Hydrodynamics & Beyond Hydrodynamics

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宋慧超

Peking University

ULtra-RelatIvistiC HEavy IoNZ 2016

CERN, Geneva, July18-June 20, 2016

July. 19, 2016

QGP-the most perfect fluid in the world?

BNL News, 2005

RHIC Scientists Serve Up "Perfect" Liquid

New state of matter more remarkable than predicted -- raising many new questions

April 18, 2005

TAMPA, FL -- The four detector groups conducting research at the Relativistic Heavy Ion Collider (RHIC) -- a giant atom "smasher" located at the U.S. Department of Energy's Brookhaven National Laboratory -- say they've created a new state of hot, dense matter out of the quarks and gluons that are the basic particles of atomic nuclei, but it is a state quite different and even more remarkable than had been predicted. In peer-reviewed papers summarizing the first three years of RHIC findings, the scientists say that instead of behaving like a gas of free quarks and gluons, as was expected, the matter created in RHIC's heavy ion collisions appears to be more like a *liquid*.

"Once again, the physics research sponsored by the Department of Energy is producing historic results," said Secretary of Energy Samuel Bodman, a trained chemical engineer. "The DOE is the principal federal funder of basic research in the physical sciences, including nuclear and high-energy physics. With today's announcement we see that investment paying off."

Secretary of Energy Samuel Bodman

"The truly stunning finding at RHIC that the new state of matter created in the collisions of gold ions is more like a liquid than a gas gives us a profound insight into the earliest moments of the universe," said Dr. Raymond L. Orbach, Director of the DOE Office of Science.

Also of great interest to many following progress at RHIC is the emerging connection between the collider's results and calculations using the methods of string theory, an approach that attempts to explain

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LHC to Restart in 2009

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Ulrich Heinz

viscous hydrodynamics

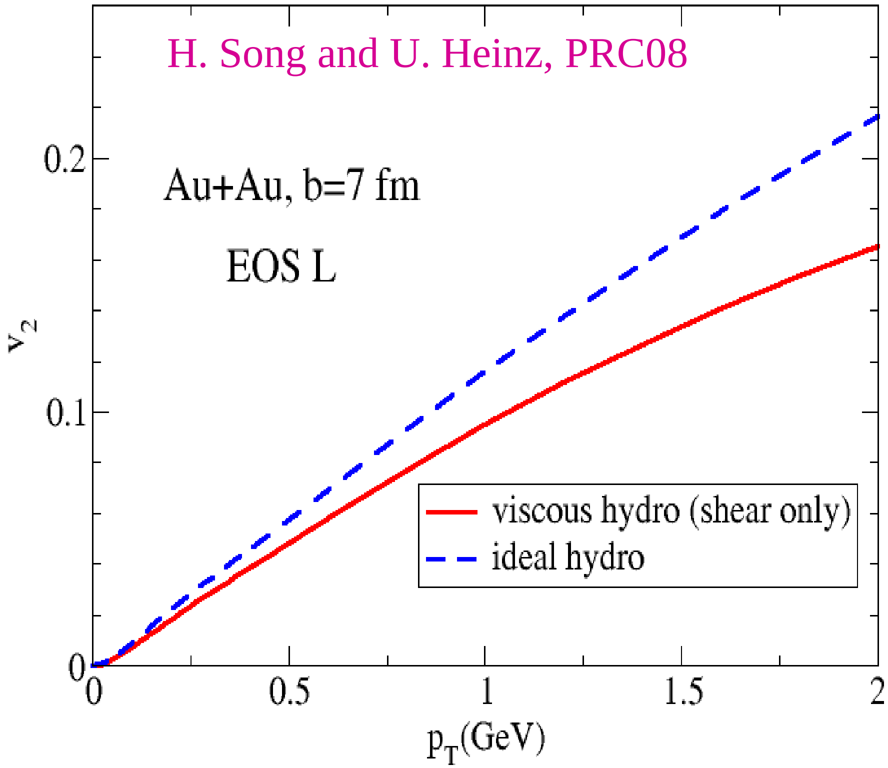
$$\partial_\mu T^{\mu\nu}(x) = 0 \qquad T^{\mu\nu} = (e + p + \Pi) u^\mu u^\nu - (p + \Pi) g^{\mu\nu} + \pi^{\mu\nu}$$

$$\tau_\pi \Delta^{\alpha\mu}\Delta^{\beta\nu}\dot{\pi}_{\alpha\beta} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} - \frac{1}{2}\pi^{\mu\nu}\frac{\eta T}{\tau_\pi}\partial_\lambda\left(\frac{\tau_\pi}{\eta T}u^\lambda\right)$$

$$\tau_\Pi \dot{\Pi} + \Pi = -\zeta(\partial \cdot u) - \frac{1}{2}\Pi\frac{\zeta T}{\tau_\Pi}\partial_\lambda\left(\frac{\tau_\Pi}{\zeta T}u^\lambda\right)$$

Input: "EOS"

$$\varepsilon = \varepsilon(p)$$



H. Song and U. Heinz, PLB08

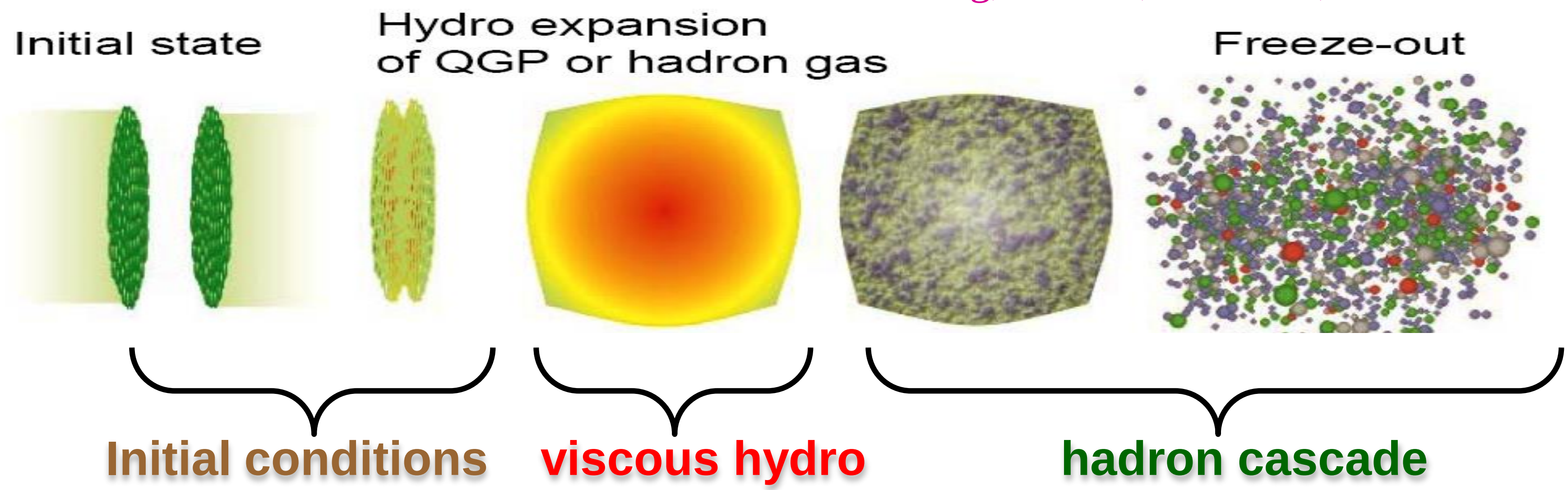
H. Song and U. Heinz, PRC08

-Elliptic flow is sensitive to the QGP shear viscosity, minimal value of η/s lead to 20-30% V_2 suppression

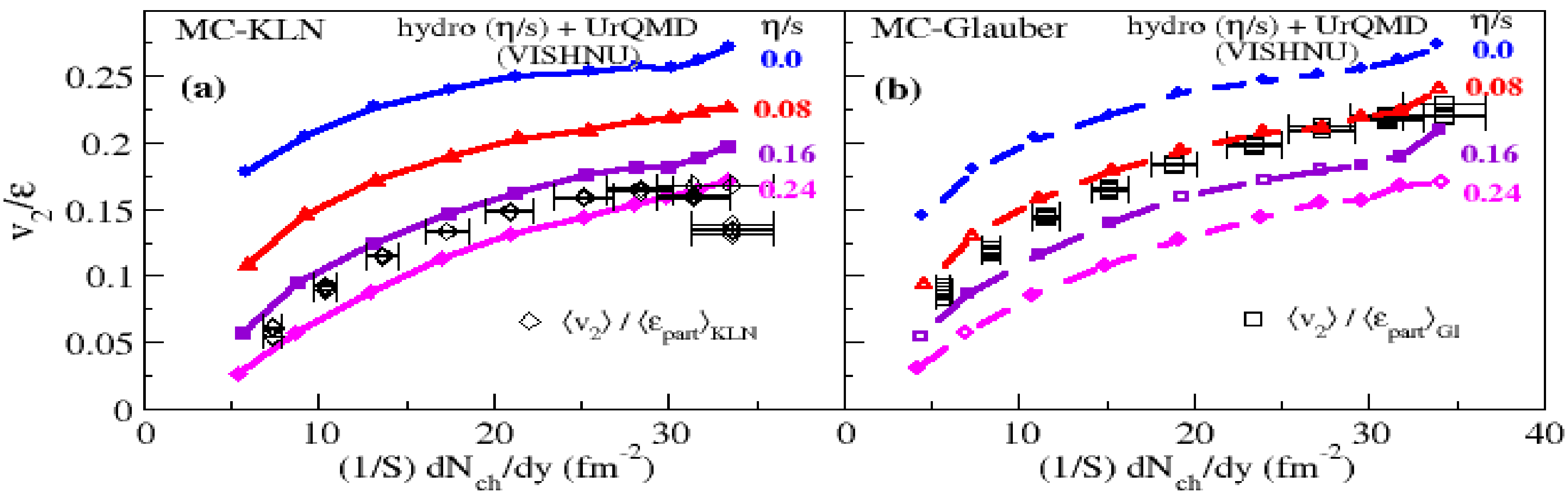
-V_2 can be used to extract the QGP shear viscosity.

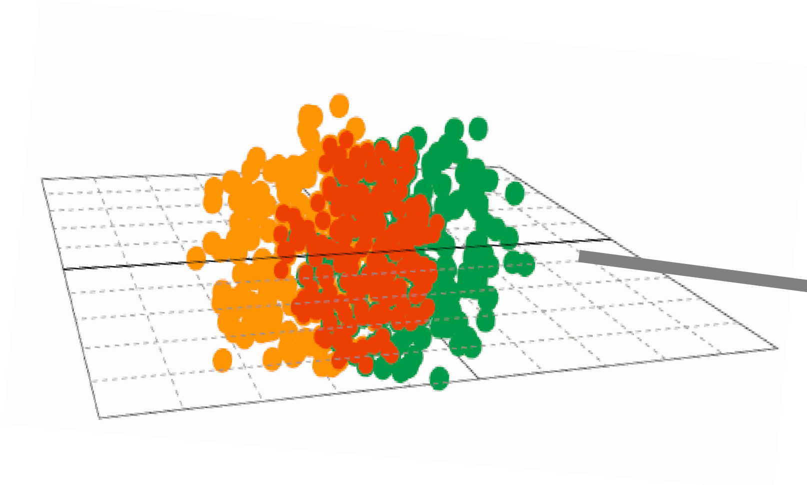
VISHNU hybrid approach

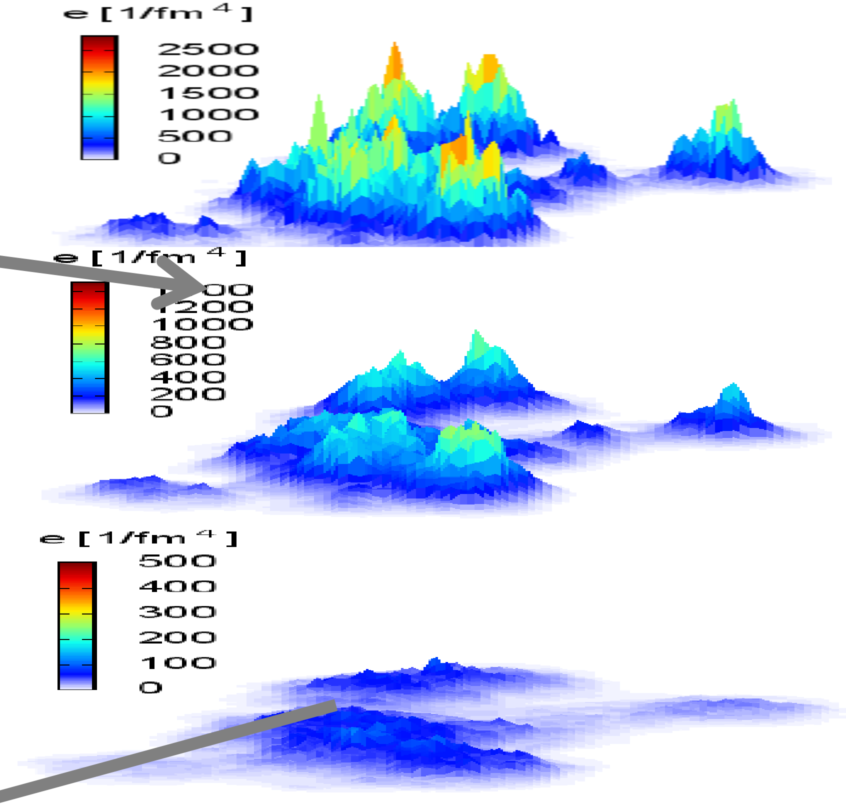
H. Song, S. Bass, U. Heinz, PRC2011



H. Song, et al, PRL2011







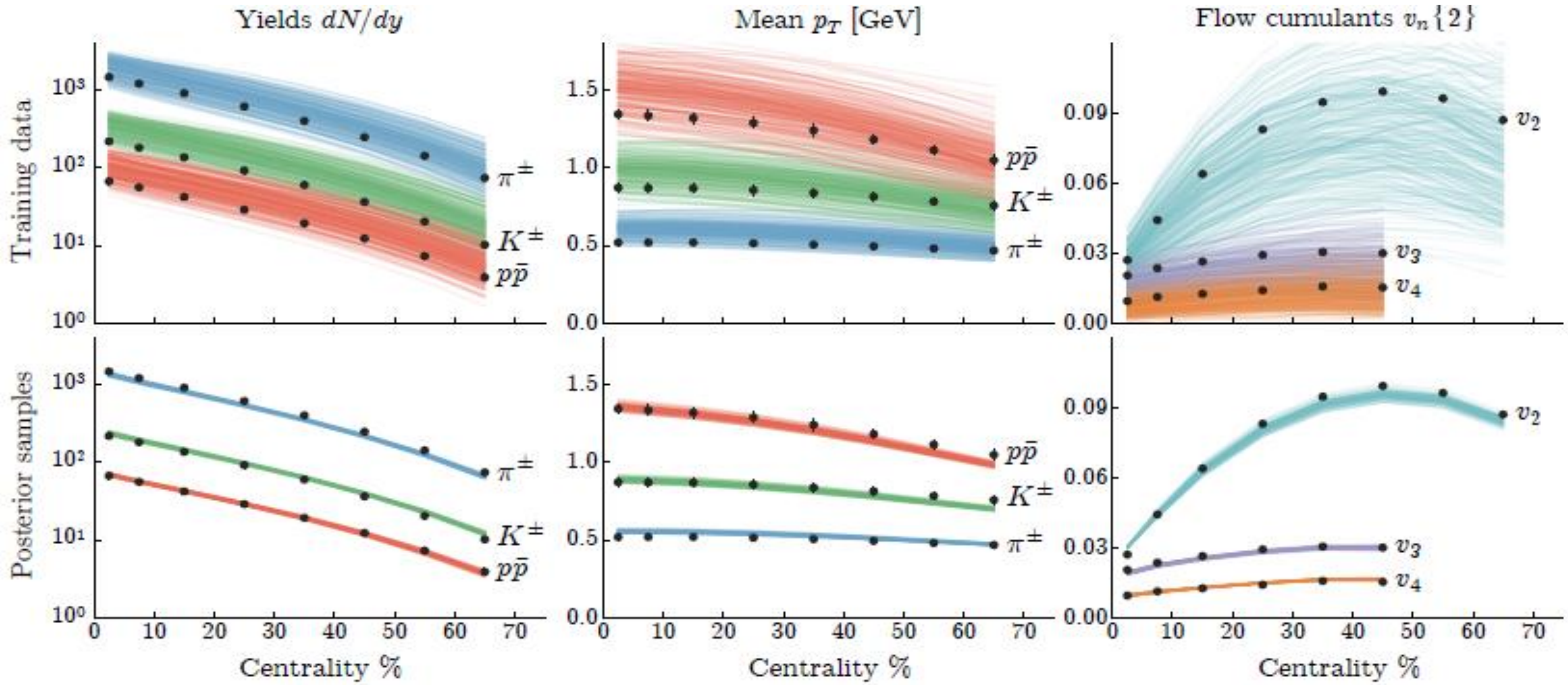
$$E\frac{dN}{d^3p} = \frac{dN}{dy p_T dp_T d\varphi}$$

