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Early Time Dynamics In Nuclear Collisions: A Semi-Analytic Approach

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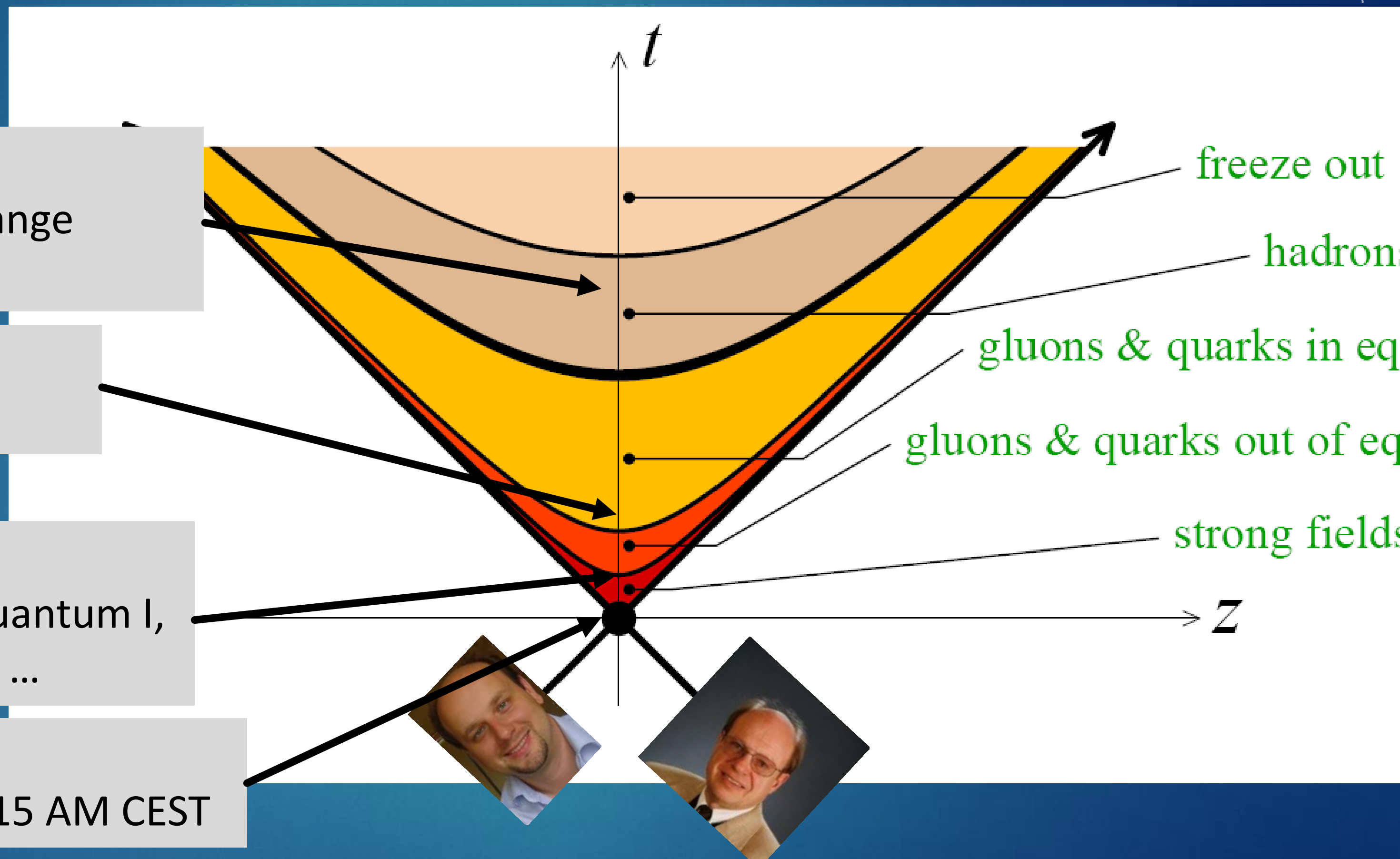
Early Time Dynamics

Late times: toward equilibrium. Long range interactions?

Era of very weak coupling.

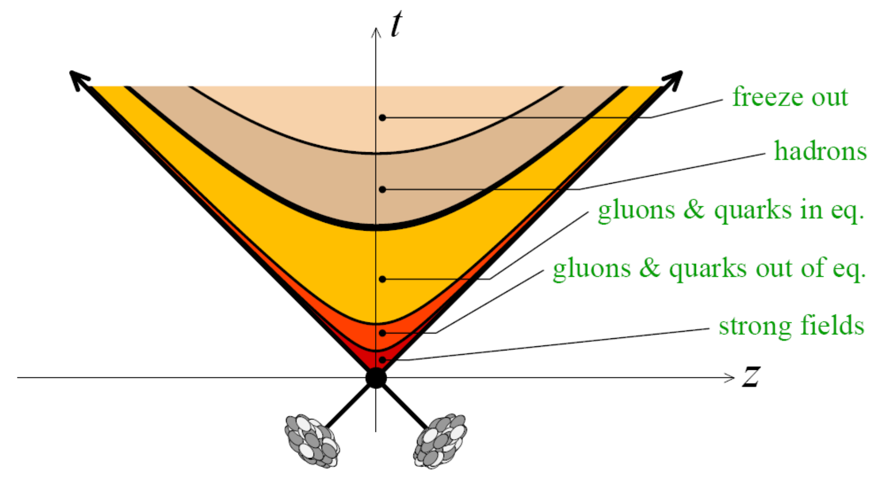
Early interactions:
Math Methods II, Quantum I, Electroweak Theory, …

Initial contact:
t_0= April 29 1992, 8.15 AM CEST



Little Bang: Macroscopic Evolution

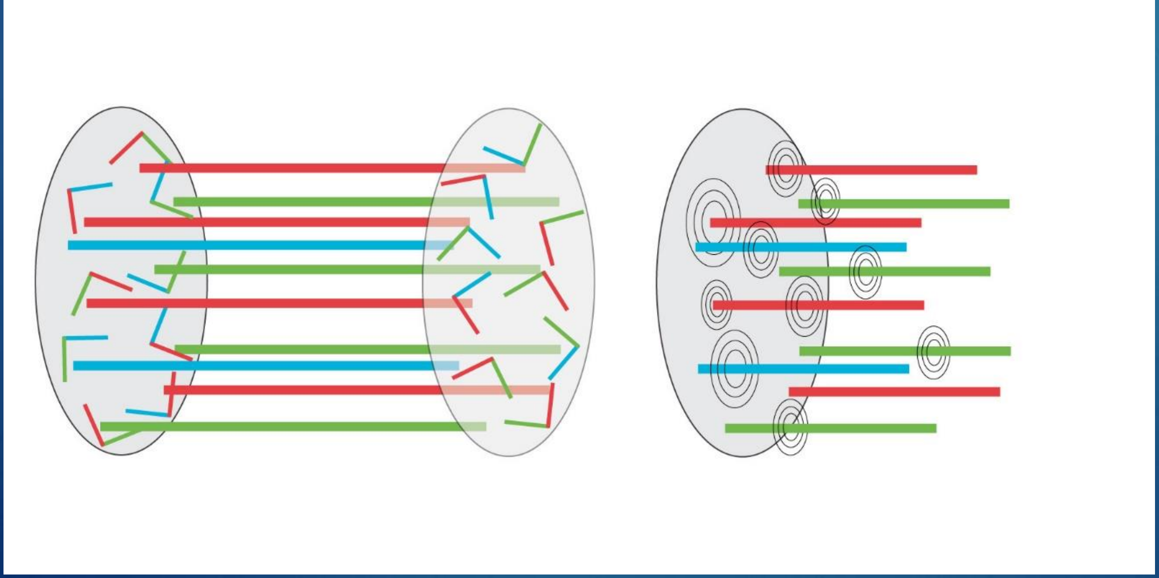
- Initial nuclear wave functions
- Strong Longitudinal gluon fields
- Local equilibration/isotropization
- QGP/HG fluid close to local equilibrium
- Hadron gas in the kinetic regime and freeze-out



- Consensus Model for *Phase II* (times ≥ τ_{th}): fluid dynamics and hadronic transport.
- No consensus on *Phase I*: early time dynamics and equilibration.

