

QCD Corrections to Weak Decays and Flavor Physics

Guido Altarelli Memorial Symposium

CERN June 10 2016



Guido Martinelli

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Sapienza & SISSA*



International School for Advanced Studies



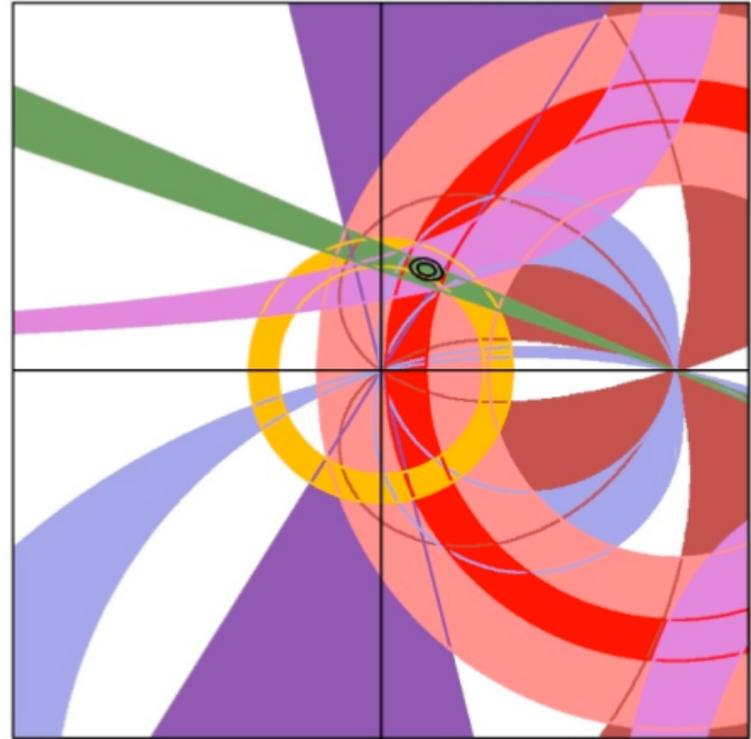
I apologize if during my talk I will be moved (as it happened already in preparing these slides).

I thank you in advance for your understanding.



PLAN OF THE TALK

- *The beginning of my collaboration with Guido;*
- *QCD and Weak Interactions: the first important steps*
 - *see also Maiani talk- ;*
- *The (first) calculation of the NLO corrections to the Effective Weak Hamiltonian;*
- *The game became more complex, where we stand now;*
- *Final remarks.*



The beginning of my collaboration with Guido
From ‘‘The Early days of QCD’’ by
Guido A. @ the
Symposium in Honour of Mario Greco 2011

after Paris and the AP paper,

Back to Rome  I met Guido Martinelli, then a post-doc with a contract for doing accelerator physics at Frascati, and I rescued him into particle physics, with a work on the transverse momentum distributions for jets in lepto-production final states [32]. In the same paper we derived an elegant formula for the longitudinal structure function F_L , also an effect of order $\alpha_s(Q^2)$, as a convolution integral over $F_2(x, Q^2)$ and the gluon density $g(x, Q^2)$. I find it surprising that it took 40 years since the start of deep inelastic scattering experiments to get meaningful data on the longitudinal structure function. The present data, recently obtained by the H1 experiment at DESY, are in agreement with this LO QCD prediction but the accuracy of the test is still far from being satisfactory for such a basic quantity.

The first papers together

The 'Ellis) Collaboration

1. Transverse Momentum of Jets in Electroproduction from Quantum Chromodynamics

Guido Altarelli (Rome U. & INFN, Rome), G. Martinelli (Frascati). Jan 1978. 6 pp.

Published in **Phys.Lett. B76 (1978) 89-94**

Print-78-1029 (ROME)

DOI: [10.1016/0370-2693\(78\)90109-0](https://doi.org/10.1016/0370-2693(78)90109-0)

[References](#) | [BibTeX](#) | [LaTeX\(US\)](#) | [LaTeX\(EU\)](#) | [Harvmac](#) | [EndNote](#)
[ADS Abstract Service](#); [Science Direct](#)

[Detailed record](#) - Cited by 323 records 250+

←
4th FIRST
NLO QCD
Calculation
($F_2^L(x)$)

2. Leptoproduction and Drell-Yan Processes Beyond the Leading Approximation in Chromodynamics

Guido Altarelli (Rome U. & INFN, Rome), R.Keith Ellis (MIT, LNS), G. Martinelli (Frascati). Jun 1978. 25 pp.

Published in **Nucl.Phys. B143 (1978) 521**, Erratum: **Nucl.Phys. B146 (1978) 544**

MIT-CTP-723

DOI: [10.1016/0550-3213\(78\)90067-6](https://doi.org/10.1016/0550-3213(78)90067-6)

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[Detailed record](#) - Cited by 450 records 250+

dimensional
regularization
from
Keith Ellis
(Mariano)

3. Large Perturbative Corrections to the Drell-Yan Process in QCD

Guido Altarelli (Rome U. & INFN, Rome), R.Keith Ellis (MIT, LNS), G. Martinelli (Frascati). Mar 1979. 37 pp.

Published in **Nucl.Phys. B157 (1979) 461-497**

MIT-CTP-776

DOI: [10.1016/0550-3213\(79\)90116-0](https://doi.org/10.1016/0550-3213(79)90116-0)

[References](#) | [BibTeX](#) | [LaTeX\(US\)](#) | [LaTeX\(EU\)](#) | [Harvmac](#) | [EndNote](#)

[Detailed record](#) - Cited by 846 records 500+

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4. Processes Involving Fragmentation Functions Beyond the Leading Order in QCD

Guido Altarelli (Rome U. & INFN, Rome), R.Keith Ellis (MIT, LNS), G. Martinelli (Frascati), So-Young Pi (MIT, LNS). Jun 1979. 29 pp.

Published in **Nucl.Phys. B160 (1979) 301-329**

MIT-CTP-793

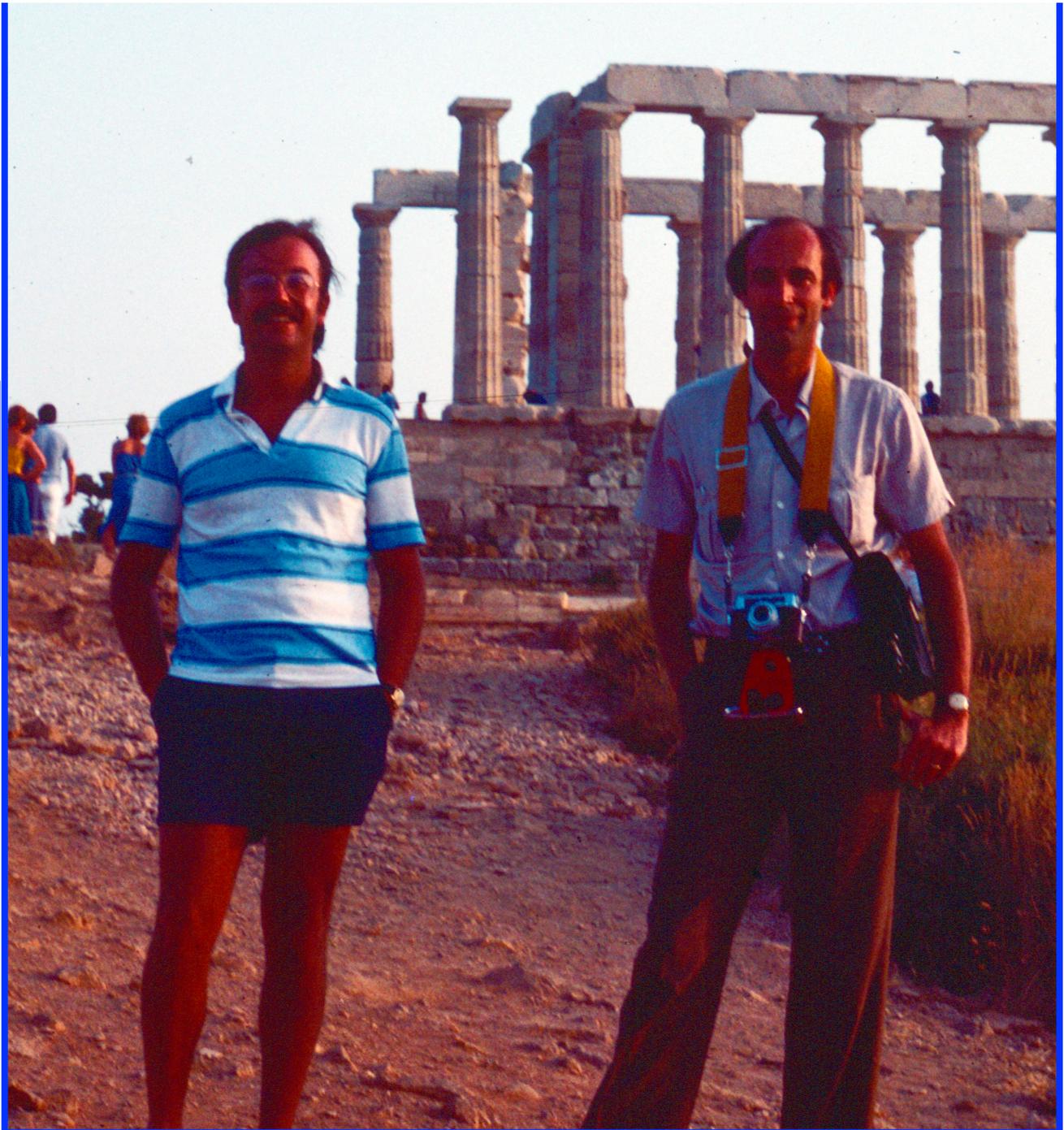
DOI: [10.1016/0550-3213\(79\)90062-2](https://doi.org/10.1016/0550-3213(79)90062-2)

[References](#) | [BibTeX](#) | [LaTeX\(US\)](#) | [LaTeX\(EU\)](#) | [Harvmac](#) | [EndNote](#)

[Detailed record](#) - Cited by 251 records 250+

←

WE WERE A
LITTLE
YOUNGER
THOUGH !!



