Semi-classical Nonlinear Gauge Field Theories, their Exact Solutions and their Usefulness for Magnetic Monopole Searches

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Phenomenology of second order phase transitions

The order parameter field and spontaneous symmetry breaking

A second order phase transition is well described phenomenologically if one identifies:

a) The order parameter field $\phi_i(x)$

b) Symmetry group G and its spontaneous breaking

L.D. Landau (1937)

Landau-Ginzburg Energy Functional describes a vast array of critical phenomena including magnetic phase transitions, condensation effects in superconductivity and superfluidity

$$E = \int d^3x \left[\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}|\nabla\phi|^2 + U(\phi, \phi^*) \right],$$

Effective free energy near the phase transition

Most general functional symmetric under $\phi \rightarrow e^{i\chi}\phi$

and space rotations, with lowest possible powers of

ϕ **and lowest number of gradients** ∇ **is**

$$F = \int d^D x \left[\vec{\nabla}\phi^* \vec{\nabla}\phi + a(T)\phi^*\phi + \frac{b(T)}{2}(\phi^*\phi)^2 \right]$$

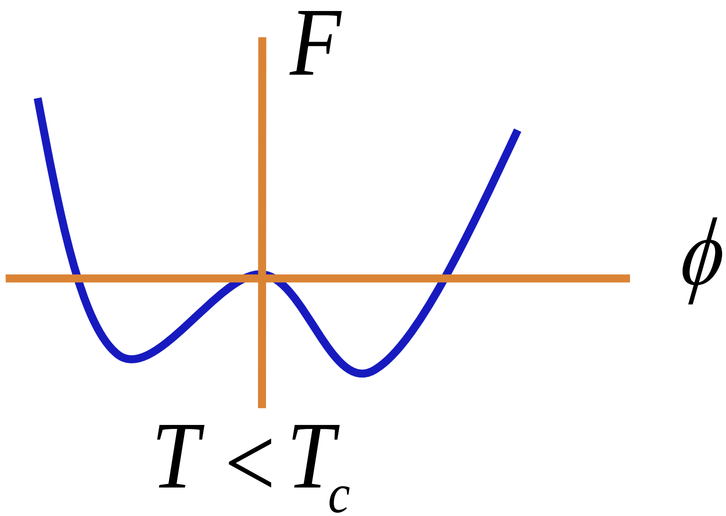
Higher order terms: $\left(\nabla\phi^*\nabla\phi\right)^2, (\phi^*\phi)^3, \ldots.$

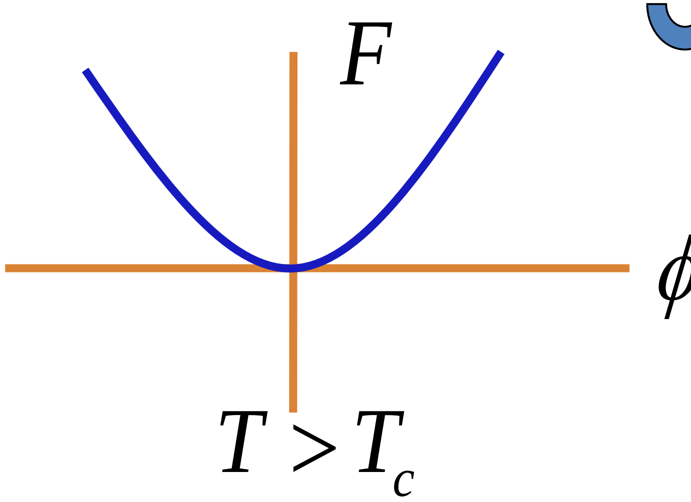
are expected to be negligible close enough to Tc.

The remaining coefficients can be expanded around Tc giving rise to a spontaneous symmetry breaking effect

$$a(T) = \alpha(T - T_c) + \ldots,$$

$$b(T) = \beta + \ldots$$





Other degrees of freedom are "irrelevant variables" sufficiently close to the critical temperature Tc. The "relevant" part, namely the symmetry breaking pattern and dimensionality defines "the universality class.

An example: paramagnetic-ferromagnetic phase transition

It requires planar classical spins of fixed length

$$\left|\vec{S}\right| = 1$$

defined on the D-dimensional lattice (the type of lattice and other microscopic details are also irrelevant).

Order parameter : average net magnetization.

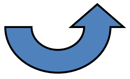
$$\vec{M} \equiv (M_x, M_y)$$

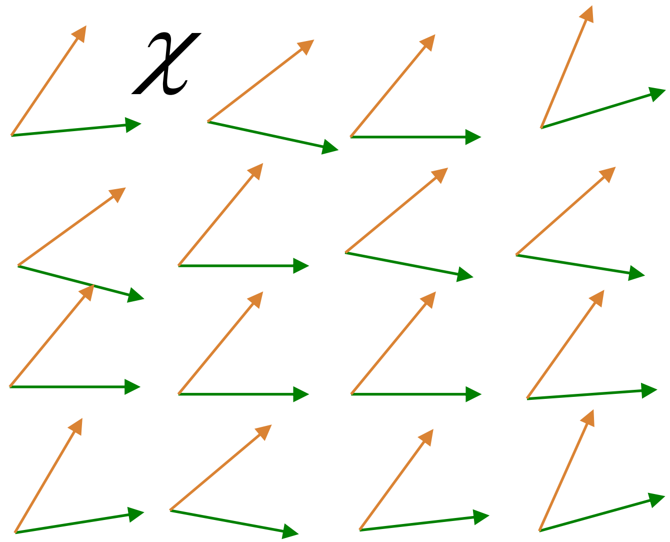
or, using complex number representation

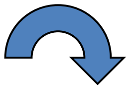
$$\phi = M_x + iM_y$$

Symmetry : 2D rotations

$$M_x \to \cos\chi\, M_x + \sin\chi\, M_y \equiv M_x'$$
$$M_y \to -\sin\chi\, M_x + \cos\chi\, M_y \equiv M_y'$$

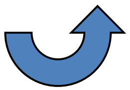






Using complex numbers the symmetry transformation becomes U(1):

$$\phi \to e^{i\chi}\phi$$

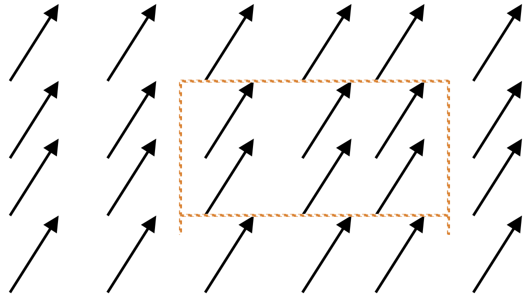


Symmetry means that the energy of the rotated state is the same as that of the original (not rotated) one

$$F(\vec{M}) = F(\vec{M}')$$

1. T=0

perfectly ordered

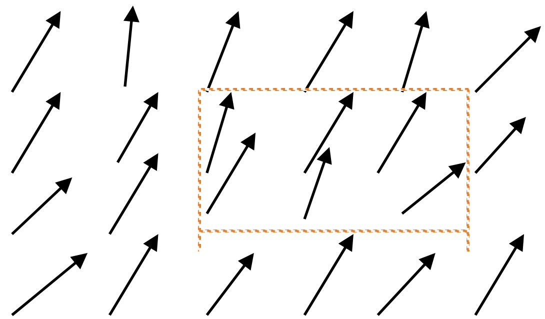


$$\left|\vec{M}\right| \equiv \left|\left\langle \vec{S} \right\rangle\right| > 0$$

large

2. 0<T<Tc

ordered

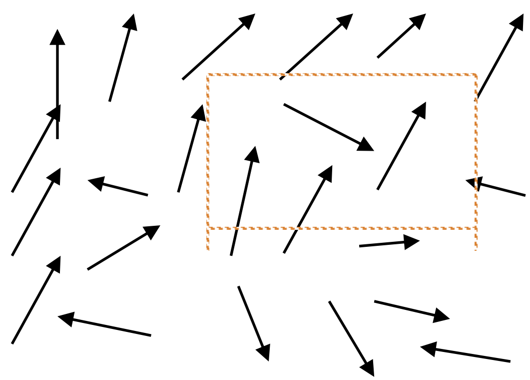


$$\left|\vec{M}\right| > 0$$

Small but nonzero

3. T>Tc

disordered



$$\left|\vec{M}\right| = \left|\left\langle \vec{S} \right\rangle\right| = 0$$

In the general case:

- Identify order parameter $\phi(x,\tau) \sim \sigma_j^z$
- Symmetries:

$$\begin{aligned} \text{Spin inversion:} \quad & \phi \to -\phi \\ \text{Time reversal} \quad & \tau \to -\tau \\ \text{Spatial inversion} \quad & x \to -x \end{aligned}$$

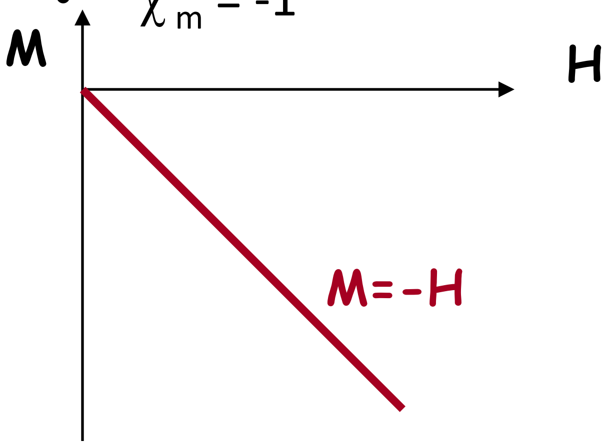
- Write down most general Lagrangian consistent with symmetries

$$\begin{aligned} \mathcal{Z} &= \int \mathcal{D}\phi(x,\tau)\exp\left(-\int d^d x \int d\tau \mathcal{L}[\phi]\right) \\ \mathcal{L}[\phi] &= \frac{1}{2}(\partial_\tau \phi)^2 + \frac{c^2}{2}(\nabla_x \phi)^2 + \frac{r}{2}\phi^2 + \frac{u}{4}\phi^4 + \dots \end{aligned}$$

- Identify phases at $r \gg 0$ and $r \ll 0$ with the paramagnet and the ferromagnet respectively.

