Anisotropy of Soft Photons
in
Relativistic Heavy Ion Collisions

Classical Bremsstrahlung Revisited

T. Koide and T. Kodama

Federal University of Rio de Janeiro

Direct Photons in Heavy Ion Collisions

The electromagnetic proves carry the information from the early stage of the collision dynamics. Many theoretical Works.

J. Kapsta (1977); J. D. Bjorken and L. McLerran (1985); J. Thiel, T. Lippert, N. Grün (*1989);* V. Koch, B. Blättel, W. Cassing, U. Mosel (1990); P. A. Ruuskanen, (1992), A. Dumitru, L. McLerran, H.Stoecker (1993); D. Srivastava (1994); N. Arbex, U. Ornik, M. Plumer, R.Weiner (1995); A. Dumitru, J. A. Marhuhn, D. H. Rischke(1995), T. Hirano, S. Muroya, M. Namiki (1995); J. Alam, S. Raha and B. Sinha(1996). S. Jeon, A. Chikanian, J. Kapusta, S. M. H. Wong (1999); U. Eichmann, C. Ernst, L.M. Satarov. W. Greiner (2000); R. Chatterjee, H. Holopainen, T. Renk, K. J. Eskola (2011); A. K. Chaudhuri and B. Sinha (2011), T.S. Biro, M. Gyulassi, Z. Schram (2012); G. Basar, D. Kharzeev, V. Skokov,........

And more recently, using the state-of-art hydro or transport theories,

C. Shen, Ulrich Heinz, J-F. Paquet, I. Kozlov, C. Gale, C. Shen, G. S. Denicol, M. Luzum, B. Schenke, S. Jeon, Y. Hidaka, S. Lin, R. D. Pisarski, D. Satow, V. V. Skokov, G. Vujanovic, O. Linnyk, E. L. Bratkovskaya, W. Cassing, C. Greiner, M. Greif, S. Endres, H. v. Hess, J. Weil, M. Bleicher ....

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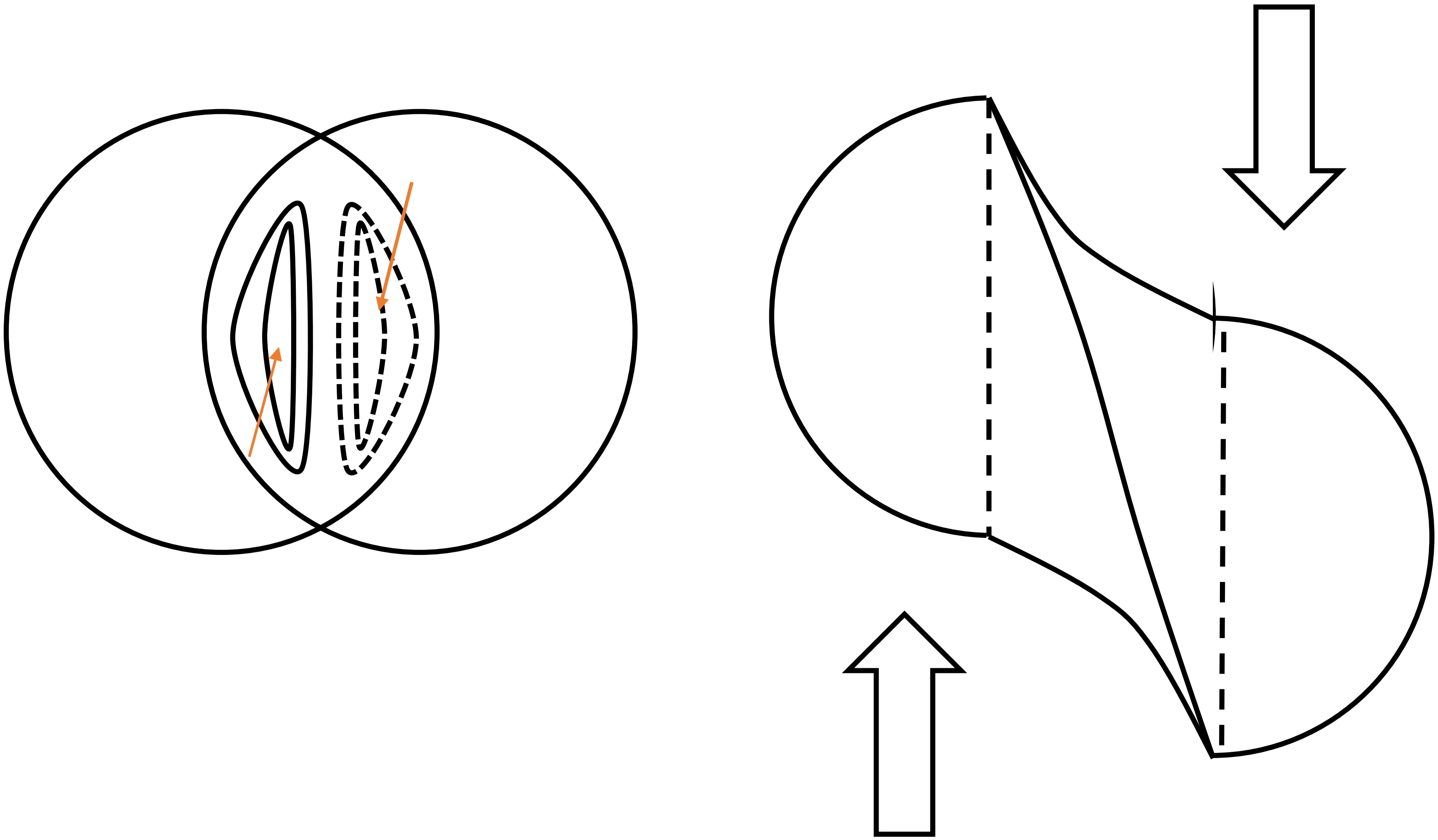
M. Bleicher ....

Early days model of photon emission

Coherent Bremsstrahlung Emission due to Deceleration of Incident Nuclei

J. Kapsta, Phys. Rev. C15, 1580 (1977), J. D. Bjorken and L. McLerran, Phys. Rev. D31, 63 (1985), J. Thiel et. al., Nucl. Phys. A504, 864 (1989). V. Koch et. al., Phys. Lett. B236, 135 (1990), T. Lippert et. al., Int. J. Mod. Phys. A29, 5249 (1991), A. Dumitru et. al., Phys. Lett. B318, 583 (1993), U. Eichmann and W. Greiner, J. Phys. G23, L65 (1997), S. Jeon et. al., Phys. Rev. C58, 1666 (1998), J. Kapusta and S. M. H. Wong, Phys. Rev. C59, 3317 (1999), U. Eichmann, C. Ernst, L.M. Satarov and W. Greiner, Phys.Rev. C62, 044902 (2000) ..