A NICE GROUP AT WORK: Manuel Greco,
myself, GUIDO, Keith Ellis, Mario Greco
Note Keith tan !



**OCTET ENHANCEMENT OF NON-LEPTONIC WEAK INTERACTIONS
IN ASYMPTOTICALLY FREE GAUGE THEORIES**

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L. MAIANI

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Received 22 June 1974

Octet enhancement of weak non leptonic amplitudes is found to occur in asymptotically free gauge theories of strong interactions, combined with unified weak and e.m. interactions. The order of magnitude of the enhancement factor for different models is discussed.

$$\mathcal{A}^{\Delta S=1}_{FI} (2\pi^4) \delta^4(p_F - p_I) = \text{tadpoles} + (\text{Higgs scalar exchange}) + \int d^4x d^4y D_{\mu\nu}(x, M_W) \langle F | T[J_\mu(y+x/2) J^\dagger_\nu(y-x/2)] | I \rangle$$

1) Tadpoles cannot give any contribution;

2) Higgs contribution suppressed as m^2/M^2_W

$$\langle F | \mathcal{H}^{\Delta S=1} | I \rangle = G_F/\sqrt{2} V_{ud} V_{us}^* \sum_i C_i(\mu) \langle F | Q_i(\mu) | I \rangle$$

WILSON OPE

$(M_W)^{di-6}$

$\Delta I = \frac{1}{2}$ Rule for Nonleptonic Decays in Asymptotically Free Field Theories

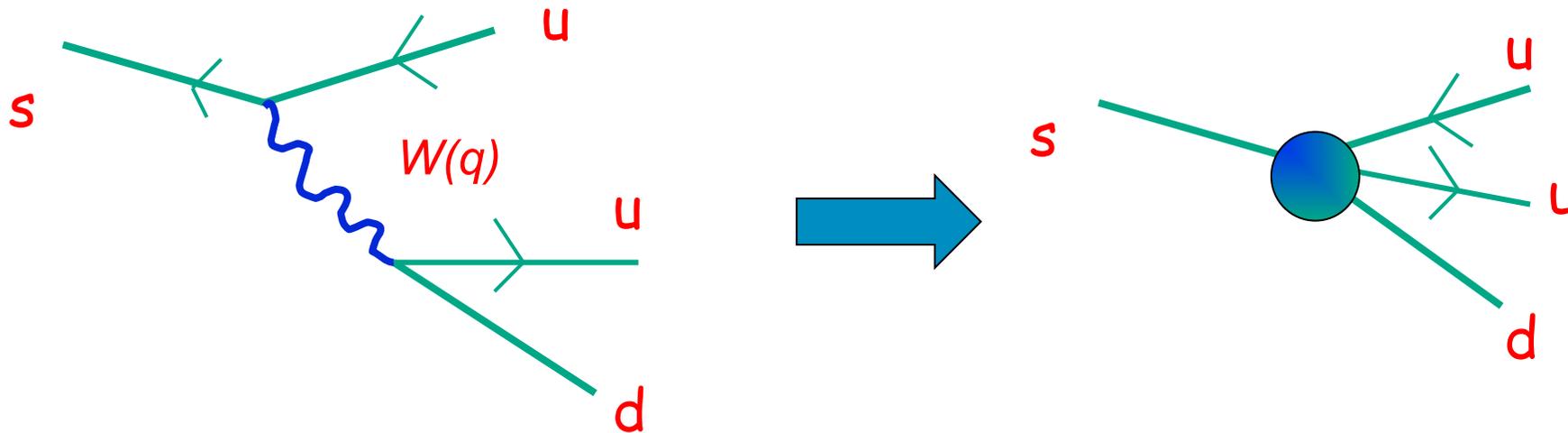
M. K. Gaillard* and Benjamin W. Lee†

National Accelerator Laboratory, Batavia, Illinois 60510

(Received 10 April 1974)

The effective nonleptonic weak interaction is examined assuming the Weinberg-Salam theory of weak interactions and an exactly-conserved-color gauge symmetry for strong interactions. It is shown that the octet part of the nonleptonic weak interaction is more singular at short distances than the 27 part. The resulting enhancement of the octet term in the effective local weak Lagrangian, together with suggested mechanisms for the suppression of matrix elements of the 27 operator, may be sufficient to account for the observed $|\Delta I| = \frac{1}{2}$ rule.

The Effective Hamiltonian



$$q \sim m_K \ll M_W$$

$$\mathcal{H}_{eff} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* (\bar{s} \gamma_\mu (1 - \gamma_5) u) (\bar{u} \gamma^\mu (1 - \gamma_5) d)$$

Wilson OPE

$$\mathcal{A}_W \approx \alpha M_W^{-2} \sum_k C_k [\ln(M_W^2/m^2)]^{dk} \langle F | Q_k(0) | I \rangle + \dots$$

Anomalous dimension
of the operator Q_k

“The OPE shows that the amplitude is dominated by the matrix elements of those operators with $dk > 0$ thus giving rise to a possible mechanism to enhance contributions with definite quantum numbers, e.g. $\Delta I = 1/2$ vs $\Delta I = 3/2$ as first suggested by Wilson”

$$O_L^1 = \bar{\psi} \gamma_\mu L^+ (1 + \gamma_5) \psi \bar{\psi} \gamma^\mu L^- (1 + \gamma_5) \psi$$

$$O_L^2 = \bar{\psi} \gamma_\mu L^+ (1 + \gamma_5) t^A \psi \bar{\psi} \gamma^\mu L^- (1 + \gamma_5) t^A \psi$$

$$O_R^1 = \bar{\psi} \gamma_\mu R^+ (1 - \gamma_5) \psi \bar{\psi} \gamma^\mu R^- (1 - \gamma_5) \psi$$

$$O_R^2 = \bar{\psi} \gamma_\mu R^+ (1 - \gamma_5) t^A \psi \bar{\psi} \gamma^\mu R^- (1 - \gamma_5) t^A \psi$$

$$O_{LR}^1 = \bar{\psi} \gamma_\mu L^+ (1 + \gamma_5) \psi \bar{\psi} \gamma^\mu R^- (1 - \gamma_5) \psi$$

$$O_{LR}^2 = \bar{\psi} \gamma_\mu L^+ (1 + \gamma_5) t^A \psi \bar{\psi} \gamma^\mu R^- (1 - \gamma_5) t^A \psi$$

(4) Definition of the Operators
 Note the perversion:
 (5) $(1 + \gamma_5)$ is left-handed

$$O_L^\pm = \frac{N \pm 1}{N} O_L^1 \pm \frac{1}{2} O_L^2; \quad d_L^\pm = \frac{1}{2b} \left(\frac{3}{8\pi^2} \right) \left(\mp \frac{N \mp 1}{N} \right) \quad (7)$$

same for O_R^\pm , $d_R^\pm = d_L^\pm$, and

$$\delta_{LR}^1 = -\frac{N^2 - 1}{N} O_{LR}^1 + \frac{1}{2} O_{LR}^2;$$

$$\delta_{LR}^2 = \frac{1}{N} O_{LR}^1 + \frac{1}{2} O_{LR}^2;$$

$$d_{LR}^1 = \frac{1}{2b} \left(\frac{3}{8\pi^2} \right) \left(-\frac{1}{N} \right);$$

$$d_{LR}^2 = \frac{1}{2b} \left(\frac{3}{8\pi^2} \right) \left(\frac{N^2 - 1}{N} \right),$$

(8)

First calculation of the LO anomalous dims:
 $\Delta I = 1/2$ dynamically enhanced
 although only qualitatively successful

WEAK INTERACTIONS PHENOMENOLOGY WAS IMPROVING AT A FAST PACE

1. Better and better data on charm production and semileptonic non-leptonic decays (1)
2. The bottom quark was discovered in 1977 and its properties & decays started to be intensively studied
3. The beginning of the Heavy Quark (Effective) Theory (2)

**ENHANCEMENT OF NON-LEPTONIC DECAYS
OF CHARMED PARTICLES**

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Laboratori di Fisica, Istituto Superiore di Sanità, Roma, Italy

Received 14 October 1974

The enhancement of non-leptonic rate due to QCD corrections improved agreement of the prediction of the semileptonic branching ratio with data

Calculations of semileptonic branching ratios were done in the “parton model” i.e. using the free particle

Search for charm

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Jonathan L. Rosner

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A systematic discussion of the phenomenology of charmed particles is presented with an eye to experimental searches for these states. We begin with an attempt to clarify the theoretical framework for charm. We then discuss the $SU(4)$ spectroscopy of the lowest lying baryon and meson states, their masses, decay modes, lifetimes, and various production mechanisms. We also present a brief discussion of searches for short-lived tracks. Our discussion is largely based on intuition gained from the familiar—but not necessarily understood—phenomenology of known hadrons, and predictions must be interpreted only as guidelines for experimenters.

