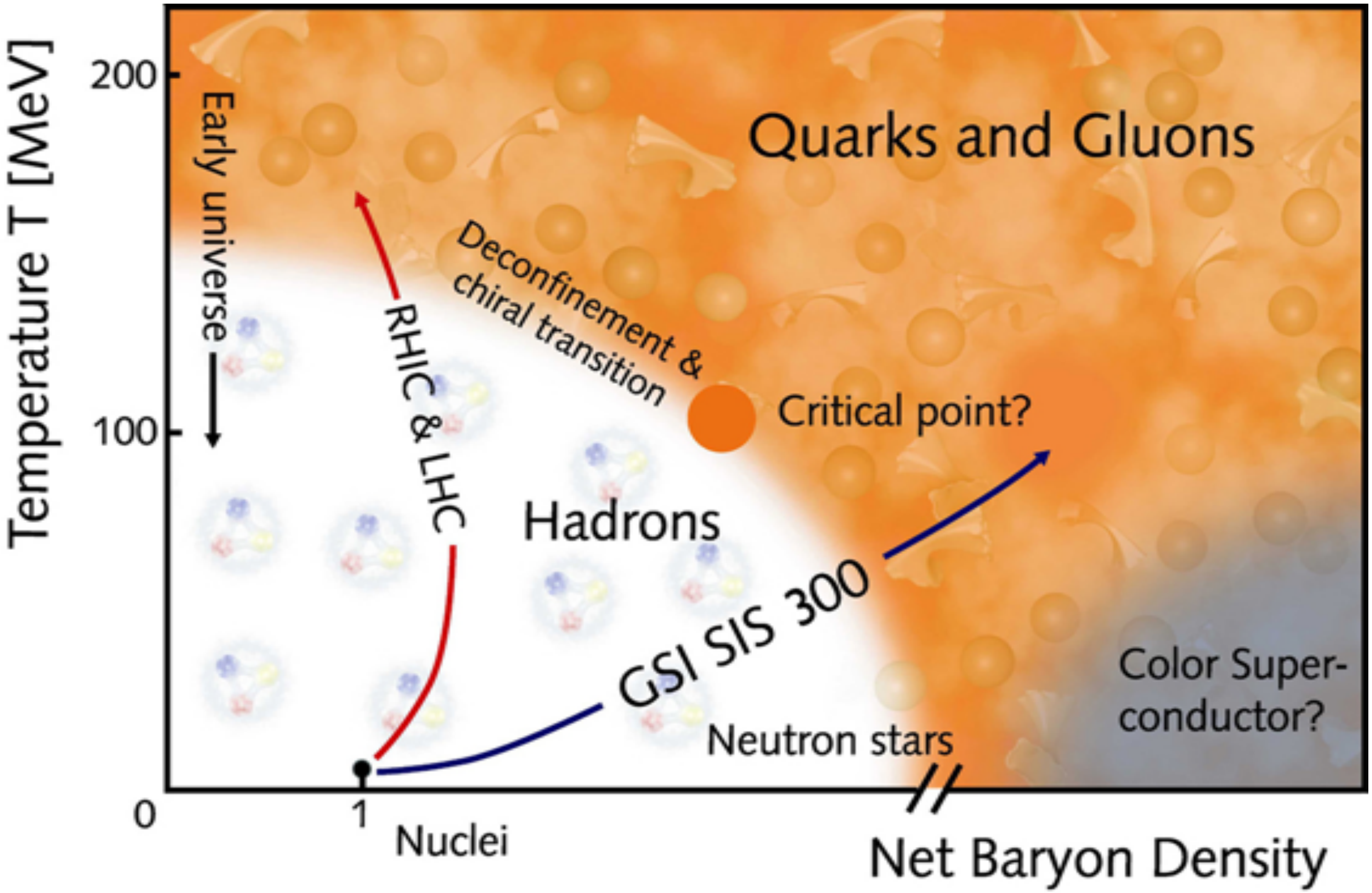
QCD phase structure and the RICH beam energy scan

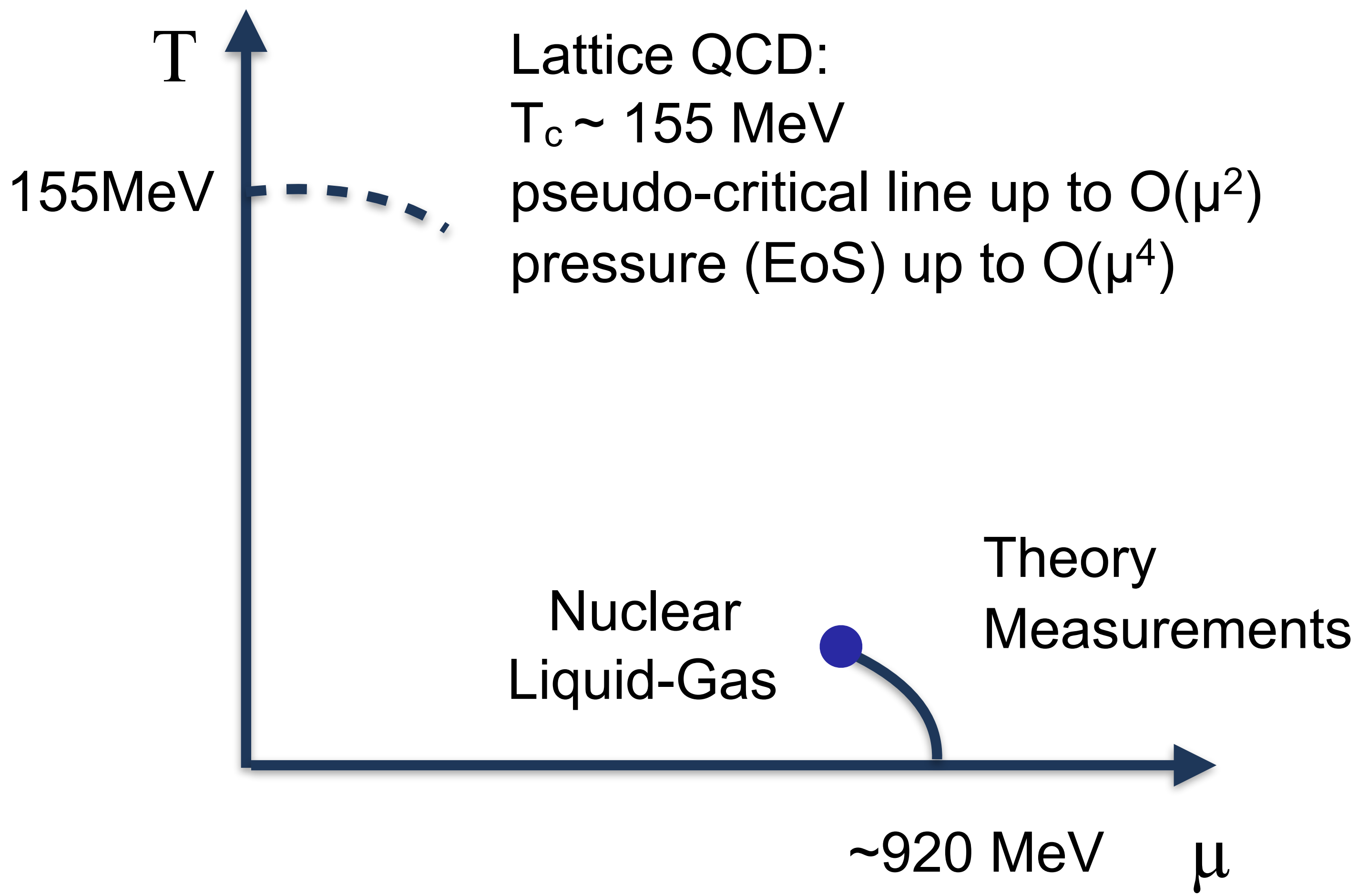
- Introduction
- Cumulants and Correlations
- Correlations in the preliminary STAR data