Decelation of incidente nuclei



Classical Electromagnetic Radiation by an accelerated point charge is given by (Liénard-Wiechert Potential),

$$\vec{E}\left(\vec{x},t\right) = \frac{e}{4\pi}\frac{1}{\left|\vec{x}-\vec{\xi}\left(t'\right)\right|}\frac{1}{1-\vec{\beta}\left(t'\right)\cdot\vec{n}}\left(1-\vec{n}\vec{n}^T\right)\frac{d\vec{\beta}(t')}{dt'},$$

$$\vec{B}\left(\vec{x},t\right) = \vec{n}\times\vec{E}\left(\vec{x},t\right),$$

where $\vec{\xi}\left(t\right)$ is the trajectory of the point charge, and $\vec{\beta}\left(t\right) = d\vec{\xi}/dt$ is the velocity. *t'* is the emission time, defined by $t' = t - \left|\vec{x}-\vec{\xi}(t')\right|$

and $$\vec{n} = \frac{\vec{x}-\vec{\xi}(t')}{\left|\vec{x}-\vec{\xi}(t')\right|}.$$

Continuum Sources (Integrate over moving charges)

$$\vec{E}\left(\vec{x},t\right) = \int d\vec{x}\,'\rho\left(\vec{x}\,'\right)\vec{E}_{\vec{x}\,'}\left(\vec{x},t\right),$$

$$\vec{B}\left(\vec{x},t\right) = \int d\vec{x}\,'\rho\left(\vec{x}\,'\right)[\vec{n}\times\vec{E}_{\vec{x}\,'}\left(\vec{x},t\right)],$$

where x' is the Lagrange coordinate of the point charge whose trajectory is $\vec{\xi}\left(\vec{x}\,',t'\right)$ and $\vec{\rho}\left(x'\right)$ is the initial distribution of the charges.

How to calculate the photon spectrum?

Equivalent Photon Method - Jackson´s book

How to calculate the photon spectrum?

Equivalent Photon Method - Jackson´s book

Or introduce the Wigner function,

$$f_W(\vec{x},\vec{p};t)=\int d^3\vec{u}\left\{e^{i\vec{p}\cdot\vec{u}/\hbar}\psi^\dagger(\vec{x}-\vec{u}/2,t)\psi(\vec{x}+\vec{u}/2,t)\right\}$$

where $\psi(\vec{x},t)$ is the wave function.

How to calculate the photon spectrum?

$$f_W(\vec{x},\vec{p};t) = \int d^3\vec{u}\left\{e^{i\vec{p}\cdot\vec{u}/\hbar}\left\langle \psi(\vec{x}-\vec{u}/2,t)\psi^\dagger(\vec{x}+\vec{u}/2,t)\right\rangle\right\}$$

Eq

Or introduce the Wign

in the QCD transport theory, T-H. Elze and U. Heinz, Phys.Rept. 183 (1989) 81-135

Identify the Classical Electromagnetic fields as the vector type wavefunction similar to the Dirac form*.

$$\vec{\psi}(\vec{x},t) \equiv \left(\vec{E} + i\vec{B}\right)/\sqrt{2},$$

then the Maxwell equations are equivalent to

$$i\left(\partial_t + \vec{\Sigma}\cdot\nabla\right)\vec{\psi} = \vec{j} \quad \text{and} \quad \nabla\cdot\vec{\psi} = \rho,$$

with $\vec{\Sigma} = \left\{ \frac{1}{i}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, \frac{1}{i}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \frac{1}{i}\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right\}$ generators of O(3) for spin 1

- ❖ L. Silberstein, Ann. d. Phys. 22, 579 (1907); 24, 783 (1907);
- ❖ H. Bateman, Cambridge 1915, Dover, New York,1955).
- ❖ I. Bialynicki-Birula, Act. Phys. Pol. A86 97 (1994).
- ❖ P. Holland, Proc. R. Soc. A461, 359 (2005).

The energy density and Poyinting Vector are expressed as

$$\varepsilon = \vec{\psi}^{\dagger}\vec{\psi},$$

$$\vec{P} = -i\left(\vec{\psi}^{\dagger}\cdot\vec{\Sigma}\right)\vec{\psi},$$

where the product $\left(\vec{\psi}^{\dagger}\cdot\vec{\Sigma}\right)$ is defined as $\left(\vec{\psi}^{\dagger}\cdot\vec{\Sigma}\right) = \sum_{i=x,y,z} \psi_i^{\dagger}\vec{\Sigma}_i$.

The time evolution after "freeze-out" ($\rho, \vec{j} = 0$) we need only

$$i\left(\partial_t + \vec{\Sigma}\cdot\nabla\right)\vec{\psi} = 0,$$

Then the corresponding Wigner function (scalar) decomposes into the electric and magnetic contributions,

$$f_W(\vec{x}, \vec{p}; t) = f_E(\vec{x}, \vec{p}; t) + f_B(\vec{x}, \vec{p}; t)$$