- [7] B.W. Lee, M.K. Gaillard and G. Rosner, *Rev. Mod. Phys.* 47 (1975) 277;
G. Altarelli, N. Cabibbo and L. Maiani, *Nucl. Phys.* B88 (1975) 285; *Phys. Lett.* 57B (1975) 277
S.R. Kingsley, S. Treiman, F. Wilczek and A. Zee, *Phys. Rev.* D11 (1975) 1914;
J. Ellis, M.K. Gaillard and D. Nanopoulos, *Nucl. Phys.* B100 (1975) 313

THE LIFETIME OF CHARMED PARTICLES

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and

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Received 10 July 1978

We present a computation of the semileptonic decay rate of charmed particles, including the first order gluon corrections and the final quark mass corrections. Taking into account these corrections, the lifetime of charmed particles is estimated to be: $\tau \approx 0.7 \times 10^{-12}$ s.

*just after I came back
from CERN – see Maiani*

**LEPTONIC DECAY OF HEAVY FLAVORS:
A theoretical update**

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Received 29 June 1982

How Guido remembered that period ...

After the Gross-Wilczek and Politzer papers we immediately turned to study the potentiality of QCD for improving the parton model. Myself and Maiani we decided to study the QCD corrections to the effective weak non-leptonic Hamiltonian, written as a Wilson expansion in terms of 4-quark operators of the $(V-A)_x(V-A)$ type obtained by integrating away the W^\pm exchange [18]. The logarithmically enhanced terms of the QCD corrections are fixed by the anomalous dimensions of these operators, much in the same way as the moments of structure functions get logarithmic corrections as computed by Gross et al [2, 3] from the anomalous dimensions of the leading-twist operators in the light-cone expansion. Our hope was to find that the QCD corrections act in the direction of enhancing the $\Delta T = 1/2$ operators with respect to those with $\Delta T = 3/2$, thus explaining, at least in part, the empirical $\Delta T = 1/2$ rule (where T is the isotopic spin). The explicit calculation turned out to lead to precisely this result, as also obtained in a simultaneous work by M. K. Gaillard and B. W. Lee [19] (actually these authors had pointed out to us the crucial role of charm in this problem). These important papers were the first calculations of the QCD corrections to the coefficients of the Wilson expansion in the product of two weak currents, an approach that, suitably generalised (by considering other weak processes) and improved (for example, by computing the anomalous dimensions beyond the leading order), still represents a basic tool in this field. In the following months we applied the method to charm decays [20], before the discovery of charm, and to weak neutral current processes [21]. To this last paper also contributed Keith Ellis, a scottish PhD student of Cabibbo, who was to stay with us in Rome for a few years, eventually speaking a very good italian and fully understanding the roman way of living. Later, in '81 myself with Curci (who, unfortunately, is no more with us), Martinelli and Petrarca [22] we computed the two-loop anomalous dimensions for the operators of the effective weak non-leptonic Hamiltonian.

The (first) calculation of the NLO corrections to the Effective Weak Hamiltonian

The physical motivations for a NLO calculation

For heavy quark decay (especially for charm) a substantial increase in the non-leptonic width is obtained, which leads to a prediction [7] for the (quark) semileptonic branching ratio B^{SL} , which is considerably smaller than the free field value. For charm, the prediction in the LLA is typically $B^{\text{SL}} \approx 13\text{--}16\%$ as compared with the free field value of $\sim 20\%$. Until recently, the results for a charm (c) quark

with real gluon emission [9]. However, the c quark decay prediction should remain essentially valid for D^+ (provided the spectator is really inert [10]) because, in D^+ , the annihilation process can only occur at the Cabibbo suppressed level. Since a value of B^{SL} for D^+ close to 20% is being currently reported [8] it is important to verify whether or not the LLA is supported by a study of the next to leading corrections.

In order to investigate these matters we computed the first non-leading QCD corrections to the effective weak non-leptonic hamiltonian (a summary of our results has already been published elsewhere [11]). The main ingredients for this calculation

Further Motivations:

$$\mathcal{A}_{\text{FI}} (2\pi^4) \delta^4 (p_{\text{F}} - p_{\text{I}}) =$$

$$\int d^4x d^4y D_{\mu\nu}(x, M_{\text{W}}) \langle F | T[J_{\mu}(y+x/2) J_{\nu}^{\dagger}(y-x/2)] | I \rangle$$


$$\langle F | \mathcal{H}^{\Delta S=1} | I \rangle = G_{\text{F}}/\sqrt{2} V_{\text{ud}} V_{\text{us}}^* \sum_i C_i(\mu) \frac{\langle F | Q_i(\mu) | I \rangle}{(M_{\text{W}})^{\text{di}-6}}$$

di= dimension of the operator $Q_i(\mu)$

$C_i(\mu)$ Wilson coefficient: it depends on M_{W}/μ and $\alpha_{\text{W}}(\mu)$

$Q_i(\mu)$ local operator renormalized at the scale μ

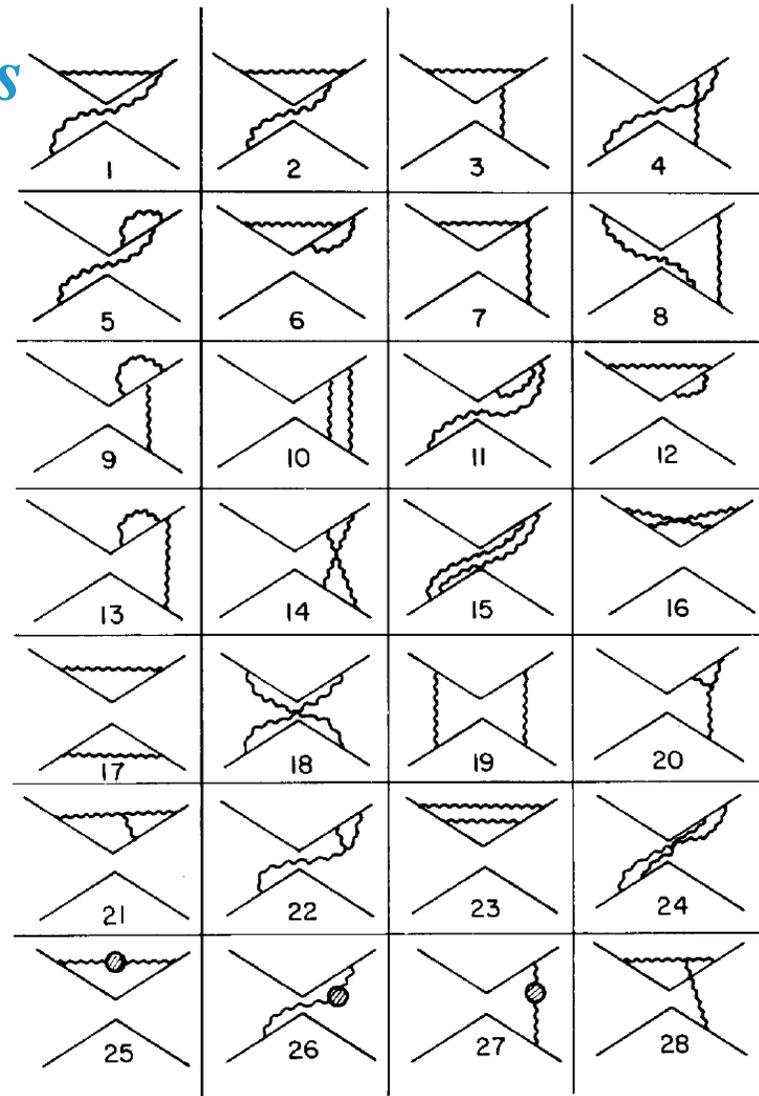
Without the next-to-leading corrections it is impossible to fix the renormalization scale and to match consistently the Wilson coefficients to the matrix elements of the (lattice) operators (see also citation from Buras *)

No penguin diagrams necessary for the charm calculation

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G. Altarelli et al. / Corrections to weak decays

*Letters exchanges
between the
CERN team
(G. Curci & GM)
And the Rome
Team
(G. Altarelli and
S. Petrarca)*



*Occasionally
some mistake
was found*

Fig. 2. The 28 independent two-loop diagrams for the anomalous dimension of the four-fermion operators of dimension six. Replicas differing by up-down, left-right reflections of diagrams are not shown. "Penguin" like diagrams are absent in the massless theory. They are irrelevant for transition involving four different flavours as in $c \rightarrow s\bar{d}u$.

