# There is strongly interacting stuff inside of everything

## Towards a better determination of the hadronic contributions to the magnetic moment of the muon

Stefan Leupold

CERN, July 2016

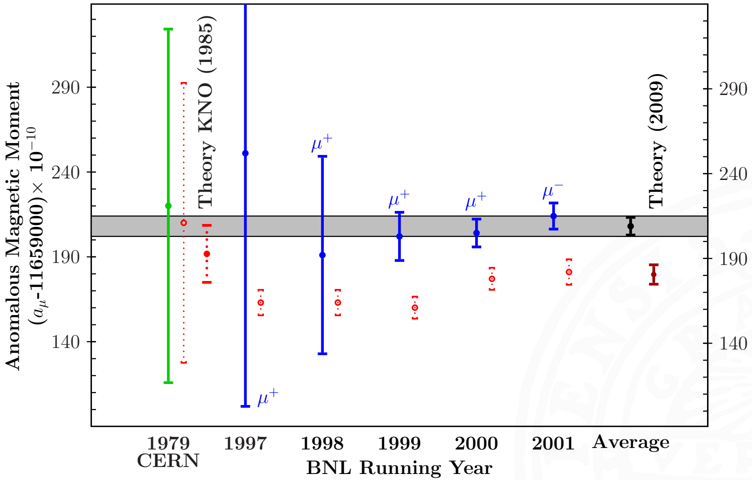
# Collaborators

- Martin Hoferichter (Seattle)
- Jonny Jansson (Uppsala)
- Bastian Kubis (Bonn)
- Bai Long (Bonn)
- Franz Niecknig (Bonn)
- Sebastian Schneider (Bonn, now industry)

related previous work of the Bonn group:

F. Niecknig, B. Kubis and S. P. Schneider, Eur. Phys. J. C 72, 2014 (2012)
S. P. Schneider, B. Kubis and F. Niecknig, Phys. Rev. D 86, 054013 (2012)
M. Hoferichter, B. Kubis and D. Sakkas, Phys. Rev. D 86, 116009 (2012)

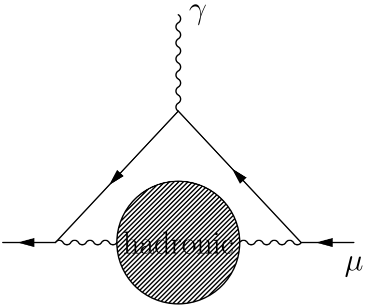
# $g-2$ of the muon — status: $\approx 3\sigma$ deviation



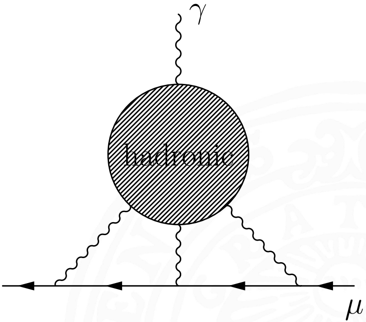

Jegerlehner/Nyffeler, Phys. Rept. 477, 1 (2009)

# $g-2$ of the muon — theory

Largest uncertainty of standard model: hadronic contributions

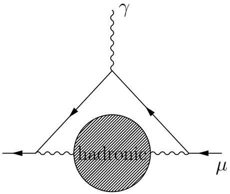



vacuum polarization
$\sim \alpha^2$

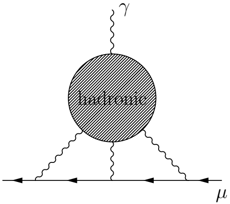



light-by-light scattering
$\sim \alpha^3$

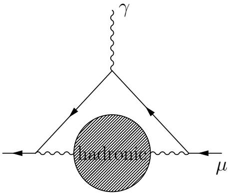
# Hadronic contribution to $g-2$ of the muon




how to determine size of hadronic fluctuations?

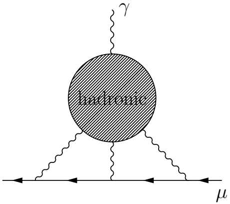


# Hadronic contribution to $g - 2$ of the muon

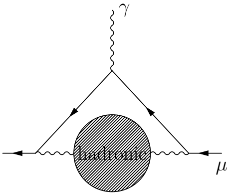



how to determine size of hadronic fluctuations?

$\hookrightarrow$ develop a phenomenological hadronic model or quark model **P**(?)