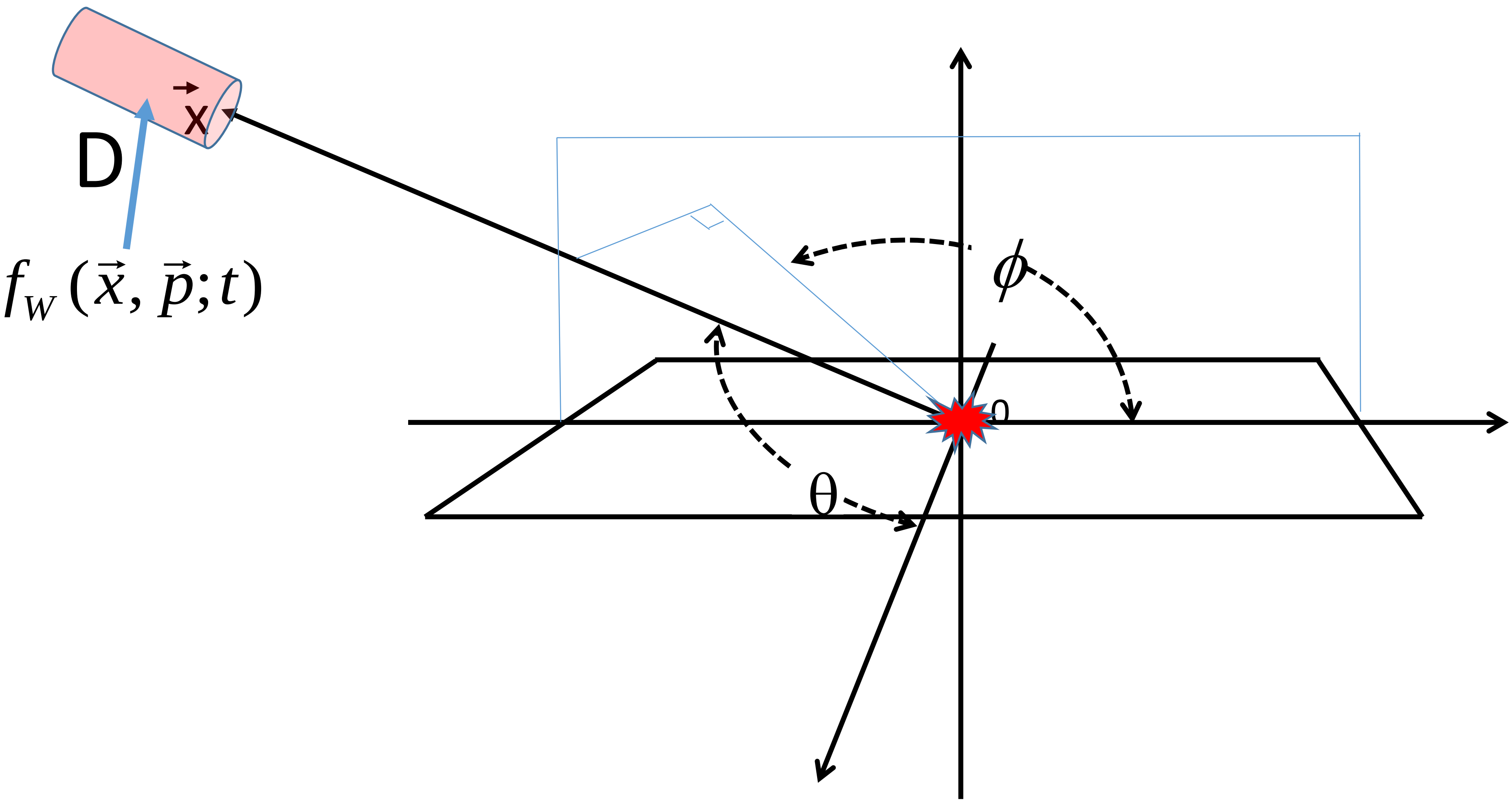
From the definition, we know that

$$\frac{d^3 n}{d^3 \vec{p}} = \int d^3\vec{r}\, f_W(\vec{x}, \vec{p}; t_{FO}) = |\psi(p)|^2,$$

where t_{FO} is the freezeout time. But it is instructive to calculate the spectrum as the energy that enters into the detector from the large distance behavior of the Wigner function as;

$$E_{Detector} = \int_{\vec{r}\in D} d^3\vec{r}\, f_W(\vec{x}, \vec{p}; t),$$

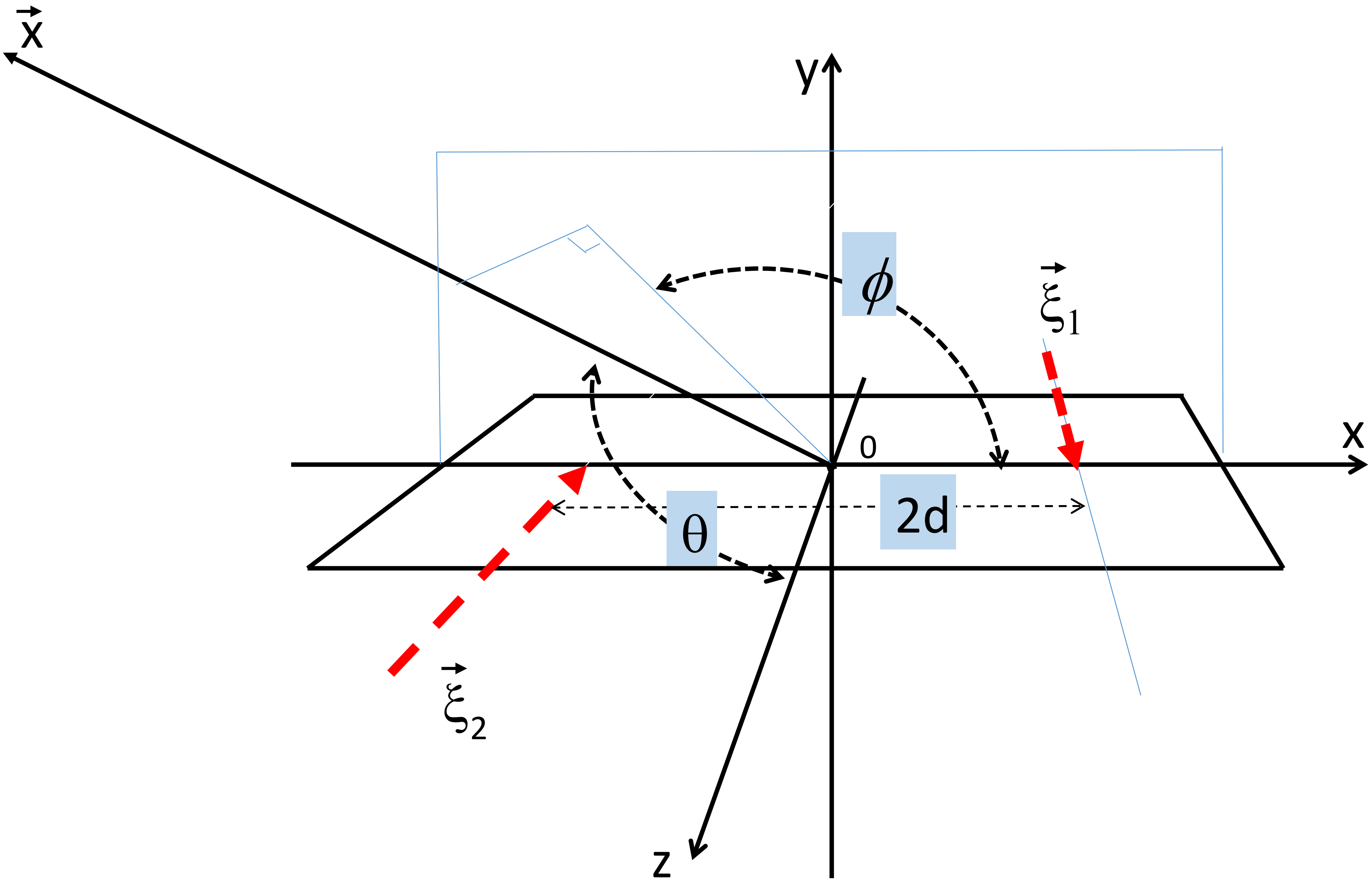
where D is the domain of the detector positioned at a solid angle $d^2\Omega$.