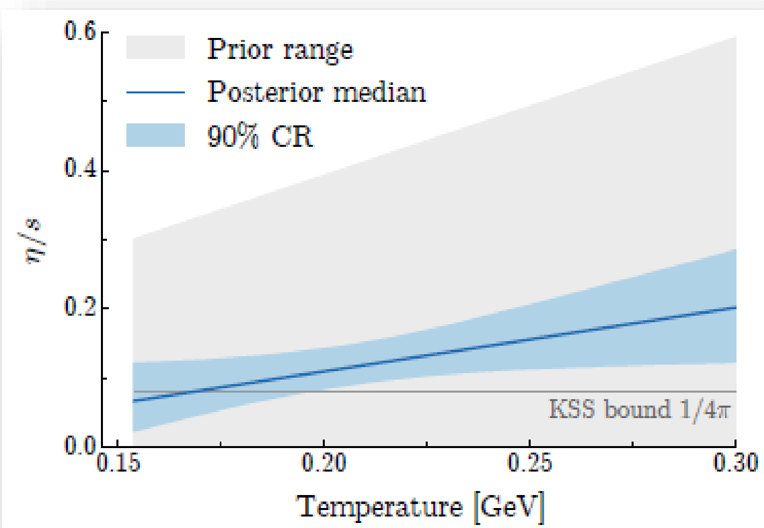
$$= \frac{1}{2\pi}\frac{dN}{dy p_T dp_T}[1 + 2v_1(p_T, b)\cos(\varphi) + 2v_2(p_T, b)\cos(2\varphi) + 2v_3(p_T, b)\cos(3\varphi)......]$$

iEBE-VISHNU

C. Shen, Z. Qiu, H. Song, J. Bernhard, S. Bass and U. Heinz, Comput. Phys. Commun. 199, 61 (2016)

An quantitatively extract the QGP viscosity





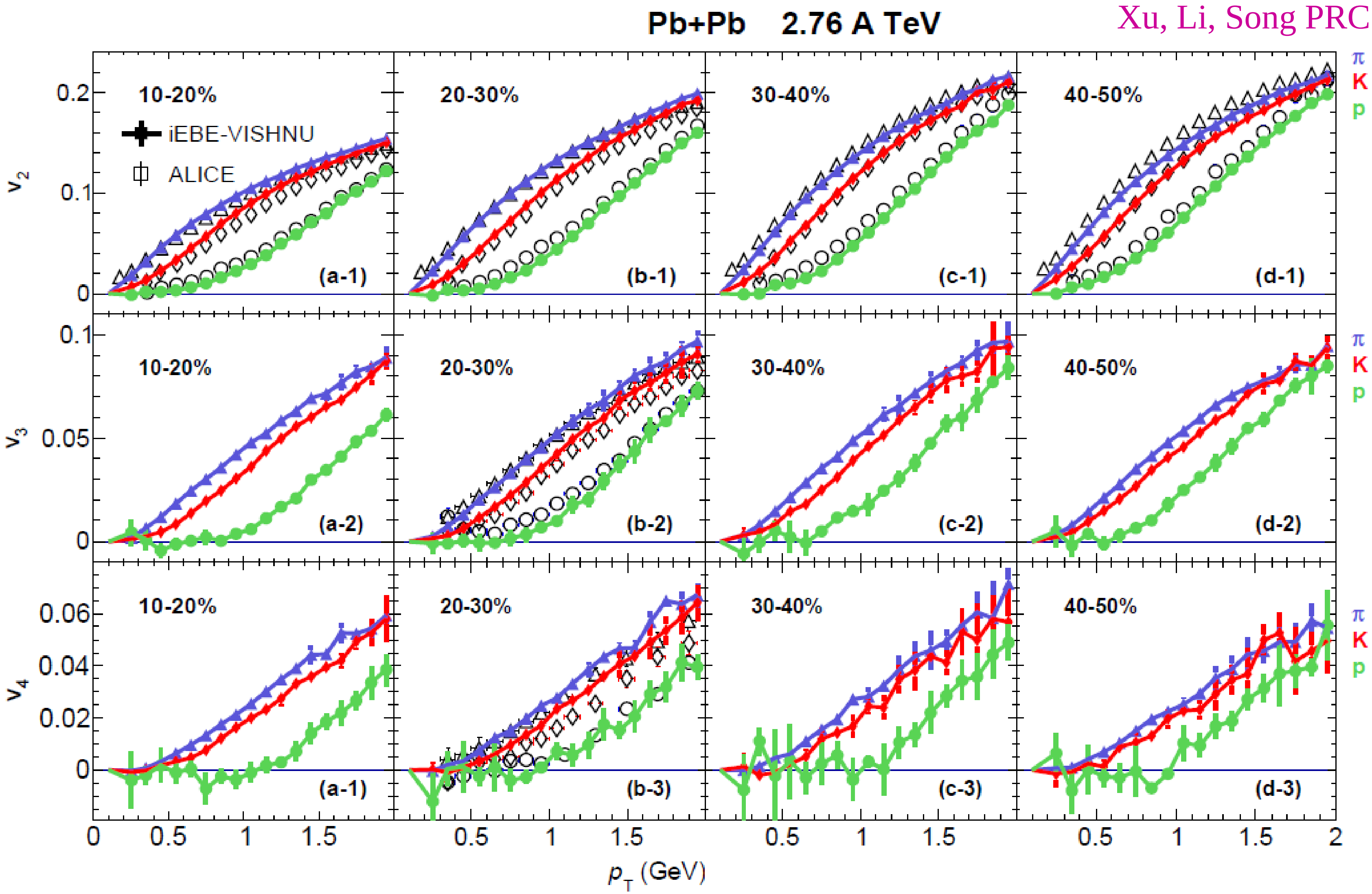
-An quantitatively extraction of the QGP viscosity with iEBE-VISHNU and the massive data evaluation

-η/s(T) is very close to the KSS bound of 1/4π

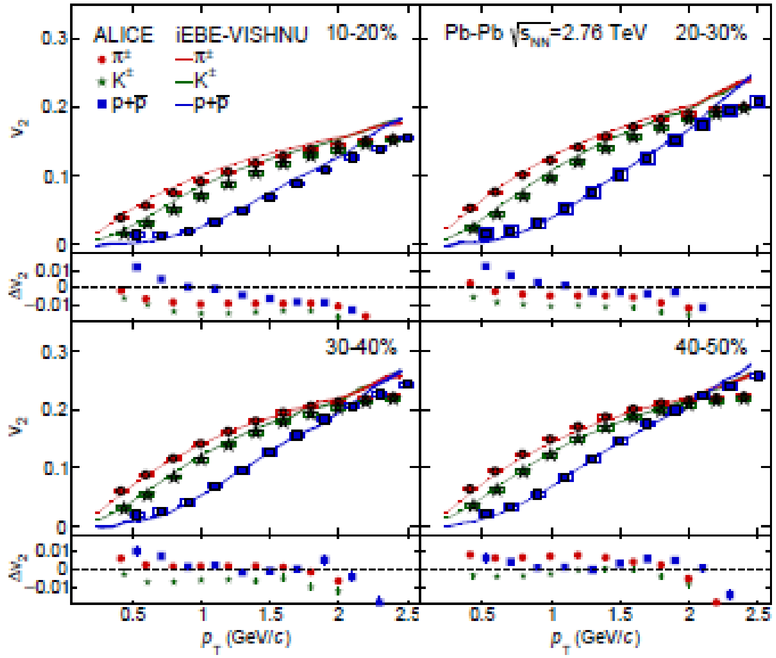
J. Bernhard, S. Moreland, S.A. Bass, J. Liu, U. Heinz, arXiv:1605.03954

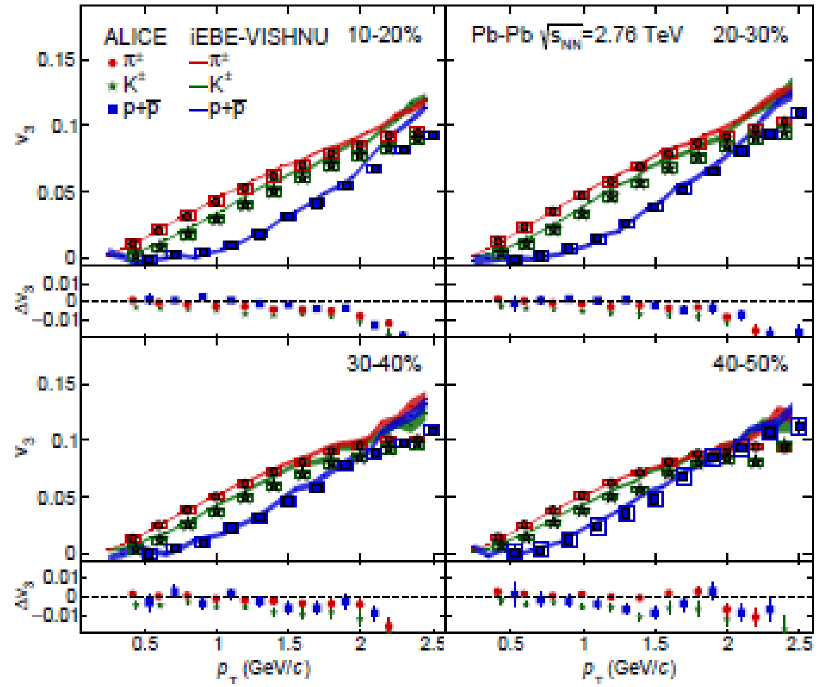
V_2, V_3, V_4 of identified hadrons

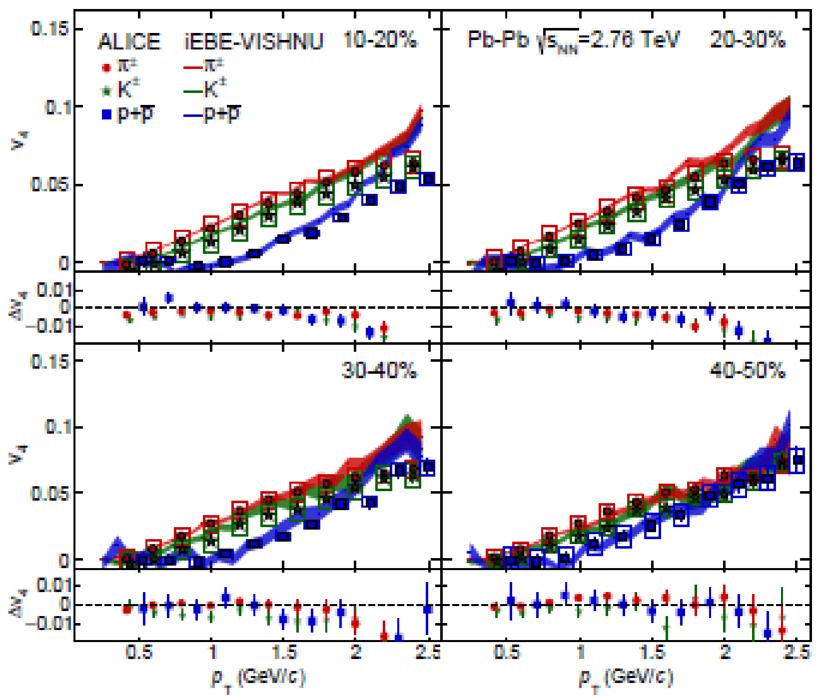
Xu, Li, Song PRC 2016



-V_3 & V_4 shows similar mass orderings as V_2 for various centrality







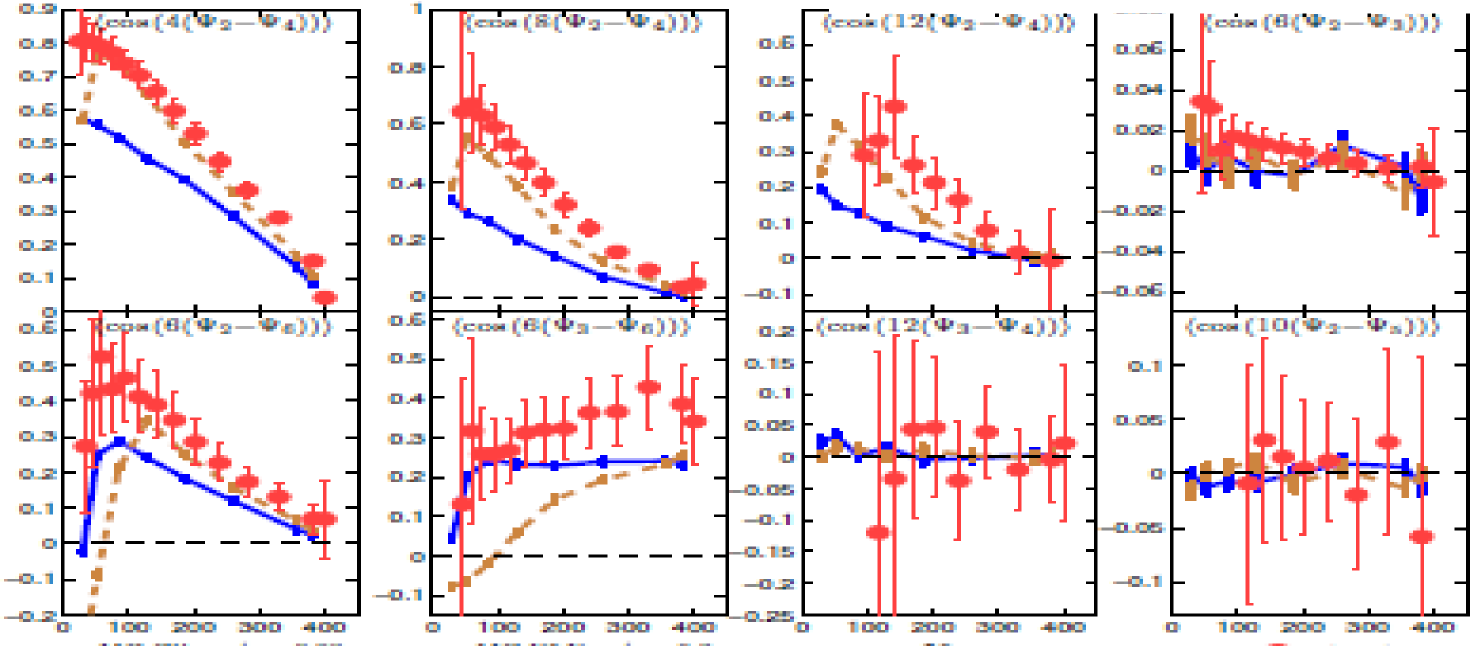
ALICE: 1606.06507

iEBE- VISHNU: Xu, Li, Song PRC 2016

-iEBE-VISHNU (AMPT initial conditions) nicely describe the ALICE V_n of pions,kaons and protons at various centralities

