

# For Ulrich

I got to know Ulrich while hiking at a conference in Banff Canada more than 20 yrs ago.

One year later, he offered me my first postdoc position, and I got to think for the first time in my life about an experimental data point.

If scientific genealogy is meaningful, then Ulrich is my scientific father.  
I owe Ulrich more than anyone else in physics.

Sons are strange: they want to differ from their parents  
but even where they succeed  
the parents recognize themselves in what they do.

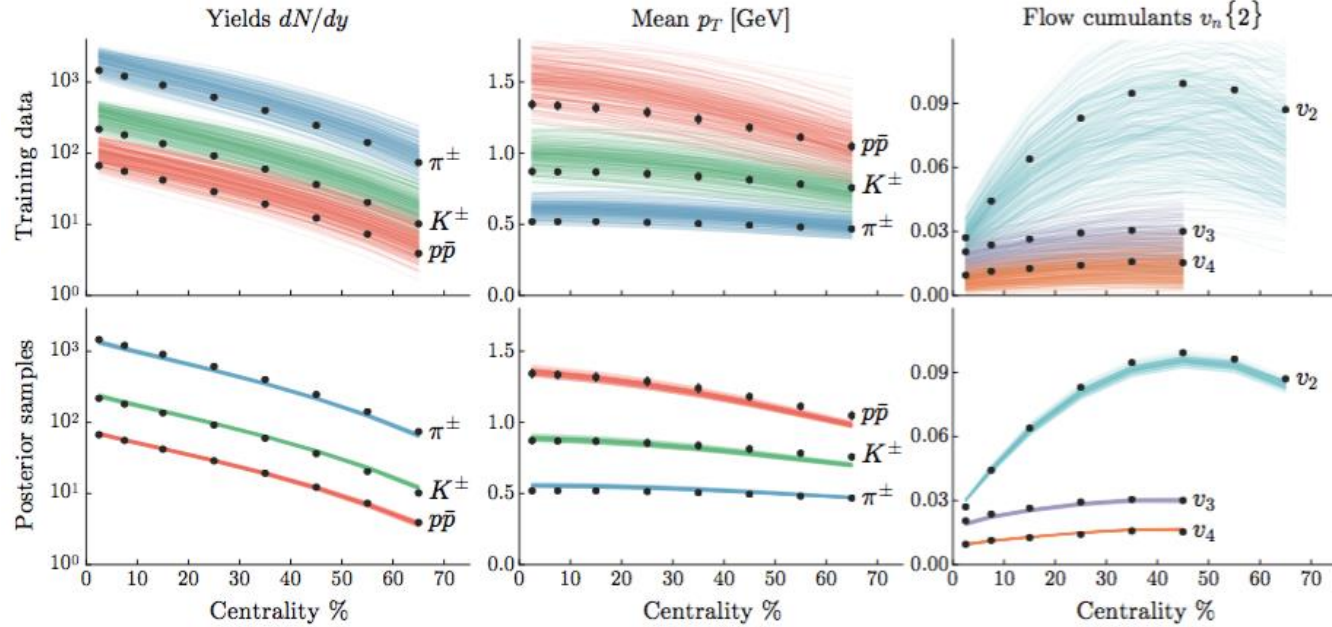
That's why I have chosen the following scientific story for Ulrich's birthday.

# Heavy Ion Phenomenology

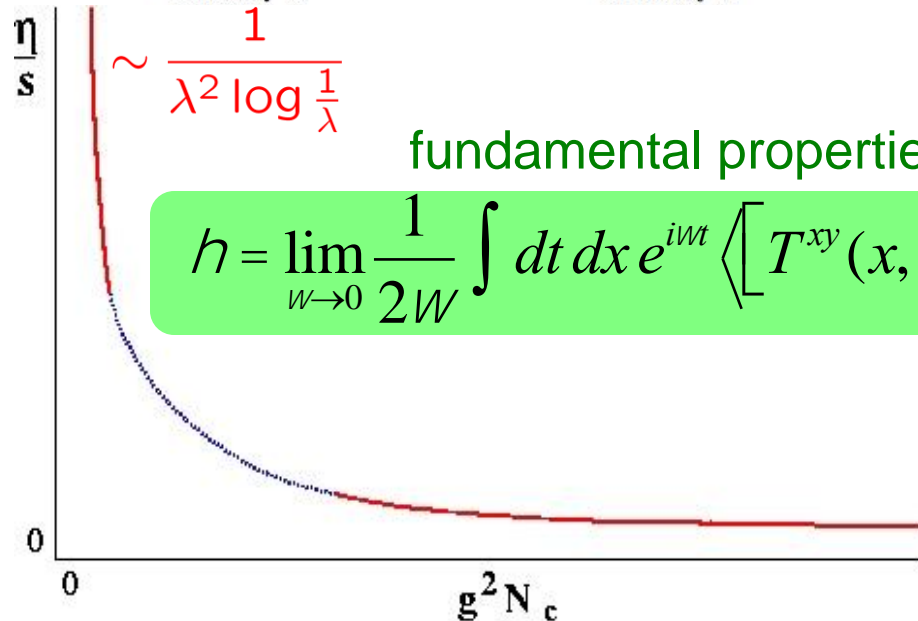
Ulrich is where the action is.

What we fit ...

Bernard, Moreland,  
Bass, Liu, Heinz,  
arXiv:1605.03954



... and what we are after ...



fundamental properties of hot QCD

$$h = \lim_{W \rightarrow 0} \frac{1}{2W} \int dt dx e^{iWt} \left\langle \left[ T^{xy}(x, t), T^{xy}(0, 0) \right] \right\rangle_{eq}$$

$$\frac{h}{s} > \frac{1}{4\rho}$$

# On the stock market of paradigms

## The Bulls:

- Hydro works!

## The Bears:

- But how can hydro work?