D
x⃗
$f_W(\vec{x}, \vec{p}; t)$
ϕ
θ
0

Minimal Model as Extreme Coherence

- Each set of participant protons of the two incidente nuclei are considered as point-like charges with the effective charge Z_{eff} .
- These two effective charges stop instantaneously as the two nuclei colide.



$\vec{x}$
y
ϕ
$\vec{\xi}_1$
x
0
2d
θ
$\vec{\xi}_2$
z

Taking the trajectories as $\vec{\xi}_1(t) = \begin{pmatrix} d \\ 0 \\ tV_0\theta(-t) \end{pmatrix}, \quad \vec{\xi}_2(t) = \begin{pmatrix} -d \\ 0 \\ -tV_0\theta(-t) \end{pmatrix}$

we get for the fields created by the trajectory-1 as

$$\vec{E}_1(\vec{x},t) = \frac{eV_0 Z_{eff}}{4\pi}\frac{1}{r_-^{\,3}}\begin{pmatrix} -(x-d)z \\ -xy \\ (x-d)^2 + y^2 \end{pmatrix}\delta(t-r_-), \qquad \vec{B}_1(\vec{x},t) = \frac{eV_0 Z_{eff}}{4\pi}\frac{1}{r_-^{\,2}}\begin{pmatrix} y \\ -(x-d) \\ 0 \end{pmatrix}\delta(t-r_-),$$

where $r_- = \sqrt{(x-d)^2 + y^2 + z^2}$ and for the trajectory-2, we get simply by the following replacement,

$$\vec{E}_1(\vec{x},t),\ \vec{B}_1(\vec{x},t) \ \Rightarrow\ \vec{E}_2(\vec{x},t),\ \vec{B}_2(\vec{x},t)$$

$$\left(d, V_0\right) \ \Rightarrow\ \left(-d, -V_0\right)$$

From these, we have

$$f_{E,B}(\vec{x},\vec{p};t) = f^{(0)}{}_{E,B}(\vec{x}-\vec{d},\vec{p};t) + f^{(0)}{}_{E,B}(\vec{x}+\vec{d},\vec{p};t)$$
$$-2\cos(2\vec{p}\cdot\vec{d})\, f^{(0)}{}_{E,B}(\vec{x},\vec{p};t)$$

where $$f^{(0)}{}_{E,B}(\vec{x},\vec{p};t) = f_{E,B}(\vec{x},\vec{p};t)\Big|_{d=0}$$

and $$f_E(\vec{x},\vec{p};t) = \int d^3\vec{u}\left\{e^{i\vec{p}\cdot\vec{q}/\hbar}\vec{E}(\vec{x}-\vec{q}/2,t)\cdot\vec{E}(\vec{x}+\vec{q}/2,t)\right\},$$

and analogously for f_B .

These integrals contain the product of two delta functions

$$\delta\left(t-\sqrt{\left(\vec{x}+\vec{q}/2\right)^2}\right)\delta\left(t-\sqrt{\left(\vec{x}-\vec{q}/2\right)^2}\right)$$

which reduces for large distance (i.e., r,t >> 1/p, d) to

$$\delta\left(t-\sqrt{\left(\vec{x}+\vec{q}/2\right)^2}\right)\delta\left(t-\sqrt{\left(\vec{x}-\vec{q}/2\right)^2}\right) \approx \delta(t-x)\delta(q_{//})$$

for arbitrary vector $\vec{q}$ with $|\vec{q}| \approx d$, and $q_{//}$ is the parallel component of $\vec{q}$ with respect to $\vec{\boldsymbol{x}}$

Finally we get

$$f_W(\vec{x},\vec{p};t) \cong 4\pi^2 \alpha_{EM} Z_{eff}{}^2 V_0{}^2 \frac{1}{p^2 r^2} \left\{ 1 - \cos\left(\vec{p} \cdot \vec{d} \right) \right\} \delta^2\left(\vec{\Omega}_{\vec{p}} - \vec{\Omega}_{\vec{x}} \right)$$

The deta function (the last term) means that for a large (macroscopic) distance, the observational direction coincides with that of the momentum detected.

The energy spectrum is given by the integral within the volume of the detecter *D*, which is equivalente to

$$\frac{d^3\mathcal{E}}{d\vec{p}^3} = \frac{1}{(2\pi)^3}\int dt \int_{\Omega\in D} d^2\Omega_{\vec{x}} R_D{}^2 f_W(\vec{R}_D, \vec{p}; t)$$

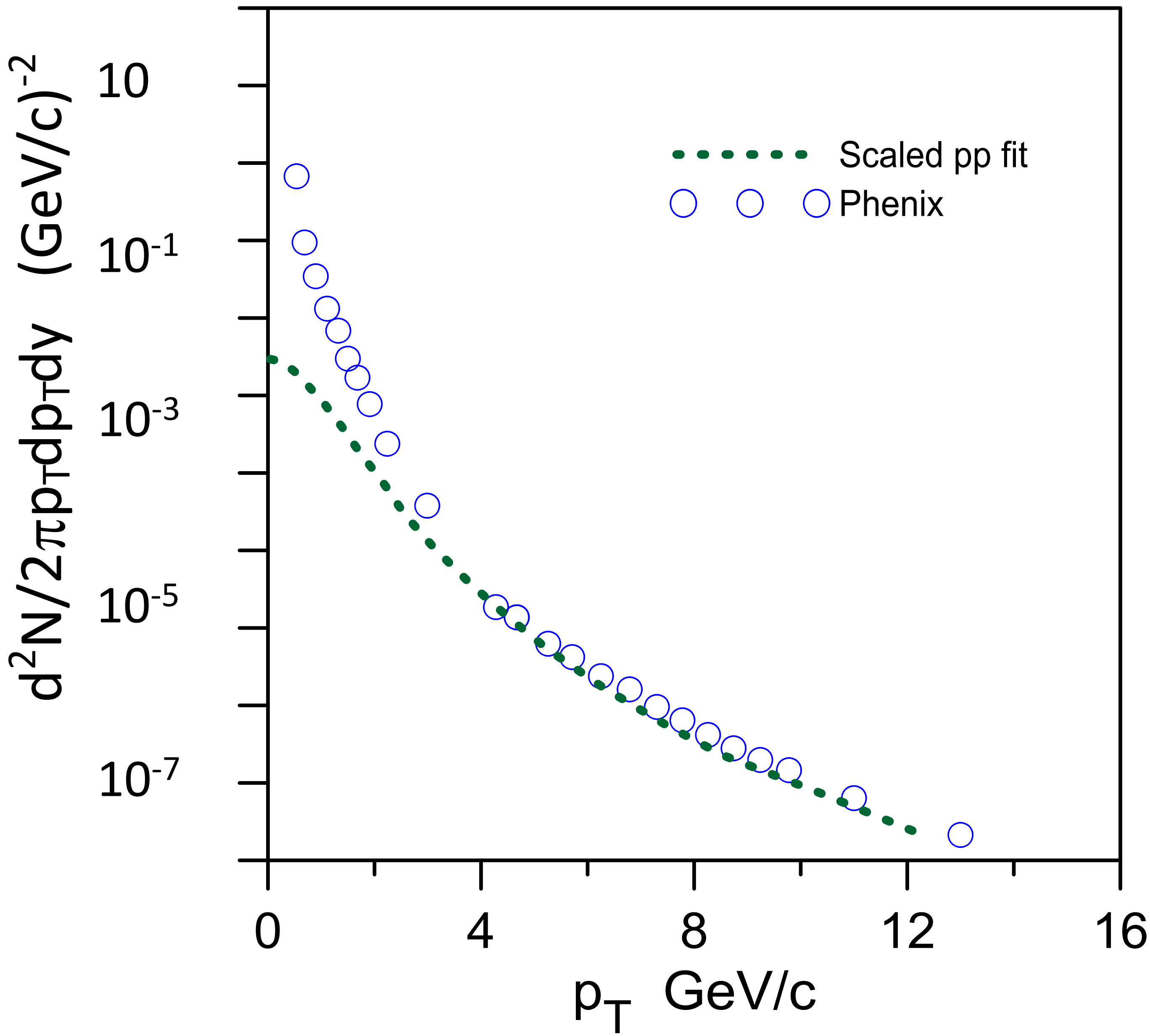
where R_D is the position of the detector D.
Photon number distribution is