We were scared of using Naive Dimensional Regularization (NDR) in the presence of chiral currents (γ_5) and decided to use Dimensional Reduction (we were really naïve!!)

Volume 148B, number 1,2,3

PHYSICS LETTERS

22 November 1984

**CONSISTENCY BETWEEN DIFFERENT DIMENSIONAL REGULARIZATIONS
IN TWO-LOOP CALCULATIONS FOR SUPERSYMMETRIC GAUGE THEORIES**

G. CURCI and G. PAFFUTI

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Received 6 August 1984

We show that dimensional regularization and dimensional reduction are consistent up to two-loop in susy gauge theories. No anomalies are found for supersymmetry at two-loop level.

Recently Van Damme and 't Hooft [1] have raised the problem of compatibility between standard dimensional regularization (DR) [2] and the dimensional reduction scheme (SDR) [3] in supersymmetric gauge theories.

A convenient device to perform calculations for the $N = 1, 2, 4$ models at once is offered by the formalism of ref. [4] used for similar computations in ref. [5].

Let us consider the Yang–Mills theory in D dimensions with fermions in the adjoint representation

Climbing NLO and NNLO Summits of Weak Decays

Andrzej J. Buras *arXiv:1102.5650v4*

In 1981 Guido (M.) took part in the pioneering calculation of the two loop anomalous dimensions of the current-current operators. This calculation done in collaboration with Guido Altarelli, Giuseppe Curci and Silvano Petrarca has been unfortunately performed in the dimensional reduction scheme (DRED) that was not familiar to most phenomenologists and its complicated structure discussed in detail by these authors most probably scared many from checking their results. Moreover it was known that the treatment of γ_5 in the DRED scheme, similarly to the dimensional regularization scheme with anticommuting γ_5 (known presently as the NDR scheme), may lead to mathematically inconsistent results.

Consequently it was not clear in 1988 whether the result of Altarelli et al. was really correct.

The calculation by Buras & Weiz, in NDR and DRED, of the NLO corrections to KK bar mixing confirmed our results and demonstrated that the calculation could have been done in NDR as well.

Further Motivations & Recent Developments

$$\mathcal{A}_{\text{FI}} (2\pi^4) \delta^4 (p_{\text{F}} - p_{\text{I}}) =$$

$$\int d^4x d^4y D_{\mu\nu}(x, M_{\text{W}}) \langle F | T[J_{\mu}(y+x/2) J_{\nu}^{\dagger}(y-x/2)] | I \rangle$$


$$\langle F | \mathcal{H}^{\Delta S=1} | I \rangle = G_{\text{F}}/\sqrt{2} V_{\text{ud}} V_{\text{us}}^* \sum_i C_i(\mu) \frac{\langle F | Q_i(\mu) | I \rangle}{(M_{\text{W}})^{\text{di}-6}}$$

di= dimension of the operator $Q_i(\mu)$

$C_i(\mu)$ Wilson coefficient: it depends on M_{W}/μ and $\alpha_{\text{W}}(\mu)$ @NLO

$Q_i(\mu)$ local operator renormalized at the scale μ FROM LATTICE

Without the next-to-leading corrections it is impossible to fix the renormalization scale and to match consistently the Wilson coefficients to the matrix elements of the (lattice) operators (see also citation from Buras *)

*Numerical Estimates of Hadronic Masses in a Pure
SU(3) Gauge Theory*

H. Hamber & G. Parisi

Phys.Rev.Lett. 47 (1981) 1792

- Weak Hamiltonian on the Lattice Cabibbo et al.
+ Gavela et al. + Bernard & Soni
- Construction and renormalization of the Weak
Hamiltonian on the Lattice Bochicchio et. al.
- Renormalization of composite operators GM et al.
- $K\pi\pi$ amplitudes on a finite volume Lellouch &
Luscher

*Leptonic, Semileptonic, $K\pi\pi$, B and K Mixing,
Radiative, ...*

Andrzej J. Buras Gospel *arXiv:1102.5650v4*

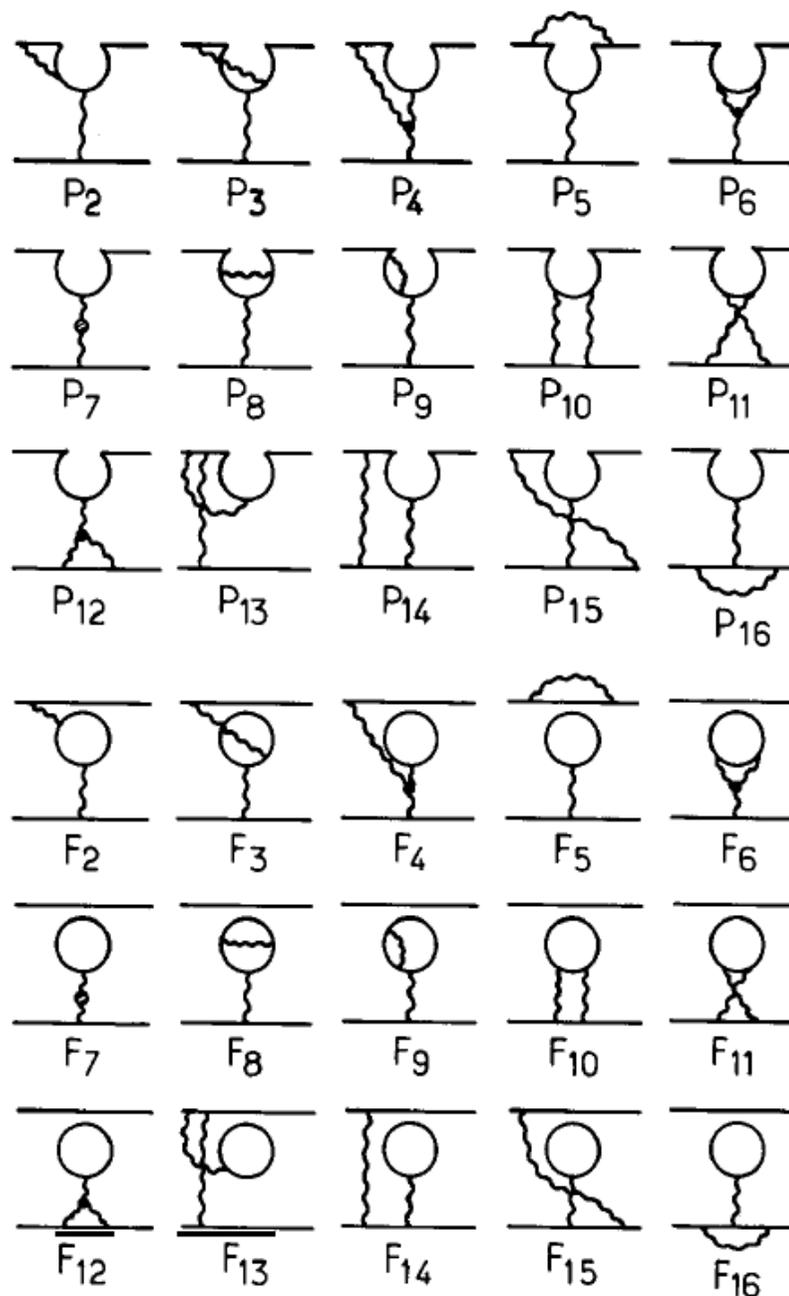
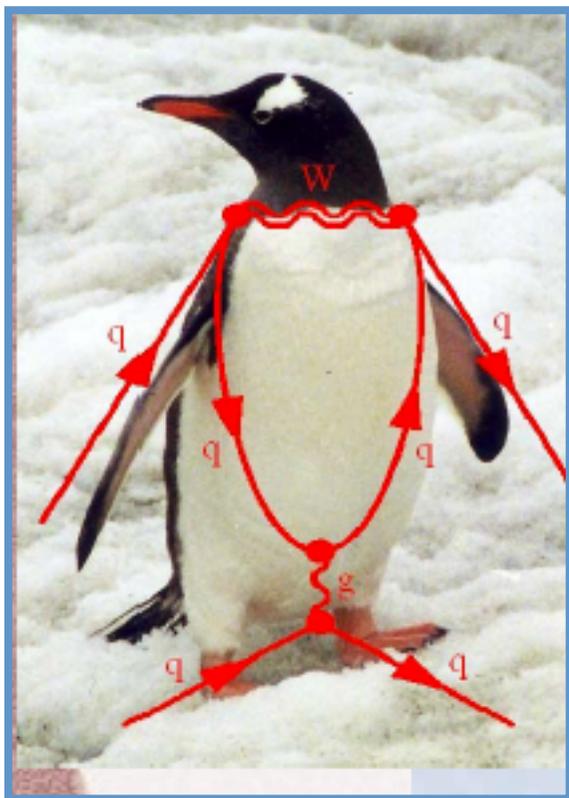
During the last supper of the Ringberg workshop ('88) Guido Martinelli and me realized that it would be important to calculate NLO QCD corrections to the Wilson coefficients of penguin operators relevant for $K \rightarrow \pi\pi$ decays

.. NLO QCD corrections to $\Delta S = 1$ and $\Delta B = 1$ non-leptonic decays... $\Delta S = 2$ & $\Delta B = 2$ transitions, rare K and B decays, in particular $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $B_{s,d} \rightarrow \mu^+ \mu^-$... the inclusive decay $B \rightarrow X_s \gamma$, $B \rightarrow X_s$ gluon, ... $K_L \rightarrow \pi^0 \ell^+ \ell^-$, $B \rightarrow X_s \ell^+ \ell^-$... $B \rightarrow K^(\rho) \ell^+ \ell^-$*

several thousands citations

still the road has been opened by Guido Altarelli

The Penguin Era Begins (J. Ellis)



M. Shifman, A.I. Vainshtein,
V. I. Zakharov
J. Flynn and L. Randall

Fig. 11. Penguin diagrams at two loops.

