For Ulrich

I got to know Ulrich while hiking at a conference in Banff Canada more than 20 yrs ago.

One year later, he offered me my first postdoc position, and I got to think for the first time in my life about experimental data.

If scientific genealogy is meaningful, then Ulrich is my scientific father.
I owe Ulrich more than anyone else in physics.

Sons are strange: they want to differ from their parents
but even where they succeed
the parents recognize themself in what they do.

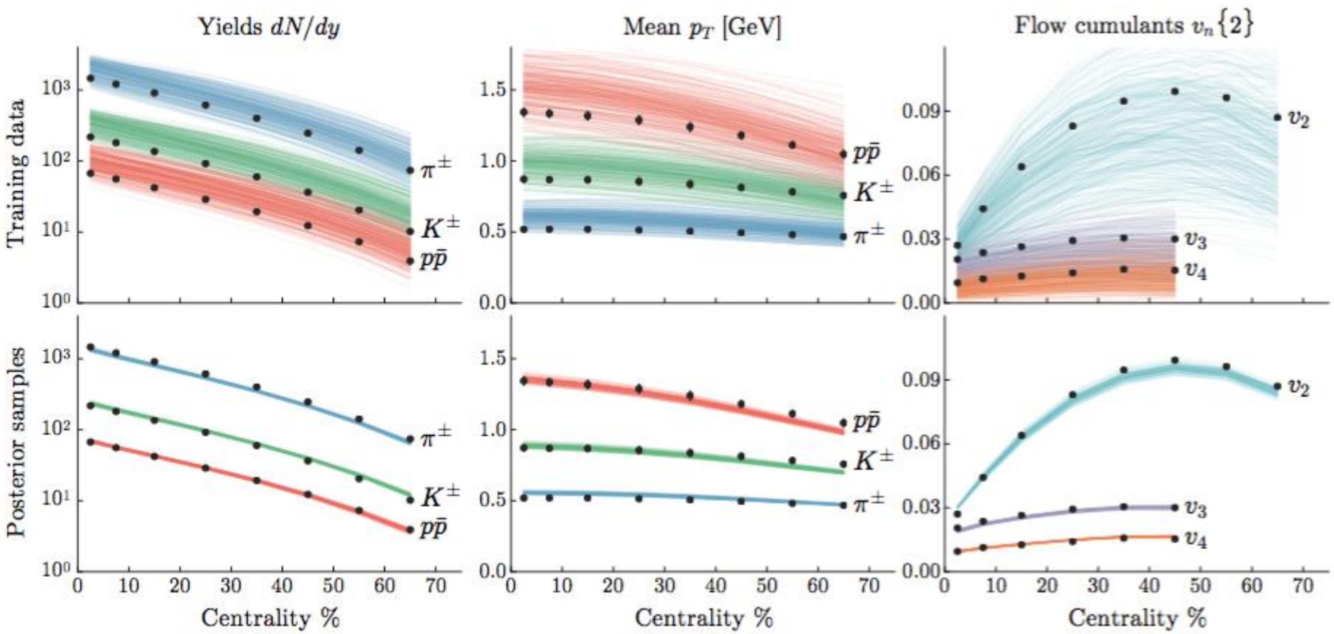
That's why I have chosen the following scientific story for Ulrich's birthday.

Heavy Ion Phenomenology

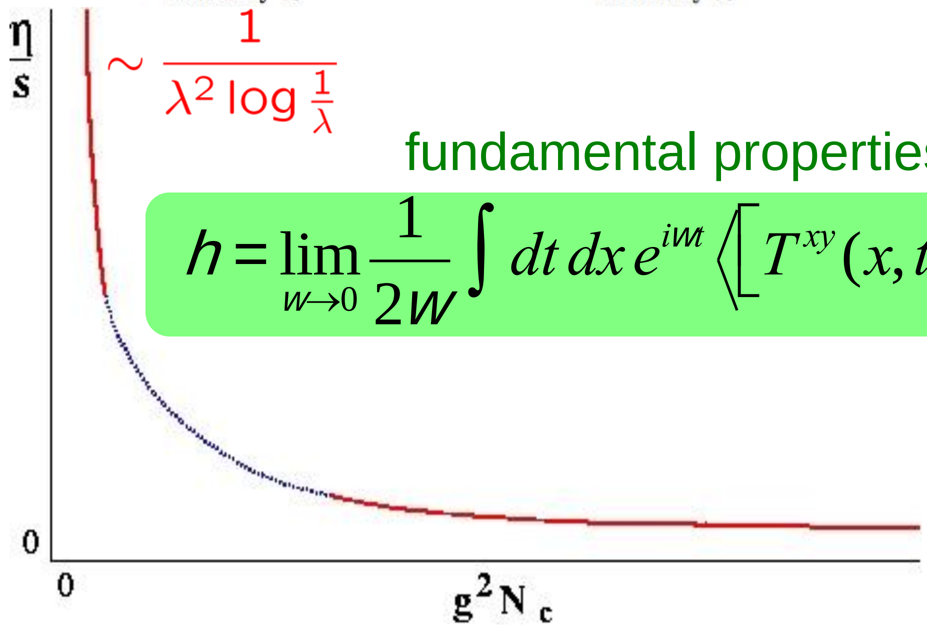
Ulrich is where the action is.

