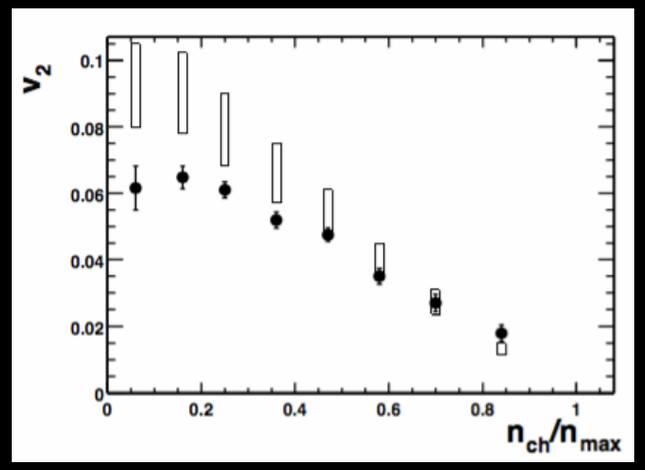
Triangular Flow in high multiplicity pp collisions
-
how did we get here?

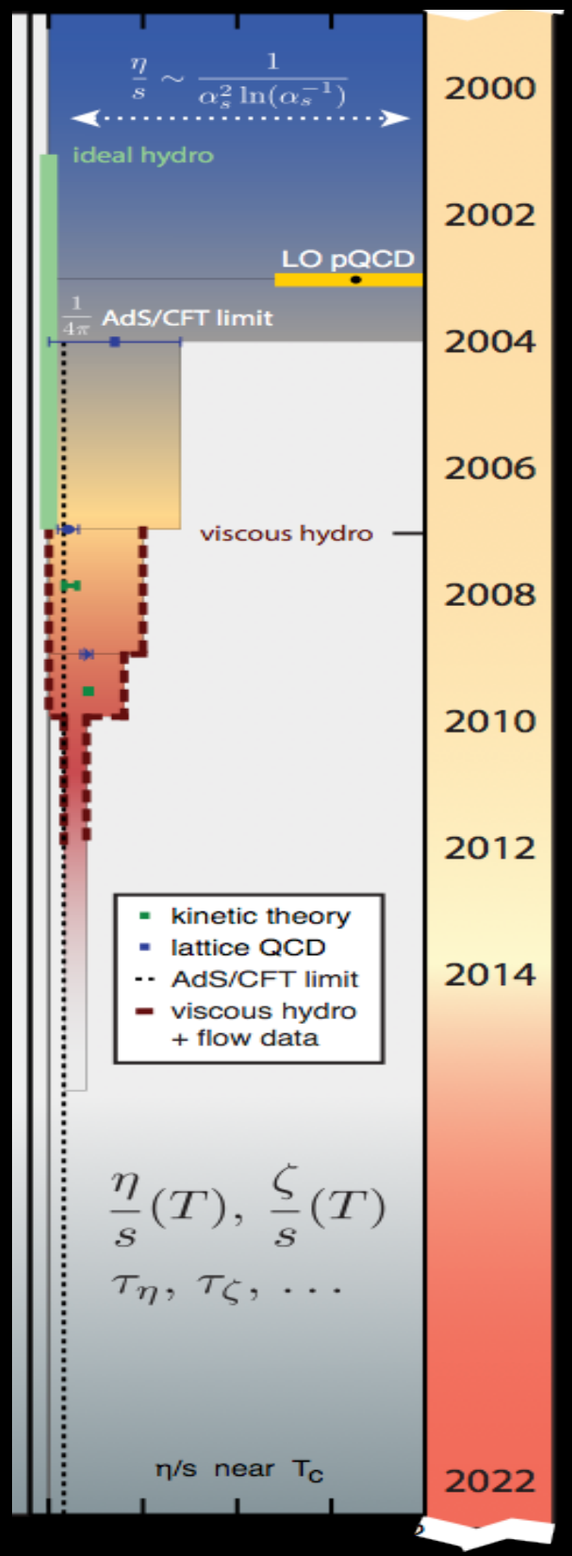
Gunther Roland

How did we get here?

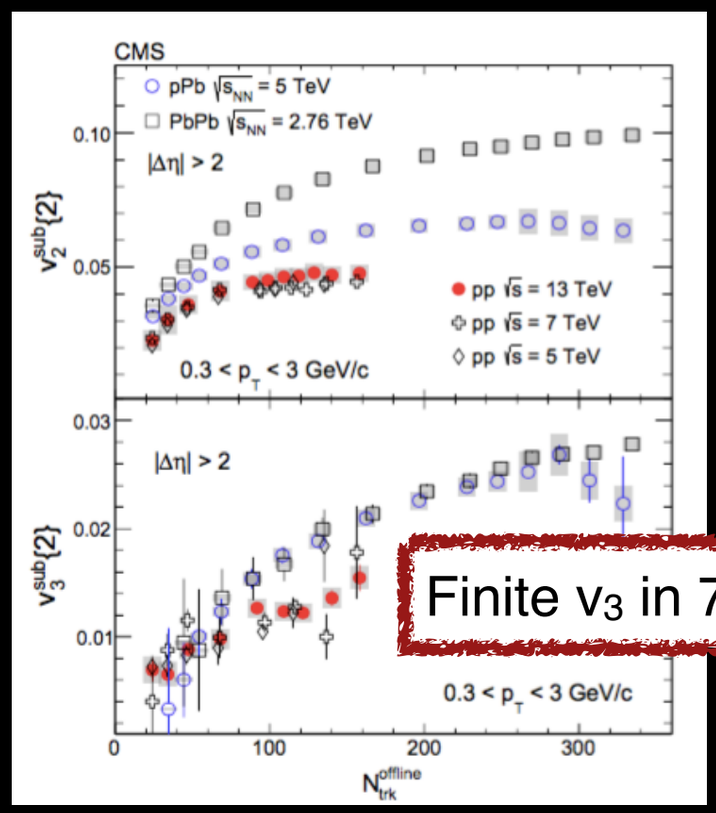
STAR AuAu, 2000



Calculations: Heinz, Kolb
Ollitraut



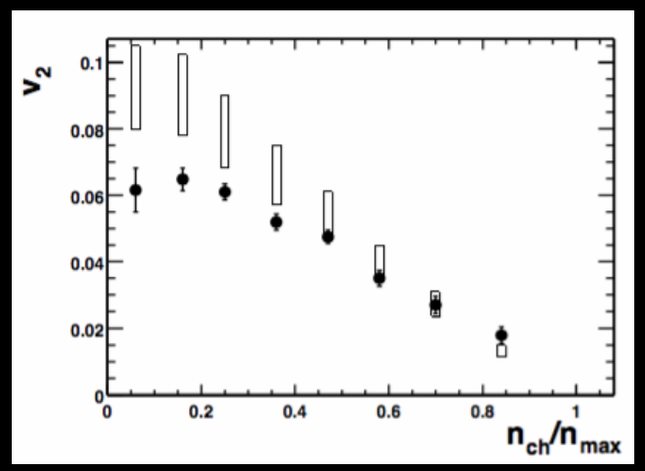
CMS, 2016



Finite v_3 in 7 TeV pp

How did we get here?

STAR AuAu, 2000



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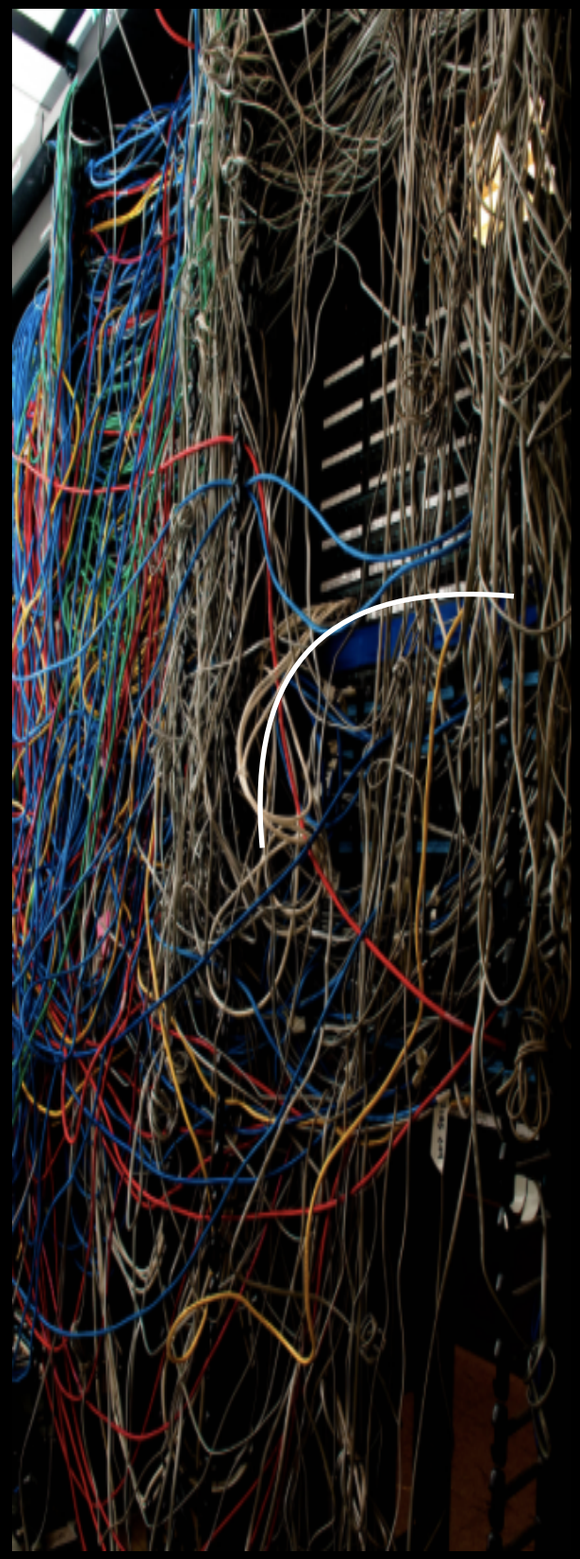
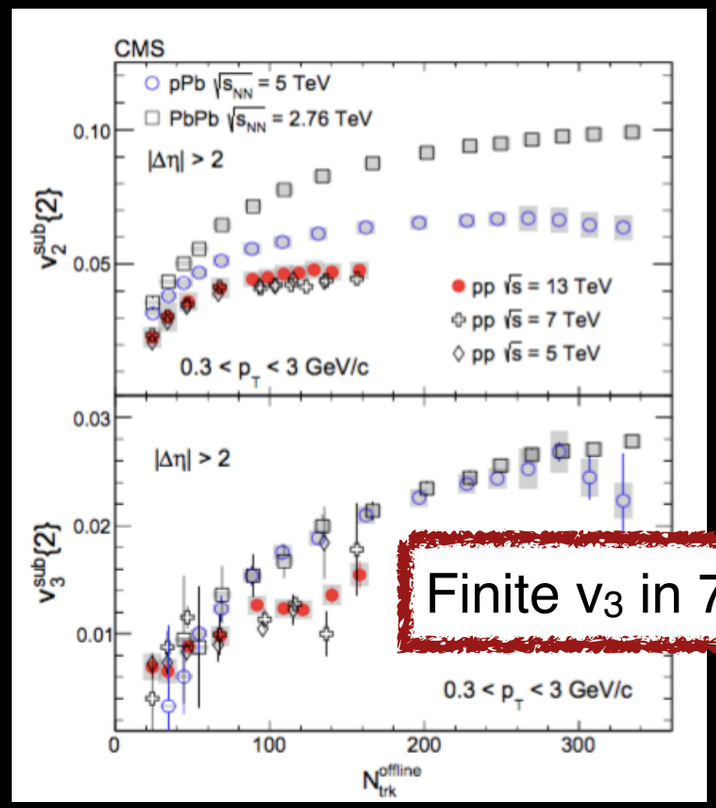


CMS, 2016



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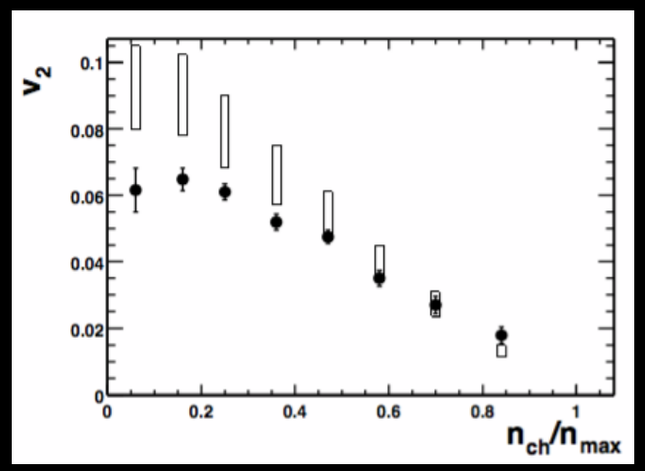

Calculations: Heinz, Kolb
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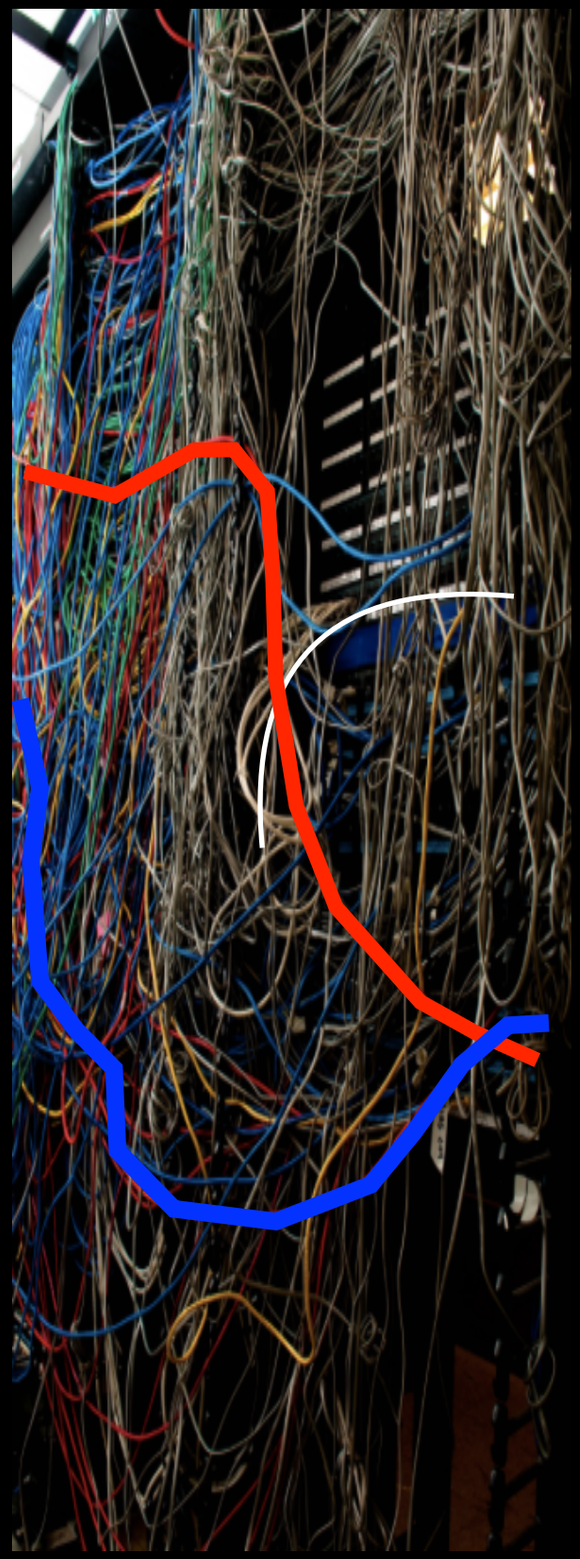
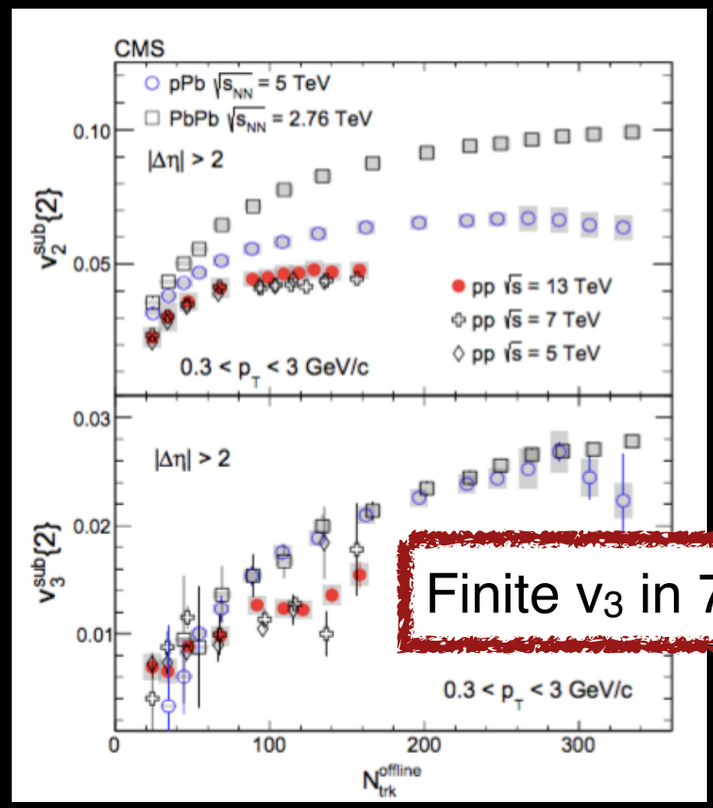
Initial State Fluctuations



Correlations in small systems

CMS, 2016



Finite v_3 in 7 TeV pp

B.Alver[4], B.B.Back[1], M.D.Baker[2], M.Ballintijn[4], D.S.Barton[2], R.R.Betts[7], R.Bindel[8], W.Busza[4], V.Chetluru[7], E.García[7], T.Gburek[3], J.Hamblen[9], U.Heinz[6], D.J.Hofman[7], R.S.Hollis[7], A.Iordanova[7], W.Li[4], C.Loizides[4], S.Manly[9], A.C.Mignerey[8], R.Nouicer[2], A.Olszewski[3], C.Reed[4], C.Roland[4], G.Roland[4], J.Sagerer[7], P.Steinberg[2], G.S.F.Stephans[4], M.B.Tonjes[8], A.Trzupek[3], G.J.van Nieuwenhuizen[4], S.S.Vaurynovich[4], R.Verdier[4], G.I.Veres[4], P.Walters[9], E.Wenger[4], B.Wosiek[3], K.Woźniak[3], B.Wysłouch[4]