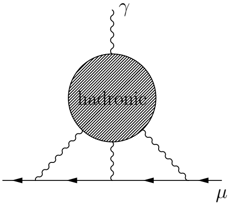
# Hadronic contribution to $g-2$ of the muon



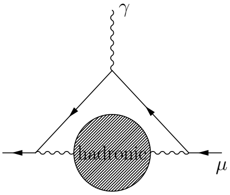

how to determine size of hadronic fluctuations?

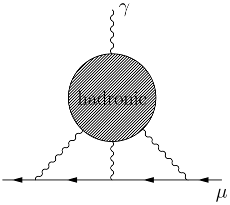
$\hookrightarrow$ develop a phenomenological hadronic model or quark model **P**(?)

$\hookrightarrow$ this would yield a **P**-model prediction



# Hadronic contribution to $g-2$ of the muon



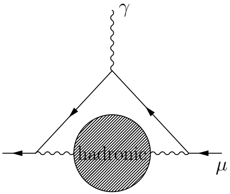

how to determine size of hadronic fluctuations?

↪ develop a phenomenological hadronic model or quark model **P**(?)

↪ this would yield a **P**-model prediction

↪ but we want a standard-model prediction

# Hadronic contribution to $g-2$ of the muon

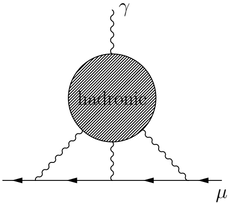



how to determine size of hadronic fluctuations?

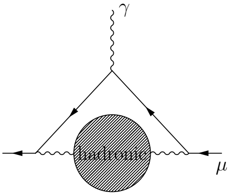
$\hookrightarrow$ develop a phenomenological hadronic model or quark model **P**(?)

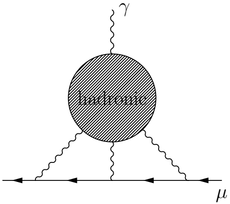
$\hookrightarrow$ this would yield a **P**-model prediction

$\hookrightarrow$ but we want a standard-model prediction and with a reliable uncertainty estimate!



# Hadronic contribution to $g - 2$ of the muon

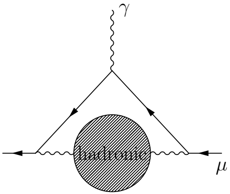


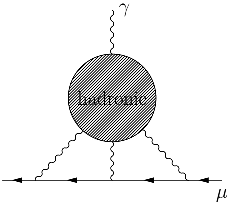




how to determine size of hadronic fluctuations?

- $\hookrightarrow$ develop a phenomenological hadronic model or quark model **P**(?)
- $\hookrightarrow$ this would yield a **P**-model prediction
- $\hookrightarrow$ but we want a standard-model prediction and with a reliable uncertainty estimate!
- $\hookrightarrow$ need a model independent approach
- $\hookrightarrow$ lattice QCD, effective field theory or "data"

# Hadronic contribution to $g - 2$ of the muon



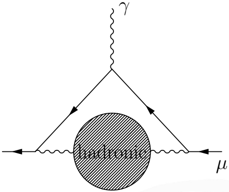




how to determine size of hadronic fluctuations?

- ↪ develop a phenomenological hadronic model or quark model **P**(?)
- ↪ this would yield a **P**-model prediction
- ↪ but we want a standard-model prediction and with a reliable uncertainty estimate!
- ↪ need a model independent approach
- ↪ lattice QCD, effective field theory or "data" (← highest accuracy so far)

# Data-driven approach

## vacuum polarization (now dominant uncertainty)



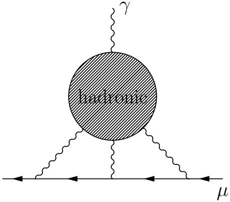

- directly related to cross sect. $e^+e^- \to$ hadrons (by dispersion relation)

$\rightsquigarrow$ measurable

$\rightsquigarrow$ ongoing improvements by international efforts

## light-by-light scattering

(soon dominant uncertainty)




- $\gamma^*\gamma^* \leftrightarrow$ hadron(s) not so easily accessible by experiment

$\hookrightarrow$ crank dispersive machinery further

Colangelo/Hoferichter/Kubis/Procura/Stoffer, Phys.Lett. B738 (2014) 6

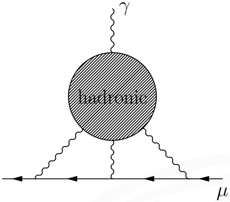
$\hookrightarrow$ defines extensive experimental and theoretical program

# Ulrich as a role model

- data-driven approach to hadronic light-by-light contribution requires close interaction between experiment and theory