What we fit …

Bernard, Moreland, Bass, Liu, Heinz, arXiv:1605.03954



… and what we are after …



fundamental properties of hot QCD

$$h = \lim_{w \to 0} \frac{1}{2w} \int dt\, dx\, e^{iwt} \left\langle \left[T^{xy}(x,t), T^{xy}(0,0) \right] \right\rangle_{eq}$$

$$\frac{h}{s} > \frac{1}{4p}$$

On the stock market of paradigms

The Bulls:
- Hydro works!

The Bears:
- But how can hydro work?

This is a story told by Ursus, the bear, about
- how hydro works for *non-interacting* (dark matter) particles in cosmology
- how *effective viscosity becomes calculable* in these systems

From Boltzmann Transport to Fluid Dynamics

- Boltzmann equation with collision term $p^{\mu}\partial_{\mu} f(x,t,p) = C$
- Momentum moments of phase space distribution:

$$\int_p p^{\mu}\partial_{\mu} f(x,t,p) \equiv \partial_{\mu} N^{\mu} = 0 = \int_p C$$

$$\int_p p^{\mu} p^{\alpha}\partial_{\mu} f(x,t,p) \equiv \partial_{\mu} T^{\mu\alpha} = 0 = \int_p p^{\alpha} C$$

$$\int_p p^{\mu} p^{\alpha} p^{\beta}\partial_{\mu} f(x,t,p) = \int_p p^{\alpha} p^{\beta} C$$

a reminder

- Fluid dynamics applies if
  - higher moments can be truncated
  - perturbations around average stay small

Gradient expansion of perturbations leads then to (dissipative) fluid dynamics

$$f = f_0 (1 + \delta f) \qquad T^{\mu\nu} = T_0^{\mu\nu} + \Pi^{\mu\nu}$$

$$\delta f(x,t,p) = \varepsilon(x,t) + \varepsilon_{\lambda}(x,t) p^{\lambda} + \varepsilon_{\lambda\nu}(x,t) p^{\lambda} p^{\nu}$$

$$\left\langle T^{\mu\nu} - T_0^{\mu\nu} \right\rangle = \left\langle \Pi^{\mu\nu} \right\rangle = \varepsilon_{\alpha\beta}(x) \int_p p^{<\mu} p^{\nu>} p^{\alpha} p^{\beta} f_0$$

from which the Israel-Stewart equations of motion can be derived.

From Boltzmann transport to the cosmological fluid

Vlasov-Boltzmann equation: free-streaming in gravitational field

$$\frac{\partial f}{\partial \eta} + \frac{\vec{p}.}{am}\vec{\nabla} f - am\vec{\nabla}\Phi.\frac{\partial f}{\partial \vec{p}} = 0$$

$$\vec{\nabla}\Phi = \frac{G_N m}{a}\int_{x',p'} \frac{\vec{x}-\vec{x}'}{\left|\vec{x}-\vec{x}'\right|^3} f\left(x',p',\eta\right)$$

- Dark matter is non-interacting but of thermal origin:
  => expand around average

$$f\left(\vec{x},\vec{p},\eta\right) = f_0\left(p,\eta\right)\left[1+\delta_f\left(\vec{x},p,\hat{p},\eta\right)\right]$$

$$\delta_f\left(\vec{k},p,\hat{p},\eta\right) = \sum_n i^n (2n+1)\delta_f^{[n]}\left(k,p,\eta\right)P_n\left(\hat{k}.\hat{p}\right)$$

- Momentum moments:

$$\delta_\Sigma \equiv m a^{-3}\int 4\pi p^2\, dp\, f_0\left(p\right)\left(\mathrm{CHF}\right)$$

Σ	density ρ	pressure $p_{ressure}$	Velocity gradient $\left(\bar{p}+\bar{\rho}\right)\theta$	Shear (scalar component) $\left(\bar{p}+\bar{\rho}\right)\sigma$
CHF	$\delta_f^{[0]}$	$\frac{1}{3}v_p^2\,\delta_f^{[0]}$	$k v_p\,\delta_f^{[1]}$	$-\frac{2}{3}v_p^2\,\delta_f^{[2]}$

- Hierarchy: $\delta_f^{[n]} \sim \left(k v_p H^{-1}\right)^{n-2}\delta_f^{[2]}$ where $v_p = p/\left(am\right) = \dot{x}$

- Truncation of Vlasov-Boltzmann hierarchy applies if

Baumann, Nicolis, Senatore, Zaldarriaga, arXiv:1004.2488

$$k\cdot \underbrace{v_p H^{-1}}_{free\,stream\,dist} << 1$$

Hydro works since lifetime of Universe is too short to fall out of equilibrium.

Calculating the growth of cosmological structure

- Truncating Vlasov-Boltzmann at lowest moments yields equations of motion

$$f_a(h,k) = \left(dr_k(h), -\frac{q_k}{H} \right)$$

$$\partial_h f_a(k) + W_{ab}(k,h) f_b(k) - \int_{p,q} g_{abc}(k,p,q,h) f_b(p) f_c(q) = 0 = (eom)_a$$

- For Einstein deSitter Universe

$$W_{ab}(k,h) = \begin{pmatrix} 0 & -1 \\ -3/2 & 1/2 \end{pmatrix}$$

Crocce, Scoccimarro, arXiv:0509419

$$g^{(ret)}_{ab}(h-h') = \frac{e^{h-h'}}{5} \begin{pmatrix} 3 & 2 \\ 3 & 2 \end{pmatrix} - \frac{e^{-3(h-h')/2}}{5} \begin{pmatrix} -2 & 2 \\ 3 & -3 \end{pmatrix}$$

Density contrast **grows exponentially** on all scales k, gravitational collapse forms structures from small perturbations.