A concrete (most difficult) example:

$K \rightarrow \pi\pi$ decays

$$\mathcal{H}^{\Delta S=1} = G_F/\sqrt{2} V_{ud} V_{us}^* \left[(1-\tau) \sum_{i=1,2} z_i (Q_i - Q_i^c) + \tau \sum_{i=1,10} (z_i + y_i) Q_i \right]$$

Where y_i and z_i are short distance coefficients, which are known
In perturbation theory at the NLO (Buras et al. + Ciuchini et al.)

$$\tau = -V_{ts}^* V_{td} / V_{us}^* V_{ud}$$

We must compute $\mathcal{A}^{I=0,2}_i = \langle (\pi\pi)_{I=0,2} | Q_i | K \rangle$
with a non perturbative technique (lattice LL,
QCD sum rules, 1/N expansion etc.)

$$\begin{aligned} \mathcal{A}^{I=0,2}_i(\mu) &= \langle (\pi \pi)_{I=0,2} | Q_i(\mu) | K \rangle \\ &= Z_{ik}(\mu a) \langle (\pi \pi)_{I=0,2} | Q_k(a) | K \rangle \end{aligned}$$

Where $Q_i(a)$ is the bare lattice operator
And a the lattice spacing.

The effective Hamiltonian can then be read as:

$$\langle F | H^{\Delta S=1} | I \rangle = G_F / \sqrt{2} V_{ud} V_{us}^* \sum_i C_i(1/a) \langle F | Q_i(a) | I \rangle$$

In practice the renormalization scale (or $1/a$) are the scales which separate short and long distance dynamics

GENERAL FRAMEWORK

$$\langle \mathcal{H}^{\Delta S=1} \rangle = G_F/\sqrt{2} V_{ud} V_{us}^* \dots \sum_i C_i(\mathbf{a}) \langle Q_i(\mathbf{a}) \rangle$$

$$M_W = 100 \text{ GeV}$$

Effective Theory - quark & gluons

$$a^{-1} = 2\text{-}5 \text{ GeV}$$

Hadronic non-perturbative region

$$\Lambda_{\text{QCD}}, M_K = 0.2\text{-}0.5 \text{ GeV}$$

perturbative regime

Chiral regime

100 GeV

perturbative region

Large mass scale: heavy degrees of freedom (m_t, M_W, M_S) are removed and their effect included in the Wilson coefficients

1-2 GeV

non-perturbative region

renormalization scale μ (inverse lattice spacing $1/a$); this is the scale where the quark theory is matched to the effective hadronic theory

Scale of the low energy process
 $\Lambda \sim M_W$

THE SCALE PROBLEM:

Effective theories prefer low scales,
Perturbation Theory prefers large scales

Where we are now?

- *non-perturbative renormalization of the relevant operators*
- *$K \rightarrow \pi\pi$ computed at the physical point using Lellouch-Lüscher (see also Lin, Sachrajda, gm, Testa)*
- *Unquenched and at (almost) physical quark masses*
- *Enormous progresses made by RBC-UKQCD*



RBC-UK QCD

$$\varepsilon'/\varepsilon = (1.4 \pm 7.0) \cdot 10^{-4}$$

$$\left(\frac{\text{Re } A_0}{\text{Re } A_2} \right) = 31.0 \pm 6.6$$

$$\left(\varepsilon'/\varepsilon \right)_{\text{exp}} = (16.6 \pm 2.3) \cdot 10^{-4}$$

$$\left(\frac{\text{Re } A_0}{\text{Re } A_2} \right)_{\text{exp}} = 22.4$$

Courtesy by A. Buras

Four dominant contributions to ε'/ε in the SM

AJB, Jamin, Lautenbacher (1993); AJB, Gorbahn, Jäger, Jamin (2015)

$$\text{Re}(\varepsilon'/\varepsilon) = \left[\frac{\text{Im}(V_{td} V_{ts}^*)}{1.4 \cdot 10^{-4}} \right] 10^{-4} \left[\overset{\text{From Re}A_0}{\downarrow} -3.7 + 21.2 \cdot B_6^{(1/2)} + \overset{\text{From Re}A_2}{\downarrow} 1.1 - 9.6 \cdot B_8^{(3/2)} \right]$$

(Q₄)

(V-A) ⊗ (V-A)
QCD Penguins

(V-A) ⊗ (V+A)
QCD Penguins

(V-A) ⊗ (V-A)
EW Penguins

(V-A) ⊗ (V+A)
EW Penguins

Assumes that $\text{Re}A_0$ and $\text{Re}A_2$ ($\Delta I=1/2$ Rule) fully described by SM (includes isospin breaking corrections)

ε'/ε from RBC-UKQCD

Calculate all contributions directly (no isospin breaking corrections)

$$\left[-(6.5 \pm 3.2) + 25.3 \cdot B_6^{(1/2)} + (1.2 \pm 0.8) - 10.2 \cdot B_8^{(3/2)} \right]$$

ε'/ε from RBC-UKQCD

Anatomy: AJB, Gorbahn, Jäger, Jamin (2015)

Calculate all contributions directly

$$\text{Re}(\varepsilon'/\varepsilon) = \left[\frac{\text{Im}(V_{td} V_{ts}^*)}{1.4 \cdot 10^{-4}} \right] 10^{-4} \left[-6.5 + 25.3 \cdot B_6^{(1/2)} + 1.2 - 10.2 \cdot B_8^{(3/2)} \right]$$

(Q₄)

(V-A) ⊗ (V-A)
QCD Penguins

(V-A) ⊗ (V+A)
QCD Penguins

(V-A) ⊗ (V-A)
EW Penguins

(V-A) ⊗ (V+A)
EW Penguins

Extracted from

RBC-UKQCD

$B_6^{(1/2)} = B_8^{(3/2)} = 1$ in the large N limit

$B_6^{(1/2)} = 0.57 \pm 0.15$

$B_8^{(3/2)} = 0.76 \pm 0.05$

EW penguins in full agreement with BGJJ but

+ third term very similar to BGJJ
 $(\text{Re}A_2)_{\text{Lattice}} \approx (\text{Re}A_2)_{\text{exp}}$

$\left[\frac{(\text{Re}A_0)}{(\text{Re}A_0)_{\text{exp}}} \approx 1.4 \right]$

The negative contribution of Q₄ overestimated

$\left(\frac{\varepsilon'}{\varepsilon} \right)_{\text{Lattice}} = (1.4 \pm 7.0) \cdot 10^{-4}$

Anatomy of ε'/ε – A new flavour anomaly?