Correlations of Event Plane Angles

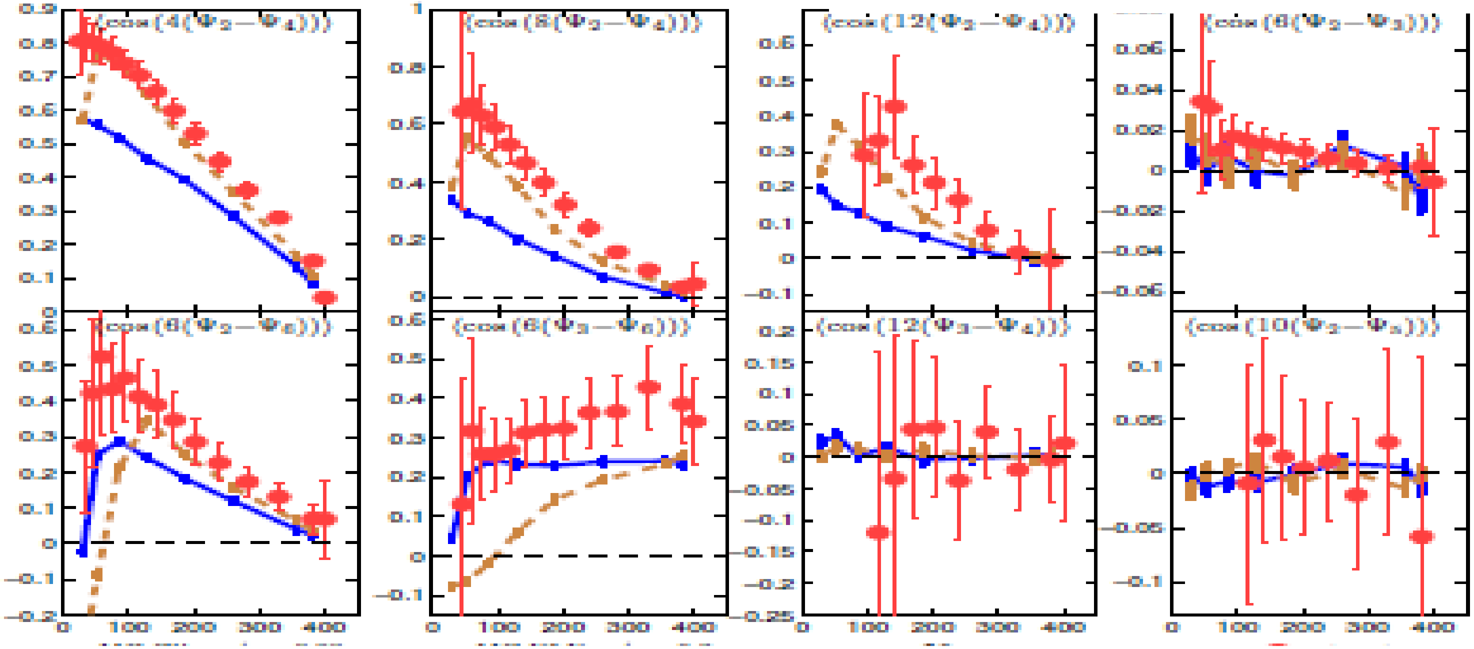
Qiu & Heinz, PLB 2012



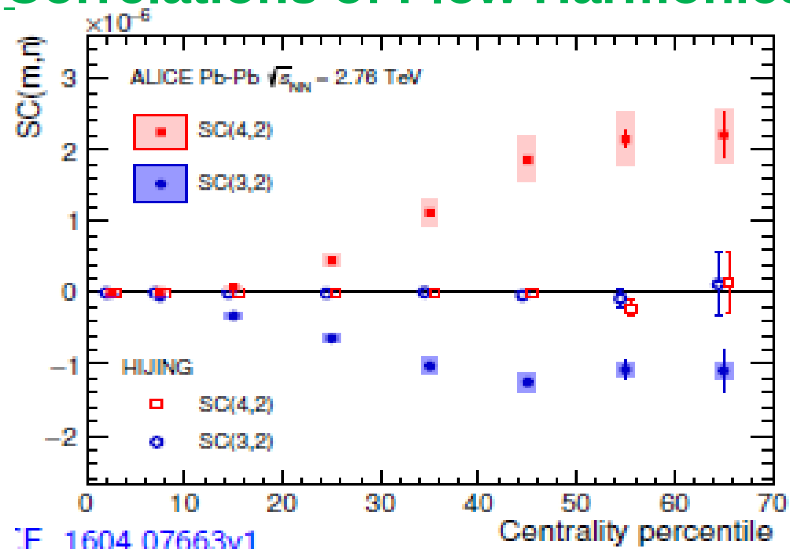
-The correlations between different flow angles are qualitatively described by e-b-e hydrodynamics with different initial conditions

Correlations of Event Plane Angles

Qiu & Heinz, PLB 2012



Correlations of Flow Harmonics



ALICE, 1604.07663

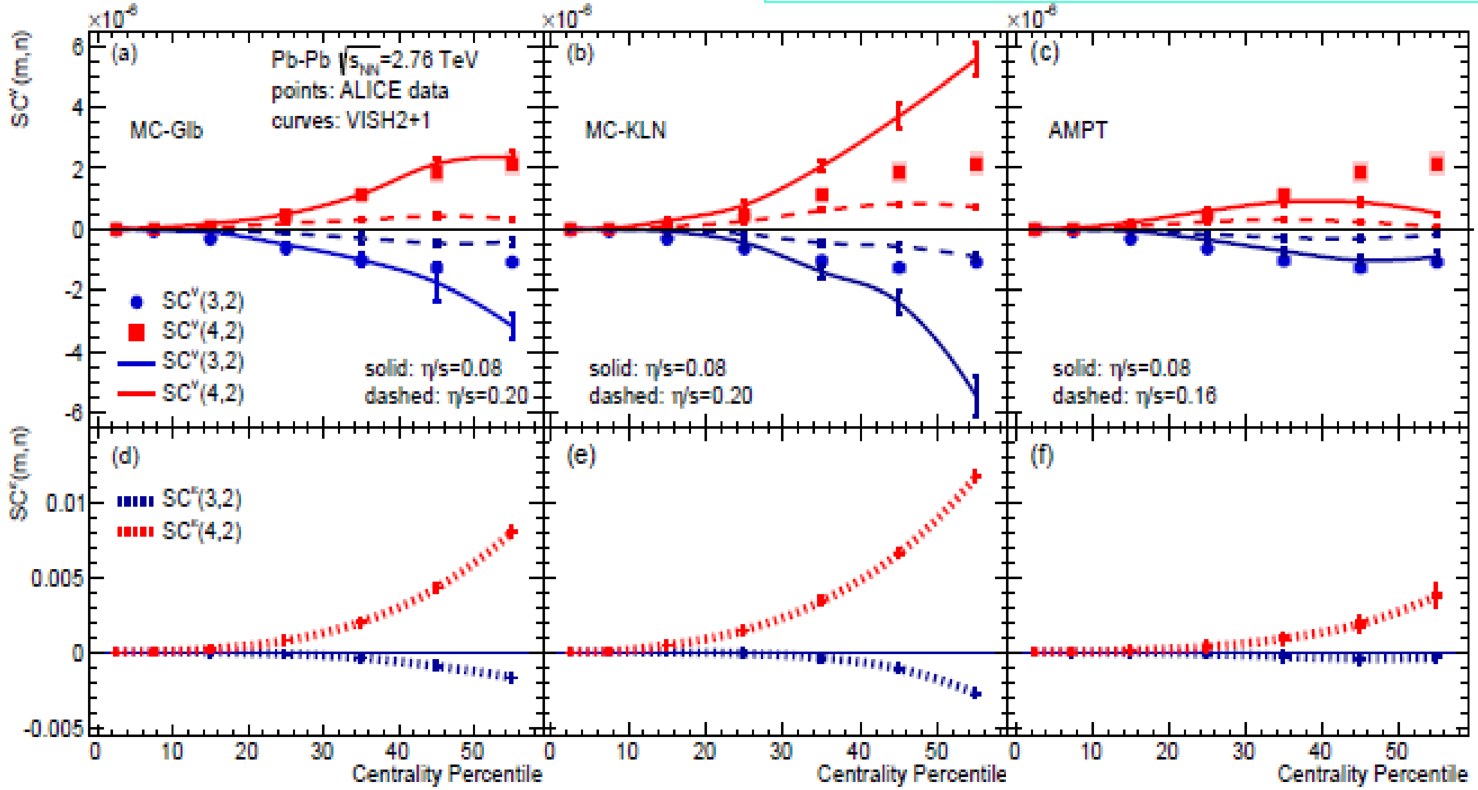
$$SC^{v}(m,n) = \langle v_m^2\, v_n^2 \rangle - \langle v_m^2 \rangle \langle v_n^2 \rangle$$

- V_2 and V_4 are correlated
- V_3 and V_4 are anti-correlated
- SC(m,n) from HIJING are compatible to zero

$SC^v(3,2)$ & $SC^v(4,2)$

$$SC^v(m,n) = \langle v_m^2\, v_n^2 \rangle - \langle v_m^2 \rangle \langle v_n^2 \rangle$$

$$SC^\varepsilon(m,n) = \langle \varepsilon_m^2 \varepsilon_n^2 \rangle - \langle \varepsilon_m^2 \rangle \langle \varepsilon_n^2 \rangle$$



Zhu, Xu, Zhou, Song, in preparation

-$SC^v(3,2)$ and $SC^v(4,2)$ are sensitive to both initial conditions and η/s

-hydrodynamic simulations correctly capture the sign of $SC^v(3,2)$ and $SC^v(4,2)$

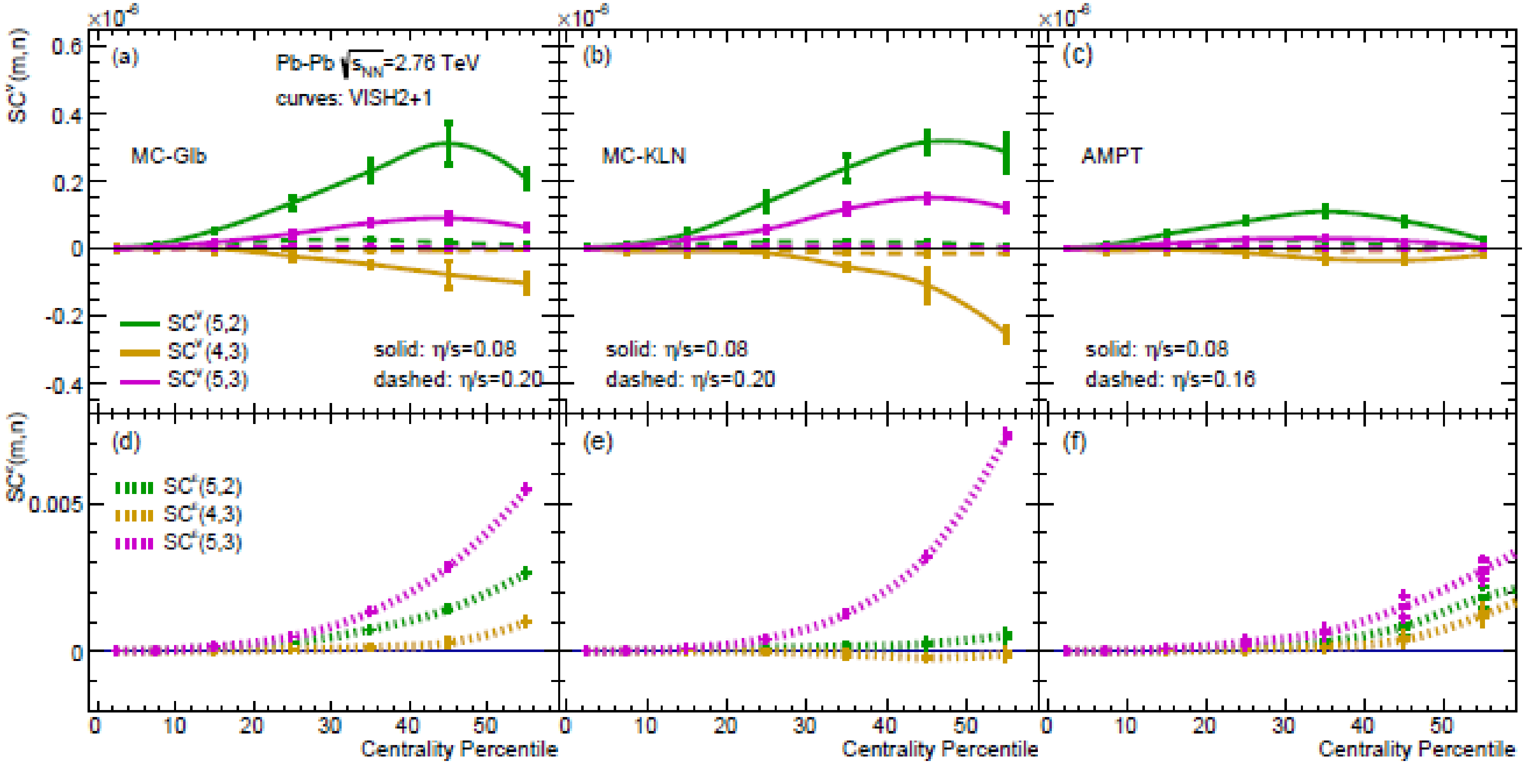
- V_2 and V_4 are correlated, V_2 and V_3 are anti-correlated

-$SC^v(3,2)$ and $SC^v(4,2)$ follow the sign of $SC^\varepsilon(3,2)$ and $SC^\varepsilon(4,2)$

$SC^v(5,2)$, $SC^v(4,3)$ & $SC^v(5,3)$

$$SC^v(m,n) = \langle v_m^2\, v_n^2 \rangle - \langle v_m^2 \rangle \langle v_n^2 \rangle$$

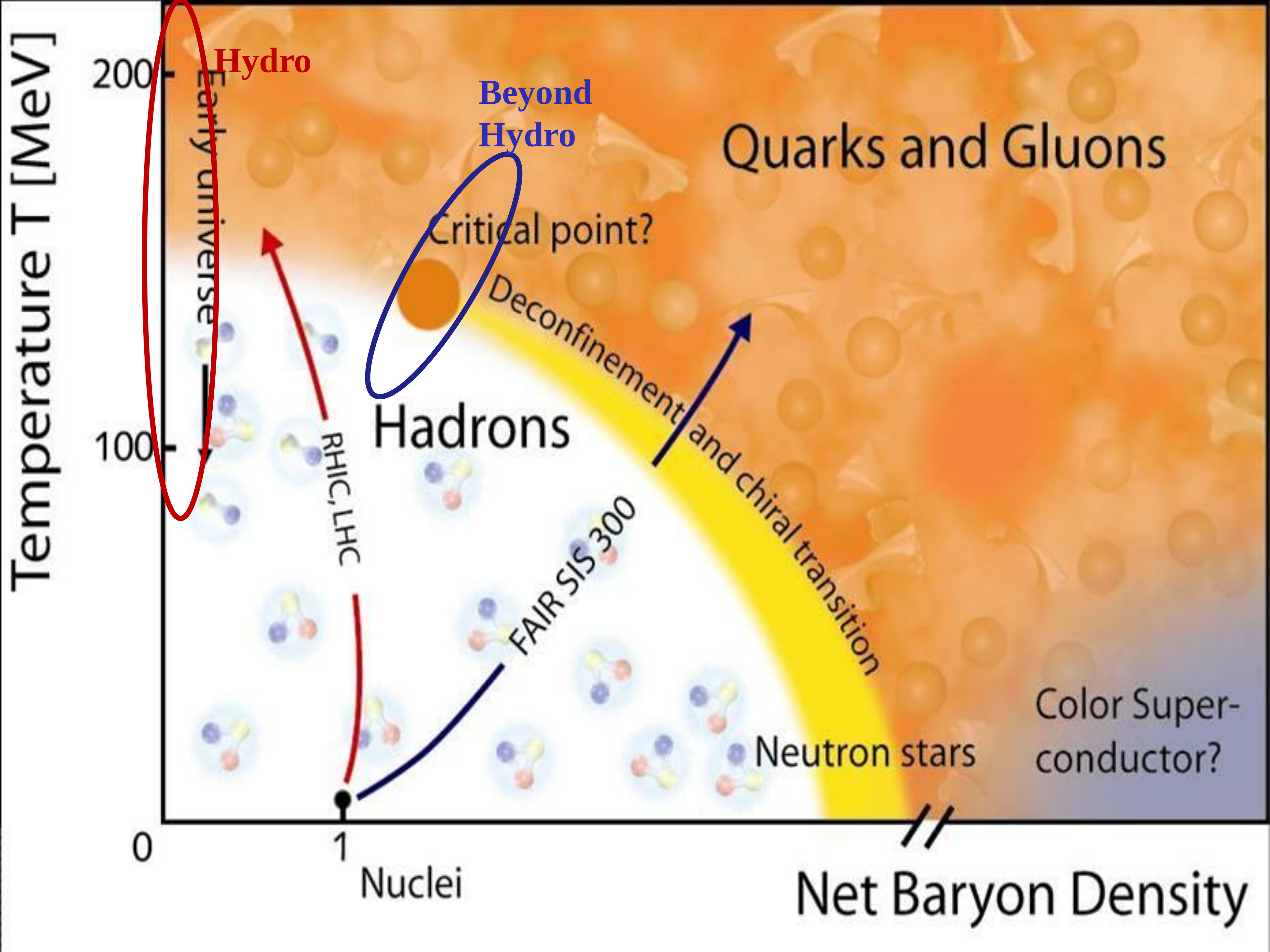
$$SC^\varepsilon(m,n) = \langle \varepsilon_m^2 \varepsilon_n^2 \rangle - \langle \varepsilon_m^2 \rangle \langle \varepsilon_n^2 \rangle$$