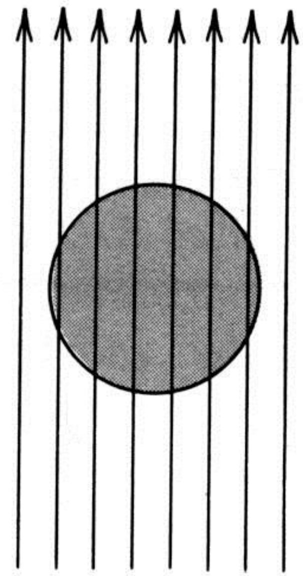
Perfect Diamagnetism in Type I Superconductors

- $\chi_m = -1$

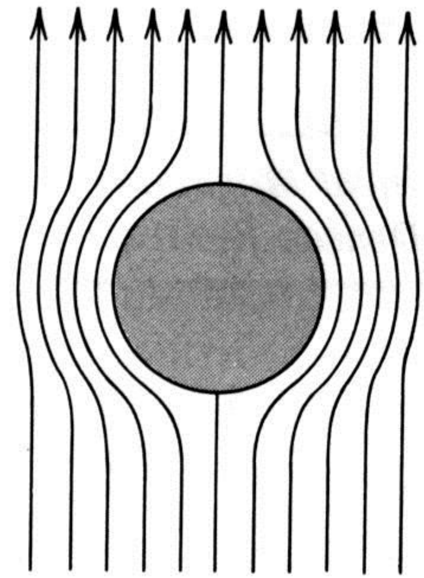


- Means:

$$B = \mu_o(H + M)$$
$$B = \mu_o(H + \chi_m H)$$
$$B = 0$$



Normal Metal

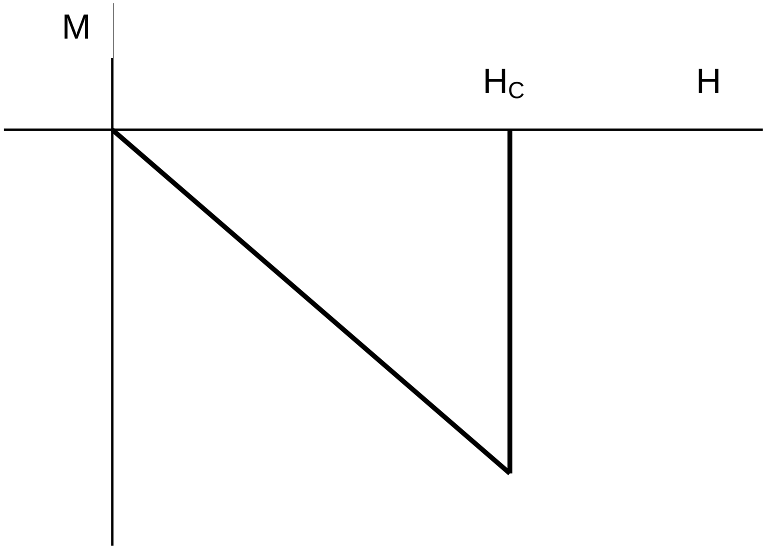


Superconductor

Flux is excluded from the bulk by supercurrents flowing at the surface to a penetration depth (λ) ~ 200-500 nm

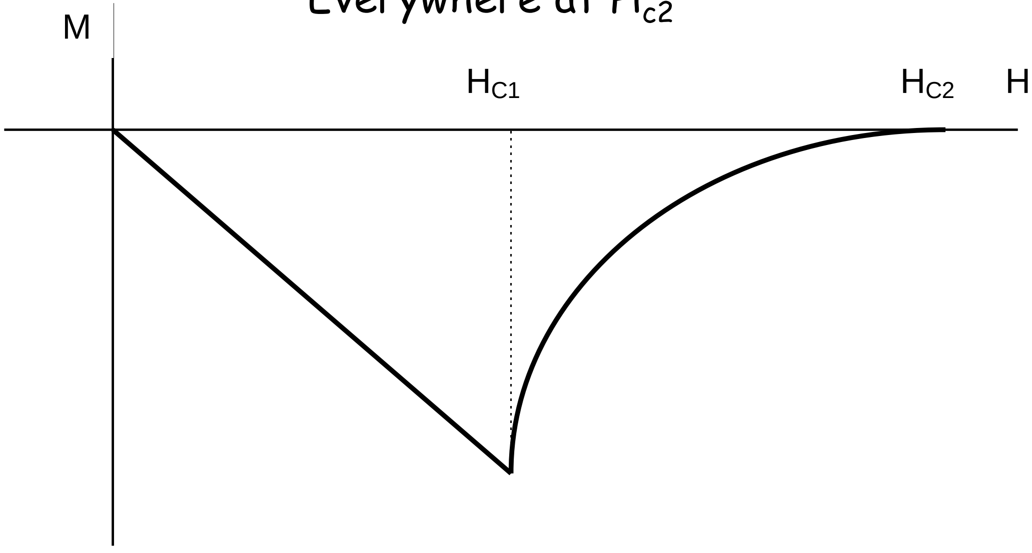
Type I and Type II Superconductors

- Type I
  - Material Goes Normal Everywhere at H_c



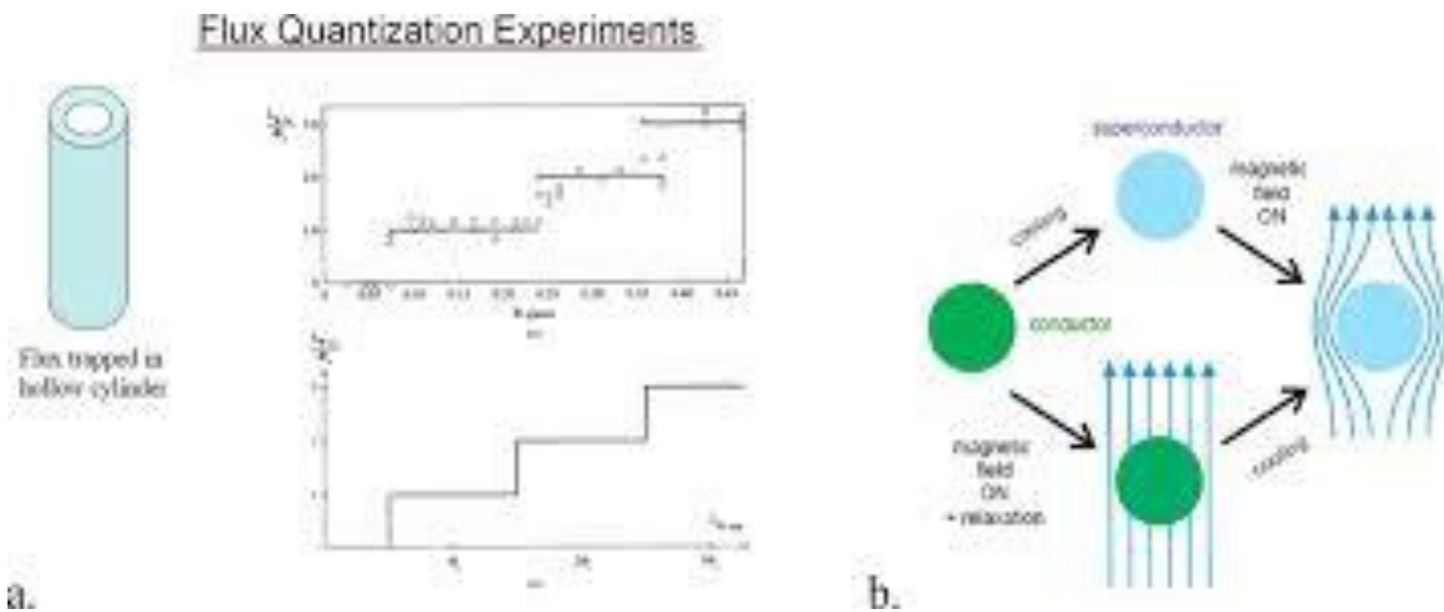
Complete flux exclusion up to H_c, then destruction of superconductivity by the field

- Type II
  - Material Goes Normal Locally at H_{c1}, Everywhere at H_{c2}



Complete flux exclusion up to H_{c1}, then partial flux penetration as vortices

Adding a gauge term describing magnetic fields leads to the Meissner effect and vortex arrays: superconductivity and magnetism mutually “avoid” each other



Landau-Ginzburg description of the superconducting-normal transition

Symmetry and order parameter

The complex order parameter is "amplitude of the Cooper pair center of mass:

$$\Psi(x) \propto \Delta(x), \quad \Delta(k) \propto \left\langle c_{\uparrow}(k)\, c_{\downarrow}(-k) \right\rangle$$

which is either the gap function of BCS or the charge density of Cooper pairs

quantum fluctuations of the ordered state (Cooper pair) are ignored and the "Bose condensate" amplitude is treated as a semi- classical field

The symmetry content of this complex field can be better specified via modulus and phase:

$$\Psi(x) \equiv \sqrt{n_s{*}(x)}\, e^{i\phi(x)}$$

$$n_s{*}(x) = \frac{n_s(x)}{2}$$

density of the Cooper pairs, the Bose condensate

$$\phi(x)$$

the superconductor (the Josephson, the global U(1)) phase

Broken Symmetry and Free Energy

The broken symmetry is that of U(1)

Without an external magnetic field the free energy near the phase transition is:

$$F[\Psi]=\int d^3x\left[\frac{\hbar^2}{2m*}\vec{\nabla}\Psi*\vec{\nabla}\Psi+\alpha(T-T_c)\,\Psi*\Psi+\frac{\beta}{2}(\Psi*\Psi)^2\right]$$

$$m*=2\,m_e$$

Ginzburg and Landau (1950) postulated to generalize this to the case of arbitrary magnetic field

$$\vec{B}(x)$$

using the local gauge invariance of electrodynamics.

This invariance dictates the charge fields coupling to the magnetic field.

To ensure local gauge invariance one replaces any derivative by a covariant derivative:

$$\vec{D}\Psi(x) = \left(\vec{\nabla} - i\frac{e^*}{\hbar c}\vec{A} \right)\Psi(x)$$

The local gauge invariance of the gradient term follows from linearity of the transformation of the covariant derivative:

$$\vec{D}\Psi(x) \rightarrow \left[\vec{\nabla} - \frac{ie^*}{\hbar c}\left(\vec{A} + \frac{c\hbar}{e^*}\vec{\nabla}\chi(x) \right) \right] \Psi(x) e^{i\chi(x)}$$

$$= \left[\vec{\nabla}\Psi + \Psi \cdot i\vec{\nabla}\chi - \frac{ie^*}{\hbar c}\vec{A} \cdot \Psi - i\vec{\nabla}\chi \cdot \Psi \right] e^{i\chi(x)}$$

$$= \vec{D}\Psi \cdot e^{i\chi(x)}$$

$$\Rightarrow \quad |\vec{D}\Psi(x)|^2 \rightarrow |\vec{D}\Psi(x)|^2$$

Magnetic field

$$\vec{B} \equiv \vec{\nabla} \times \vec{A} \Leftrightarrow B_i = \varepsilon_{ijk}\partial_j A_k$$

is also gauge invariant

Ginzburg – Landau equations

Minimizing the free energy with covariant derivatives one arrives at the set of GL equations: the nonlinear Schrödinger equation (variation with respect to Ψ)

$$-\frac{\hbar^2}{2m^*}\left(\vec{\nabla}-i\frac{e^*}{\hbar c}\vec{A}\right)^2\Psi+\alpha(T-T_c)\Psi+\beta\Psi\left|\Psi\right|^2=0$$

and the supercurrent equation (variation of A):