↪ for instance in defining quantities of central interest and in translating uncertainties from input to output

↪ what I learned from Ulrich:

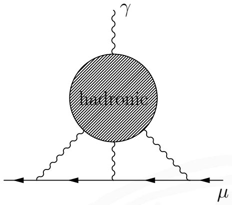


# Ulrich as a role model

- data-driven approach to hadronic light-by-light contribution requires close interaction between experiment and theory

↪ for instance in defining quantities of central interest and in translating uncertainties from input to output

↪ what I learned from Ulrich:

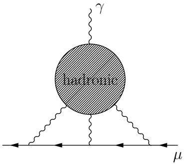



make an effort to talk "experimentalish"

copyright for "experimentalish" by Karin Schönning, Uppsala

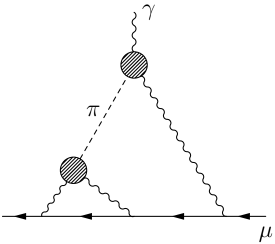
# Hadronic light-by-light contribution

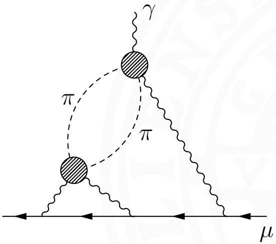
true for all hadronic contributions:




- the lighter the hadronic system, the more important
  (though high-energy contributions not unimportant for light-by-light)

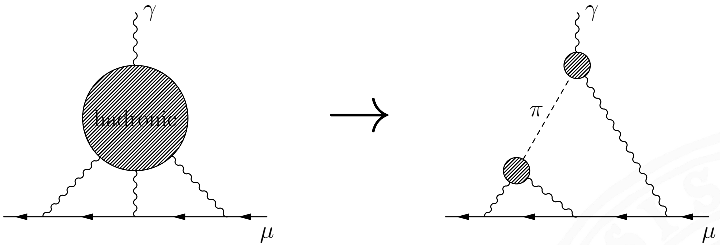
$\hookrightarrow$ $\gamma^{(*)}\gamma^{(*)} \leftrightarrow \pi^0$ $\qquad$ $\gamma^{(*)}\gamma^{(*)} \leftrightarrow 2\pi, \ldots$



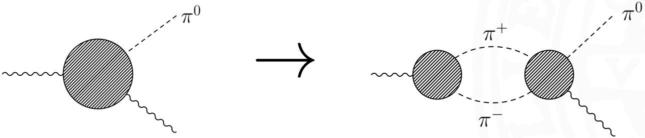



# Using lowest-mass states

hadronic light-by-light contribution

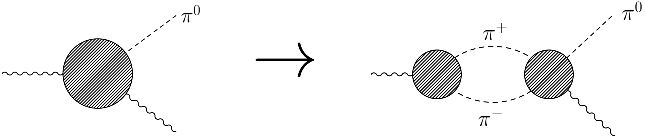



$\rightsquigarrow$ need pion transition form factor

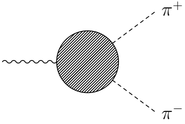


# Dispersive reconstruction I

pion transition form factor

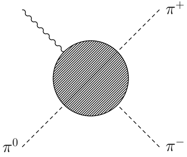



⇝ need pion vector form factor

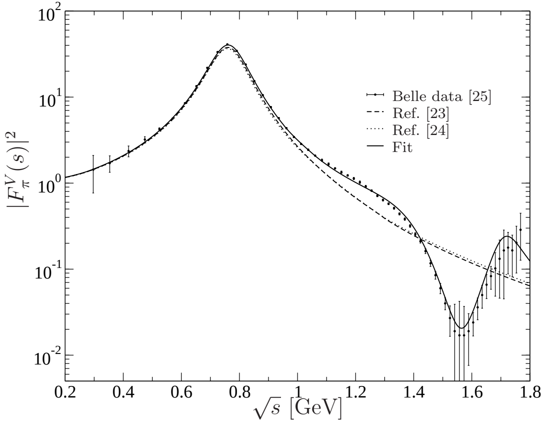


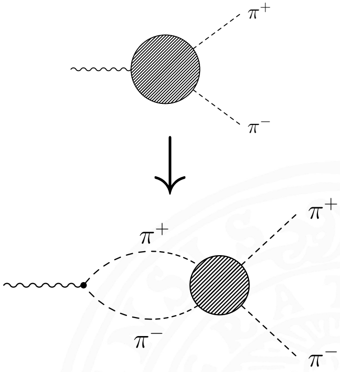

→ very well measured

and amplitude $\gamma^*$–3-pion



# Pion vector form factor



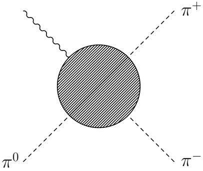




pion phase shift very well known; fits to pion vector form factor

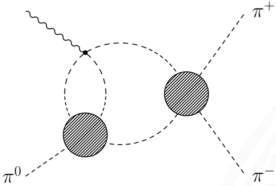
Sebastian P. Schneider, Bastian Kubis, Franz Niecknig, Phys.Rev.D86:054013,2012

