

# Effects of hadronic rescatterings in small system

Tetsufumi Hirano

Sophia Univ.

Ultra-Relativistic Heavy Ion Collisions 2016

18-20 July 2016, CERN

# Contents

1. My early career and collaboration with Ulrich

2. Integrated dynamical model

3. Initial longitudinal structure

4. Results in d+Au

5. Hydro + mini-jet model

6. Summary

# Ulrich's paper triggered one of my first studies



ELSEVIER

Nuclear Physics A638 (1998) 357c–364c

NUCLEAR  
PHYSICS A

Hadronic Observables: Theoretical Highlights\*

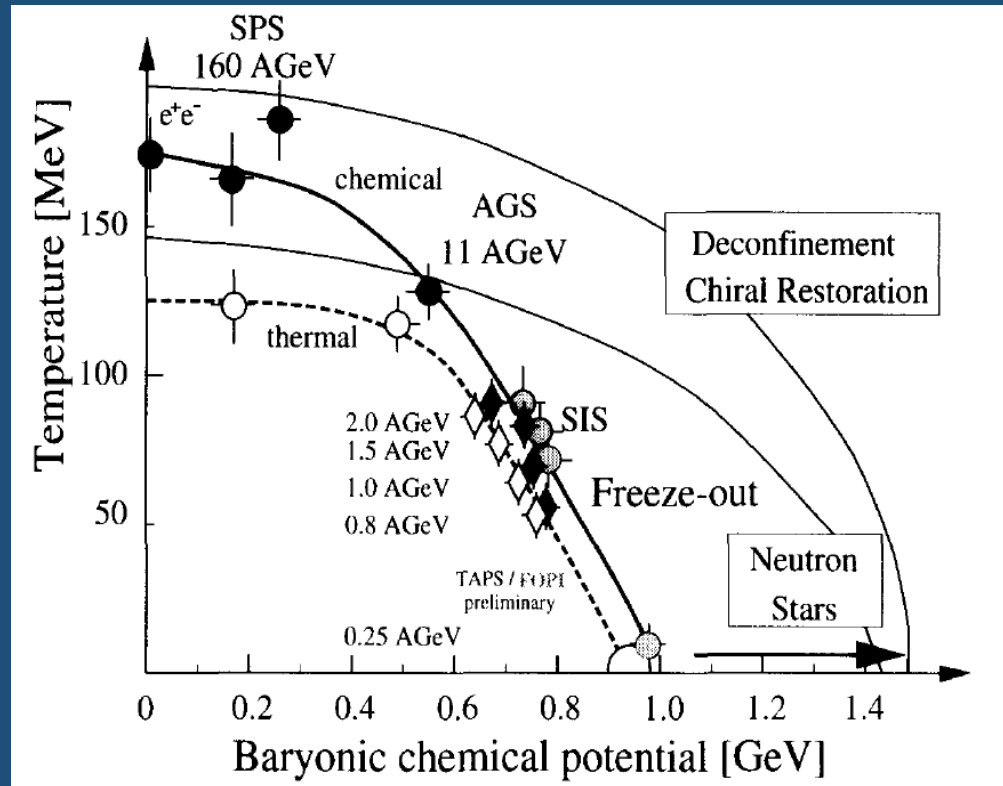
Ulrich Heinz

Inst. für Theoretische Physik, Universität Regensburg, D-93040 Regensburg, Germany

I present highlights from the parallel sessions on the theory of hadronic observables in  $e^+e^-$ , hadronic and nuclear reactions.

Proceedings of QM97 at Tsukuba  
(My very first QM attendance)

$$T_{\text{ch f.o.}} > T_{\text{th f.o.}}$$



### 2.3. Rescattering – yes or no?

In his overview of the beam energy dependence of flow phenomena Ollitrault showed that all three types of flow appear in the data simultaneously, pointing to rescattering among the secondary hadrons as a common origin. The difference between the chemical and thermal freeze-out points in Fig. 1 suggests significant elastic rescattering between hadronization and decoupling, causing expansion and cooling of the momentum distributions. (Elastic collisions include resonance channels like  $\pi + N \rightarrow \Delta \rightarrow \pi + N$  which do not change particle abundances.) While present SPS data are consistent with a common

# Idea of partial chemical equilibrium in hydro

PHYSICAL REVIEW C **66**, 054905 (2002)

**Collective flow and two-pion correlations from a relativistic hydrodynamic model  
with early chemical freeze-out**

Tetsufumi Hirano<sup>1,\*</sup> and Keiichi Tsuda<sup>2</sup>

<sup>1</sup>*Physics Department, University of Tokyo, Tokyo 113-0033, Japan*

<sup>2</sup>*Department of Physics, Waseda University, Tokyo 169-8555, Japan*

(Received 8 August 2002; published 27 November 2002)

Yield and slope of  $p_T$  spectra  
at once within hydro

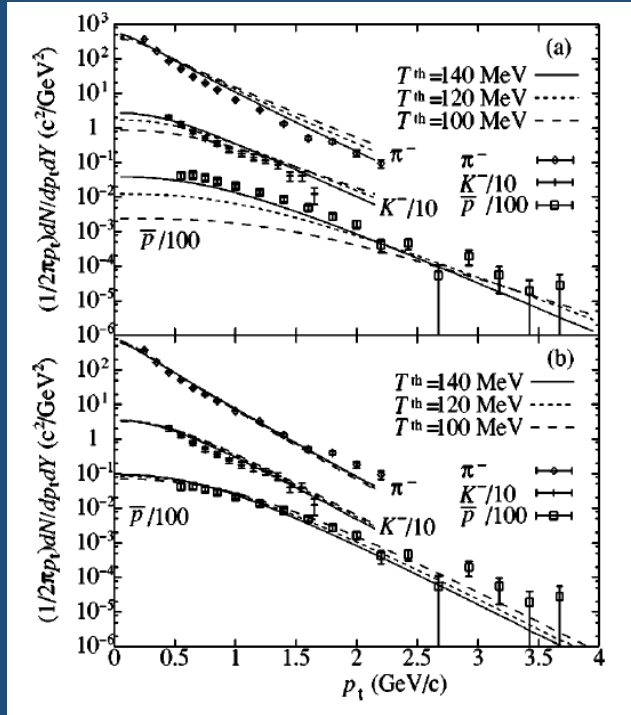
Similar ideas:

D.Teaney (2002)

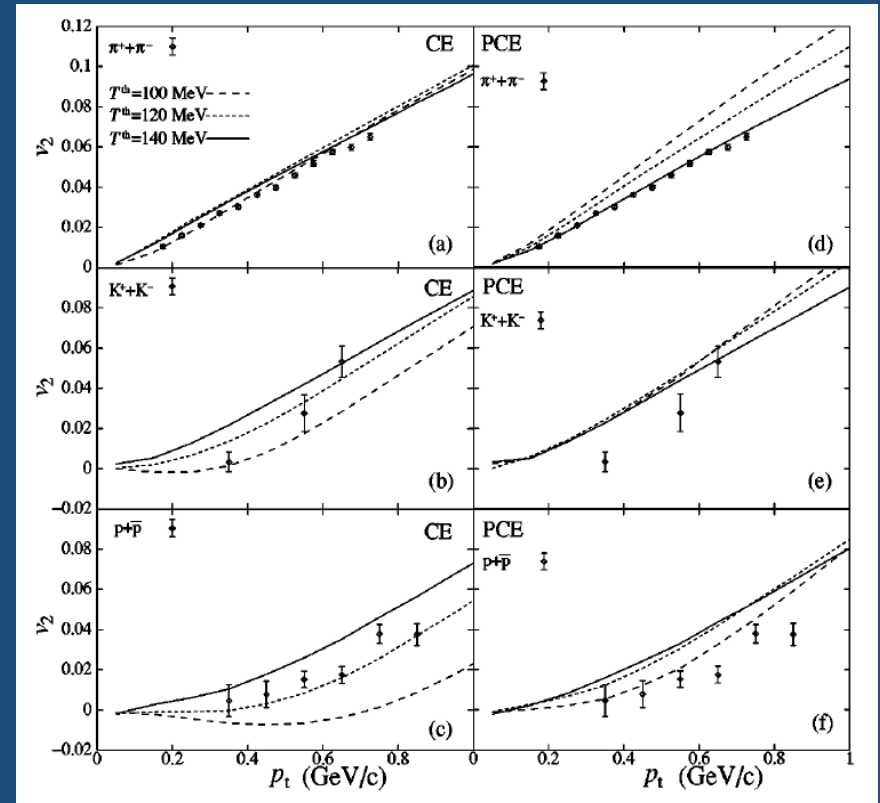
P.Kolb and R.Rapp (2003)

P.Huovinen (2008)

# Slope of $v_2(p_T)$ ?



Upper: Chem. Eq.  
Lower: Partial Chem. Eq.



PCE spoils success of hydro!

Most people: Why is  $v_2$  sensitive to final stage? Simply wrong!  
It seemed to me Ulrich was at least interested in these results.