- Fun for theorists: action principle

$$S[f,c] = \grave{O} dh \, \grave{O}_k \, c_a(-k,h) \, (eom)_a(k,h)$$

  - Eom follows from variation: $dS[f,c] / dc_a(k,h) = 0$
  - Stochastic initial conditions $w[f_a(0)] = \exp\left[-\tfrac{1}{2} \int_k f_a(-k,0) C_{ab}(k) f_b(k,0) \right]$

  Initial spectrum of density & velocity fluctuations: $C^{-1}_{ab}(k) = P^0_{ab}(k)$

Path integrals for a classical stochastic field

- Generating functional for stochastic initial conditions

Matarrese, Pietroni, arXiv:0703563

$$Z\left[J,K;P^0\right]=\int Df_a(0)\,w\left[f_a(0),P^0\right]\int Df(h_f)\,Df\,Dc\,\exp\left[iS[f,c]+Jf+Kc\right]$$

- Generating functional of connected n-point functions

$$W\left[J,K;P^0\right]=-Log Z\left[J,K;P^0\right]$$

$$\frac{d^2W}{dJ_a(p,h)\,dK_b(p',h')}=id\left(p-p'\right)G_{ab}^{(ret)}(p,h,h')$$

Full propagator describes how mode k evolves in a spectrum of perturbations.

- The effective dynamics of the mode k is given by the effective action

$$G\left[f,c,P^0\right]=\int\left(Jf+Kc\right)-W\left[J,K;P^0\right]$$

- Coarse graining the effective dynamics

$$P^0_{k,ab}(q)=q\left(|q|-k\right)P^0_{ab}(q)$$

$$W_k\left[J,K\right]=W_k\left[J,K;P_k^0\right]$$

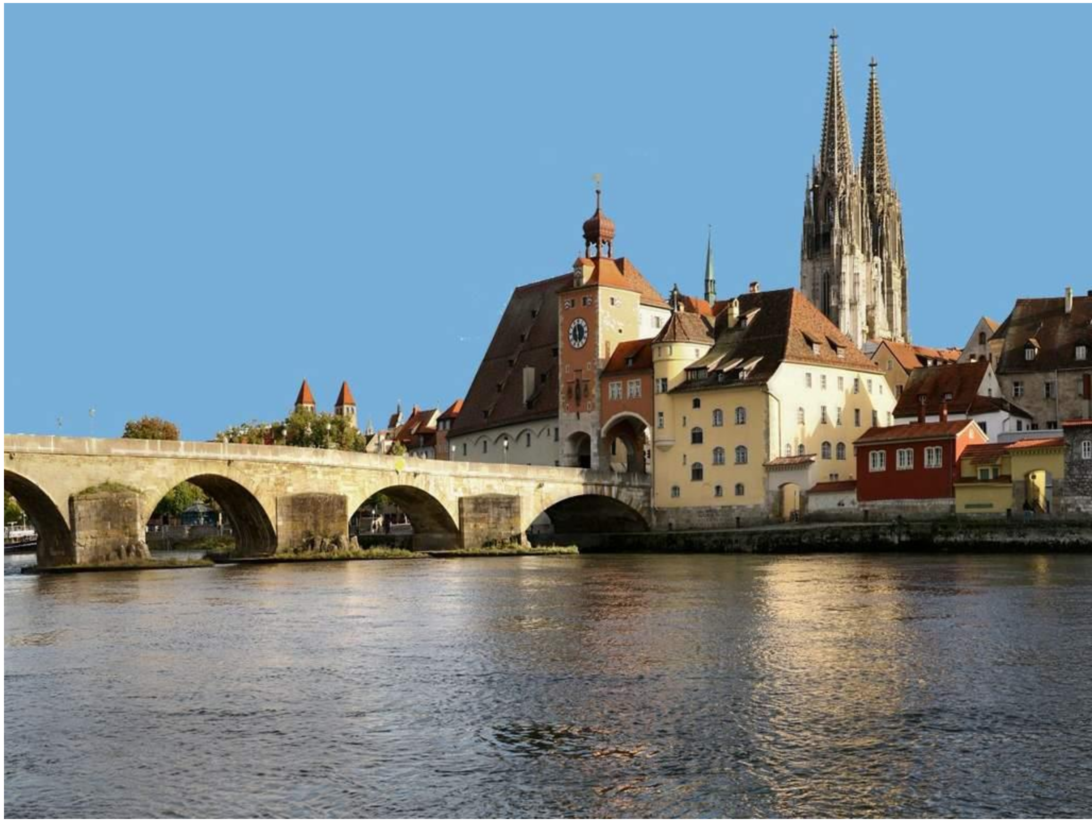
Floerchinger, Garny, Tetradis, Wiedemann, arXiv:1607.03453

$$\boxed{\partial_k G_k\left[f,c\right]=\tfrac{1}{2}Tr\left[\left\{G_k^{(2)}\left[f,c\right]-\left(P_k^0-P^0\right)\right\}^{-1}\left(\partial_k P_k^0\right)\right]}$$