This is a story told by Ursus, the bear, about

- how hydro works for *non-interacting* (dark matter) particles in cosmology
- how *effective viscosity becomes calculable* in these systems

# From Boltzmann Transport to Fluid Dynamics

- Boltzmann equation with collision term  $p^m d_m f(x, t, p) = C$
- Momentum moments of phase space distribution:
 
$$\dot{\int}_p p^m d_m f(x, t, p) \circ d_m N^m = 0 = \dot{\int}_p C$$

$$\dot{\int}_p p^m p^a d_m f(x, t, p) \circ d_m T^{ma} = 0 = \dot{\int}_p p^a C$$

$$\dot{\int}_p p^m p^a p^b d_m f(x, t, p) = \dot{\int}_p p^a p^b C$$
- Fluid dynamics applies if
  - higher moments can be truncated
  - perturbations around average stay small

Gradient expansion  
of perturbations  
leads then to  
(dissipative) fluid  
dynamics

$$f = f_0 (1 + df) \quad T^{mn} = T_0^{mn} + P^{mn}$$

$$df(x, t, p) = e(x, t) + e_l(x, t) p^l + e_{ln}(x, t) p^l p^n$$

$$\langle T^{mn} - T_0^{mn} \rangle = \langle P^{mn} \rangle = e_{ab}(x) \int_p p^{<m} p^{n>} p^a p^b f_0$$

from which the Israel-Stewart equations of motion can be derived.

# From Boltzmann transport to the cosmological fluid

Vlasov-Boltzmann equation: free-streaming in gravitational field

$$\frac{\partial f}{\partial \eta} + \frac{\vec{p} \cdot \vec{\nabla} f}{am} - am \vec{\nabla} \Phi \cdot \frac{\partial f}{\partial \vec{p}} = 0$$

$$\vec{\nabla} \Phi = \frac{G_N m}{a} \int_{x', p'} \frac{\vec{x} - \vec{x}'}{|\vec{x} - \vec{x}'|^3} f(x', p', \eta)$$

- Dark matter is non-interacting but of thermal origin:  
=> expand around average

$$f(\vec{x}, \vec{p}, \eta) = f_0(p, \eta) [1 + \delta_f(\vec{x}, p, \hat{p}, \eta)]$$

$$\delta_f(\vec{k}, p, \hat{p}, \eta) = \sum_n i^n (2n+1) \delta_f^{[n]}(k, p, \eta) P_n(\hat{k} \cdot \hat{p})$$

- Momentum moments:

$$d \text{ \$ } \circ ma^{-3} \int 4 \rho p^2 dp f_0(p) \text{ (CHF)}$$

	density	pressure	Velocity gradient	Shear (scalar component)
\$	$r$	$p_{\text{ressure}}$	$(\bar{p} + \bar{r}) q$	$(\bar{p} + \bar{r}) S$

<b>CHF</b>	$d_f^{[0]}$	$\frac{1}{3} v_p^2 d_f^{[0]}$	$k v_p d_f^{[1]}$	$-\frac{2}{3} v_p^2 d_f^{[2]}$
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- Hierarchy:  $d_f^{[n]} \sim (k v_p H^{-1})^{n-2} d_f^{[2]}$  where  $v_p = p/(am) = \dot{x}$

- Truncation of Vlasov-Boltzmann hierarchy applies if

Matarrese, Pietroni,  
arXiv:0703563

$$k \cdot \underbrace{v_p H^{-1}}_{\text{free stream dist}} \ll 1$$

**Hydro works since lifetime of Universe is too short to fall out of equilibrium.**

# Calculating the growth of cosmological structure

- Truncating Vlasov-Boltzmann at lowest moments yields equations of motion  $f_a(h, k) = \left( dr_k(h), -\frac{q_k}{H} \right)$

$$\partial_h f_a(k) + W_{ab}(k, h) f_b(k) - \int_{p,q} g_{abc}(k, p, q, h) f_b(p) f_c(q) = 0 = (eom)_a$$

- For Einstein deSitter Universe  $W_{ab}(k, h) = \begin{pmatrix} 0 & -1 \\ -3/2 & 1/2 \end{pmatrix}$  Crocce, Scoccimarro, arXiv:0509419

$$g_{ab}^{(ret)}(h-h') = \frac{e^{h-h'}}{5} \begin{pmatrix} 3 & 2 \\ 3 & 2 \end{pmatrix} - \frac{e^{-3(h-h')/2}}{5} \begin{pmatrix} -2 & 2 \\ 3 & -3 \end{pmatrix}$$

Density contrast **grows exponentially** on all scales  $k$ , gravitational collapse forms structures from small perturbations.

- Fun for theorists: action principle  $S[f, c] = \int_k dh \int c_a(-k, h) (eom)_a(k, h)$ 
  - Eom follows from variation:  $dS[f, c] / dc_a(k, h) = 0$
  - Stochastic initial conditions  $w[f_a(0)] = \exp \left[ -\frac{1}{2} \int_k f_a(-k, 0) C_{ab}(k) f_b(k, 0) \right]$   
Initial spectrum of density & velocity fluctuations:  $C_{ab}^{-1}(k) = P_{ab}^0(k)$

# Path integrals for a classical stochastic field

- Generating functional for stochastic initial conditions

Matarrese, Pietroni,  
arXiv:0703563

$$Z[J, K; P^0] = \int Df_a(0) w[f_a(0), P^0] \int Df(h_f) Df Dc \exp[iS[f, c] + Jf + Kc]$$

- Generating functional of connected n-point functions

$$W[J, K; P^0] = -\text{Log} Z[J, K; P^0]$$

Full propagator describes  
how mode k evolves in a  
spectrum of perturbations.