$$\frac{d^3N}{d\vec{p}_T{}^2 dy} = \frac{1}{2\pi^2}\frac{\alpha_{EM} Z_{eff}{}^2 V_0{}^2}{p^2\cosh^2 y}\left\{1-\cos\left(2\vec{p}\cdot\vec{d}\right)\right\}$$

Angle integrated p_T spectrum is given by ($V_0 \sim 1$)

$$\left.\frac{d^3N}{2\pi p_T dp_T dy}\right|_{y=0} \simeq 0.37\times 10^{-3}\frac{Z_{eff}{}^2}{p_T{}^2}\left\{1-J_0\left(2pd\right)\right\}$$

p_T Spectrum

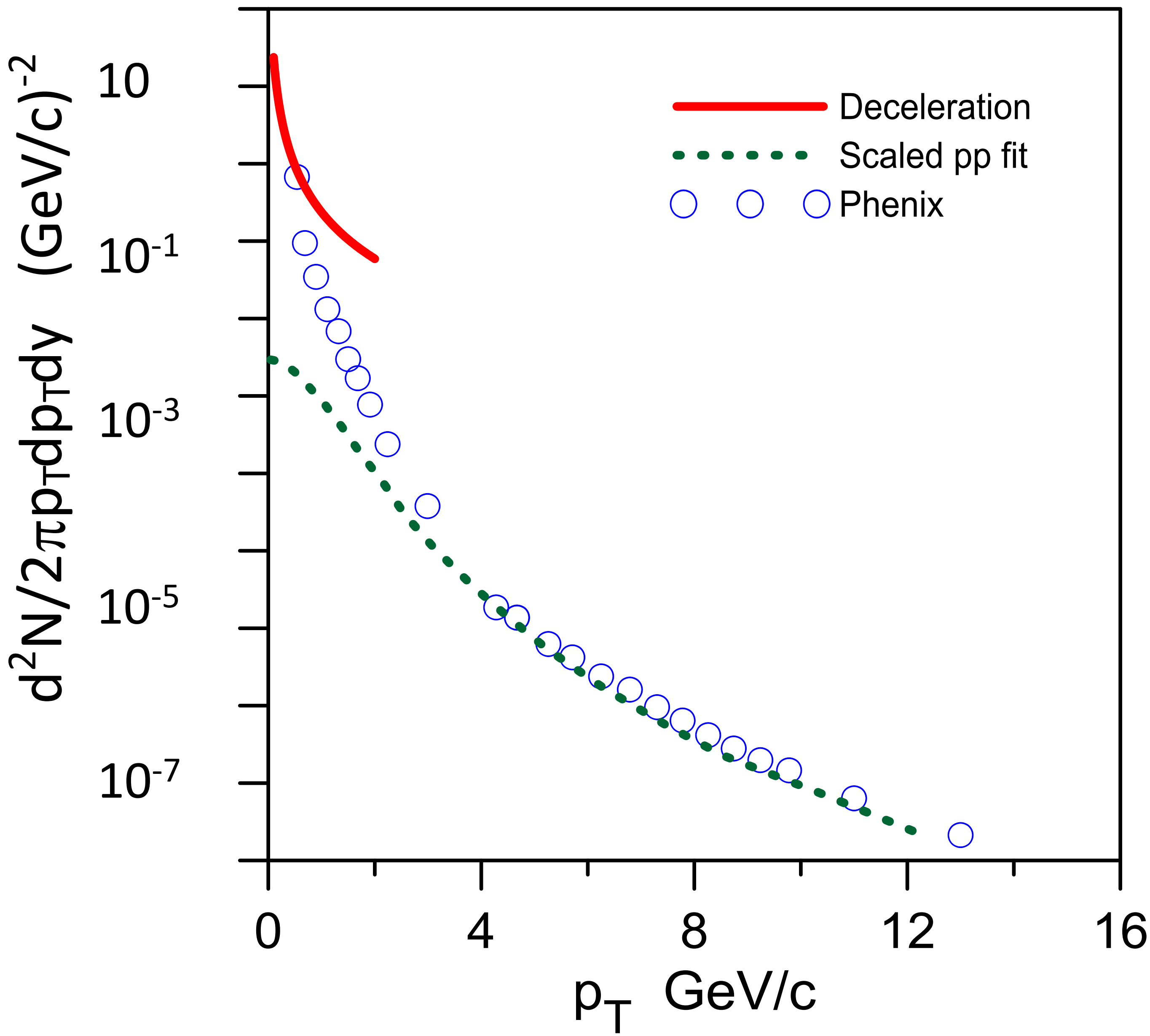


