Heavy Ion Collisions Beyond the "Dilute Projectile" Limit

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based on work in collaboration with G. Chirilli and D. Wertepny, arXiv:1501.03106 **[hep-ph]**

Outline

- Introduction: classical gluon production problem in AA collisions and why it is important.
- Background: gluon production in pA has been known for the last ~20 years.
- Heavy-Light Ion collisions: striving to get to AA from pA.
- Results (scary).
- Outlook.

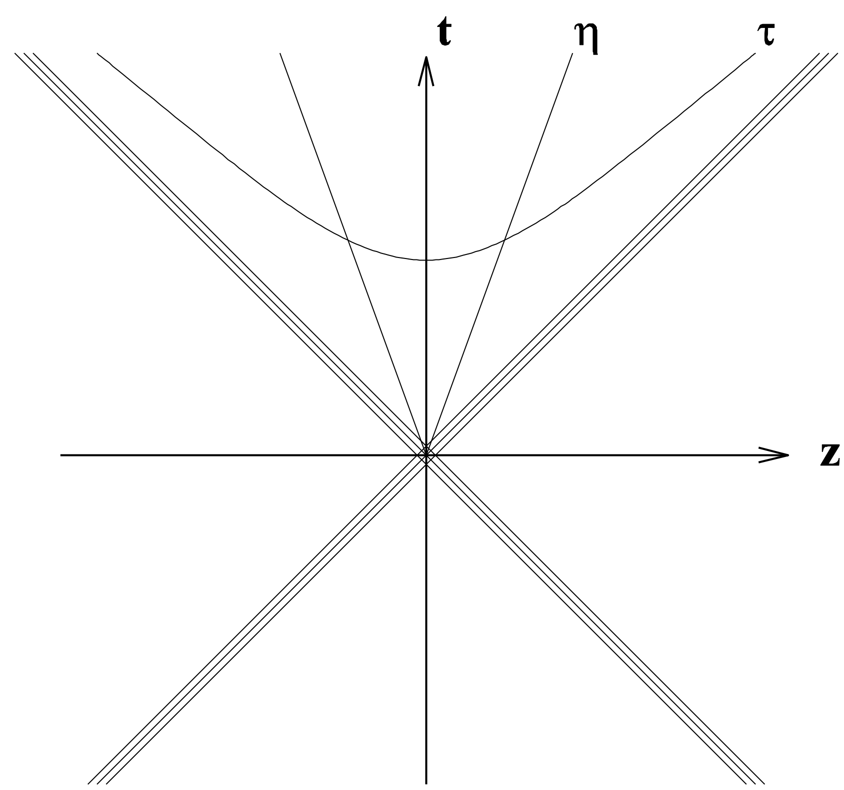
Classical Gluon Production in AA Collisions

Classical Gluon Production

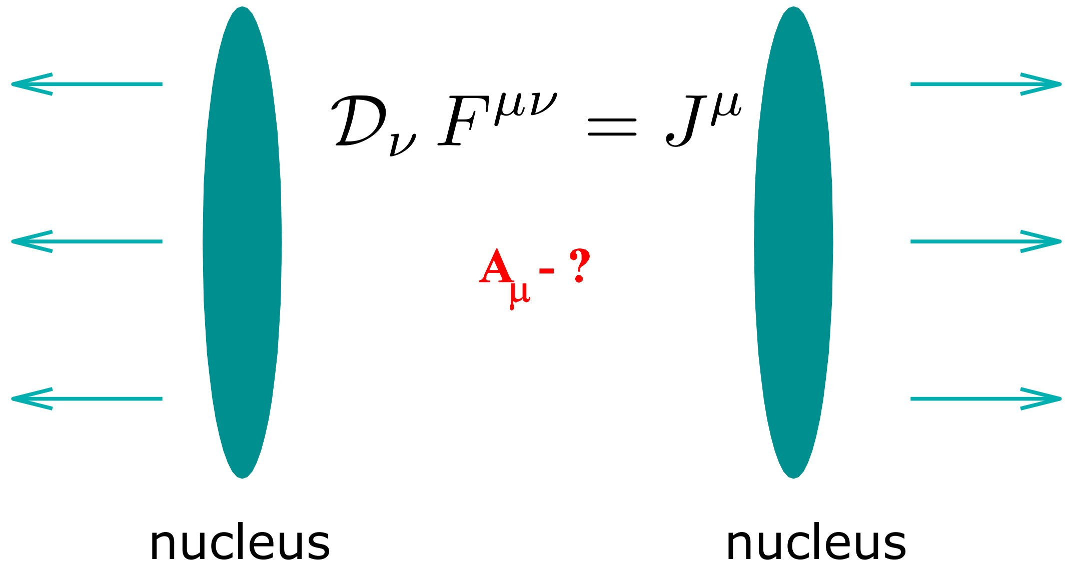
- To understand heavy ion collisions in saturation/CGC network one has to first find the classical gluon production cross section (McLerran-Venugopalan model).