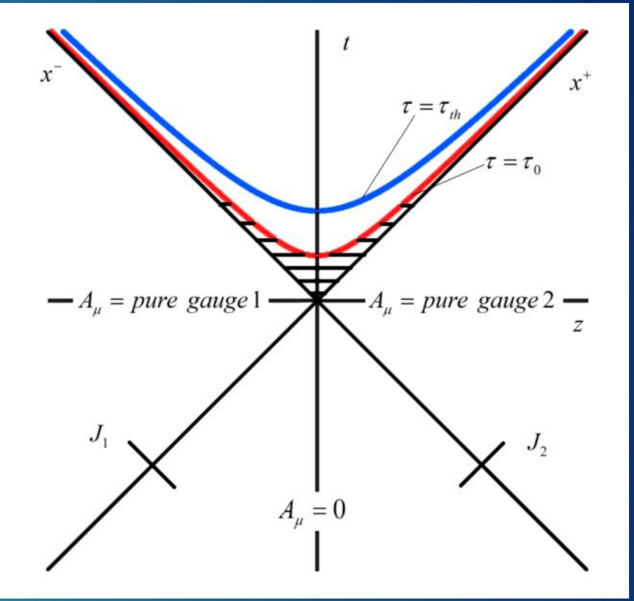
Early Time Evolution

- Initial wave function can be calculated approximately using Color Glass picture.
- Overlap mechanism of nuclear fields known (in the light cone limit) = boundary conditions for further time evolution. Kovner, McLerran, Weigert (1995)
- Dominance of longitudinal (chromo) electric and magnetic fields ("flux tubes") McLerran, Lappi (2006) RJF, Kapusta, Li (2006)



Classical MV Approximation

- Solve Yang-Mills equations $[D^\mu, F^{\mu\nu}] = J^\nu$ for gluon field $A^\mu(\rho)$.
  - Source = light cone current *J* (2-D color charge distributions ρ).
- Calculate gluon field $A^\mu(\rho)$ and any observables $O(\rho)$ determined by the gluon field.
- ρ from *Gaussian* color fluctuations of a color-neutral nucleus.



$$\langle \rho_i^a(x) \rangle = 0$$

$$\langle \rho_i^a(x_1) \rho_j^b(x_2) \rangle = \frac{g^2}{N_c^2 - 1} \delta_{ij} \delta^{ab} \lambda_i(x_1^\mp) \delta(x_1^\mp - x_2^\mp) \delta^2(\mathbf{x}_{1T} - \mathbf{x}_{2T}) \qquad \mu_i = \int dx^\mp \lambda_i(x^\mp)$$

Solving Yang-Mills for $\tau \geq 0$

- Numerical Solutions → Successful Phenomenology
  Krasnitz, Nara, Venugopalan (2003); Lappi (2003)
  … ; Schenke, Tribedy, Venugopalan (2012)

- Here: Analytic approach, expansion of gauge field in time τ
  RJF, Kapusta, Li (2006)
  Chen, Fries, Kapusta, Li (2015)

$$A(\tau, x_\perp) = \sum_{n=0}^{\infty} \tau^n A_{(n)}(x_\perp)$$

$$A_\perp^i(\tau, x_\perp) = \sum_{n=0}^{\infty} \tau^n A_{\perp(n)}^i(x_\perp)$$

- Boundary conditions

$$A_{\perp(0)}^i(x_\perp) = A_1^i(x_\perp) + A_2^i(x_\perp)$$

$$A_{(0)}(x_\perp) = -\frac{ig}{2}\left[A_1^i(x_\perp), A_2^i(x_\perp)\right]$$

- Recursive solution:

$$A_{(n)} = \frac{1}{n(n+2)} \sum_{k+l+m=n-2} \left[D_{(k)}^i, \left[D_{(l)}^i, A_{(m)}\right]\right]$$

$$A_{\perp(n)}^i = \frac{1}{n^2}\left(\sum_{k+l=n-2}\left[D_{(k)}^j, F_{(l)}^{ji}\right] + ig \sum_{k+l+m=n-4}\left[A_{(k)}, \left[D_{(l)}^i, A_{(m)}\right]\right]\right)$$

Solving Yang-Mills for $\tau \geq 0$

- Gauge field in terms of colliding fields

$$A(\tau, x_\perp) = A_{(0)} + \frac{\tau^2}{8}[D^j, [D^j, A_{(0)}]] + \frac{\tau^4}{192}[D^k, [D^k, [D^j, [D^j, A_{(0)}]]]] + \frac{ig\tau^4}{48}\epsilon^{ij}[D^i A_{(0)}, D^j B_0] + \mathcal{O}(\tau^6)\,,$$

$$A_\perp^i(\tau, x_\perp) = A_{\perp(0)}^i + \frac{\tau^2}{4}\epsilon^{ij}[D^j, B_0] + \frac{\tau^4}{64}\epsilon^{ij} D^j D^k D^k B_0 - \frac{ig\tau^4}{64}[B_0, D^i B_0] + \frac{ig\tau^4}{16}[A_{(0)}, [D^i, A_{(0)}]] + \mathcal{O}(\tau^6)$$

- 0th order (boundary conditions at $\tau = 0$): Immediately leads to the known initial longitudinal fields

$$F_{(0)}^{+-} = ig\left[A_1^i, A_2^i\right]$$

$$F_{(0)}^{21} = ig\varepsilon^{ij}\left[A_1^i, A_2^j\right]$$

Energy Momentum Tensor at $O(\tau^2)$

- Minkowski Components

$\vec{S} = \vec{E} \times \vec{B}$

$$T_{\mathrm{f}}^{\mu\nu} = \begin{pmatrix} \varepsilon_0 + O(\tau^2) & \alpha^1 \cosh\eta + \beta^1 \sinh\eta & \alpha^2 \cosh\eta + \beta^2 \sinh\eta & O(\tau^2) \\ \alpha^1 \cosh\eta + \beta^1 \sinh\eta & \varepsilon_0 + O(\tau^2) & O(\tau^2) & \alpha^1 \sinh\eta + \beta^1 \cosh\eta \\ \alpha^2 \cosh\eta + \beta^2 \sinh\eta & O(\tau^2) & \varepsilon_0 + O(\tau^2) & \alpha^2 \sinh\eta + \beta^2 \cosh\eta \\ O(\tau^2) & \alpha^1 \sinh\eta + \beta^1 \cosh\eta & \alpha^2 \sinh\eta + \beta^2 \cosh\eta & -\varepsilon_0 + O(\tau^2) \end{pmatrix}$$

- 0th order: Color Capacitor $\varepsilon_0 = \frac{1}{2}(E_0^2 + B_0^2)$
- 1st order: Poynting vector → Transverse Flow

RJF, Kapusta, Li (2006)
Chen, RJF (2013)
Chen, RJF, Kapusta, Li (2015)

$$\alpha^i = -\frac{\tau}{2}\nabla^i \varepsilon_0$$

$$\beta^i = \frac{\tau}{2}\varepsilon^{ij}\left([D^j, B_0]E_0 - [D^j, E_0]B_0\right)$$

Like hydrodynamic flow, gradient of transverse pressure $p_T = \varepsilon_0$; even in rapidity.

Beyond hydro; odd in rapidity

Early Energy Density and Pressure

- Event-by-event calculations (semi-analytic) $\rho_1, \rho_2 \rightarrow A_1^i, A_2^i$
- For now: event averages with MV model, $\langle\rho_1^2\rangle \sim \mu_1, \langle\rho_2^2\rangle \sim \mu_2$
  - Initial energy density $\varepsilon_0 = \frac{g^6 N_c \left(N_c^2 - 1\right)}{8\pi} \mu_1 \mu_2 \ln^2 \frac{Q^2}{m^2}$

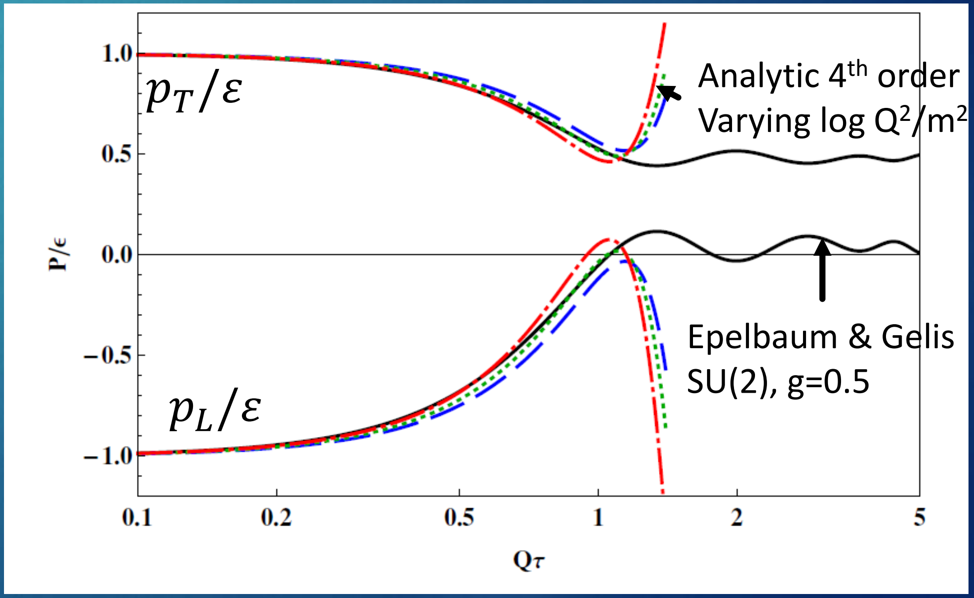
Lappi (2006)