$V_0 \approx 1,$

$Z_{eff} \approx 80,$

$d \approx 1\,fm$

p_T Spectrum



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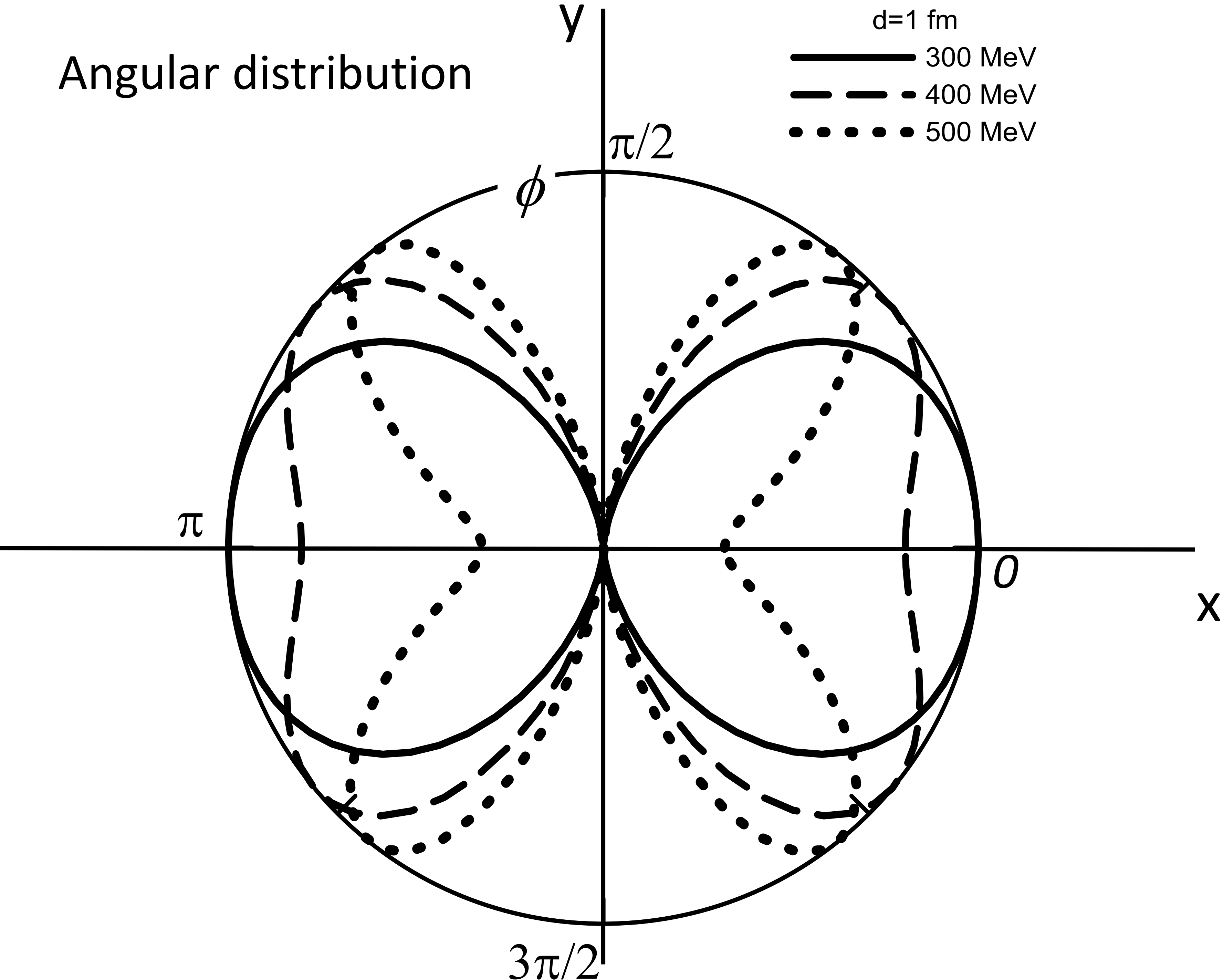
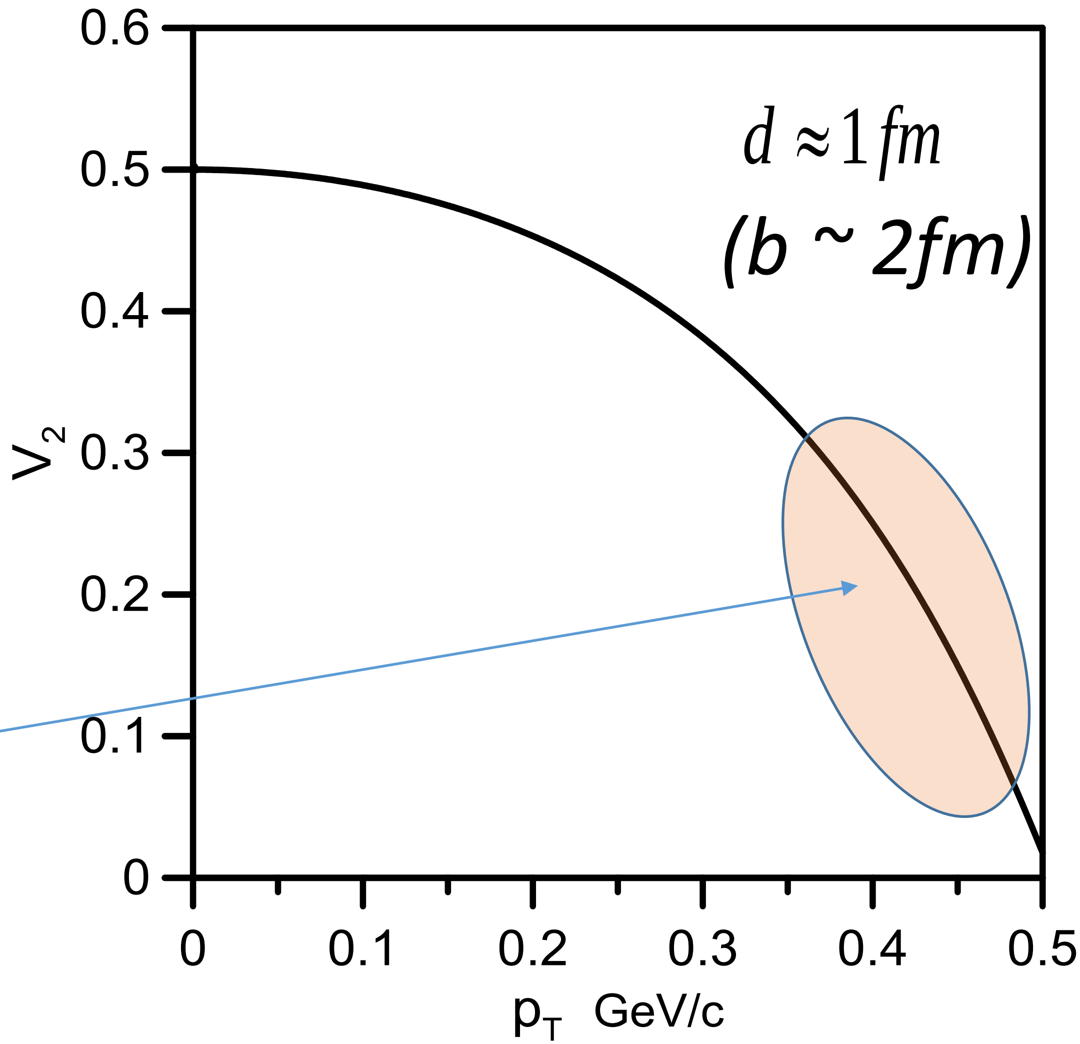

Angular distribution
d=1 fm
300 MeV
400 MeV
500 MeV
y
x
π/2
π
0
3π/2
ϕ

Anisotropy Parameter v_2

$$v_2 = \frac{J_2(2p_T d)}{1 - J_0(2p_T d)}$$



Large increase of v_2 for lower p_T !