- Then one should include quantum correction, etc.

$$\mathcal{D}_\nu F^{\mu\nu} = J^\mu$$

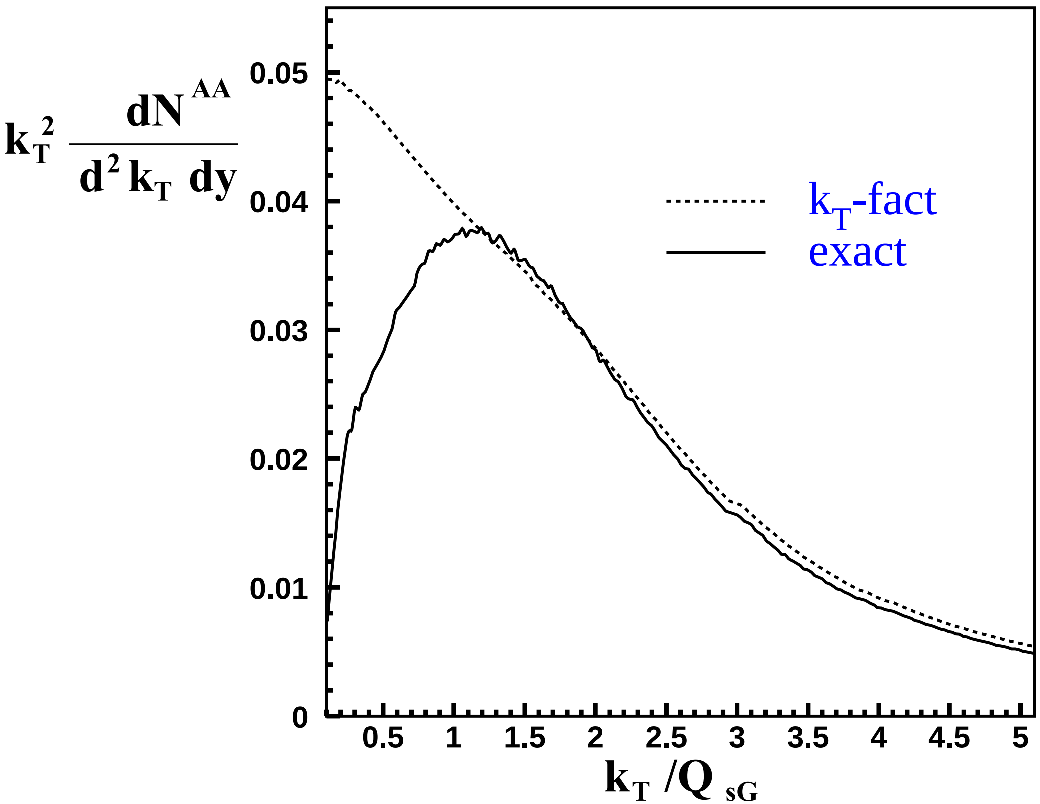


Heavy Ion Collisions in CGC: Classical Gluon Field



- To construct initial conditions for quark-gluon plasma formation in McLerran-Venugopalan model one has to find the classical gluon field left behind by the colliding nuclei.
- No analytical solution exists.
- Perturbative calculations by Kovner, McLerran, Weigert '97; Rischke, Yu.K.,'97; Gyulassy, McLerran '97; Balitsky '04.
- Numerical simulations by Krasnitz, Nara and Venugopalan '00 on, Lappi '02, Schenke et al '13.

Numerical Solution



Numerical solution exists: Krasnitz & Venugopalan '00, same + Nara '03, Lappi '02, + more recent solutions.

Plot on left: a comparison of the k_T-factorization formula with the exact numerical simulation.

End of story?