PHOBOS collaboration

Uli's career as an experimentalist

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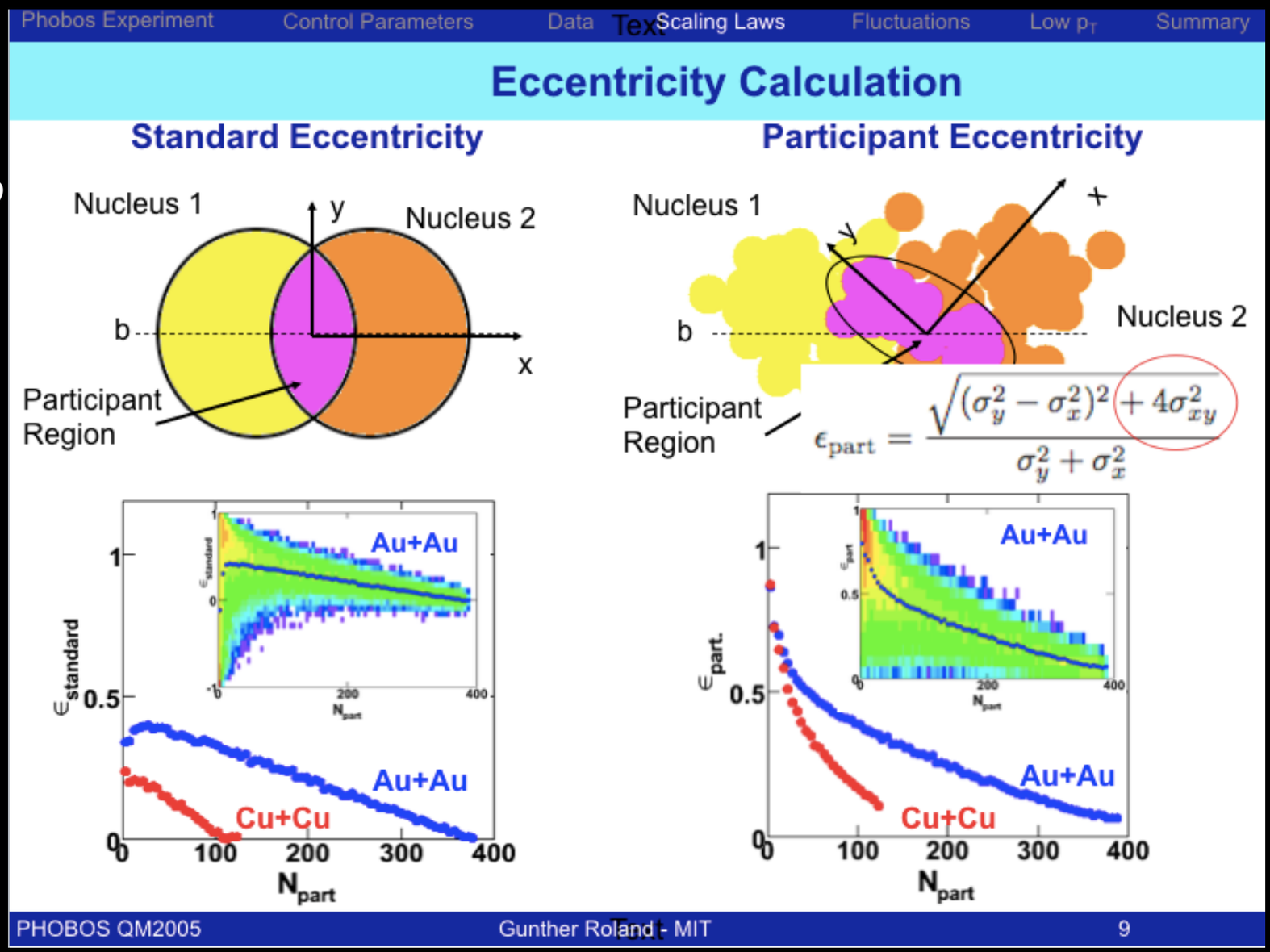
Aguiar, Hama et al
Nucl.Phys.A698 (2002) 639

Miller, Snellings
nucl-ex/0312008

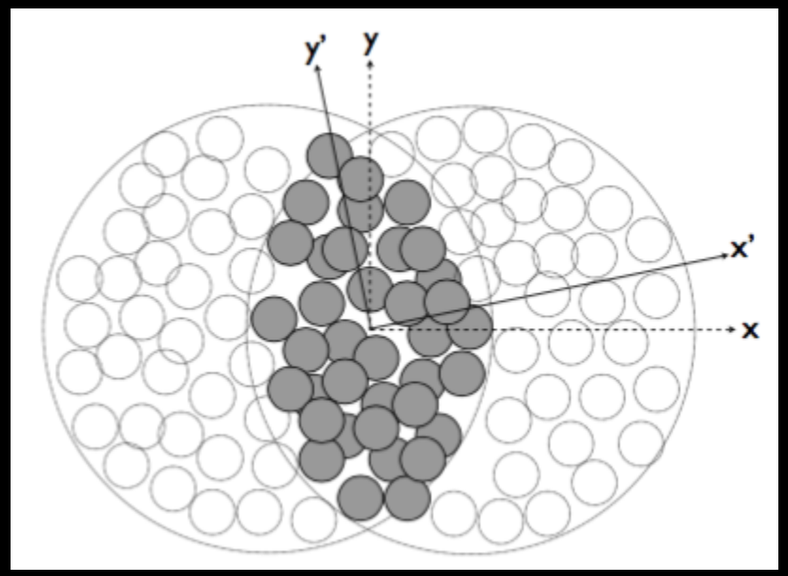
PHOBOS QM'05

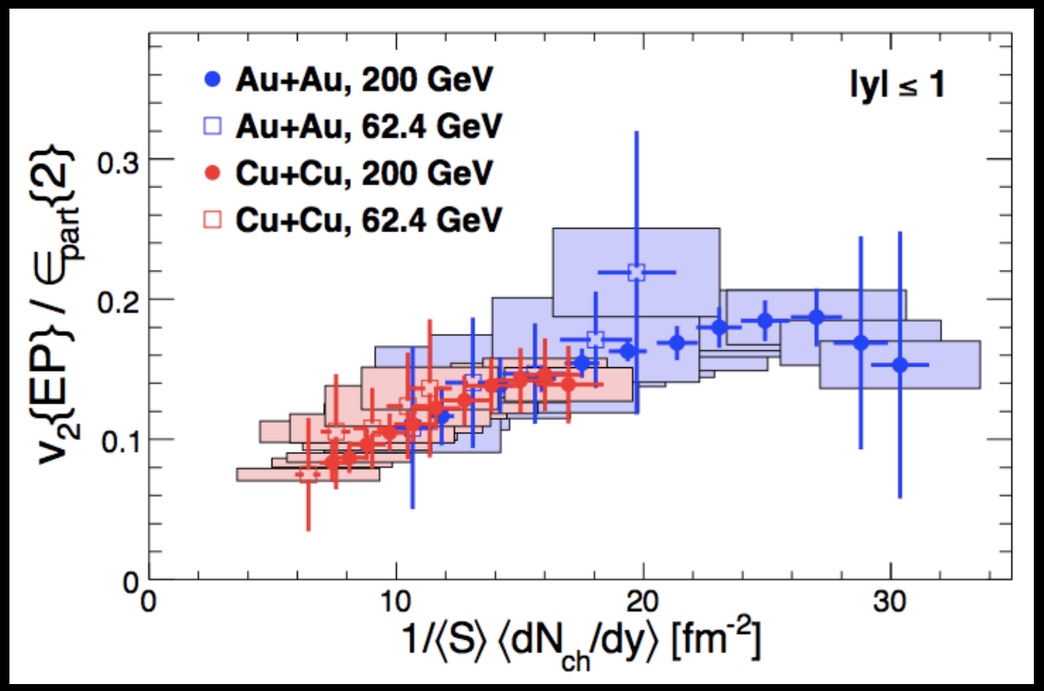


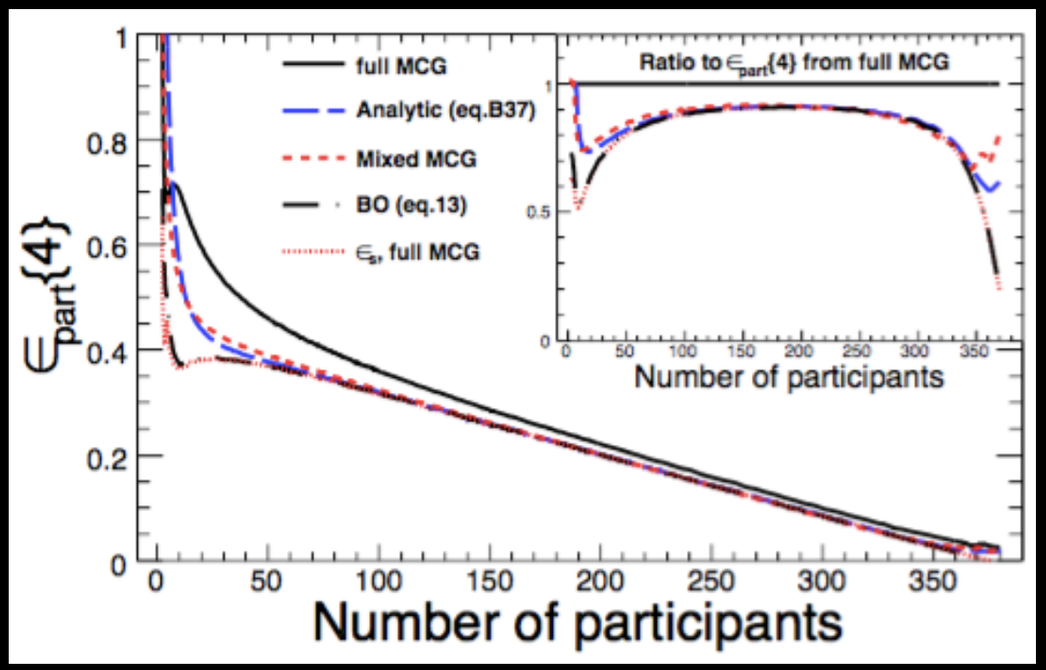
not met with universal approval by nuclear theory community

The Importance of Correlations and Fluctuations on the Initial Source Eccentricity in High-Energy Nucleus–Nucleus Collisions

B.Alver[4], B.B.Back[1], M.D.Baker[2], M.Ballintijn[4], D.S.Barton[2], R.R.Betts[7], R.Bindel[8], W.Busza[4], V.Chetluru[7], E.García[7], T.Gburek[3], J.Hamblen[9], U.Heinz[6], D.J.Hofman[7], R.S.Hollis[7], A.Iordanova[7], W.Li[4], C.Loizides[4], S.Manly[9], A.C.Mignerey[8], R.Nouicer[2], A.Olszewski[3], C.Reed[4], C.Roland[4], G.Roland[4], J.Sagerer[7], P.Steinberg[2], G.S.F.Stephans[4], M.B.Tonjes[8], A.Trzupek[3], G.J.van Nieuwenhuizen[4], S.S.Vaurynovich[4], R.Verdier[4], G.I.Veres[4], P.Walters[9], E.Wenger[4], B.Wosiek[3], K.Woźniak[3], B.Wysłouch[4]







Showed robustness of ε_{part} definition and unification of CuCu and AuAu results

arXiv:0711.3724v1 (~200 citations)

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Compared to the δ^2 terms, Eq. (B5), this is suppressed by a factor $1/N$. Starting with the δ^4 term, the expressions begin to get more complicated. In particular,

$$\langle \delta_f \delta_g \delta_h \delta_u \rangle = \frac{\langle \hat{f}\hat{g}\hat{h}\hat{u} \rangle}{N^3} + \frac{(N-1)}{N^3}\Big(\langle \hat{f}\hat{g} \rangle \langle \hat{h}\hat{u} \rangle + \langle \hat{f}\hat{h} \rangle \langle \hat{g}\hat{u} \rangle + \langle \hat{f}\hat{u} \rangle \langle \hat{g}\hat{h} \rangle \Big) , \quad \text{(B7)}$$

which is actually $\mathcal{O}\left(1/N^2\right)$, i.e. the same order in the number of participants as the δ^3 term.

The fifth and sixth order terms can be calculated similarly. Any terms involving single powers like $\langle \hat{f} \rangle$ will again vanish. The nonzero terms are

$$\begin{aligned}\langle \delta_f \delta_g \delta_h \delta_u \delta_v \rangle = {} & \frac{1}{N^4}\Big\langle \hat{f}\hat{g}\hat{h}\hat{u}\hat{v} \Big\rangle + \frac{N-1}{N^4}\Big(\langle \hat{f}\hat{g}\hat{h} \rangle \langle \hat{u}\hat{v} \rangle \\ & + \langle \hat{f}\hat{g}\hat{u} \rangle \langle \hat{h}\hat{v} \rangle + \langle \hat{f}\hat{g}\hat{v} \rangle \langle \hat{h}\hat{u} \rangle + \langle \hat{f}\hat{h}\hat{u} \rangle \langle \hat{g}\hat{v} \rangle \\ & + \langle \hat{f}\hat{h}\hat{v} \rangle \langle \hat{g}\hat{u} \rangle + \langle \hat{f}\hat{u}\hat{v} \rangle \langle \hat{g}\hat{h} \rangle + \langle \hat{g}\hat{h}\hat{u} \rangle \langle \hat{f}\hat{v} \rangle \\ & + \langle \hat{g}\hat{h}\hat{v} \rangle \langle \hat{f}\hat{u} \rangle + \langle \hat{g}\hat{u}\hat{v} \rangle \langle \hat{f}\hat{h} \rangle + \langle \hat{h}\hat{u}\hat{v} \rangle \langle \hat{f}\hat{g} \rangle \Big)\end{aligned} \quad \text{(B8)}$$

and

$$\begin{aligned}\langle \delta_f \delta_g \delta_h \delta_u \delta_v \delta_w \rangle = {} & \frac{1}{N^5}\Big[\Big\langle \hat{f}\hat{g}\hat{h}\hat{u}\hat{v}\hat{w} \Big\rangle \\ & + (N-1)\Big(\langle \hat{f}\hat{g}\hat{h}\hat{u} \rangle \langle \hat{v}\hat{w} \rangle + 14 \text{ perms.} \Big) \\ & + (N-1)\Big(\langle \hat{f}\hat{g}\hat{h} \rangle \langle \hat{u}\hat{v}\hat{w} \rangle + 9 \text{ perms.} \Big) \\ & + (N-1)(N-2) \times \\ & \Big(\langle \hat{f}\hat{g} \rangle \langle \hat{h}\hat{u} \rangle \langle \hat{v}\hat{w} \rangle + 14 \text{ perms.} \Big) \Big] . \end{aligned} \quad \text{(B9)}$$

In general, the dominant terms in the $1/N$ expansion should be those composed of products of bilinears like $\left\langle \hat{f}\hat{g} \right\rangle$ in the case of even powers of δ, and those with bilinears and one trilinear in the case of odd powers of δ. So, we have $\mathcal{O}\left(\delta^{2n-1}\right) = \mathcal{O}\left(1/N^n\right)$ for $n > 1$ and $\mathcal{O}\left(\delta^{2n}\right) = \mathcal{O}\left(1/N^n\right)$ for $n \geq 1$. This means that we must consider terms up to $\mathcal{O}\left(\delta^6\right)$ if we want to find all $\mathcal{O}\left(1/N^2\right)$ terms in the series truncated by B&O, and terms up to $\mathcal{O}\left(\delta^6\right)$ in order to capture the leading behavior in the limit $\epsilon_s \to 0$ (see below).

2. B&O Equation 8

The Taylor expansion which leads to Eq. (8) in B&O is not applicable everywhere as it implicitly assumes that $1/(N\epsilon_s^2) \ll 1$. Since $\epsilon_s \to 0$ for central collisions, this quantity is not guaranteed to be small, and this expansion is poorly behaved and formally divergent for central collisions. Fortunately, Eq. (8) of B&O is not actually needed in order to derive and generalize their Eqs. (12)–(14).

3. B&O Equation 12: Calculating $\epsilon_{\text{part}}\{2\}^2$

In order to calculate $\epsilon_{\text{part}}\{2\}^2 = \langle \epsilon_{\text{part}}^2 \rangle$, we must first express ϵ_{part}^2 in terms of the δ's. We start with the definition, Eq. (A6):

$$\epsilon_{\text{part}}^2 = \frac{\langle \sigma_y^2 - \sigma_x^2 \rangle^2 + 4\sigma_{xy}^2}{\langle \sigma_y^2 + \sigma_x^2 \rangle^2} . \quad \text{(B10)}$$

Following B&O we have

$$\sigma_x^2 = \langle x^2 \rangle - \langle x \rangle^2 = \langle x^2 \rangle + \delta_{x^2} - \delta_x^2 \quad \text{(B11)}$$

$$\sigma_y^2 = \langle y^2 \rangle - \langle y \rangle^2 = \langle y^2 \rangle + \delta_{y^2} - \delta_y^2 \quad \text{(B12)}$$

$$\sigma_{xy} = \langle xy \rangle - \langle x \rangle \langle y \rangle = \delta_{xy} - \delta_x \delta_y \quad \text{(B13)}$$

using $\langle x \rangle = \langle y \rangle = \langle xy \rangle = \langle xy^2 \rangle = \langle x^2 y \rangle = 0$. This leads to the exact result:

$$\begin{aligned}\epsilon_{\text{part}}^2 = {} & \left[\epsilon_s^2 + \frac{2\epsilon_s \delta_{y^2-x^2}}{\langle r^2 \rangle} + \frac{\delta_{y^2-x^2}^2}{\langle r^2 \rangle^2} + \frac{4\delta_{xy}^2}{\langle r^2 \rangle^2} \right. \\ & - \frac{2\epsilon_s(\delta_y^2 - \delta_x^2)}{\langle r^2 \rangle} - \frac{2\delta_{y^2-x^2}(\delta_y^2 - \delta_x^2)}{\langle r^2 \rangle^2} \\ & \left. - \frac{8\delta_{xy}\delta_x\delta_y}{\langle r^2 \rangle^2} + \frac{(\delta_y^2 - \delta_x^2)^2}{\langle r^2 \rangle^2} + \frac{4\delta_x^2\delta_y^2}{\langle r^2 \rangle^2} \right] \times \\ & \left[1 + \frac{\delta_{r^2}}{\langle r^2 \rangle} - \frac{(\delta_y^2 + \delta_x^2)}{\langle r^2 \rangle} \right]^{-2} . \end{aligned} \quad \text{(B14)}$$