- Analytic results for early time pressure and energy density, consistent with numerical evaluations

Chen, RJF, Kapusta, Li (2015)

- Simplified pocket formulas
  - $\varepsilon = \varepsilon_0 \left(1 - \frac{1}{2}(Q\tau)^2 + O(\tau^4)\right)$
  - $p_T = \varepsilon_0 \left(1 - (Q\tau)^2 + O(\tau^4)\right)$
  - $p_L = \varepsilon_0 \left(1 - \frac{3}{2}(Q\tau)^2 + O(\tau^4)\right)$
- Resumming $(Q\tau)^k$ terms

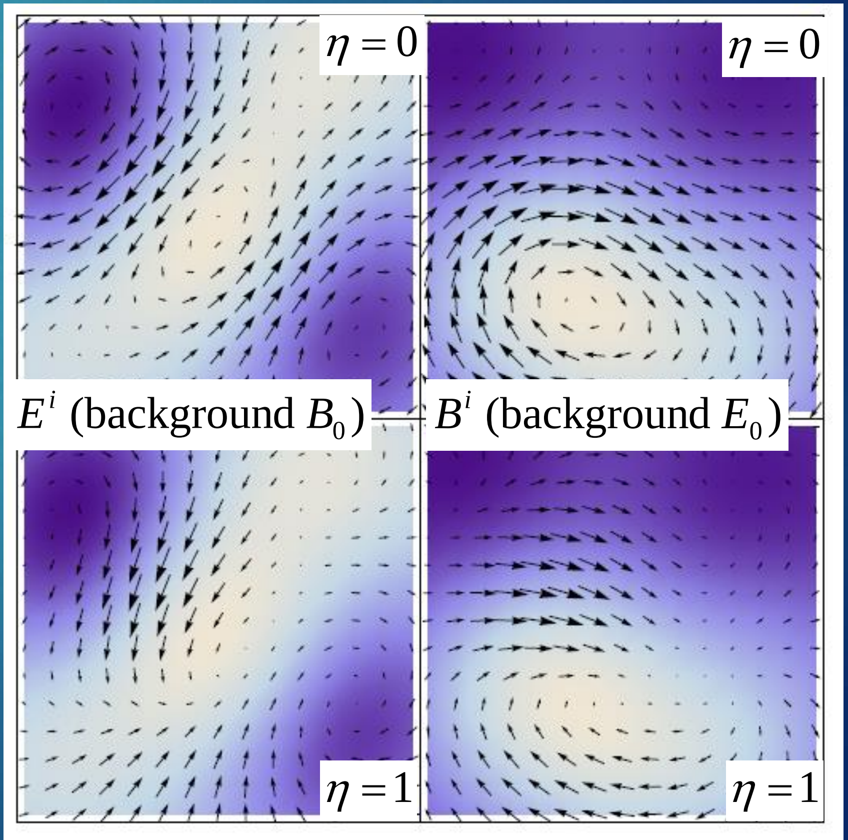
Li, Kapusta, 1602:09060



Field And Flow Patterns

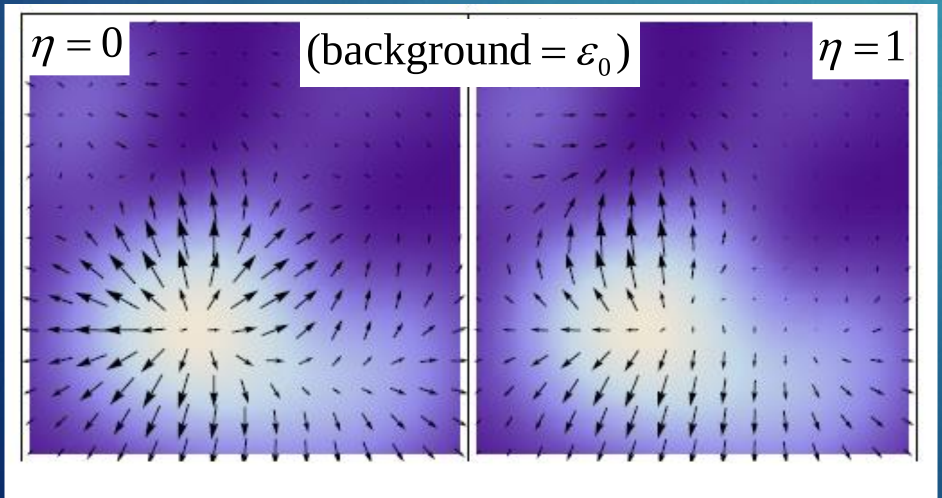
- Transverse Poynting vector for random initial fields (abelian example).
- $\eta = 0$: “Hydro-like” flow from large to small energy density
- $\eta \neq 0$: Quenching/amplification of flow due to the underlying field structure (from Gauss Law),

Transverse Fields



Energy Flow

Chen, RJF (2013)

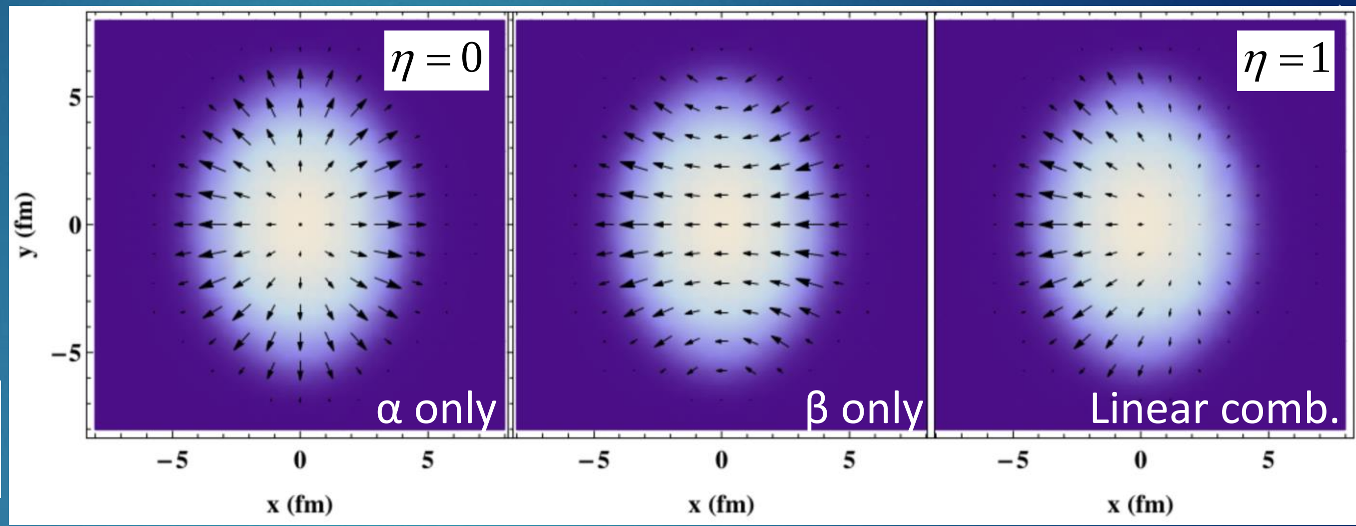


Transverse Flow

- Transverse Poynting vector: Transverse plane

Chen, RJF (2013)