# 8 papers with Ulrich

The screenshot shows a web browser window with the URL `inspirehep.net/search?ln=ja&p=f+a+Hirano+and+Heinz&of=hb&action_search=検索&sf=earliestdate&so=d`. The browser displays a list of 8 search results, each corresponding to a paper by Tetsufumi Hirano and Ulrich Heinz. The results are numbered 1 through 8 and include the following information for each entry:

- 1. Thoughts on heavy-ion physics in the high luminosity era: the soft sector**  
Fedenco Antinori (INFN, Padua) *et al.*, Apr 12, 2016, 19 pp.  
CERN-TH-2016-086  
e-Print: [arXiv:1604.03310 \[hep-ph\]](#) | [PDF](#)  
[References](#) | [BibTeX](#) | [LaTeX\(US\)](#) | [LaTeX\(EU\)](#) | [Harvmac](#) | [EndNote](#)  
[CERN Document Server](#); [ADS Abstract Service](#)  
[このページの詳細](#)
- 2. The QGP shear viscosity: Elusive goal or just around the corner?**  
Chun Shen (Ohio State U.), Steffen A. Bass (Duke U.), Tetsufumi Hirano (Sophia U. & Tokyo U.), Pasi Huovinen (Frankfurt U.), Zhi Qiu (Ohio State U.), Huichao Song (LBL, Berkeley), Ulrich Heinz (Ohio State U.), Jun 2011, 4 pp.  
Published in *J.Phys.* **G38** (2011) 124045  
DOI: [10.1088/0954-3899/38/12/124045](#)  
Conference: [C110523.2 Proceedings](#)  
e-Print: [arXiv:1106.6350 \[nucl-th\]](#) | [PDF](#)  
[References](#) | [BibTeX](#) | [LaTeX\(US\)](#) | [LaTeX\(EU\)](#) | [Harvmac](#) | [EndNote](#)  
[ADS Abstract Service](#)  
[このページの詳細](#) - Cited by 40 records
- 3. Hadron spectra and elliptic flow for 200 A GeV Au+Au collisions from viscous hydrodynamics coupled to a Boltzmann cascade**  
Huichao Song (LBL, Berkeley), Steffen A. Bass (Duke U.), Ulrich Heinz (Ohio State U.), Tetsufumi Hirano (LBL, Berkeley & Tokyo U.), Chun Shen (Ohio State U.), Jan 2011, 12 pp.  
Published in *Phys.Rev.C* **83** (2011) 054910, Erratum: *Phys.Rev.C* **86** (2012) 059903  
DOI: [10.1103/PhysRevC.86.059903](#), [10.1103/PhysRevC.83.054910](#)  
e-Print: [arXiv:1101.4638 \[nucl-th\]](#) | [PDF](#)  
[References](#) | [BibTeX](#) | [LaTeX\(US\)](#) | [LaTeX\(EU\)](#) | [Harvmac](#) | [EndNote](#)  
[ADS Abstract Service](#)  
[このページの詳細](#) - Cited by 105 records
- 4. 200 A GeV Au+Au collisions serve a nearly perfect quark-gluon liquid**  
Huichao Song (LBL, Berkeley & Ohio State U.), Steffen A. Bass (Duke U.), Ulrich Heinz (Ohio State U.), Tetsufumi Hirano (Tokyo U. & LBL, Berkeley), Chun Shen (Ohio State U.), Nov 2010, 4 pp.  
Published in *Phys.Rev.Lett.* **106** (2011) 192301, Erratum: *Phys.Rev.Lett.* **109** (2012) 139904  
DOI: [10.1103/PhysRevLett.109.139904](#), [10.1103/PhysRevLett.106.192301](#)  
e-Print: [arXiv:1011.2783 \[nucl-th\]](#) | [PDF](#)  
[References](#) | [BibTeX](#) | [LaTeX\(US\)](#) | [LaTeX\(EU\)](#) | [Harvmac](#) | [EndNote](#)  
[ADS Abstract Service](#)  
[このページの詳細](#) - Cited by 242 records
- 5. Hadronic dissipative effects on transverse dynamics at RHIC**  
T. Hirano (Tokyo U.), U.W. Heinz (Ohio State U. & CERN), D. Kharzeev (Brookhaven), R. Lacey (SUNY, Stony Brook), Y. Nara (Akita Intl. U.), May 2008, 4 pp.  
Published in *J.Phys.* **G35** (2008) 104124  
DOI: [10.1088/0954-3899/35/10/104124](#)  
To appear in the proceedings of Conference: [C08-02-04.2 Proceedings](#)  
e-Print: [arXiv:0805.0064 \[nucl-th\]](#) | [PDF](#)  
[References](#) | [BibTeX](#) | [LaTeX\(US\)](#) | [LaTeX\(EU\)](#) | [Harvmac](#) | [EndNote](#)  
[CERN Document Server](#); [ADS Abstract Service](#)  
[このページの詳細](#) - Cited by 2 records
- 6. Mass ordering of differential elliptic flow and its violation for phi mesons**  
Tetsufumi Hirano (Tokyo U.), Ulrich W. Heinz (Ohio State U. & CERN), Dmitri Kharzeev (Brookhaven), Roy Lacey (SUNY, Stony Brook, Chem. Dept.), Yasushi Nara (Akita Intl. U.), Nov 2007, 14 pp.  
Published in *Phys.Rev.* **C77** (2008) 044909  
CERN-PH-TH-2007-209  
DOI: [10.1103/PhysRevC.77.044909](#)  
e-Print: [arXiv:0710.3795 \[nucl-th\]](#) | [PDF](#)  
[References](#) | [BibTeX](#) | [LaTeX\(US\)](#) | [LaTeX\(EU\)](#) | [Harvmac](#) | [EndNote](#)  
[CERN Document Server](#); [ADS Abstract Service](#); [CERN Server](#)  
[このページの詳細](#) - Cited by 99 records
- 7. Elliptic Flow from a Hybrid CGC, Full 3D Hydro and Hadronic Cascade Model**  
Tetsufumi Hirano (Tokyo U.), Ulrich W. Heinz (Ohio State U.), Dmitri Kharzeev (Brookhaven), Roy Lacey (SUNY, Stony Brook, Chem. Dept.), Yasushi Nara (Frankfurt U.), Jan 2007, 4 pp.  
Published in *J.Phys.* **G34** (2007) S879-882  
DOI: [10.1088/0954-3899/34/9/S117](#)  
Presented at Conference: [C06-11-14 Proceedings](#)  
e-Print: [nucl-th/0701075](#) | [PDF](#)  
[References](#) | [BibTeX](#) | [LaTeX\(US\)](#) | [LaTeX\(EU\)](#) | [Harvmac](#) | [EndNote](#)  
[ADS Abstract Service](#)  
[このページの詳細](#) - Cited by 42 records
- 8. Hadronic dissipative effects on elliptic flow in ultrarelativistic heavy-ion collisions**  
Tetsufumi Hirano (Columbia U.), Ulrich W. Heinz (Ohio State U.), Dmitri Kharzeev (Brookhaven), Roy Lacey (SUNY, Stony Brook), Yasushi Nara (Frankfurt U.), Nov 2005, 6 pp.  
Published in *Phys.Lett.* **B636** (2006) 299-304  
DOI: [10.1016/j.physletb.2006.03.080](#)  
e-Print: [nucl-th/0511046](#) | [PDF](#)  
[References](#) | [BibTeX](#) | [LaTeX\(US\)](#) | [LaTeX\(EU\)](#) | [Harvmac](#) | [EndNote](#)  
[ADS Abstract Service](#)  
[このページの詳細](#) - Cited by 380 records

# Very first full 3D hydro + cascade



Available online at [www.sciencedirect.com](http://www.sciencedirect.com)

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Physics Letters B 636 (2006) 299–304

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PHYSICS LETTERS B

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[www.elsevier.com/locate/physletb](http://www.elsevier.com/locate/physletb)

## Hadronic dissipative effects on elliptic flow in ultrarelativistic heavy-ion collisions

Tetsufumi Hirano<sup>a,\*</sup>, Ulrich Heinz<sup>b</sup>, Dmitri Kharzeev<sup>c</sup>, Roy Lacey<sup>d</sup>, Yasushi Nara<sup>e</sup>

<sup>a</sup> Department of Physics, Columbia University, 538 West 120th Street, New York, NY 10027, USA

<sup>b</sup> Department of Physics, Ohio State University, 191 West Woodruff Avenue, Columbus, OH 43210, USA

<sup>c</sup> Nuclear Theory Group, Physics Department, Brookhaven National Laboratory, Upton, NY 11973-5000, USA

<sup>d</sup> Department of Chemistry, SUNY Stony Brook, Stony Brook, NY 11794-3400, USA

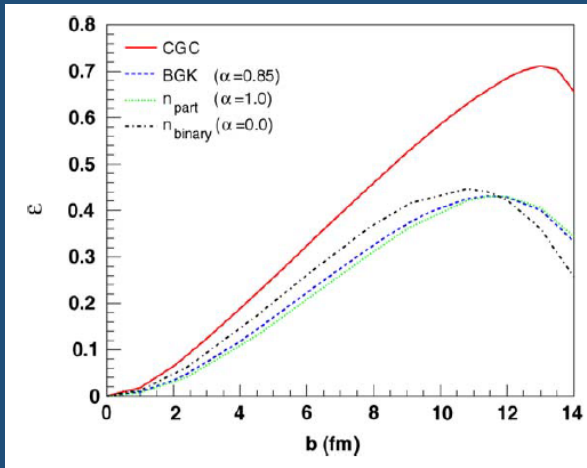
<sup>e</sup> Institut für Theoretische Physik, J.W. Goethe-Universität, Max v. Laue Str. 1, D-60438 Frankfurt, Germany

Received 17 November 2005; received in revised form 27 January 2006; accepted 27 March 2006