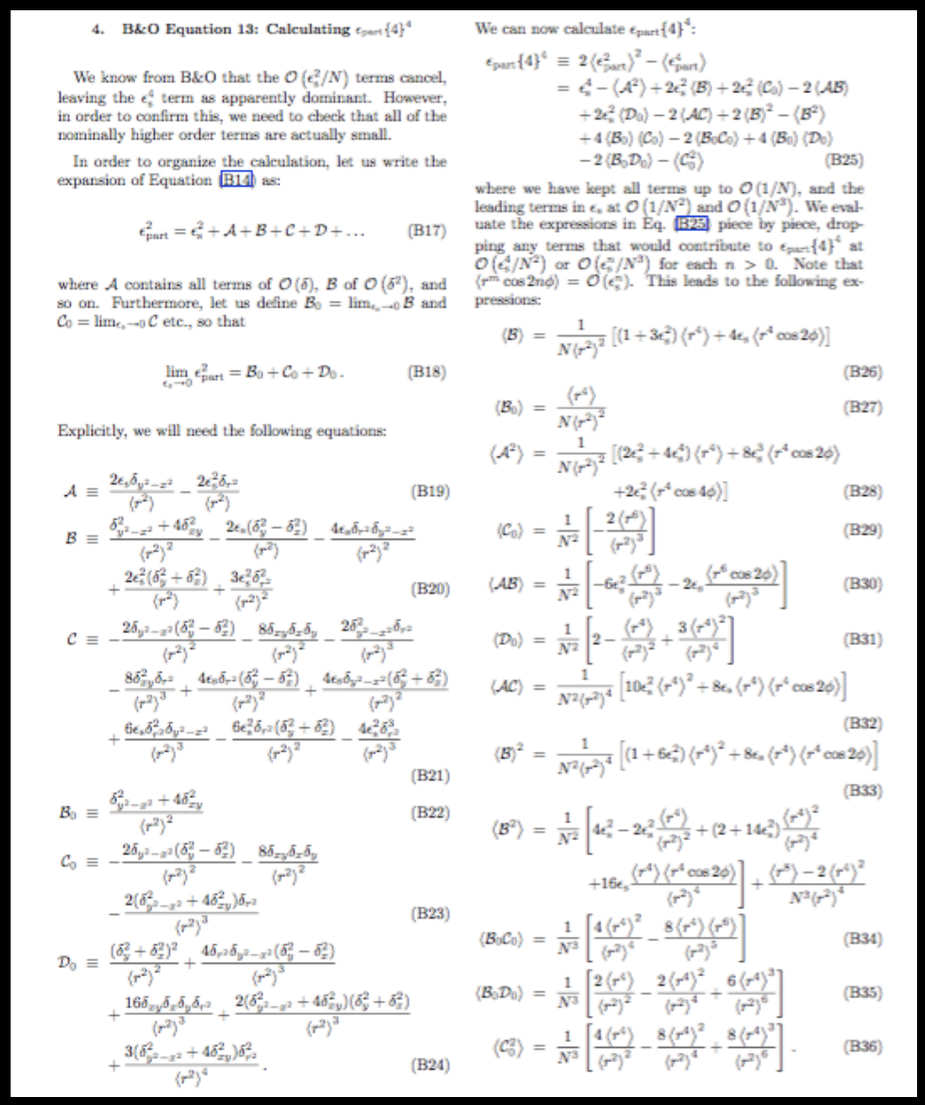
where $\langle r^2 \rangle = \langle x^2 \rangle + \langle y^2 \rangle$. The second and third terms in the denominator are genuinely $\ll 1$, so it can be safely Taylor expanded. The resulting polynomial series is well behaved. The leading terms are ϵ_s^2 and $\mathcal{O}\left(\epsilon_s^2/N\right)$. All terms of $\mathcal{O}\left(\delta^3\right)$ and higher are $\mathcal{O}\left(\epsilon_s^2/N^2\right)$ and can be dropped since they are at least a full power of $1/N$ down without any compensating $1/\epsilon_s$ factors. So, we obtain

$$\begin{aligned}\epsilon_{\text{part}}^2 = {} & \epsilon_s^2 + \frac{2\epsilon_s \delta_{y^2-x^2}}{\langle r^2 \rangle} - \frac{2\epsilon_s^2 \delta_{r^2}}{\langle r^2 \rangle} + \frac{\delta_{y^2-x^2}^2}{\langle r^2 \rangle^2} + \frac{4\delta_{xy}^2}{\langle r^2 \rangle^2} \\ & - \frac{2\epsilon_s(\delta_y^2 - \delta_x^2)}{\langle r^2 \rangle} - \frac{4\epsilon_s \delta_{r^2}\delta_{y^2-x^2}}{\langle r^2 \rangle^2} + \frac{2\epsilon_s^2(\delta_y^2 + \delta_x^2)}{\langle r^2 \rangle} \\ & + \frac{3\epsilon_s^2 \delta_{r^2}^2}{\langle r^2 \rangle^2} + \mathcal{O}\left(\epsilon_s^2 \delta^3\right) + \mathcal{O}\left(\epsilon_s^3 \delta^3\right) \\ & + \mathcal{O}\left(\epsilon_s^2 \delta^4\right) + \mathcal{O}\left(\epsilon_s^3 \delta^4\right) + \ldots . \end{aligned} \quad \text{(B15)}$$

This leads to the same result as B&O Eq. (12), except that we have also shown that all further terms are subdominant:

$$\begin{aligned}\left\langle \epsilon_{\text{part}}^2 \right\rangle = {} & \epsilon_s^2 + \frac{1}{N\langle r^2 \rangle^2}\left[(1 + 3\epsilon_s^2)\langle r^4 \rangle + 4\epsilon_s \langle r^4 \cos 2\phi \rangle \right] \\ & + \mathcal{O}\left(\frac{\epsilon_s^3}{N^2}\right) + \ldots . \end{aligned} \quad \text{(B16)}$$

Similarly, Eq. (14) in B&O is well-behaved and correctly contains all of the leading terms. As noted above, this is different for the expansion of Eq. (8) in B&O, which does not converge in the limit $\epsilon_s \to 0$.



4. B&O Equation 13: Calculating $\epsilon_{\text{part}}\{4\}^4$

We know from B&O that the $\mathcal{O}\left(\epsilon_s^2/N\right)$ terms cancel, leaving the ϵ_s^4 term as apparently dominant. However, in order to confirm this, we need to check that all of the nominally higher order terms are actually small.

In order to organize the calculation, let us write the expansion of Equation B14 as:

$$\epsilon_{\text{part}}^2 = \epsilon_s^2 + \mathcal{A} + \mathcal{B} + \mathcal{C} + \mathcal{D} + \ldots \quad \text{(B17)}$$

where $\mathcal{A}$ contains all terms of $\mathcal{O}\left(\delta\right)$, $\mathcal{B}$ of $\mathcal{O}\left(\delta^2\right)$, and so on. Furthermore, let us define $\mathcal{B}_0 = \lim_{\epsilon_s \to 0} \mathcal{B}$ and $\mathcal{C}_0 = \lim_{\epsilon_s \to 0} \mathcal{C}$ etc., so that

$$\lim_{\epsilon_s \to 0} \epsilon_{\text{part}}^2 = \mathcal{B}_0 + \mathcal{C}_0 + \mathcal{D}_0 . \quad \text{(B18)}$$

Explicitly, we will need the following equations:

$$\mathcal{A} \equiv \frac{2\epsilon_s \delta_{y^2-x^2}}{\langle r^2 \rangle} - \frac{2\epsilon_s^2 \delta_{r^2}}{\langle r^2 \rangle} \quad \text{(B19)}$$

$$\begin{aligned}\mathcal{B} \equiv {} & \frac{\delta_{y^2-x^2}^2 + 4\delta_{xy}^2}{\langle r^2 \rangle^2} - \frac{2\epsilon_s(\delta_y^2 - \delta_x^2)}{\langle r^2 \rangle} - \frac{4\epsilon_s \delta_{r^2}\delta_{y^2-x^2}}{\langle r^2 \rangle^2} \\ & + \frac{2\epsilon_s^2(\delta_y^2 + \delta_x^2)}{\langle r^2 \rangle} + \frac{3\epsilon_s^2 \delta_{r^2}^2}{\langle r^2 \rangle^2} \end{aligned} \quad \text{(B20)}$$

$$\begin{aligned}\mathcal{C} \equiv {} & - \frac{2\delta_{y^2-x^2}(\delta_y^2 - \delta_x^2)}{\langle r^2 \rangle^2} - \frac{8\delta_{xy}\delta_x\delta_y}{\langle r^2 \rangle^2} - \frac{2\delta_{y^2-x^2}\delta_{r^2}}{\langle r^2 \rangle^3} \\ & - \frac{8\delta_{xy}^2\delta_{r^2}}{\langle r^2 \rangle^3} + \frac{4\epsilon_s \delta_{r^2}(\delta_y^2 - \delta_x^2)}{\langle r^2 \rangle^2} + \frac{4\epsilon_s \delta_{y^2-x^2}(\delta_y^2 + \delta_x^2)}{\langle r^2 \rangle^2} \\ & + \frac{6\epsilon_s \delta_{r^2}^2 \delta_{y^2-x^2}}{\langle r^2 \rangle^3} - \frac{6\epsilon_s^2 \delta_{r^2}(\delta_y^2 + \delta_x^2)}{\langle r^2 \rangle^2} - \frac{4\epsilon_s^2 \delta_{r^2}^3}{\langle r^2 \rangle^3} \end{aligned} \quad \text{(B21)}$$

$$\mathcal{B}_0 \equiv \frac{\delta_{y^2-x^2}^2 + 4\delta_{xy}^2}{\langle r^2 \rangle^2} \quad \text{(B22)}$$

$$\begin{aligned}\mathcal{C}_0 \equiv {} & - \frac{2\delta_{y^2-x^2}(\delta_y^2 - \delta_x^2)}{\langle r^2 \rangle^2} - \frac{8\delta_{xy}\delta_x\delta_y}{\langle r^2 \rangle^2} \\ & - \frac{2(\delta_{y^2-x^2}^2 + 4\delta_{xy}^2)\delta_{r^2}}{\langle r^2 \rangle^3} \end{aligned} \quad \text{(B23)}$$

$$\begin{aligned}\mathcal{D}_0 \equiv {} & \frac{(\delta_y^2 + \delta_x^2)^2}{\langle r^2 \rangle^2} + \frac{4\delta_{r^2}\delta_{y^2-x^2}(\delta_y^2 - \delta_x^2)}{\langle r^2 \rangle^3} \\ & + \frac{16\delta_{xy}\delta_x\delta_y\delta_{r^2}}{\langle r^2 \rangle^3} + \frac{2(\delta_{y^2-x^2}^2 + 4\delta_{xy}^2)(\delta_y^2 + \delta_x^2)}{\langle r^2 \rangle^3} \\ & + \frac{3(\delta_{y^2-x^2}^2 + 4\delta_{xy}^2)\delta_{r^2}^2}{\langle r^2 \rangle^4} . \end{aligned} \quad \text{(B24)}$$

We can now calculate $\epsilon_{\text{part}}\{4\}^4$:

$$\begin{aligned}\epsilon_{\text{part}}\{4\}^4 \equiv {} & 2\left\langle \epsilon_{\text{part}}^2 \right\rangle^2 - \left\langle \epsilon_{\text{part}}^4 \right\rangle \\ = {} & \epsilon_s^4 - \langle \mathcal{A}^2 \rangle + 2\epsilon_s^2 \langle \mathcal{B} \rangle + 2\epsilon_s^2 \langle \mathcal{C}_0 \rangle - 2\langle \mathcal{A}\mathcal{B} \rangle \\ & + 2\epsilon_s^2 \langle \mathcal{D}_0 \rangle - 2\langle \mathcal{A}\mathcal{C} \rangle + 2\langle \mathcal{B} \rangle^2 - \langle \mathcal{B}^2 \rangle \\ & + 4\langle \mathcal{B}_0 \rangle \langle \mathcal{C}_0 \rangle - 2\langle \mathcal{B}_0 \mathcal{C}_0 \rangle + 4\langle \mathcal{B}_0 \rangle \langle \mathcal{D}_0 \rangle \\ & - 2\langle \mathcal{B}_0 \mathcal{D}_0 \rangle - \langle \mathcal{C}_0^2 \rangle \end{aligned} \quad \text{(B25)}$$

where we have kept all terms up to $\mathcal{O}\left(1/N\right)$, and the leading terms in ϵ_s at $\mathcal{O}\left(1/N^2\right)$ and $\mathcal{O}\left(1/N^3\right)$. We evaluate the expressions in Eq. B25 piece by piece, dropping any terms that would contribute to $\epsilon_{\text{part}}\{4\}^4$ at $\mathcal{O}\left(\epsilon_s^4/N^2\right)$ or $\mathcal{O}\left(\epsilon_s^n/N^3\right)$ for each $n > 0$. Note that $\langle r^m \cos 2n\phi \rangle = \mathcal{O}\left(\epsilon_s^n\right)$. This leads to the following expressions:

$$\langle \mathcal{B} \rangle = \frac{1}{N\langle r^2 \rangle^2}\left[(1 + 3\epsilon_s^2)\langle r^4 \rangle + 4\epsilon_s \langle r^4 \cos 2\phi \rangle \right] \quad \text{(B26)}$$

$$\langle \mathcal{B}_0 \rangle = \frac{\langle r^4 \rangle}{N\langle r^2 \rangle^2} \quad \text{(B27)}$$

$$\langle \mathcal{A}^2 \rangle = \frac{1}{N\langle r^2 \rangle^2}\left[(2\epsilon_s^2 + 4\epsilon_s^4)\langle r^4 \rangle + 8\epsilon_s^3 \langle r^4 \cos 2\phi \rangle + 2\epsilon_s^2 \langle r^4 \cos 4\phi \rangle \right] \quad \text{(B28)}$$

$$\langle \mathcal{C}_0 \rangle = \frac{1}{N^2}\left[-\frac{2\langle r^6 \rangle}{\langle r^2 \rangle^3} \right] \quad \text{(B29)}$$

$$\langle \mathcal{A}\mathcal{B} \rangle = \frac{1}{N^2}\left[-6\epsilon_s^2 \frac{\langle r^6 \rangle}{\langle r^2 \rangle^3} - 2\epsilon_s \frac{\langle r^6 \cos 2\phi \rangle}{\langle r^2 \rangle^3} \right] \quad \text{(B30)}$$

$$\langle \mathcal{D}_0 \rangle = \frac{1}{N^2}\left[2 - \frac{\langle r^4 \rangle}{\langle r^2 \rangle^2} + \frac{3\langle r^4 \rangle^2}{\langle r^2 \rangle^4} \right] \quad \text{(B31)}$$

$$\langle \mathcal{A}\mathcal{C} \rangle = \frac{1}{N^2\langle r^2 \rangle^4}\left[10\epsilon_s^2 \langle r^4 \rangle^2 + 8\epsilon_s \langle r^4 \rangle \langle r^4 \cos 2\phi \rangle \right] \quad \text{(B32)}$$

$$\langle \mathcal{B} \rangle^2 = \frac{1}{N^2\langle r^2 \rangle^4}\left[(1 + 6\epsilon_s^2)\langle r^4 \rangle^2 + 8\epsilon_s \langle r^4 \rangle \langle r^4 \cos 2\phi \rangle \right] \quad \text{(B33)}$$

$$\begin{aligned}\langle \mathcal{B}^2 \rangle = {} & \frac{1}{N^2}\left[4\epsilon_s^2 - 2\epsilon_s^2 \frac{\langle r^4 \rangle}{\langle r^2 \rangle^2} + (2 + 14\epsilon_s^2)\frac{\langle r^4 \rangle^2}{\langle r^2 \rangle^4} \right. \\ & \left. + 16\epsilon_s \frac{\langle r^4 \rangle \langle r^4 \cos 2\phi \rangle}{\langle r^2 \rangle^4} \right] + \frac{\langle r^8 \rangle - 2\langle r^4 \rangle^2}{N^3\langle r^2 \rangle^4} \end{aligned}$$

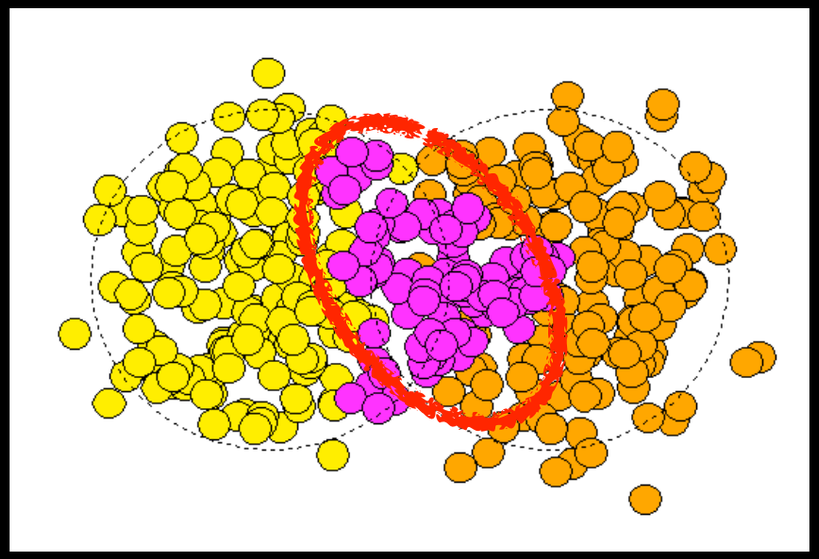
$$\langle \mathcal{B}_0 \mathcal{C}_0 \rangle = \frac{1}{N^3}\left[\frac{4\langle r^4 \rangle^2}{\langle r^2 \rangle^4} - \frac{8\langle r^4 \rangle \langle r^6 \rangle}{\langle r^2 \rangle^5} \right] \quad \text{(B34)}$$

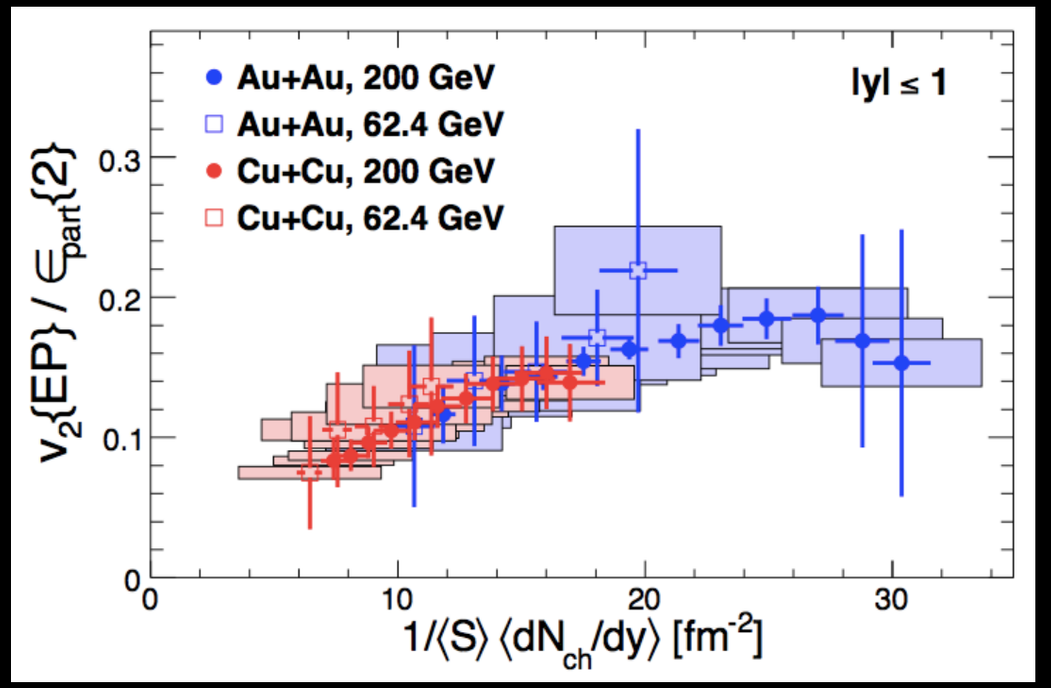
$$\langle \mathcal{B}_0 \mathcal{D}_0 \rangle = \frac{1}{N^3}\left[\frac{2\langle r^4 \rangle}{\langle r^2 \rangle^2} - \frac{2\langle r^4 \rangle^2}{\langle r^2 \rangle^4} + \frac{6\langle r^4 \rangle^3}{\langle r^2 \rangle^6} \right] \quad \text{(B35)}$$

$$\langle \mathcal{C}_0^2 \rangle = \frac{1}{N^3}\left[\frac{4\langle r^4 \rangle}{\langle r^2 \rangle^2} - \frac{8\langle r^4 \rangle^2}{\langle r^2 \rangle^4} + \frac{8\langle r^4 \rangle^3}{\langle r^2 \rangle^6} \right] . \quad \text{(B36)}$$