Pb+Pb, b=6 fm

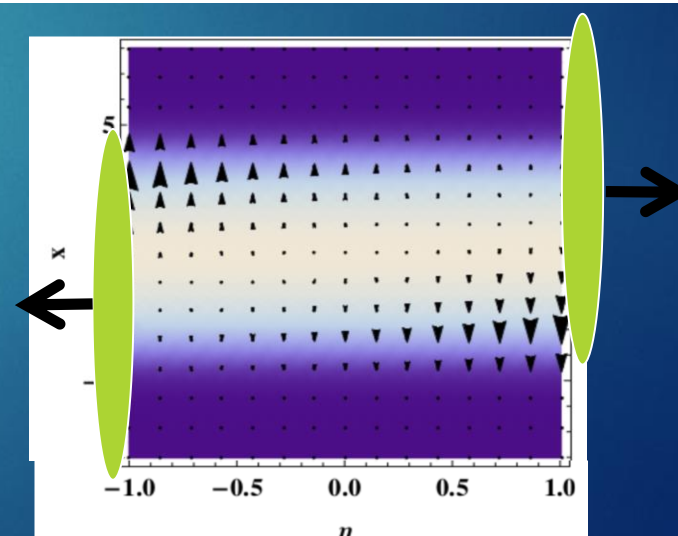


$$\alpha^i = -\varepsilon_0 \frac{\nabla^i (\mu_1 \mu_2)}{\mu_1 \mu_2}$$

$$\beta^i = -\varepsilon_0 \frac{\mu_2 \nabla^i \mu_1 - \mu_1 \nabla^i \mu_2}{\mu_1 \mu_2}$$

- Transverse Poynting vector: Event Plane

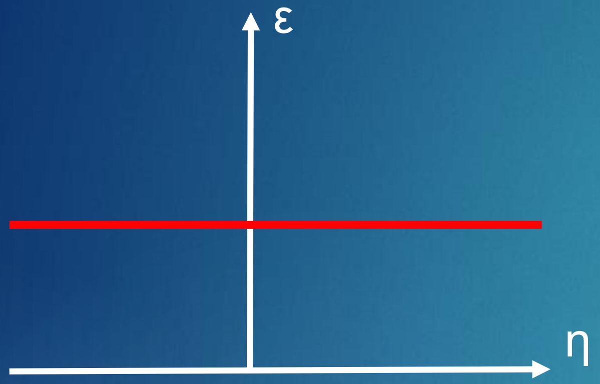
  (long. component suppressed)

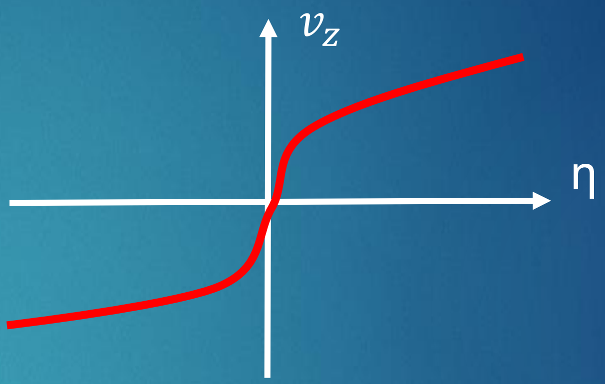


- Radial and elliptic flow
- Rapidity-odd directed flow (from Gauss Law)
- Angular momentum

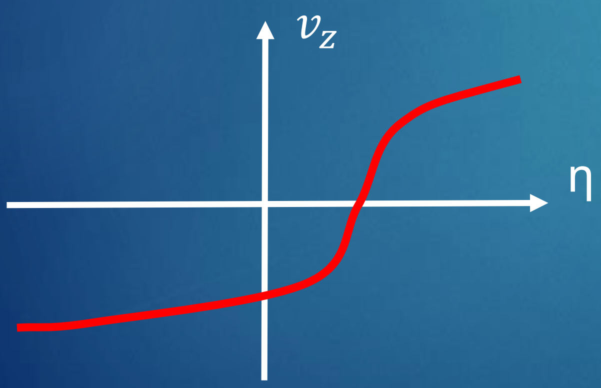
On Boost Invariance

- Naïve boost invariance:





- This is also allowed:



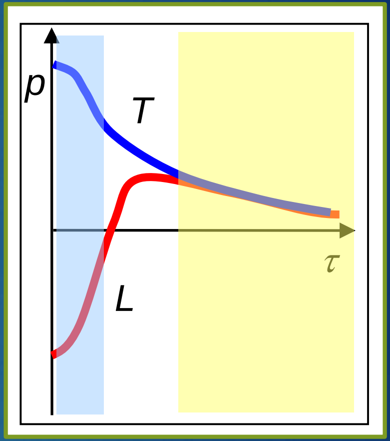
- Boost invariance does not have to pick the lab frame for the node $v_z = 0$.
- An asymmetry between ρ_1 and ρ_2 can lead to a shift of the node away from the lab frame.
- Locally determined for each point in the transverse plane.

Switching to Hydro

- No equilibration in classical YM.
- Pragmatic solution: interpolate between clYM and viscous fluid dynamics, enforce conservation laws.

$$T^{\mu\nu} = T_{\mathrm{f}}^{\mu\nu} r(\tau) + T_{\mathrm{pl}}^{\mu\nu}\left(1 - r(\tau)\right)$$

$$\partial_\mu T^{\mu\nu} = 0 \qquad \partial_\mu M^{\mu\nu\lambda} = 0 \qquad M^{\mu\nu\lambda} = x^\mu T^{\nu\lambda} - x^\nu T^{\mu\lambda}$$



- Direct matching = rapid thermalization assumption

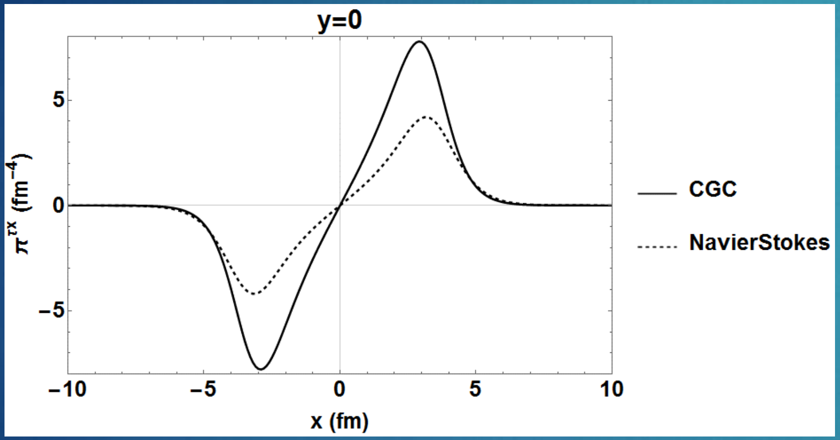
$$r(\tau) = \Theta(\tau_0 - \tau)$$

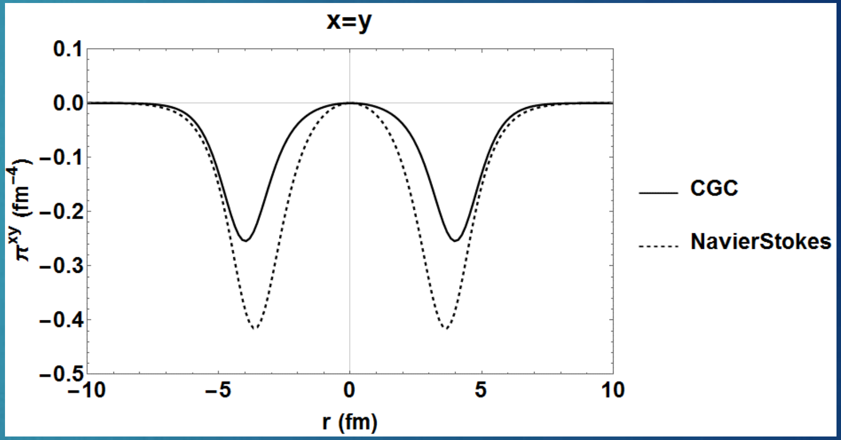
  - Mathematically equivalent to imposing smoothness condition on all components of $T_{\mu\nu}$ (a la Schenke et al.).
  - Decompose YM energy momentum tensor into hydro fields at some matching time τ_0

$$T_f^{\mu\nu} = (\varepsilon + p + \Pi)u^\mu u^\nu - (p + \Pi)g^{\mu\nu} + \pi^{\mu\nu}$$

Yang Mills in Terms of Hydro Fields

- We keep viscous stress even if it is large (otherwise we violate conservation laws at $\tau = \tau_0$).
- Viscous stress extracted from YM generally follows Navier-Stokes behavior.
- Samples