$$\frac{d^2 W}{dJ_a(p, h) dK_b(p', h')} = id(p - p') G_{ab}^{(ret)}(p, h, h')$$

- The effective dynamics of the mode k is given by the effective action

$$G[f, c, P^0] = \int (Jf + Kc) - W[J, K; P^0]$$

- Coarse graining the effective dynamics

$$P_{k,ab}^0(q) = q(|q| - k) P_{ab}^0(q)$$

$$W_k[J, K] = W_k[J, K; P_k^0]$$

Floerchinger, Garny, Tetradis,  
Wiedemann, arXiv:1607.03453

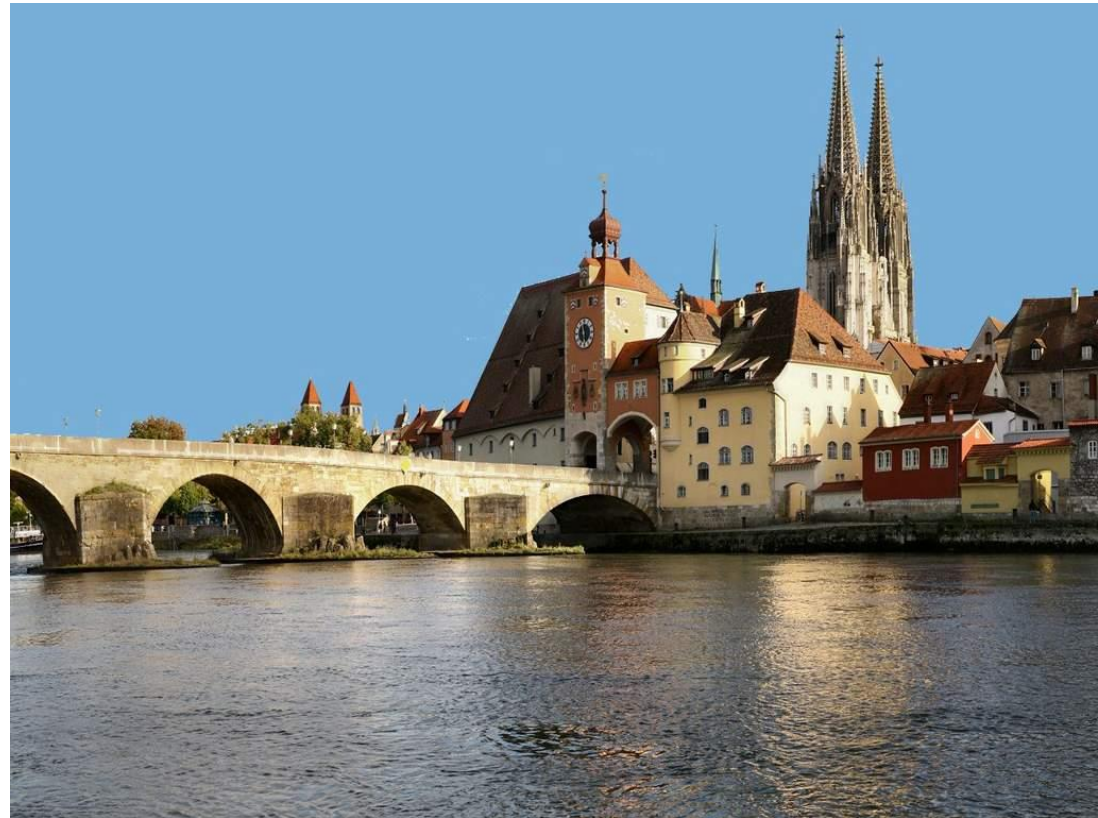
$$\partial_k G_k[f, c] = \frac{1}{2} \text{Tr} \left[ \left\{ G_k^{(2)}[f, c] - (P_k^0 - P^0) \right\}^{-1} (\partial_k P_k^0) \right]$$

**Renormalization  
group flow of the  
effective action**



# Effective dynamics vs. fundamental dynamics

- Task: model flowing river for wave modes  $k < 1/(10\text{m})$
- Simulation fails if you use the fundamental textbook viscosity of water. Why?
- The dominant dissipative process is momentum transfer from long wavelength ( $q < k$ ) caused by turbulent eddies. (eddy viscosity, **Boussinesq 1877**)
- Eddy viscosity is an example of an effective viscosity:
  - it depends on the spectrum of excitations in the medium (it is state dependent, not a fundamental property of matter)
  - it depends on the scale  $k$



What I showed on the previous slide is a framework for

- identifying effective viscosity and other dissipative processes
- calculating the scale-dependence of the effective viscosity via an RG flow equation.



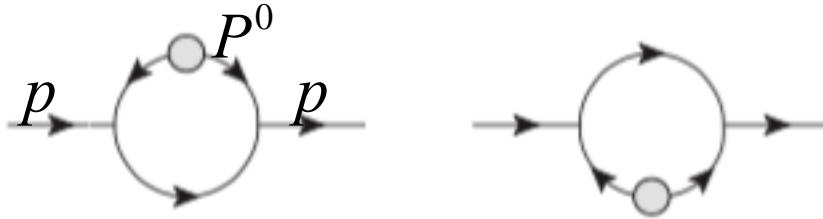
# Dissipation from mode-mode-coupling

- The inverse of the retarded propagator is

$$\frac{d^2 G_k}{dc_a(-q, h) df_b(q', h')} = -d(q - q') D_{k,ab}^{(ret)}(q, h, h')$$

$$= -d(q - q') (d_{ab} \partial_m + W_{ab}(q, h)) d(h - h') - S_{k,ab}(q, h; q', h')$$

- Perturbatively, the **'self-energy'** can be calculated



Describes perturbatively how mode  $p$  evolves in spectrum  $P^0$ .

- Eom of viscous fluid dynamics are obtained by

$$W_{ab}(k, h) \Rightarrow \hat{W}_{ab}(k, h) = W_{ab}(k, h) + \begin{pmatrix} 0 & 0 \\ g_s(h)q^2 & g_n(h)q^2 \end{pmatrix}$$

$$g_s \circ \frac{dp/dr}{H^2}$$

$$g_n \circ \frac{\frac{4}{3}h + \chi}{(\bar{r} + \bar{p})Ha}$$

- Matching viscous coefficients to perturbative 1-loop **'self-energy'**

**Effective viscosities**

$$g_n = \frac{78}{35} e^{2h} S_{v,k}^2$$

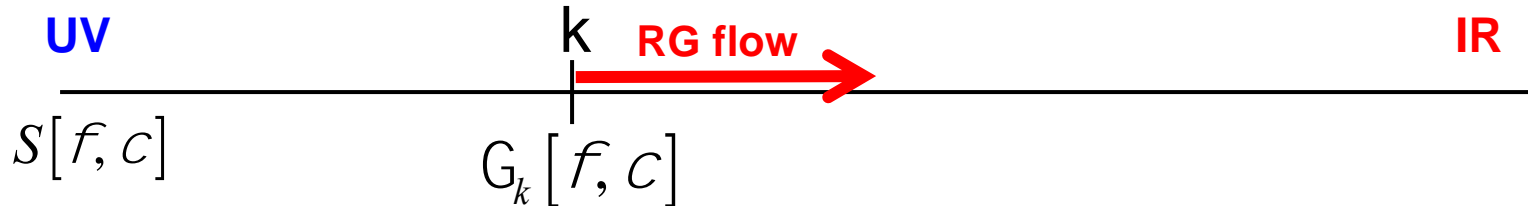
$$g_s = \frac{31}{70} e^{2h} S_{v,k}^2$$