Zhu, Xu, Zhou, Song, in preparation

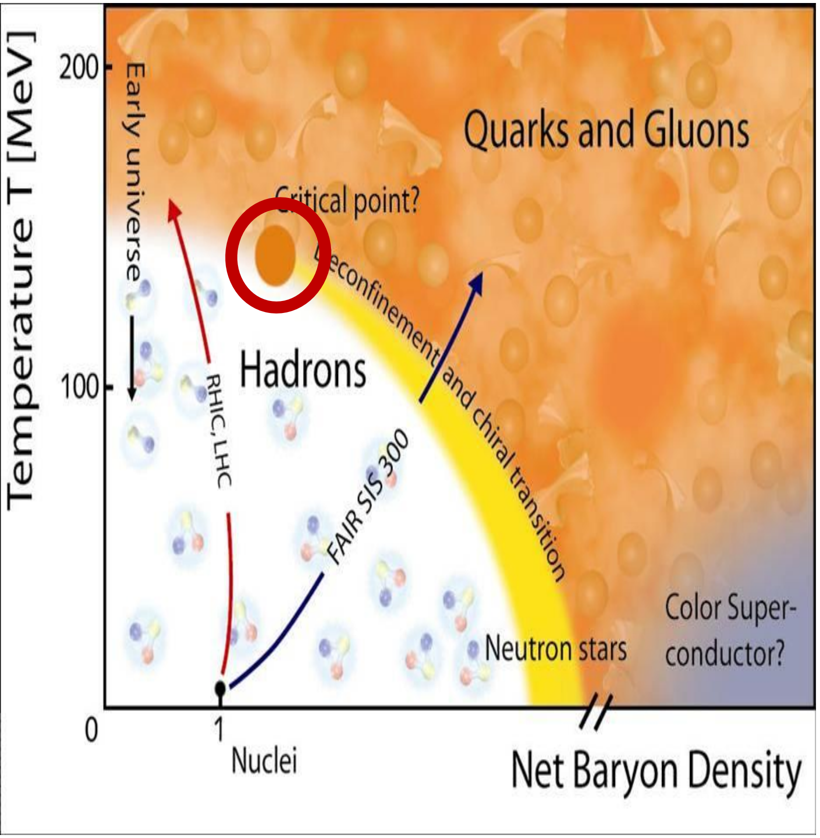
- V_2 and V_5 ; V_3 and V_5 are correlated, V_3 and V_4 are anti-correlated
- $SC^v(5,2)$ and $SC^v(5,3)$ respectively follow the sign of $SC^\varepsilon(5,2)$ and $SC^\varepsilon(5,3)$
- $SC^v(4,3)$ & $SC^\varepsilon(4,3)$ show opposite signs

$$v_4 e^{i4\Psi} = a_0 \varepsilon_4 e^{i4\Phi_4} + a_1 (\varepsilon_2 e^{i2\Phi_2})^2$$



Hydro
Beyond
Hydro
Quarks and Gluons
Critical point?
Early universe
Deconfinement and chiral transition
Hadrons
RHIC, LHC
FAIR SIS 300
Neutron stars
Color Super-
conductor?
Temperature T [MeV]
200
100
0
1
Nuclei
Net Baryon Density

Correlated fluctuations near the QCD critical point



<u>Initial State Fluctuations</u>

-QGP fireball evolutions smear-out the initial fluctuations

-uncorrelated (in general)

<u>Fluctuations near the critical point</u>

-dramatically increase near Tc

-Strongly correlated

Theoretical predictions on critical fluctuations

Stephanov PRL 2009

$$P[\sigma] \sim \exp\{-\Omega[\sigma]/T\}, \qquad \Omega = \int d^3x \left[\frac{1}{2}(\nabla\sigma)^2 + \frac{m_\sigma^2}{2}\sigma^2 + \frac{\lambda_3}{3}\sigma^3 + \frac{\lambda_4}{4}\sigma^4 + \cdots\right]$$

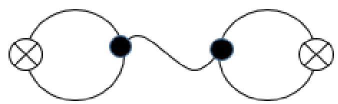
$$\langle\sigma_0^2\rangle = \frac{T}{V}\xi^2 \qquad \langle\sigma_0^3\rangle = \frac{2\lambda_3 T}{V}\xi^6; \qquad \langle\sigma_0^4\rangle_c = \frac{6T}{V}[2(\lambda_3\xi)^2 - \lambda_4]\xi^8.$$

<u>Critical Fluctuations of particles</u> :

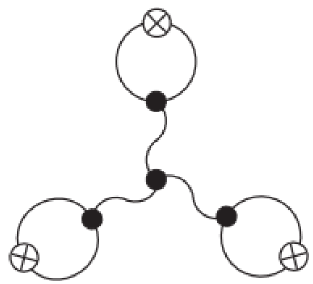
$$\langle(\delta N)^2\rangle \sim \xi^2$$

$$\langle(\delta N)^3\rangle \sim \xi^{4.5}$$

$$\langle(\delta N)^4\rangle \sim \xi^7$$

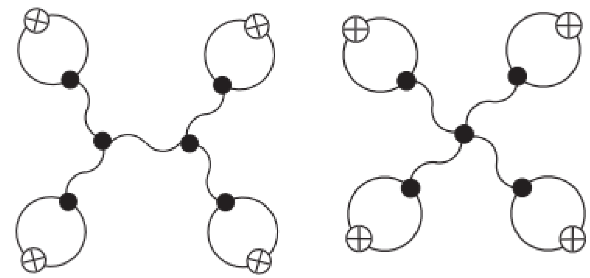


$\sim \xi^2$



$\sim \xi^{4.5}$

3×



$\sim \xi^7$

At critical point : $\xi \sim \infty$ **(infinite medium)**

Finite size & finite evolution time: $\xi < \mathrm{O(2-3fm)}$

It is important to address the effects from dynamical evolutions

Dynamical Modeling near the QCD critical point

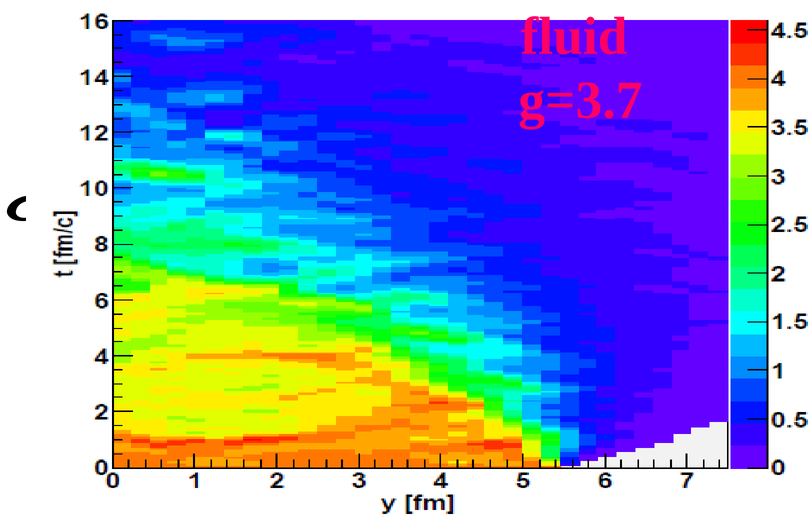
-Chiral Hydrodynamics

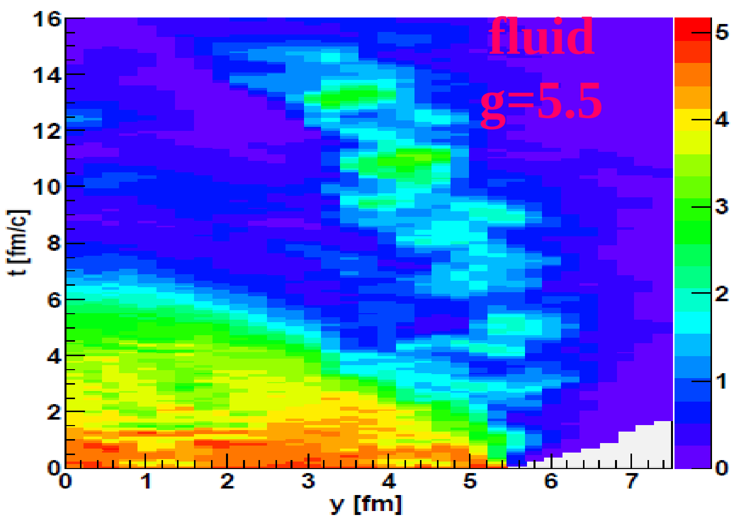
$$L=\bar{q}[i\gamma-g(\sigma+i\gamma_5\tau\pi)]q+\frac{1}{2}[\partial_\mu\sigma\partial^\mu\sigma+\partial_\mu\pi\partial^\mu\pi]-U(\sigma,\pi)$$

$$\begin{cases}\partial_\mu\partial^\mu\sigma+\dfrac{\partial U_{eff}}{\partial\sigma}+g<\bar{q}q>=0 \\ \partial_\mu T^{\mu\nu}_{fluid}=S^\nu \qquad S^\nu=-(\partial^2 u+\dfrac{\partial U_{eff}}{\partial u})\partial^\nu\sigma\end{cases}$$

Chiral Hydrodynamics

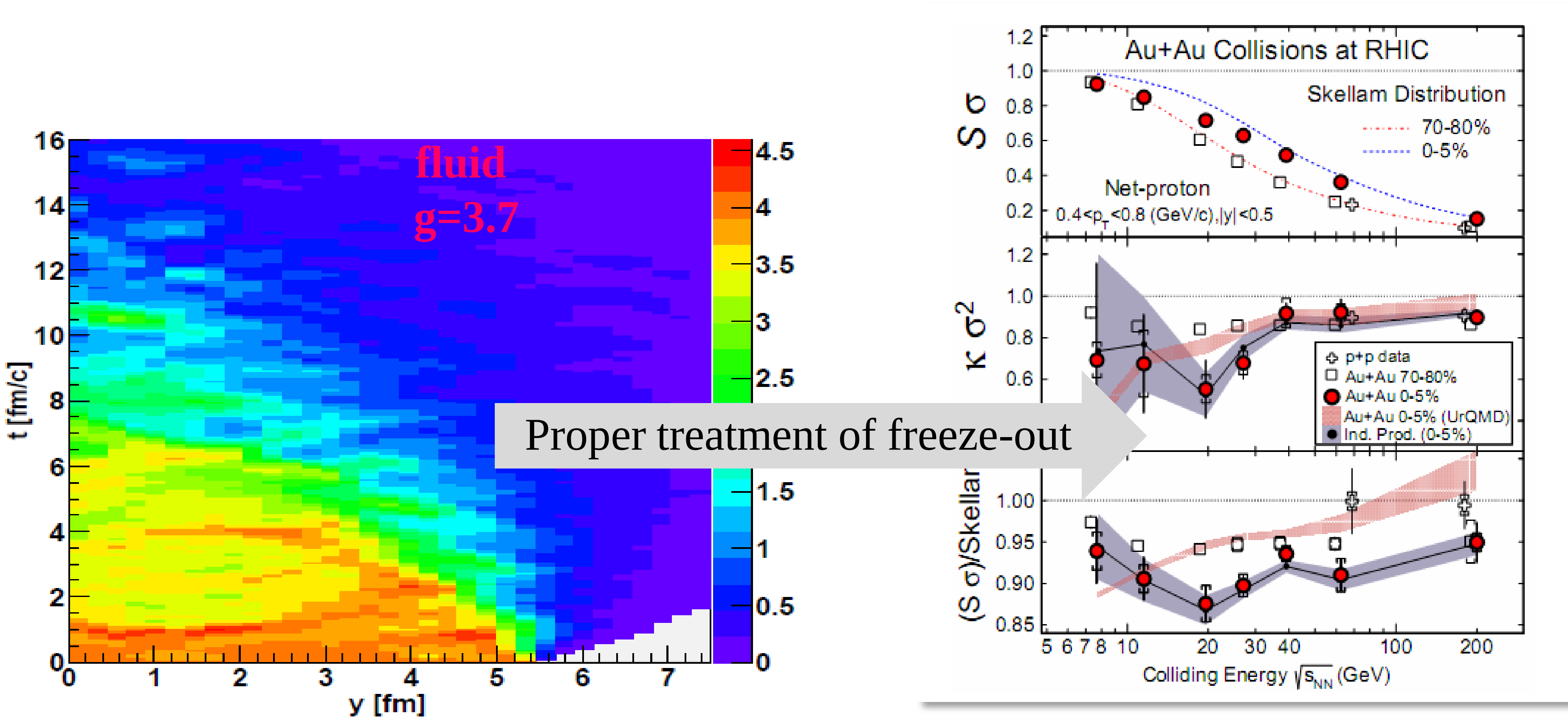
K. Paech, H. Stocker and A. Dumitru, PRC2003





-Chiral fluid dynamics with dissipation & noise Nahrgang,et al., PRC 2011

-Chiral fluid dynamics with a Polyakov loop (PNJL) Herold, et al., PRC 2013

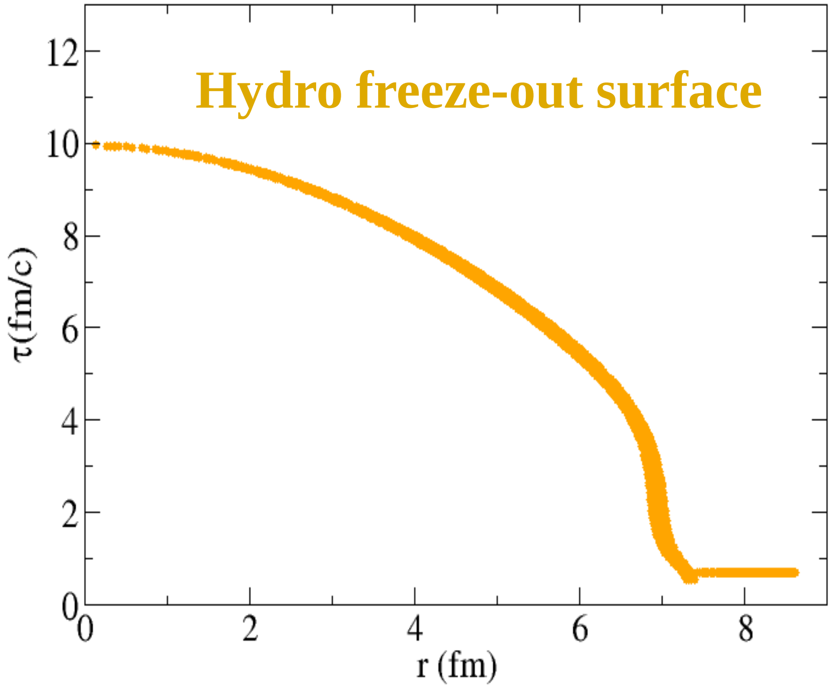


From dynamical evolution to experimental observables, it is important to properly treat the freeze-out procedure with an external field

Freeze-out scheme near T_{cr}
& static critical fluctuations

Jiang, Li & Song, arXiv:1512.06164[nucl-th]

Particle emissions near T_{cr} with external field



Jiang, Li & Song, arXiv: 1512.06164[nucl-th]

Particle emissions in traditional hydro

$$E\frac{dN}{d^3p} = \int_\Sigma \frac{p_\mu d\sigma^\mu}{2\pi^3} f(x,p)$$

Particle emissions near T_{cr}

$$M \longrightarrow g\sigma(x)$$

$$f(x,p) = f_0(x,p)[1 - g\sigma(x)/(\gamma T)]$$
$$= f_0 + \delta f$$

$$\langle \delta f_1 \delta f_2 \rangle_\sigma = f_{01} f_{02} f_{03} \left(\frac{g^2}{\gamma_1 \gamma_2} \frac{1}{T^3} \right) \langle \sigma_1 \sigma_2 \rangle_c ,$$

$$\langle \delta f_1 \delta f_2 \delta f_3 \rangle_\sigma = f_{01} f_{02} f_{03} \left(-\frac{g^3}{\gamma_1 \gamma_2 \gamma_3} \frac{1}{T^3} \right) \langle \sigma_1 \sigma_2 \sigma_3 \rangle_c ,$$

$$\langle \delta f_1 \delta f_2 \delta f_3 \delta f_4 \rangle_\sigma = f_{01} f_{02} f_{03} f_{04} \left(\frac{g^4}{\gamma_1 \gamma_2 \gamma_3 \gamma_4} \frac{1}{T^4} \right) \langle \sigma_1 \sigma_2 \sigma_3 \sigma_4 \rangle_c .$$