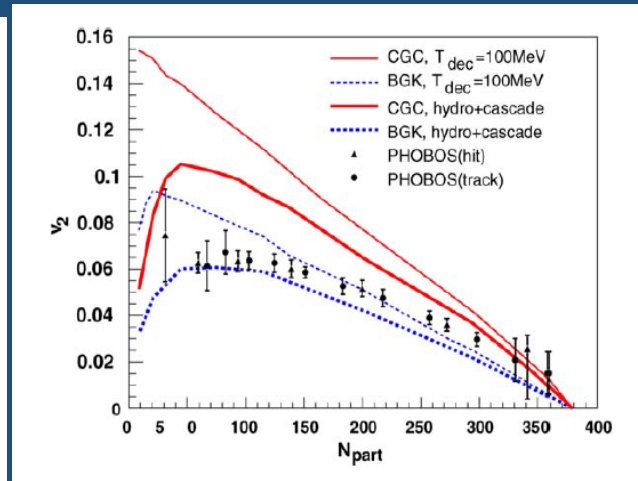
Available online 7 April 2006

Editor: J.-P. Blaizot

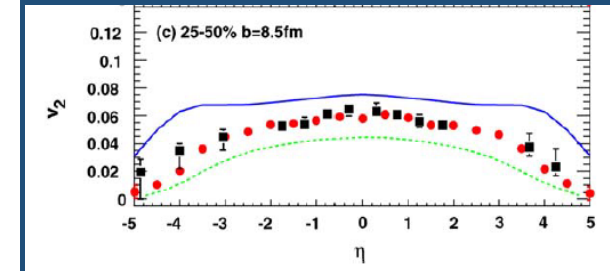
# Main results



$$\varepsilon_{2,CGC} > \varepsilon_{2,Glauber}$$



Initial condition:  
CGC vs. Glauber



$v_2(\eta)$ : Crucial  
roll played by  
hadronic  
rescatterings

These ideas survive in recent hydro analysis

# Contents

1. My early career and collaboration with Ulrich

2. Integrated dynamical model

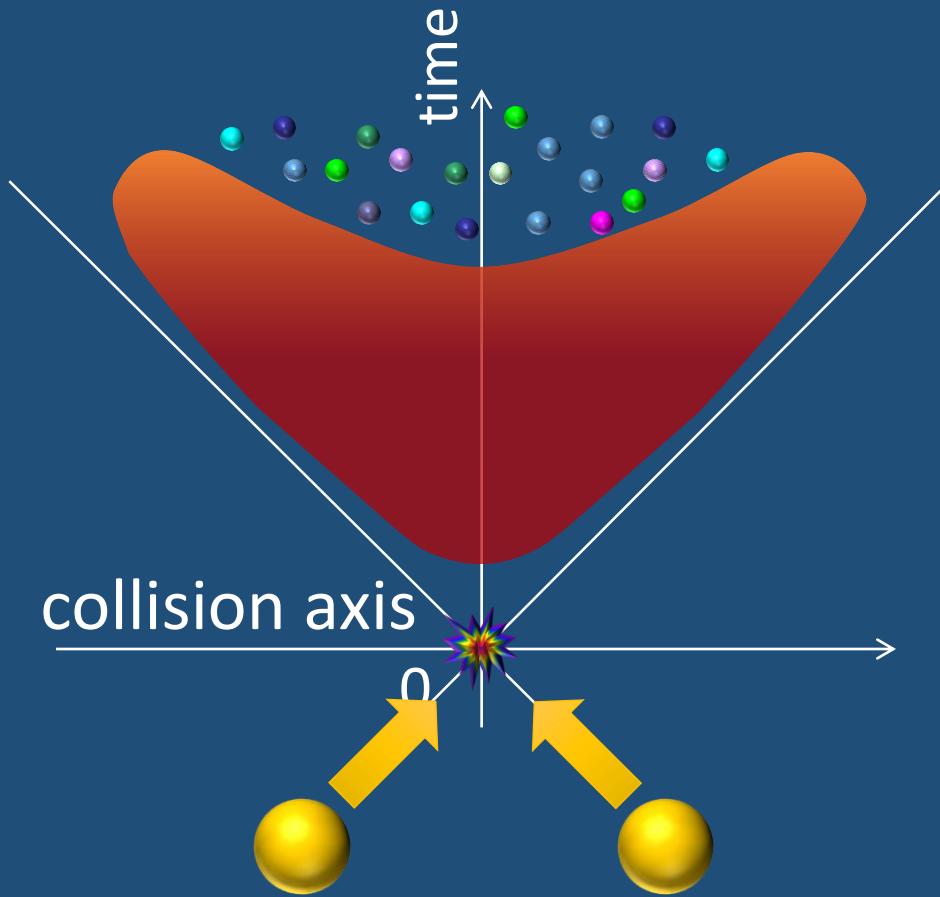
3. Initial longitudinal structure

4. Results in d+Au

5. Hydro + mini-jet model

6. Summary

# Integrated dynamical model



Hadronic cascade model  
Microscopic description of  
Hadron gases (JAM)

Relativistic hydrodynamics  
Space-time evolution of  
QGP+hadronic fluids

**Lattice EOS**

Monte-Carlo Glauber/KLN  
model

Initial profile of matter

Hirano, Huovinen, Murase, Nara (2013)  
Takeuchi, MHHN (2015)

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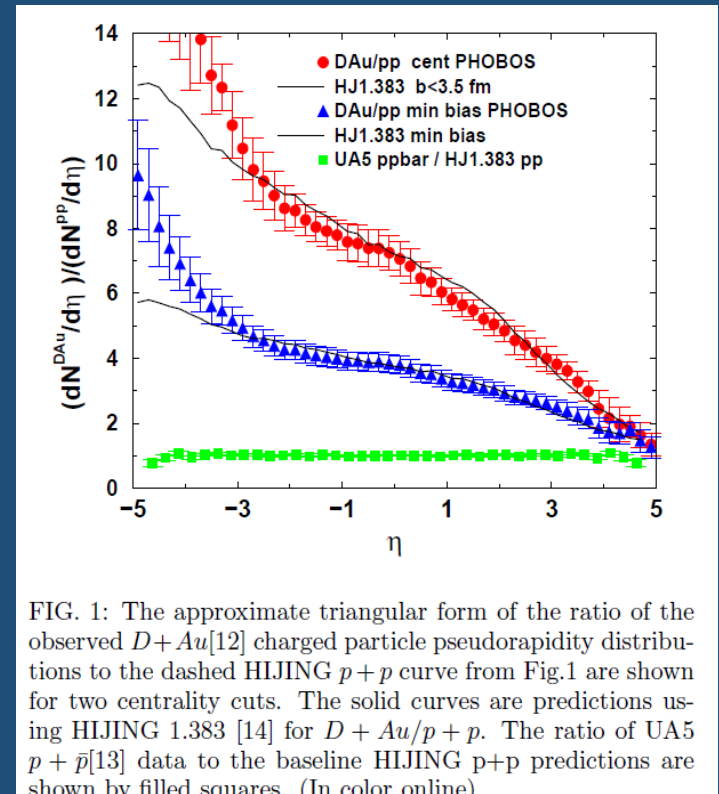
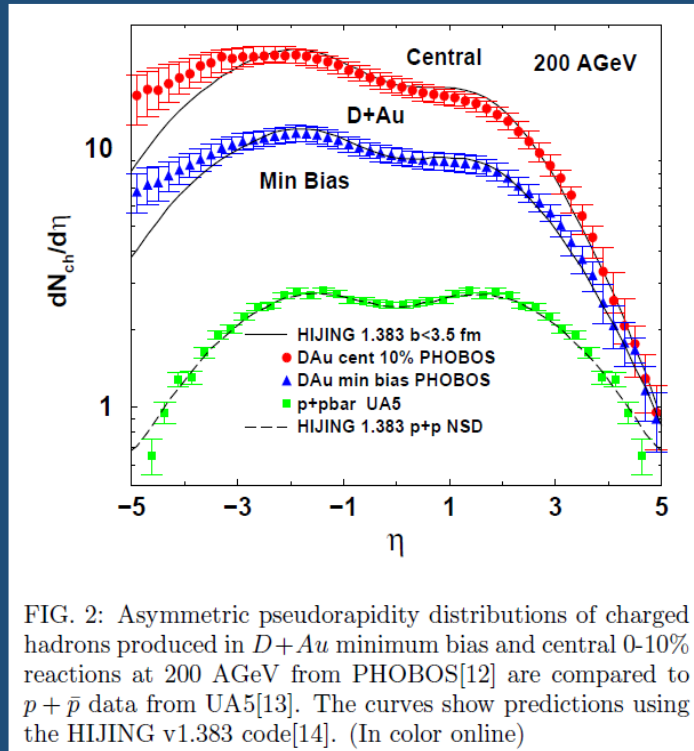
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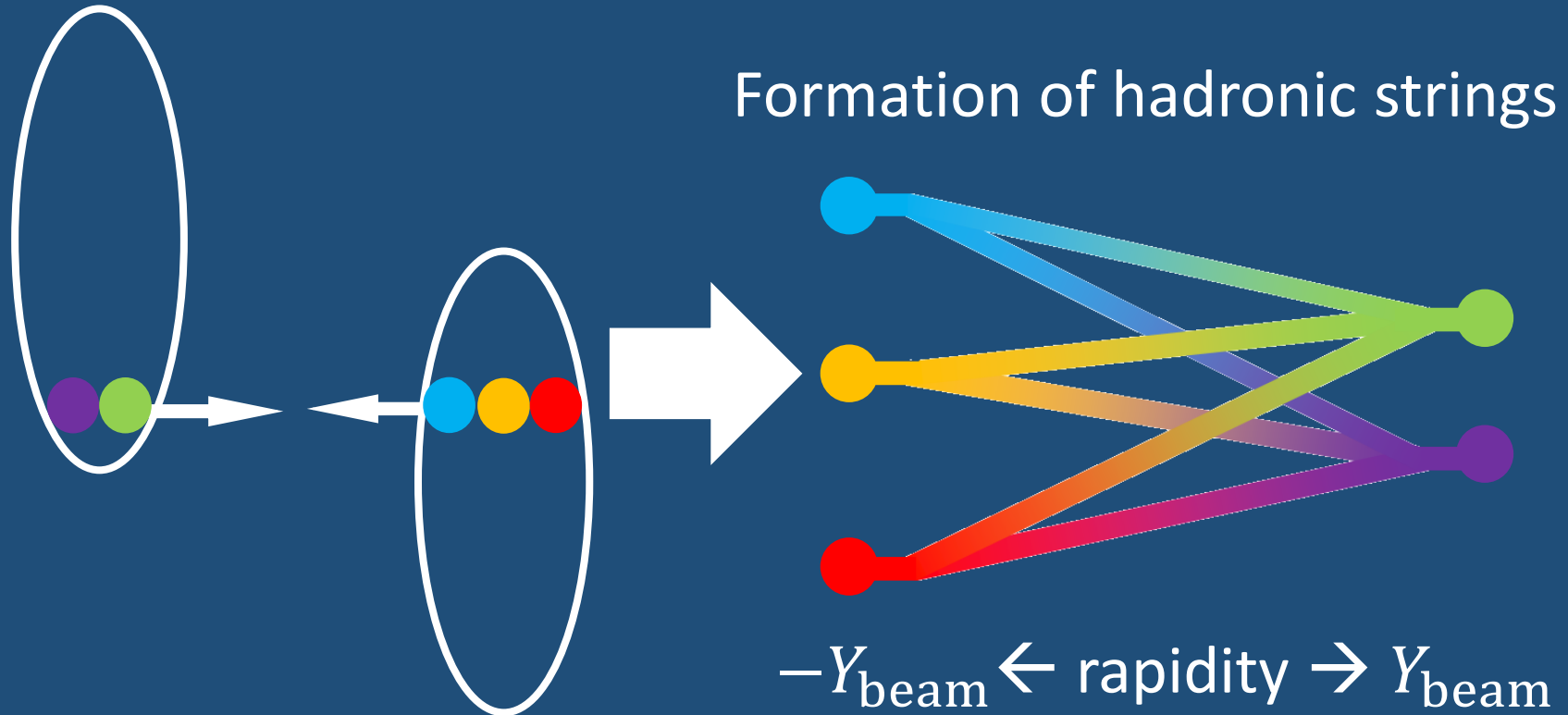
6. Summary

# Rapidity triangle/trapezoid of $(dN_{dAu}/d\eta)/(dN_{pp}/d\eta)$



Figures from  
Adil and Gyulassy (2005)

