DGLAP, PDFs and all that

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When Guido and myself were both in Paris (Guido in the ENS and myself in the IHES of Bures), we wrote a the paper *Asymototic freedom in parton language*.

Guido liked to remark that this is the most quoted French paper in high energy physics.

The motivations of the paper were clearly stated in the introduction.

Speaking of scaling violations in QCD we wrote:

In spite of the relative simplicity of the final results, their derivation, although theoretically rigorous, is somewhat abstract and formal, being formulated in the language of renormalization group equations for the coefficient functions of the local operators which appear in the light cone expansion for the product of two currents.

In this paper we show that an alternative derivation of all results of current interest for the Q^2 behaviour of deep inelastic structure functions is possible. In this approach all stages of the calculation refer to parton concepts and offer a very illuminating physical interpretation of the scaling violations. In our opinion the present approach,although less general, is remarkably simpler than the usual one since all relevant results can be derived in a direct way from the basic vertices of QCD, with no loop calculations being involved (the only exception is the lowest order expression for the running coupling constant which we do not rederive).

This method can be described as an appropriate generalization of the equivalent photon approximation in quantum electrodynamics (Weizsaker-Williams .... Cabibbo, Rocca).

In order to explain the content of the paper I have to speak of the light cone expansion for the product of two currents. Let me first make a few remarks:

- Very few things were known on strongly interacting quantum field.
- People were very careful in making assumptions that were not proved and nearly nothing was proved.
- Wilson (Poliakov) operator product expansion was considered to be a solid result

$$A(x)B(0) \to_{x\to 0} \sum_C C(0)|x|^{-d_A-d_B+d_C}$$

  The dimensions of the operators were the canonical one in free theory. They could be different from the canonical ones in an interacting theory.

- The total cross section for "virtual gamma"+ proton (i.e. deep inelastic scattering) can be written in terms of the function

$$G(x^2, x_0) = \langle p | J(x) J(0) | p \rangle \ . \tag{1}$$

  Kinematical considerations imply that the region relevant for deep inelastic scattering is $x^2 \approx 0$.

- Biorken scaling for deep inelastic scattering was transformed in the naive light cone expansion by Brandt and Preparata (1969)

$$J(x) J(0) \to_{x^2 \to 0} \frac{O(x_0, 0)}{x^2} \tag{2}$$

  $O(x_0, 0)$ is a bylocal operator.

- The naive light cone expansion can be derived in free field theory by the Wilson expansion by doing a Taylor expansion at $x_0 = 0$.

  An infinite number of terms in the Taylor expansion have to be considered.

- Strong interaction are not a free theory. In the case of a non-free theory we have to use the renormalization group. The Wison expansion had been proved in one particular case by Symanzik, where the dimensions were not the naive one: the dimensions contained a non trivial coupling constant dependent part: anomalous dimensions. The renormalization group with a running coupling constant has to be used.
- Norman Christ Mueller and Al Muller (1971) computed the coupling dependent anomalous dimensions of the operators relevant for deep inelastic scattering (twist two operators).

- If we neglect the dependence of the running coupling constant on the momenta, one has something like

$$M_n(q^2) \equiv \int_0^1 dx x^{n-1} F(x,q^2) \; ; \quad M_n(q^2) = C_n \exp(\gamma_n(\alpha) \log(q^2)) \tag{3}$$

- If we specialize to QCD, we consider only valence quark and we take care of the running coupling constant, we get the final formulae

$$\frac{\partial M_n(q^2)}{\partial \log(q^2)} = \gamma_n(\alpha(q^2)) \, .$$

If we use Mellin trasform we finally get at the first order in $\alpha(q^2)$

$$\frac{\partial F(x,q^2)}{\partial \log(q^2)} = \int_x^1 \frac{dy}{y} F(x,q^2) P_{q,q}(x/y)$$

$$P_{q,q}(z) = \frac{\alpha(q^2)}{4\pi} \left(\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(z-1) \right)$$

When we wrote our work (spring ’77), all that was known. Similar formule could be written for the gluon and see contribution using inverse Mellin transform.

Parton model

The deep inelastic structure functions measure the number of charged partons (quarks) carrying a fraction of momentum x in the infinite momentum frame. These leads to Bjorken scaling.

Equivalent photon approximation in quantum electrodynamics (Weizsaker-Williams .... Cabibbo, Rocca).

In QED one often finds that the net effect of radiative corrections is proportional to $\alpha \log(E/m_e)$ or $(\alpha \log(E/m_e))^k$. Various techniques can be used.

Cabibbo and Rocca (Cern Preprint, May 1974) wrote formulae like:

$$P_{e \to e\gamma}(\eta) = \frac{\alpha}{\pi} \frac{1 + (1-\eta)^2}{\eta} \log(E/m_e)$$

where η is the fraction of longitudinal momentum carried by the photon (the electron carries $1-\eta$).

$$P_{\gamma \to e^+e^-}(\epsilon) = \frac{\alpha}{2\pi}(1 + (1-2\epsilon)^2) \log(E/m_e)$$

where ϵ is the fraction of longitudinal momentum carried by the e^+

These probabilities can be combined. They compute the probability of finding inside a γ a triplet γ, e^+, e^- proportional to $P_{\gamma \to e^+e^-} P_{e \to e\gamma} \log(E/m_e)^2$.

All the computations were done in the infinite momentum frame.

In spring '77 Guido and I discussed about QCD scaling violations. Guido suggested that it would be pedagogically useful to derive the equations for scaling violations using the same tecniques of Cabibbo Rocca; no loops: only the evaluation of the vertices in the infinite momentum frame.

Only the gluon splitting into two gluon function was missing. The computations where much simpler that the original computations base on the one loop corrections to the vertices of the operator entering in the Wilson expansion.

The computation were particular transparent and simple when we extended it to the case of polarized partons.

The paper was very successfully.

The paper was very clearly written: it was written by Guido, not by myself ;-) It was really pedagogic.

Personally I think that the most important result of the paper was not the construction of a practical way to compute scaling violations in deep inelastic scattering. It could have be done (it was already done) using the Mellin transformation.

The importan point was to shift the focus from Wilson operator expansion to resolution (energy) dependent effective number of partons.

It was more than a computation: it was a shift in the language we use.

The Drell-Yan process $pp \to l^+l^-$ could be not studied by a Wilson operator expansion. This is also true for jet production in hadronic collisions.

However it was possible to study these process by factorizing the amplitude for the process in a part containing the effective parton distribution at the relevant energy and the hard scattering that could be treated in perturbation theory.

This opportunity was immediately taken by Guido. All the next papers are published in '78:

Leptoproduction and Drell-Yan processes beyond the leading approximation in chromodynamics, G Altarelli, RK Ellis, G Martinelli.

Transverse momentum in Drell-Yan processes, G Altarelli, G Parisi, R Petronzio.

Transverse momentum of jets in electroproduction from quantum chromodynamics, G Altarelli, G Martinelli .

Processes involving fragmentation functions beyond the leading order in QCD, G Altarelli, RK Ellis, G Martinelli, S Y Pi.