For a stationary & infinite medium:

$$\left\langle (\delta N)^2 \right\rangle_c = \left(\frac{g_i}{(2\pi)^3}\right)^2 \int d^3p_1 d^3x_1 \int d^3p_2 d^3x_2 \frac{f_{01} f_{02}}{\gamma_1\gamma_2} \frac{g^2}{T^2} \langle \sigma_1 \sigma_2 \rangle_c ,$$

$$\left\langle (\delta N)^3 \right\rangle_c = \left(\frac{g_i}{(2\pi)^3}\right)^3 \int d^3p_1 d^3x_1 \int d^3p_2 d^3x_2 \int d^3p_3 d^3x_3 \frac{f_{01} f_{02} f_{03}}{\gamma_1\gamma_2\gamma_3} \left(-\frac{g^3}{T^3} \langle \sigma_1 \sigma_2 \sigma_3 \rangle_c \right),$$

$$\left\langle (\delta N)^4 \right\rangle_c = \left(\frac{g_i}{(2\pi)^3}\right)^4 \int d^3p_1 d^3x_1 \int d^3p_2 d^3x_2 \int d^3p_3 d^3x_3 \int d^3p_4 d^3x_4 \frac{f_{01} f_{02} f_{03} f_{04}}{\gamma_1\gamma_2\gamma_3\gamma_4} \frac{g^4}{T^4} \langle \sigma_1 \sigma_2 \sigma_3 \sigma_4 \rangle_c .$$

$$P[\sigma] \sim \exp\{-\Omega[\sigma]/T\}, \qquad \Omega[\sigma] = \int d^3x \left[\frac{1}{2}(\nabla\sigma)^2 + \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{\lambda_3}{3}\sigma^3 + \frac{\lambda_4}{4}\sigma^4 \right],$$

$$\langle \sigma_1 \sigma_2 \rangle_c = T D(x_1 - x_2),$$

$$\langle \sigma_1 \sigma_2 \sigma_3 \rangle_c = -2T^2 \lambda_3 \int d^3z D(x_1 - z) D(x_2 - z) D(x_3 - z),$$

$$\langle \sigma_1 \sigma_2 \sigma_3 \sigma_4 \rangle_c = -6T^3 \lambda_4 \int d^3z D(x_1 - z) D(x_2 - z) D(x_3 - z) D(x_4 - z)$$
$$+12T^3 \lambda_3^2 \int d^3u \int d^3v D(x_1 - u) D(x_2 - u) D(x_3 - v) D(x_4 - v) D(u - v).$$

$$\left\langle \delta n_{p_1} \delta n_{p_2} \right\rangle_c = \frac{f_{01} f_{02}}{\omega_{p_1}\omega_{p_2}} \frac{G^2}{T} \frac{V}{m_\sigma^2}. \qquad \left\langle \delta n_{p_1} \delta n_{p_2} \delta n_{p_3} \right\rangle_c = \frac{2\lambda_3}{V^2 T} \frac{f_{01} f_{02} f_{03}}{\omega_{p_1}\omega_{p_2}\omega_{p_3}} \left(\frac{G}{m_\sigma^2}\right)^3 .$$

$$\left\langle \delta n_{p_1} \delta n_{p_2} \delta n_{p_3} \delta n_{p_4} \right\rangle_c = \frac{6}{V^3 T} \frac{f_{01} f_{02} f_{03} f_{04}}{\omega_{p_1}\omega_{p_2}\omega_{p_3}\omega_{p_4}} \left(\frac{G}{m_\sigma^2}\right)^4 \left[2\left(\frac{\lambda_3}{m_\sigma}\right)^2 - \lambda_4 \right].$$

--the results in Stephanov PRL09 are reproduced

CORRELATED particle emissions along the freeze-out surface

$$\left\langle (\delta N)^2 \right\rangle_c = \left(\frac{g_i}{(2\pi)^3} \right)^2 \left(\prod_{i=1,2} \left(\frac{1}{E_i} \int d^3p_i \int_{\Sigma_i} p_{i\mu} d\sigma_i^\mu d\eta_i \right) \right) \frac{f_{01} f_{02}}{\gamma_1 \gamma_2} \frac{g^2}{T^2} \langle \sigma_1 \sigma_2 \rangle_c \,,$$

$$\left\langle (\delta N)^3 \right\rangle_c = \left(\frac{g_i}{(2\pi)^3} \right)^3 \left(\prod_{i=1,2,3} \left(\frac{1}{E_i} \int d^3p_i \int_{\Sigma_i} p_{i\mu} d\sigma_i^\mu d\eta_i \right) \right) \frac{f_{01} f_{02} f_{03}}{\gamma_1 \gamma_2 \gamma_3} \left(-\frac{g^3}{T^3} \langle \sigma_1 \sigma_2 \sigma_3 \rangle_c \right) ,$$

$$\left\langle (\delta N)^4 \right\rangle_c = \left(\frac{g_i}{(2\pi)^3} \right)^4 \left(\prod_{i=1,2,3,4} \left(\frac{1}{E_i} \int d^3p_i \int_{\Sigma_i} p_{i\mu} d\sigma_i^\mu d\eta_i \right) \right) \frac{f_{01} f_{02} f_{03} f_{04}}{\gamma_1 \gamma_2 \gamma_3 \gamma_4} \frac{g^4}{T^4} \langle \sigma_1 \sigma_2 \sigma_3 \sigma_4 \rangle_c$$

CORRELATED particle emissions along the freeze-out surface

$$\left\langle (\delta N)^2 \right\rangle_c = \left(\frac{g_i}{(2\pi)^3}\right)^2 \left(\prod_{i=1,2} \left(\frac{1}{E_i}\int d^3p_i \int_{\Sigma_i} p_{i\mu} d\sigma_i^\mu d\eta_i\right)\right) \frac{f_{01} f_{02}}{\gamma_1\gamma_2} \frac{g^2}{T^2} \langle \sigma_1 \sigma_2 \rangle_c \,,$$

$$\left\langle (\delta N)^3 \right\rangle_c = \left(\frac{g_i}{(2\pi)^3}\right)^3 \left(\prod_{i=1,2,3} \left(\frac{1}{E_i}\int d^3p_i \int_{\Sigma_i} p_{i\mu} d\sigma_i^\mu d\eta_i\right)\right) \frac{f_{01} f_{02} f_{03}}{\gamma_1\gamma_2\gamma_3} \left(-\frac{g^3}{T^3} \langle \sigma_1 \sigma_2 \sigma_3 \rangle_c\right) ,$$

$$\left\langle (\delta N)^4 \right\rangle_c = \left(\frac{g_i}{(2\pi)^3}\right)^4 \left(\prod_{i=1,2,3,4} \left(\frac{1}{E_i}\int d^3p_i \int_{\Sigma_i} p_{i\mu} d\sigma_i^\mu d\eta_i\right)\right) \frac{f_{01} f_{02} f_{03} f_{04}}{\gamma_1\gamma_2\gamma_3\gamma_4} \frac{g^4}{T^4} \langle \sigma_1 \sigma_2 \sigma_3 \sigma_4 \rangle_c$$

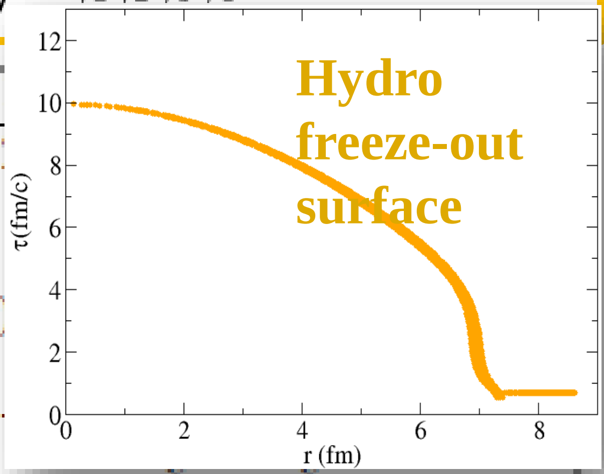
$$P[\sigma] \sim \exp\{-\Omega[\sigma]/T\}, \qquad \Omega[\sigma] = \int d^3x \left[\frac{1}{2}(\nabla\sigma)^2 + \ldots\right.$$

$$\langle \sigma_1 \sigma_2 \rangle_c = T D(x_1 - x_2),$$

$$\langle \sigma_1 \sigma_2 \sigma_3 \rangle_c = -2T^2 \lambda_3 \int d^3z D(x_1 - z) D(x_2 - z) D(x_3 - z \ldots$$

$$\langle \sigma_1 \sigma_2 \sigma_3 \sigma_4 \rangle_c = -6T^3 \lambda_4 \int d^3z D(x_1 - z) D(x_2 - z) D(x_3 \ldots$$

$$+12T^3 \lambda_3^2 \int d^3u \int d^3v D(x_1 - u) D(x_2 - u) D(x_3 - v) D(x_4 - v) D(u - v).$$



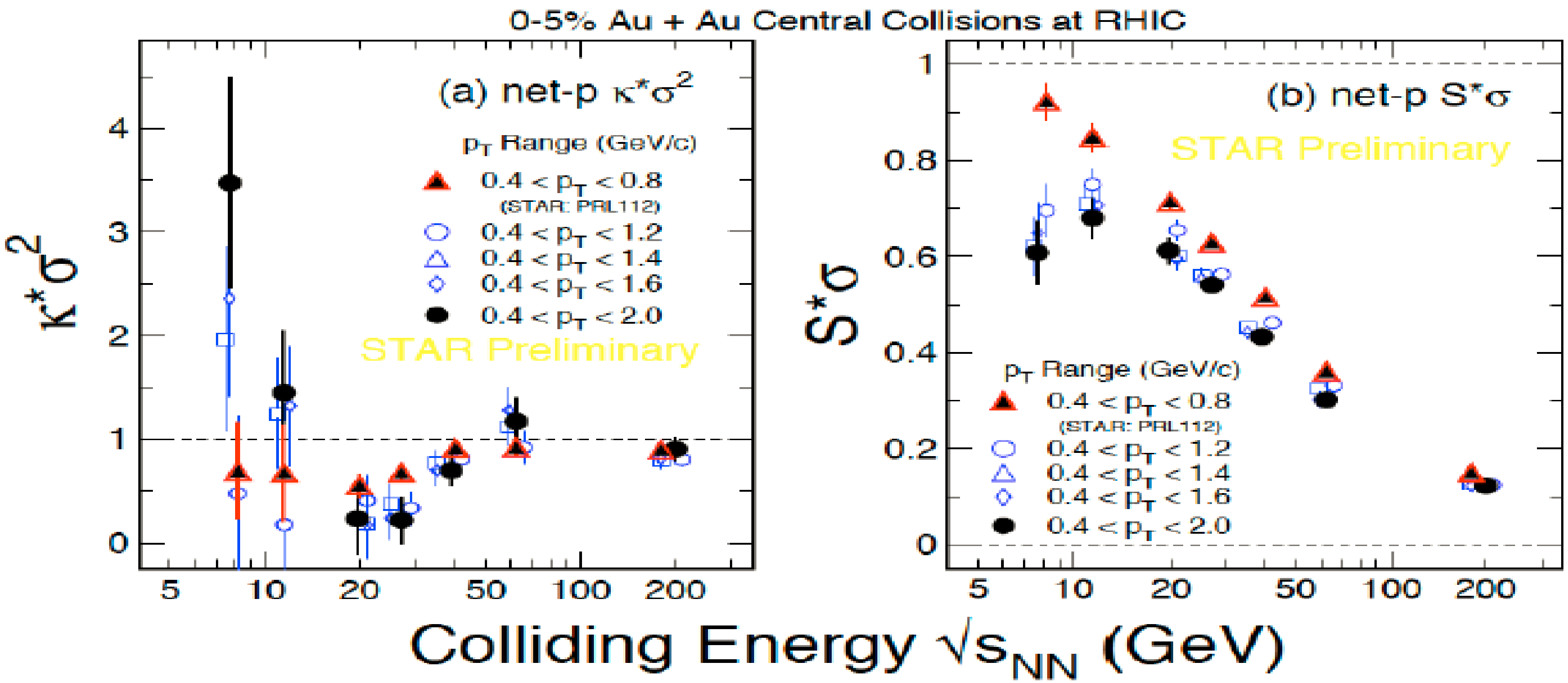
For simplicity: We assume that the correlated sigma field only influence the particle emissions near Tc, which does not influence the evolution of the bulk matter

-- Static critical fluctuations along the freeze-out surface

Comparison with the experimental data

STAR data (acceptance dependence)

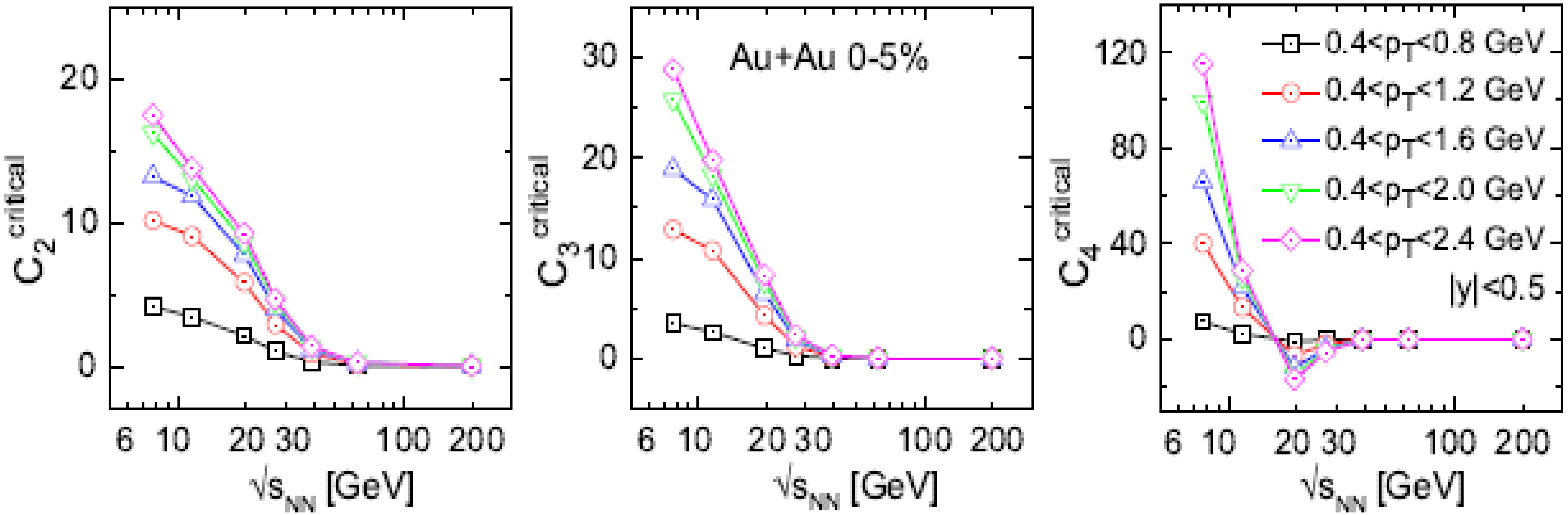
Xiaofeng Luo CPOD 2014



-Wider p_T range lead to more pronounced fluctuation signals

Transverse momentum acceptance dependence

Jiang, Li & Song, arXiv: 1512.06164



-The critical fluctuations are significantly enhanced with the p_T ranges increased to 0.4-2.0 GeV