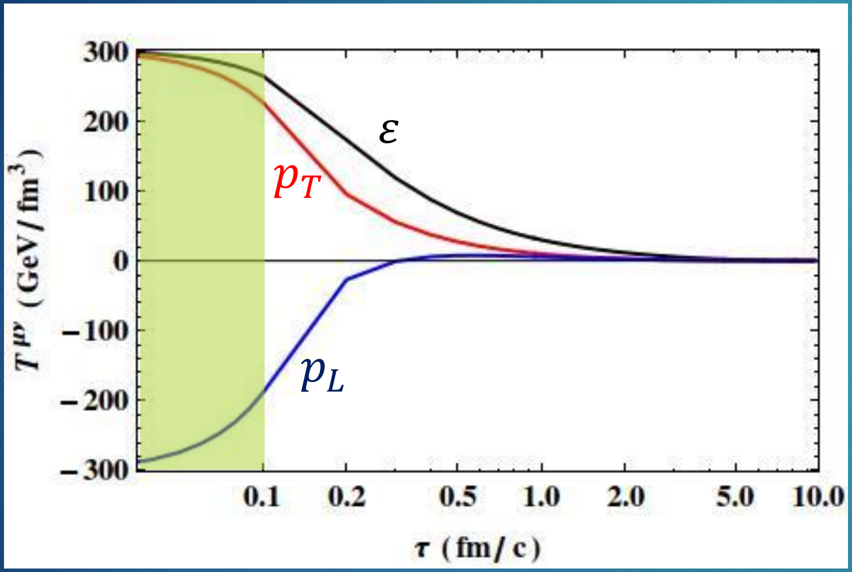


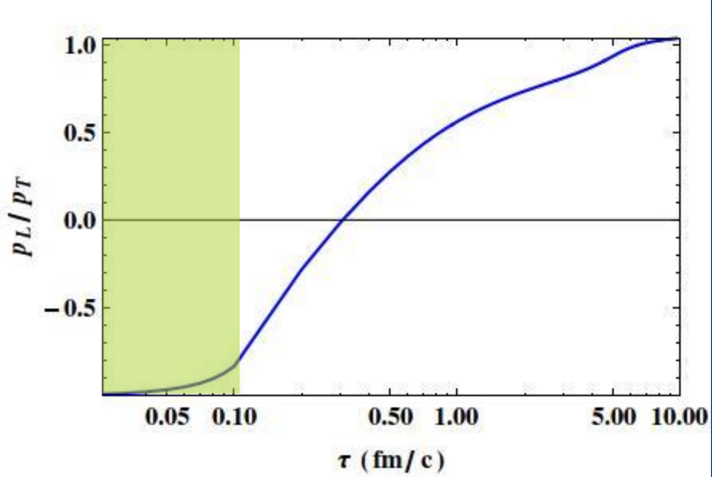


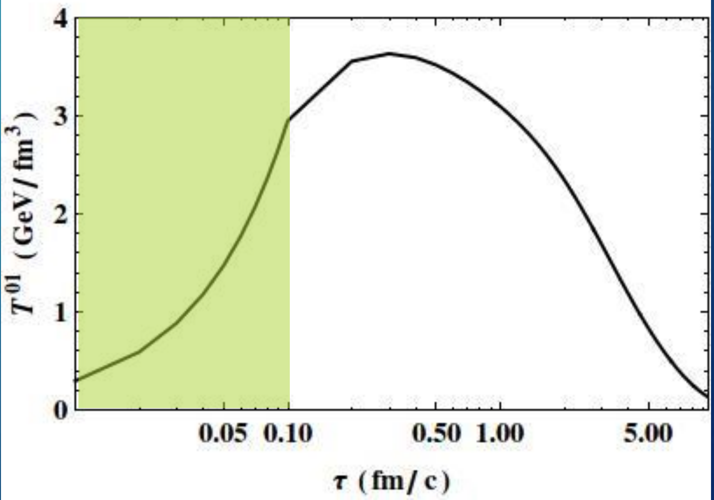
- Expected?
- Now feed into viscous fluid dynamics

Hydro Evolution

- Pressure and flow fields evolve smoothly
  - Here Pb+Pb, *b* = 6 fm, not fitted to a particular energy, $\eta/s = 1/4\pi$.
  - Match at $\tau = 0.1$ fm.





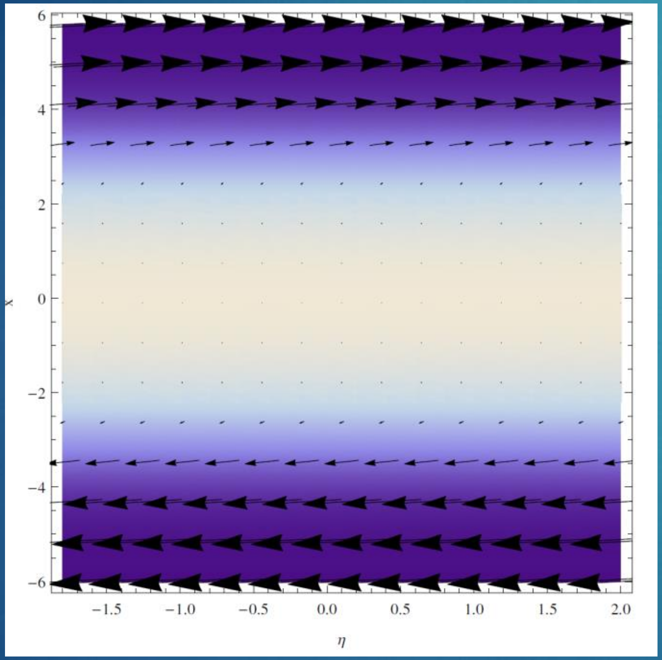


- Not tuned to data.

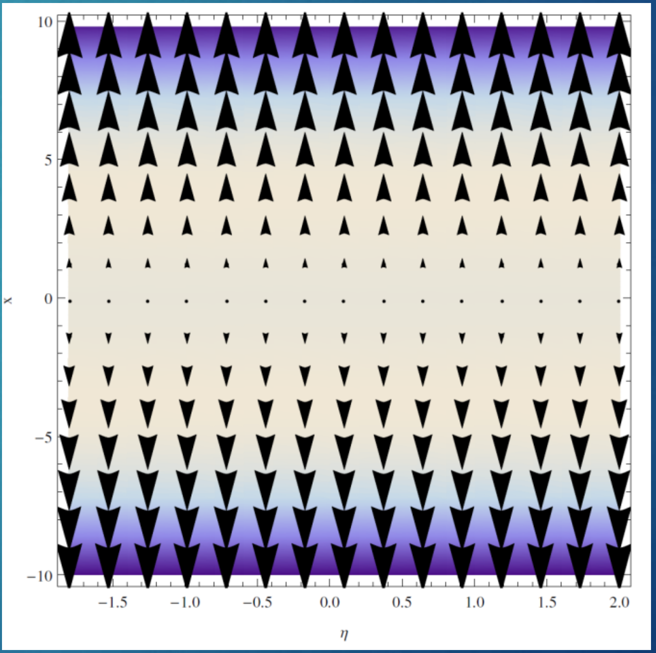
Gauss Law vs Shear Viscosity

- Angular momentum realized as shear flow, not rotation (due to boost invariance)
- $\eta - x$ (event plane) velocity vector in Milne coordinates:

$\tau = 0.1$ fm



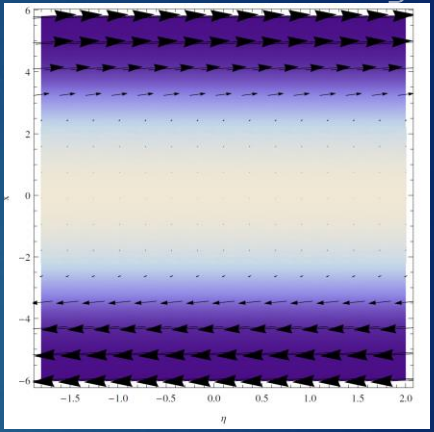
$\tau = 9.6$ fm



- Shear flow decreases rapidly in the hydro evolution.

Gauss Law vs Shear Viscosity

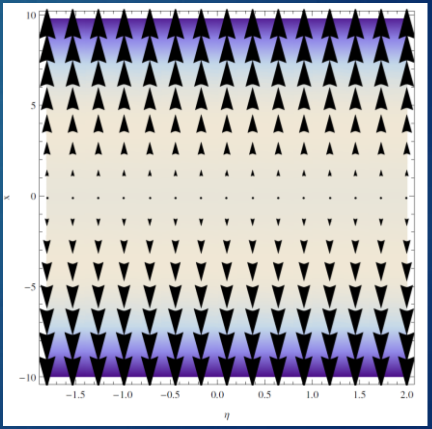
- Yang-Mills phase: Gauss Law builds up shear flow.