With Adam Bzdak and Nils Strodthoff,   1607.nnnn

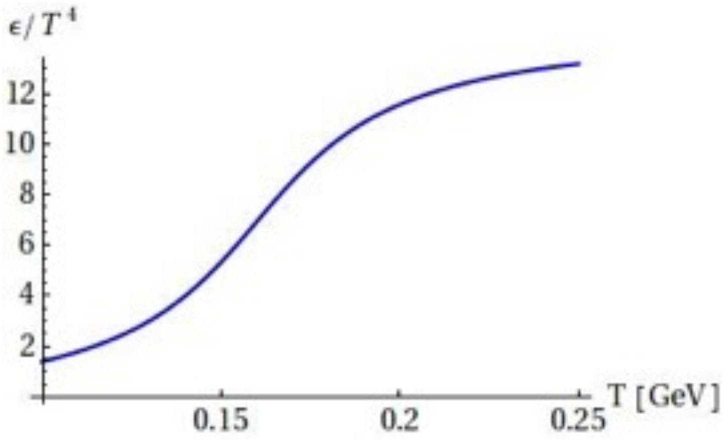
The QCD Phase diagram



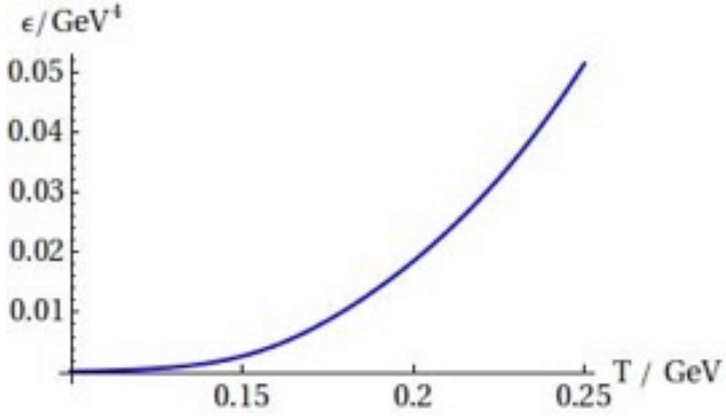
What we know about the Phase Diagram



The Lattice EOS



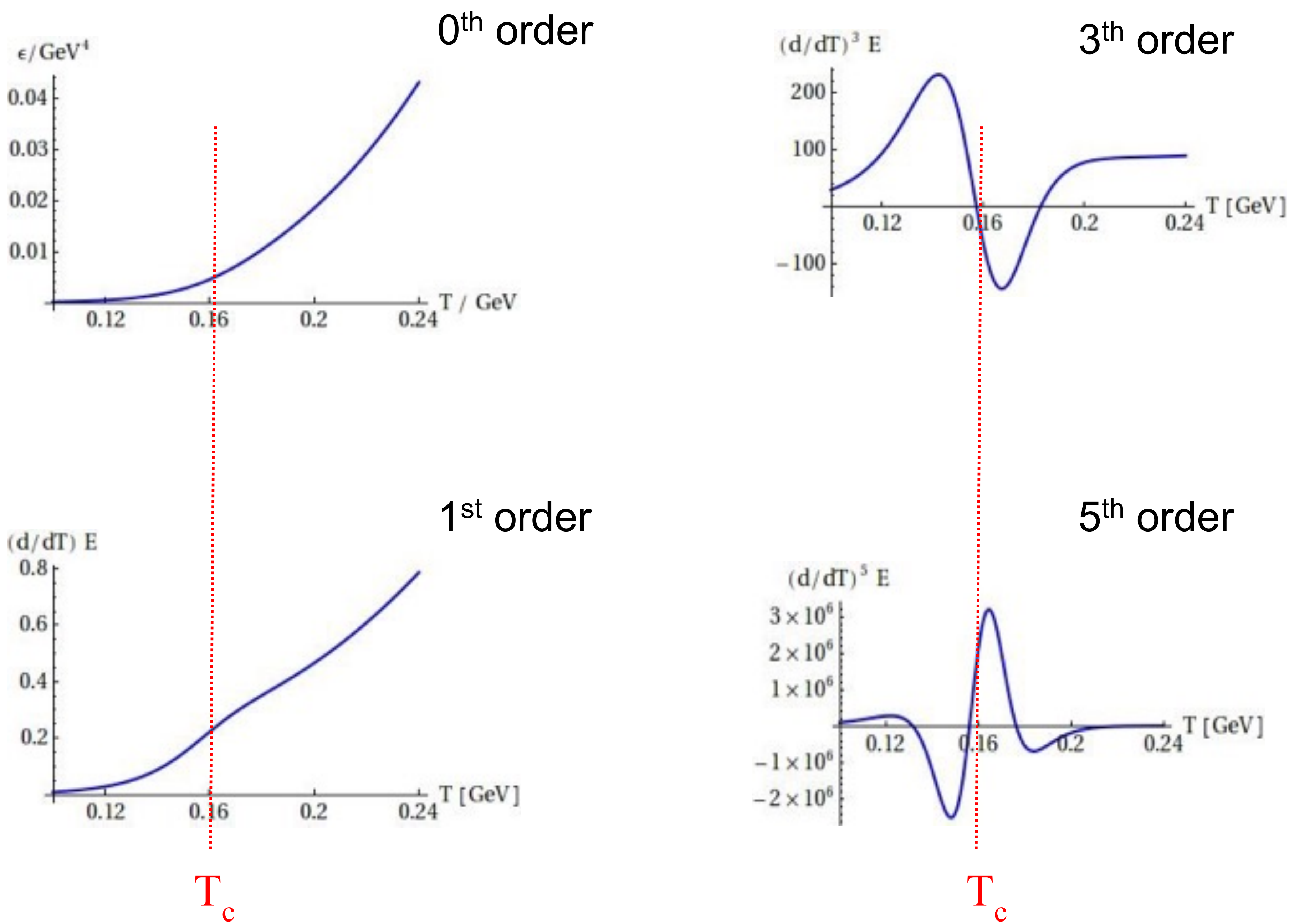
What we always see....



What it really means....

"T_c" ~ 160 MeV

Derivatives



How to measure derivatives

At μ = 0:

$$Z = tr\, e^{-\hat{E}/T + \mu/T \hat{N}_B}$$

$$\langle E \rangle = \frac{1}{Z}\, tr\, \hat{E}\, e^{-\hat{E}/T + \mu/T \hat{N}_B} = -\frac{\partial}{\partial 1/T} \ln(Z)$$

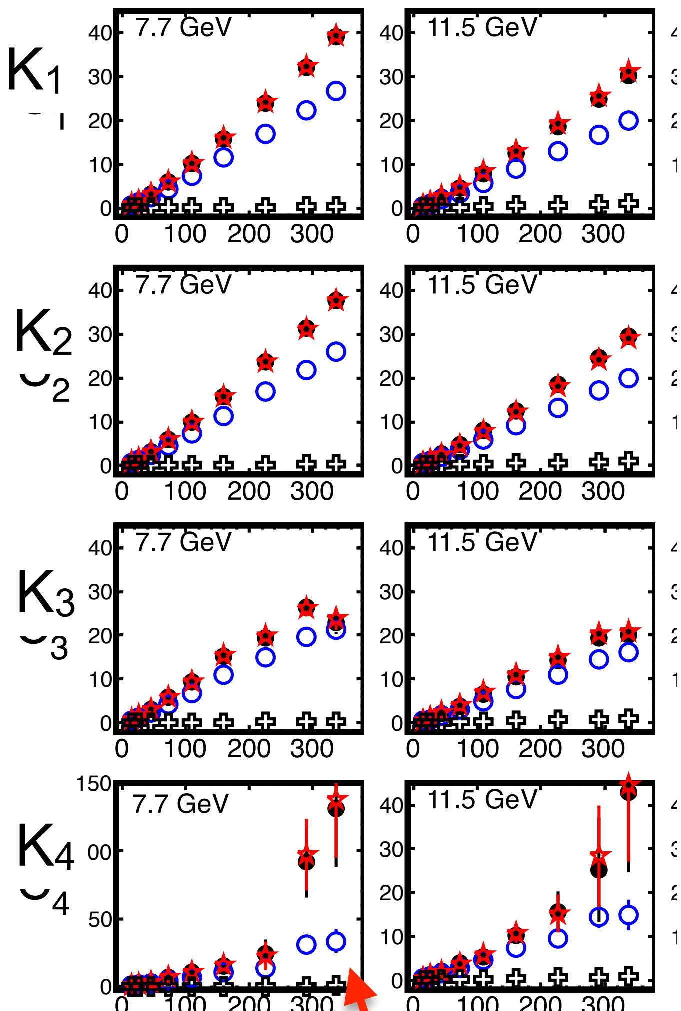
$$\langle (\delta E)^2 \rangle = \langle E^2 \rangle - \langle E \rangle^2 = \left(-\frac{\partial}{\partial 1/T} \right)^2 \ln(Z) = \left(-\frac{\partial}{\partial 1/T} \right) \langle E \rangle$$

$$\langle (\delta E)^n \rangle = \left(-\frac{\partial}{\partial 1/T} \right)^{n-1} \langle E \rangle$$

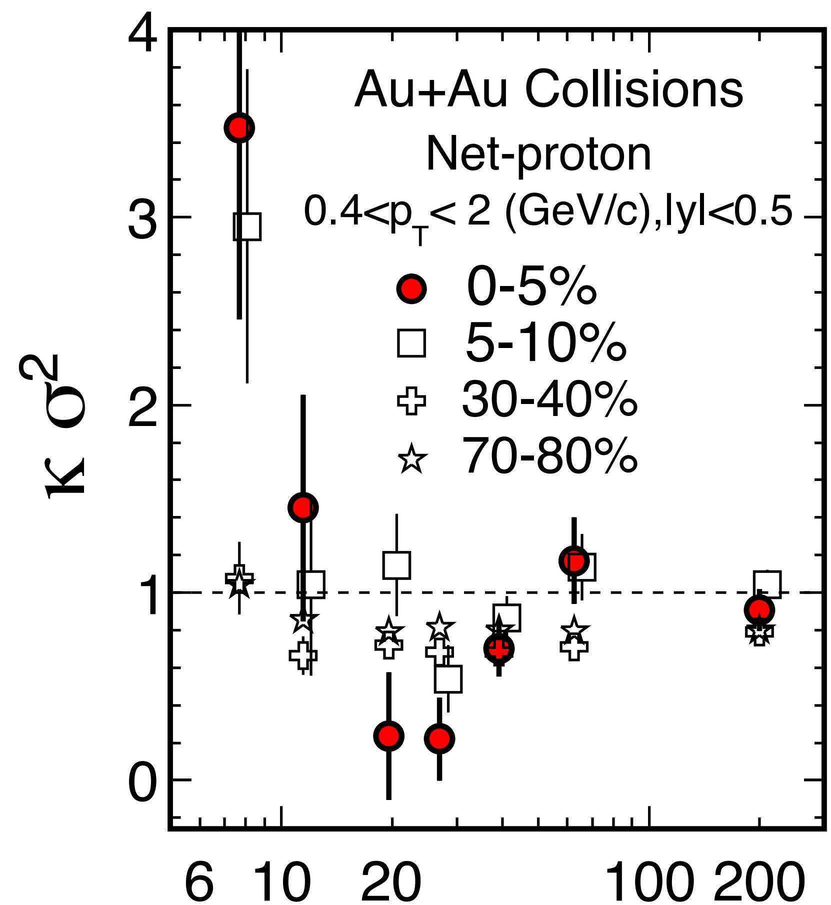
Cumulants of Energy measure the temperature derivatives of the EOS

Cumulants of Baryon number measure the chem. pot. derivatives of the EOS

Latest STAR result on net-proton cumulants



X. Luo, arXiv:1503.02558



Unfolding makes huge difference in new STAR data!

