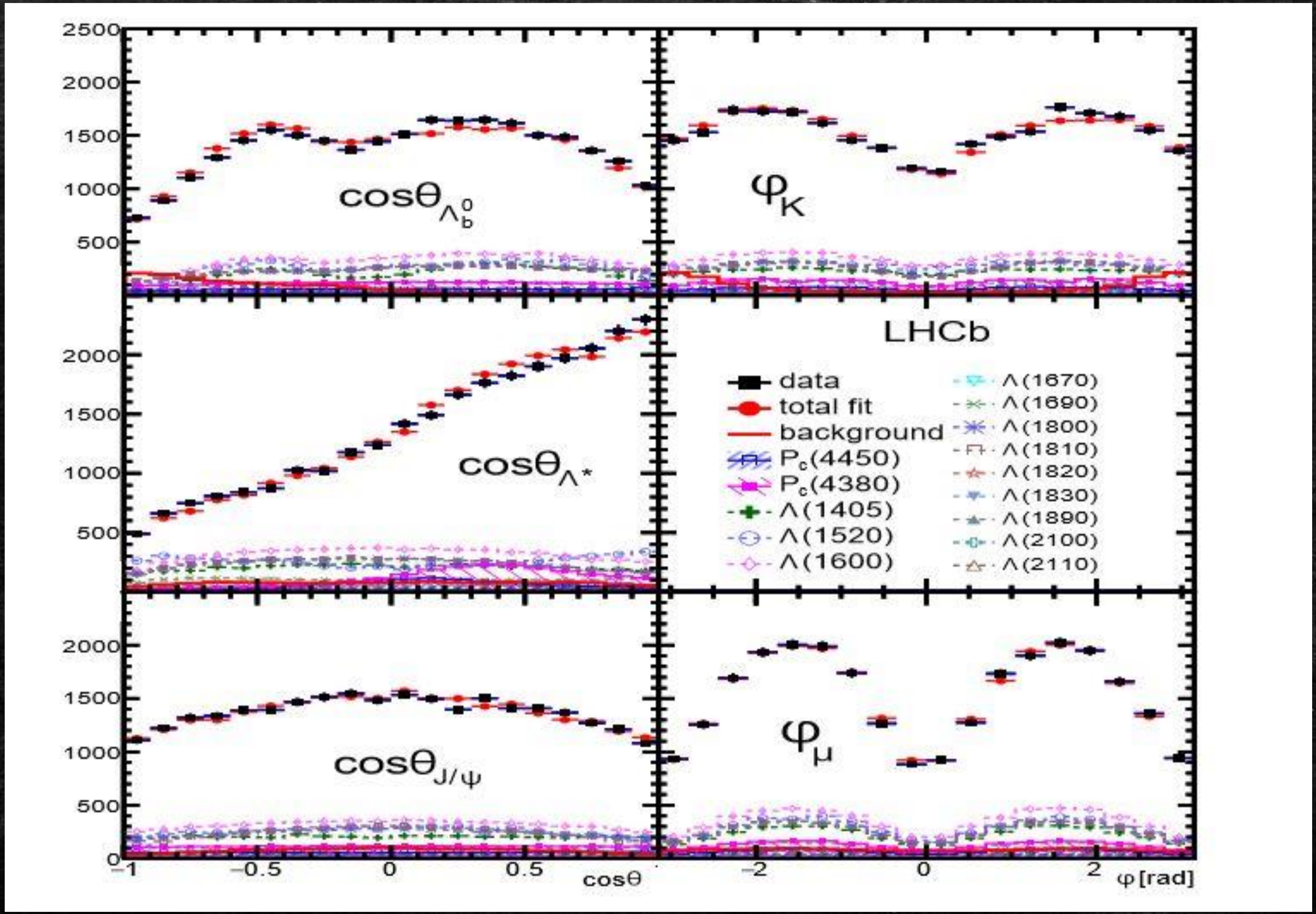


Figure 13: Results of the fit with one $J^P = 5/2^+$ P_c^+ candidate. (a) Projection of the invariant mass of K^-p combinations from $\Lambda_b^0 \to J/\psi K^- p$ candidates. The data are shown as (black) squares with error bars, while the (red) circles show the results of the fit; (b) the corresponding $J/\psi\, p$ mass projection. The (blue) shaded plot shows the P_c^+ projection, the other curves represent individual Λ^* states.