AJB, Gorbahn, Jäger, Jamin,, 1507.xxxx

RBC-UKQCD

$$\varepsilon'/\varepsilon = (1.4 \pm 7.0) \cdot 10^{-4}$$

(3.2 σ) $\varepsilon'/\varepsilon = (2.2 \pm 3.8) \cdot 10^{-4}$

$$\varepsilon'/\varepsilon = (6.3 \pm 2.5) \cdot 10^{-4}$$

$$\varepsilon'/\varepsilon = (9.1 \pm 3.3) \cdot 10^{-4}$$

exp: $\varepsilon'/\varepsilon = (16.6 \pm 3.3) \cdot 10^{-4}$

RBC-QCD values

$$B_6^{(1/2)} = 0.57 \pm 0.15$$

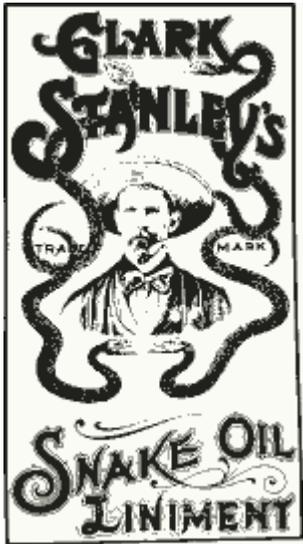
$$B_8^{(3/2)} = 0.76 \pm 0.05$$

large N bounds (AJB, Gérard)

$$B_6^{(1/2)} = B_8^{(3/2)} = 0.76$$

large N bounds (AJB, Gérard)

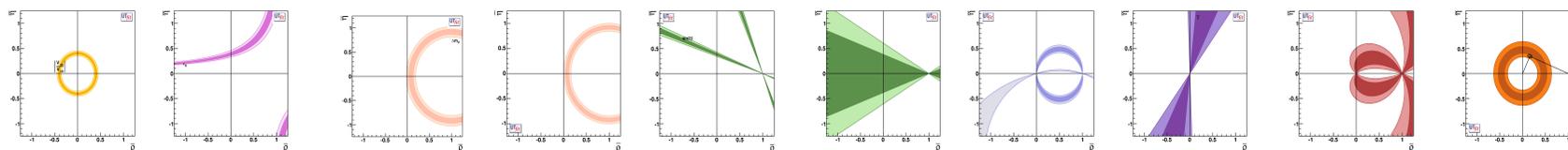
$$B_6^{(1/2)} = B_8^{(3/2)} = 1.0$$



M.Bona *et al.*, UTfit
JHEP0507:028, 2005

www.utfit.org

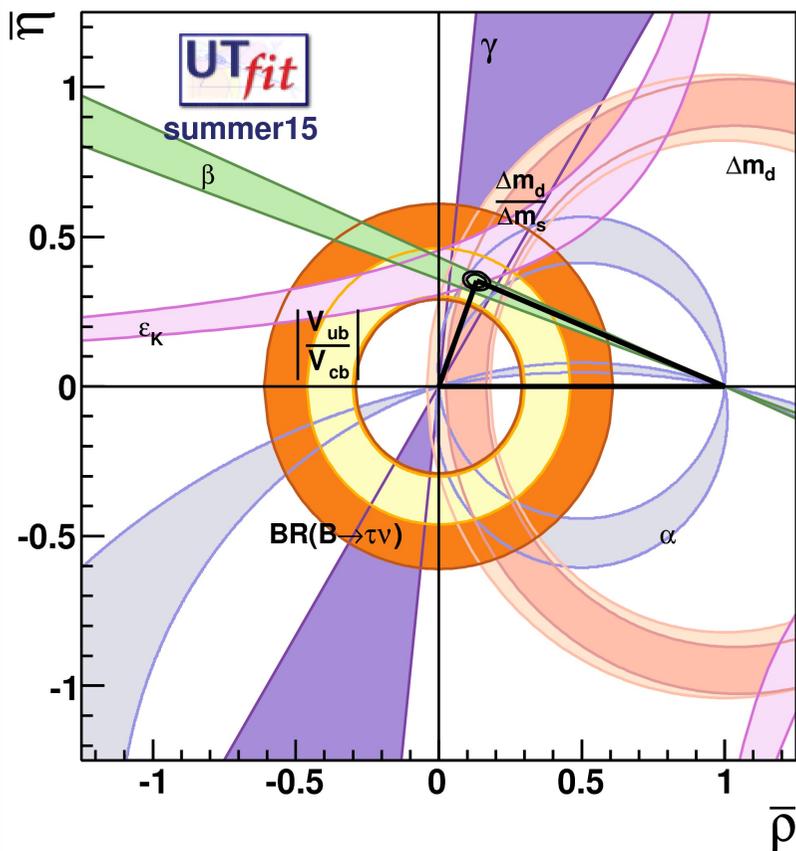
A. Bevan, M. Bona, M. Ciuchini,
D. Derkach, E. Franco, V. Lubicz,
G. Martinelli, F. Parodi, M. Pierini,
C. Schiavi, L. Silvestrini, A. Stocchi,
V. Sordini, C. Tarantino and V. Vagnoni



2015 results

$$\bar{\rho} = 0.142 \pm 0.019 \quad \bar{\eta} = 0.348 \pm 0.013$$

In the hadronic sector, the SM CKM pattern represents the principal part of the flavor structure and of CP violation



$$\alpha = (90.5 \pm 2.6)^\circ$$

$$\sin 2\beta = 0.691 \pm 0.018$$

$$\beta = (21.82 \pm 0.72)^\circ$$

$$\gamma = (67.4 \pm 2.7)^\circ$$

$$A = 0.828 \pm 0.012$$

$$\lambda = 0.22549 \pm 0.00066$$

Consistence on an over constrained fit of the CKM parameters

CKM matrix is the dominant source of flavour mixing and CP violation

Uppsala, July 1, 1987

Schubert

SUMMARY & PERSPECTIVE

G. ALTARELLI



$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} =$$

$$= \begin{pmatrix} c_\theta c_\beta & s_\theta c_\beta & s_\beta e^{-i\delta} \\ -s_\theta c_\gamma - c_\theta s_\beta s_\gamma e^{i\delta} & c_\theta c_\gamma - s_\theta s_\beta s_\gamma e^{i\delta} & c_\beta s_\gamma \\ s_\theta s_\gamma - c_\theta s_\beta c_\gamma e^{i\delta} & -c_\theta s_\gamma - s_\theta s_\beta c_\gamma e^{i\delta} & c_\beta c_\gamma \end{pmatrix}$$

$$V = \begin{pmatrix} .9754 \pm .0004 & .2206 \pm .0018 & 0.000 \pm .0076 \\ -.2203 \pm .0019 & .9743 \pm .0005 & .0474 \pm .0066 \\ .0104 \pm .0075 & -.0462 \pm .0067 & .9989 \pm .0003 \end{pmatrix}$$

$$+i \begin{pmatrix} 0 & 0 & 0 \pm .0076 \\ 0 \pm .0004 & 0 \pm .0001 & 0 \\ 0 \pm .0075 & 0 \pm .0017 & 0 \end{pmatrix}$$

$$\theta = (12.74 \pm 0.11)^\circ$$

$$\beta = (0 \pm 0.43)^\circ$$

$$\gamma = (2.72 \pm 0.38)^\circ$$

$$\left| \frac{V_{ub}}{V_{cb}} \right| < 0.20 \quad 90\% \quad \text{CLEO.}$$

CKM Matrix in the SM

The fit results for all the nine CKM elements are

$$V_{CKM} = \begin{pmatrix} (0.9743 \pm 0.00014) & (0.22509 \pm 0.00061) & (0.00366 \pm 0.00012)e^{i(-67.8 \pm 2.8)^\circ} \\ (-0.22498 \pm 0.00066)e^{i(0.0353 \pm 0.00095)^\circ} & (0.97343 \pm 0.00015)e^{i(-0.00188333 \pm 5 \times 10^{-5})^\circ} & (0.04206 \pm 0.00053) \\ (0.00876 \pm 0.00015)e^{i(-22.03 \pm 0.83)^\circ} & (-0.04129 \pm 0.00054)e^{i(1.054 \pm 0.039)^\circ} & (0.999107 \pm 2.235 \times 10^{-5}) \end{pmatrix}$$

Standard Parametrization (PDG)