Renormalization group flow of the effective action

Effective dynamics vs. fundamental dynamics

- Task: model flowing river for wave modes k< 1/(10m)
- Simulation fails if you use the fundamental textbook viscosity of water. Why?
- The dominant dissipative process is momentum transfer from long wavelength (q < k) caused by turbulent eddies.
  (eddy viscosity, **Boussinesq 1877**)
- Eddy viscosity is an example of an *effective* viscosity:
  - it depends on the spectrum of exitations in the medium
    (it is state dependent, not a fundamental property of matter)
  - it depends on the scale k



What I showed on the previous slide is a framework for

- identifying effective viscosity and other dissipative processes
- calculating the scale-dependence of the effective viscosity via an RG flow equation.

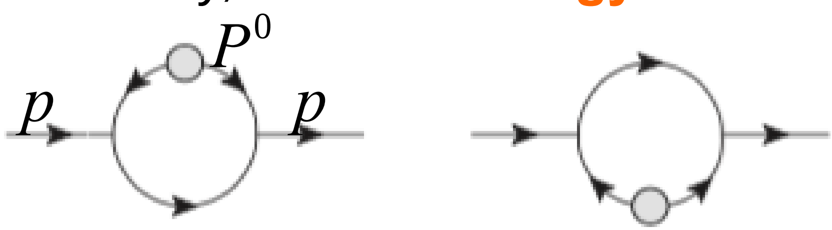
Dissipation from mode-mode-coupling

- The inverse of the retarded propagator is

$$\frac{\delta^2\Gamma_k}{\delta\chi_a(-q,\eta)\,\delta\phi_b(q',\eta')} = -\delta(q-q')\,D^{(ret)}_{k,ab}(q,\eta,\eta')$$

$$= -\delta(q-q')\left(\delta_{ab}\partial_\mu + \Omega_{ab}(q,\eta)\right)\delta(\eta-\eta') - \boxed{\Sigma_{k,ab}(q,\eta;q',\eta')}$$

- Perturbatively, the **'self-energy'** can be calculated



Describes perturbatively how mode p evolves in spectrum P^0.

- Eom of viscous fluid dynamics are obtained by

$$\Omega_{ab}(k,\eta) \Rightarrow \hat{\Omega}_{ab}(k,\eta) = \Omega_{ab}(k,\eta) + \boxed{\begin{pmatrix} 0 & 0 \\ \gamma_s(\eta)q^2 & \gamma_\eta(\eta)q^2 \end{pmatrix}}$$

$$\gamma_s \equiv \frac{dp/d\rho}{H^2}$$

$$\gamma_\eta \equiv \frac{\frac{4}{3}\eta + \zeta}{(\bar{\rho} + \bar{p})Ha}$$

- Matching viscous coefficients to perturbative 1-loop **'self-energy'**

Effective viscosities

$$\boxed{\begin{aligned} \gamma_\eta &= \frac{78}{35} e^{2\eta} \sigma^2_{v,k} \\ \gamma_s &= \frac{31}{70} e^{2\eta} \sigma^2_{v,k} \end{aligned}}$$

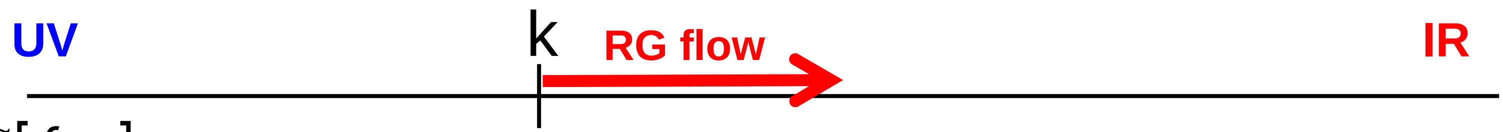
depend on spectrum of initial fluctations

$$\boxed{\sigma^2_{v,k} = \frac{4\pi}{3}\int_k^\infty dq\, P^0(q)}$$

Floerchinger, Garny, Tetradis, Wiedemann, arXiv:1607.03453

Can we do better?

- Consider scale-dependence of the effective dynamics



$S[f, c]$ $\qquad$ $G_k[f, c]$

- We look for a solution to

$$\boxed{\partial_k G_k[f, c] = \tfrac{1}{2} Tr\left[\left\{ G_k^{(2)}[f, c] - \left(P_k^0 - P^0 \right) \right\}^{-1} \left(\partial_k P_k^0 \right) \right]}$$

- If we assume that $G_k[f, c]$ yields viscous fluid dynamic eom at scale k, then the functional RG equation reduces to a set of coupled ordinary differential equations for

(for quite a few technical details needed at this step, see **arXiv:1607.03453**)