# Modified BGK model

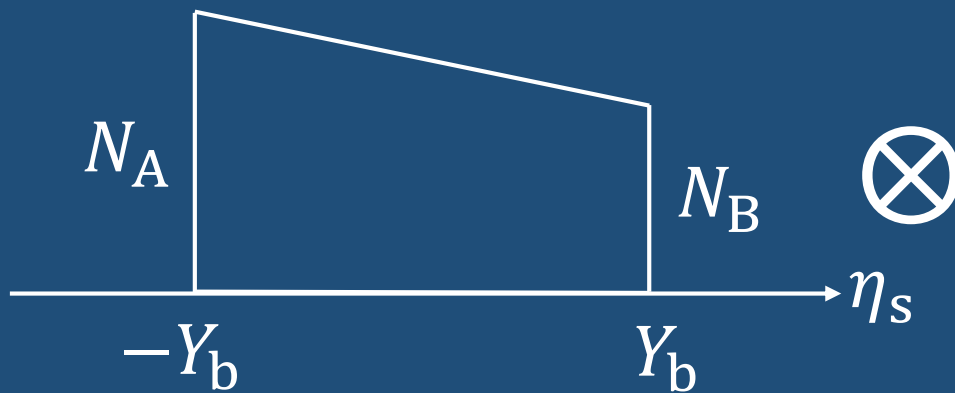


Number of participants  
from MC-Glauber model

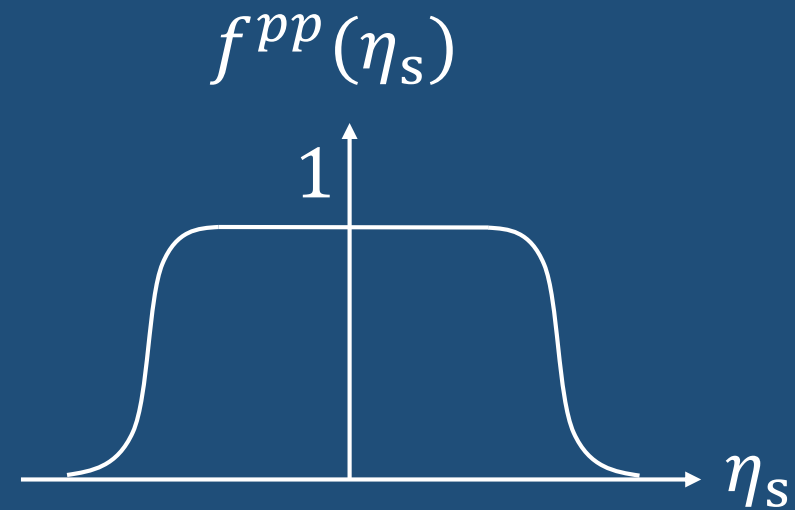
Parametrization of  
entropy density dist.

# Rapidity dependence of initial entropy density

$$s_0(\tau_0, \eta_s, x_\perp) \propto$$

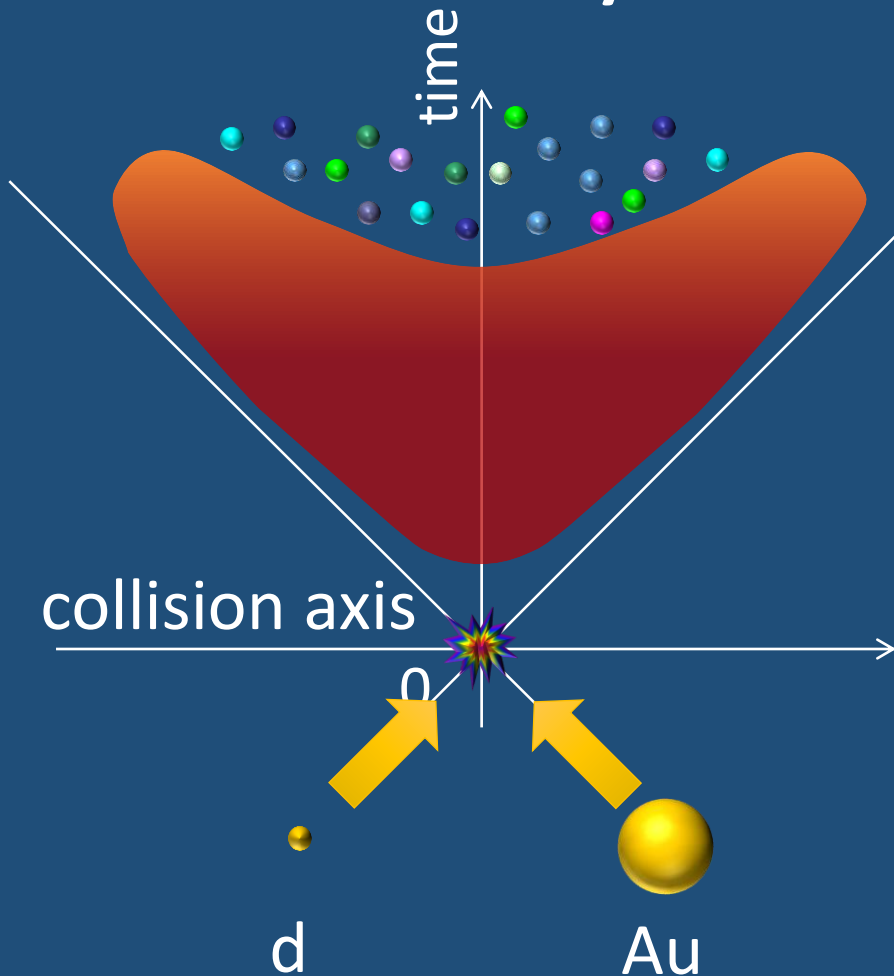


Rapidity  
triangle/trapezoid



Rapidity profile in pp  
collisions  
→ Boost inv. near  
midrapidity

# Integrated dynamical model for small systems



Hadronic cascade model

Microscopic description of  
Hadron gases

Relativistic hydrodynamics

Space-time evolution of  
QGP+hadronic fluids

Monte-Carlo Glauber +  
modified BGK + PYTHIA

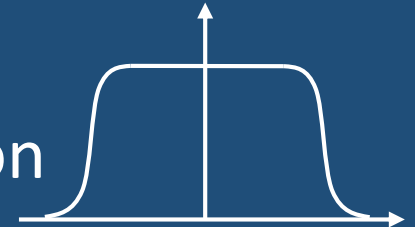
Initial profile of matter

NEW

# New version of Modified BGK Model

Issues in  $f^{pp}(\eta_s)$

- No multiplicity fluctuation
  - Multiplicity does not fluctuate in pp collisions.
- No longitudinal fluctuation
  - Smooth profile in longitudinal direction



➔ PYTHIA8.1

Multiplicity fluctuation even in pp collisions

Fluctuation of longitudinal matter profile

➔ New baseline initial condition based on Glauber picture

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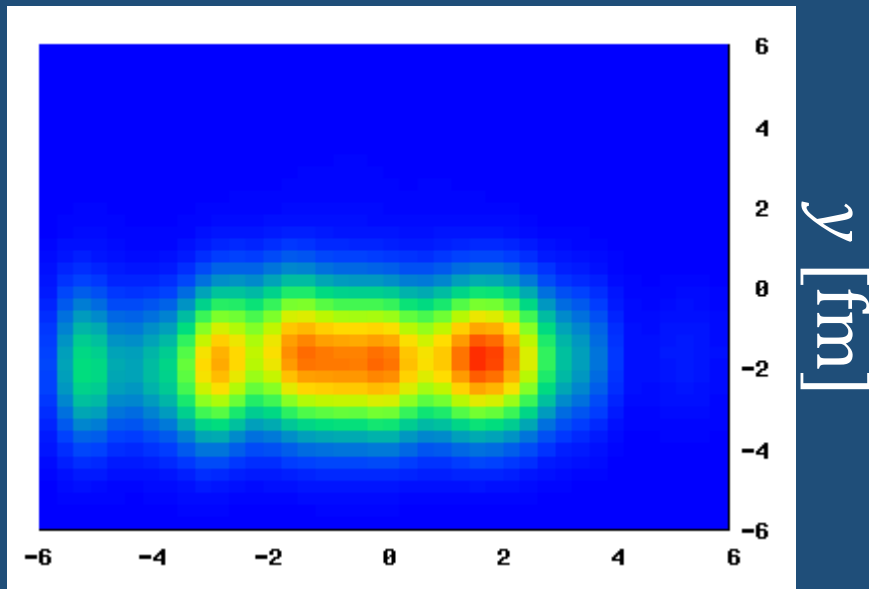
4. Results in d+Au

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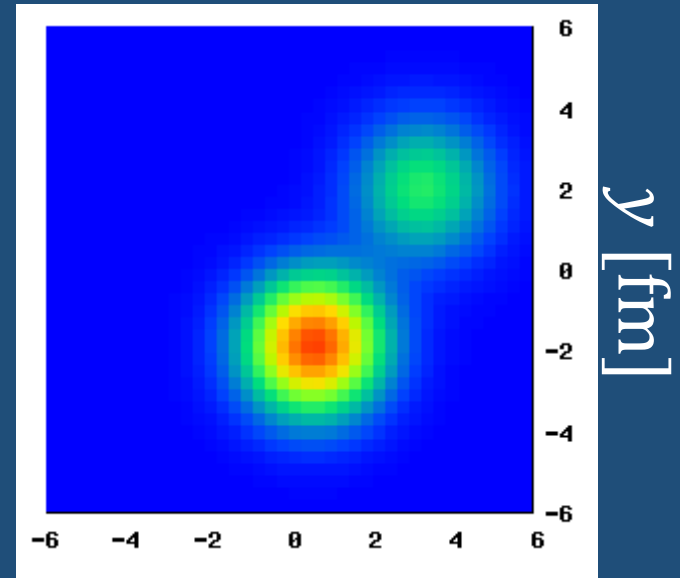
# Matter Profile in d+Au Collision

Longitudinal profile ( $x = 0$ )



Au-going  $\eta_s$  d-going  
side side

Transverse profile ( $\eta_s = 0$ )