Things to consider

- Fluctuations of conserved charges ?!
- Volume Fluctuations
- Net-protons different from net-baryons
  - Isospin fluctuations
- “Stopping” fluctuations
- Higher cumulants probe the tails. Statistics!
- The detector “fluctuates” !
  - Efficiency effects
- …….

From Cumulants to Correlations

Cumulants $$K_n = \frac{\partial^n}{\partial \hat{\mu}^n} P/T^4$$

$$K_2 = \langle N - \langle N \rangle \rangle^2 = \left\langle (\delta N)^2 \right\rangle$$

$$\rho_2(p_1, p_2) = \rho_1(p_1)\rho_1(p_2) + C_2(p_1, p_2),$$

Correlation Function

$$K_3 = \left\langle (\delta N)^3 \right\rangle$$

$$\begin{aligned}\rho_3(p_1, p_2, p_3) = \rho_1(p_1)\rho_1(p_2)\rho_1(p_3) + \rho_1(p_1)\underline{C_2(p_2, p_3)} + \rho_1(p_2)\underline{C_2(p_1, p_3)} \\ + \rho_1(p_3)\underline{C_2(p_1, p_2)} + \underline{C_3(p_1, p_2, p_3)}\end{aligned}$$

From Cumulants to Correlations
(no anti-protons)

Factorial moments:

$$F_n = \langle N(N-1)\ldots(N-n+1)\rangle = \int dp_1 \ldots dp_2 \rho_n(p_1,\ldots,p_n)$$

$$F_1 = \int dp\rho_1(p) = \langle N \rangle$$

$$F_2 = \int dp_1 dp_2 \rho_2(p_1,p_2) = \langle N \rangle^2 + C_2$$

$$F_3 = \int dp_1 dp_2 dp_3 \rho_3(p_1,p_2,p_3) = \langle N \rangle^3 + 3\langle N \rangle C_2 + C_3$$

and so on…

Integrated correlations function

$$C_n = \int dp_1 \ldots dp_n C_n(p_1,\ldots,p_n)$$

From cumulants to correlations

$$F_1 = \int dp \rho_1(p) = \langle N \rangle$$
$$F_2 = \int dp_1 dp_2 \rho_2(p_1, p_2) = \langle N \rangle^2 + C_2$$
$$F_3 = \int dp_1 dp_2 dp_3 \rho_3(p_1, p_2, p_3) = \langle N \rangle^3 + 3 \langle N \rangle C_2 + C_3$$

$$K_1 \equiv \langle N \rangle = F_1,$$
$$K_2 \equiv \langle (\delta N)^2 \rangle = F_1 - F_1^2 + F_2,$$
$$K_3 \equiv \langle (\delta N)^3 \rangle = F_1 + 2F_1^3 + 3F_2 + F_3 - 3F_1(F_1 + F_2),$$

Can express correlations C_n in terms of cumulants K_n

$$C_2 = -K_1 + K_2,$$
$$C_3 = 2K_1 - 3K_2 + K_3,$$
$$C_4 = -6K_1 + 11K_2 - 6K_3 + K_4, .$$

Correlations near the critical point

M. Stephanov, 0809.3450, PRL 102

Scaling of Cumulants K_n with correlation length ξ

$$K_2 \sim \xi^2, \;\; K_3 \sim \xi^6, \;\; K_4 \sim \xi^8$$

Cumulants from Correlations

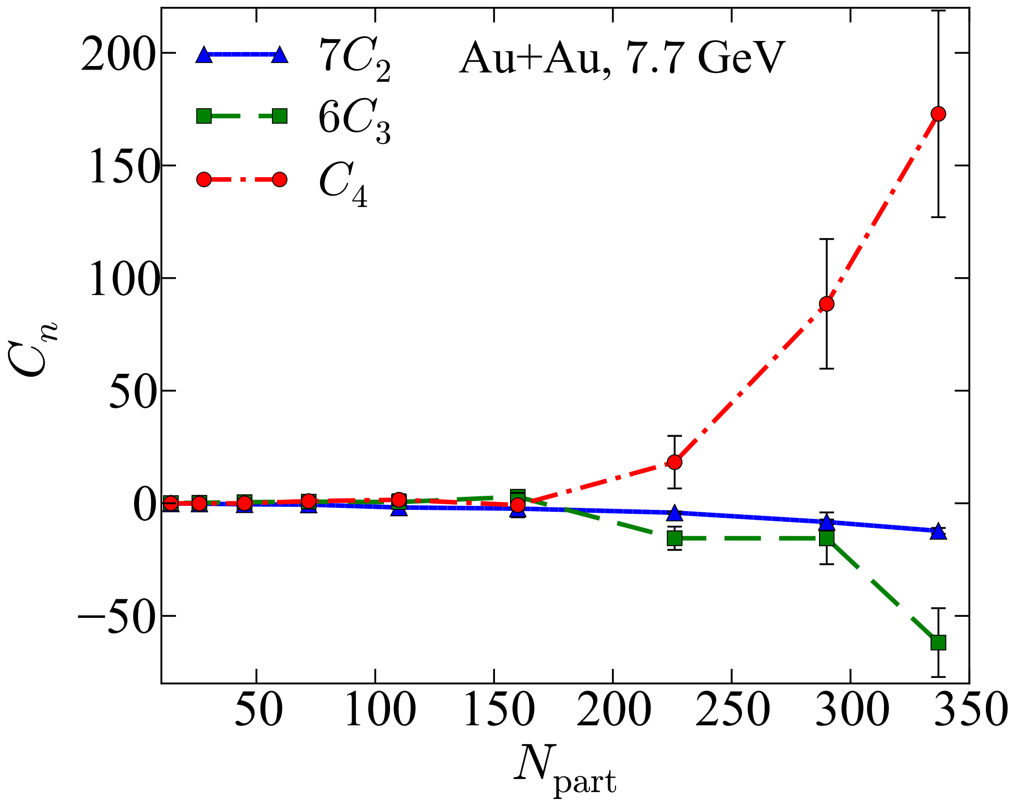
$$K_2 = \langle N \rangle + C_2$$
$$K_3 = \langle N \rangle + 3C_2 + C_3$$
$$K_4 = \langle N \rangle + 7C_2 + 6C_3 + C_4$$

Consequently: $C_2 \sim \xi^2, \;\; C_3 \sim \xi^6, \;\; C_4 \sim \xi^8$

Correlations C_n pick up the most divergent pieces of cumulants K_n!

Preliminary Star Data

(X. Luo, PoS Cpod 2014 (019))



Significant four particle correlations!

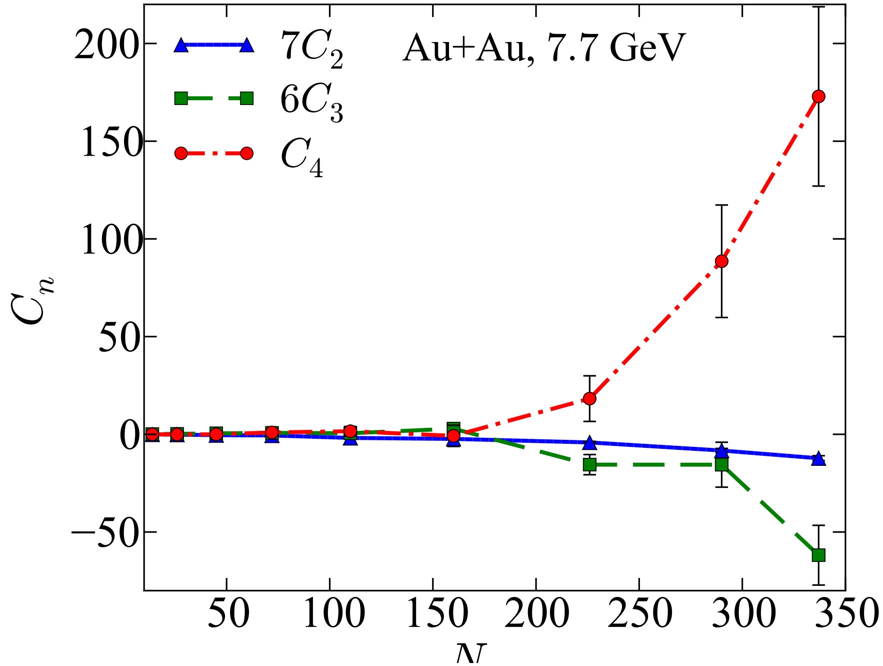
Four particle correlation dominate K_4 for central collisions