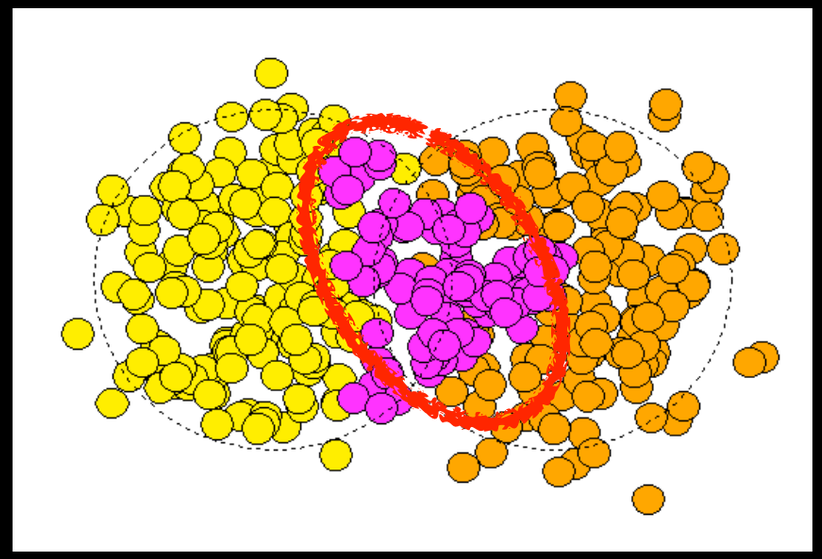
Lifetime supply of equations for PHOBOS

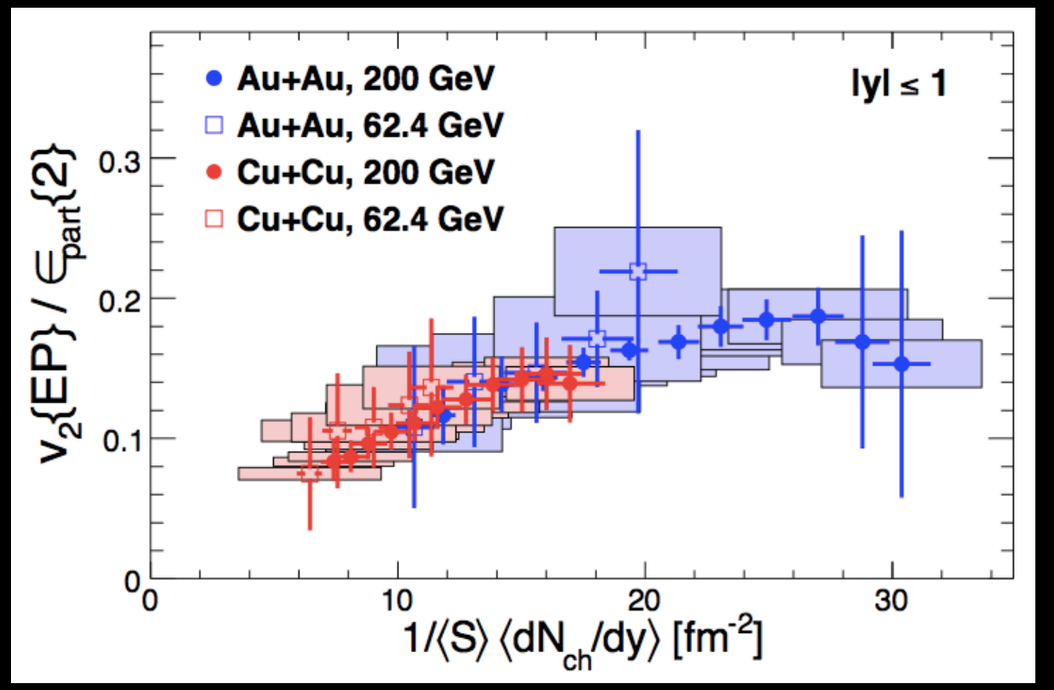
2005



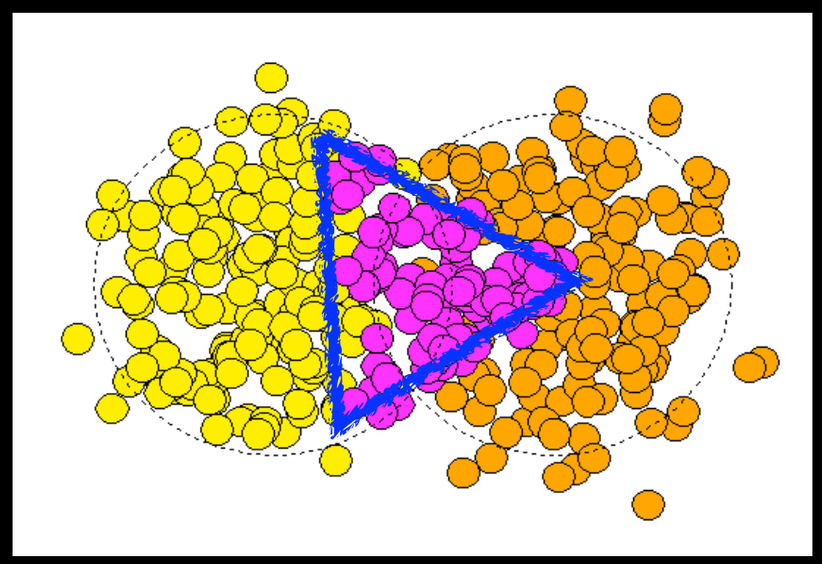

Au+Au, 200 GeV
Au+Au, 62.4 GeV
Cu+Cu, 200 GeV
Cu+Cu, 62.4 GeV
|y| ≤ 1
$v_2\{EP\} / \epsilon_{part}\{2\}$
$1/\langle S \rangle \langle dN_{ch}/dy \rangle$ [fm^{-2}]
0
0.1
0.2
0.3
10
20
30

2005



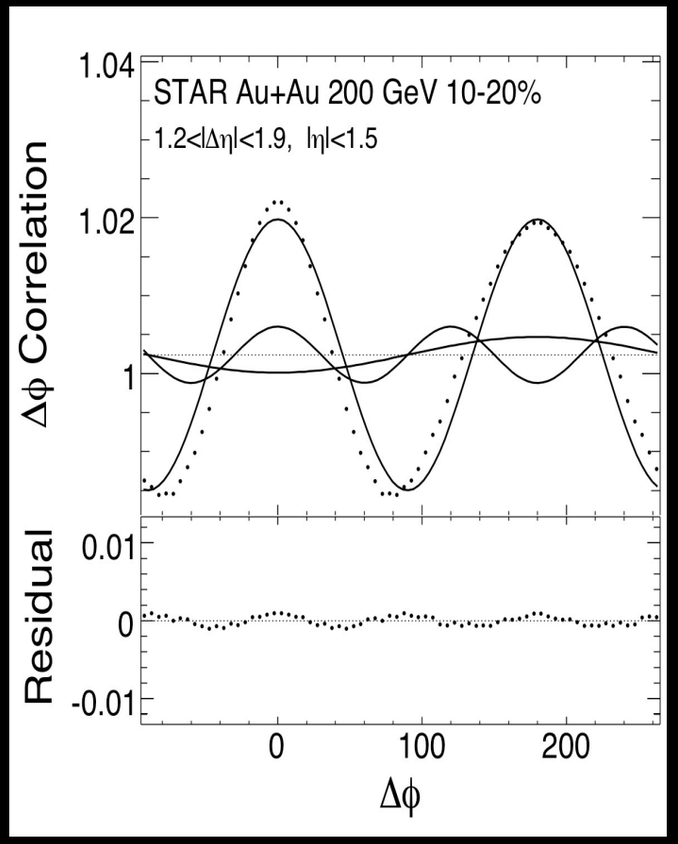


2010

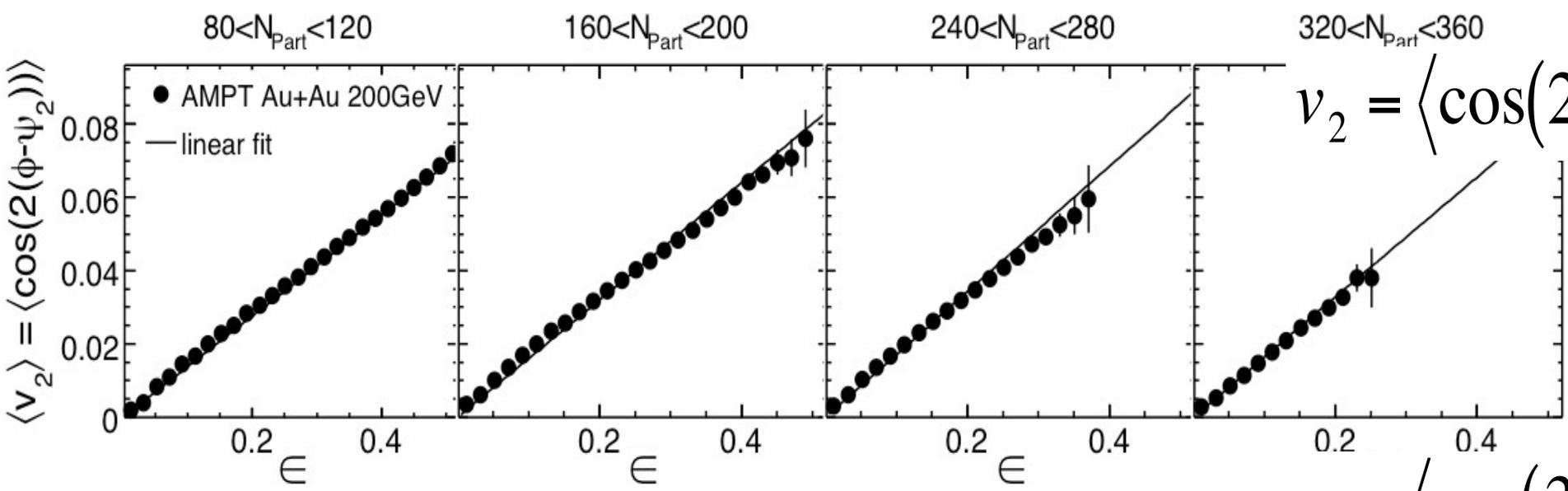


Geometry fluctuations and final state correlations:

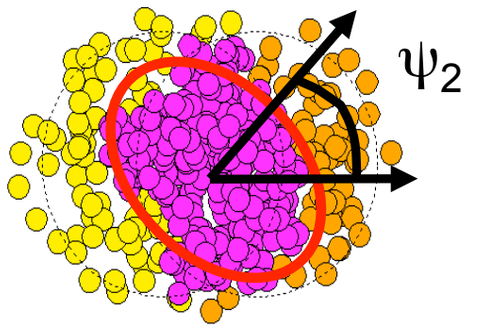
Mishra et al arXiv:0711.1323
Takahashi et al, arXiv:0902.4870
Sorensen, arXiv:1002.4878
Alver, GR, arXiv:1003.0194

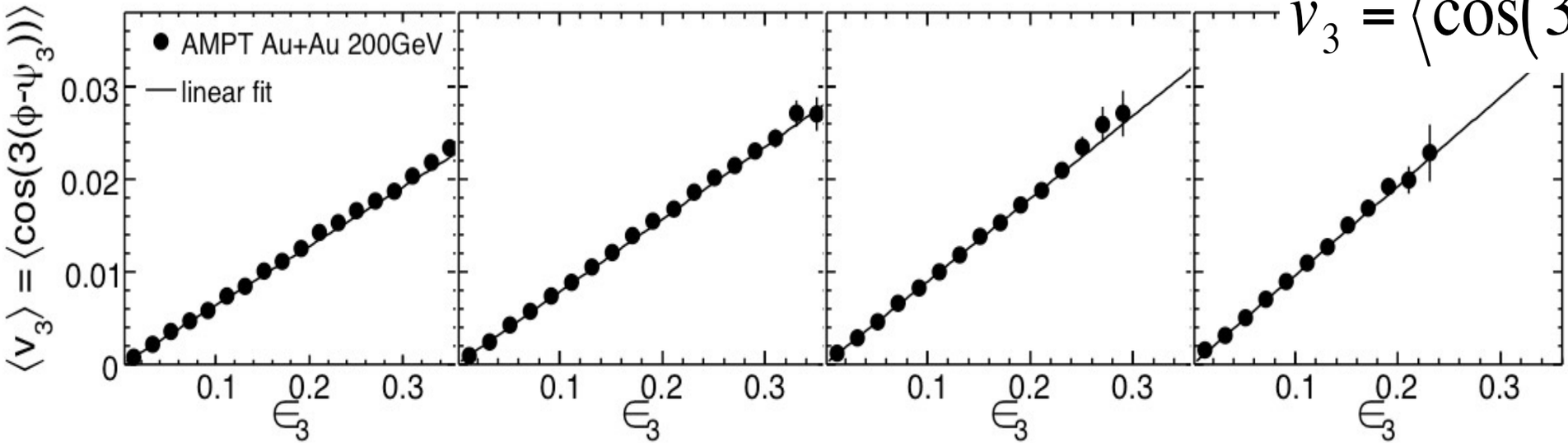


Elliptic and triangular flow

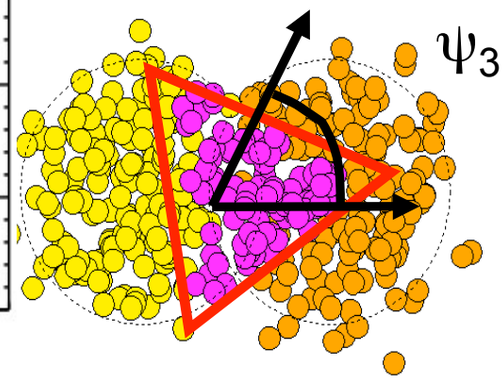


$$v_2 = \left\langle \cos\left(2(\phi - \psi_2)\right)\right\rangle \propto \varepsilon$$





$$v_3 = \left\langle \cos\left(3(\phi - \psi_3)\right)\right\rangle \propto \varepsilon_3$$



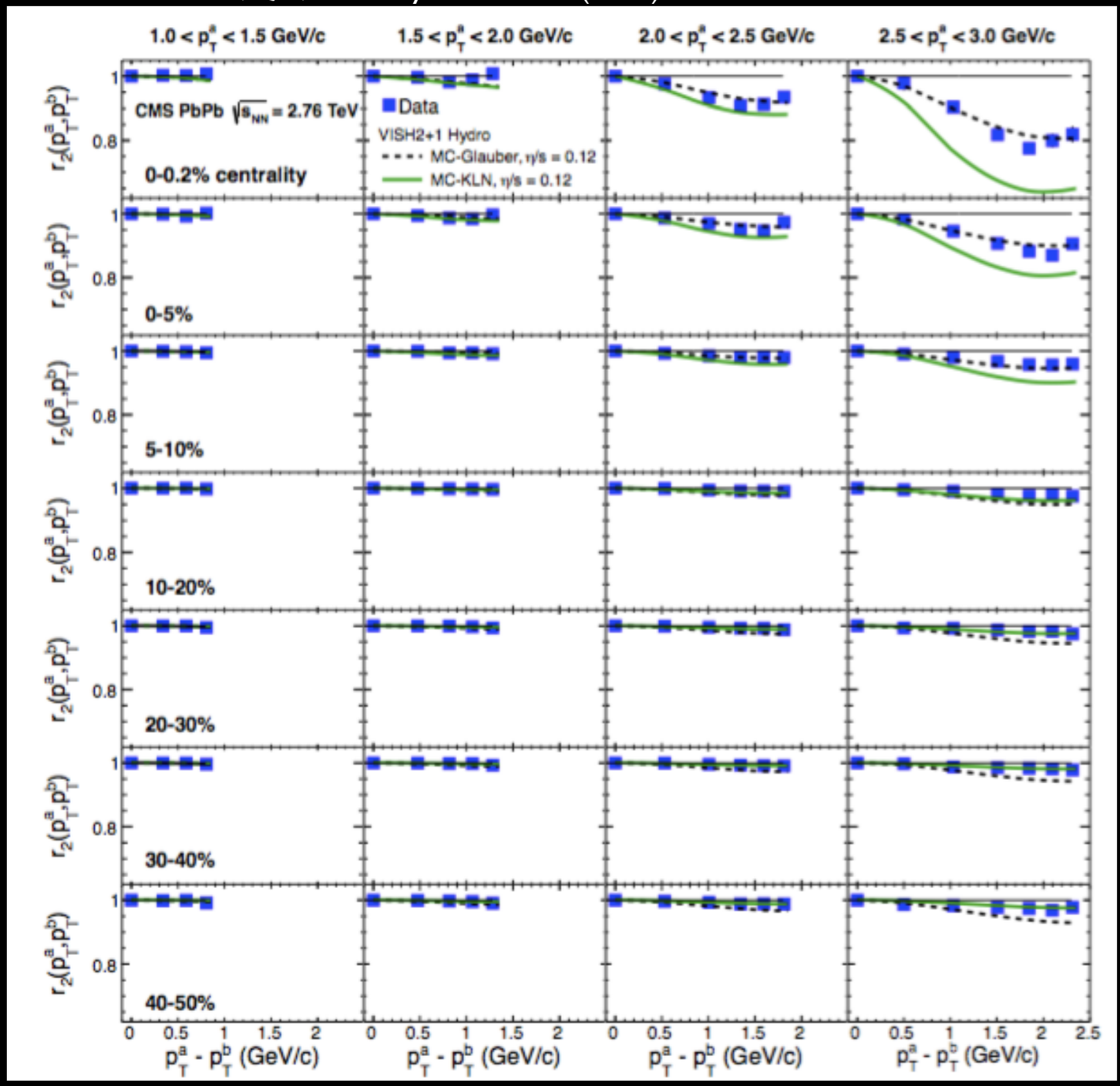
Just like elliptic flow reflects event-by-event eccentricity, "triangular flow" (v_3) reflects event-by-event "triangularity" (ε_3)

Burak Alver, GR, arXiv:1003.0194

"When has hydro ever predicted something?"

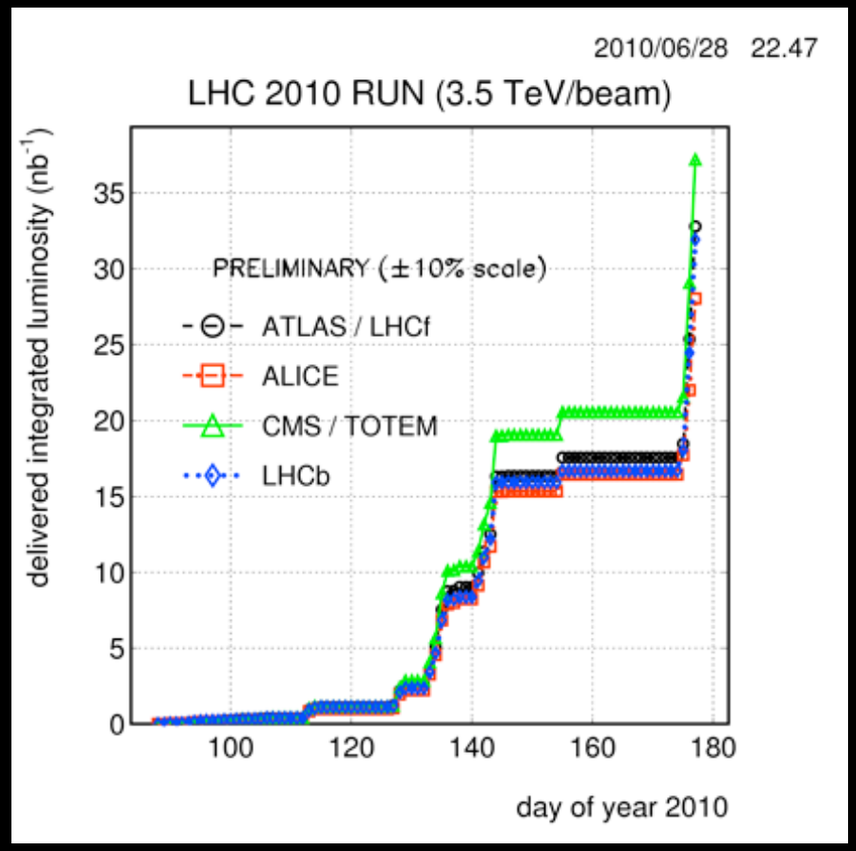
"When has hydro ever predicted something?"