Higher number (>=3) of P_c states does not improve the quality of fit

Other lineshape from the fit



Efficiency and acceptance

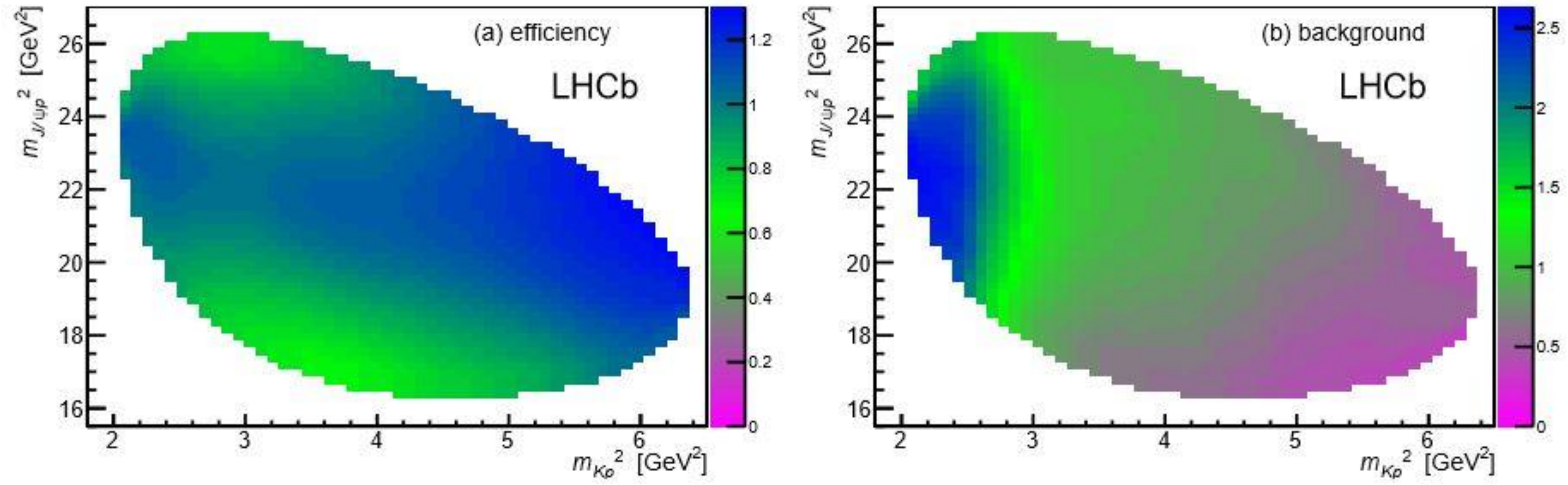


Figure: Parametrized signal efficiency (a) and background density (b) on the Dalitz plane.