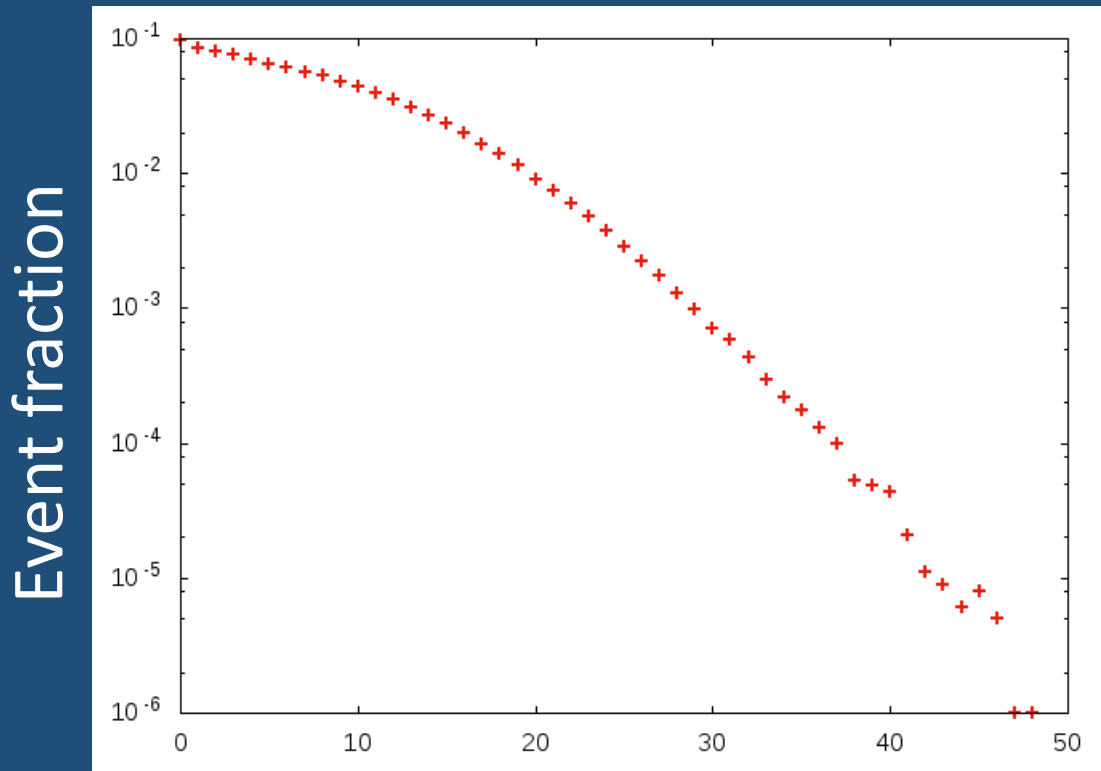


$x$  [fm]

Smearing parameters

$$\delta_x = \delta_y = 0.3 \text{ [fm]}, \delta_{\eta_s} = 0.3$$

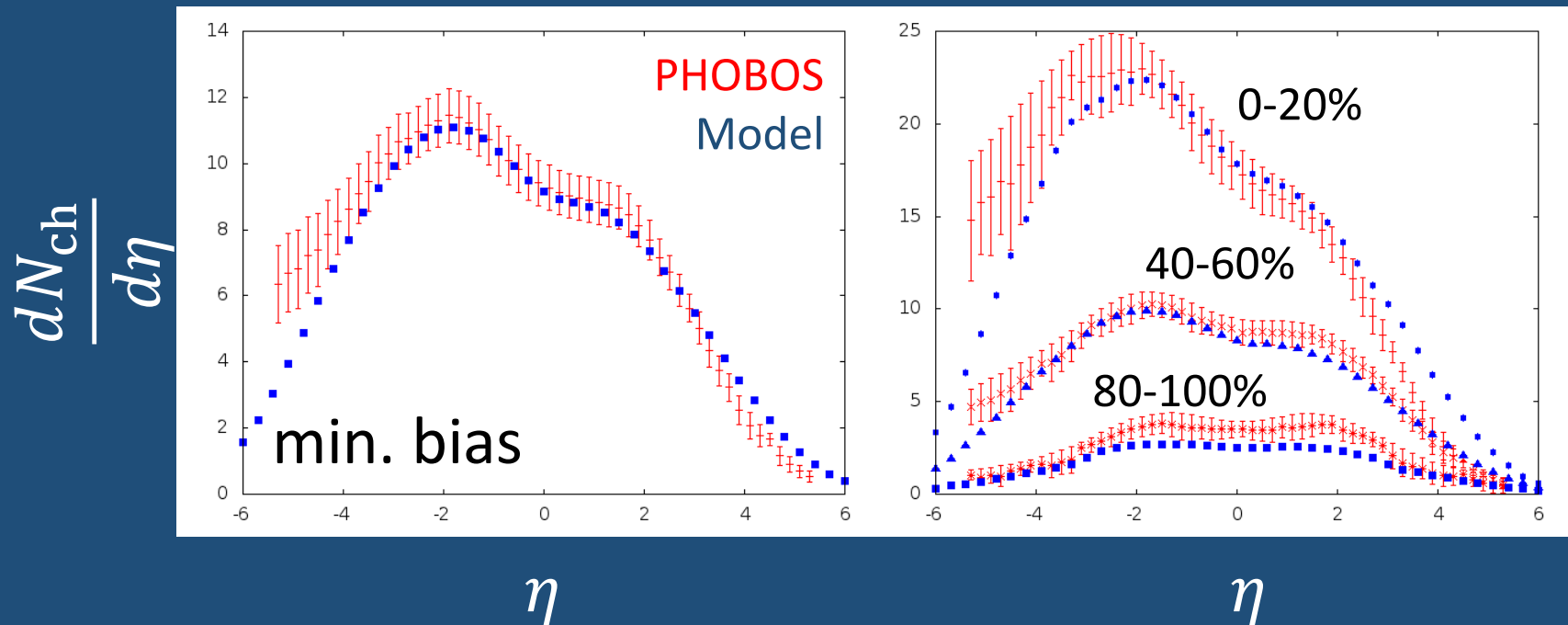
# Multiplicity distribution in d+Au collisions



$N_{ch}(-3.9 < \eta < -3.0)$   
Au-going side

Utilize for centrality cut

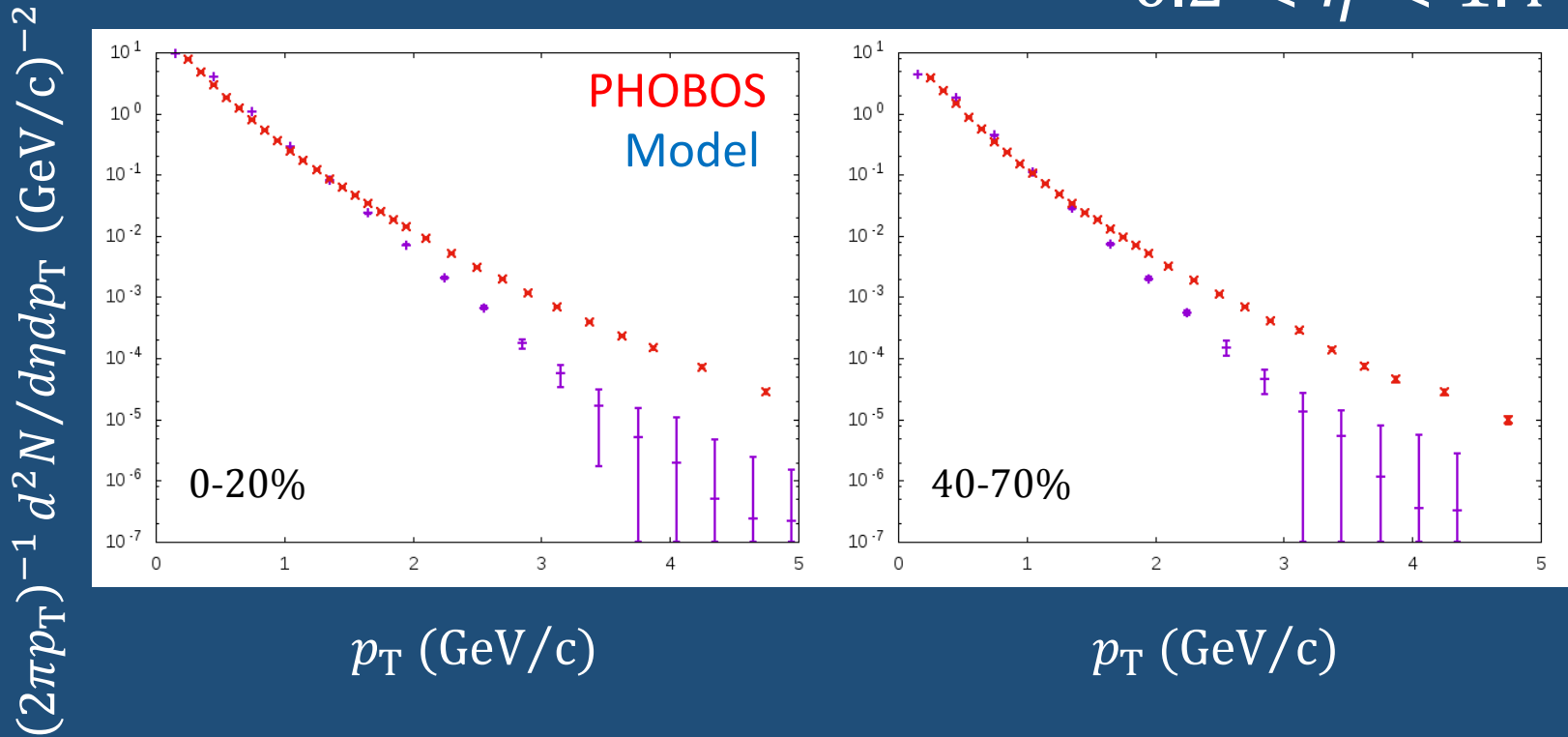
# Pseudorapidity distribution in d+Au collisions



Reasonable description of pseudorapidity and centrality dependences in d+Au collisions