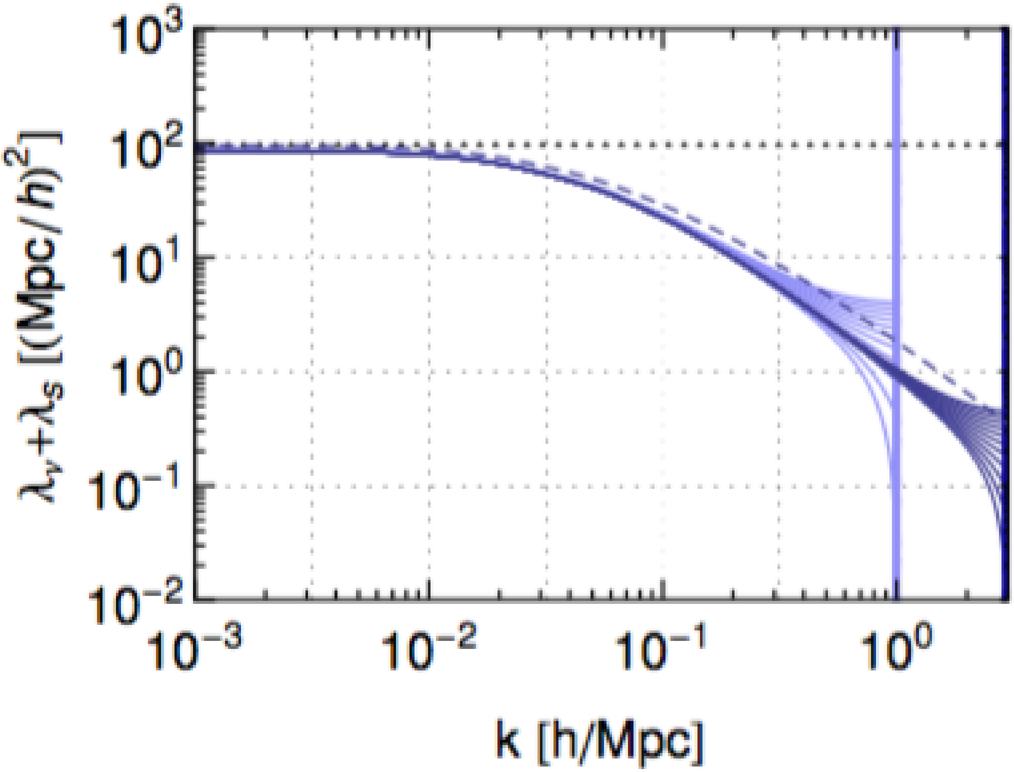
Floerchinger, Garny, Tetradis, Wiedemann, arXiv:1607.03453

$$g_{s,k}(h) = l_s(k) e^{k(k)h}$$

$$g_{n,k}(h) = l_n(k) e^{k(k)h}$$

Numerical solution of RG flow

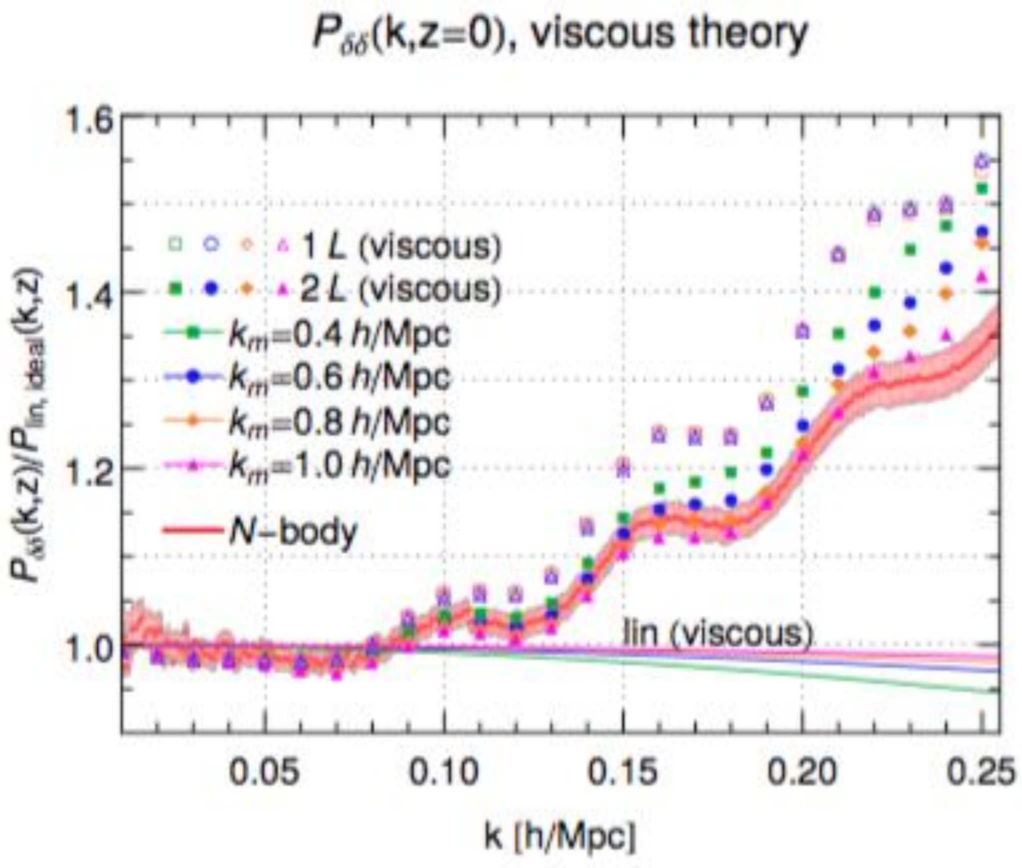
- RG flow shows an IR attractor behavior:
  almost irrespective of the value taken by fundamental viscosity at the UV scale, one obtains almost the same phenomenologically relevant effective viscosity at intermediate scale.