**depend on spectrum of initial fluctuations**

$$S_{v,k}^2 = \frac{4\rho}{3} \int_0^{\infty} dq P^0(q)$$

# Can we do better?

- Consider scale-dependence of the effective dynamics



- We look for a solution to

$$\partial_k G_k[f, c] = \frac{1}{2} \text{Tr} \left[ \left\{ G_k^{(2)}[f, c] - (P_k^0 - P^0) \right\}^{-1} (\partial_k P_k^0) \right]$$

- If we assume that  $G_k[f, c]$  yields viscous fluid dynamic eom at scale  $k$ , then the **functional RG equation** reduces to a set of coupled ordinary differential equations for

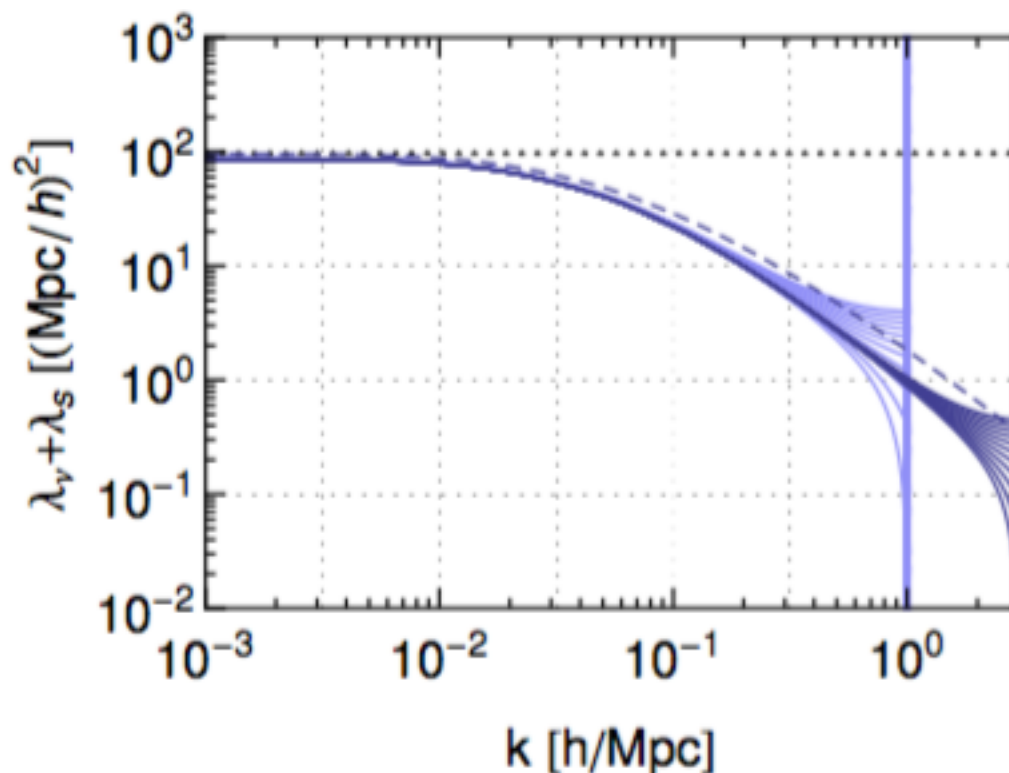
(for quite a few technical details needed at this step, see [arXiv:1607.03453](https://arxiv.org/abs/1607.03453) )

$$g_{s,k}(h) = I_s(k) e^{k(k)h}$$

$$g_{n,k}(h) = I_n(k) e^{k(k)h}$$

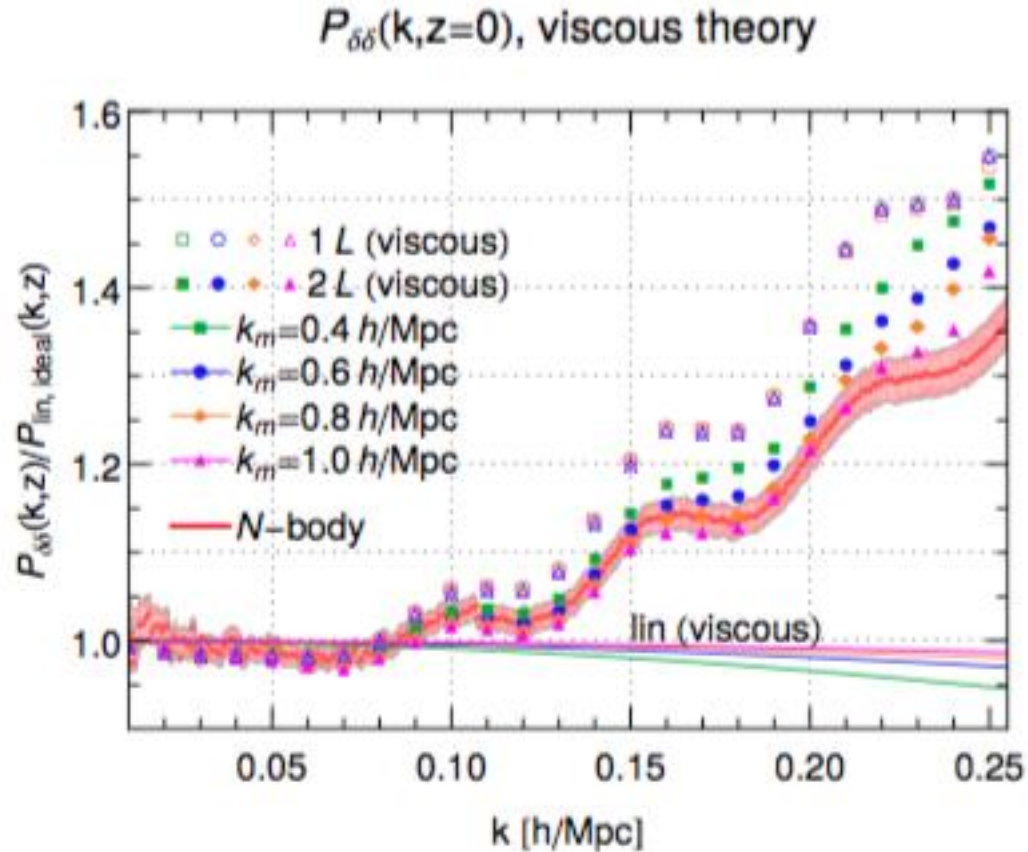
# Numerical solution of RG flow

- RG flow shows an IR attractor behavior: almost irrespective of the value taken by fundamental viscosity at the UV scale, one obtains almost the same phenomenologically relevant effective viscosity at intermediate scale.