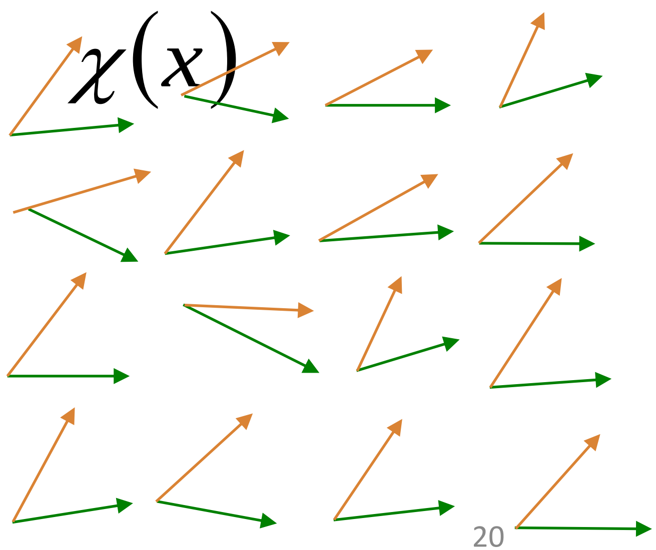
$$\frac{c}{4\pi}\vec{\nabla}\times\vec{B}=\vec{J}_s\equiv-\frac{ie^*\hbar}{2m^*}(\Psi^*\vec{\nabla}\Psi-\Psi\vec{\nabla}\Psi^*)-\frac{e^{*2}}{m^*c}\left|\Psi\right|^2\vec{A}$$

Influence of the magnetic field

Electrodynamics is invariant under local gauge transformations:

$$\begin{cases} \Psi(x) \to e^{i\chi(x)}\Psi(x) \\ \vec{A}(x) \to \vec{A}(x) + \dfrac{\hbar c}{e^*}\vec{\nabla}\chi(x) \end{cases}$$

$$e^* = 2e$$



Two characteristic length scales

GL equations possess two scales. Coherence length ξ **characterizes variations of** $\Psi(x)$ **, while the penetration depth λ characterizes variations of** $B(x)$

$$\xi(T) = \frac{\hbar}{\sqrt{2m^*\alpha(T_c - T)}},$$

$$\lambda(T) = \frac{c}{e^*}\sqrt{\frac{m^*\beta}{4\pi\alpha\left(T_c - T\right)}}$$

Both diverge at T=Tc.

Ginzburg – Landau parameter

The only dimensionless parameter is the ratio of the two lengths which is temperature independent:

$$\kappa = \frac{\lambda(T)}{\xi(T)} = \frac{m^* c}{e^* \hbar}\sqrt{\frac{\beta}{2\pi}}$$

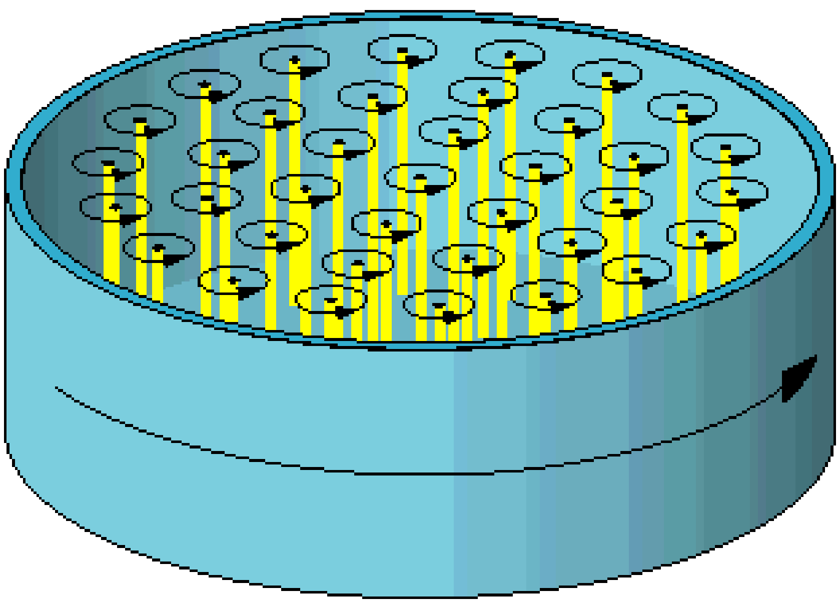
Properties of solutions crucially depend on the GL parameter. When

$$\kappa > \frac{1}{\sqrt{2}}$$

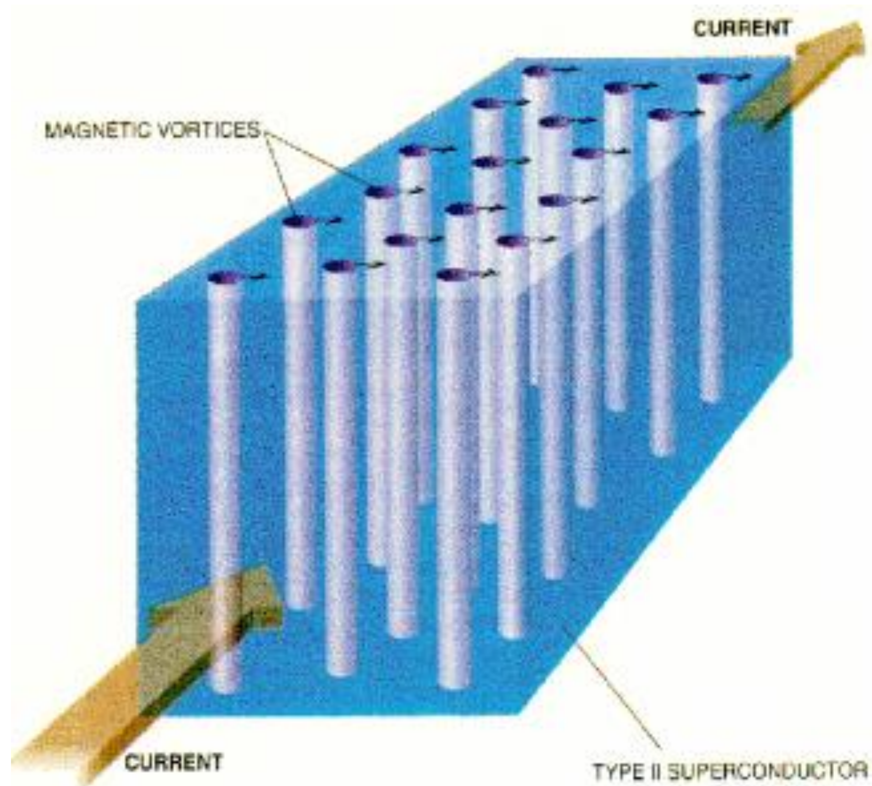
(type II superconductivity) there exist "topologically nontrivial" solutions – the Abrikosov vortices.

Abrikosov (1957)

Abrikosov vortex lattice

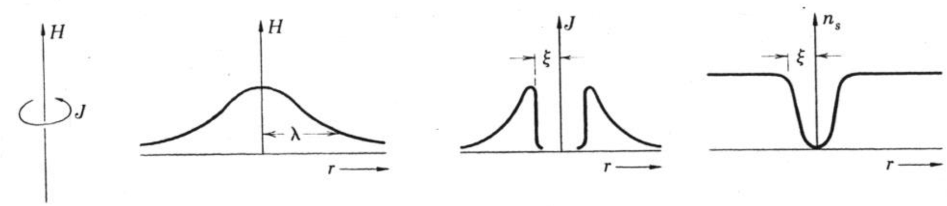


Quantized Vortices in Helium-4

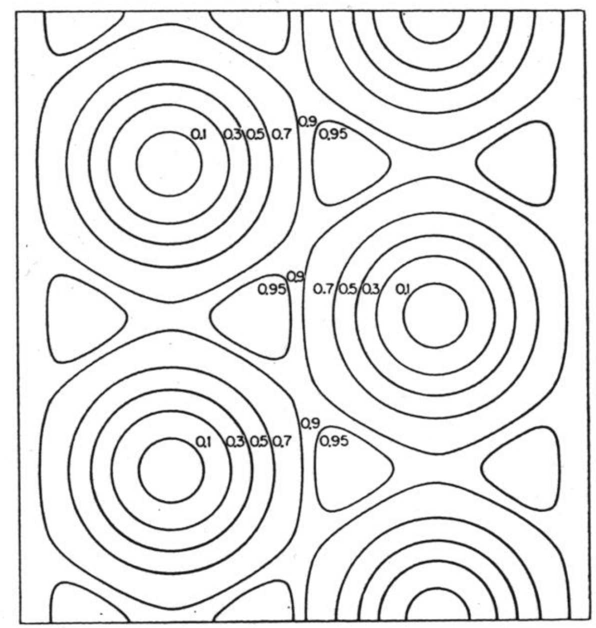


Vortices in type-II superconductors

Vortex properties



- Two characteristic lengths
  - **coherence length ξ,** the pairing length of the superconducting pair
  - **penetration depth λ**, the length over which the screening currents for the vortex flow
- Vortices have defined properties in superconductors
  - normal core diameter, ~2ξ
  - each vortex contains a flux quantum ϕ_0 currents flow at J_d over diameter of 2λ
  - vortex separation $a_0 = 1.08(\phi_0/B)^{0.5}$

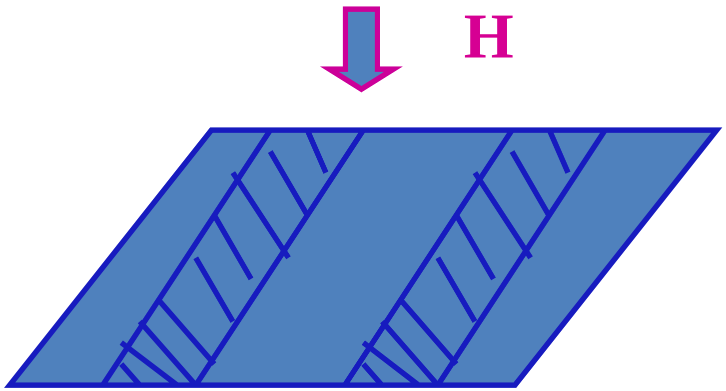


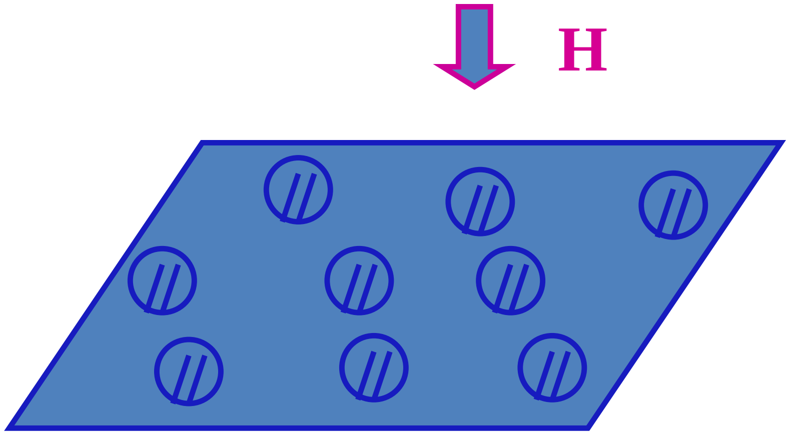
$H_{c2} = \phi/2\pi\xi^2$

$\phi_0 = h/2e = 2.07 \times 10^{-15}$ Wb

Interface energy is negative for type II superconductors, while positive for the type I.