Floerchinger, Garny, Tetradis, Wiedemann, arXiv:1607.03453

Comparison to data

- The dynamics derived from this RG-improved effective action compares well to data:

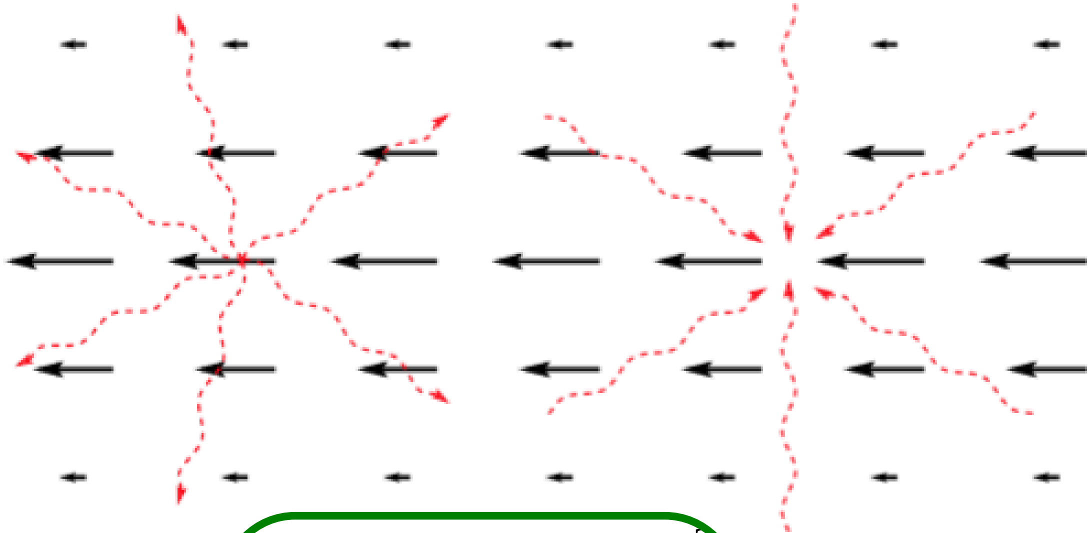


Floerchinger, Garny, Tetradis, Wiedemann, arXiv:1607.03453

Back to UlRHICs: are there similar phenomena?

- Thermal sound modes are an additional (calculable!) source of viscosity

Kovtun, Moore, Romatschke, arXiv:1104.1586, Phys.Rev. D84 (2011) 025006

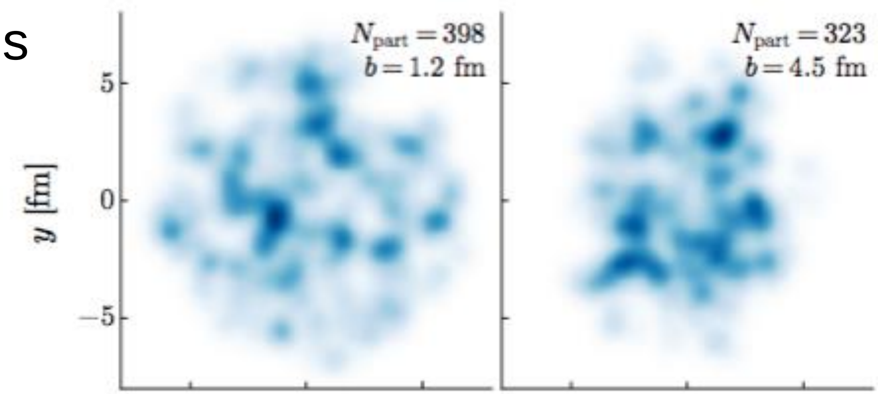


"Stickiness of sound"

$$h = h_{cl} + \boxed{\frac{17\, p_{\max} g_h T (e+p)^2}{120\, p^2 h_{cl}^2}}$$

Extra contribution from sound modes is parametrically subdominant.

- But what about viscous corrections from non-thermal fluctuations?

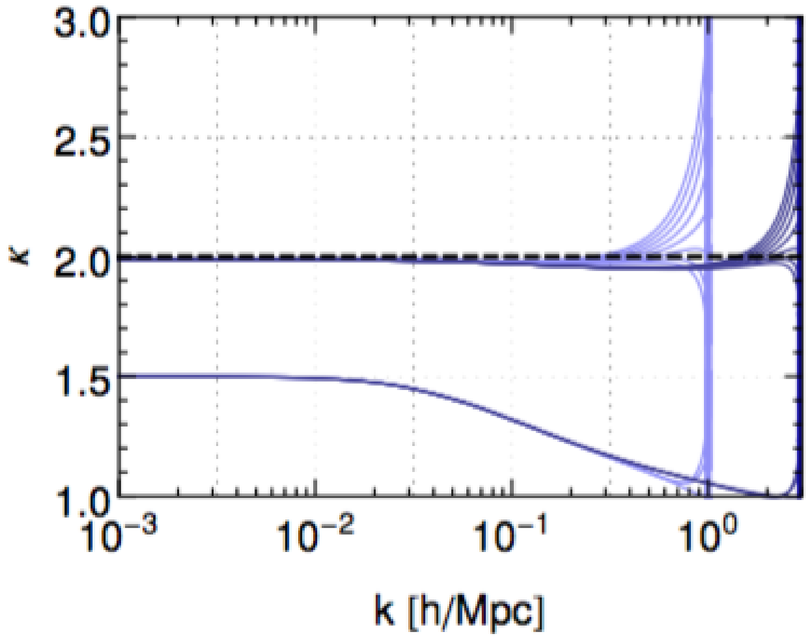


Happy Birthday!