Calculation: Heinz, Qiu, Shen Phys. Rev. C **87** (2013) 034913



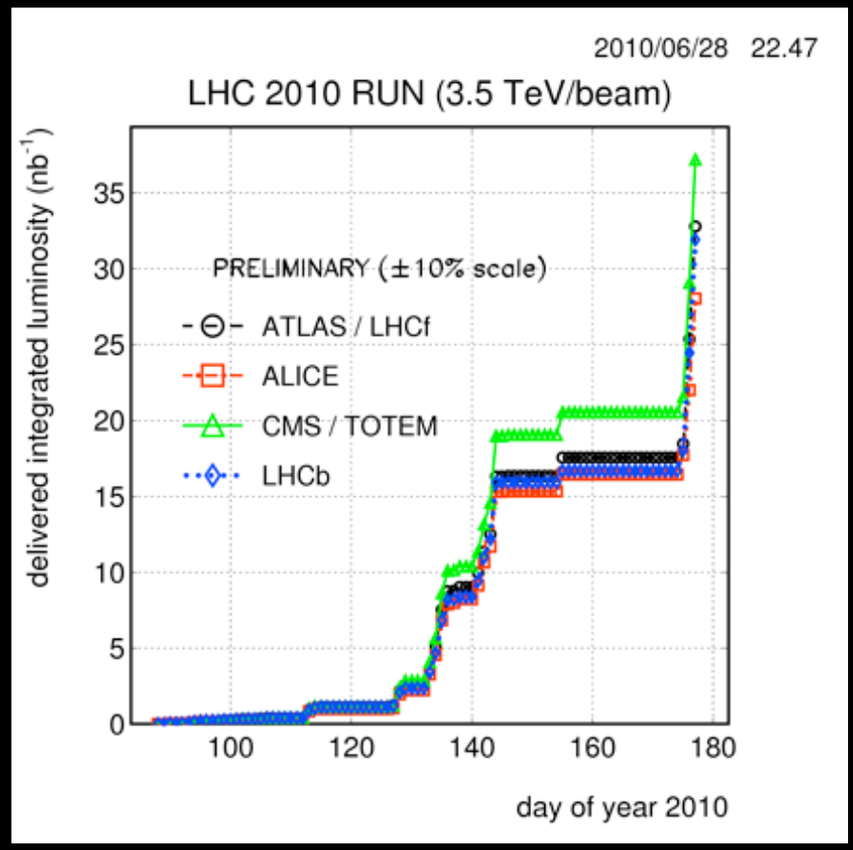
Factorization breaking: <u>prediction</u> and data

Back to 2010…



Successful start of LHC pp program at 7TeV, rampup of luminosity - waiting for heavy-ion collisions in November 2010

Back to 2010…

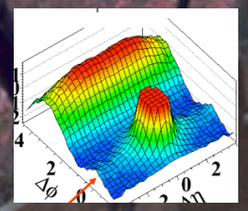


Successful start of LHC pp program at 7TeV, rampup of luminosity - waiting for heavy-ion collisions in November 2010

idle hands are the devil's workshop…

….designed and implemented CMS high multiplicity pp trigger, ran in summer 2010

Return of the Ridge



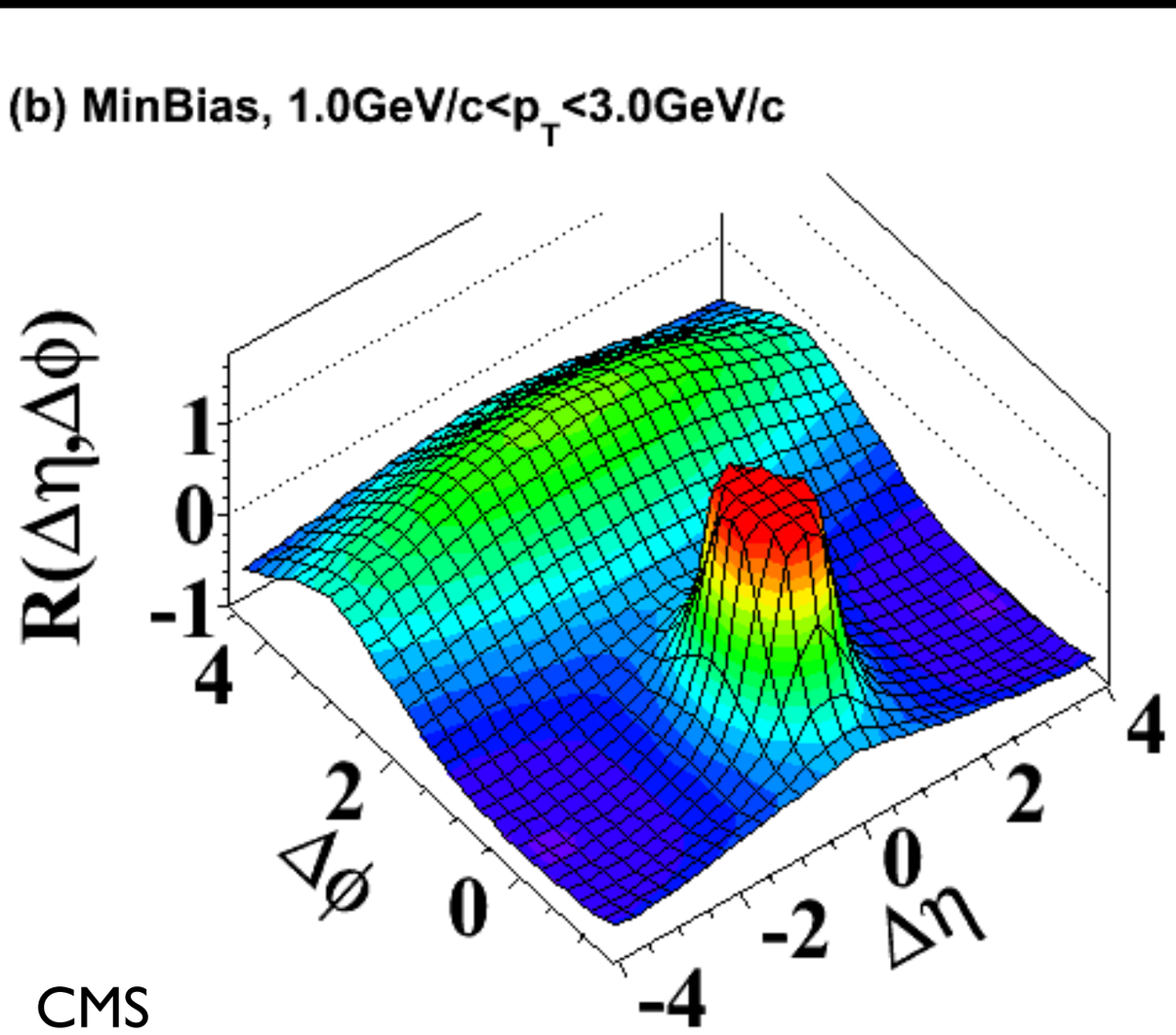
Two-Particle Correlations in pp (Sep 2010)

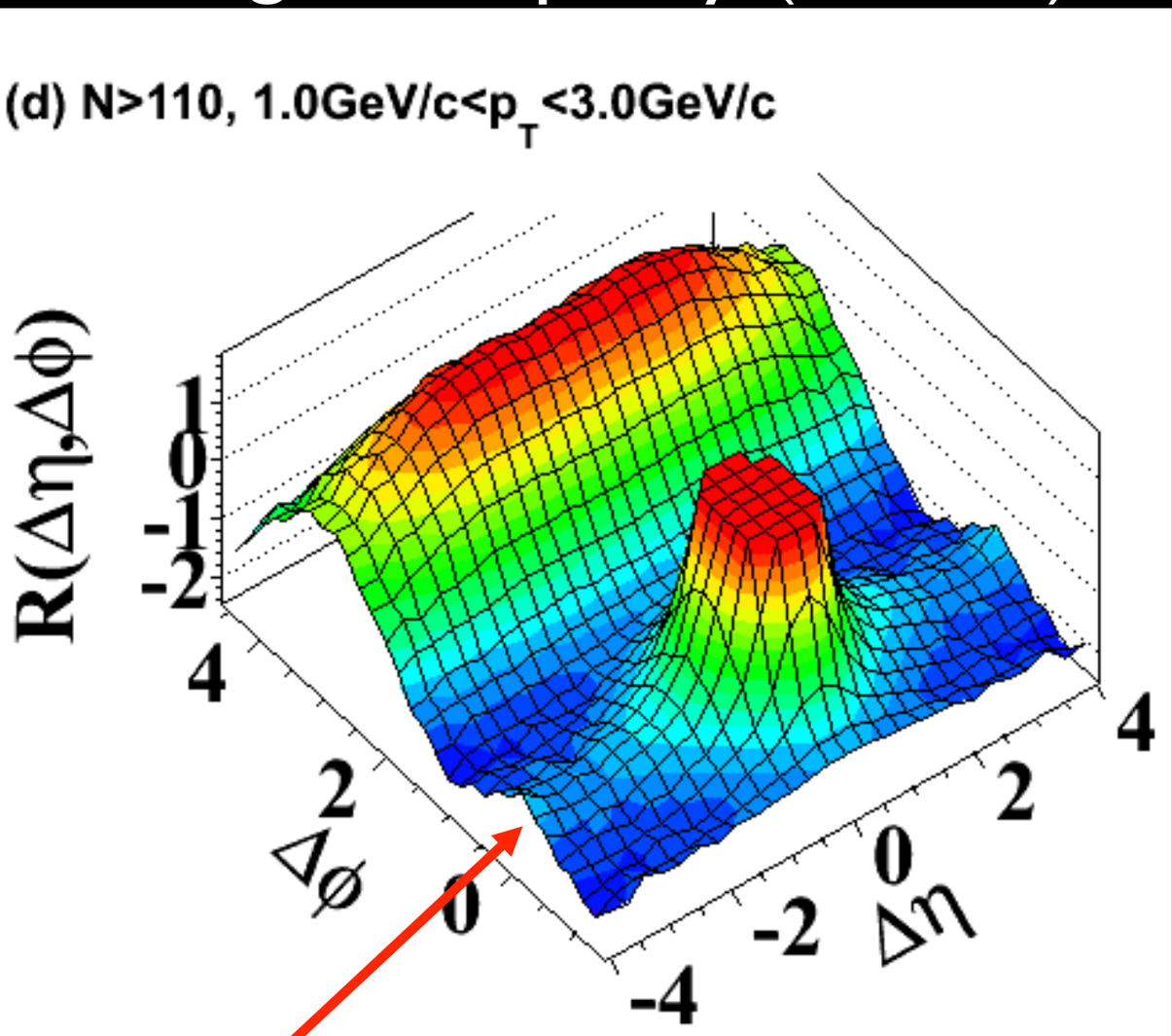
Results based on 1fb^{-1},
i.e. sampling 50billion pp events
with high multiplicity trigger

Intermediate p_T: 1-3 GeV/c

MinBias

high multiplicity (N>110)





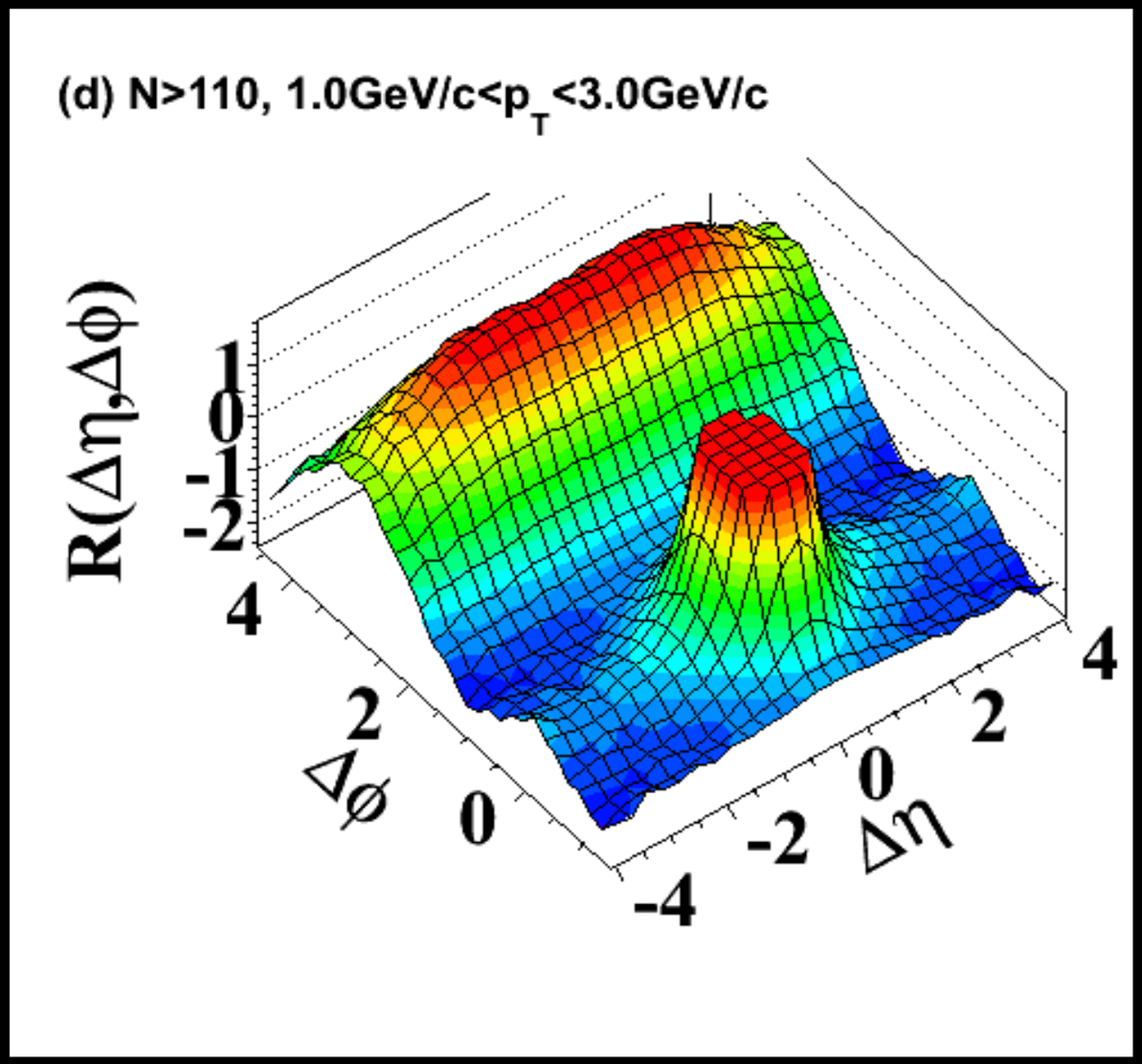
CMS

JHEP 1009 (2010) 091

Pronounced structure at large $\delta\eta$ around $\delta\phi \sim 0$!

Two-Particle Correlations in 7TeV pp

High multiplicity (N>110)



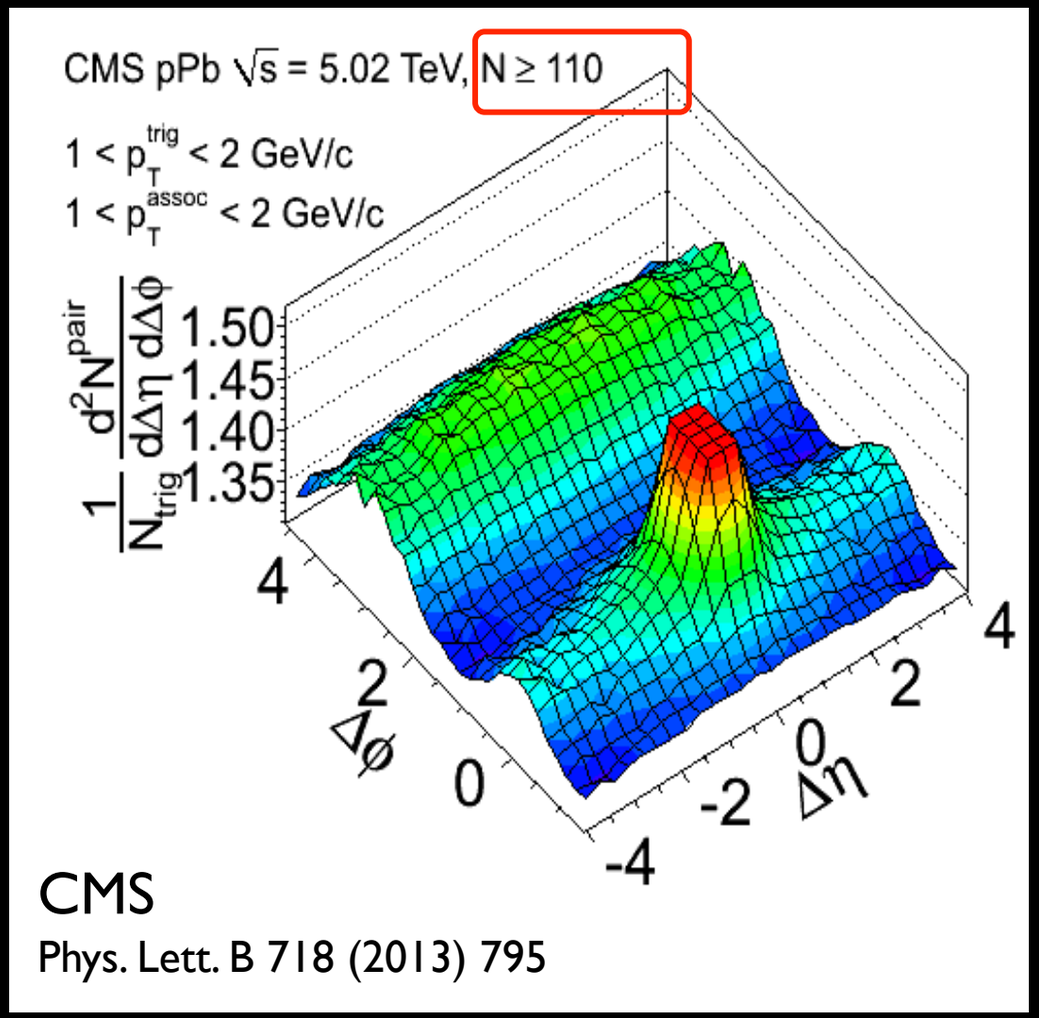
Multiplicity in these events is dominated by jet contribution.