# $p_T$ spectra of charged hadrons

$0.2 < \eta < 1.4$

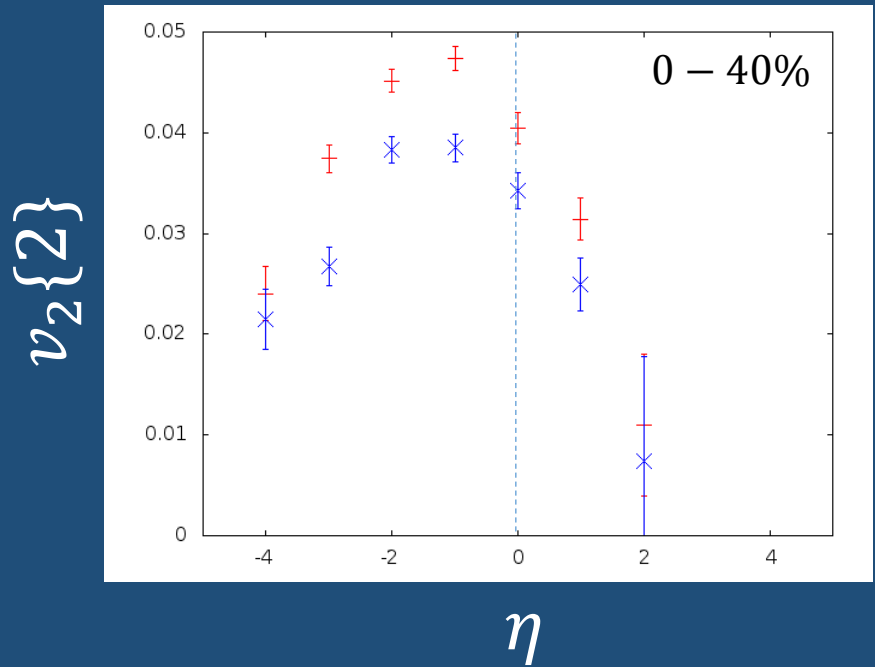
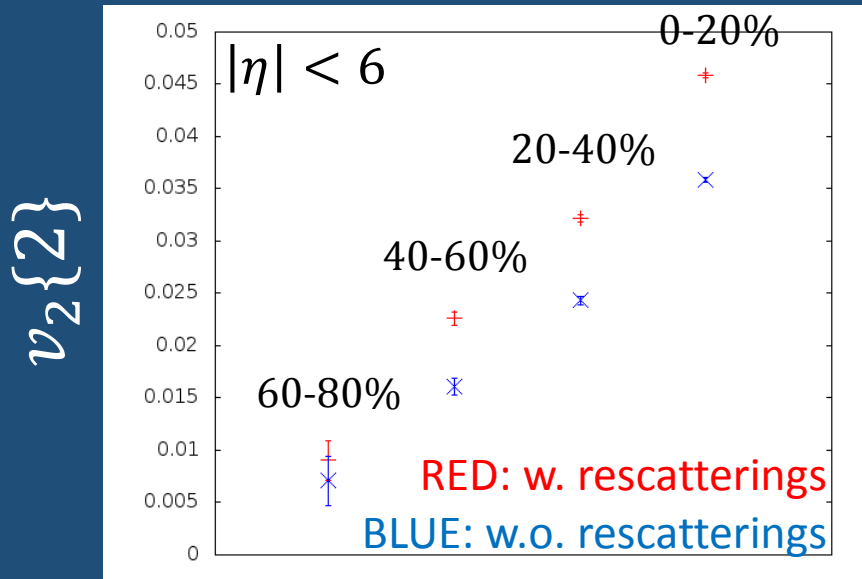


Low  $p_T$  well described by this model

# Effects of hadronic rescatterings on $v_2$ in d+Au collisions

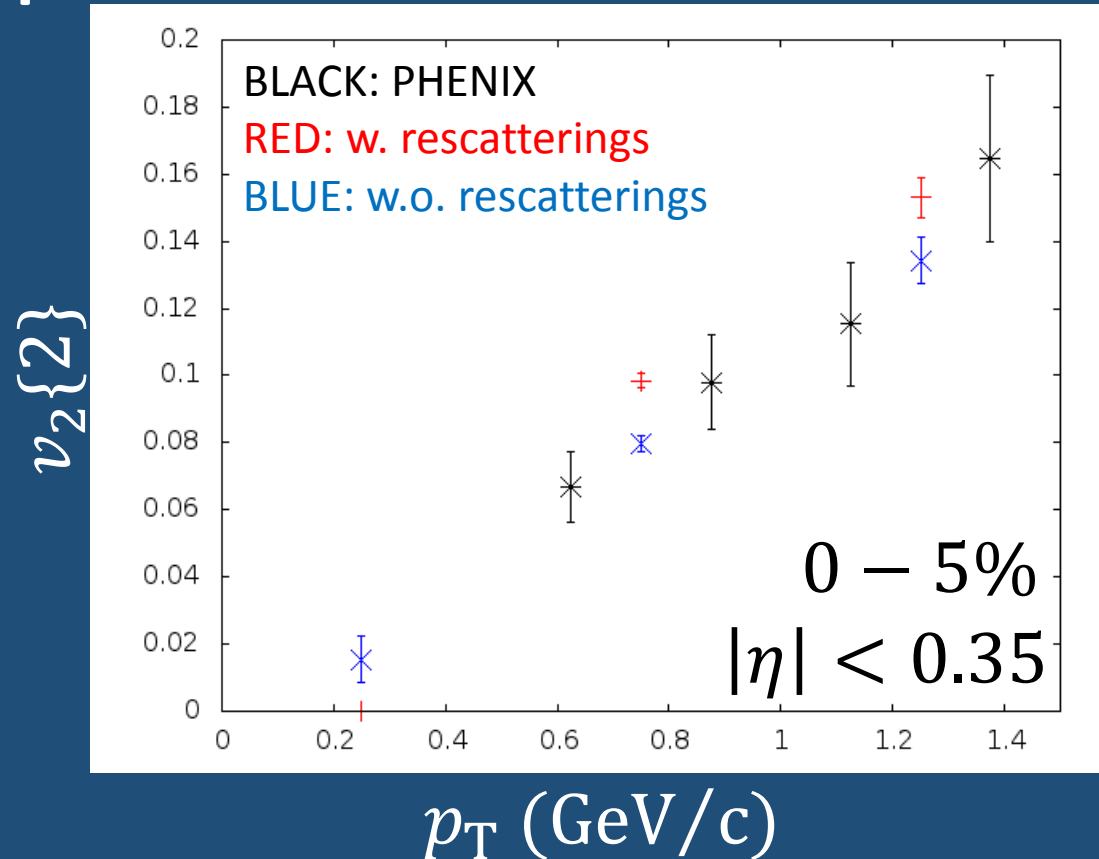
Centrality dependence

Pseudorapidity dependence



~80% of total  $v_2$  from fluid-dynamical stage  
→ Similar to AA collisions

# Comparison with PHENIX data



Not significant effects of rescatterings on  $v_2(p_T)$  slope  
 → Partly canceled by increase of mean  $p_T$

Analysis of  $v_2\{EP\}$ : work in progress

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# New source of higher harmonics?

Fluctuations appear everywhere!

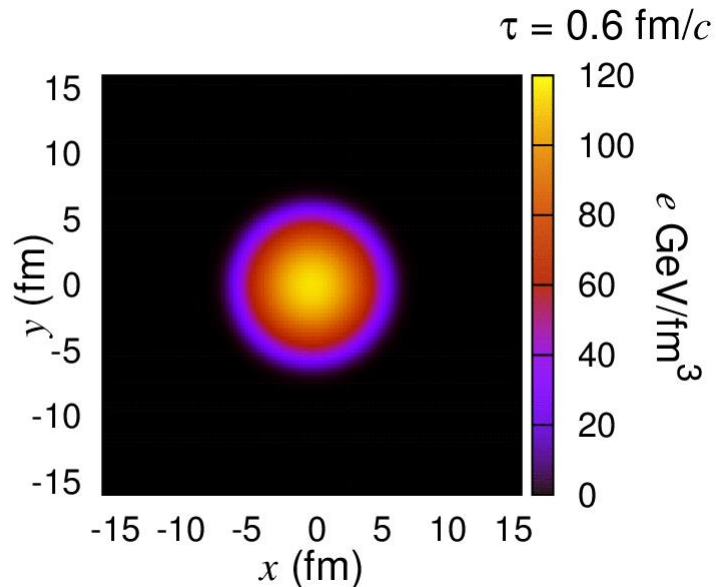
- Initial state fluctuations
  - Fluctuations of transverse position of nucleons in colliding nuclei
  - Multiplicity fluctuations
- Hydrodynamic fluctuations
  - Thermal noises
  - Critical fluctuations
- Disturbance by (mini-)jet propagation

Schulk, Tomášik(2014)

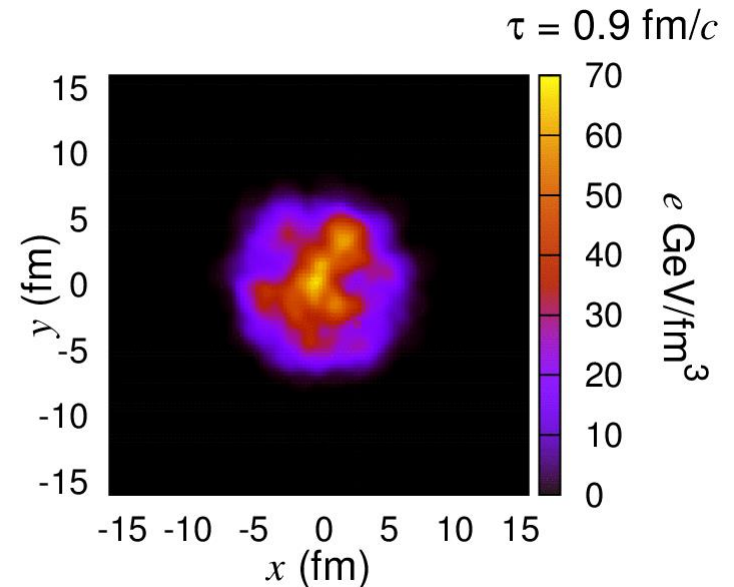
# Jet propagation in $\gamma$ -jet event

Full 3D ideal hydrodynamics

+ energy-momentum deposition by a parton



Smooth profile case



Bumpy profile case

Without linearization

Chaudhuri, Heinz (2006), Betz, Noronha, Torrieri, Gyulassy, Rischke (2010),  
Tachibana, Hirano (2014, 2016), ...