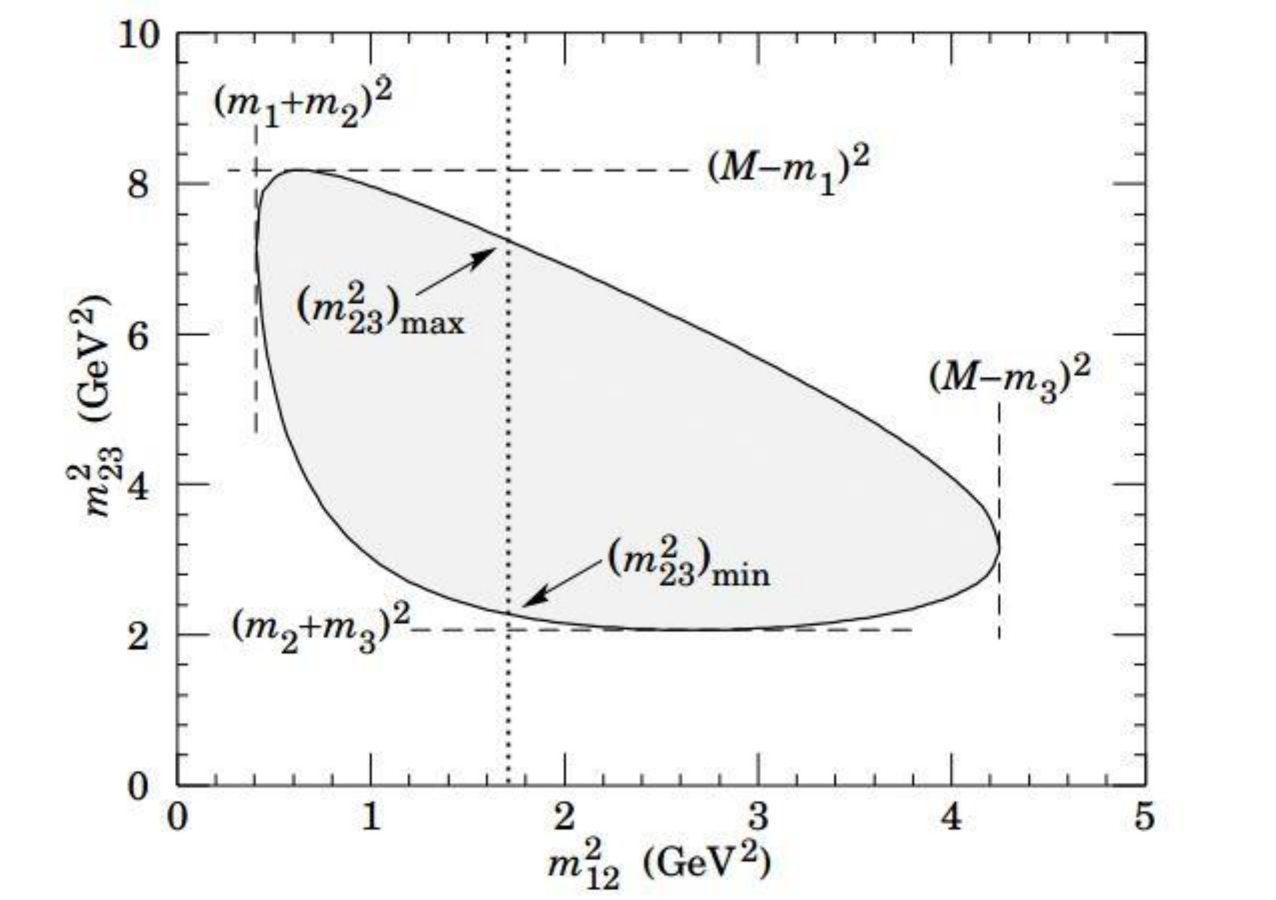
Dalitz plot

$$(m_{23}^2)_{\max} =$$

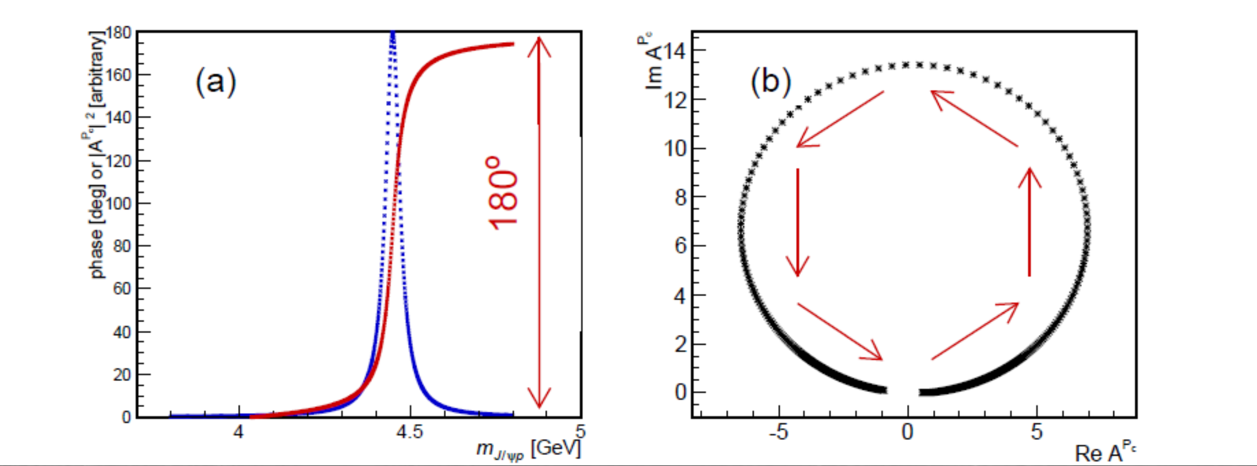
$$(E_2^* + E_3^*)^2 - \left(\sqrt{E_2^{*2} - m_2^2} - \sqrt{E_3^{*2} - m_3^2}\right)^2 ,$$

$$(m_{23}^2)_{\min} =$$

$$(E_2^* + E_3^*)^2 - \left(\sqrt{E_2^{*2} - m_2^2} + \sqrt{E_3^{*2} - m_3^2}\right)^2 .$$



Argand plot



$$\mathrm{BW}(m|M_0,\Gamma_0) = \frac{1}{M_0^2 - m^2 - iM_0\Gamma(m)}\,,$$

$$\Gamma(m) = \Gamma_0 \left(\frac{q}{q_0}\right)^{2\,L_{\Lambda*}+1} \frac{M_0}{m} B'_{L_{\Lambda*}}(q,q_0,d)^2\,.$$

Expected phase motion of a resonance which follows a Breit-Wigner line shape with M_0=4449.8MeV and Γ_0=39MeV.

- Breit-Wigner represents the expected behavior of a true resonance.
- Circle can be rotated by arbitrary phase.

Argand plot – theoretical approach

The non-relativistic Breit-Wigner

$$A = \frac{\Gamma_e/2}{E_0 - E - i\Gamma_t/2}$$

$$\mathrm{Re}A = \frac{\Gamma_e(E_0 - E)}{2[(E_0 - E)^2 + \Gamma_t^2/4]}$$

$$\mathrm{Im}A = \frac{\Gamma_e \Gamma_t}{4[(E_0 - E)^2 + \Gamma_t^2/4]}$$

Real and imaginary part

Get rid of E (energy)

$$(\mathrm{Re}A)^2 + (\mathrm{Im}A - \frac{\Gamma_e}{2\Gamma_t})^2 = (\frac{\Gamma_e}{2\Gamma_t})^2$$

Equation of a circle with center $(0, \Gamma_e/2\Gamma_t)$ and radius $\Gamma_e/2\Gamma_t$