Mixed state under applied magnetic field





Type I :

Minimal area of domain walls.

Type II:

Maximal area of domain walls.

Magnetic Flux quantization.

To minimize the potential term far from an isolated vortex where B=0, one has to minimize the modulus of the order parameter:

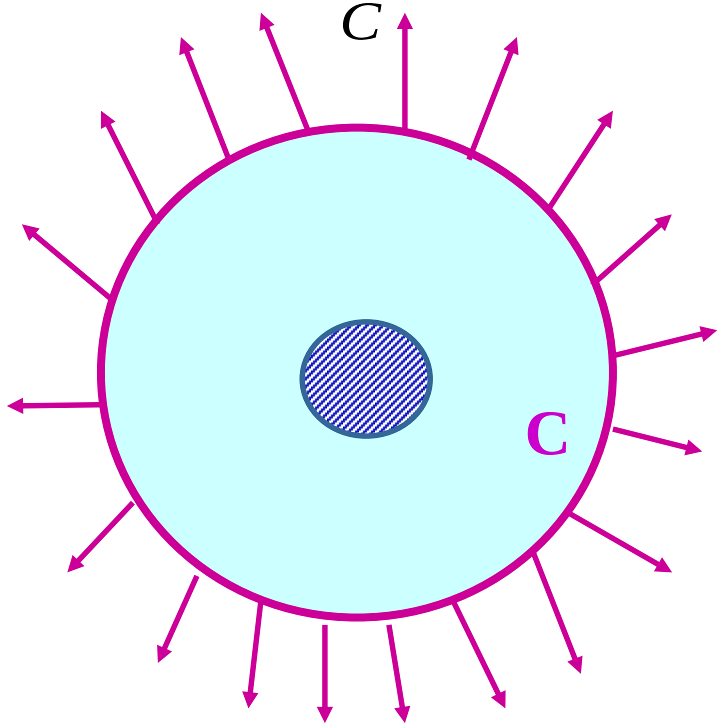
$$|\Psi| = \Psi_0 \equiv \sqrt{\frac{\alpha(T_c - T)}{\beta}}$$

The phase is free to vary. In order to minimize the (positive) gradient term, one demands:

$$\vec{D}\Psi(x) = \Psi_0 \vec{D} e^{i\phi(x)} = \Psi_0 \left(\vec{\nabla} - i\frac{e^*}{\hbar c}\vec{A} \right) e^{i\phi(x)} = 0$$

$$\vec{\nabla}\phi = \frac{e^*}{\hbar c}\vec{A} \quad \Rightarrow \quad F_{grad} = 0$$

$$\Phi = \oint_C d\vec{s}\cdot\vec{A} = \frac{\hbar c}{e^*}\oint_C \vec{\nabla}\phi\cdot d\vec{s} = n\,\Phi_0$$



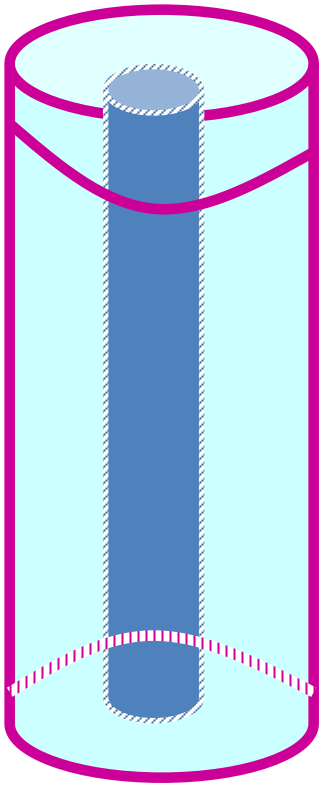
n – integer

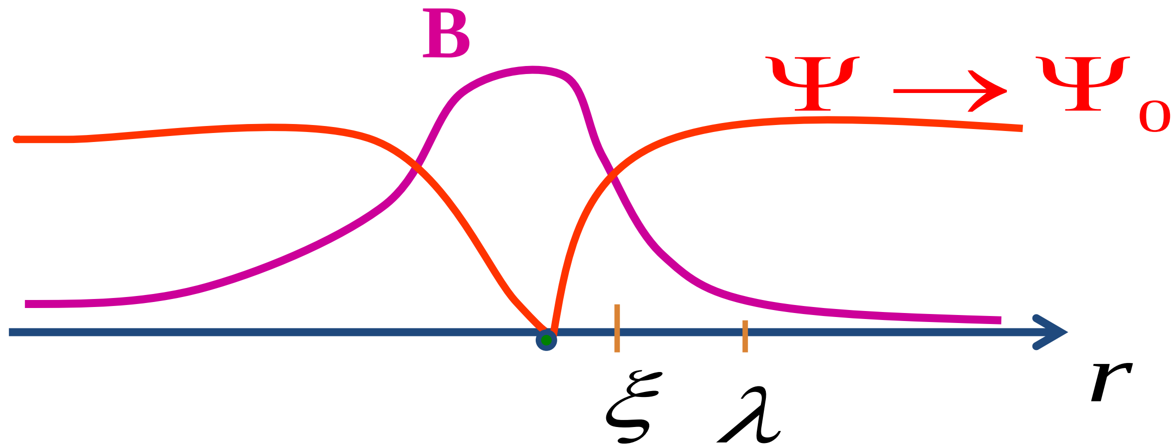
$$\Phi_0 = \frac{hc}{e^*} = \frac{hc}{2e}$$

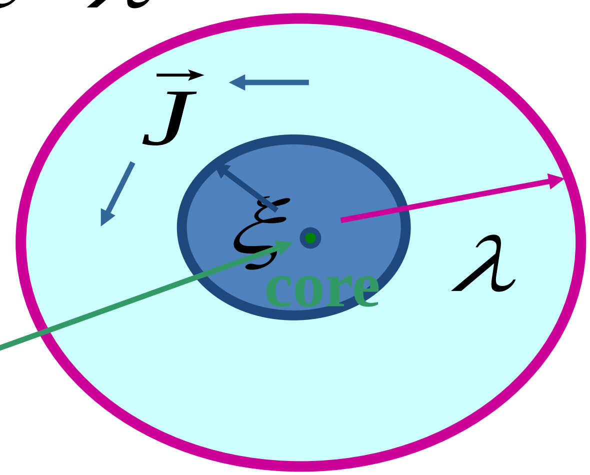
n=1 is energetically favored over n>2

The normal core $\Psi = 0$ ***region shrinks to a point.***

Shape of the vortex solution







Vortex – a linear topological defect.

The "singularity" line $\Psi = 0$

Vortices and systems of vortices (vortex lattice)

Inter-vortex repulsion and the Abrikosov flux line lattice

Line energy

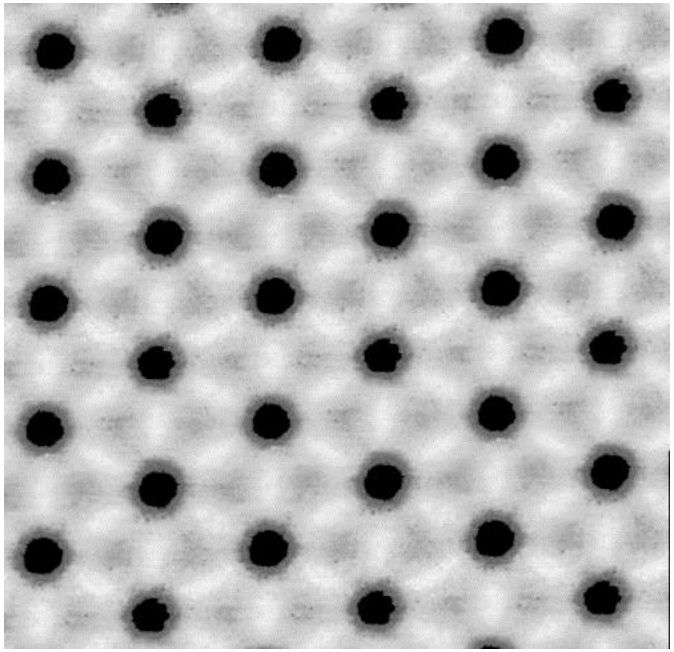
To create a vortex, one has to provide energy per unit length (line tension)

$$\varepsilon \approx \left(\frac{\Phi_0}{4\pi\lambda} \right)^2 \log\left(\frac{\lambda}{\xi} \right)$$

Therefore vortices enter an infinite sample only when the magnetic field exceeds certain value

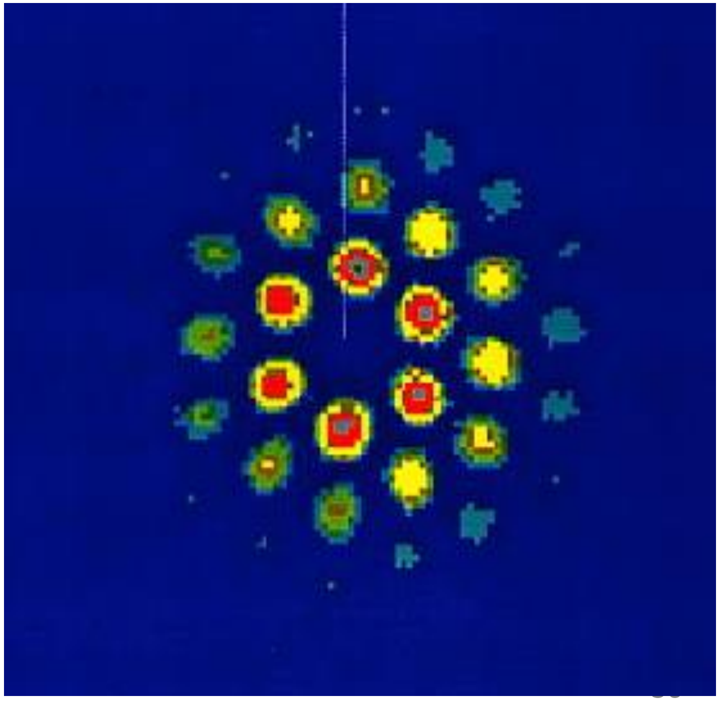
Interactions between vortices

They interact with each other via a complicated vector-vector force. Parallel straight vortices repel each other forming highly ordered structures like flux line lattice (as seen by STM and neutron scattering).