Movies: Courtesy of Y. Tachibana

# Many mini-jet propagation

Pb + Pb 2.76 TeV

$b = 0$  fm

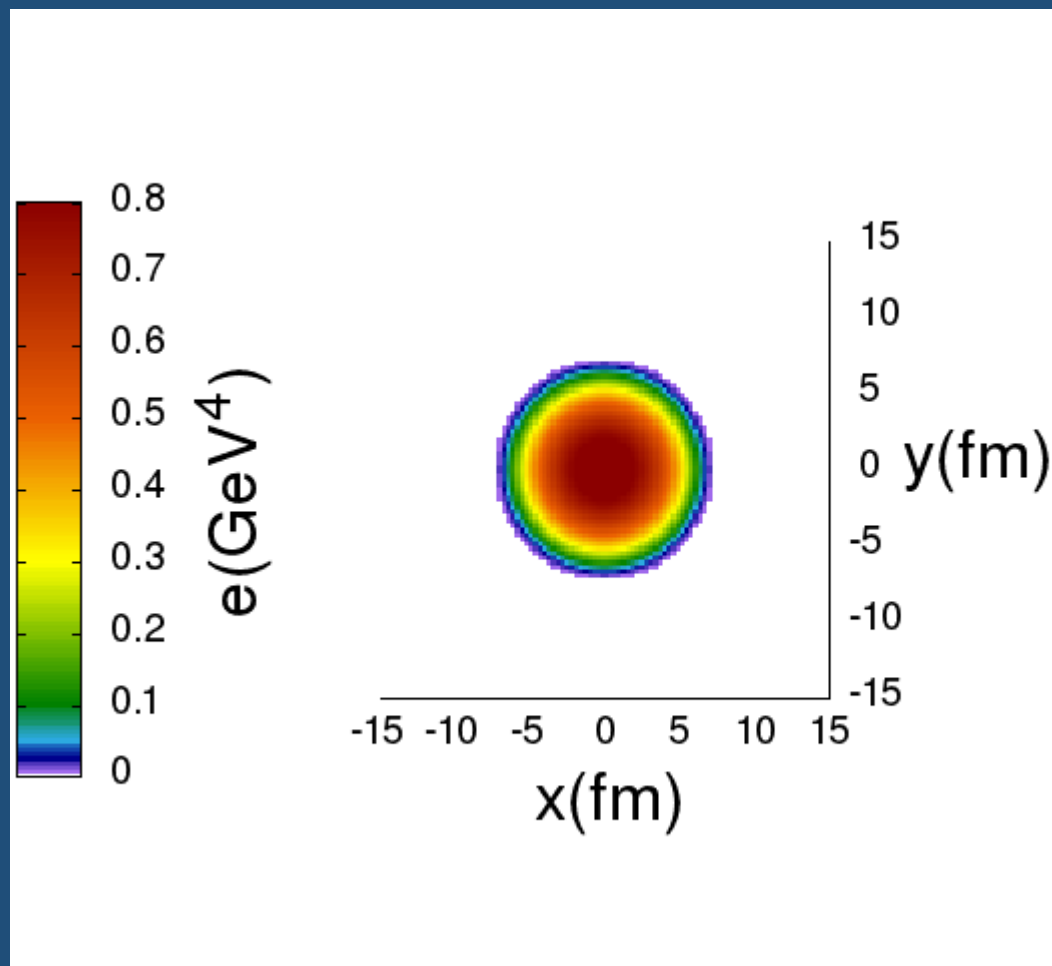
$N_{\text{coll}}: 1983, N_{\text{part}}: 411$

# of mini-jet ( $p_{\text{T}} > 2\text{GeV}$ ):  
~600 (at midrapidity)

Smooth initial  
condition (optical  
Glauber)

→ Bumpy freezeout  
hypersurface

→ Higher harmonics?



Okai, Tachibana, Hirano (work in progress)

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# Summary

## Happy 60<sup>th</sup> Birthday to Ulrich!

Effects of hadronic rescatterings (Calibration of a lens to look at the QGP) investigated together with Ulrich must be always important in high-energy nuclear collisions towards extracting transport properties of the QGP





Photos taken in  
QM2015 at Kobe  
To successful  
QM2017 at Chicago!



# Backups

# Violation of mass ordering in $v_2(p_T)$

PHYSICAL REVIEW C **77**, 044909 (2008)

## Mass ordering of differential elliptic flow and its violation for $\phi$ mesons

Tetsufumi Hirano,<sup>1,\*</sup> Ulrich Heinz,<sup>2,3</sup> Dmitri Kharzeev,<sup>4</sup> Roy Lacey,<sup>5</sup> and Yasushi Nara<sup>6</sup>

<sup>1</sup>*Department of Physics, The University of Tokyo, Tokyo 113-0033, Japan*

<sup>2</sup>*Department of Physics, The Ohio State University, Columbus, Ohio 43210, USA*

<sup>3</sup>*CERN, Physics Department, Theory Division, CH-1211 Geneva 23, Switzerland*

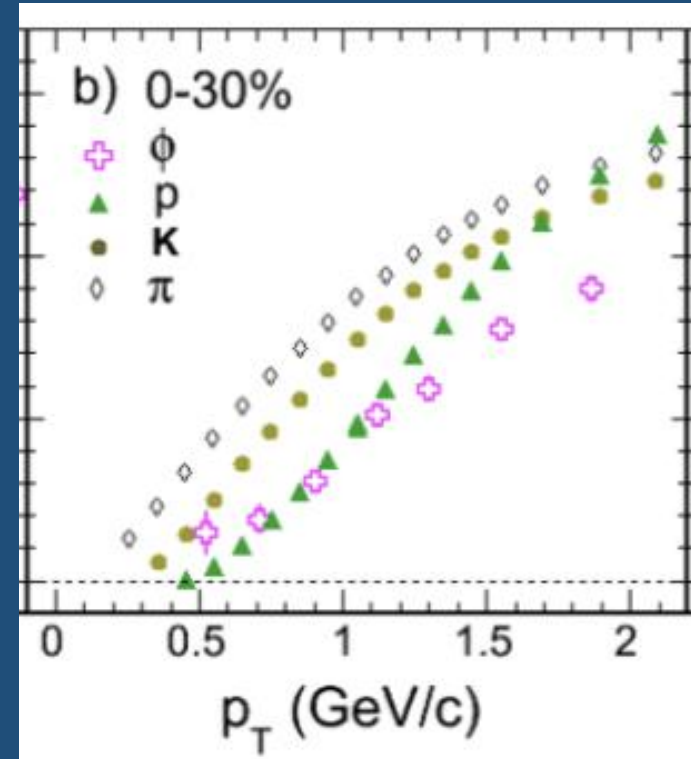
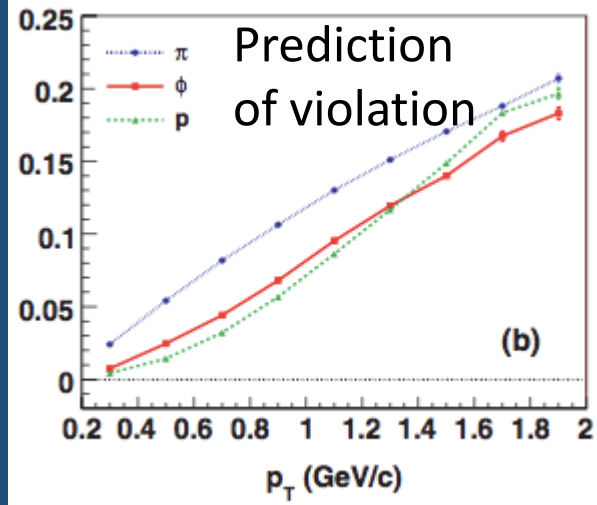
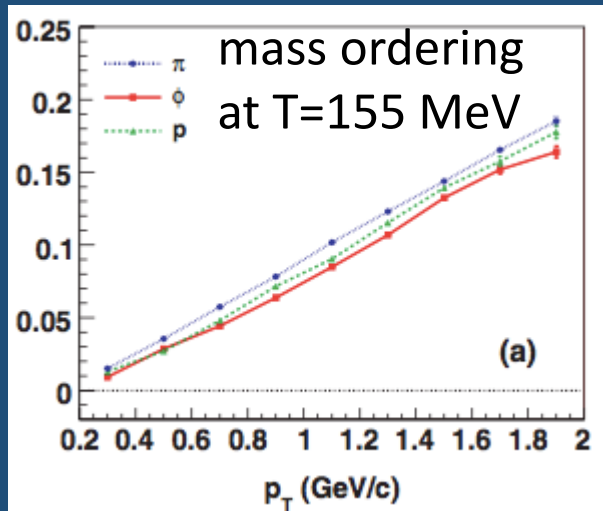
<sup>4</sup>*Nuclear Theory Group, Physics Department, Brookhaven National Laboratory, Upton, New York 11973-5000, USA*

<sup>5</sup>*Department of Chemistry, SUNY Stony Brook, Stony Brook, New York 11794-3400, USA*

<sup>6</sup>*Akita International University, 193-2 Okutsubakidai, Yuwa-Tsubakigawa, Akita 010-1211, Japan*

(Received 31 October 2007; published 28 April 2008)

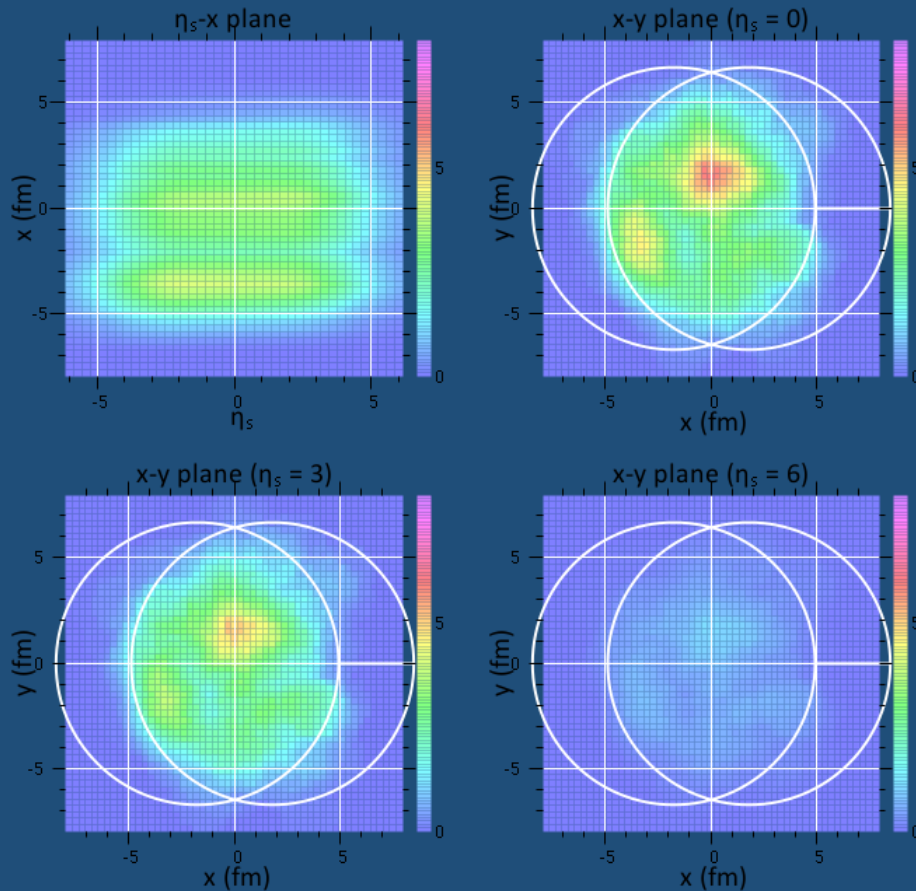
# Observed!