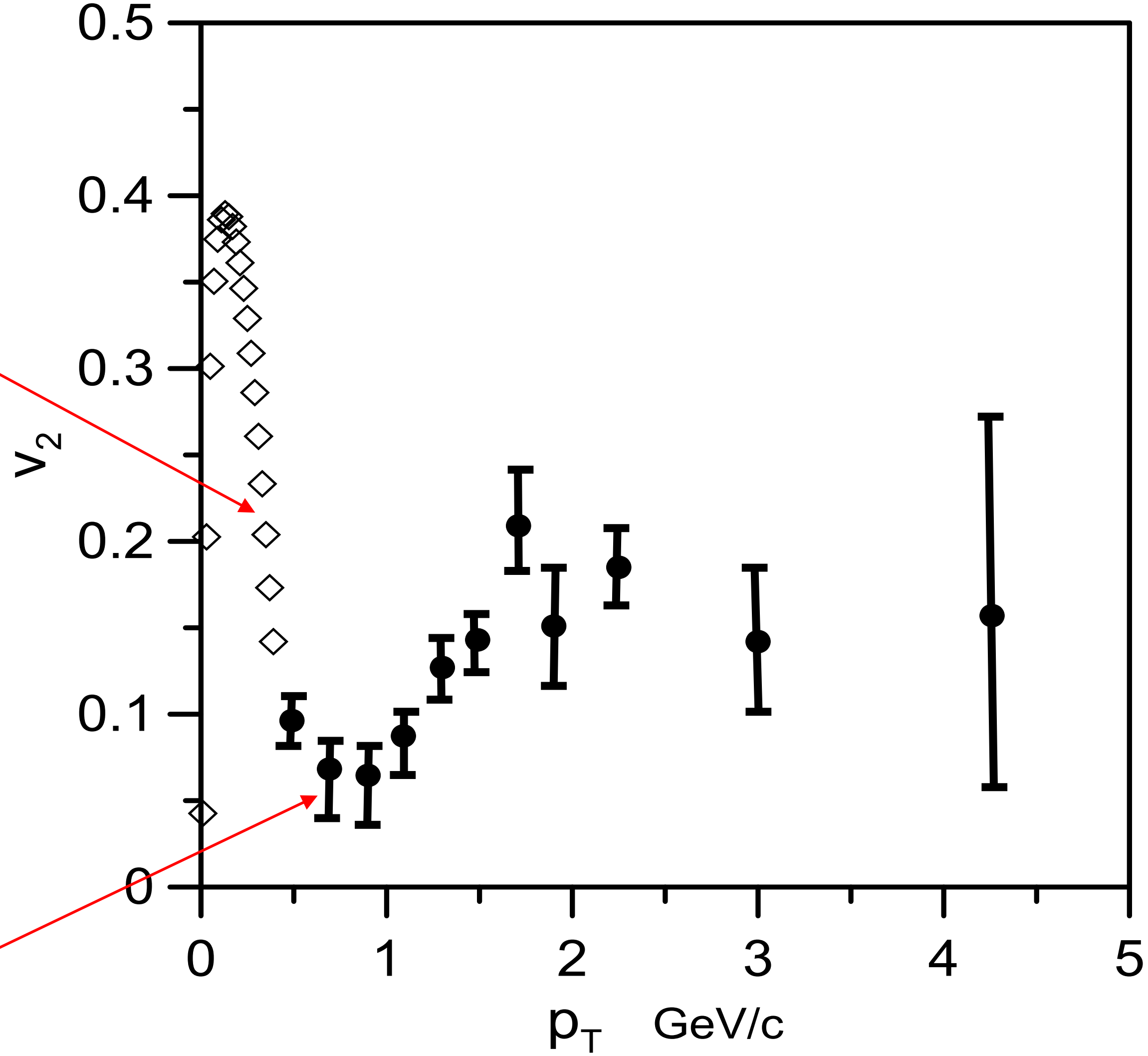
If we suppose incoherence occurs between electro-magnetic fields generated by the incidente nuclei as exponetially with p_T, we may expect*

$$v_2 \approx \frac{J_2(2p_T d)}{1+2e^{2p_T/\Delta p}/Z_{eff} - J_0(2p_T d)}$$

* T.S. Biró, Z. Szendi and Z. Schram, Euro. Phys.J A50, 62 (2014)

Curiously,...