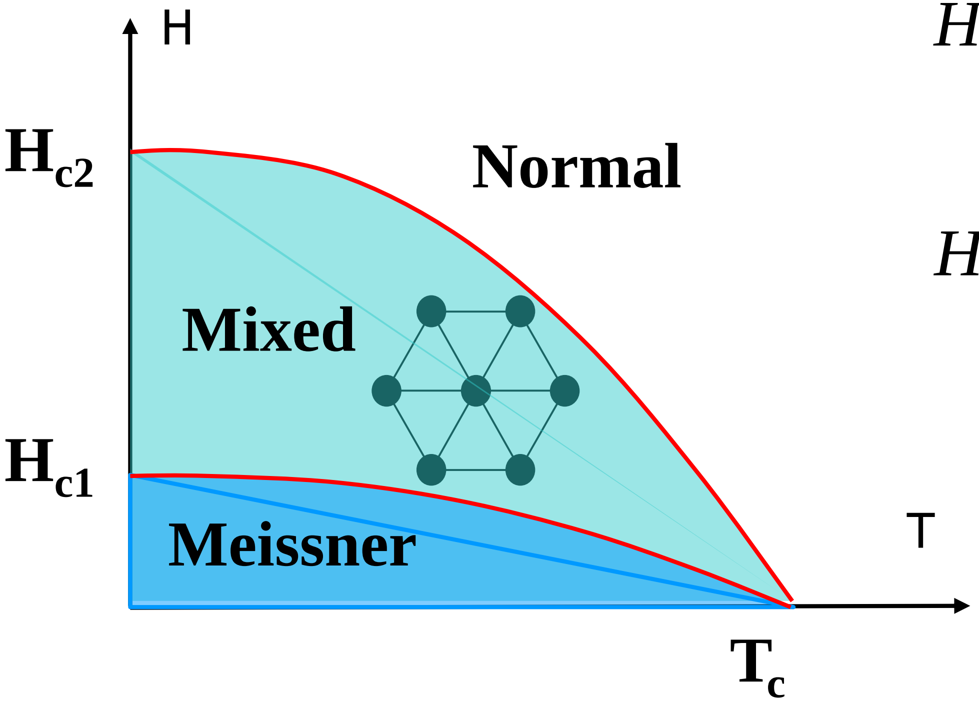
Pan et al (2002)

S.R.Park et al (2000)



Two critical fields

As a result the phase diagram of type II SC is richer than that of the two-phase type I



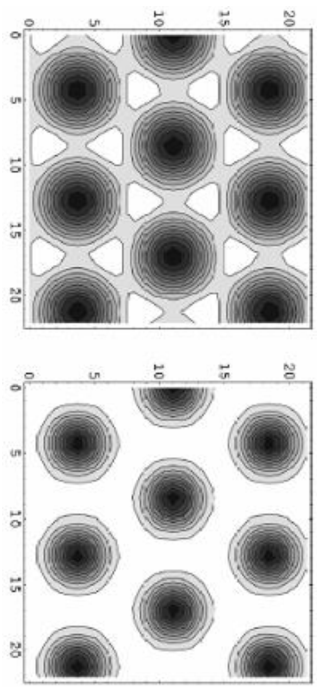
$$H_{c1} \approx \frac{\Phi_0}{4\pi\lambda^2}$$ **first vortex penetrates.**

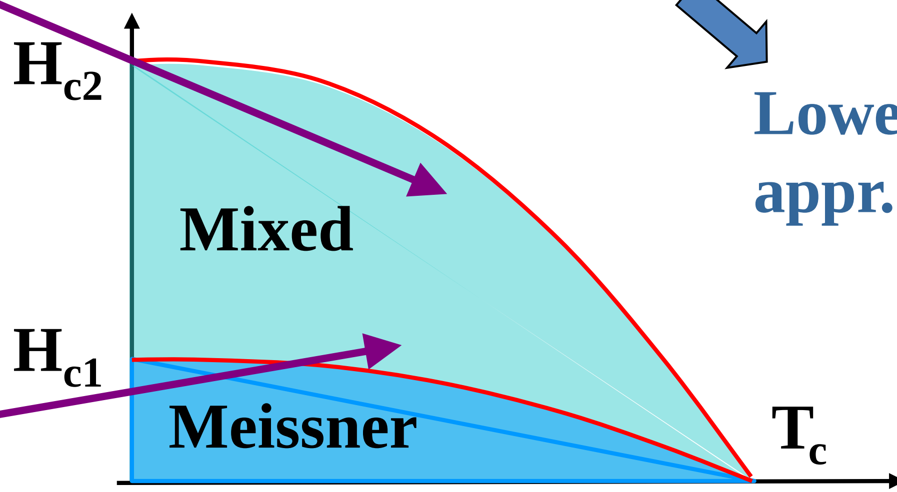
$$H_{c2} = \frac{\Phi_0}{2\pi\xi^2}$$ **cores overlap**

$$\frac{H_{c2}}{H_{c1}} \approx \frac{2\lambda^2}{\xi^2} = 2\kappa^2 > 1$$

Two theoretical approaches to the mixed state

Just below H_{c2} vortex cores almost overlap. Instead of lines one just sees array of superconducting "islands"





Lowest Landau level appr. for constant B

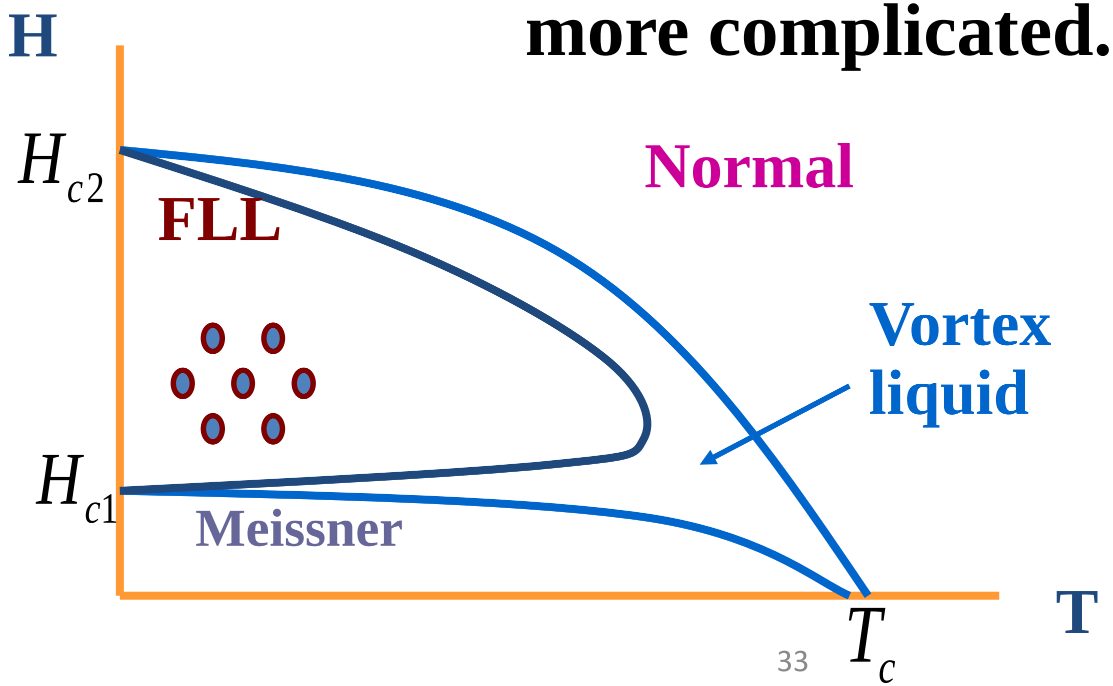
Just above H_{c1} vortices are well separated and have very thin cores

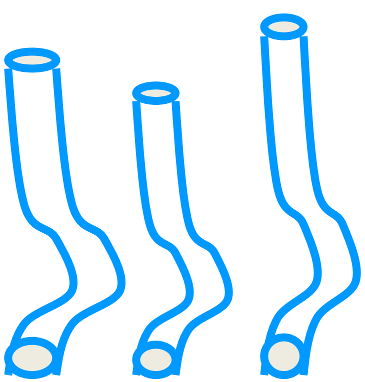
London appr. for infinitely thin lines

Thermal fluctuations and the vortex liquid

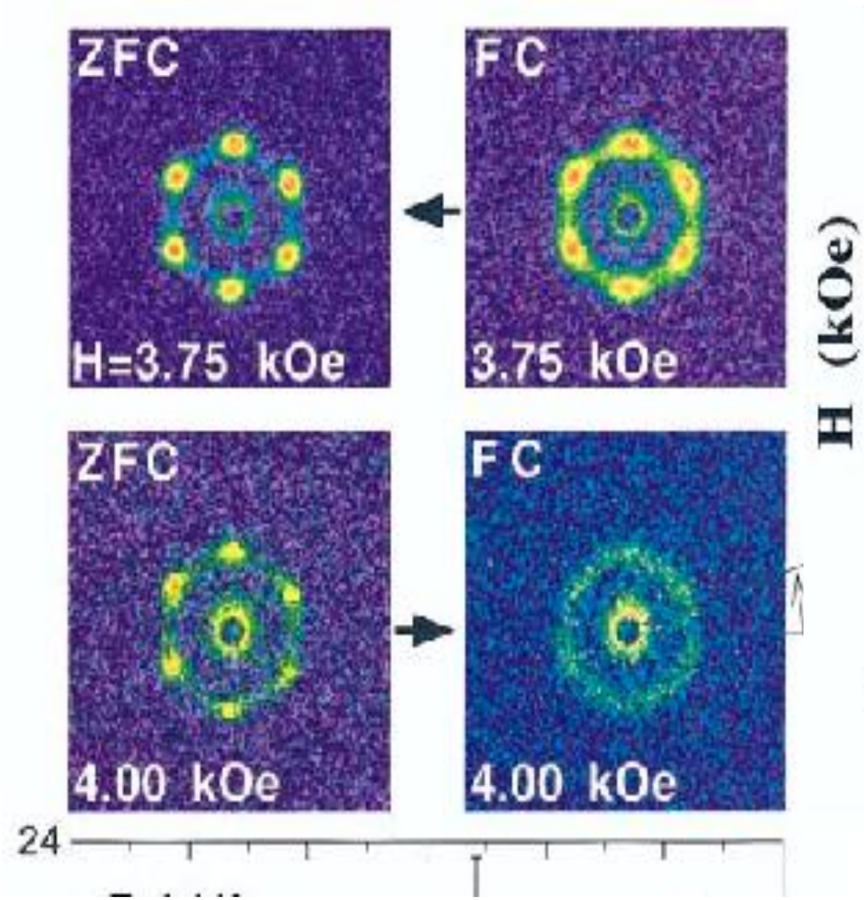
In high Tc SC due to higher Tc, smaller ξ and high anisotropy thermal fluctuations are not negligible. Thermally induced vibrations of the flux lattice can melt it into a 'vortex liquid'