# Comparison to data

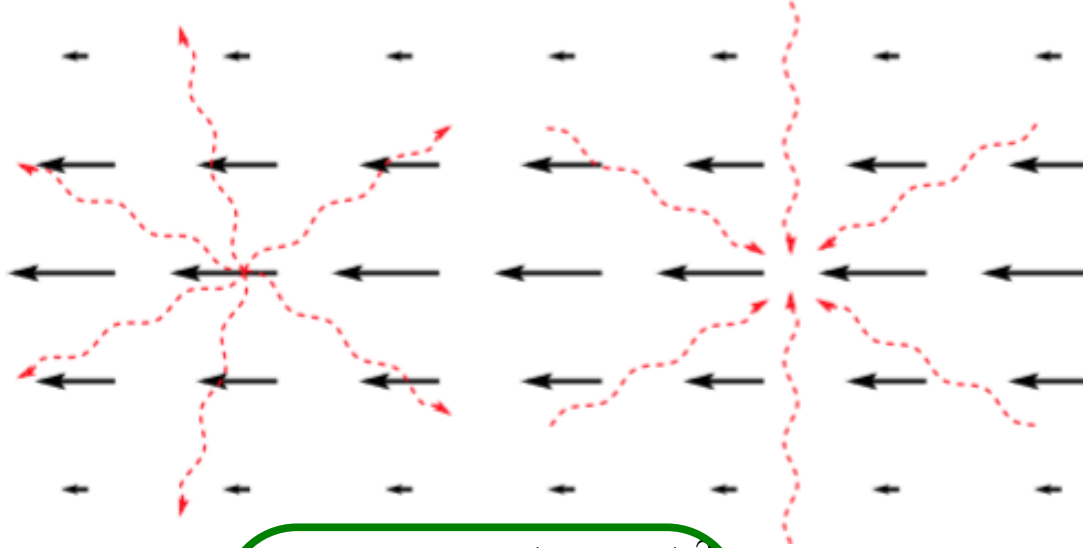
- The dynamics derived from this RG-improved effective action compares well to data:



Floerchinger, Garny, Tetradis, Wiedemann, arXiv:1607.03453

# Back to UIRHICs: are there similar phenomena?

- Thermal sound modes are an additional (calculable!) source of viscosity  
Kovtun, Moore, Romatschke, arXiv:1104.1586, Phys.Rev. D84 (2011) 025006

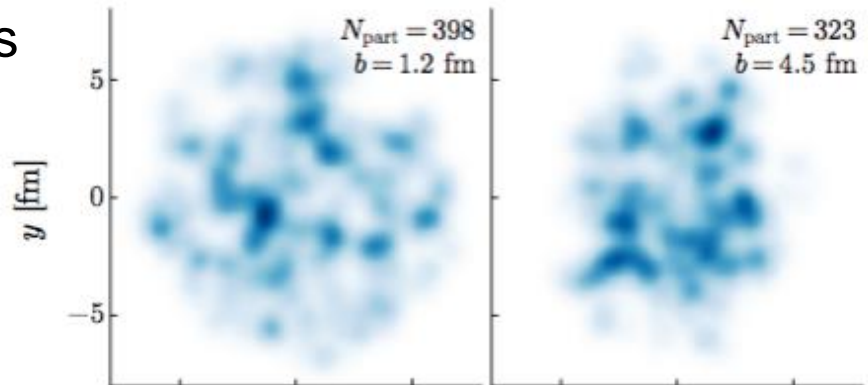


**“Stickiness  
of sound”**

$$h = h_{cl} + \frac{17 p_{\max} g_h T (e + p)^2}{120 p^2 h_{cl}^2}$$

Extra contribution from sound modes is parametrically subdominant.

- But what about viscous corrections from non-thermal fluctuations?

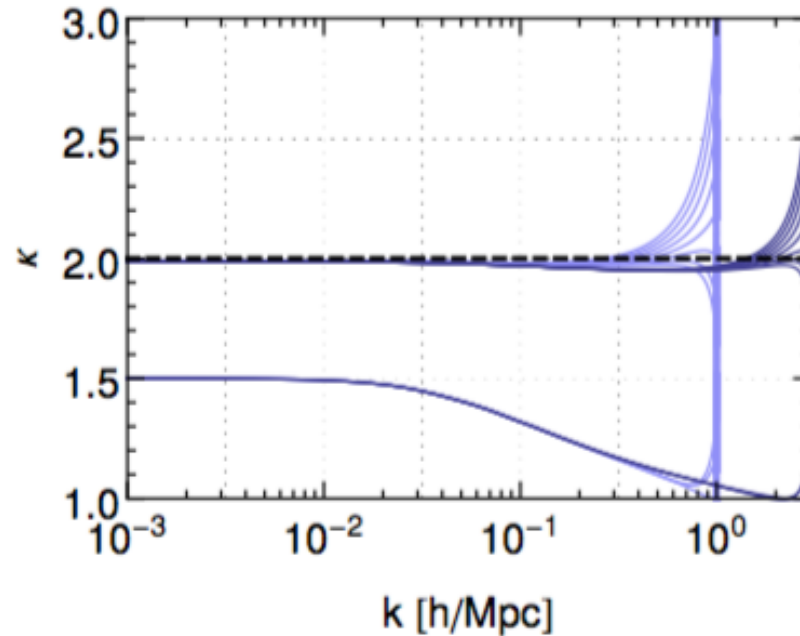


Happy Birthday!



# Numerical solution of RG flow

- Time dependence is close to the perturbative result



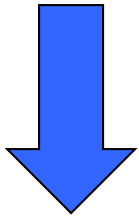
# How are fluid fluctuations described?

## In Cosmology:

**Perturbatively**, on top of homogeneous background fields.

$$de/e \sim 10^{-5}$$

(at photon decoupling)



Gravitational  
collapse

$$de/e \sim 1$$

(at time of structure formation)

Perturbative methods and N-body simulations are applied.

Early .. => .. Late

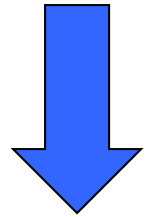
## In heavy ion physics:

**Non-perturbatively**, via numerical codes.

$$de/e \sim 1$$

(at hydrodynamization)

Dissipation in  
near-ideal fluid



$$de/e \sim 10^{-1}$$

(at freeze-out)

Can perturbative methods apply?  
P.T.O.