Back-up

Numerical solution of RG flow

- Time dependence is close to the perturbative result



How are fluid fluctuations described?

In Cosmology:

Perturbatively, on top of homogeneous background fields.

$$\delta\epsilon/\epsilon \sim 10^{-5}$$

(at photon decoupling)

Gravitational collapse

$$\delta\epsilon/\epsilon \sim 1$$

(at time of structure formation)

Perturbative methods and N-body simulations are applied.

Early .. => .. Late

In heavy ion physics:

Non-perturbatively, via numerical codes.

$$\delta\epsilon/\epsilon \sim 1$$

(at hydrodynamization)

Dissipation in near-ideal fluid

$$\delta\epsilon/\epsilon \sim 10^{-1}$$

(at freeze-out)

Can perturbative methods apply?
P.T.O.

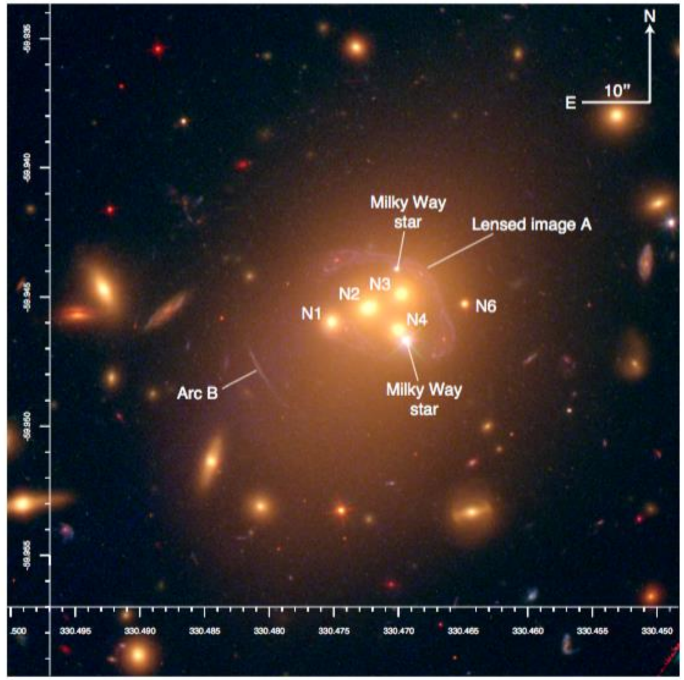
Dark Matter Properties

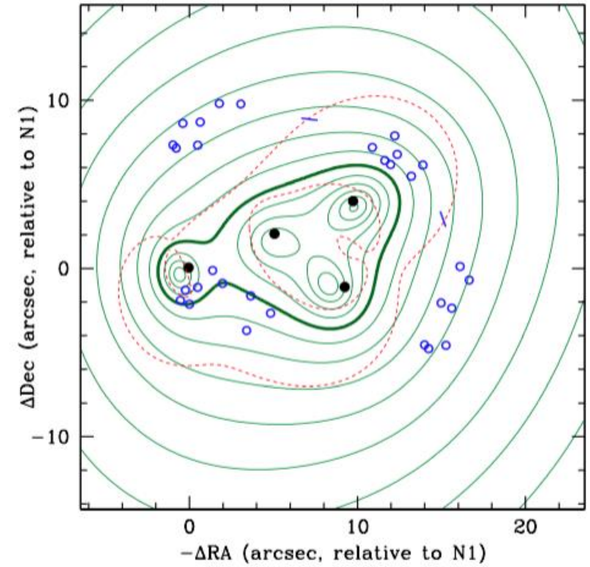
Is Dark Matter self-interacting?

Analyzing offset between DM and stars falling into cluster

Kahlhoefer et al MNRAS 452, 1 (2015) suggest

$$\frac{\sigma_{el}}{m} \approx 3\frac{cm^2}{g} \approx 0.5\frac{b}{GeV}$$





Galaxy cluster Abell 3827

[Massey *et al.*, MNRAS 449, 3393 (2015)]

For non-relativistic particles of mass m and mean velocity $\bar{v}$, the **shear viscosity**

$$\eta = \frac{m\,\bar{v}}{3\,\sigma_{el}}$$

This would sets lower bound for shear viscosity of dark matter.

Consequences for dark matter fluid dynamics!