From Blaizot, Mehtar-Tani, Lappi '11

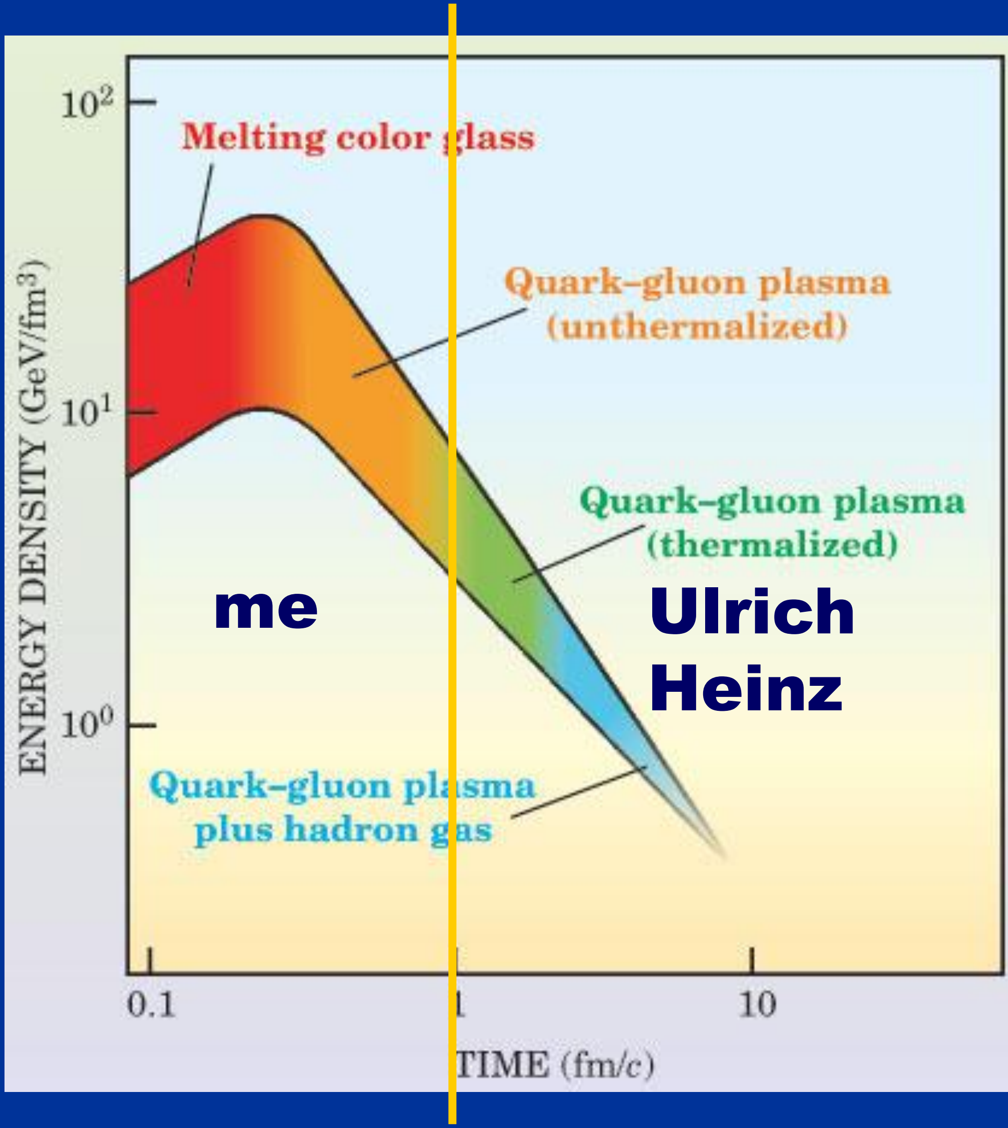
What more do we need?

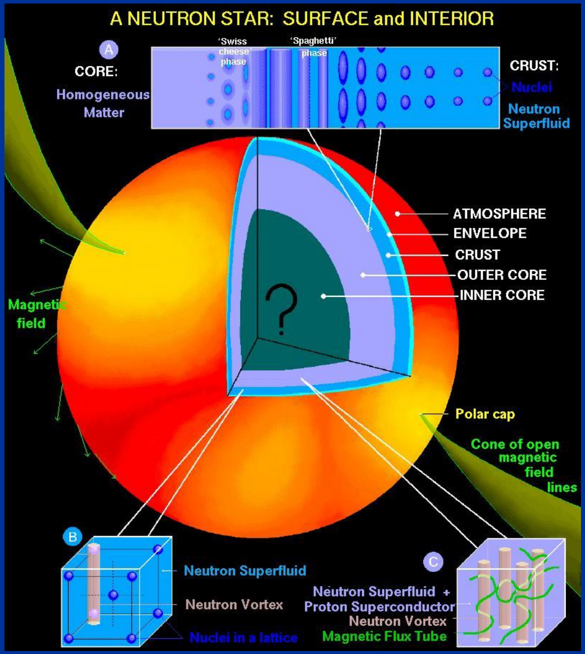
- Comparison with phenomenology often requires higher-order corrections (e.g. small-x evolution equations need rc and NLO corrections). Hard (or impossible) to include numerically.