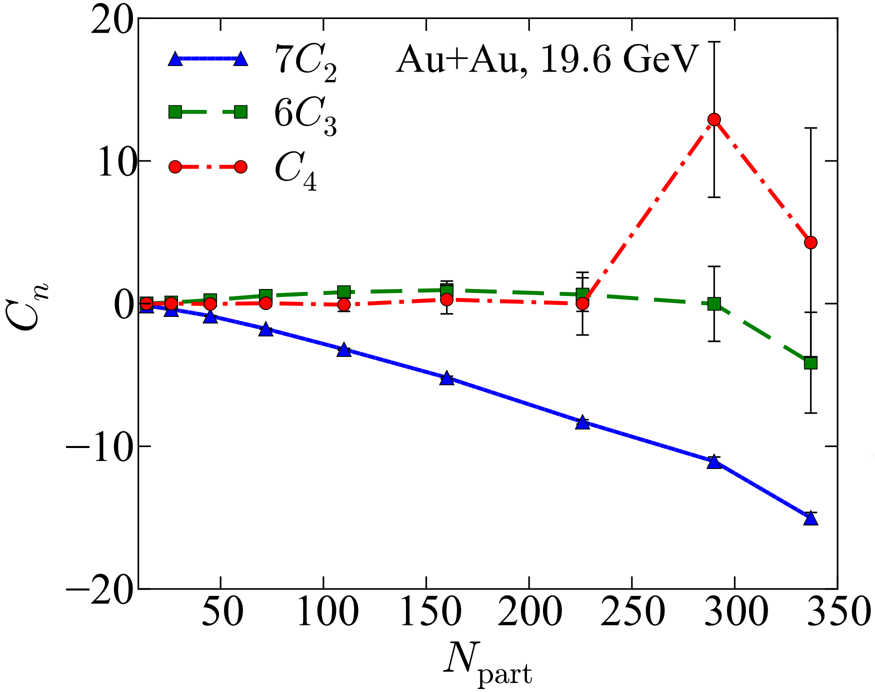
$$K_2 = \langle N \rangle + C_2$$

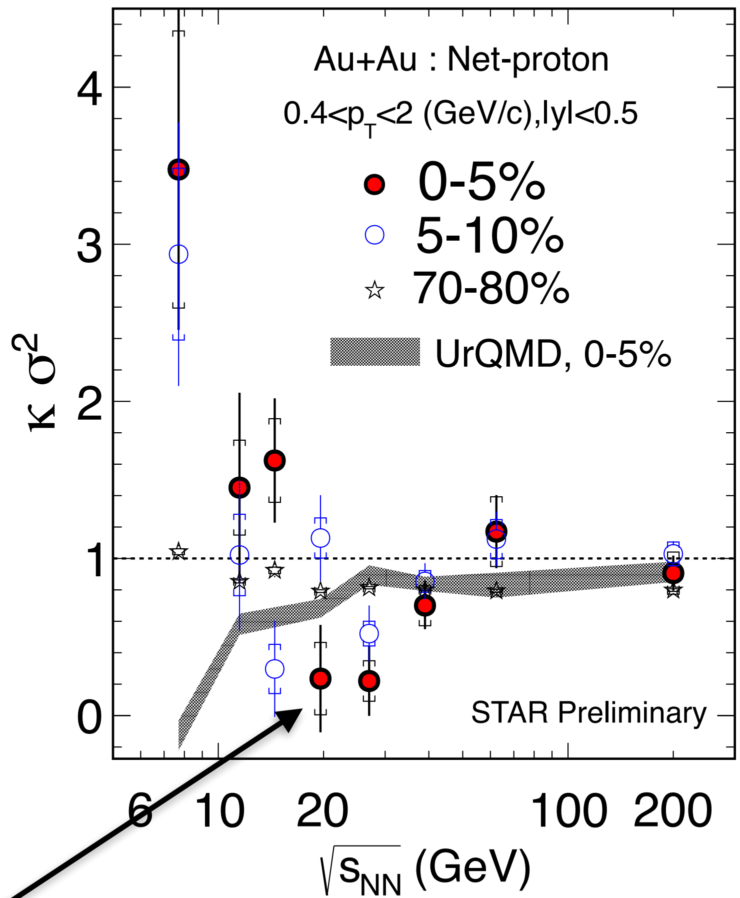
$$K_3 = \langle N \rangle + 3C_2 + C_3$$

$$K_4 = \langle N \rangle + 7C_2 + 6C_3 + C_4$$

Correlations







Dip at 19.6 GeV from NEGATIVE C_2 !

Reduced correlation function

Reduced correlation function

$$c_k = \frac{\int \rho_1(y_1)\cdots\rho_1(y_k)\, c_k(y_1,...,y_k)\, dy_1\cdots dy_k}{\int \rho_1(y_1)\cdots\rho_1(y_k)\, dy_1\cdots dy_k} \qquad C_k = \langle N\rangle^k c_k$$

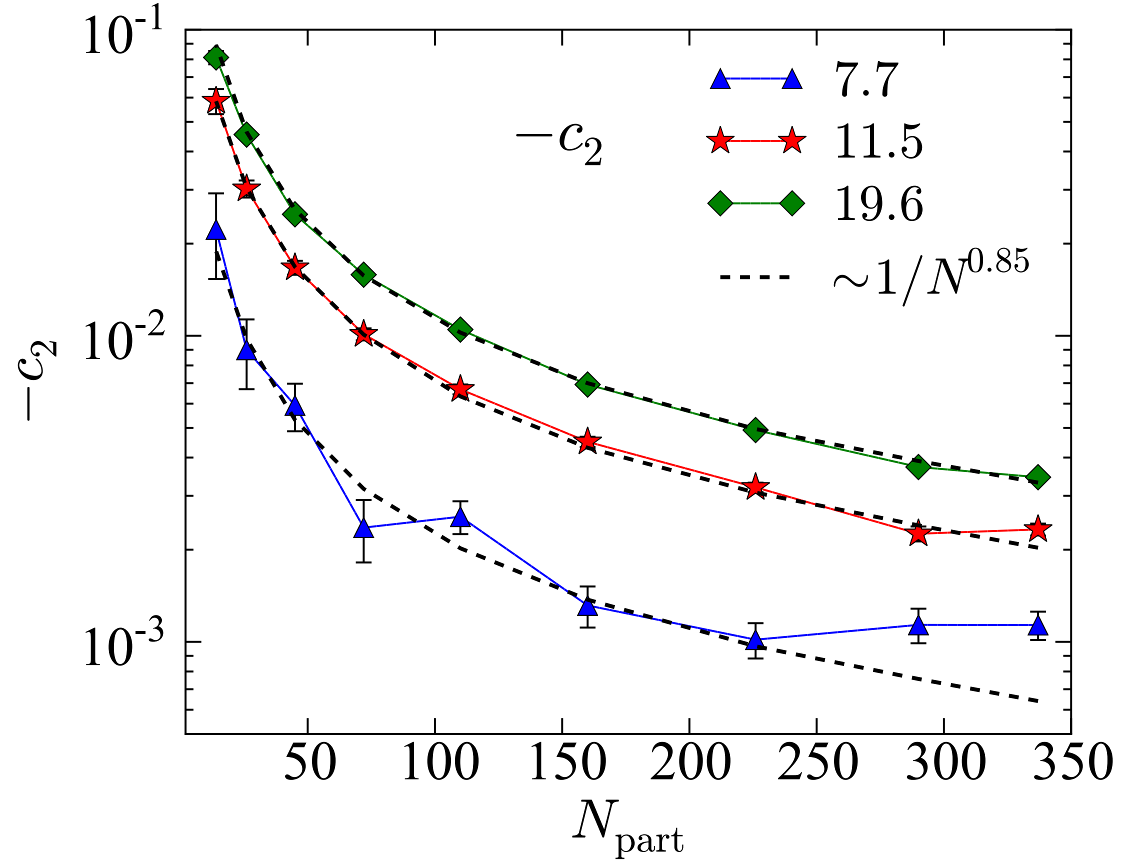
Independent sources such as resonances, cluster, p+p:

$$c_k \sim \frac{\langle N_s\rangle}{\langle N\rangle^k} \sim \frac{1}{\langle N\rangle^{k-1}}$$

For example two particle correlations:

$$c_2 \sim \frac{\text{Number of sources}}{\text{Number of all pairs}} = \frac{\text{Number of correlated pairs}}{\text{Number of all pairs}} = \frac{1}{\langle N\rangle}$$