Interpretation:

Multi-jet correlations
Jet-Jet color connections
Jet-proton remnant color connections
Jet-remnant connections + medium
Glasma correlations
Quantum entanglement
Angular momentum conservation
Angular momentum conservation + medium
Hydrodynamic flow

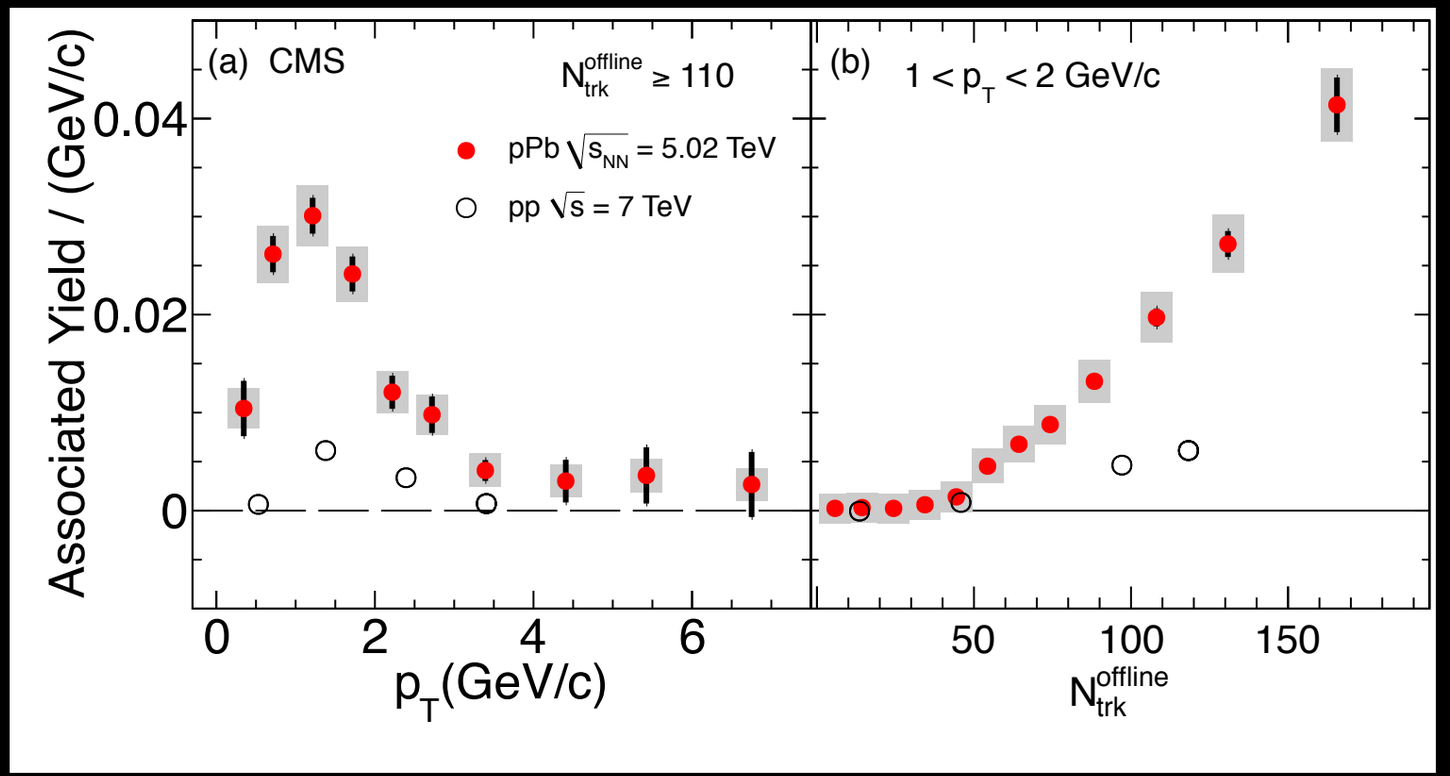
?

Two-Particle Correlations in pPb (2012)



CMS

Phys. Lett. B 718 (2013) 795

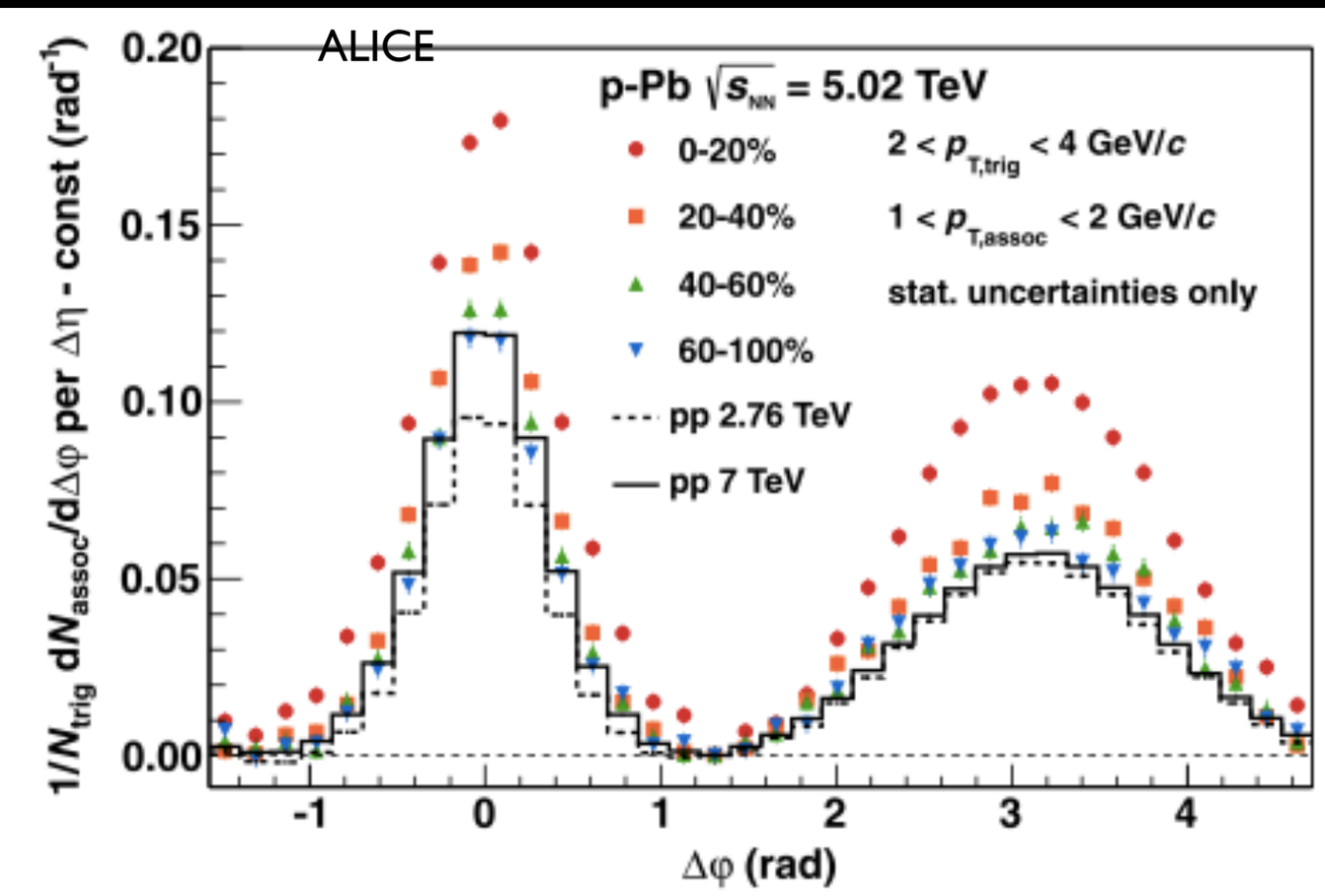


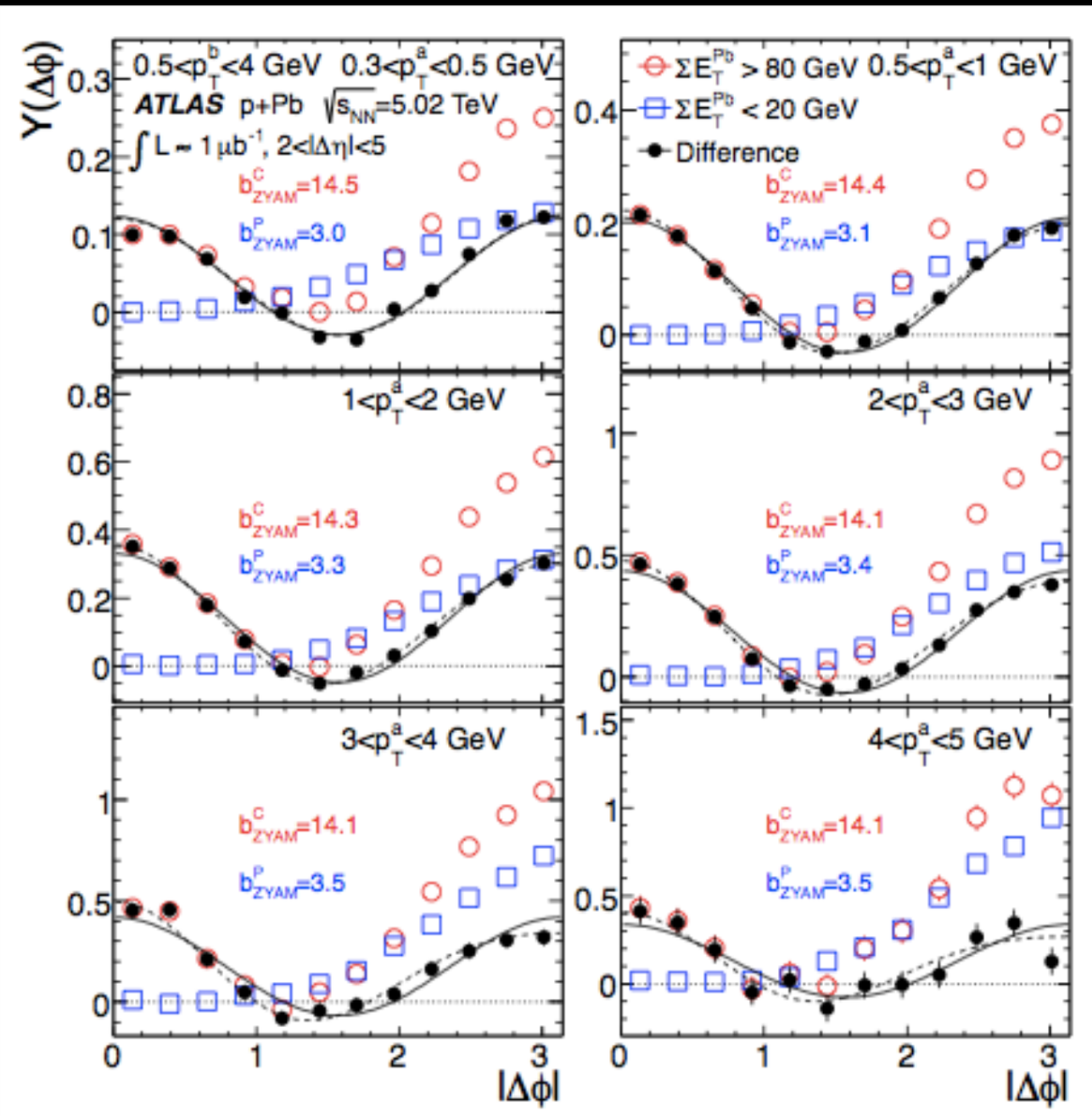
Similar correlations as in high-multiplicity pp, but larger strength (associated yield)

Peripheral subtraction in ALICE and ATLAS

ALICE Phys.Lett. B719 (2013) 29-41

ATLAS Phys. Rev. Lett. 110, 182302 (2013)

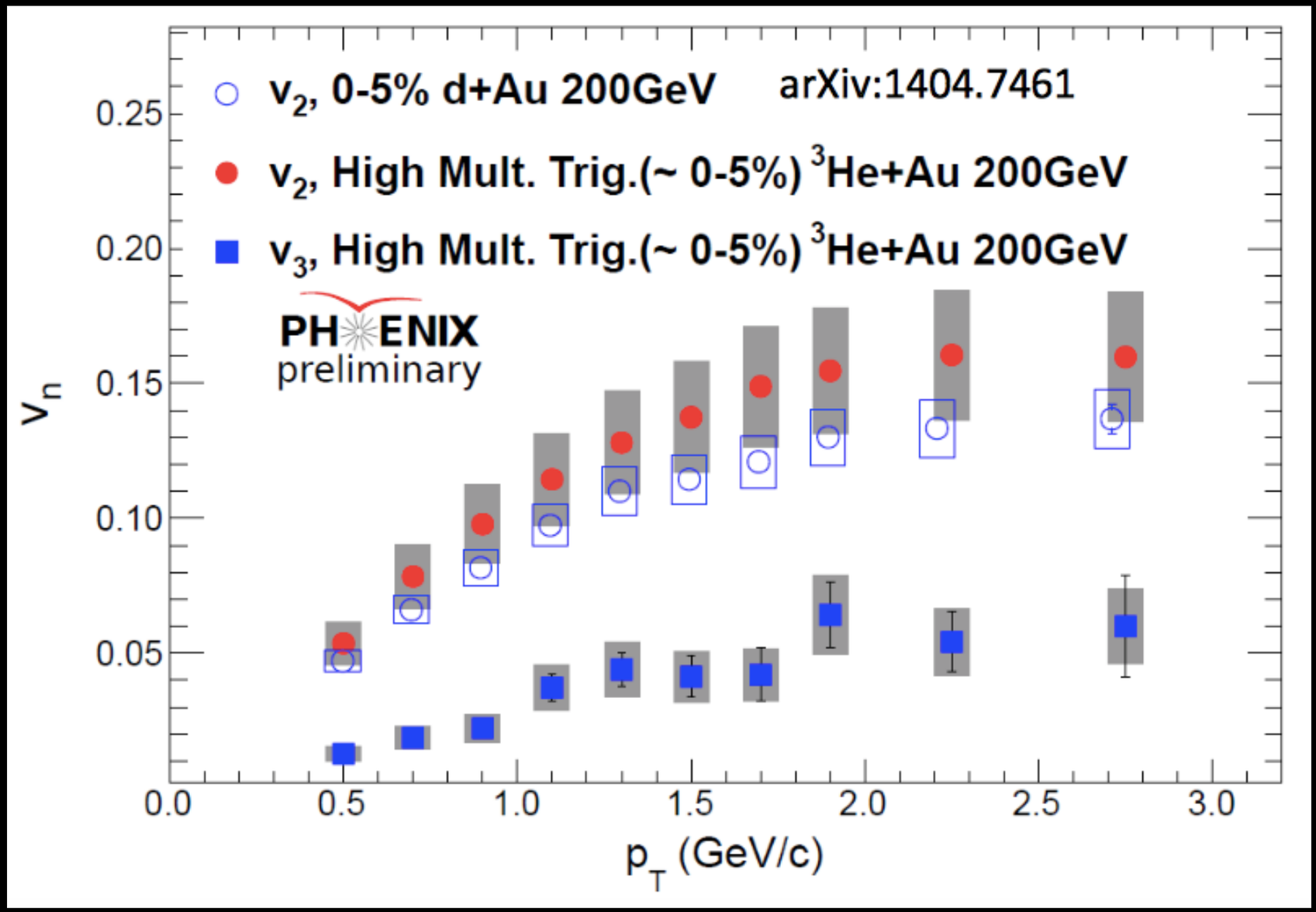




Away side yield in pp and peripheral pPb is very similar (away-side jet)

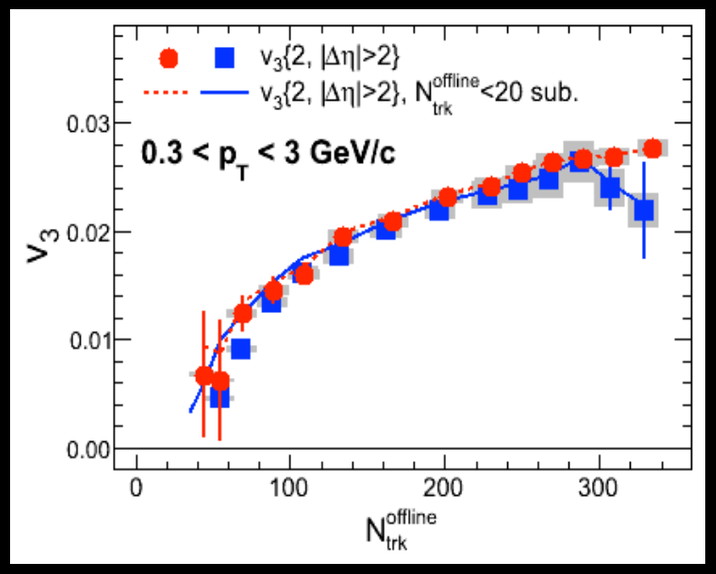
Subtraction of peripheral pPb correlations reveals nearly symmetric "double-ridge" structure