- There are many scenarios in the literature in which some of these corrections lead to thermalization of the produced medium. No analytic calculation supporting this statement exists, not even at the lowest non-trivial order.

- Hence, to connect to hydro, and to Ulrich, need to study classical gluon production in AA.

Structure of Heavy Ion Collisions





⇦ Neutron Star Core

⇦ Nuclear Matter – the matter inside atomic nucleus

T. Ludlam and L. McLerran, Physics Today, Oct. '03

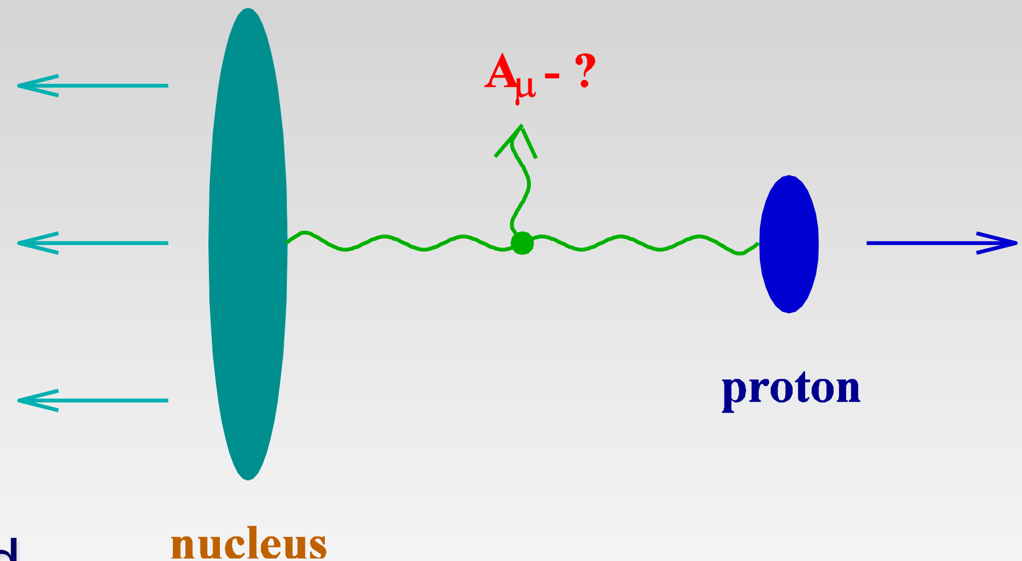
Classical Gluon Production in pA Collisions

Gluon Production in Proton-Nucleus Collisions (pA): Classical Field

To find the gluon production cross section in pA one has to solve the same classical Yang-Mills equations

$$\mathcal{D}_\nu F^{\mu\nu} = J^\mu$$

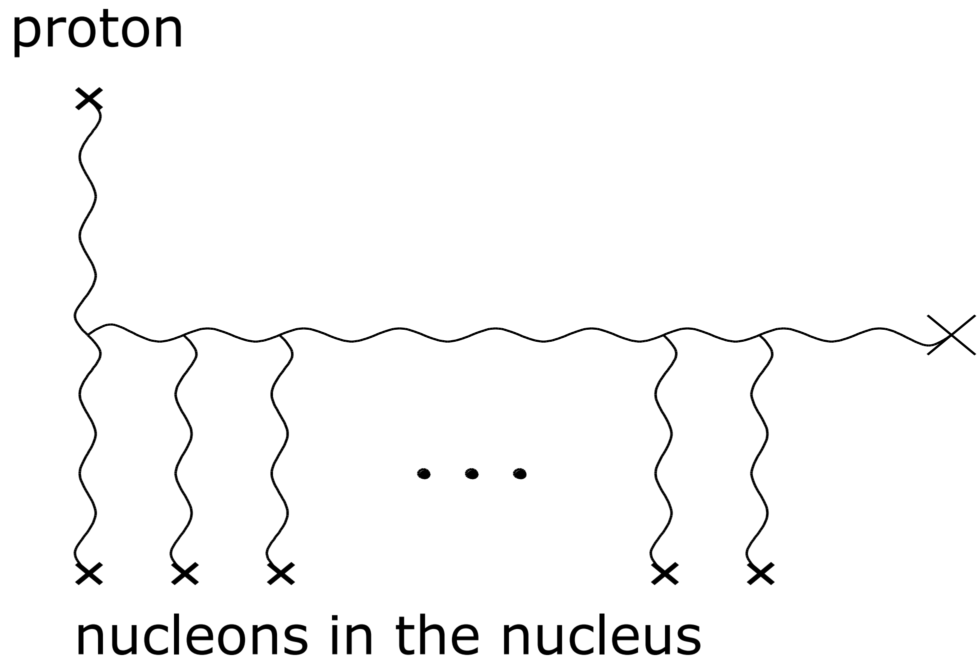
for two sources – proton and nucleus.