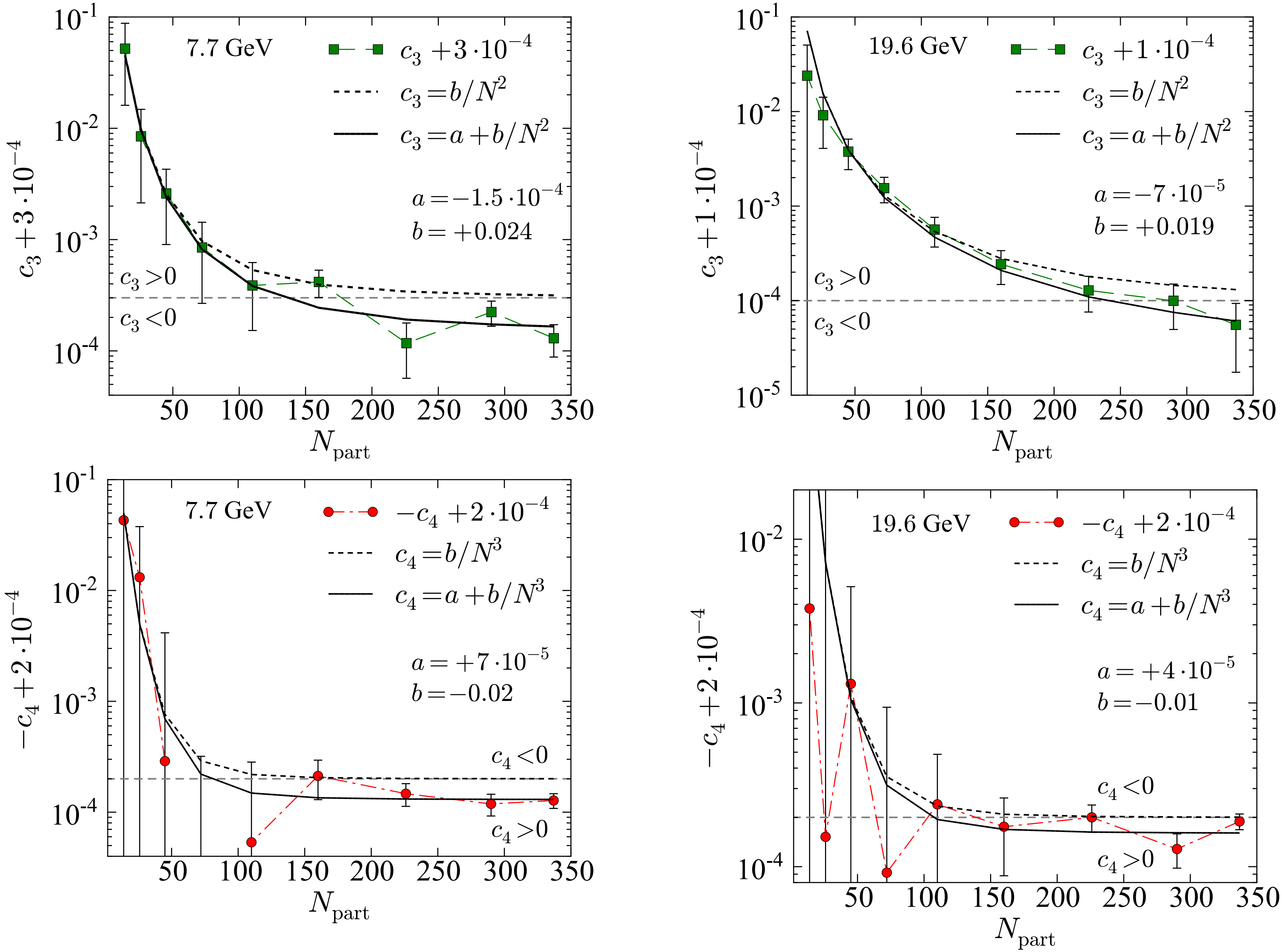
Centrality dependence



Centrality dependence

C_3

C_4



7.7 GeV 19.6 GeV

Rapidity dependence

$$C_k(\Delta Y) = \int_{\Delta Y} dy_1 \dots dy_k \rho_1(y_1) \dots \rho_1(y_k) c_k(y_1, \dots, y_k)$$

Assume: $\rho_1(y) \simeq const.$

short range correlations:

$$c_k(y_1, \dots, y_k) \sim \delta(y_1 - y_2) \dots \delta(y_{n-1} - y_k)$$

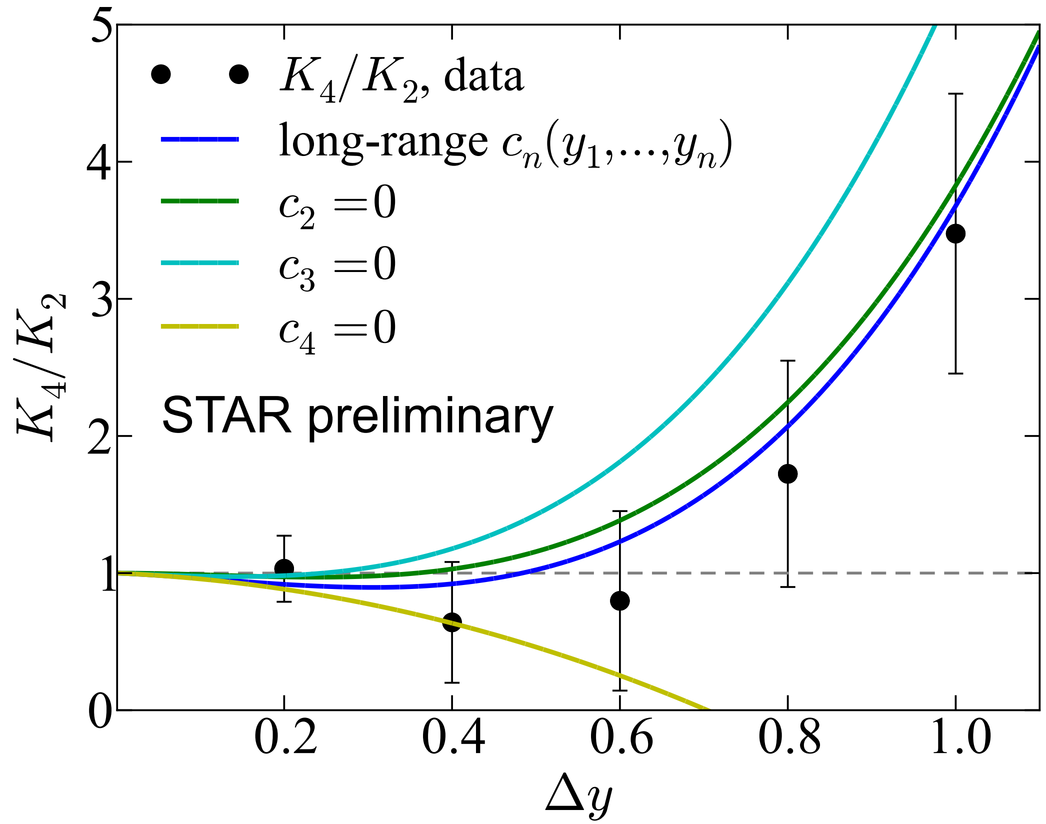
$$C_k(\Delta Y) \sim \Delta Y \rightarrow K_k \sim \Delta Y$$

Long range correlations:

$$c_k(y_1, \dots, y_k) = const.$$

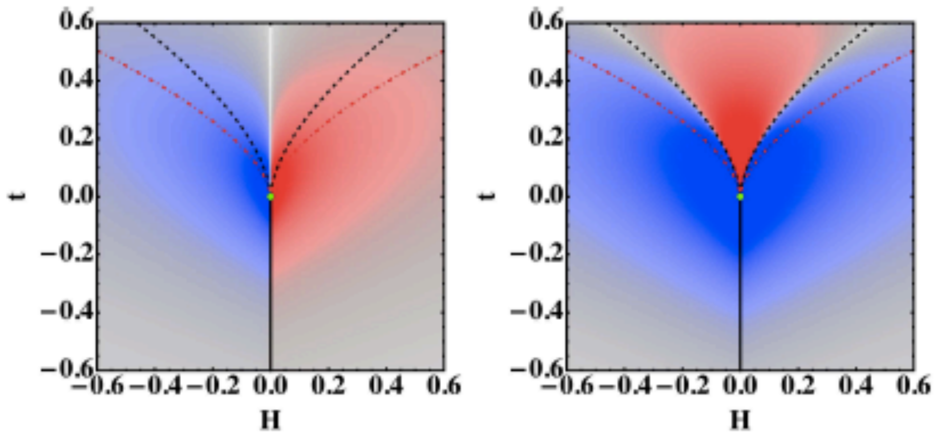
$$C_k(\Delta Y) \sim (\Delta Y)^k$$

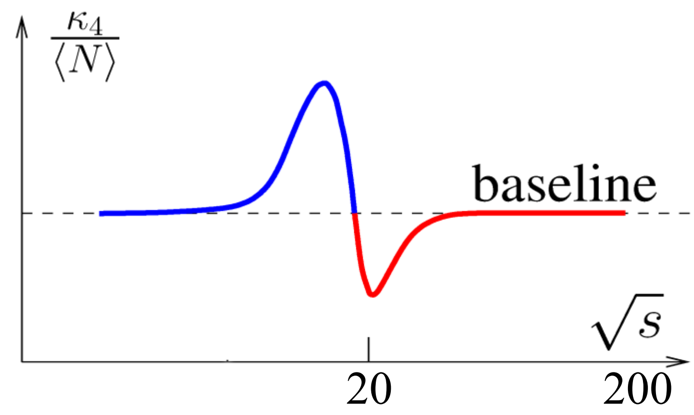
Preliminary Star data are consistent with long range correlations