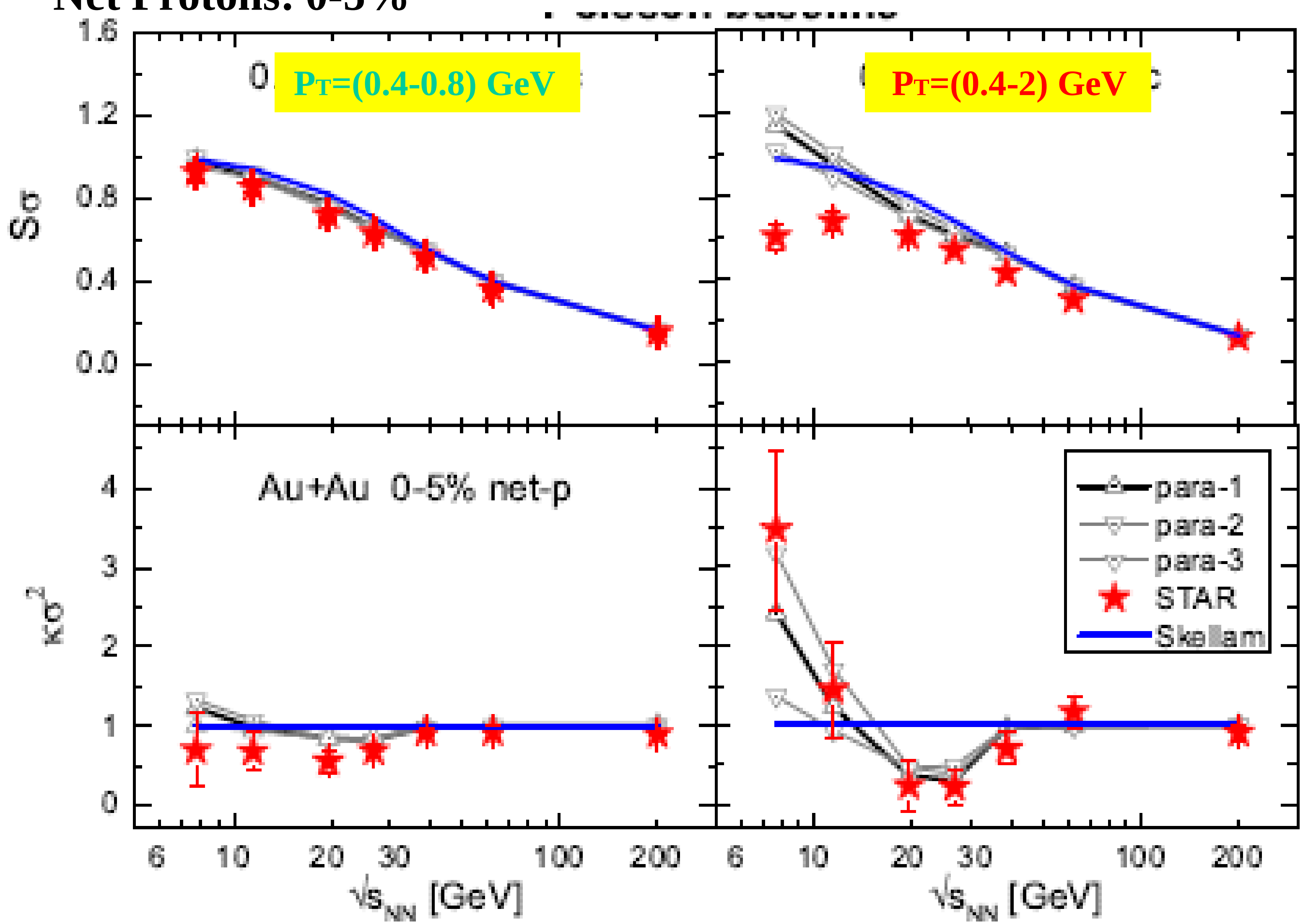
- At lower collision energies, the dramatically increased mean value of net protons also leads to dramatically enhanced critical fluctuations

-Critical fluctuations are influenced by both the mean value (average number) of net protons within specific acceptance window and the correlation length.

$\kappa\sigma^2, S\sigma$: (Model + Poisson baselines)

Jiang, Li & Song, arXiv: 1512.06164

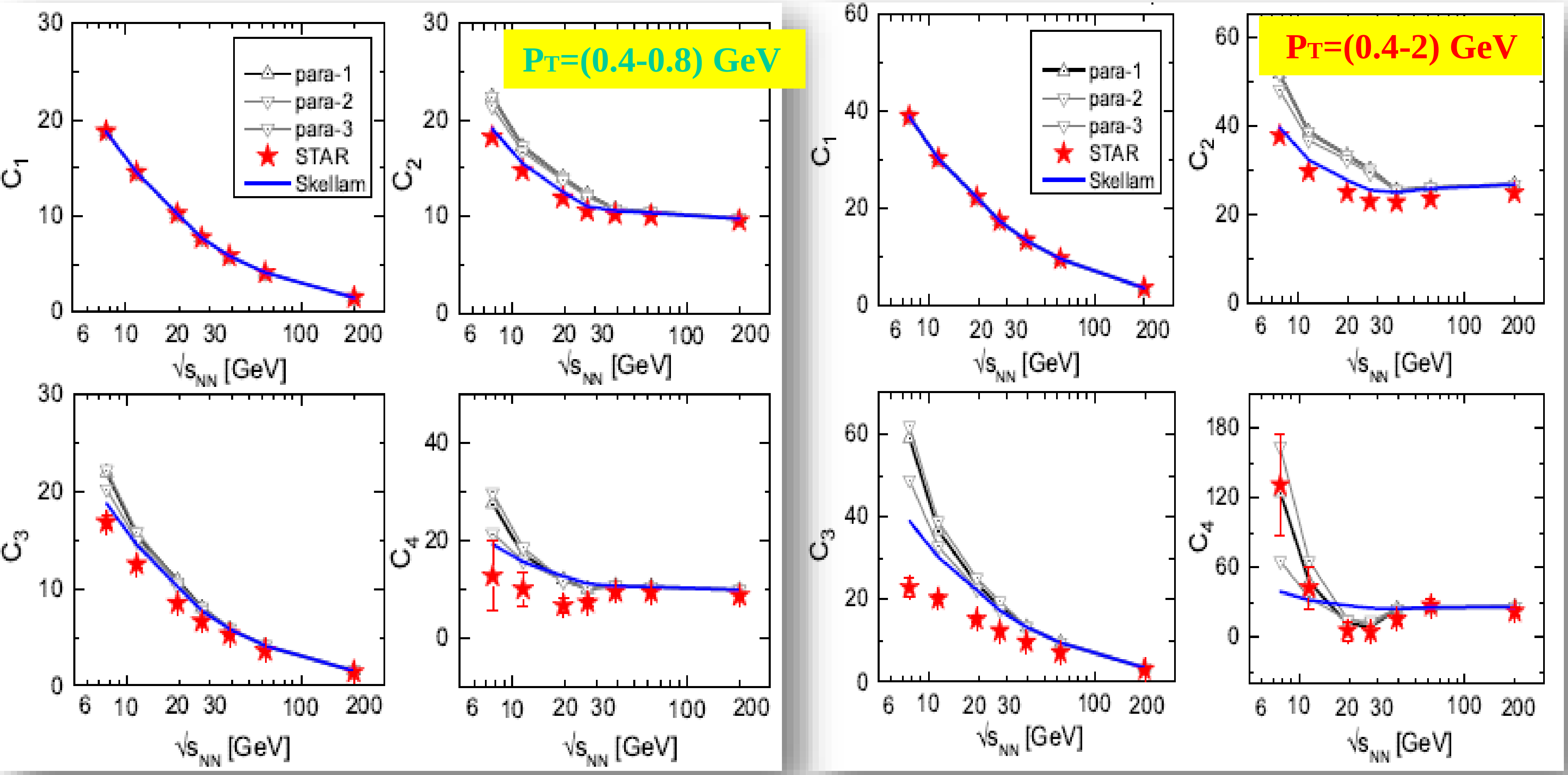
Net Protons: 0-5%



C_1 C_2 C_3 C_4: (Model + Poisson baselines)

Net Protons 0-5%

Jiang, Li & Song, arXiv: 1512.06164



Critical fluctuations give positive contribution to C_2 , C_3; well above the poisson baselines, can NOT explain/describe the C_2 , C_3 data

C_1 C_2 C_3 C_4: Pt-(0.4-2) GeV (Model + Poisson baselines)

PT=(0.4-2) GeV

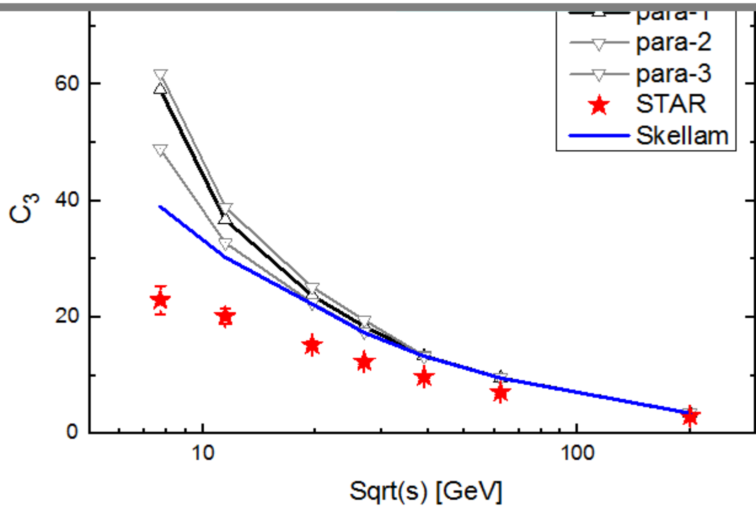
$$P[\sigma] \sim \exp\{-\Omega[\sigma]/T\}, \qquad \Omega[\sigma] = \int d^3x \left[\frac{1}{2}(\nabla\sigma)^2 + \frac{1}{2}m_\sigma^2\sigma^2 + \frac{\lambda_3}{3}\sigma^3 + \frac{\lambda_4}{4}\sigma^4\right],$$

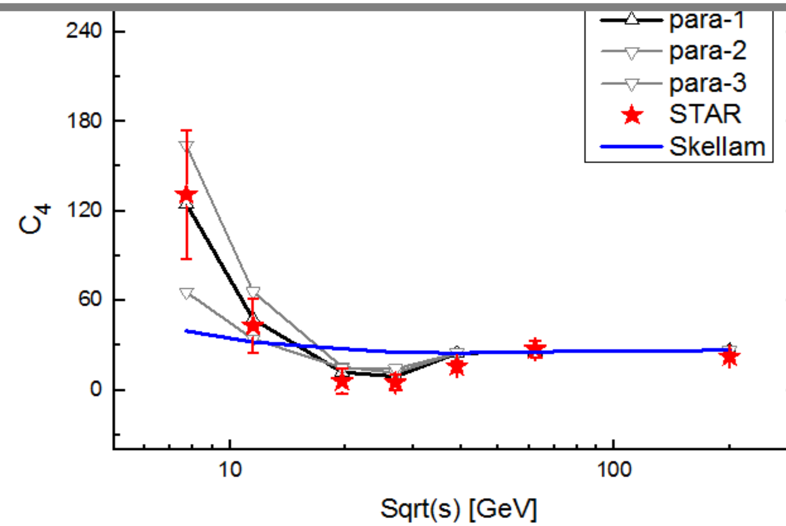
$$\langle\sigma_1\sigma_2\rangle_c = TD(x_1 - x_2),$$

$$\langle\sigma_1\sigma_2\sigma_3\rangle_c = -2T^2\lambda_3 \int d^3z D(x_1 - z)\, D(x_2 - z) D(x_3 - z),$$

$$\langle\sigma_1 \ldots P[\sigma] \sim \exp\{-\Omega[\sigma]/T\},\ \ldots d^3z D(x_1 - z)\, D(x_2 - z) D(x_3 - z)\, D(x_4 - z)$$

$$+12T^3\lambda_3^2 \int d^3u \int d^3v D(x_1 - u)\, D(x_2 - u) D(x_3 - v)\, D(x_4 - v)\, D(u - v).$$





The contributions from STATIC critical fluctuations to C_2 , C_3 are always positive **(Both this model & early Stephanov PRL09 framework)**

Dynamical Critical Fluctuations

Real time evolution of non-Gaussian cumulants

Mukherjee, Venugopalan & Yin PRC 2015

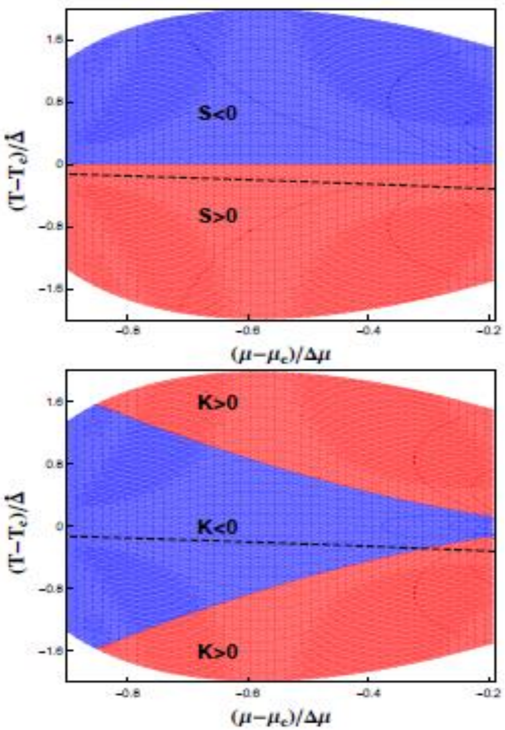
Coupled equations for higher order cumulants:

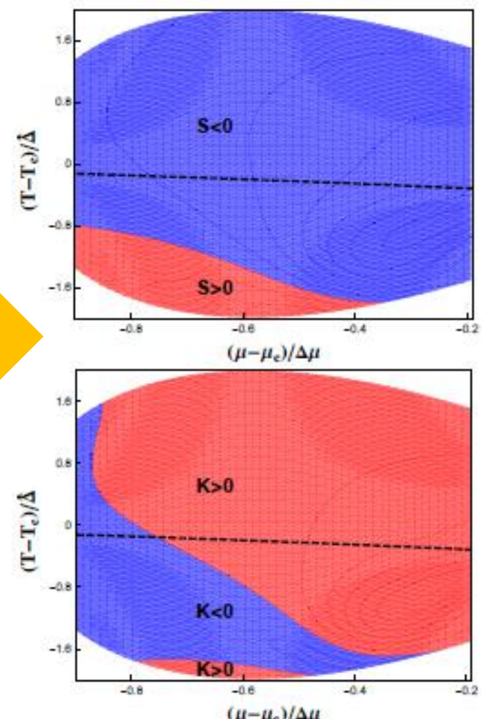
$$\sigma \equiv \frac{1}{V}\int d^3x\, \sigma(x)\,,$$

$$\partial_\tau \kappa_2(\tau) = -2\,\tau_{\text{eff}}^{-1}\,(b^2)\left[\left(\frac{\kappa_2}{b^2}\right)F_2(M) - 1\right]\left[1 + \mathcal{O}(\epsilon^2)\right]\,,$$

$$\partial_\tau \kappa_3(\tau) = -3\,\tau_{\text{eff}}^{-1}\,(\epsilon\, b^3)\left[\left(\frac{\kappa_3}{\epsilon\, b^3}\right)F_2(M) + \left(\frac{\kappa_2}{b^2}\right)^2 F_3(M)\right]\left[1 + \mathcal{O}(\epsilon^2)\right]$$

$$\partial_\tau \kappa_4(\tau) = -4\,\tau_{\text{eff}}^{-1}\,(\epsilon^2\, b^4)\left\{\left(\frac{\kappa_4}{\epsilon^2\, b^4}\right)F_2(M) + 3\left(\frac{\kappa_2}{b^2}\right)\left(\frac{\kappa_3}{\epsilon\, b^3}\right)F_3(M) + \left(\frac{\kappa_2}{b^2}\right)^3 F_4\right\} \times \left[1 + \mathcal{O}(\epsilon^2)\right]$$