Md. Nasim [STAR Collaboration],  
Nucl. Phys. A 904-905 (2013) 413c

Quantitatively?

# MC-Glauber to hydro



Initial time:  $\tau_0 = 0.6$  fm  
Soft/hard fraction:  $\alpha = 0.18$   
+ overall normalization  
+ rapidity shape in  $pp$

Entropy density



The other thermodynamic  
variables through  
equation of state

One example of initial condition

# Relativistic hydrodynamics

## Hydrodynamic equations

$$\partial_{\mu} T^{\mu\nu} = 0$$

$$T^{\mu\nu} = (e + p)u^{\mu}u^{\nu} - pg^{\mu\nu}$$

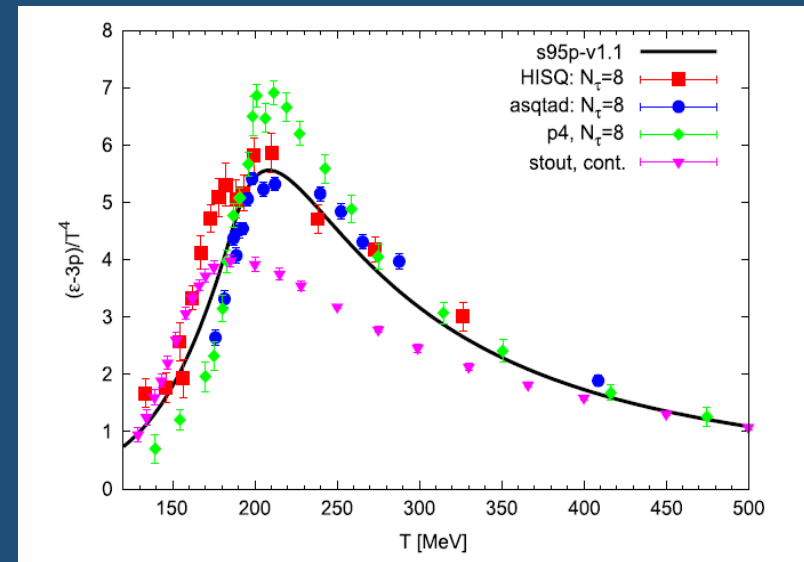
$e$ : energy density

$p$ : pressure

$u^{\mu}$ : four flow velocity

\*No dissipation in this study

## Equation of state



High  $T$

→ Lattice result (hot QCD)

Low  $T$

→ Hadron resonance gas

See also, P.Huovinen and P.Petreczky,  
NPA837, 26 (2010)

# Hydro to cascade

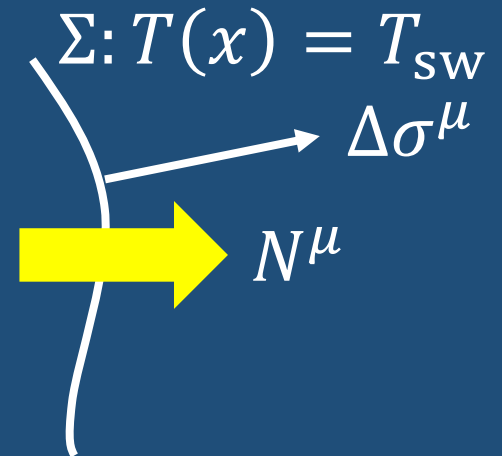
One-particle distribution from a fluid element

F.Cooper, G.Frye (1974)

$$E \frac{d^3 \Delta N}{d^3 k} = \frac{g}{(2\pi)^3} \frac{k \cdot \Delta \sigma}{\exp[k \cdot u / T_{sw}] \pm 1}$$

$$T_{sw} = 155 \text{ MeV}$$

Thermal dist. in switching hypersurface  $\Sigma$   
boosted by fluid velocity  $u^\mu$



Output from hydro as initial condition for cascade

# Hadronic cascade model (JAM)

Incoherent convolution of hadron-hadron scattering and resonance decays

Some remarks on strangeness sector

- $\bar{K}N$  and  $KN$  incoming channel



\*Inverse process through detailed balance

- Other processes  $\rightarrow$  Additive quark model


$$\sigma_{\text{tot}} = \sigma_{NN} \frac{n_1}{3} \frac{n_2}{3} \left( 1 - 0.4 \frac{n_{s1}}{n_1} \right) \left( 1 - 0.4 \frac{n_{s2}}{n_2} \right)$$

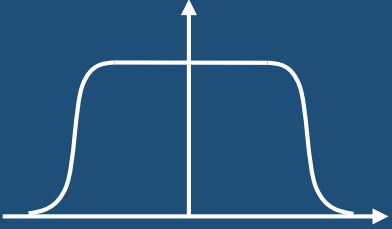
See also, Y.Nara *et al.*, PRC61, 024901 (2000),  
TH and Y.Nara, PTEP 01A203 (2012).

# Some details of initial conditions in longitudinal direction

$$s_0(\tau_0, \eta_s, x_\perp) = \frac{dS}{\tau_0 d\eta_s d^2x_\perp}$$
$$= \frac{C}{\tau_0} f^{pp}(\eta_s) \left[ \frac{1-\alpha}{2} \left( \frac{Y_b - \eta_s}{Y_b} \rho_A(x_\perp) + \frac{Y_b + \eta_s}{Y_b} \rho_B(x_\perp) \right) + \alpha \rho_{\text{coll}}(x_\perp) \right]$$

Triangle/Trapezoid



$$f^{pp}(\eta_s) = \exp \left[ -\theta (|\eta_s| - \Delta\eta) \frac{(|\eta_s| - \Delta\eta)^2}{\sigma_\eta^2} \right]$$


RHIC energy:

$$C = 15.0, \alpha = 0.18, \tau_0 = 0.6 \text{ fm}, \Delta\eta = 1.3, \sigma_\eta = 2.1$$

# Rejection sampling

One AA event

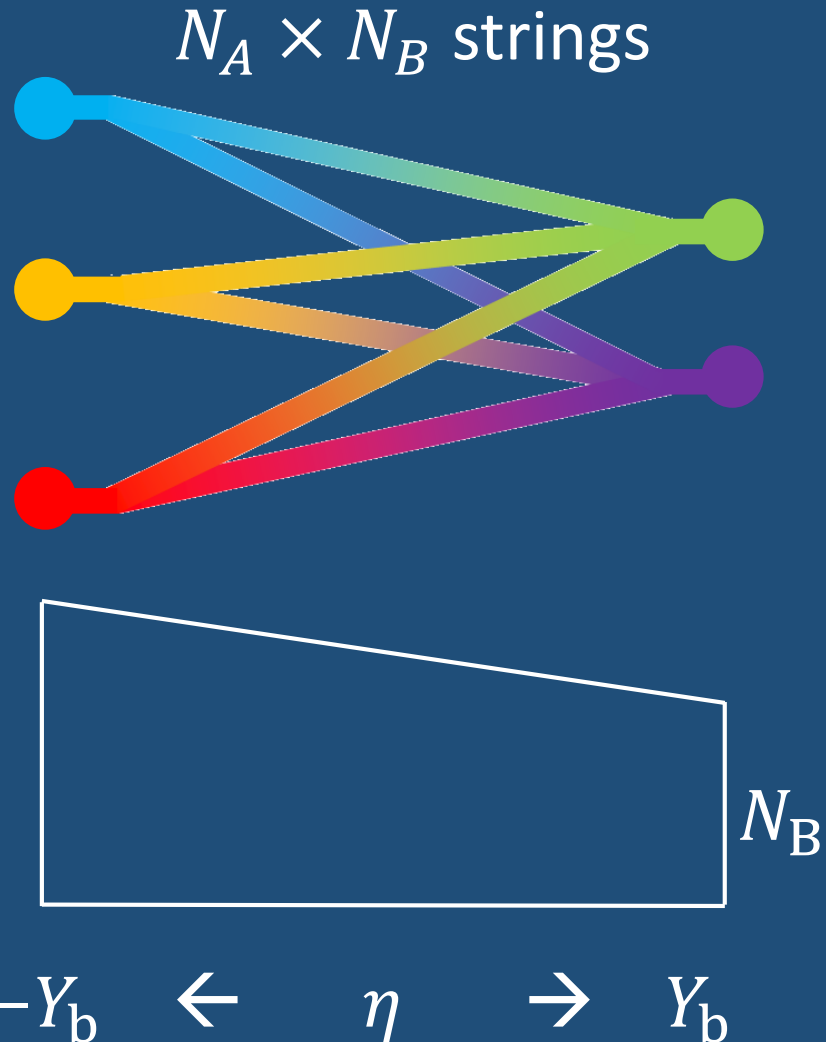
→  $N_A \times N_B$  PYTHIA events

Weight in one PYTHIA event

$w(\eta)$

$$= \frac{1}{2} \left( \frac{Y_b + \eta}{Y_b} \frac{1}{N_A} + \frac{Y_b - \eta}{Y_b} \frac{1}{N_B} \right)$$

$$N_A N_B w(\eta) = \begin{cases} N_A & (\eta = -Y_b) \\ N_B & (\eta = Y_b) \end{cases}$$



# $p_T$ dependent option in rejection sampling

$$w(\eta, p_T) = w(\eta) \times \frac{1}{2} \left( 1 - \tanh \frac{p_T - p_{T0}}{\Delta p_T} \right) + 1 \times \frac{1}{2} \left( 1 + \tanh \frac{p_T - p_{T0}}{\Delta p_T} \right)$$

$$N_A N_B w(\eta = 0, p_T)$$