- Viscous fluid dynamics works against the gradient.

$$\frac{\partial v_z}{\partial t} = \frac{\eta}{\varepsilon + p}\frac{\partial^2 v_z}{\partial x^2}$$

- Discontinuity in the time evolution.
- Is this something that has observable consequences?



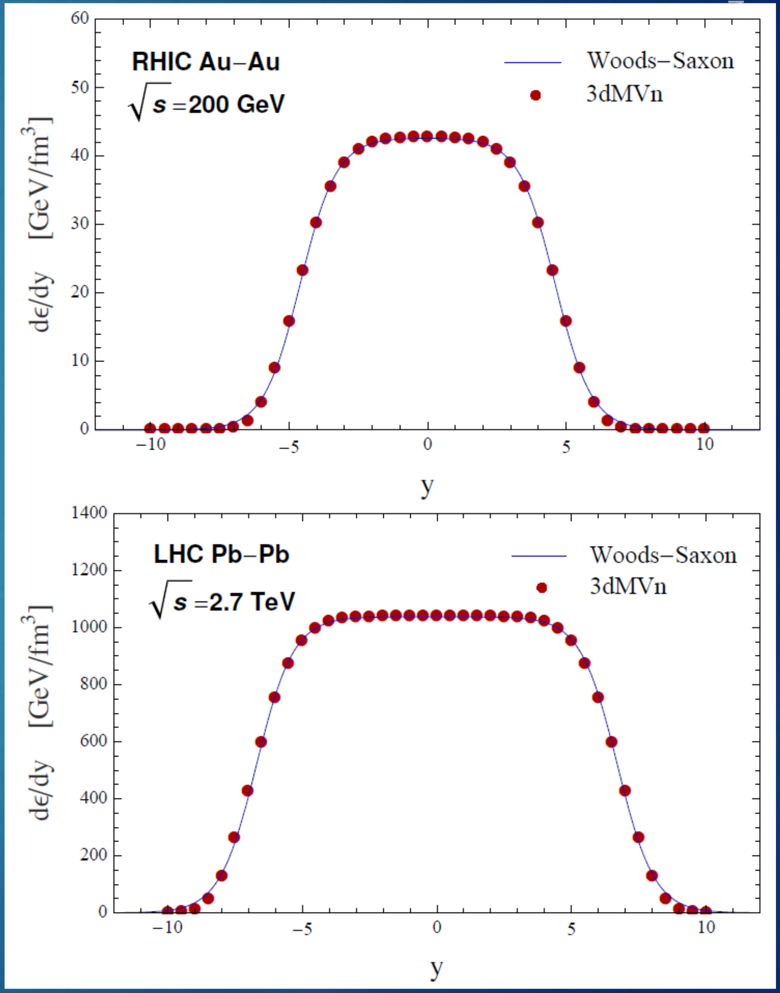
- Angular momentum conserved locally but not globally (sources on the light cone!)

Beyond Boost Invariance

- Real nuclei are slightly off the light cone.
- Need to break boost-invariance in a controlled way.
- Using approximations valid for $R_A/\gamma << 1/Q_s$ we estimated the rapidity dependence of the initial energy density ε_0.

Ozonder, RJF (2014)

- It would enable us to study angular momentum in the initial color glass. Stay tuned.



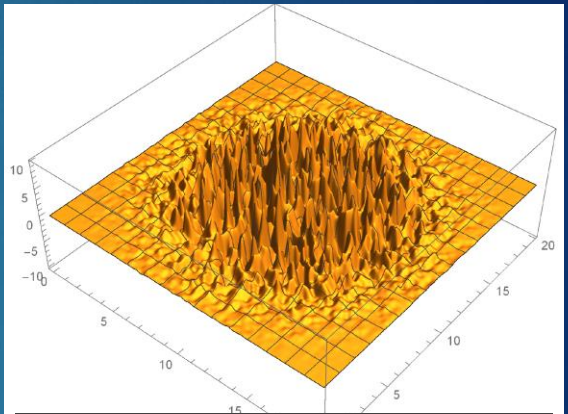
Event-By-Event Calculations

- Event-by-event calculations (semi-analytic) $\rho_1, \rho_2 \rightarrow A_1^i, A_2^i$
- Use recursion relation: no need to solve differential equations for the time evolution
- MC Sampling of charge distributions ρ_1, ρ_2
  - Set UV scale at this step through coarse graining
- Solve for gauge fields in covariant gauge
  - Semi-analytically with Greens function, set IR scale.
- Gauge transformation to physical gauge A_1^i, A_2^i

$$A^j(x^-, \vec{x}_\perp) = \frac{i}{g} U(x^-, \vec{x}_\perp) \partial^j U^\dagger(x^-, \vec{x}_\perp)$$

$$U(x^-, x_\perp) = \mathcal{P} \exp\left[-ig \int_{-\infty}^{x^-} \alpha(z^-, \vec{x}_\perp) dz^- \right]$$

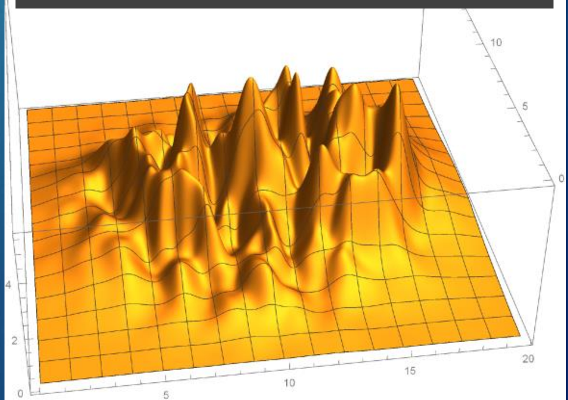
- Calculate coefficients in our power series.



ρ^1

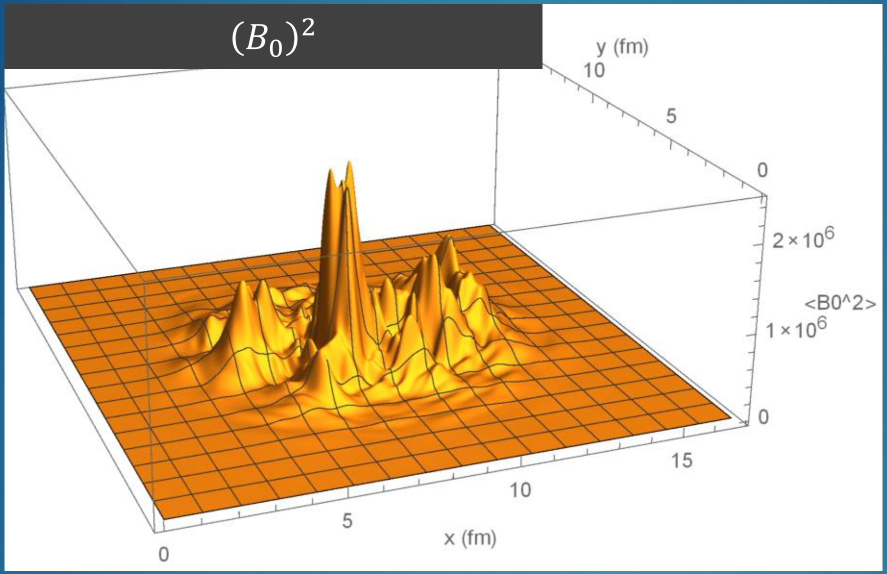
Woods-Saxon; no sub structure; fixed resolution 0.2 fm

$(A^x)^2$



Event-By-Event Calculations

- Fields and energy momentum tensor after nuclear overlap



- Work in progress. Need to
  - compare correlation functions to analytic results.
  - study influence of scales (can be dialed here).
  - compare to purely numerical solutions.

Summary

- (Semi)-analytic results available for clQCD/MV early time evolution.
- Order by order in powers of time, usually good up to ~ $1/Q_s$.

- On the field level flow driven by QCD Gauss/Ampere/Faraday.
- Rapidity odd (but boost invariant) contributions
- Angular momentum; need to go beyond boost-invariance to study rotation.
- Matching to fluid dynamics: smooth evolution of energy density and pressure, discontinuities of microscopic mechanisms.