-Critical slowing down limits growth of correlation length

-Non-Gaussian cumulants do not follow growth of the correlation length

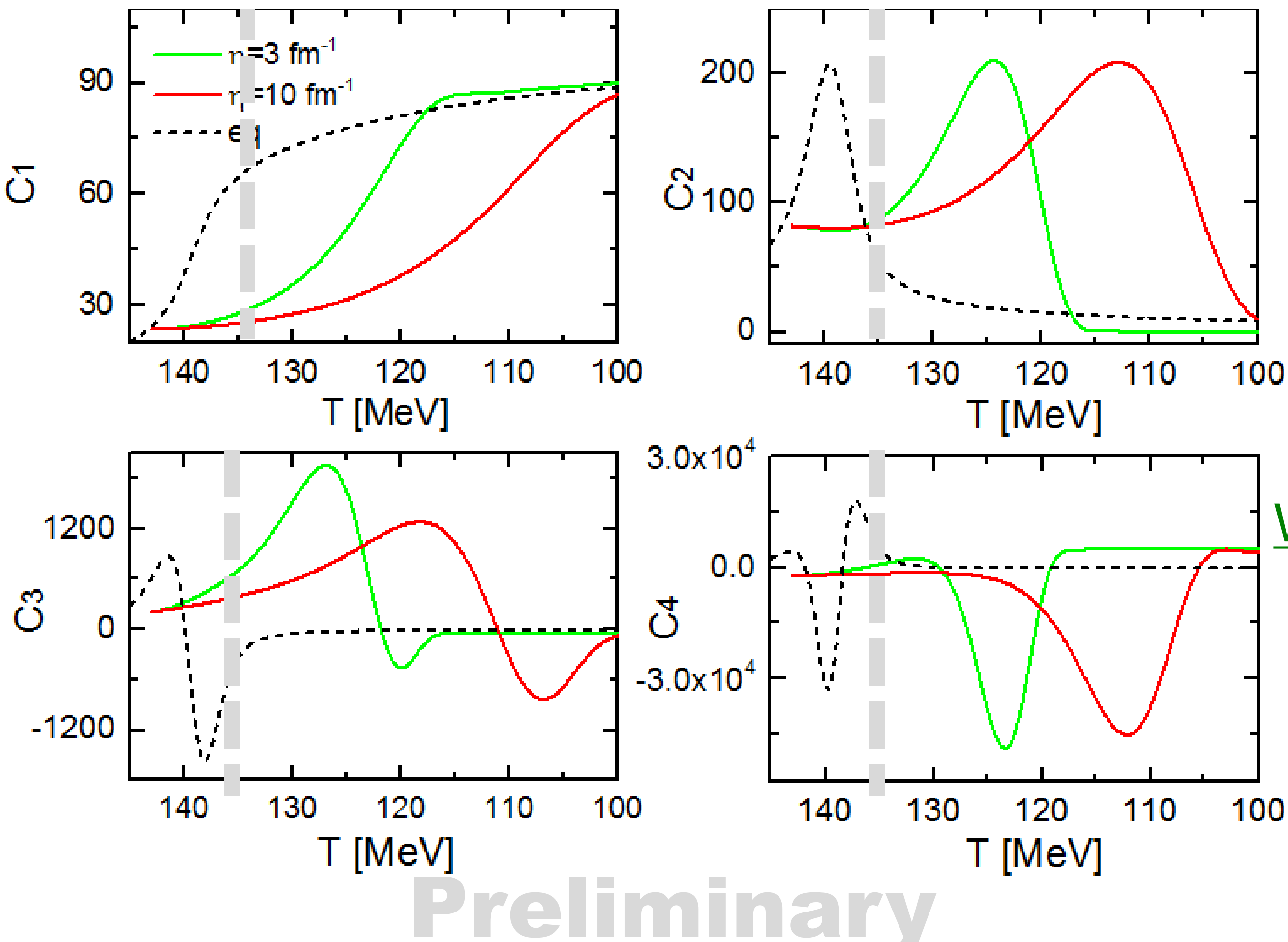
-Sign of the non -Gaussian cumulants can be different from the equilibrium one

Dynamical critical fluctuations of the sigma field

Langevin dynamics: $\partial^\mu \partial_\mu \sigma(t,x) + \eta \partial_t \sigma(t,x) + V'_{eff}(\sigma) = \xi(t,x)$

with effective potential from linear sigma model with constituent quarks

$$V_{eff}(\sigma) = U(\sigma) + \Omega_{\bar{q}q}(T,\sigma) = \frac{\lambda^2}{4}\left(\sigma^2 - \nu^2\right)^2 - h_q\sigma - U_0 - 2d_q T \int \frac{d^3p}{(2\pi)^3} \ln\left(1 + \exp\left(-\frac{E}{T}\right)\right)$$



-The sign of C_3 is different from the equilibrium one due to the memory effects

Work in the near future

Coupling sigma field with particles; Study the dynamical critical fluctuations of net protons

Summary and outlook

Static critical fluctuations:

-qualitatively explain the acceptance dependence of critical fluctuations

-C_4 and $\kappa\sigma^2$ can be reproduced through tuning the parameters of the model

-However C_2 , C_3 are well above the poisson/BN baselines, which can NOT explain/describe the data

Dynamical critical fluctuations:

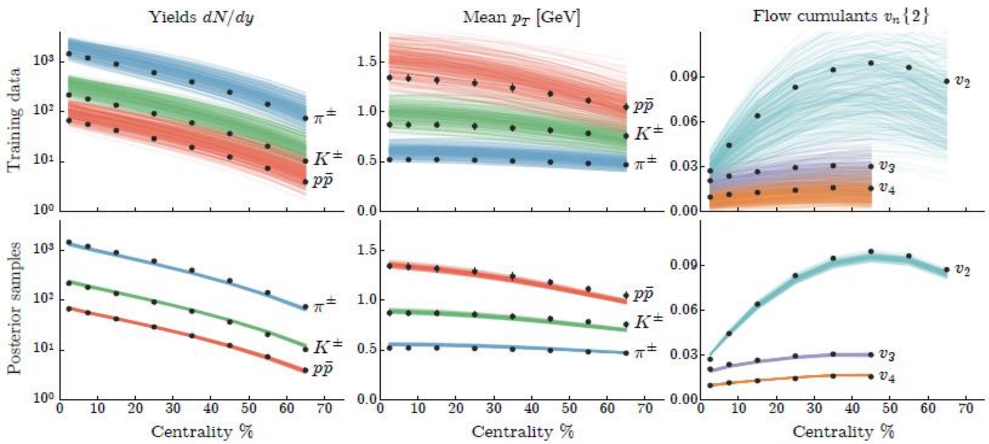
-Sign of the C_3, C_4 cumulants can be different from the equilibrium one due to the memory effects

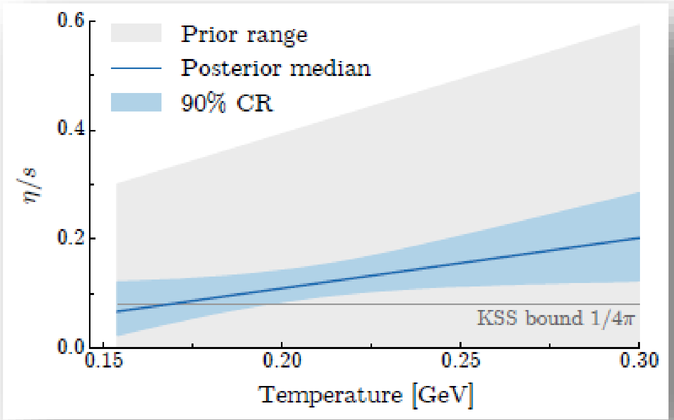
Initial State Fluctuations

- Hydrodynamics and hybrid model has been fully developed

-Lots of efforts from both exp and theory to study the initial state fluctuations and final state correlations

-the QGP shear viscosity has been extracted with massive data evaluations!





Critical Fluctuations

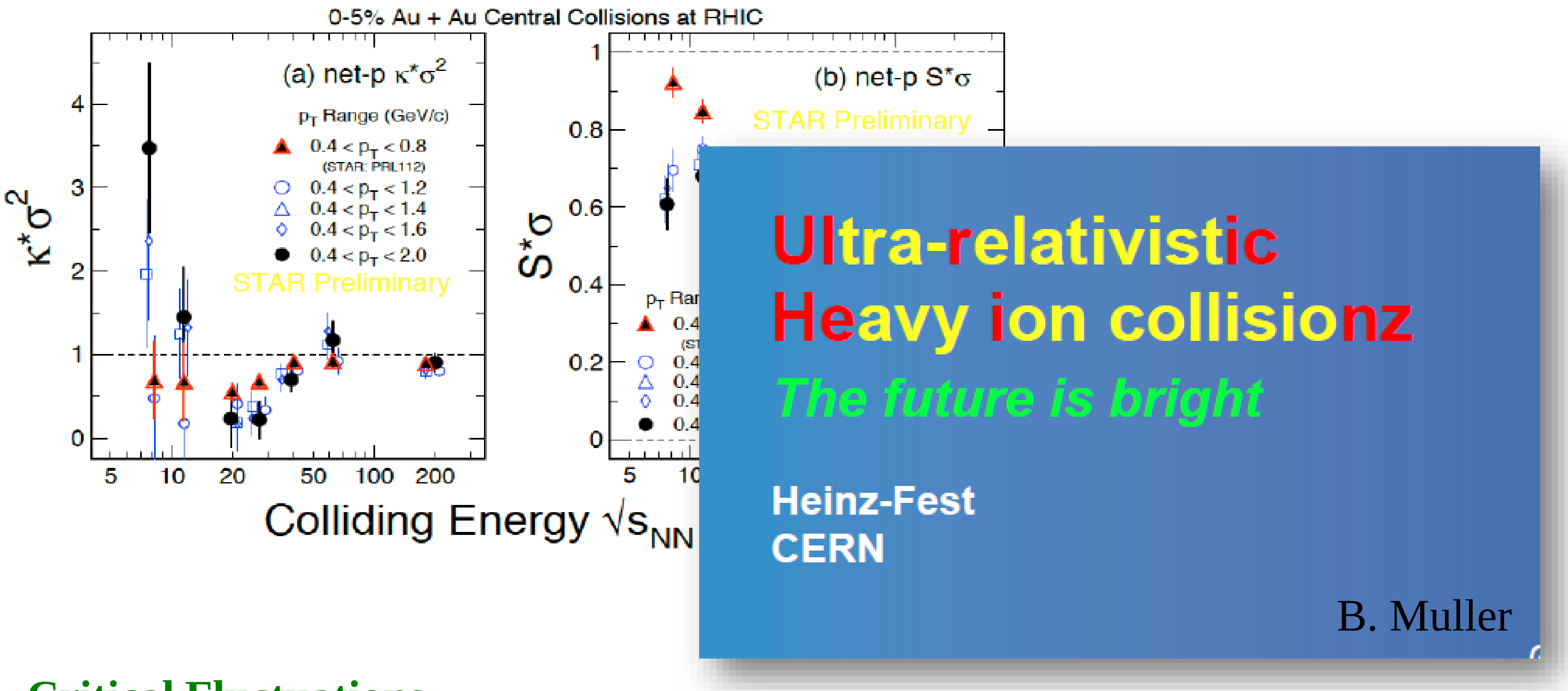
-full development of the dynamical model near the critical point is needed

-microscopic/ macroscopic evolution of the bulk of matter, together with the evolution of the order parameter field

-proper treatment of freeze-out with the external field

-interactions between thermal & critical fluctuations

… … … …



Critical Fluctuations

-full development of the dynamical model near the critical point is needed

-microscopic/ macroscopic evolution of the bulk of matter, together with the evolution of the order parameter field

-proper treatment of freeze-out with the external field

-interactions between thermal & critical fluctuations

-where is the critical points located in the (T μ) plane ?

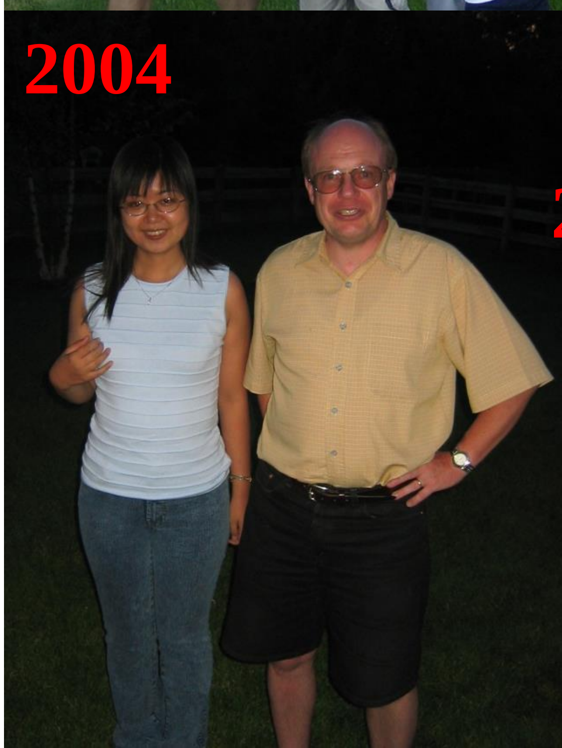
-what is the effective correlation length ξ ?

… … … …

2004 Party at Ulrich's Home

2009 Graduation

2004



2009 workshop at Trento

Thanksgiving Turkey

Happy 60th Birthday

Thank You

Boltzmann approach with external field

Stephanov PRD 2010

$$\mathcal{S} = \int d^3x \frac{1}{2}(\partial_\mu \sigma \partial^\mu \sigma - U(\sigma)) - \int ds M(\sigma),$$

$$\begin{cases} \partial^2\sigma + dU/d\sigma + (dM/d\sigma)\int_p f/\gamma = 0. \\ \frac{p^\mu}{M}\frac{\partial f}{\partial x^\mu} + \partial^\mu M \frac{\partial f}{\partial p^\mu} + \mathcal{C}[f] = 0, \end{cases}$$

-analytical solution with perturbative expansion, please refer to Stephanov PRD 2010

Stationary solution for the Boltamann equation with external field

$$f_\sigma(\boldsymbol{p}) = e^{\mu/T} e^{-\gamma(\boldsymbol{p})M/T}.$$

Effective particle mass: $M = M(\sigma) = g\sigma$

Freeze-out scheme for the dynamical models near T_{cr}

Method A:

$$\left\langle (\delta N)^2 \right\rangle_c = \left(\frac{g_i}{(2\pi)^3}\right)^2 \left(\prod_{i=1,2} \left(\frac{1}{E_i}\int d^3p_i \int_{\Sigma_i} p_{i\mu} d\sigma_i^\mu d\eta_i\right)\right) \frac{f_{01}f_{02}}{\gamma_1\gamma_2}\frac{g^2}{T^2}\langle\sigma_1\sigma_2\rangle_c\,,$$

$$\left\langle (\delta N)^3 \right\rangle_c = \left(\frac{g_i}{(2\pi)^3}\right)^3 \left(\prod_{i=1,2,3} \left(\frac{1}{E_i}\int d^3p_i \int_{\Sigma_i} p_{i\mu} d\sigma_i^\mu d\eta_i\right)\right) \frac{f_{01}f_{02}f_{03}}{\gamma_1\gamma_2\gamma_3}\left(-\frac{g^3}{T^3}\langle\sigma_1\sigma_2\sigma_3\rangle_c\right),$$

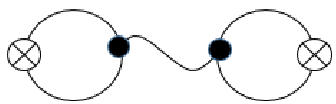
$$\left\langle (\delta N)^4 \right\rangle_c = \left(\frac{g_i}{(2\pi)^3}\right)^4 \left(\prod_{i=1,2,3,4} \left(\frac{1}{E_i}\int d^3p_i \int_{\Sigma_i} p_{i\mu} d\sigma_i^\mu d\eta_i\right)\right) \frac{f_{01}f_{02}f_{03}f_{04}}{\gamma_1\gamma_2\gamma_3\gamma_4}\frac{g^4}{T^4}\langle\sigma_1\sigma_2\sigma_3\sigma_4\rangle_c$$

Method B:

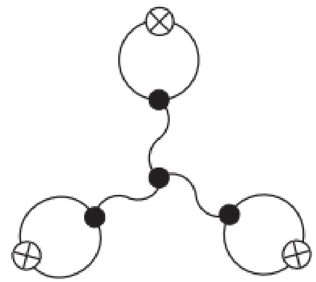
$$E\frac{dN}{d^3p} = \int_\Sigma \frac{p_\mu d\sigma^\mu}{2\pi^3} f(x,p)$$

With Modified $f(x,p)$ $M \longrightarrow g\sigma(x)$

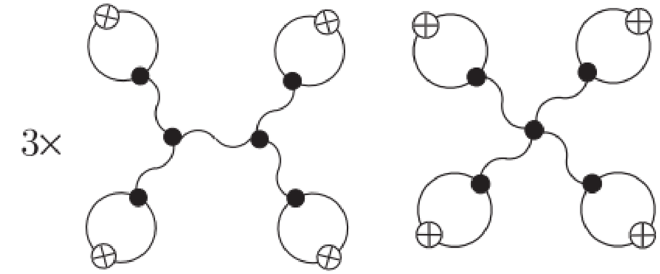
The choice of input parameters



$g_{\sigma pp}$ ξ



$g_{\sigma pp}$ ξ λ_3



$g_{\sigma pp}$ ξ λ_3 λ_4

- $g_{\sigma pp} \sim (0, 10)$

  phenomenological model

- $\xi \sim$ 3fm (max value)

  near the critical point, critical s

- $\lambda_3 \sim (0, 8)$, $\lambda_4 \sim (4, 20)$

  lattice simulation of the effective
  point.