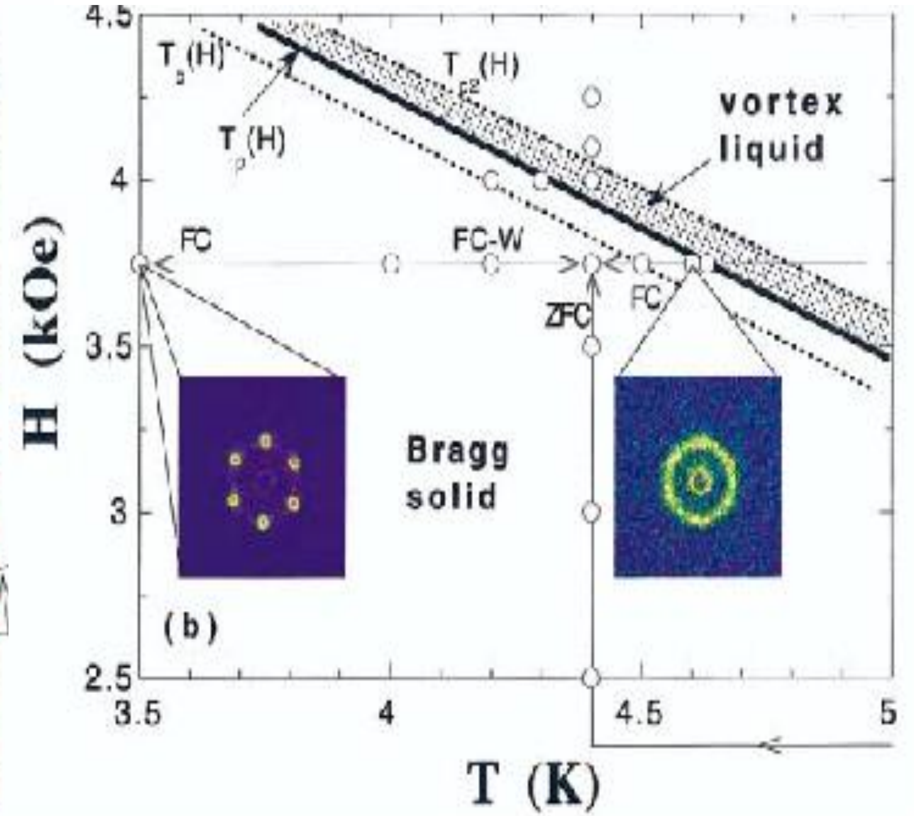
The phase diagram becomes more complicated.





Metastable states: zero field cooled and field cooled protocols result in different states.



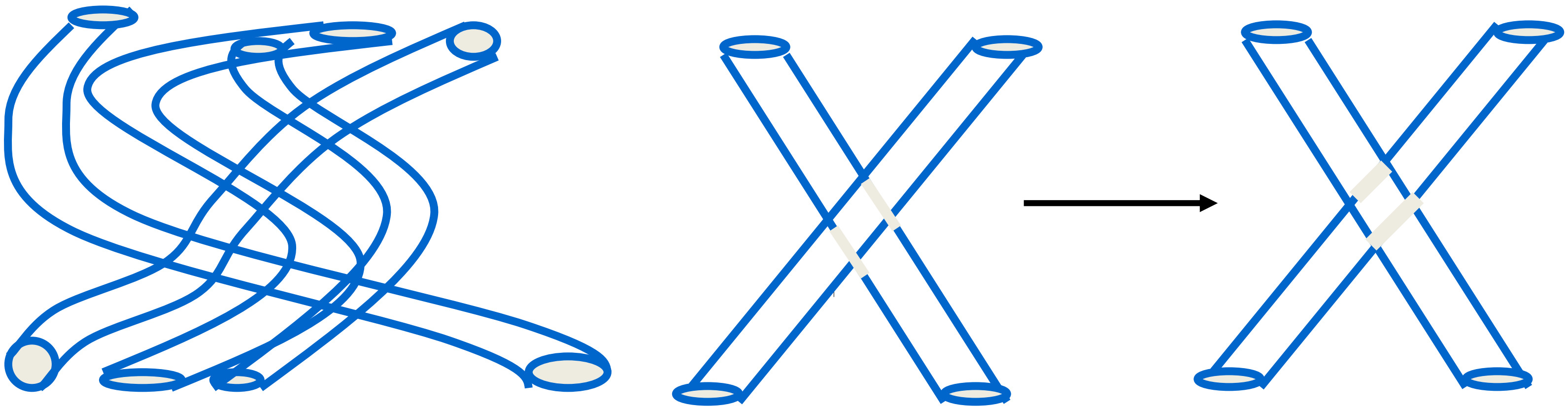


Neutron scatering in Nb

Ling et al (2000)

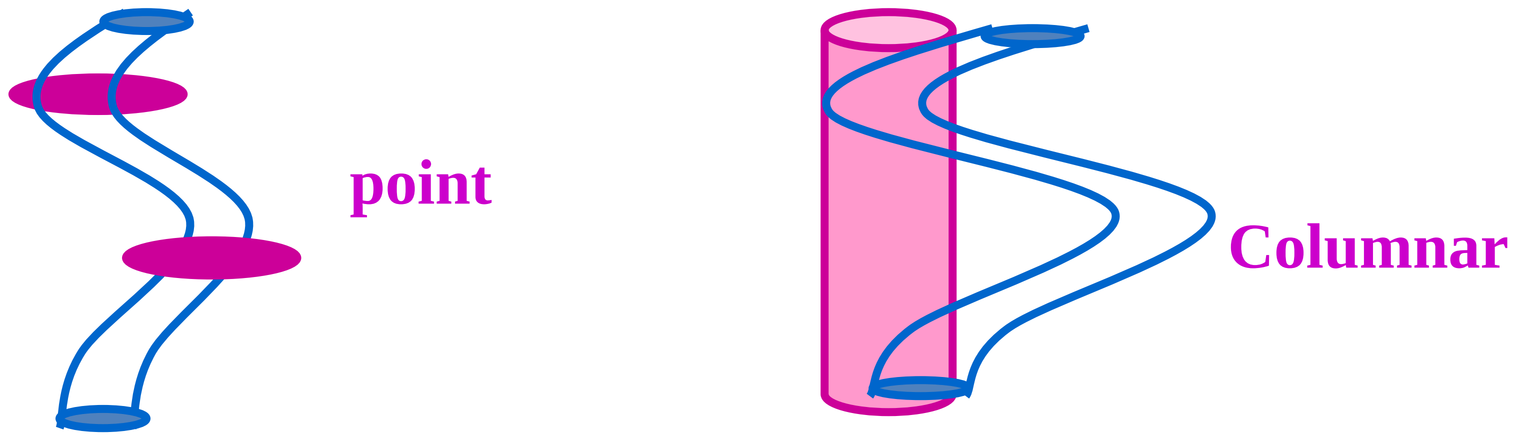
Vortex “cutting” and entanglement

Vortices can entangle around each other like polymers, however due to vectorial nature of their interaction they can also “disentangle” or ‘cut each other’, monopole-like?



There are profound differences compared to the physics of polymers

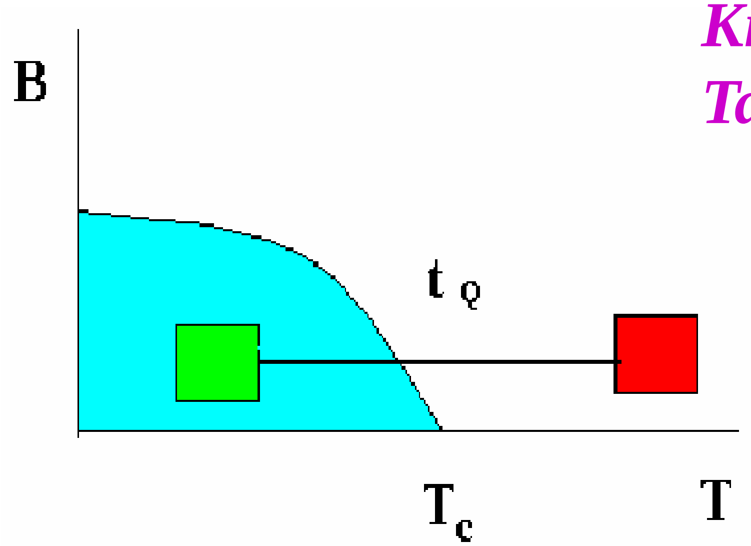
Disorder and the vortex glass



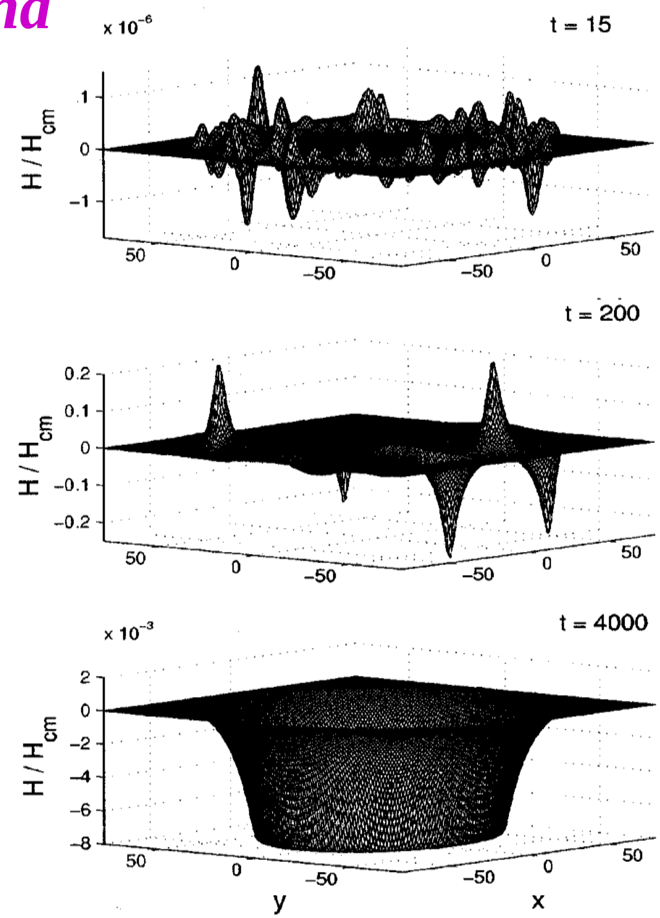
Vortices are pinned by disorder creating a glassy state or viscous entangled liquid. In the glass phase material becomes superconducting (zero resistance) below a certain critical current value Jc.