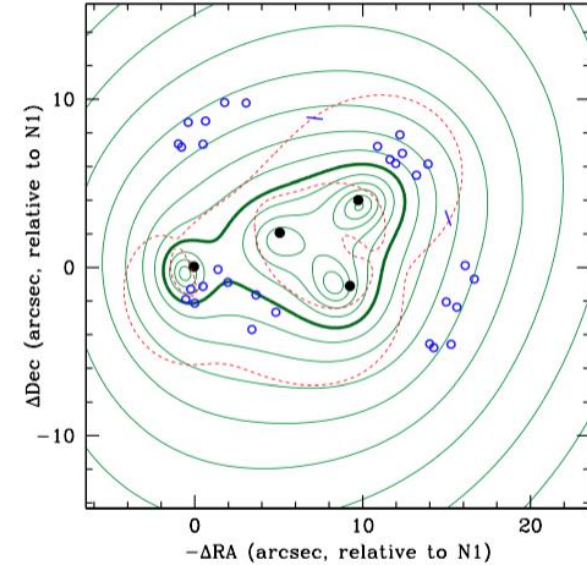
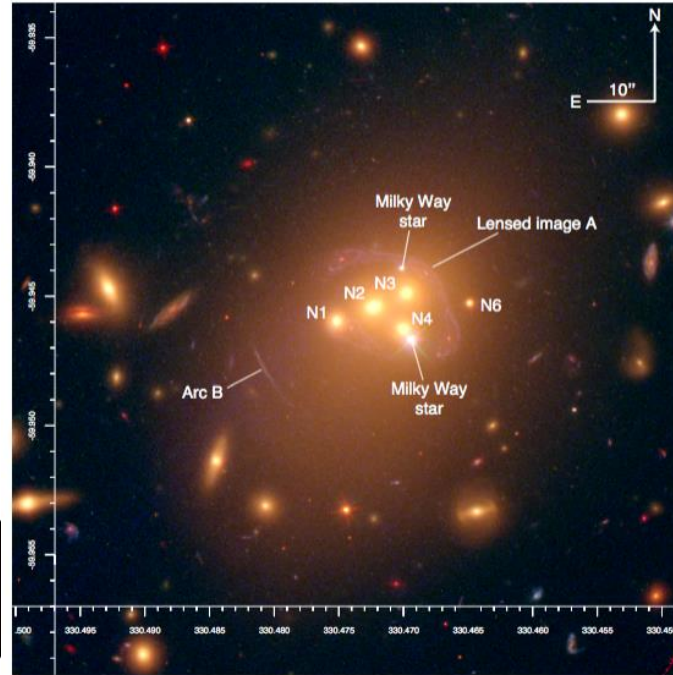
# Dark Matter Properties

Is Dark Matter self-interacting?

Analyzing offset between DM and stars falling into cluster

Kahlhoefer et al  
MNRAS 452, 1 (2015)  
suggest

$$\frac{S_{el}}{m} \gg 3 \frac{cm^2}{g} \gg 0.5 \frac{b}{GeV}$$



Galaxy cluster Abell 3827

[Massey et al., MNRAS 449, 3393 (2015)]

For non-relativistic particles of mass  $m$  and mean velocity  $\bar{v}$ , the **shear viscosity**

$$h = \frac{m \bar{v}}{3 S_{el}}$$

This would set **lower bound for shear viscosity of dark matter.**

Consequences for dark matter fluid dynamics!

But if viscosity too large, then one should not applying fluid dynamics

# Cosmological Structure Formation

- Described by energy conservation and Navier-Stokes, for  $d^\circ de/e \ll 1$

$$\delta\dot{\varepsilon} + 3H\delta\varepsilon + \bar{\varepsilon}\theta = 0, \quad H = \frac{\dot{a}}{a}, \quad \theta = \vec{\nabla} \cdot \vec{v}$$

$$\bar{\varepsilon} \left[ \dot{\theta} + H\theta - q^2\psi \right] + \frac{(\xi + \frac{4}{3}\eta)}{a} q^2\theta = 0,$$

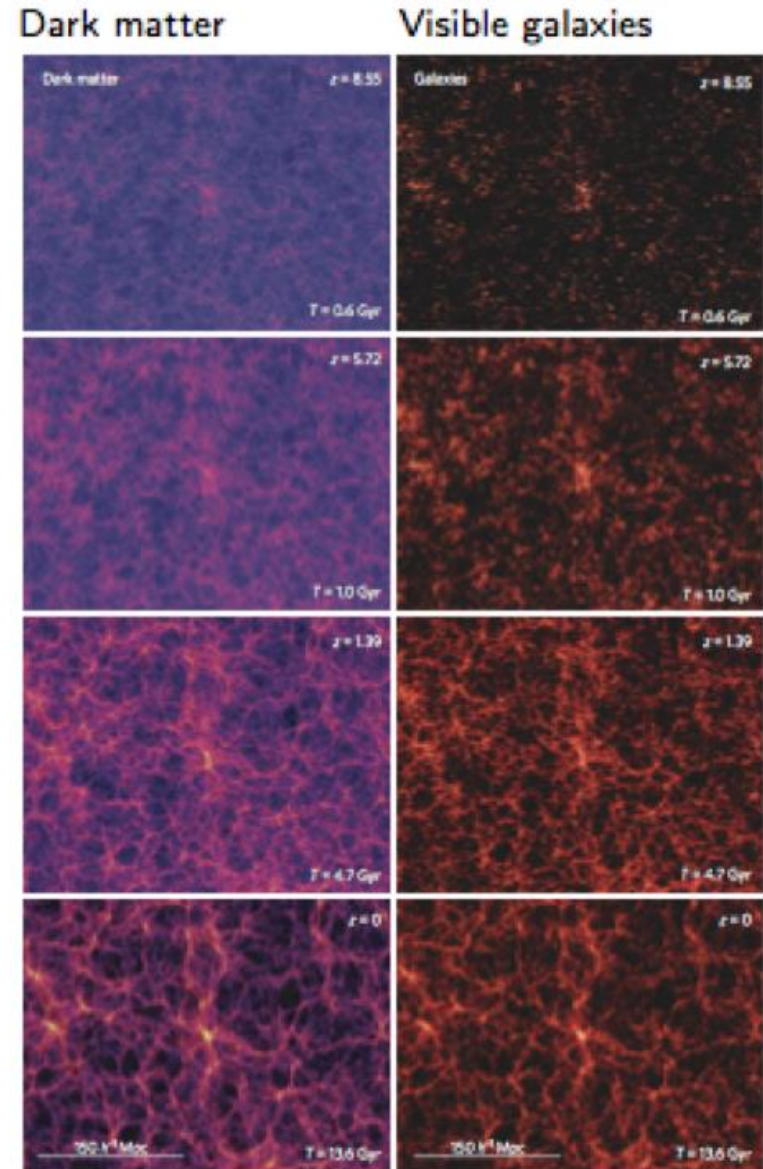
- Closed by Poisson equation for Newtonian potential  $\psi$