$$\sin \theta_{12} = 0.22504 \pm 0.00065$$

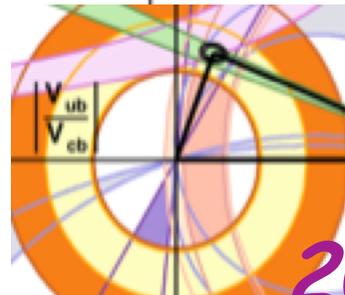
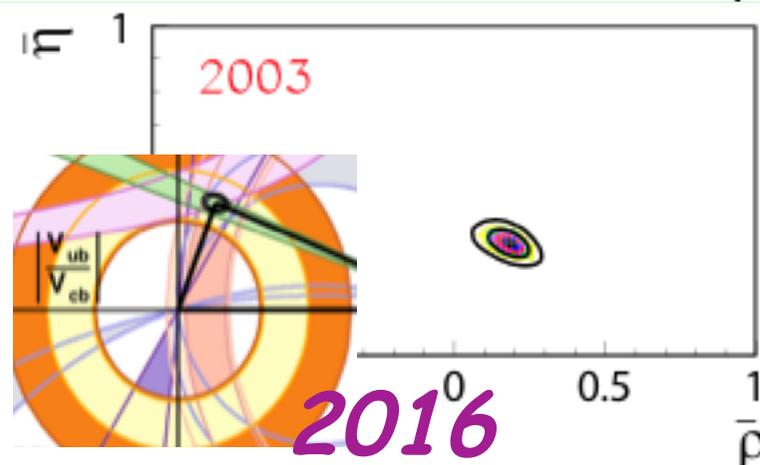
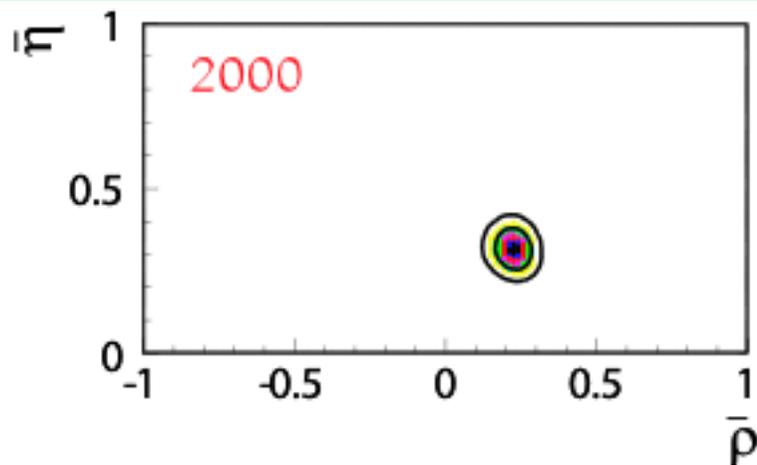
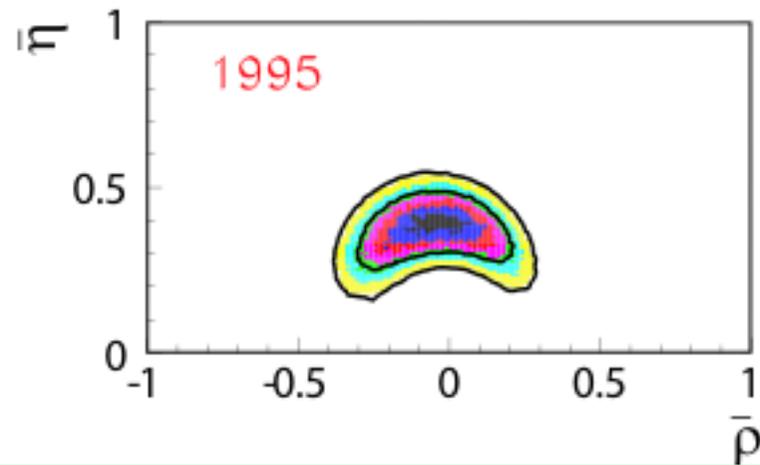
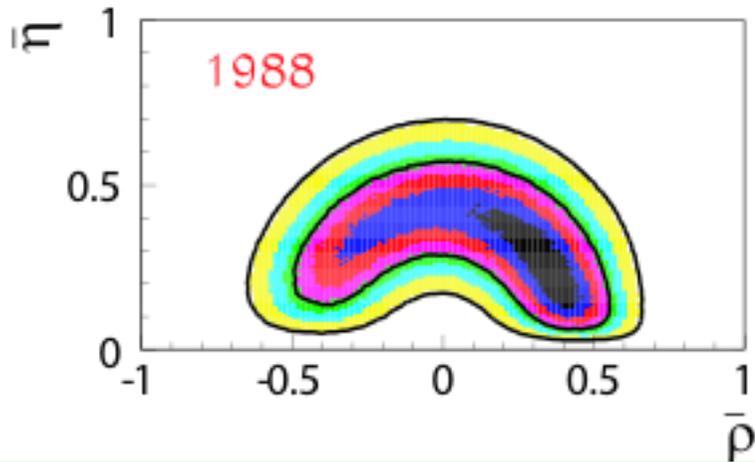
$$\sin \theta_{23} = 0.04206 \pm 0.00054$$

$$\sin \theta_{13} = 0.00366 \pm 0.00012 \quad \delta = 67.8 \pm 2.8$$

Wolfenstein Parametrization (PDG)

$$\lambda = 0.22514 \pm 0.00066 \quad A = 0.828 \pm 0.012$$

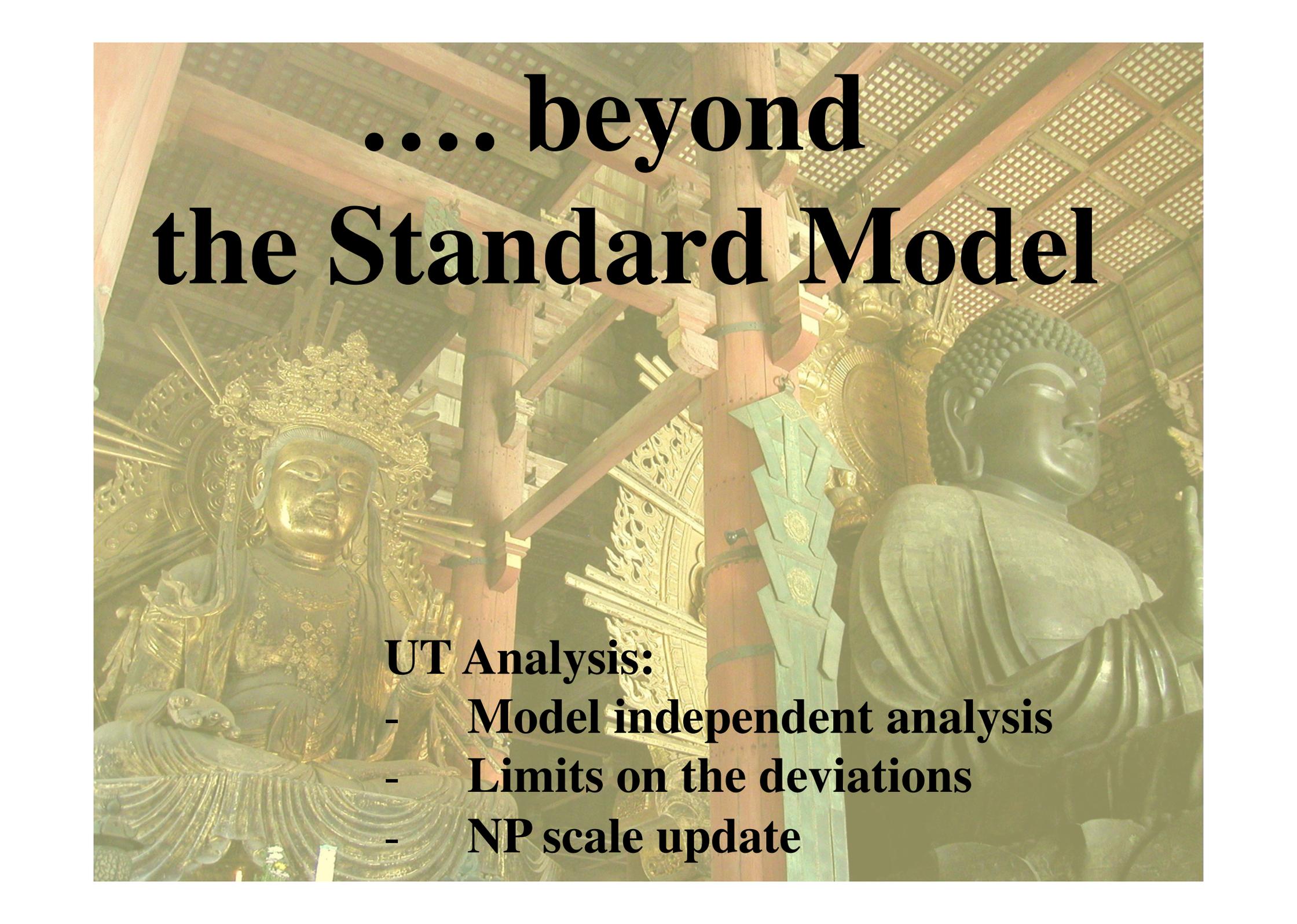
PROGRESS SINCE 1988



2016

LATTICE PARAMETERS

	Lattice	Prediction	Pull
\hat{B}_K	0.766 ± 0.010 1.3 %	0.84 ± 0.07 8.3 %	0.9
f_{B_s}	0.226 ± 0.005 2.2 %	0.2256 ± 0.0039 2.7 %	0.0
f_{B_s}/f_{B_d}	1.204 ± 0.016 1.3 %	1.197 ± 0.056 0.4 %	0.0
B_s	0.875 ± 0.040 1.3 %	0.875 ± 0.030 0.4 %	0.0
B_s/B_d	1.03 ± 0.08 7.8 %	1.096 ± 0.062 5.7 %	0.7



.... beyond the Standard Model

UT Analysis:

- **Model independent analysis**
- **Limits on the deviations**
- **NP scale update**