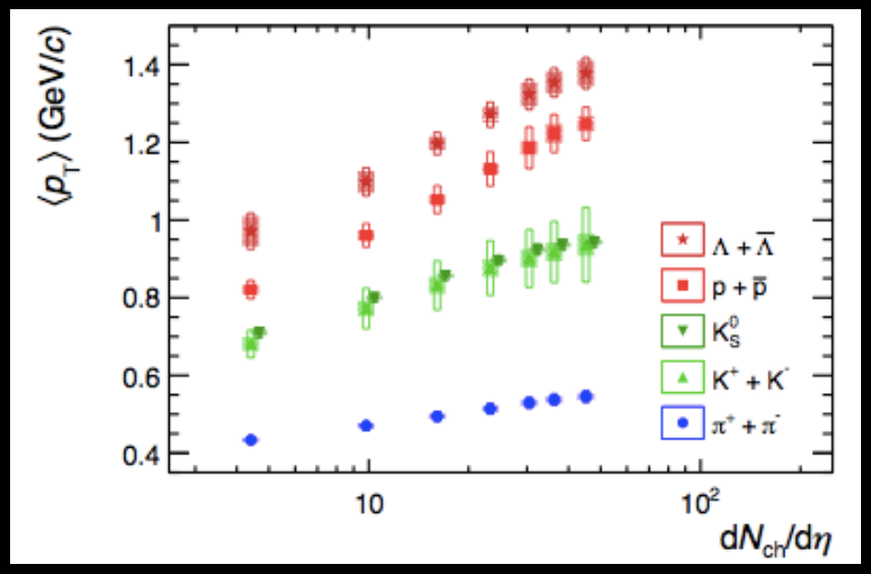
Connection to initial geometry

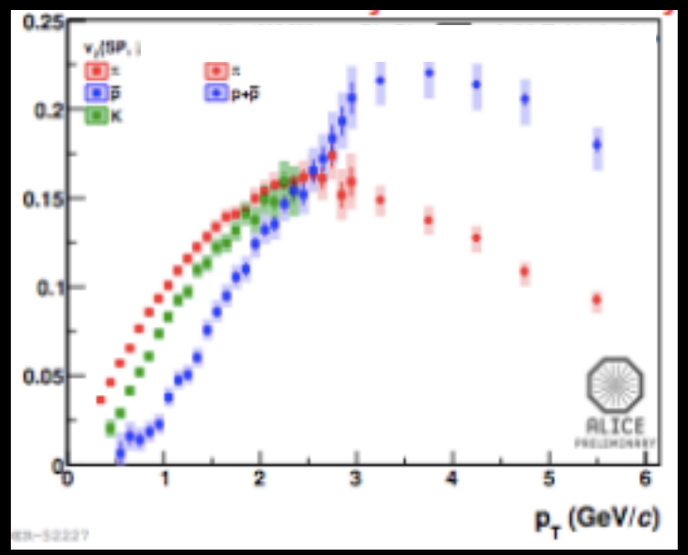


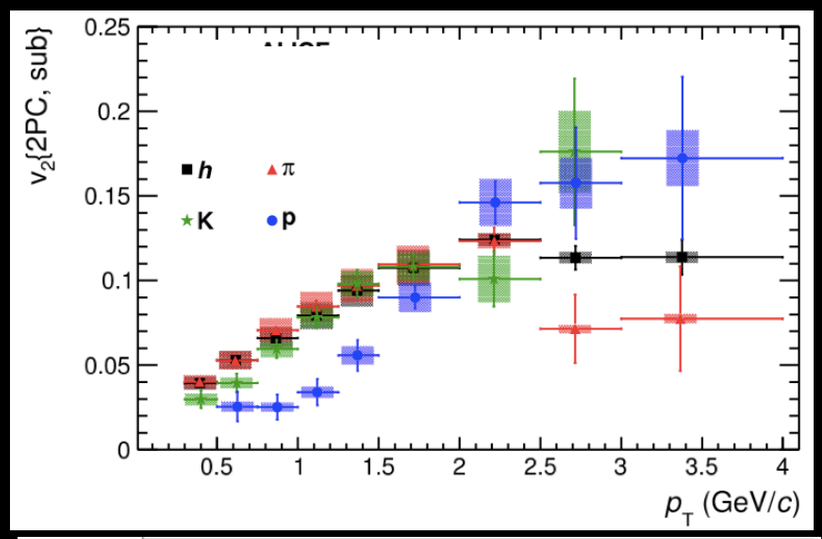
d+Au vs 3He+Au flow harmonics

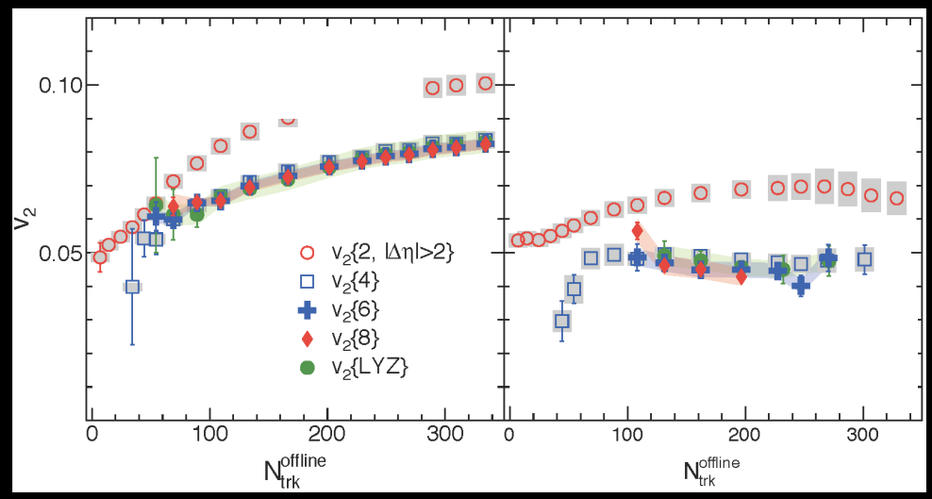
Quiz: Find pPb vs PbPb

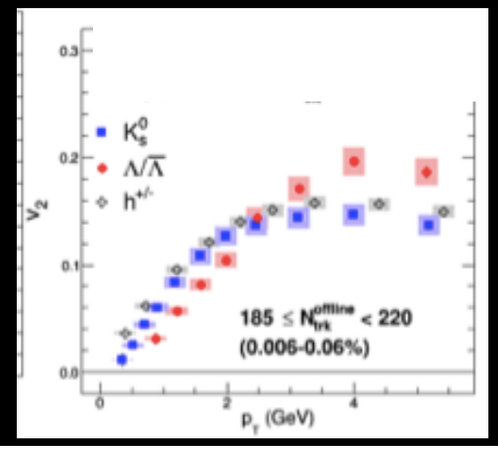


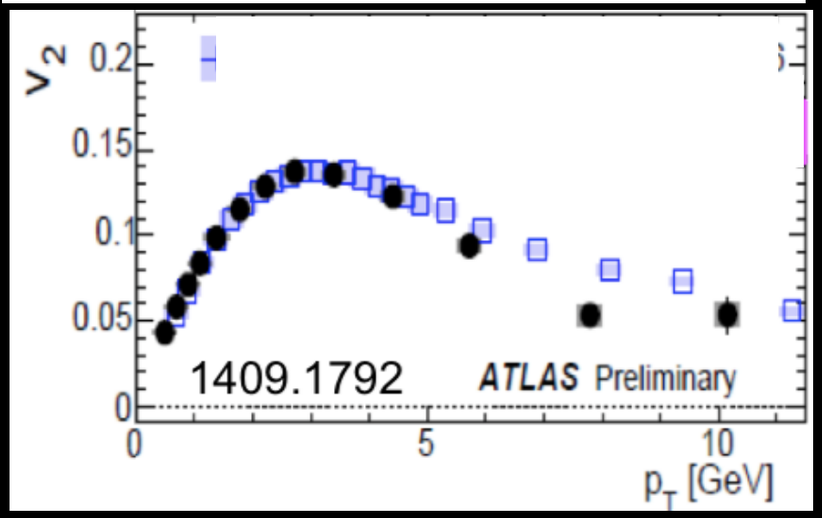


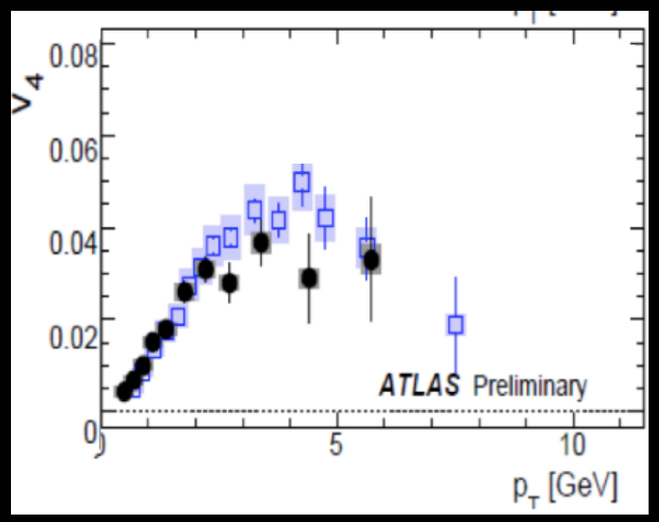


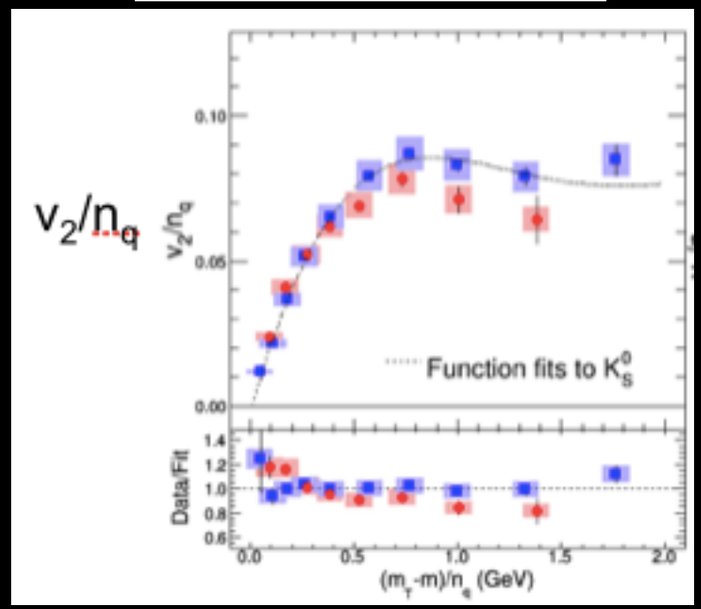












Gunther Roland

ULtra-RelatIvistiCH HEavy IoNZ 2016

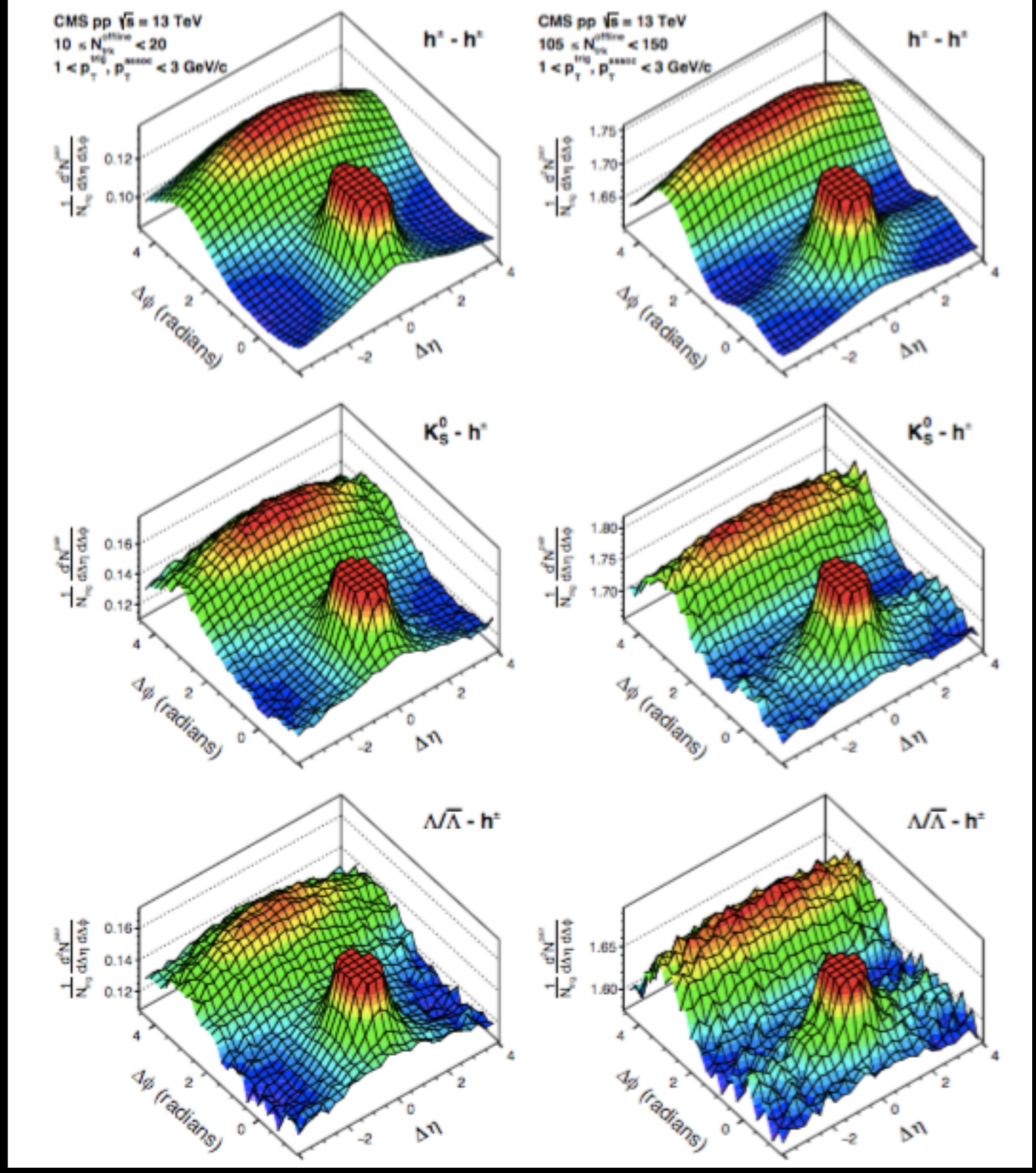
CERN July 2016

But what about pp?

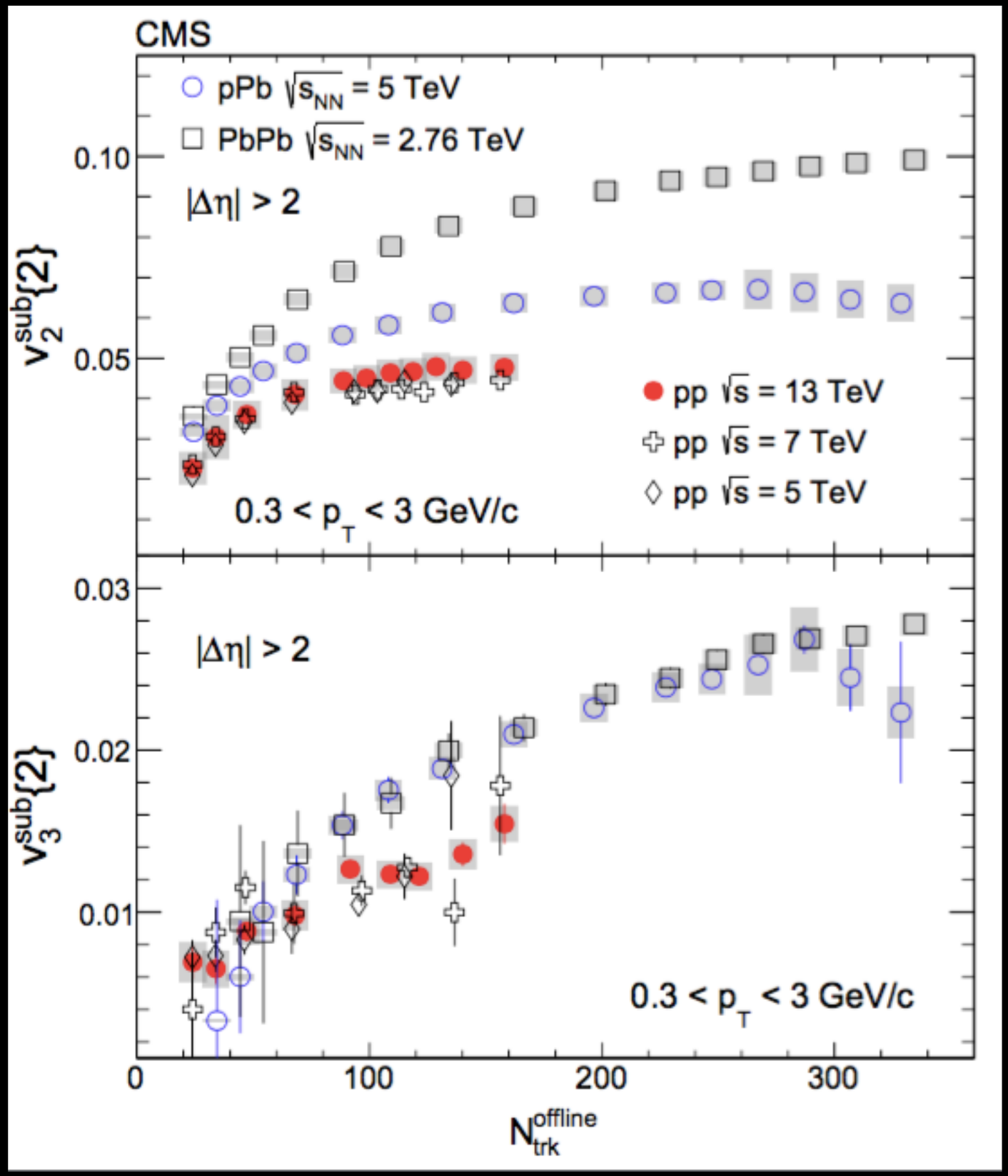
CMS, arXiv:1606.06198

$N_{trk}^{offline}$	Fraction			$\langle N_{trk}^{offline} \rangle$			$\langle N_{trk}^{corrected} \rangle$		
	5 TeV	7 TeV	13 TeV	5 TeV	7 TeV	13 TeV	5 TeV	7 TeV	13 TeV
MB	1.0	1.0	1.0	13	15	16	16±1	17±1	19±1
[0, 10)	0.48	0.44	0.43	4.8	4.8	4.8	5.8±0.3	5.5±0.2	5.9±0.3
[10, 20)	0.29	0.28	0.26	14	14	14	17±1	16±1	17±1
[20, 30)	0.14	0.15	0.15	24	24	24	28±1	28±1	30±1
[30, 40)	0.06	0.08	0.08	34	34	34	41±2	40±2	42±2
[40, 60)	0.03	0.05	0.07	47	47	47	56±2	54±2	58±2
[60, 85)	3×10^{-3}	7×10^{-3}	0.02	66	67	68	80±3	78±3	83±3
[85, 95)	9×10^{-5}	3×10^{-4}	1×10^{-3}	88	89	89	106±4	103±4	109±4
[95, 105)	2×10^{-5}	9×10^{-5}	5×10^{-4}	98	99	99	118±5	114±4	121±5
[105, 115)	5×10^{-6}	2×10^{-5}	2×10^{-4}	108	109	109	130±5	126±5	133±5
[115, 125)	1×10^{-6}	8×10^{-6}	6×10^{-5}	118	118	119	142±6	137±5	145±6
[125, 135)	2×10^{-7}	2×10^{-6}	2×10^{-5}	126	128	129	153±6	149±6	157±6
[135, 150)	5×10^{-8}	4×10^{-7}	8×10^{-6}	139	140	140	167±7	162±6	171±7
[150, ∞)	5×10^{-9}	8×10^{-8}	2×10^{-6}	155	156	158	186±8	181±7	193±8