$V_0 \approx 1,\ Z_{eff} \approx 80,$
$d \approx 1\,fm,$
$\Delta p = 0.2GeV,$



PHENIX data
Centrality 20-40%
arXiv:1509.07758

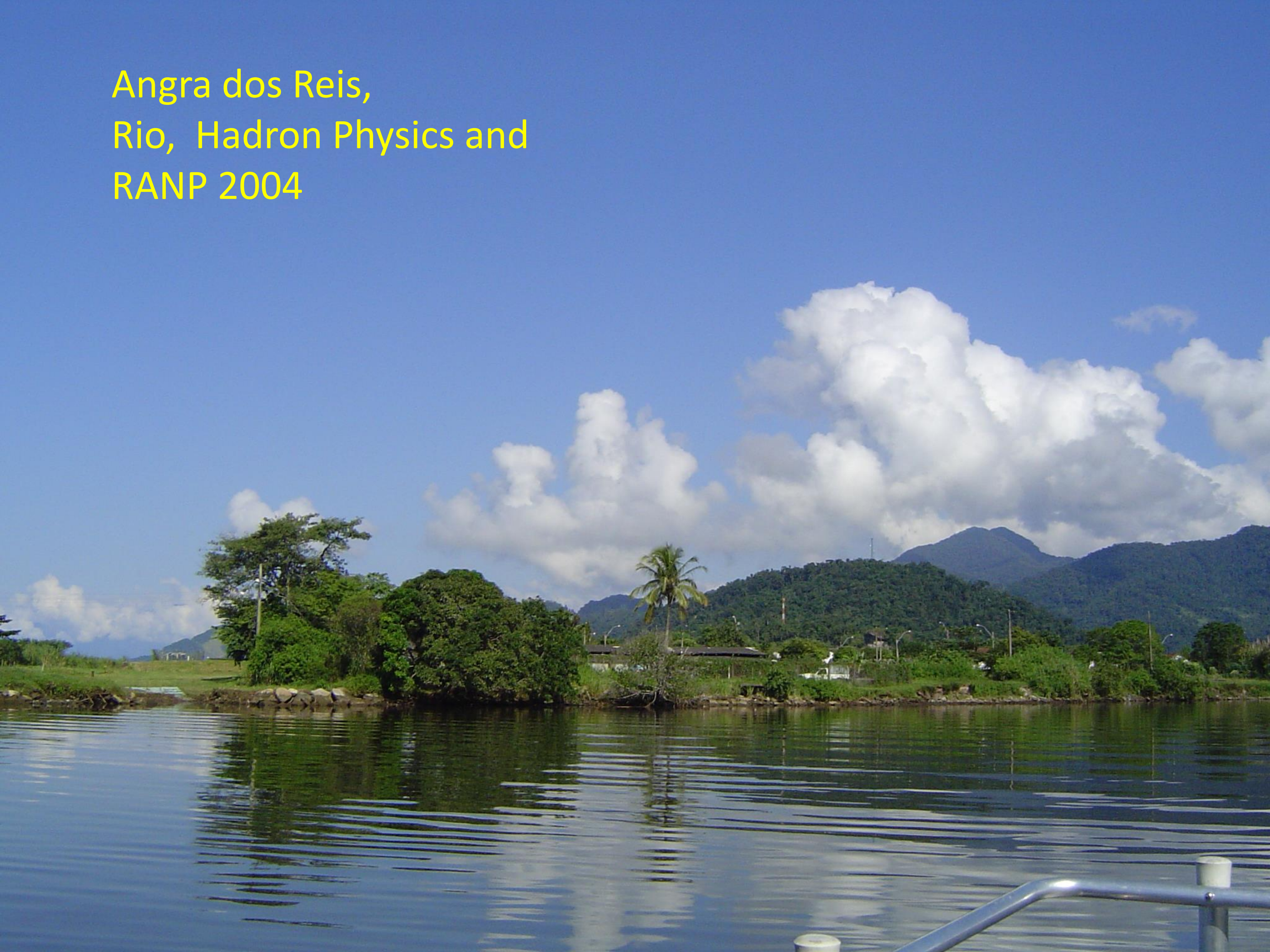
Summary

- The classical electromagnetic fields radiated by the deceleration process of relativistic heavy ions were re-examined in a very (probably not realistic for LHC, but for FAIR/NICA programs, may not be so bad..) simplified scenario.
- If the increase of v_2 in PHENIX data for lower p_T really tells something, we may think of some coherent (collective) deceleration mechanism of the incidente charges.
- If this happens, then we may have a very interesting approach to determine the collision geometry.....
- Of course we are aware of experimental difficulties, and also many serious questions of the present approach. To be examined more in detail, especially how to deal with the decoherent mechanism.

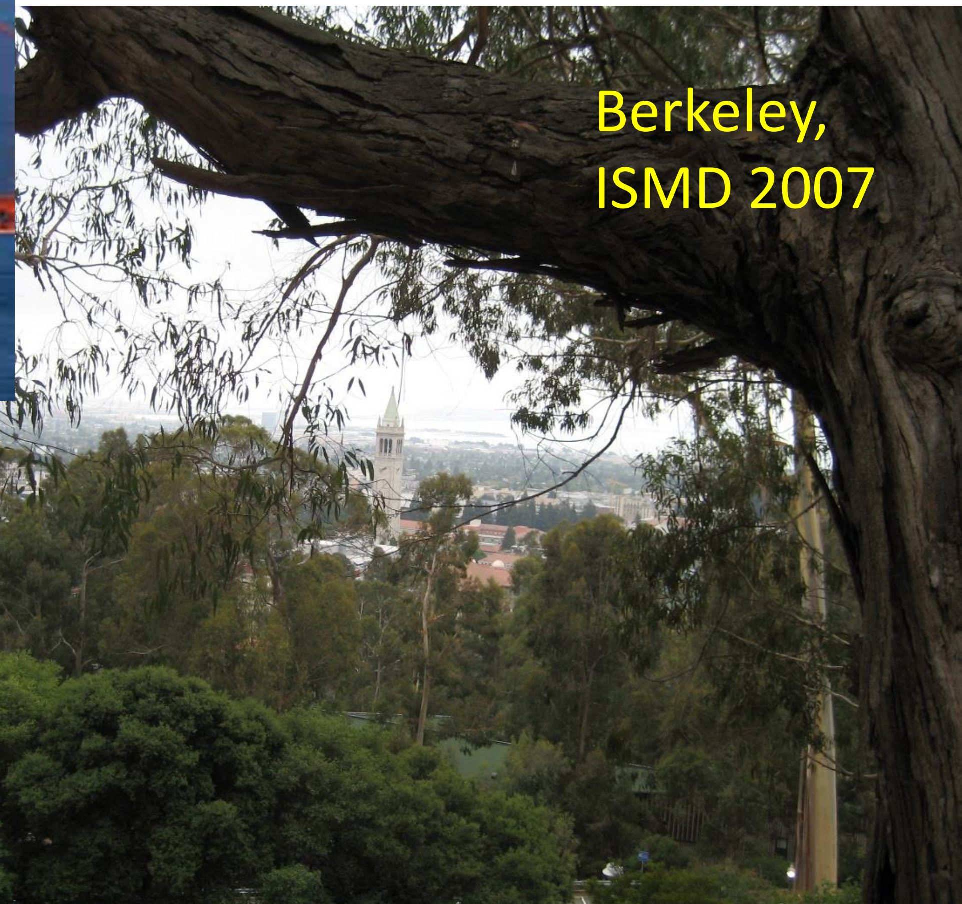
Talking about “Observation”,..

Talking about “Observation”,..

We observe frequently Ulrich’s wavefunction in many nice places with nice foods and drinks,...


Angra dos Reis,
Rio, Hadron Physics and
RANP 2004


Kindly collaborates in helping
Brazilian community by accepting
hard works, such as selection of posters for
the best poster award.



Berkeley,
ISMD 2007

Of course after hard works, we deserve for


a happy hour..


Cape Town
DM 2010


You are always nice and cordial,...

until foods and drinks get scarce,...


Creta, TURIC2011


nice moments with our common good old friends...


but professionally, never miss to ask hard questions,...

At the Capital of US ..


more works, comissions,...
Steffen A. Bass
Barbara Jacak
Victoria Greene


But in Rio de Janeiro,


You will never miss good
foods and drinks, and
are always welcome…

Congratulations for your accomplishments and Happy the Second Turn of the Life!

Stay as the key person for the another turn!

Thank you !