Results from a fit to the Wilson Coefficients

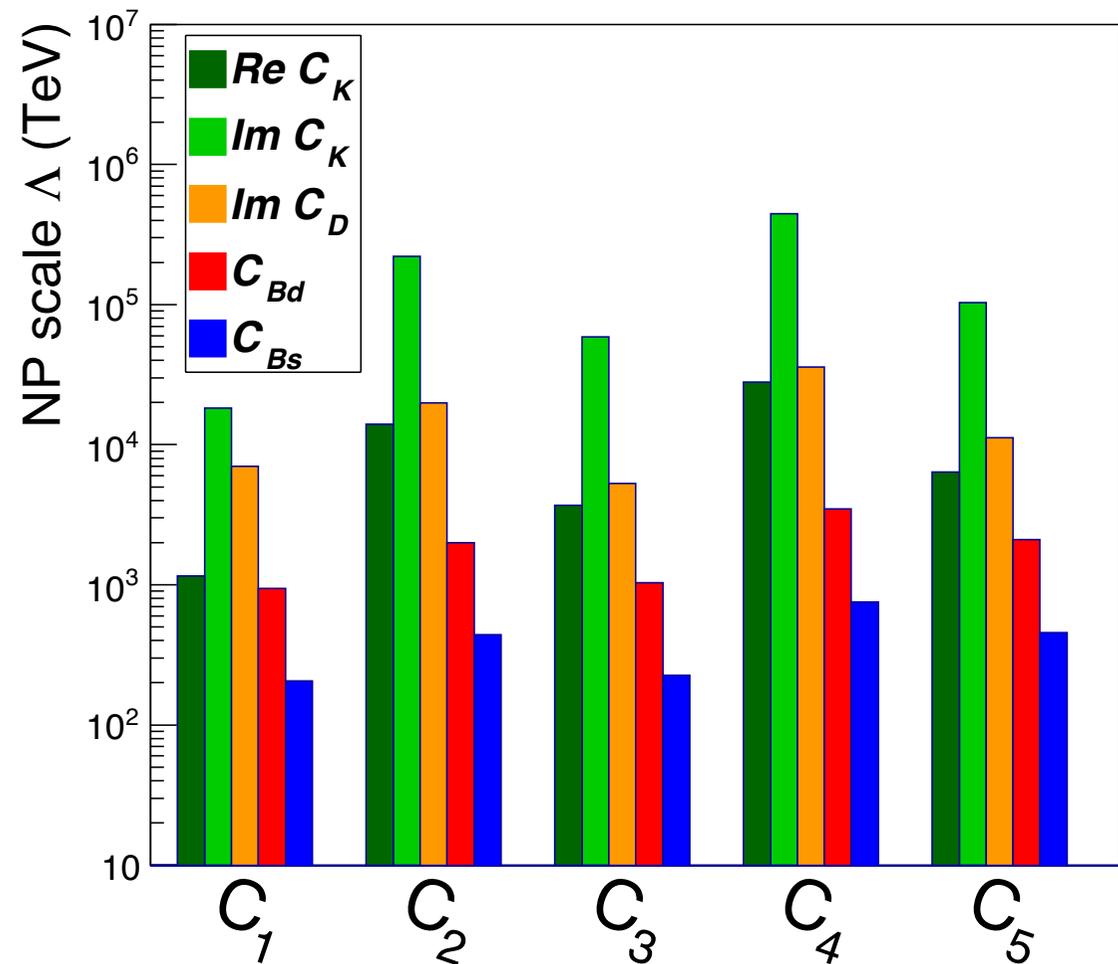
Results obtained with $L=1$ corresponding to tree level NP effects and an arbitrary flavor structure

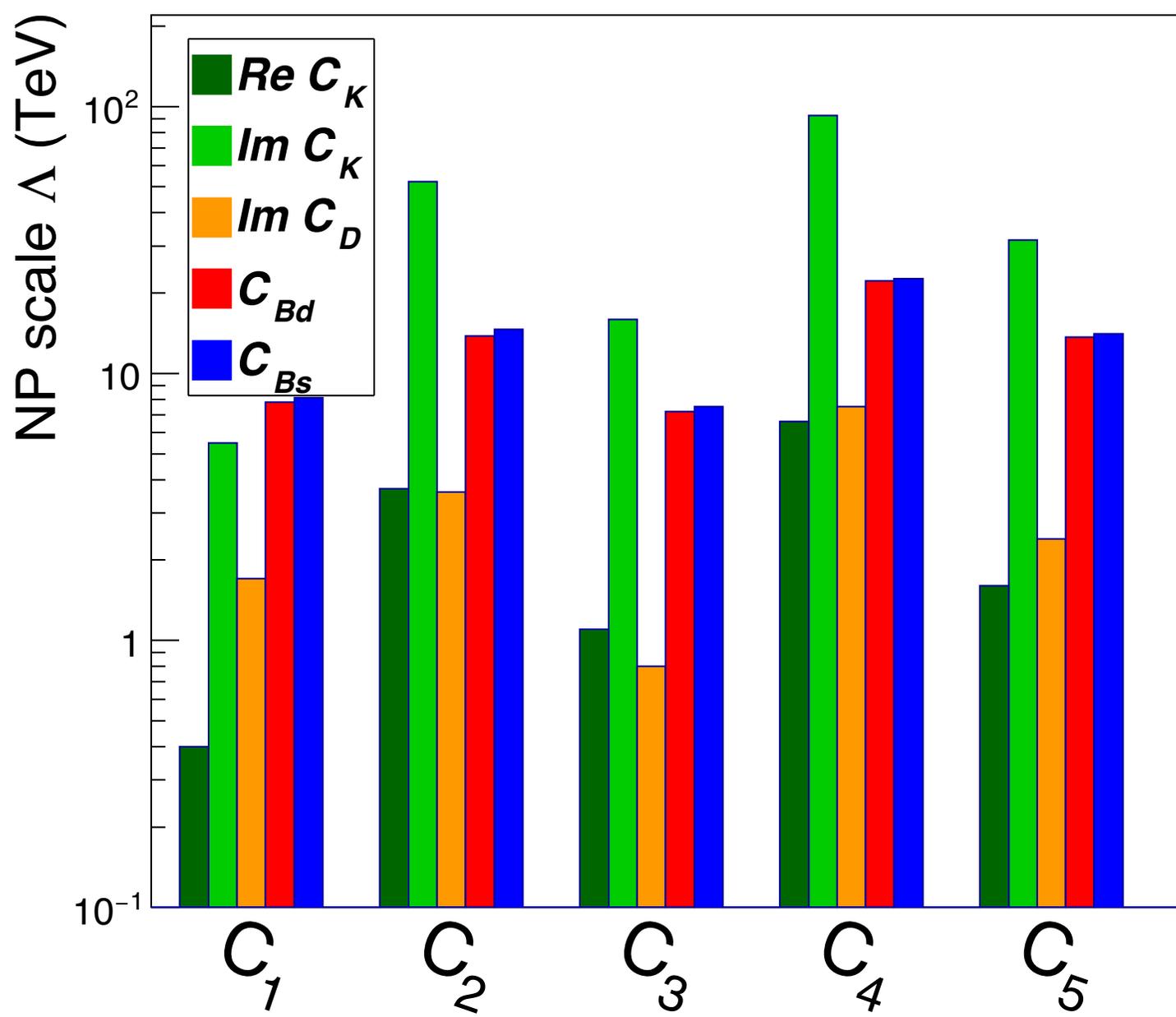
$$\varepsilon_K \quad \Lambda = 5 \cdot 10^5 \text{ TeV}$$

$$D \quad \Lambda = 10^4 \text{ TeV}$$

$$B_d \quad \Lambda = 3 \cdot 10^3 \text{ TeV}$$

$$B_s \quad \Lambda = 8 \cdot 10^2 \text{ TeV}$$





NMFV



This is my last paper with Guido

1. Failure of local duality in inclusive nonleptonic heavy flavor decays

Guido Altarelli (CERN & Rome III U.), G. Martinelli, S. Petrarca, F. Rapuano (Rome U. & INFN, Rome). Mar 1996. 9 pp.

Published in **Phys.Lett. B382 (1996) 409-414**

CERN-TH-96-77, ROME1-1143-96

DOI: [10.1016/0370-2693\(96\)00637-5](https://doi.org/10.1016/0370-2693(96)00637-5)

e-Print: [hep-ph/9604202](#) | [PDF](#)

[References](#) | [BibTeX](#) | [LaTeX\(US\)](#) | [LaTeX\(EU\)](#) | [Harvmac](#) | [EndNote](#)

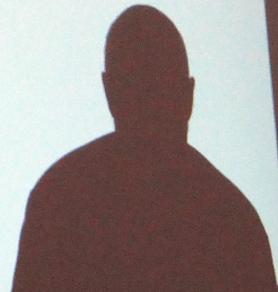
[CERN Document Server](#); [ADS Abstract Service](#)

[Detailed record](#) - [Cited by 62 records](#) 50+

but our friendship continued untouched

Roma 2012

Happy Birthday Guido!!





Singapore 2014





Tutti noi abbiamo ammirato la dignità e il coraggio con cui Guido ha lasciato questo mondo. Chi ha avuto il privilegio di collaborarci o semplicemente di conoscerlo continuerà a ricordarlo con ammirazione e rispetto. Noi, che di Guido siamo stati amici e gli abbiamo voluto bene, non lo dimenticheremo e lo piangeremo ancora molto a lungo.

We all admired the dignity and courage with which Guido left this world. Who had the privilege of working with him or just to know him will continue to remember him with admiration and respect. We, who have been friends of Guido and loved him, we will not forget him and we will mourn for him much longer.



THANKS FOR YOUR ATTENTION



International School for Advanced Studies