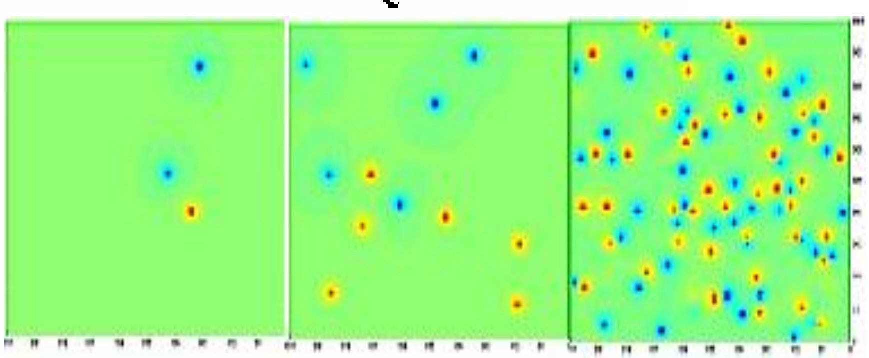
In 2D thermal fluctuations generate a curious Kosterlitz – Thouless (KT) vortex plasma exhibiting many unique features

Unstable normal domain splits into vortex-antivortex (KT) plasma



Kirtley,Tsuei and Tafuri (2003)





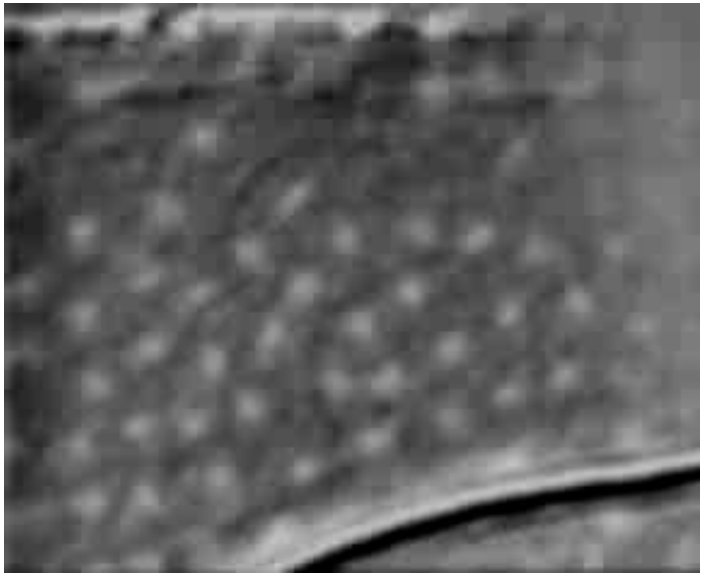
Polturak, Maniv (2004)

Scanning SQUID magnetometer

Vortex dynamics in the presence of disorder

Disorder profoundly affects dynamics leading to the truly superconducting vortex glass state in which exhibits irreversible and memory dependent phenomena

Magneto-optics in Nb
Johansson et al (2004)



It is a rich playground to study vortex glass dynamics

Summary (so far)

1. In extreme type II superconductors the "topological" vortex degrees of freedom dominate most of the macroscopic magnetic and transport properties.

2. One can use the GL theory to describe these degrees of freedom.

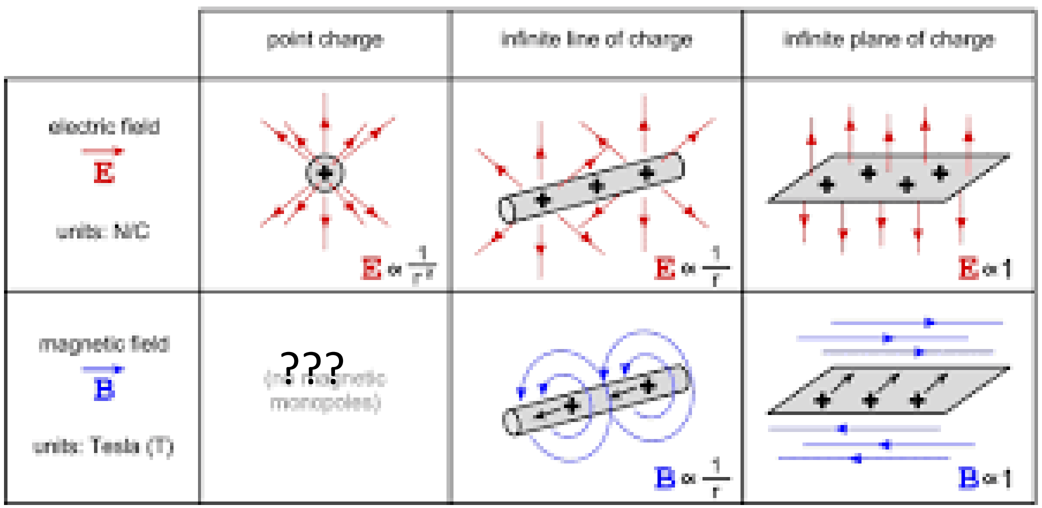
3. Experiments suggest that in new high Tc SC thermal fluctuations are important as well as disorder.

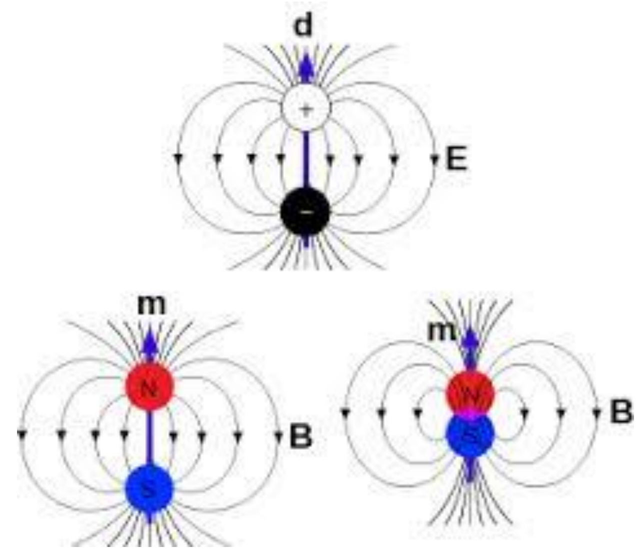
4. The vortex matter physics is quite unique, well controlled experimentally and may serve as a "laboratory" to test a great variety of theoretical ideas.

5. It can improve our understanding of magnetic monopoles

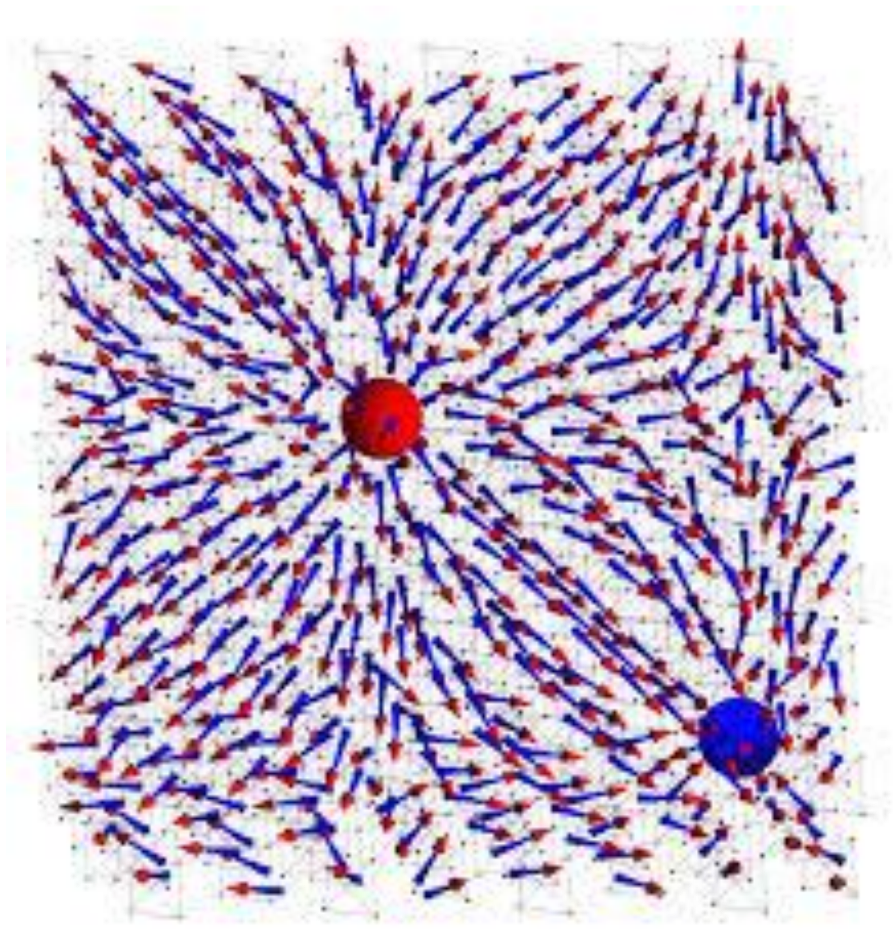
Electric and magnetic monopoles and dipoles

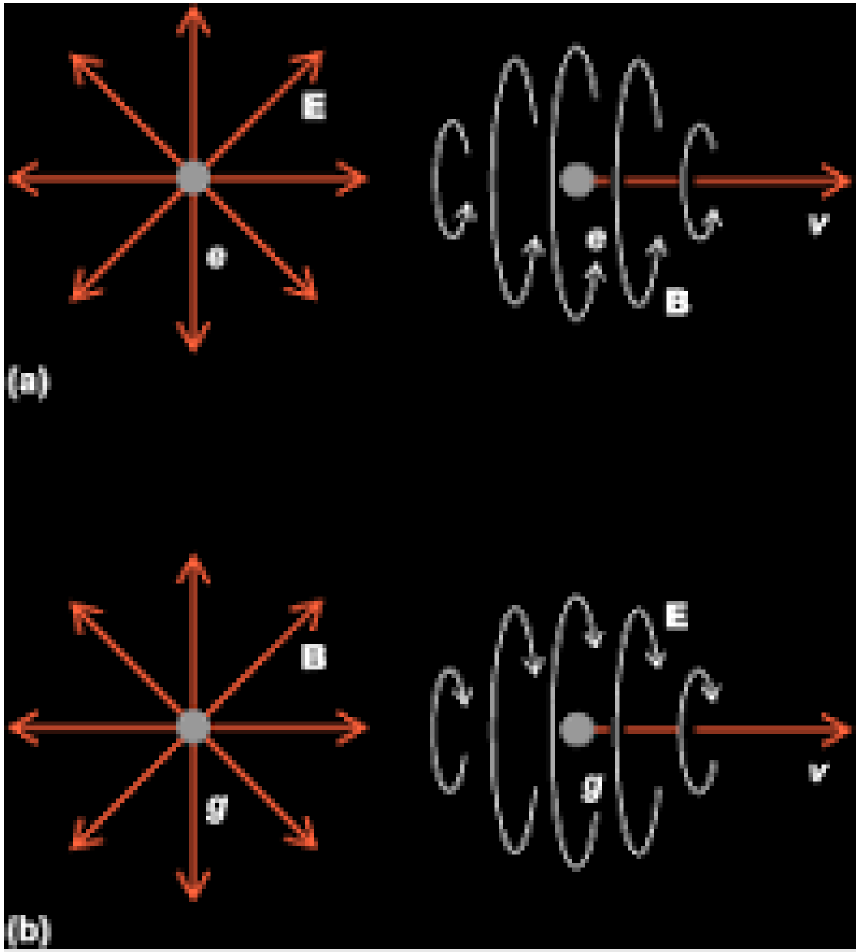
Fields Monopoles monopole assemblies dipoles