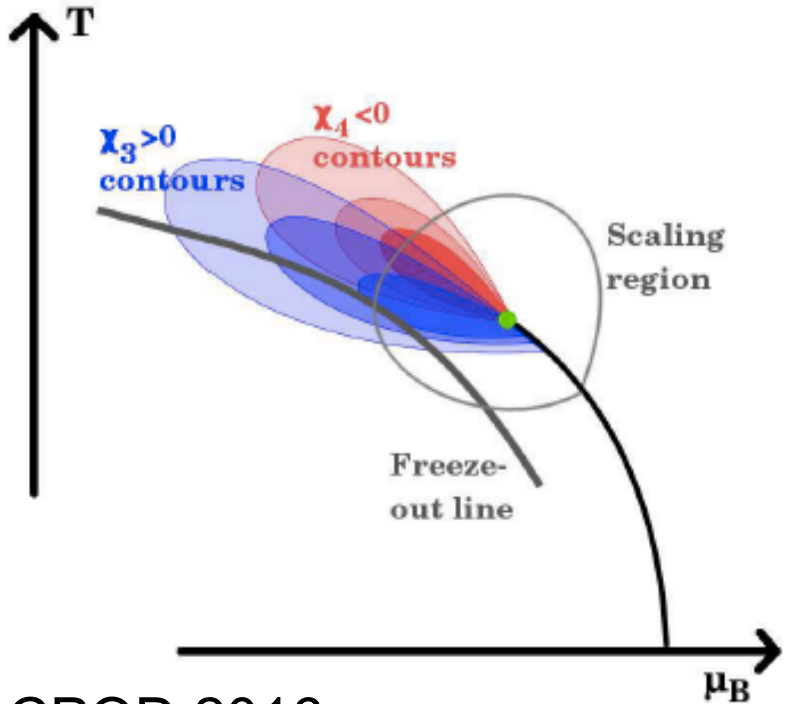


7.7 GeV
central

Expectation from Calculations





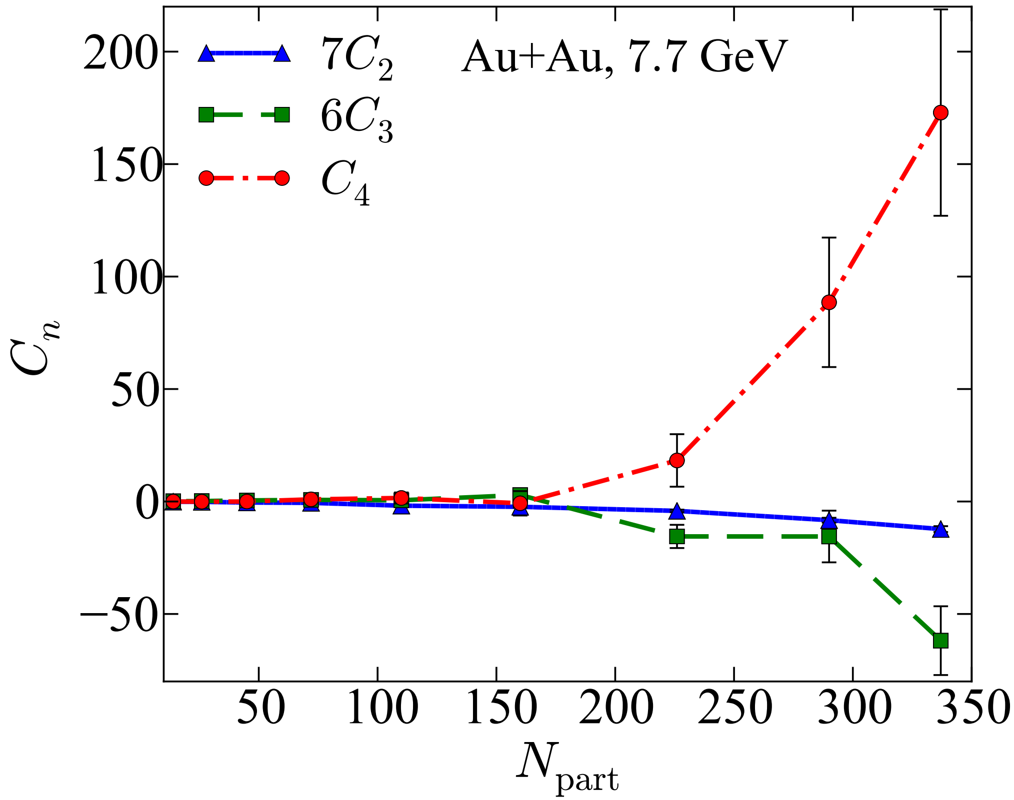


N. Xu, CPOD 2016

Characteristic "Oscillating pattern" is expected for the QCD critical point but *the exact shape depends on the location of freeze-out with respect to the location of CP*

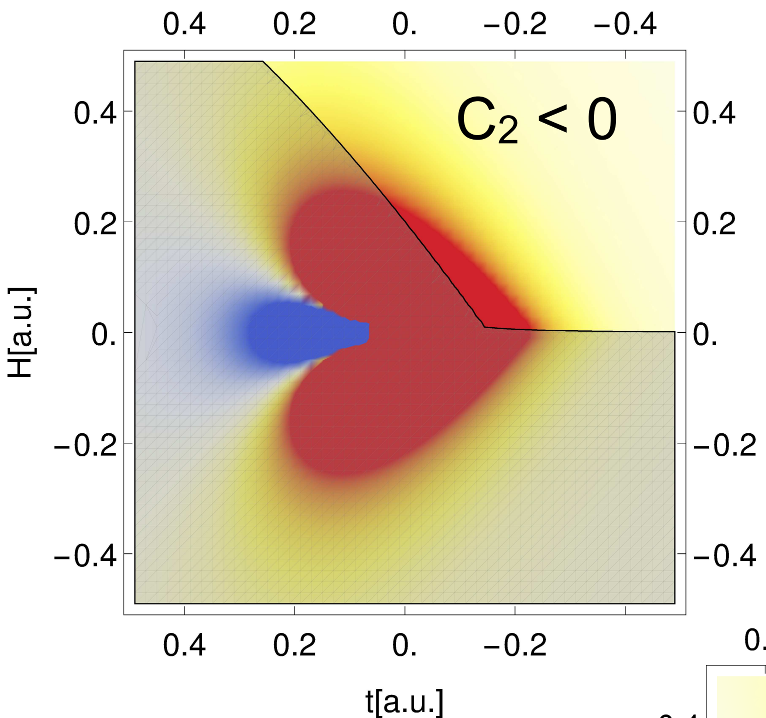
- M. Stephanov, ***PRL107***, 052301(2011)
- V. Skokov, Quark Matter 2012
- J.W. Chen, J. Deng, H. Kohyyama, arXiv: 1603.05198, Phys. Rev. **D93** (2016) 034037