2015 5TeV and 13TeV pp data provide high statistics of high multiplicity events



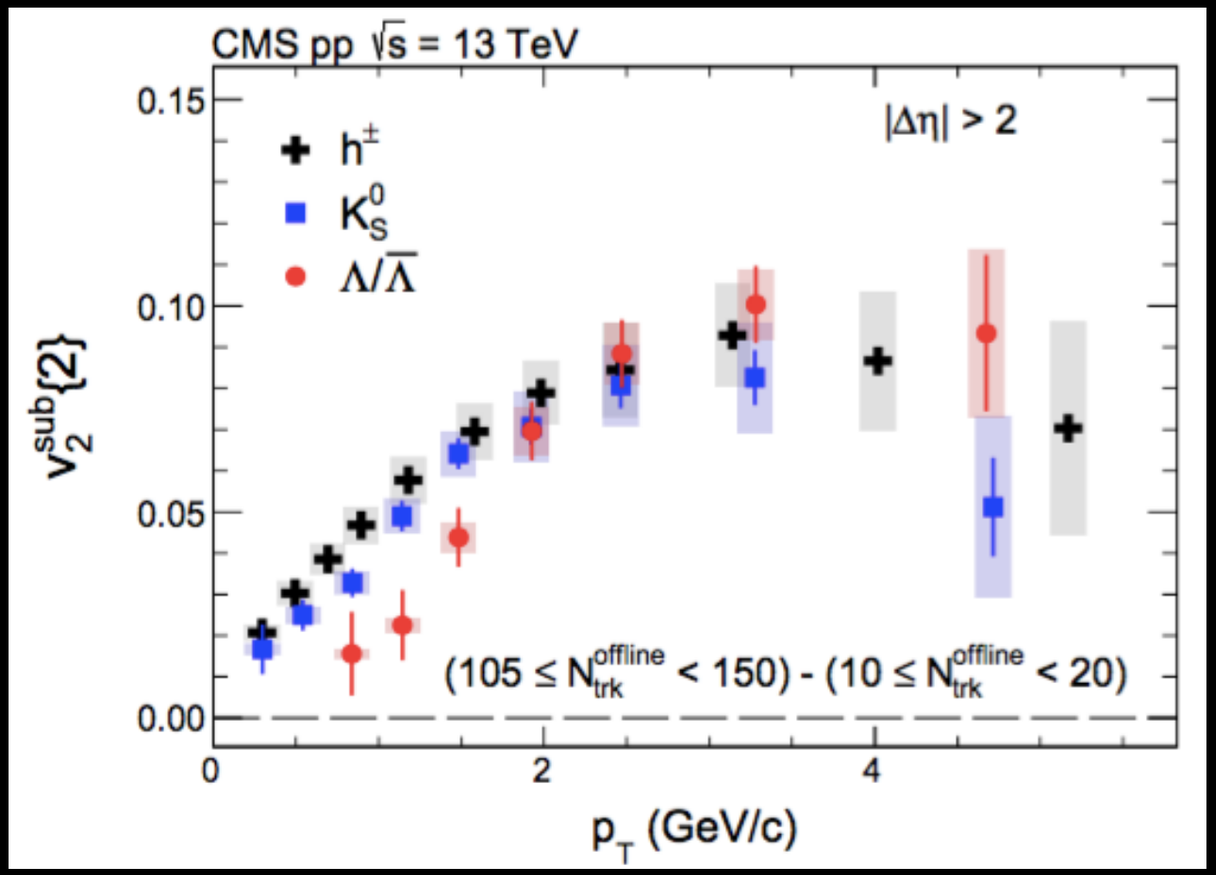
Triangular flow in pp

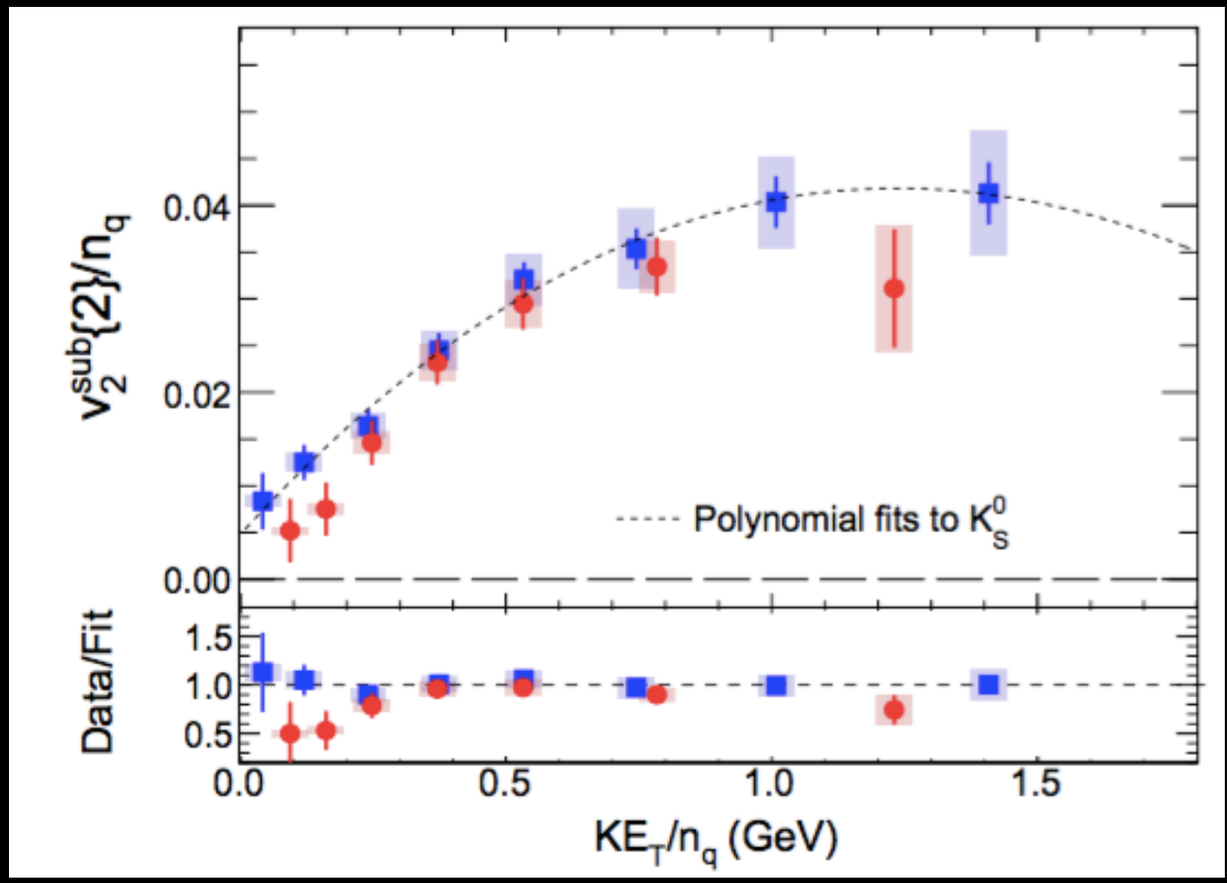


CMS, arXiv:1606.06198

Mass ordering in pp

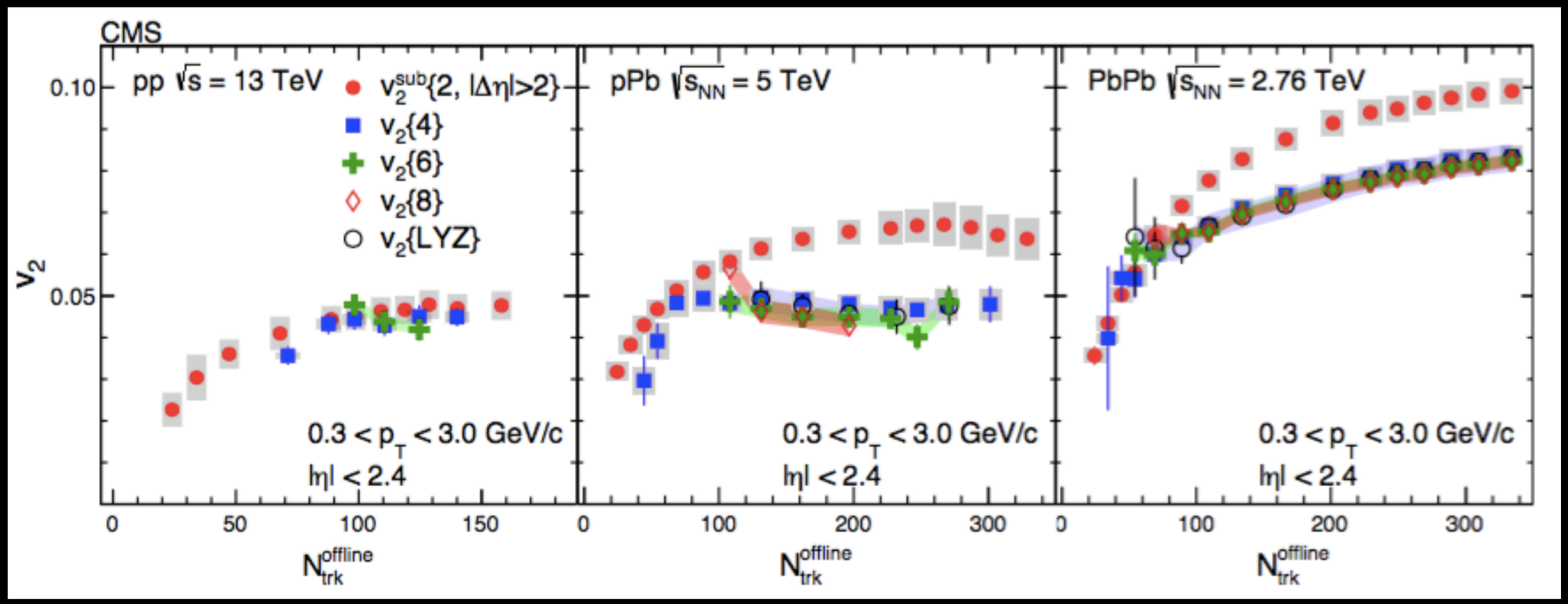
CMS, arXiv:1606.06198





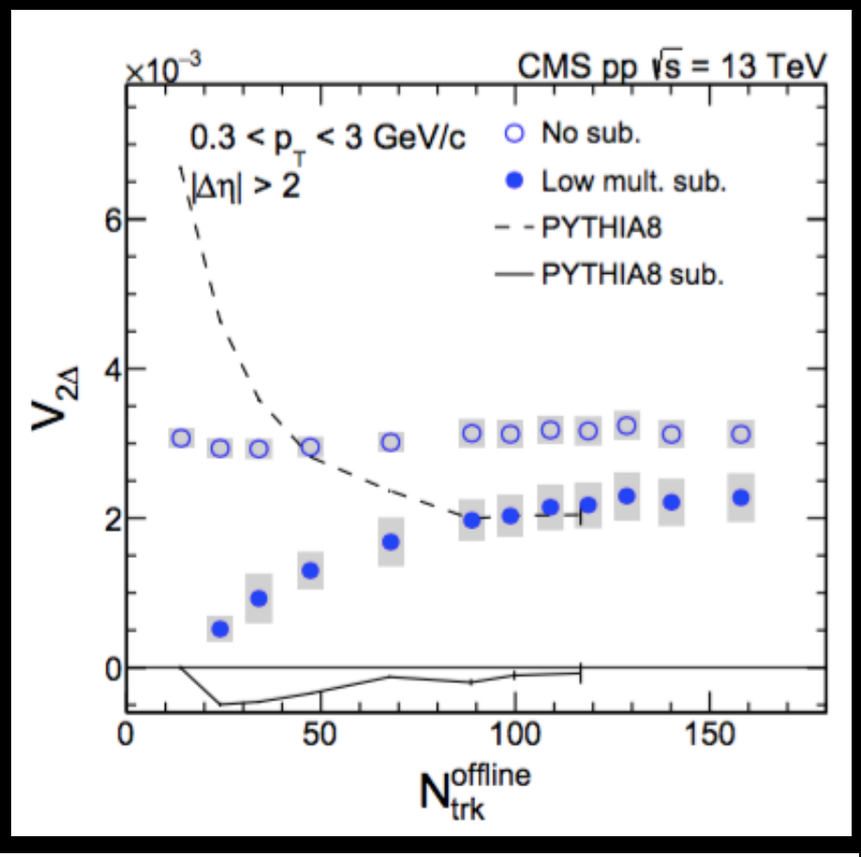
Collectivity in pp

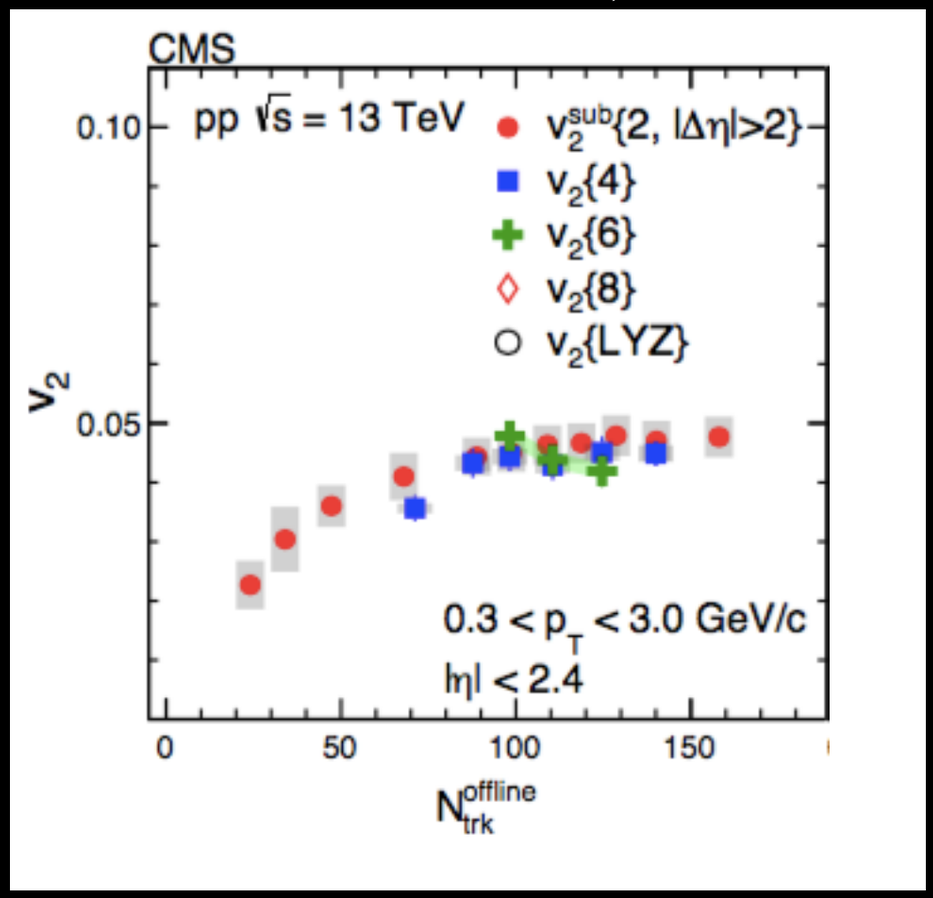
CMS, arXiv:1606.06198

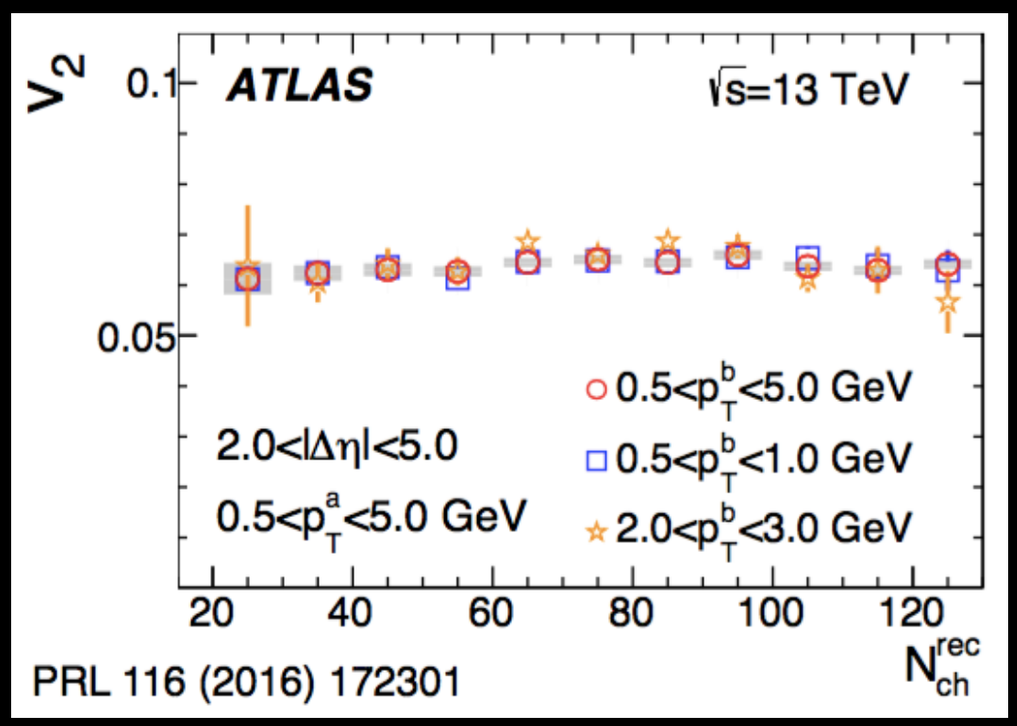


In lieu of a summary

CMS, arXiv:1606.06198







Wherever we can measure, hadronic systems of all sizes show hallmarks of hydrodynamic expansion…

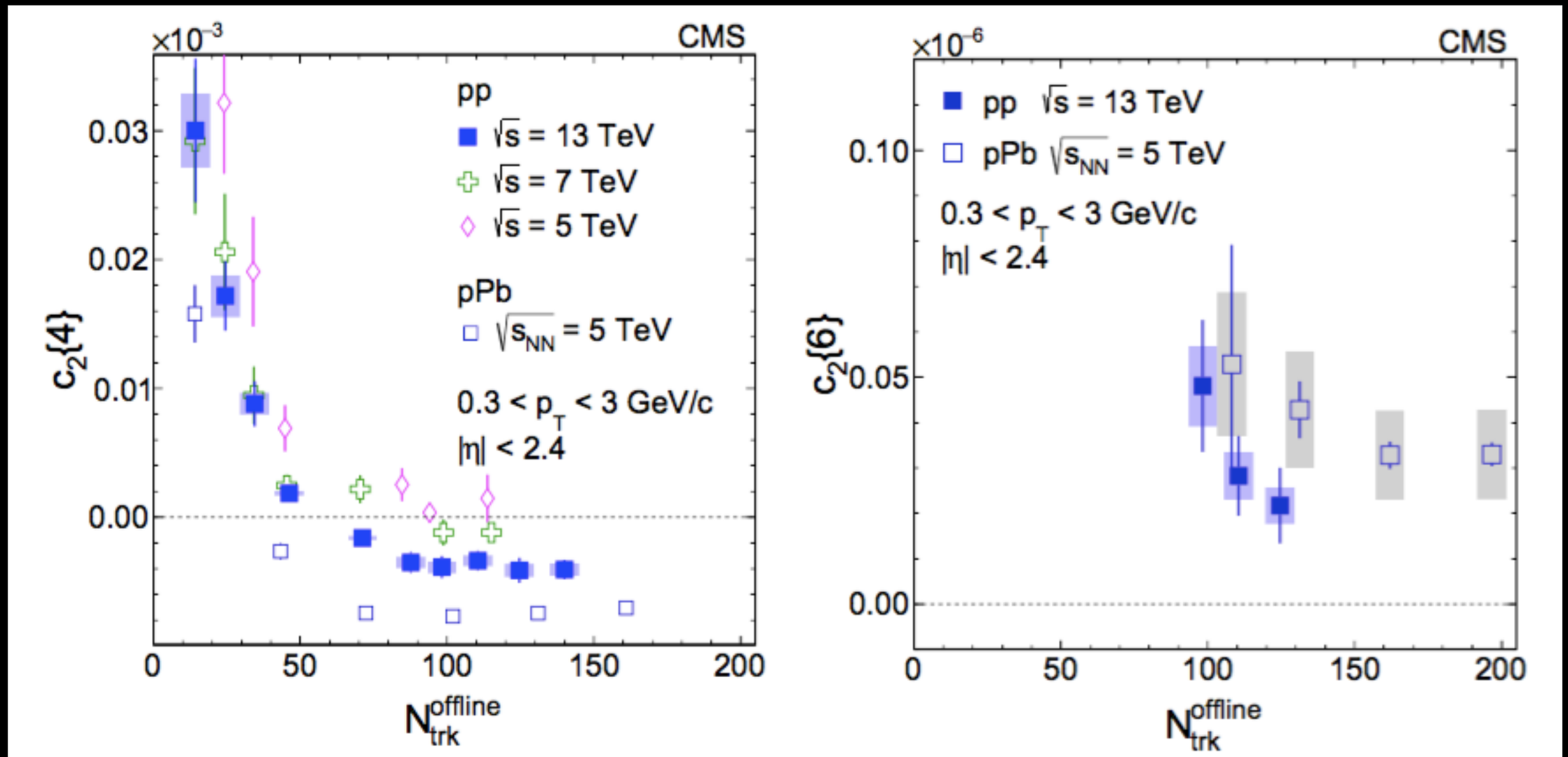
…may not even turn off in min bias pp

Happy Birthday…

Happy Birthday…

…but beware of experimentalists bearing gifts

Cumulants


×10⁻³
CMS
pp
√s = 13 TeV
√s = 7 TeV
√s = 5 TeV
pPb
√s_NN = 5 TeV
0.3 < p_T < 3 GeV/c
|η| < 2.4
c₂{4}
0.03
0.02
0.01
0.00
0
50
100
150
200
N_trk^offline
×10⁻⁶
CMS
pp √s = 13 TeV
pPb √s_NN = 5 TeV
0.3 < p_T < 3 GeV/c
|η| < 2.4
c₂{6}
0.10
0.05
0.00
0
50
100
150
200
N_trk^offline