# Dispersive reconstruction II

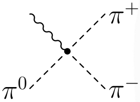
amplitude $\gamma^*$–3-pion




contains two-body correlations
(depend on $s$, $t$, $u$), e.g. $\rightsquigarrow$

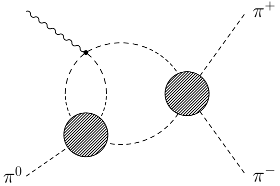



and genuine three-body correlations
(depend on $m^2_{3\pi} = m^2_{\gamma^*}$)

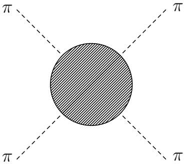


# Required input

for

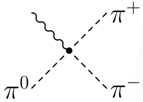



need pion phase shift




$\rightsquigarrow$ very well measured

and genuine three-body correlations
(one-parameter function!)



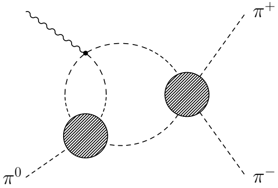

$\rightsquigarrow$ fit to cross section of $e^+e^- \to \pi^+\pi^-\pi^0$

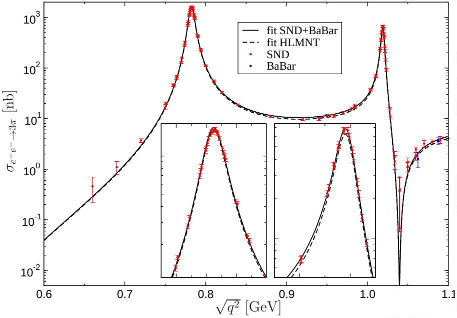
# Fit to $e^+e^- \to \pi^+\pi^-\pi^0$

- dominated by narrow resonances $\omega$, $\phi$

$\hookrightarrow$ use Breit-Wigners plus background for genuine three-body correlations

$\hookrightarrow$ fully include cross-channel rescattering of pion pairs (two-body correlations)



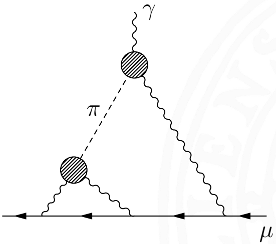

M. Hoferichter, B. Kubis, S.L., F. Niecknig, S. P. Schneider, Eur.Phys.J. C74 (2014) 11, 3180

# Results

- so far: single-virtual pion transition form factor
  - time-like: cross section $e^+e^- \to \pi^0\gamma$
    $\hookrightarrow$ compare to experimental data (postdiction)
  - space-like: reaction $\gamma^*\gamma \to \pi^0$
    $\hookrightarrow$ prediction for low energies

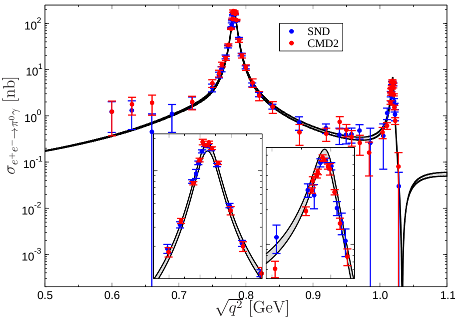
- final aim: double-virtual pion transition form factor

$\hookrightarrow$ relevant for $g-2$




Bai Long, Martin Hoferichter, Bastian Kubis, S.L.; work in progress

# Time-like pion transition form factor



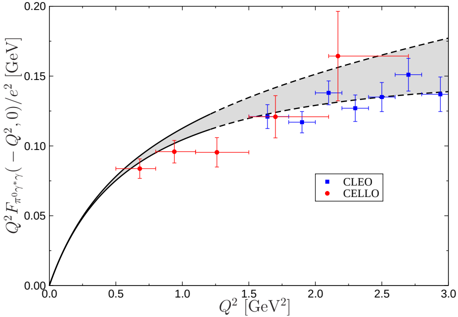

theory uncertainties from

- different data sets for $e^+e^- \to 3\pi$
- different pion phase shifts
- other intermediate states than $2\pi$ neglected
  - $\hookrightarrow$ explored by different cutoff for range where $2\pi$ dominates