- Semi-analytic event-by-event calculations
- Limited to very early times but simple and numerically inexpensive
- Extensive testing ahead

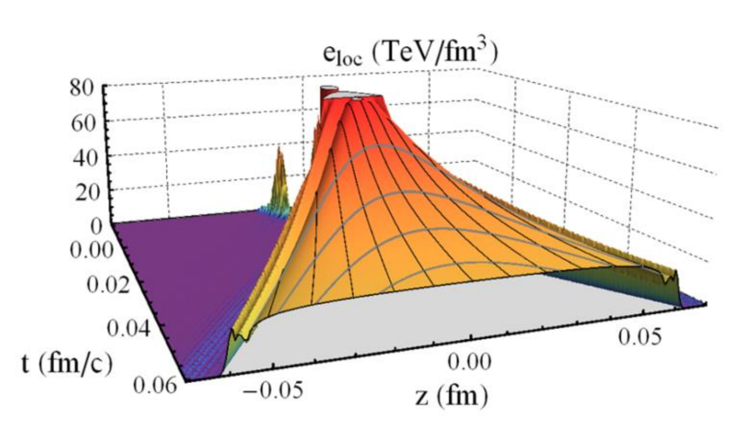
Happy Birthday Uli!

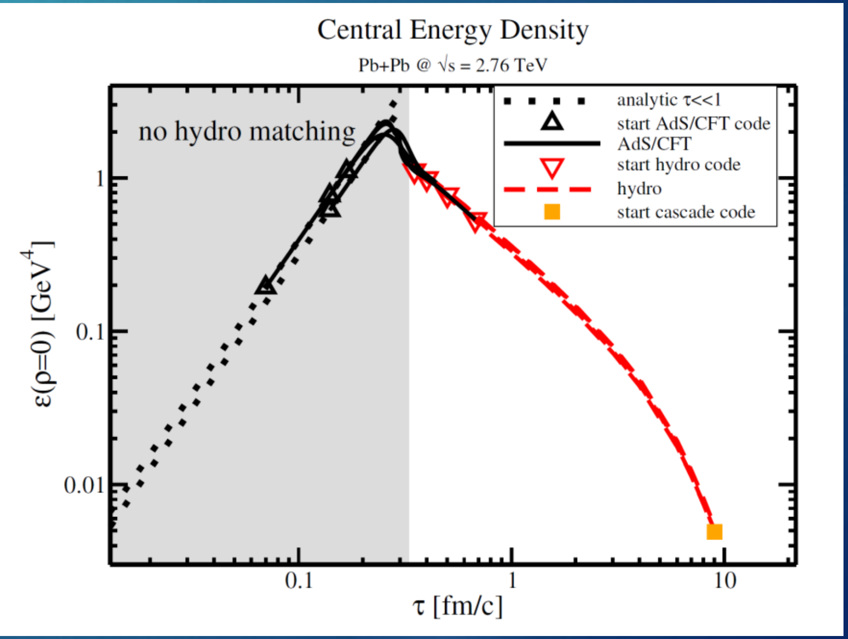
Backup

Early Time Evolution

- Large body of work on early time dynamics
  - Phenomenological models
  - Weak coupling limit
  - Color glass condensate
- Recently much progress on strong coupling scenarios

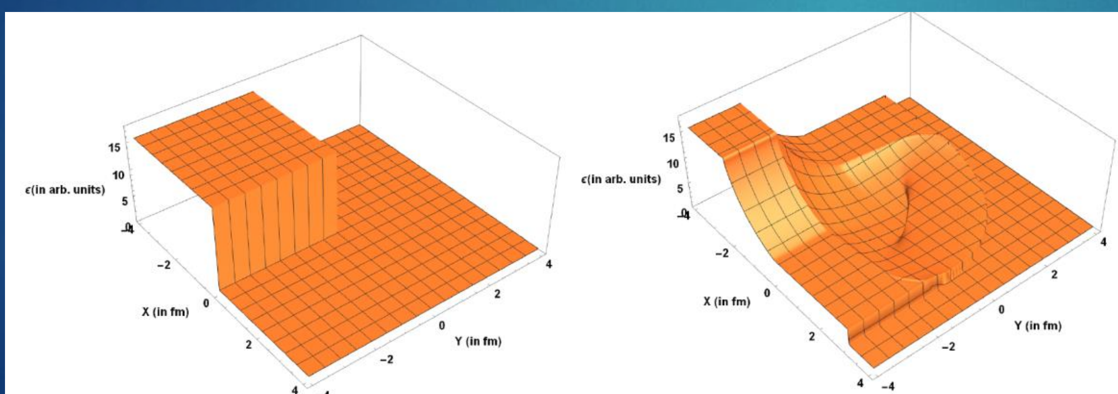
Chesler, Kilbertus, van der Schee (2015)
van der Schee, Schenke (2015)
van der Schee, Romatschke, Pratt (2013)
…

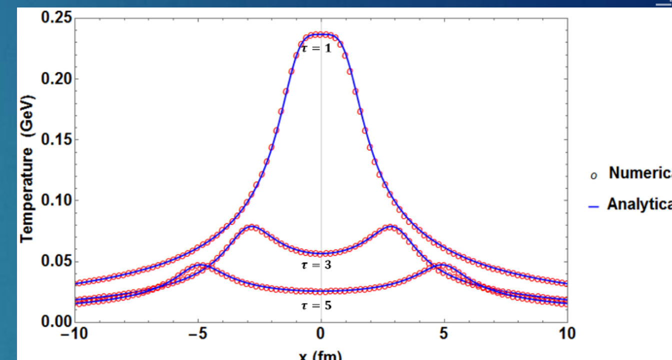


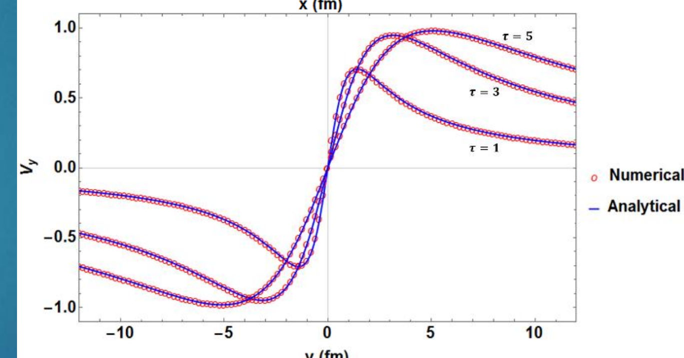


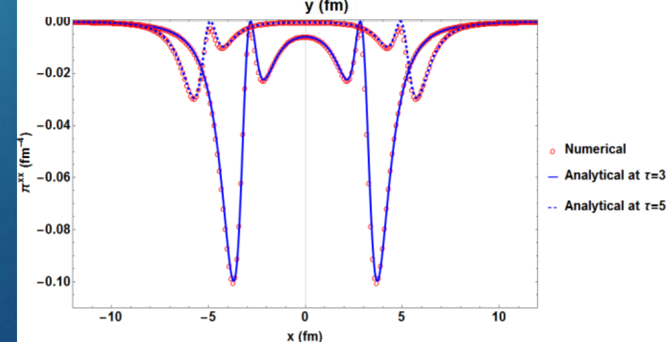
Texas 3+1 D Fluid Code

- KT for fluxes, 5th order WENO for spatial derivatives, 3rd order TVD Runge Kutta for time integration.
- Bulk and shear stress, vorticity
- Gubser and Sod-type tests:



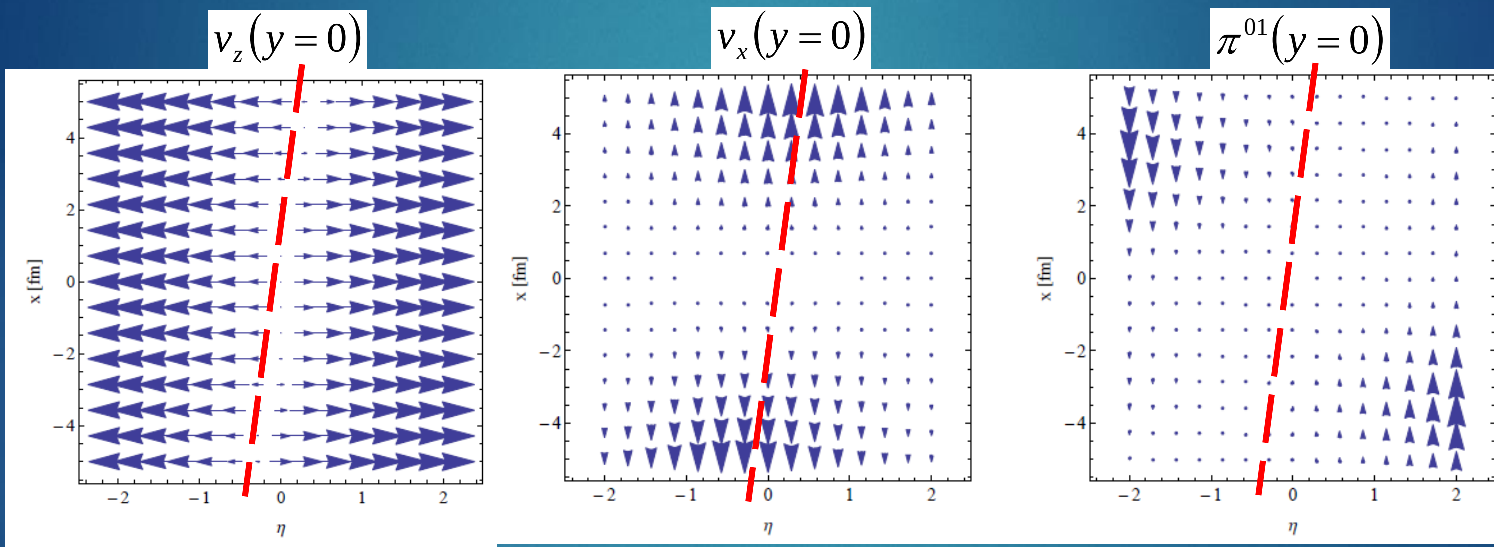






Matching to Hydrodynamics

- Hydro fields in Minkowski components



Origin of Flow in YM

- Initial longitudinal fields E_0, B_0 → transverse fields through QCD versions of Ampere's, Faraday's and Gauss' Law.
  - Here *abelian* version for simplicity.
- Gauss' Law at fixed time *t*
  - Difference in long. flux ⟶ transverse flux
  - *rapidity-odd* and *radial*
- Ampere/Faraday as function of *t*:
  - Decreasing long. flux ⟶ transverse field
  - *rapidity-even* and *curling* field
- Full classical QCD at $O(\tau^1)$:

$$E^i = -\frac{\tau}{2}\left(\sinh\eta\left[D^i, E_0\right] + \cosh\eta\,\varepsilon^{ij}\left[D^j, B_0\right]\right)$$

$$B^i = \frac{\tau}{2}\left(\cosh\eta\,\varepsilon^{ij}\left[D^j, E_0\right] - \sinh\eta\left[D^i, B_0\right]\right)$$

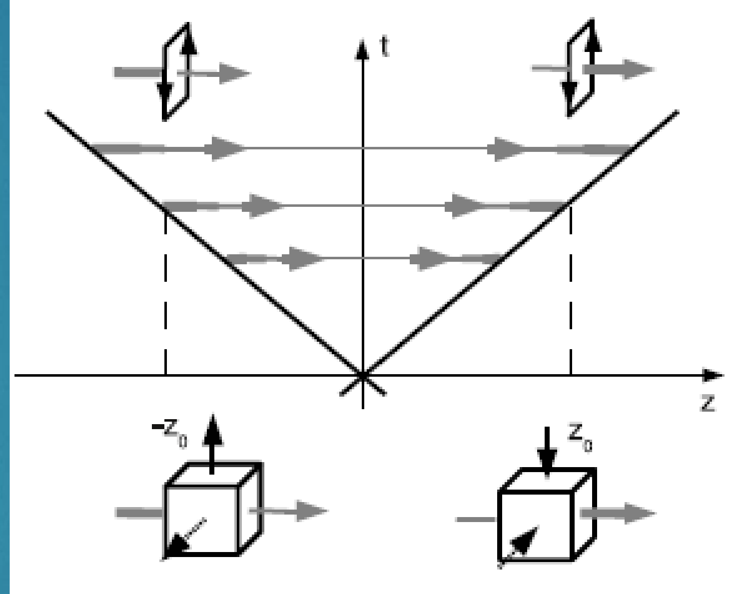


Figure 1: Two observers at $z = z_0$ and $z = -z_0$ test Ampère's and Faraday's Laws with areas a^2 in the transverse plane and Gauss' Law with a cube of volume a^3. The transverse fields from Ampère's and Faraday's Laws (black solid arrows) are the same in both cases, while the transverse fields from Gauss' Law (black dashed arrows) are observed with opposite signs. Initial longitudinal fields are indicated by solid grey arrows, thickness reflects field strength.

Chen, RJF (2013)

Resummation of the Time Evolution

- Generic arguments: convergence radius of the recursive solution ~1/Q_s.
- "Weak field" approximation to Yang Mills

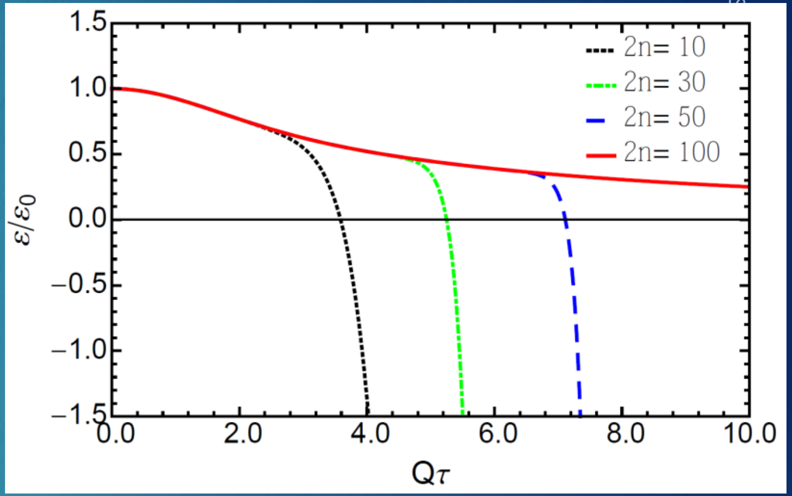
$$A^{\mathrm{LO}}(\tau,\mathbf{k}_\perp)=\frac{2A_{(0)}(\mathbf{k}_\perp)}{k_\perp\tau}J_1(k_\perp\tau)$$

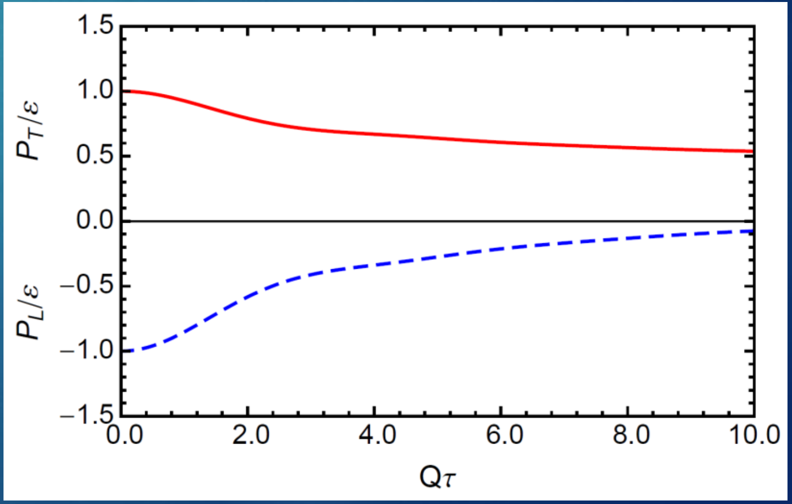
$$A_\perp^{i\,\mathrm{LO}}(\tau,\mathbf{k}_\perp)=A_{\perp(0)}^{i}(\mathbf{k}_\perp)J_0(k_\perp\tau)$$

  - Can be rederived from the recursive solution.
- Resumming $(Q\tau)^k$ terms: semi-closed form

$$\mathcal{A}=\varepsilon_0+\frac{2\varepsilon_0}{\ln^2(Q^2/m^2)}\mathsf{G}_A(Q\tau)-\frac{\varepsilon_0}{\ln(Q^2/m^2)}(Q\tau)^2\left[{}_3F_4\left(1,1,\frac{3}{2};2,2,2,2;-(Q\tau)^2\right)\right]$$

Li, Kapusta, 1602:09060





Correlation Functions

- Event-by-event calculations (semi-analytic) $\rho_1, \rho_2 \rightarrow A_1^i, A_2^i$
- Use recursion relation: no need to solve differential equations for the time evolution