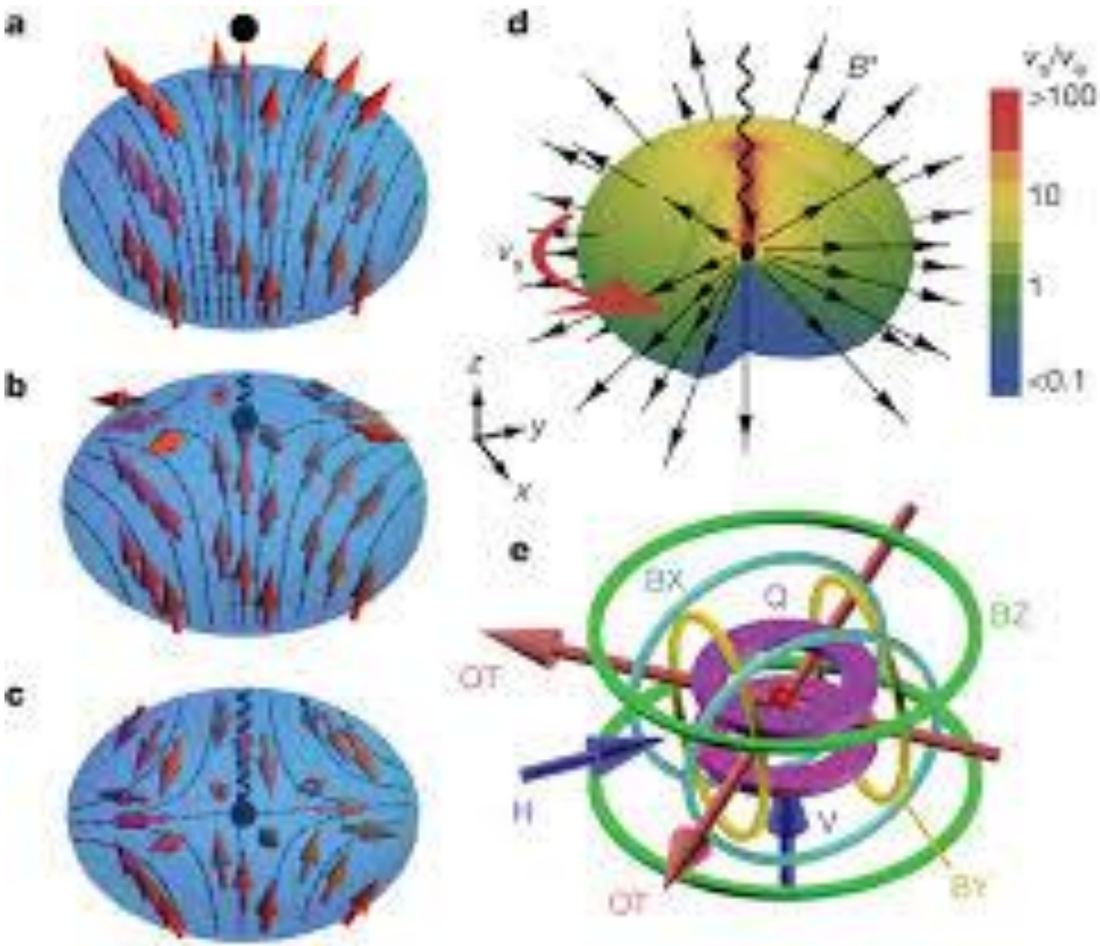


field lines around dipoles and monopoles





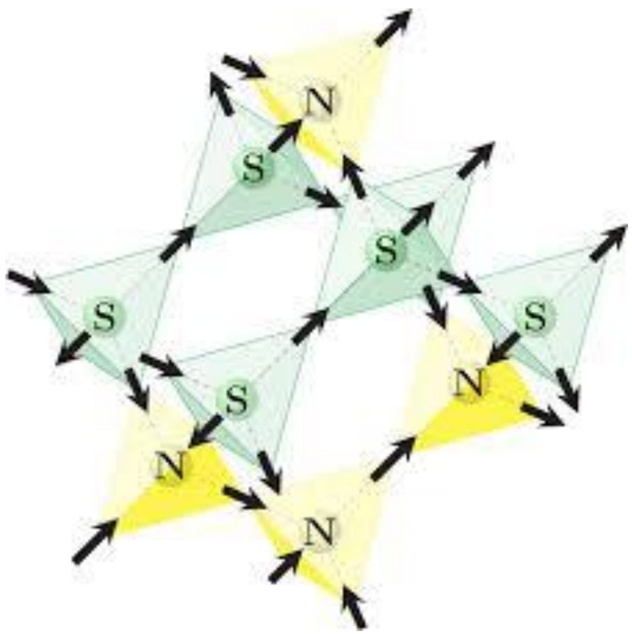
Monopole isolation?

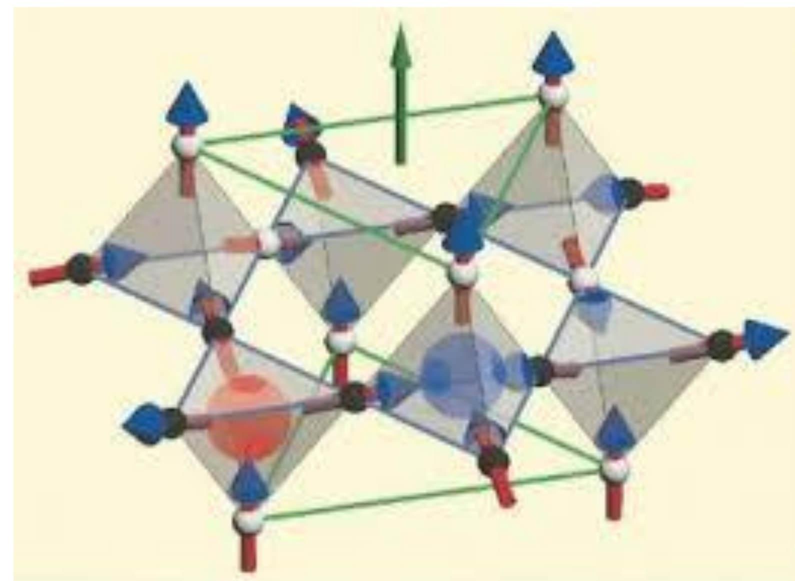


Spin Ice Representation of a Magnetic Monopole

- H. Kadowaki, N. Doi, Y. Aoki, Y. Tabata, T.J. Sato, J.W. Lynn, K. Matsuhira and Z. Hiroi. Observation of magnetic monopoles in spin ice. Journal of the Physical Society of Japan,78, No. 10, Oct. 13, 2009.
- Morris, David Jonathan Pryce, et al. "Dirac strings and magnetic monopoles in the spin ice $Dy_2Ti_2O_7$." *Science* 326.5951 (2009): 411-414
- Bramwell, S. T., et al. "Measurement of the charge and current of magnetic monopoles in spin ice." *Nature* 461.7266 (2009): 956-959.

This has been accomplished in a spin lattice by knocking one spin off its direction

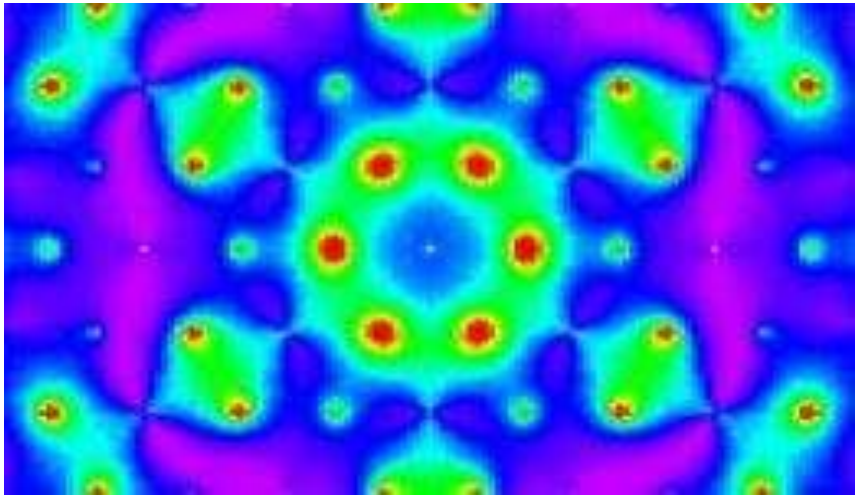


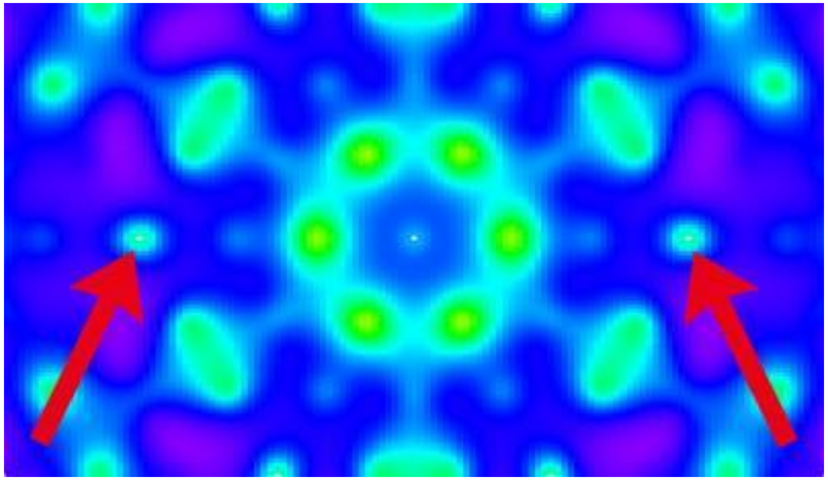


The material freezes into four-sided crystals (a pyramid with a triangular base) and the magnetic orientation, or "spin," of the ions at each of the four tips align so that their spins are balanced—two spins point inward and two outward. But using neutron beams at the NCNR, the team found they could knock one of the spins askew so that instead three point in, one out … "creating a monopole, or at least its mathematical equivalent” .

Because every crystal pyramid shares its four tips with adjacent pyramids, flipping the spin of one tip creates an "anti-monopole" in the next pyramid over. The team has created monopole-antimonopole pairs repeatedly in a relatively large chunk of the spin ice, allowing them to confirm the monopoles' existence through advanced imaging techniques such as neutron scattering.

Magnetic monopoles are created (top) when the spin of an ion in one corner of a spin ice crystal is knocked askew, creating a monopole (red sphere) and adjacent antimonopole (blue sphere). Neutron scattering shows the spin ice's transition from its normal state (center) to the monopole state. Monopoles scatter neutrons in a telltale fashion indicated by the red arrows in (bottom.)





Pietilä, Ville; Möttönen, Mikko (2009). "Creation of Dirac Monopoles in Spinor Bose–Einstein Condensates". Phys. Rev. Lett. ***103****: 030401. arXiv:0903.4732. Bibcode:2009PhRvL.103c0401P.*

- In superfluids, there is a field **B***, related to superfluid vorticity, which is mathematically analogous to the magnetic **B**-field. Because of the similarity, the field **B*** is called a "synthetic magnetic field".
- In January 2014, it was reported that monopole quasiparticles for the **B*** field were created and studied in a spinor Bose–Einstein condensate.
- This constitutes the first example of a quasi-magnetic monopole observed within a system governed by quantum field theory.

Physics beyond the standard model

- Magnetic monopoles
- Q-balls

G.R. de Melo, M. de Montigny, J. Pinfold and J.A. Tuszynski, Symmetries and soliton solutions of the Galilean complex Sine-Gordon equation, **Physics Letters A** (accepted January 28, 2016),

- Nuclearites
- strangelets