$$-q^2\psi = 4pG_N a^2 de$$

- Viscosity slows down grav. collapse but does not wash out structure

Blas, Floerchinger, Garny, Tetradis, UAW,  
arXiv:1507.06665, **JCAP 1511 (2015) 049**

$$\ddot{\delta} + \left[ \frac{\dot{a}}{a} + \frac{\xi + \frac{4}{3}\eta}{a\bar{\varepsilon}} q^2 \right] \dot{\delta} - 4\pi G_N \bar{\varepsilon} \delta = 0$$



[Springel, Frenk & White,  
Nature 440, 1137 (2006)]

# Growth of structure depends on viscosity

- Consider subhorizon perturbations,  $k \gg H$ , with  $d_k \sim q_k / H \in O(1)$   
Corrections to perturbative evolution are  $O(H^2 / k^2)$
- Assume that viscosity and pressure do not disrupt scale hierarchy

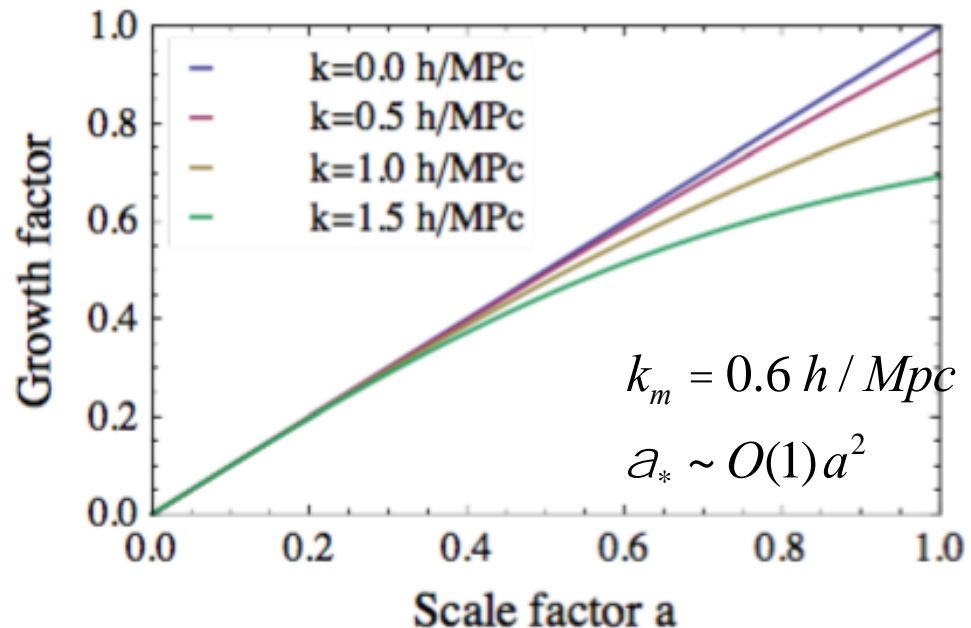
$$c_s^2 = \frac{\rho_p}{\rho_e} = a_s \frac{k^2}{k_m^2} \quad nH = \frac{hH}{(\bar{e}_m + p)a} = \frac{3}{4} a_n \frac{k^2}{k_m^2} \quad \frac{1}{k_m} \text{ shortest length scale of (effective) viscous description}$$

which leads to

$$d_k'' + \left(1 + \frac{H'}{H} + a_n \frac{k^2}{k_m^2}\right) d_k' - \left(\frac{3}{2} W_m - a_s \frac{k^2}{k_m^2}\right) d_k = 0, \quad d' = \frac{dd}{d \ln a}$$

- Viscosity affects sufficiently small length scales only

Blas, Floerchinger, Garny,  
Tetradis, UAW,  
JCAP 1511 (2015) 049





# Effective and fundamental viscosity

Viscosity parametrizes momentum transport from fluid to smaller scales:

## 1. Fundamental origin

- Transport via particles/radiation
- calculable from linear response (Green-Kubo formalism)

## 1. Effective origin

- coarse-graining: limiting fluid description e.g. to wave lengths  $> 1/k_m$ , where evolution is 'laminar'
- Effective viscosity describes then momentum transport via non-linear mode-mode coupling to 'fluid' scales  $k > k_m$
- Examples include: eddy viscosity, anomalous plasma viscosity  
(in heavy ion physics: romatschke 2008, Asakawa, Bass, Mueller 2006)
- In the following, we use effective viscosity by matching to perturbative results  
(Blas, Floerchinger, Garny, Tetradis, Wiedemann 2015)