$\rightsquigarrow$ excellent agreement

M. Hoferichter, B. Kubis, S.L., F. Niecknig, S. P. Schneider, Eur.Phys.J. C74 (2014) 11, 3180

# Space-like pion transition form factor




- this is a prediction, no data yet at low energies
- expect new measurements from BESIII
- final aim: double virtual transition form factor

$\hookrightarrow$ relevant for $g-2$

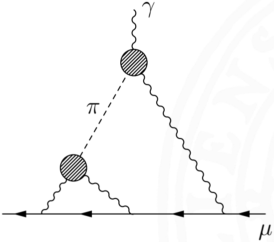
M. Hoferichter, B. Kubis, S.L., F. Niecknig, S. P. Schneider, Eur.Phys.J. C74 (2014) 11, 3180

# Results

- so far: single-virtual pion transition form factor
  - time-like: cross section $e^+e^- \to \pi^0\gamma$
    $\hookrightarrow$ compare to experimental data (postdiction)
  - space-like: reaction $\gamma^*\gamma \to \pi^0$
    $\hookrightarrow$ prediction for low energies

- final aim: double-virtual pion transition form factor

$\hookrightarrow$ relevant for $g-2$




Bai Long, Martin Hoferichter, Bastian Kubis, S.L.; work in progress

# backup slides

# Unitarity and analyticity

- constraints from quantum field theory:
  partial-wave amplitudes for reactions/decays must be
  - unitary:

$$S\,S^\dagger = 1\,, \quad S = 1 + iT \quad \Rightarrow \quad 2\,\mathrm{Im}\,T = T\,T^\dagger$$

↪ note that this is a matrix equation:
$\mathrm{Im}\,T_{A\to B} = \sum_X T_{A\to X}\, T^\dagger_{X\to B}$
⇝ in practice: use most relevant intermediate states $X$

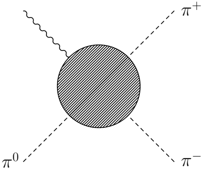
  - analytical (dispersion relations):

$$T(s) = T(0) + \frac{s}{\pi}\int\limits_{-\infty}^{\infty} ds'\, \frac{\mathrm{Im}\,T(s')}{s'\,(s'-s-i\epsilon)}\,,$$

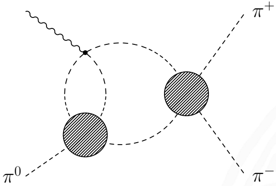
⇝ can be used to calculate whole amplitude from imaginary part

# Dispersive reconstruction II

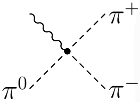
amplitude $\gamma^*$–3-pion




contains two-body correlations
(depend on $s$, $t$, $u$), e.g. $\rightsquigarrow$

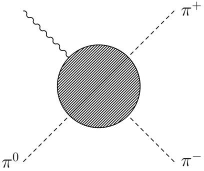



and genuine three-body correlations
(depend on $m^2_{3\pi} = m^2_{\gamma^*}$)

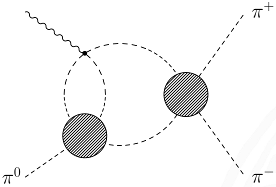


# Dispersive reconstruction II

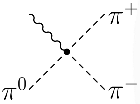
amplitude $\gamma^*$–3-pion




contains two-body correlations
(depend on $s$, $t$, $u$), e.g. $\rightsquigarrow$




and genuine three-body correlations
(depend on $m^2_{3\pi} = m^2_{\gamma^*}$)



# $g-2$ of the muon — status

Standard model theory and experiment comparison [in units $10^{-11}$].

| Contribution | Value | Error |
|---|---|---|
| QED incl. 4-loops + LO 5-loops | 116 584 718.1 | 0.2 |
| Leading hadronic vacuum polarization | 6 903.0 | 52.6 |
| Subleading hadronic vacuum polarization | −100.3 | 1.1 |
| Hadronic light-by-light | 116.0 | 39.0 |
| Weak incl. 2-loops | 153.2 | 1.8 |
| Theory | 116 591 790.0 | 64.6 |
| Experiment | 116 592 080.0 | 63.0 |
| Exp. - The. 3.2 standard deviations | 290.0 | 90.3 |

Jegerlehner/Nyffeler, Phys. Rept. 477, 1 (2009)