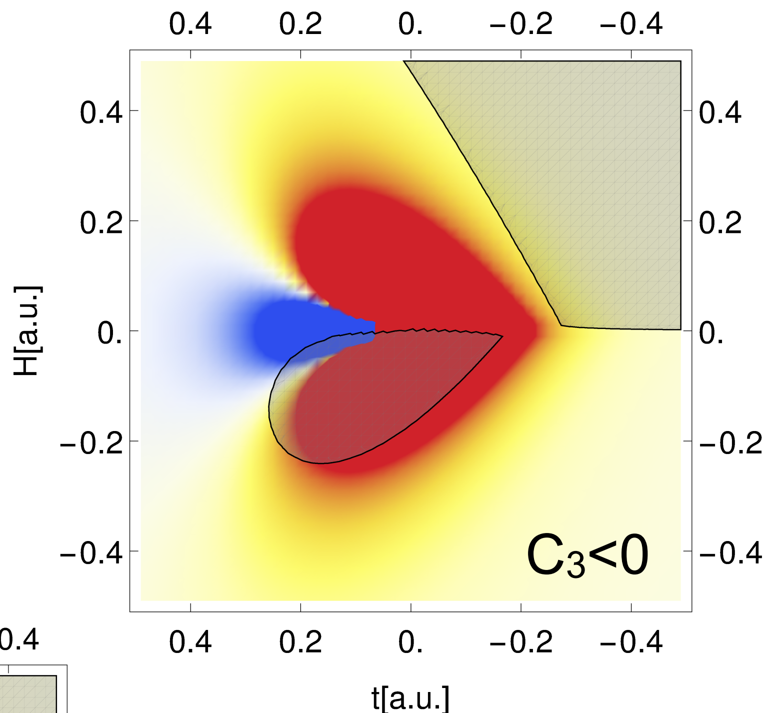
Sign of C_n

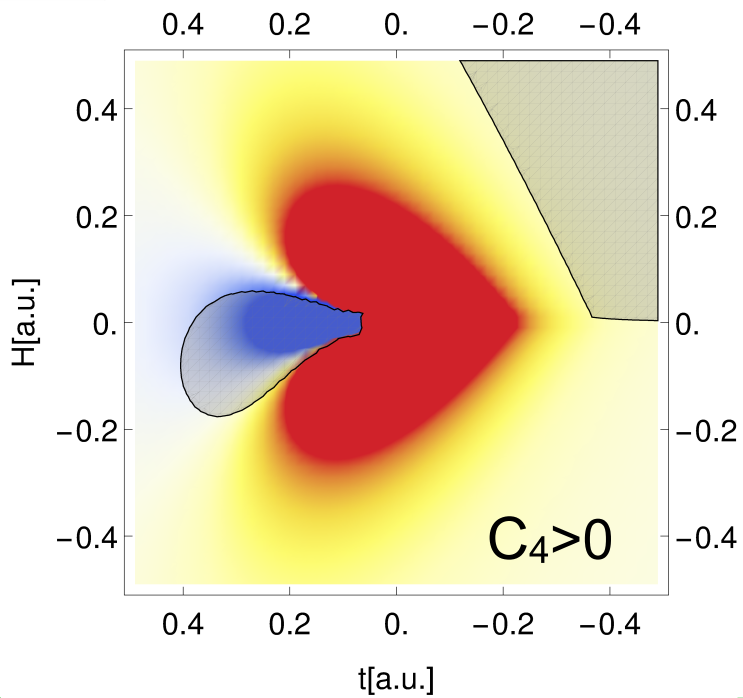


$C_2 < 0$
$C_3 < 0$
$C_4 > 0$

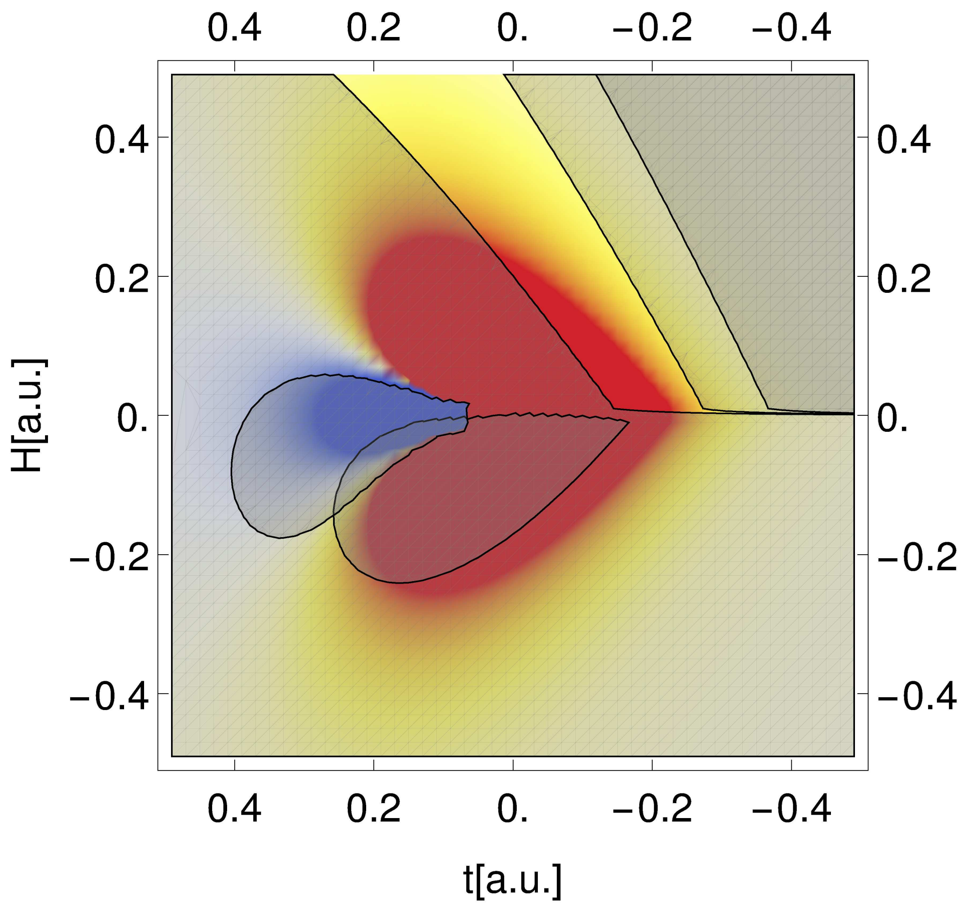
Exclusion plots





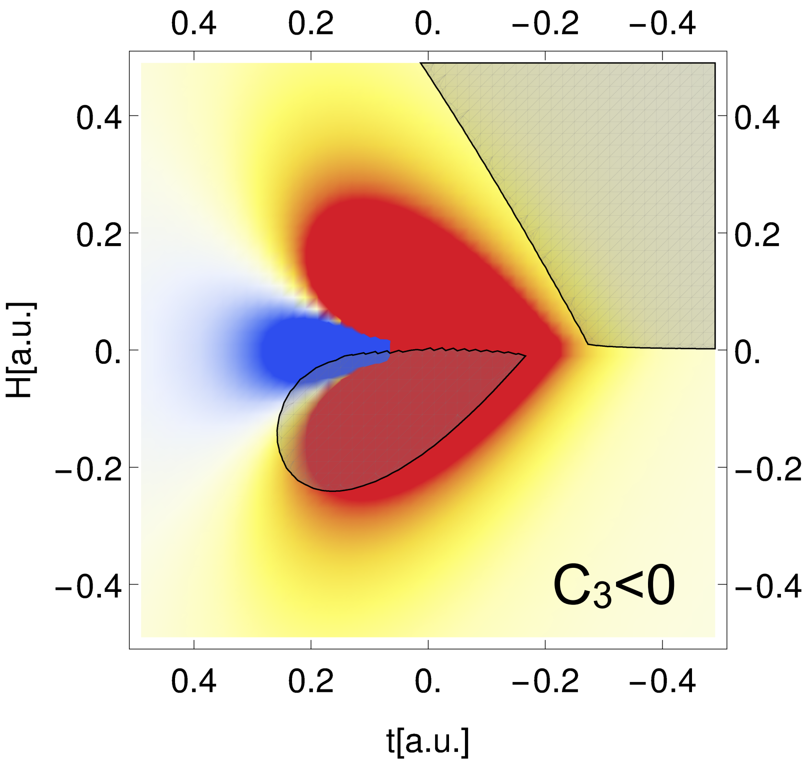


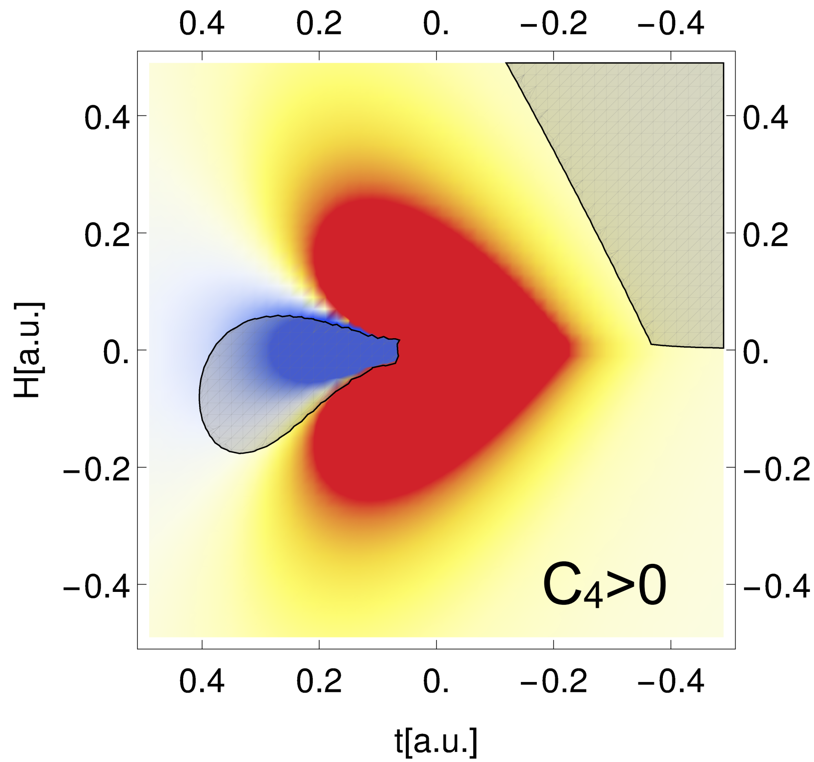
Excluding regions of the phase diagram



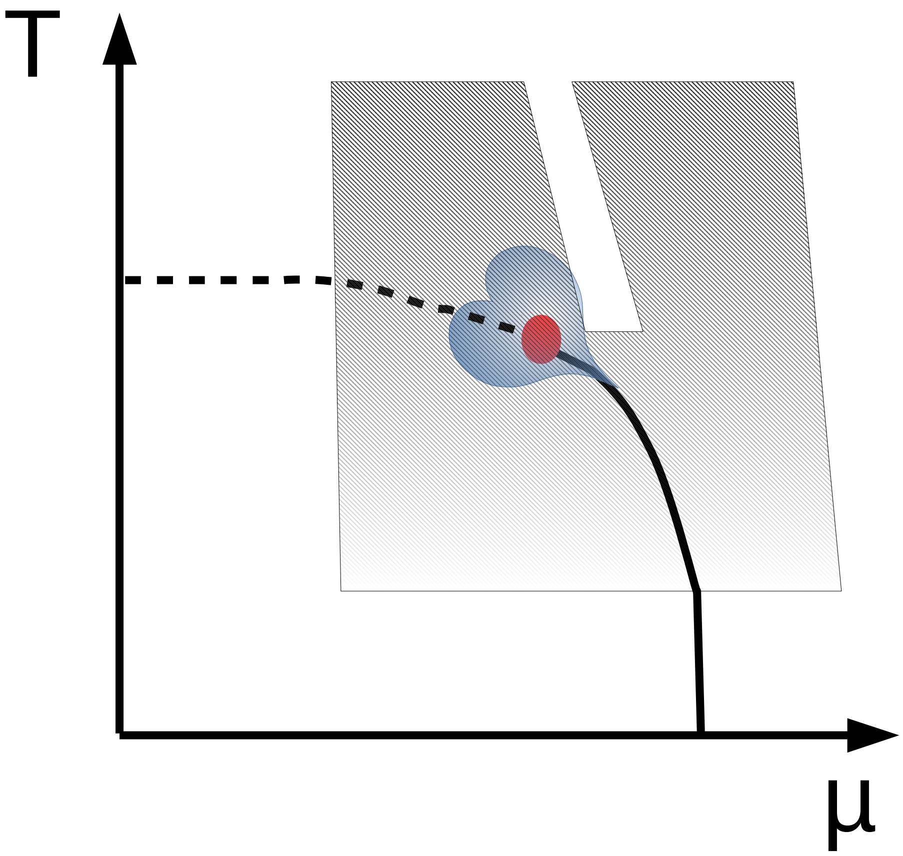
$C_2<0$, $C_3<0$, $C_4>0$

Ignore C_2

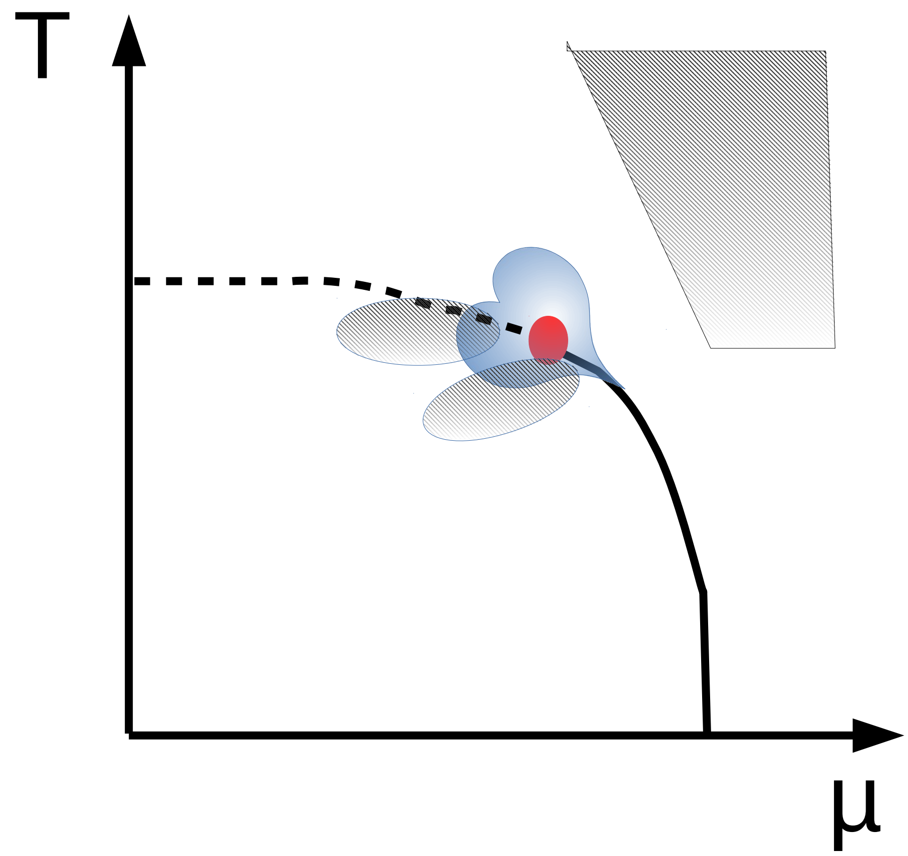




Map onto QCD phase diagram



$C_2<0$, $C_3<0$, $C_4>0$



$C_3<0$, $C_4>0$

Summary

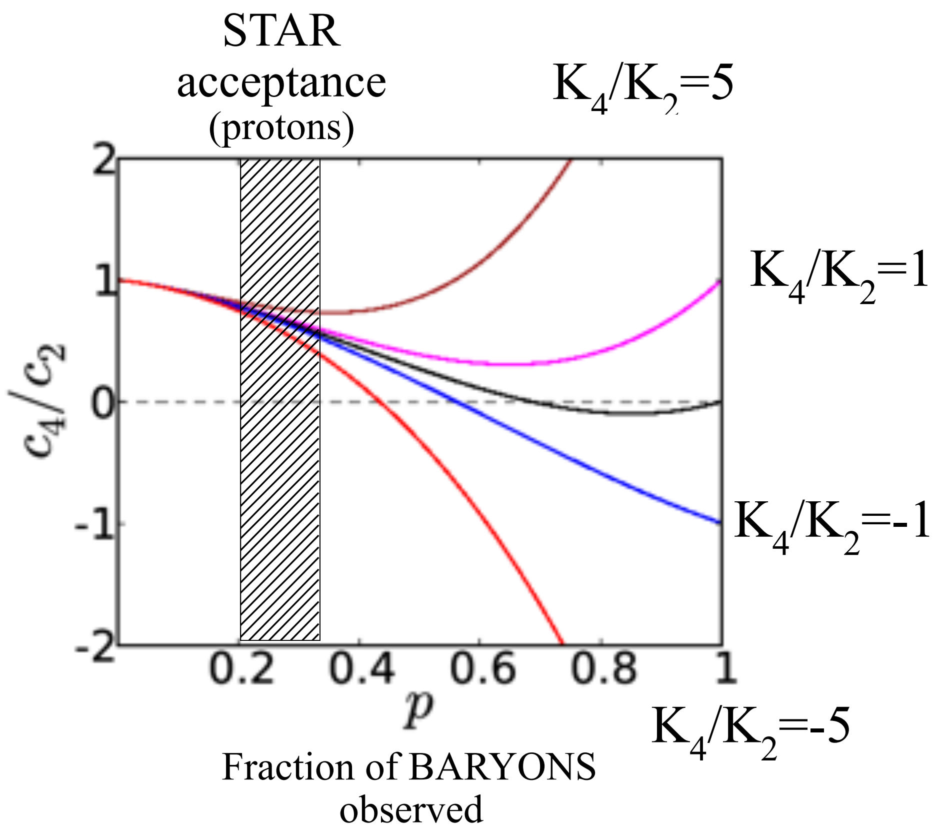
- Fluctuations sensitive to phase structure:
  - measure “derivatives” of EOS
- Detector contributes to fluctuations; need to be removed
- Cumulants contain information about correlations
- Preliminary STAR data:
  - Significant four particle correlations at 7.7 and 11.5 GeV
  - Dip in K_4/K_2 at 19.6 GeV is due to negative two-particle correlations
  - Centrality dependence (at 7.7 GeV) indicates independent sources for $N_{part} < 150$ and “collective” correlations for $N_{part} > 200$.