# Effective viscosity and sound velocity from perturbative matching

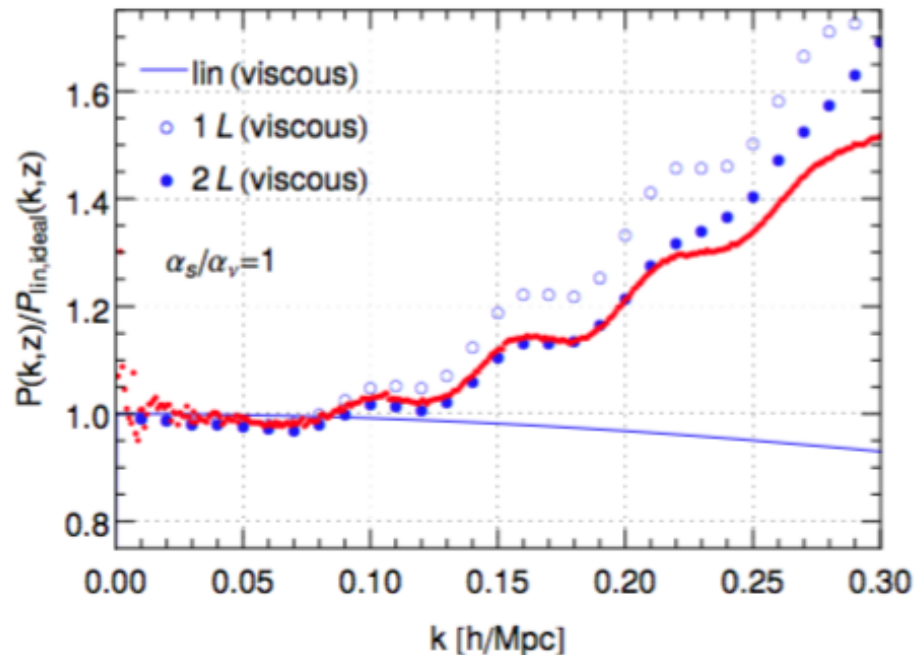
[Blas, Floerchinger, Garny, Tetradis & Wiedemann, 1507.06665]

- Consider theory with a coarse-graining scale  $k_m$
- Modes with wavenumber  $k > k_m$  modify effective propagator for long-wavelength modes by loop corrections (statistical fluctuations)
- Leading correction for growing mode can be matched to good approximation to effective viscosity and sound velocity terms.
- Perturbative calculation with no free parameter except matching scale.
- Similar to one-particle irreducible scheme.
- Some similarities to “EFT of LSS” but no parameter fitting (except  $k_m$ ) and effective viscosity and sound velocity fully considered in equations of motion

# Effective viscosity and sound velocity from perturbative matching

Dark matter density power spectrum in the BAO range

$$P_{\delta\delta}(k, z=0), k_m = 0.6h/\text{Mpc}$$



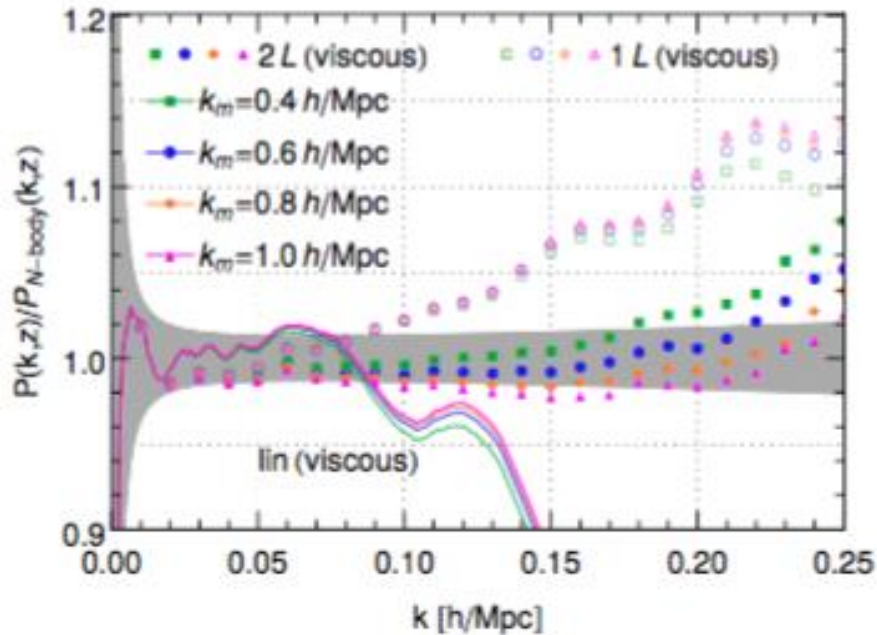
[Blas, Floerchinger, Garny, Tetradis & Wiedemann (2015)]

- 2 Loop perturbative calculation with **effective viscosity and sound velocity**, determined perturbatively
- agrees with  $N$ -body simulations up to  $k = 0.2 h/\text{Mpc}$

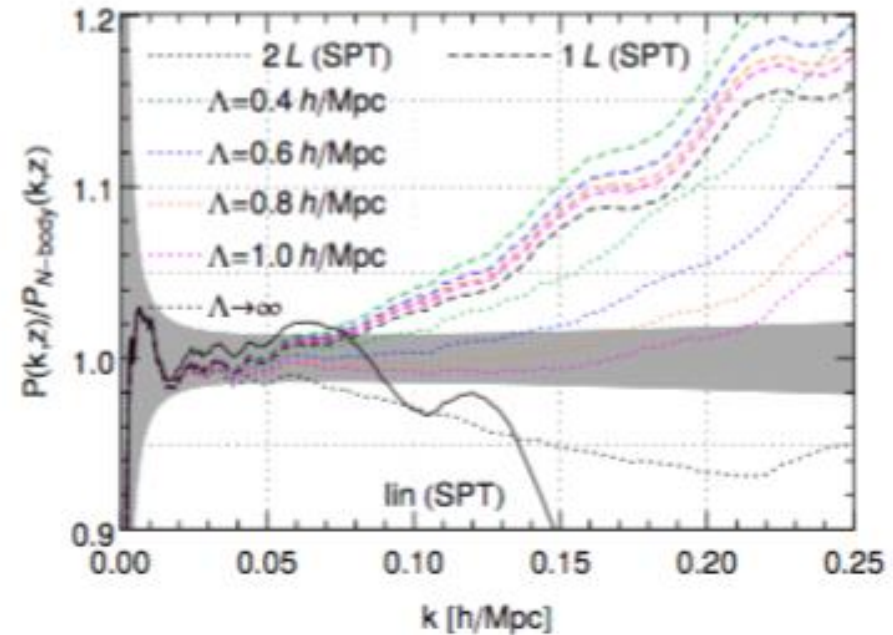
[related: Effective field theory of LSS, Baumann, Nicolis, Senatore & Zaldarriaga (2012), Carrasco, Hertzberg & Senatore (2012), ... ]

# Effective viscous vs standard cosmological perturbation theory

$P_{\delta\delta}(k, z=0)/P_{N\text{-body}}$ , viscous theory



$P_{\delta\delta}(k, z=0)/P_{N\text{-body}}$ , SPT with cutoff



- Theory with effective viscosities shows better convergence properties
- No fitting to non-perturbative information (N-body simulations) needed
- No free parameter except matching scale  $k_m$
- Weaker sensitivity to matching scale