But if viscosity too large, then one should not applying fluid dynamics

Cosmological Structure Formation

- Described by energy conservation and Navier-Stokes, for $\delta \equiv \delta\varepsilon/\varepsilon \leq 1$

$$\delta\dot{\varepsilon} + 3H\delta\varepsilon + \bar{\varepsilon}\theta = 0, \quad H = \tfrac{\dot{a}}{a}, \quad \theta = \vec{\nabla}.\vec{v}$$

$$\bar{\varepsilon}\left[\dot{\theta} + H\theta - q^2\psi\right] + \frac{\left(\zeta + \frac{4}{3}\eta\right)}{a} q^2\theta = 0,$$

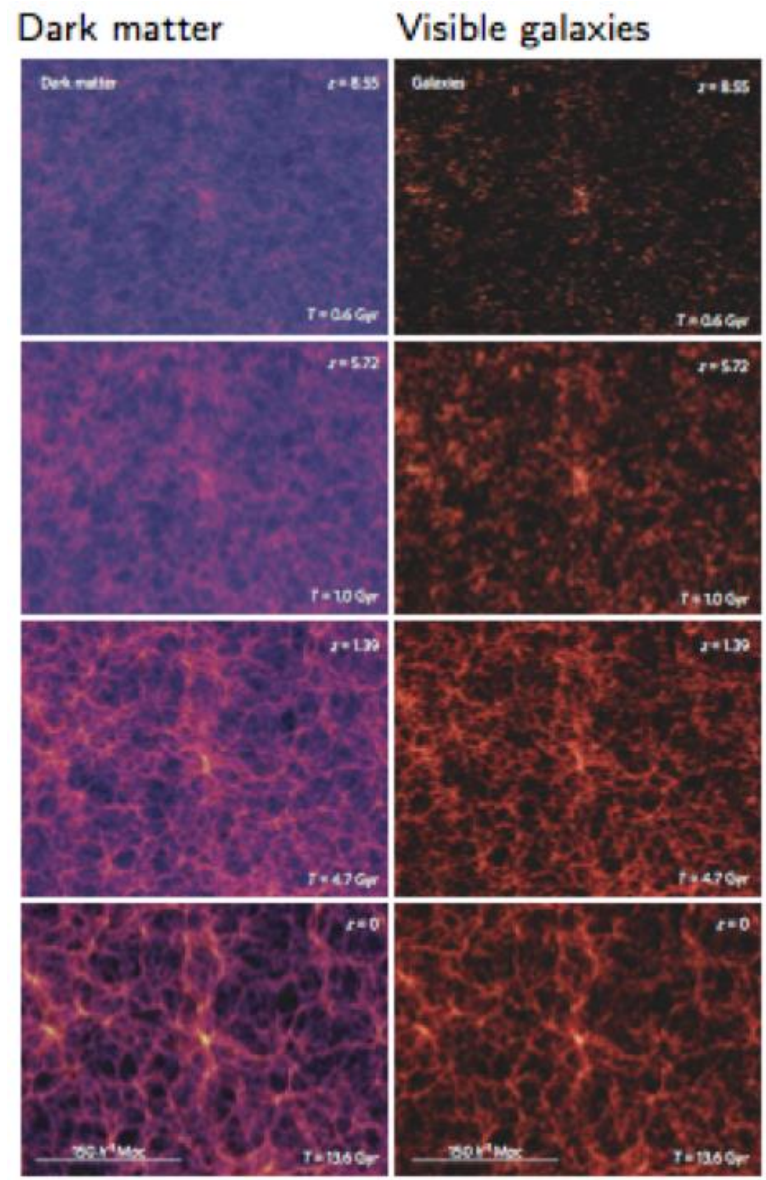
- Closed by Poisson equation for Newtonian potential ψ

$$-q^2\psi = 4\pi G_N a^2 \delta\varepsilon$$

- Viscosity slows down grav. collapse but does not wash out structure

Blas, Floerchinger, Garny, Tetradis, UAW, arXiv:1507.06665, **JCAP 1511 (2015) 049**

$$\ddot{\delta} + \left[\frac{\dot{a}}{a} + \frac{\zeta + \frac{4}{3\eta}}{a\bar{\varepsilon}} q^2\right]\dot{\delta} - 4\pi G_N \bar{\varepsilon}\,\dot{\delta} = 0$$



[Springel, Frenk & White, Nature 440, 1137 (2006)]

Growth of structure depends on viscosity

- Consider subhorizon perturbations, $k \gg H$, with $\delta_k \sim \theta_k / H \leq O(1)$
  Corrections to perturbative evolution are $O(H^2/k^2)$

- *Assume* that viscosity and pressure do not disrupt scale hierarchy

$$c_s^2 = \frac{\partial p}{\partial \varepsilon} = \alpha_s \boxed{\frac{k^2}{k_m^2}} \qquad \nu H = \frac{\eta H}{(\bar{\varepsilon}_m + p)a} = \frac{3}{4}\alpha_\nu \boxed{\frac{k^2}{k_m^2}}$$

$1/k_m$ shortest length scale of (effective) viscous description

which leads to

$$\boxed{\delta_k'' + \left(1 + \frac{H'}{H} + \alpha_\nu \frac{k^2}{k_m^2}\right)\delta_k' - \left(\frac{3}{2}\Omega_m - \alpha_s \frac{k^2}{k_m^2}\right)\delta_k = 0, \quad \delta' = \frac{d\delta}{d\ln a}}$$

- Viscosity affects <u>sufficiently small length scales only</u>

Blas, Floerchinger, Garny, Tetradis, UAW,
JCAP 1511 (2015) 049