$\sqrt{s_{NN}}[GeV]$		7.7	11.5	19.6	27	39	62.4	200
para-I	g	3.2	2.5	2.3	2.2	2	1.8	1
	λ_3	6	4	3	2	0	0	0
	λ_4	14	13	12	11	10	9	8
	ξ	1	2	3	3	2	1	0.5
para-II	g	3.2	2.5	2.3	2.2	2	1.8	1
	λ_3	6	4	3	2	2	1.5	1
	λ_4	14	13	12	11	10	9	8
	ξ	1	2.5	4	4	3	2	1
para-III	g	2.8	1.8	1.7	1.6	1	0.5	0.1
	λ_3	6	4	3	2	2	1.5	1
	λ_4	14	13	12	11	10	9	8
	ξ	1	2	3	3	2	1	0.5

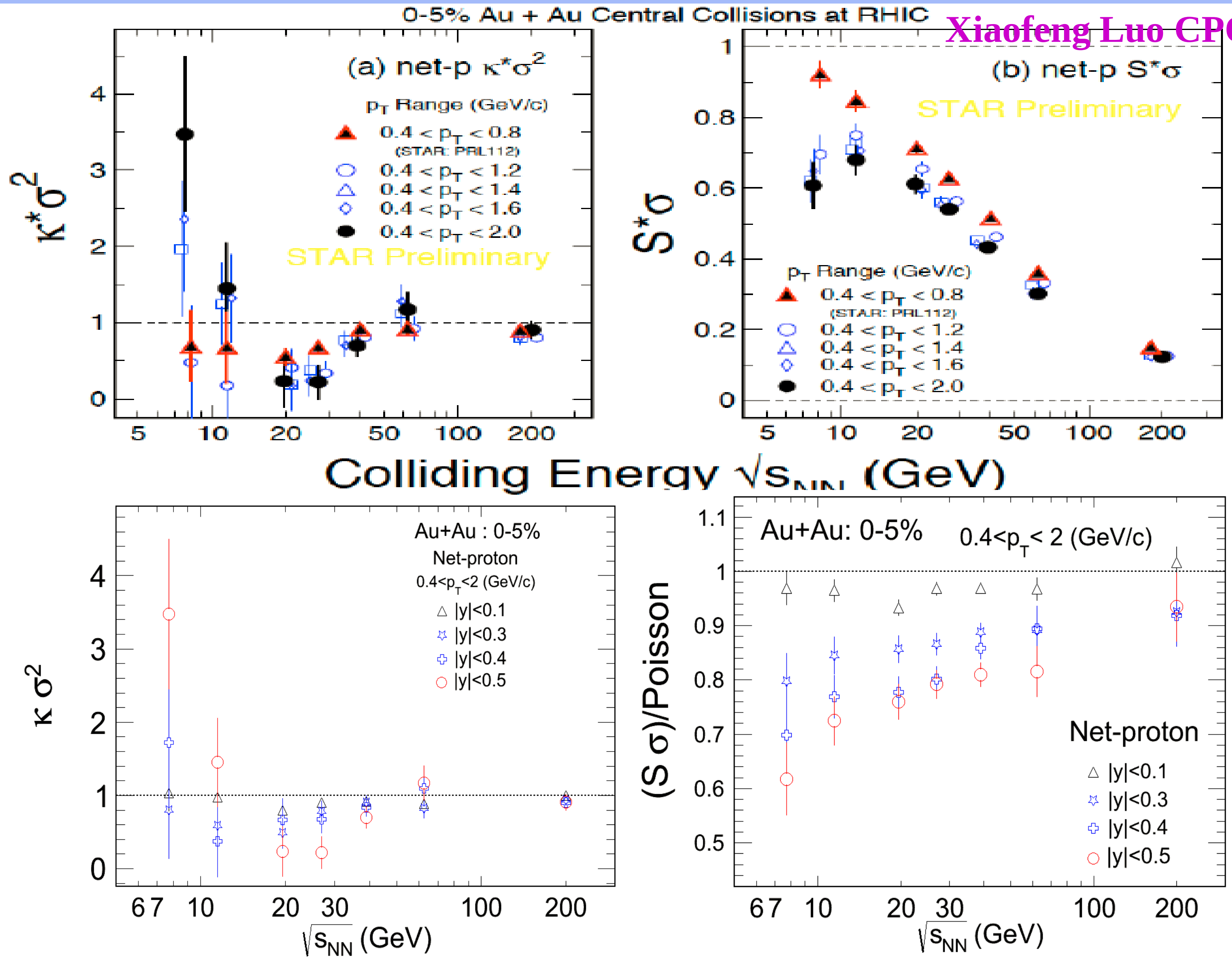
pin,

61,

Jiang, Li & Song, arXiv: 1512.06164

STAR data (acceptance dependence)

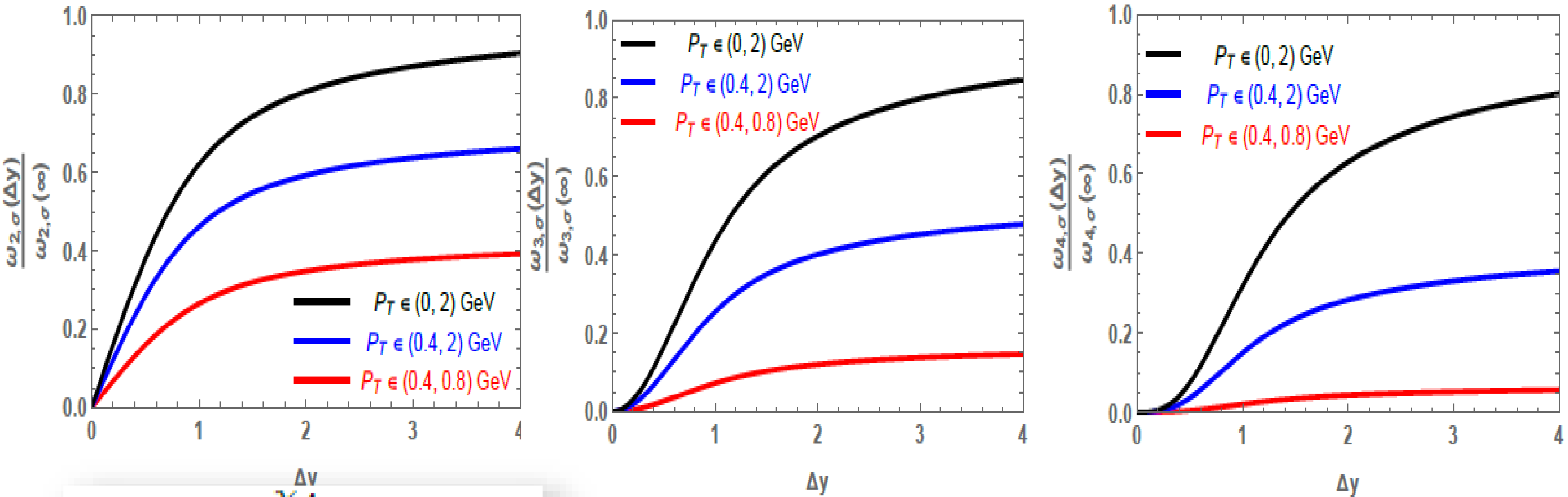
Xiaofeng Luo CPOD 2014



-Wider p_T or y acceptance lead to more pronounce fluctuation signals

Rapidity acceptance dependence

Ling & Stephanov PRC2016



$$(\delta f_A)_\sigma = -\frac{\chi_A}{\gamma_A} g\sigma(x_A),$$

$$\langle \sigma(x)\sigma(y)\rangle \to T\xi^2\delta^3(x-y)$$

$$\langle \sigma(x)\sigma(y)\sigma(z)\rangle \to -2\tilde{\lambda}_3 T^{3/2}\xi^{9/2}\delta^6(x,y,z)$$

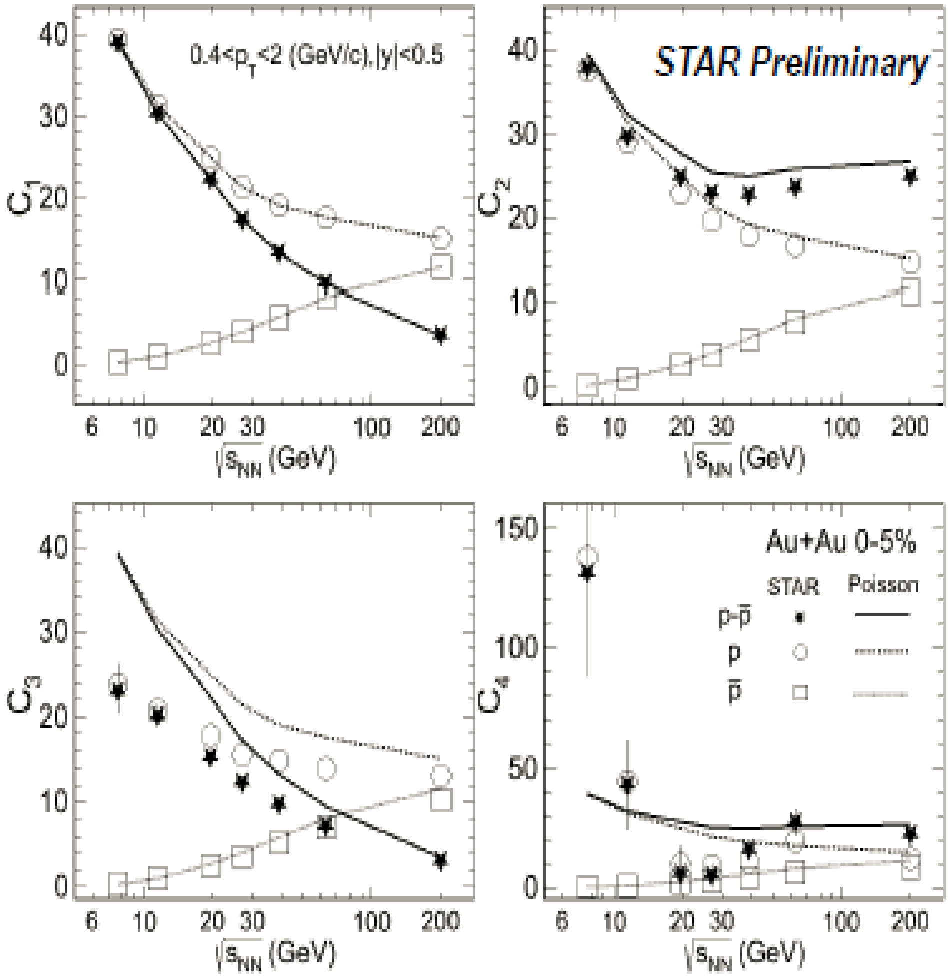
$$\langle \sigma(x)\sigma(y)\sigma(z)\sigma(w)\rangle_c \to 6(2\tilde{\lambda}_3^2 - \tilde{\lambda}_4)T^2\xi^7\delta^9(x,y,z,w)$$

- freeze-out surface: Blast Wave model:

-The dependence on transverse momentum acceptance is very significant

- extension the rapidity coverage will significantly increase the magnitude of critical fluctuations

Cumulants vs. Poisson



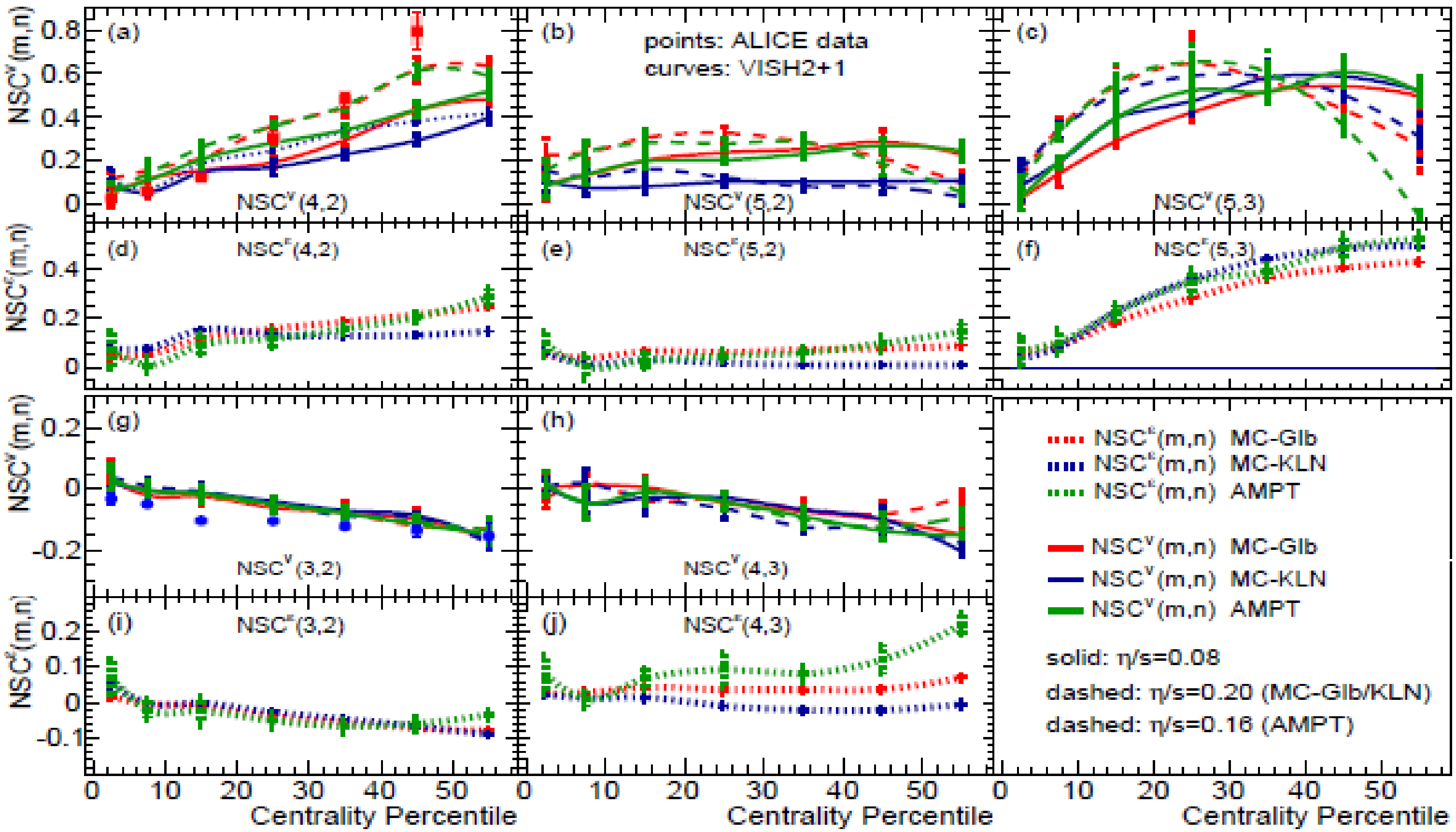
Fluctuations measured in experiment:

critical fluct. + non-critical (thermal) fluct. + …

Normalized Symmetric Cumulants NSC^v(m,n)

$$NSC^v(m,n) = \frac{SC^v(m,n)}{\langle v_m^2\rangle\langle v_n^2\rangle} = \frac{\langle v_m^2 v_n^2\rangle - \langle v_m^2\rangle\langle v_n^2\rangle}{\langle v_m^2\rangle\langle v_n^2\rangle}$$

$$NSC^\varepsilon(m,n) = \frac{SC^\varepsilon(m,n)}{\langle \varepsilon_m^2\rangle\langle \varepsilon_n^2\rangle} = \frac{\langle \varepsilon_m^2 \varepsilon_n^2\rangle - \langle \varepsilon_m^2\rangle\langle \varepsilon_n^2\rangle}{\langle \varepsilon_m^2\rangle\langle \varepsilon_n^2\rangle}$$



Zhu, Xu, Zhou, Song, in preparation

-NSC^v (3,2): insensitive to η/s and initial conditions, roughly fit the ALICE data

-NSC^v (4,2), NSC^v (5,2) & NSC^v (5,3): sensitive to η/s and initial conditions