  - For central 7.7 and 11.5 GeV two and three particle correlations are negative and four particle are positive.T
    - This would rule out a large area around the critical point

Summary

- The STAR data are still preliminary!
- Other more mundane effects may contribute
- Correlations help chasing these effects down.

Happy 40th, Ulrich!

Finite efficiency

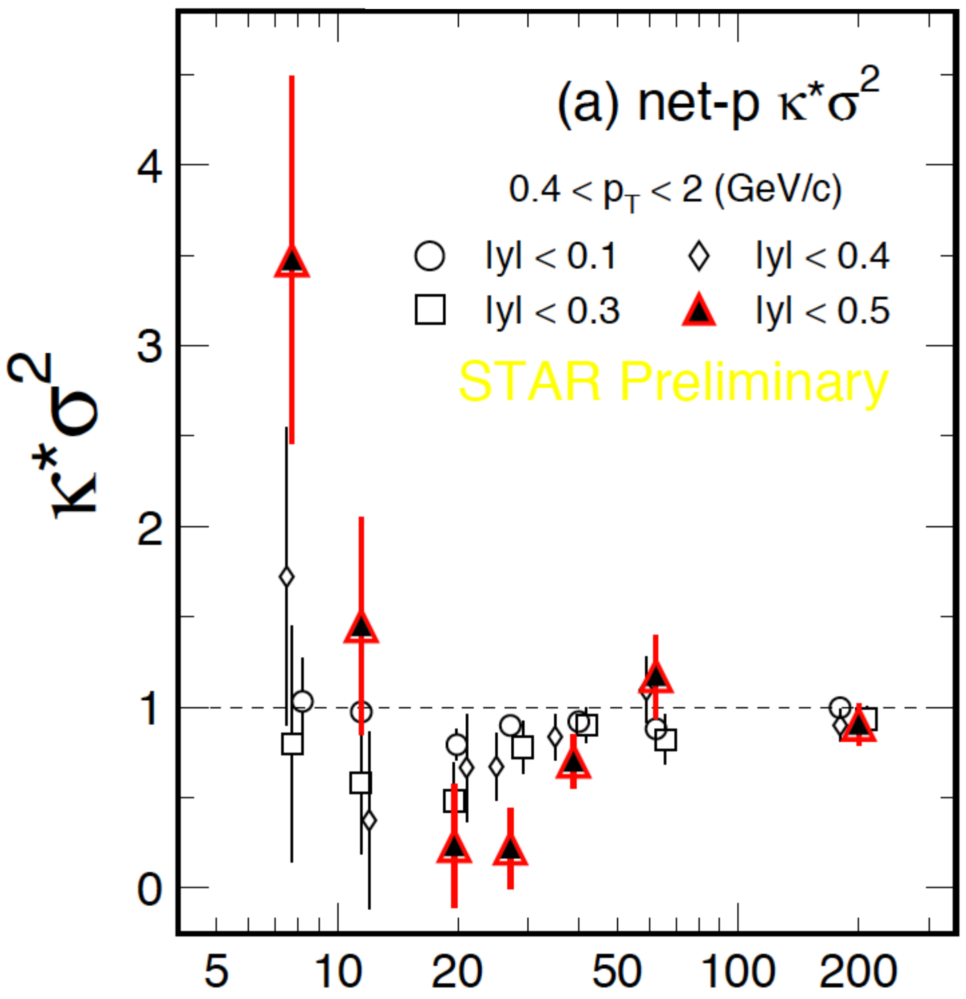


Unfolding needed if we want to know what the true cumulants are
Increases Errors!

Dependence on Rapidity window

- Kurtosis depends strongly on Rapidity window
- Comparison with Lattice:
  - Lattice catches the full correlation length
  - need to expand rapidity window until signal saturates

X. Luo, RBRC Workshop, Feb. 2015



Correlations: Lattice vs Data

$$\langle n(y_1)(\, n(y_2) - \delta(y_1 - y_2)\,)\rangle = \langle n(y_1)\rangle\langle n(y_2)\rangle\left(1 + C(y_1, y_2)\right)$$

$$C(y_{1,} y_2) \sim \exp\left(\frac{-(y_1 - y_2)^2}{2\sigma^2}\right)$$

$$\frac{\langle(\delta N)^2\rangle}{\langle N\rangle} = 1 + \langle N\rangle \int_{\Delta/2}^{\Delta/2} C(y1, y2)\, dy1\, dy_2$$

Alice Charge Flucts