The gluon production corresponding to this classical field has been found by Yu. K., Mueller '98; Kopeliovich, Tarasov, Schafer '99; Dumitru and McLerran '02

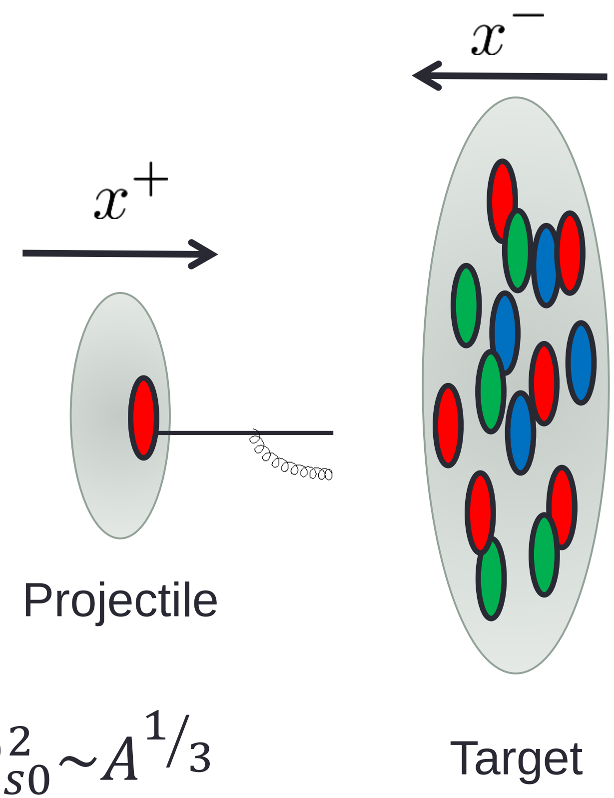
CGC in pA: the diagrams

- Again classical gluon fields correspond to tree-level (no loops) gluon production diagrams:



Proton-Nucleus Collison Case

- Due to high energy of the collision the projectile and target are Lorentz contracted.
- The interaction happens instantaneously compared to gluon emission time.
- View the projectile as emitting gluons which interact with the target instantaneously.
- Use the light-cone gauge, $A^+ = 0$.
- In the dilute projectile case:

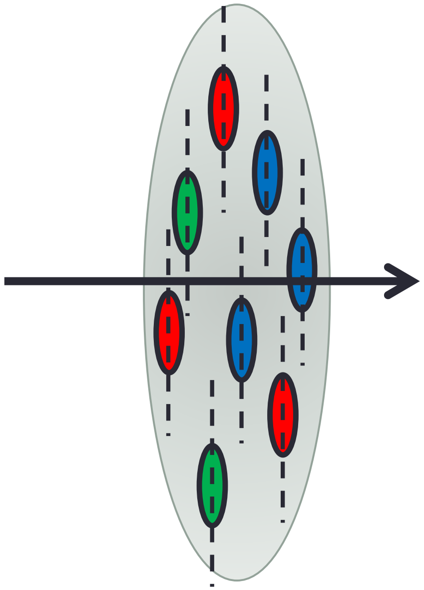


$$Q_{s0}^2 \sim A^{1/3}$$

$$A_P \ll A_T \quad \rightarrow \quad Q_{s\,P} \ll Q_{s\,T}$$

Eikonal Interaction as a Wilson Line

- A quark/gluon propagating through a nucleus at high energy can be represented as a Wilson line. Recoilless in transverse spatial coordinate it interacts with many different “color patches”.



$Q_{s0}^2 \sim A^{1/3}$

Eikonal gluon --> adjoint Wilson line

$$U_{\boldsymbol{x}} = \mathrm{P}\exp\left\{ i\,g \int\limits_{-\infty}^{\infty} dx^+\, \mathcal{A}^-(x^+, x^- = 0, \boldsymbol{x}) \right\}$$

$$\mathcal{A}^- = \sum_i T^a A_i^{a\,-}$$

Forward quark dipole amplitude

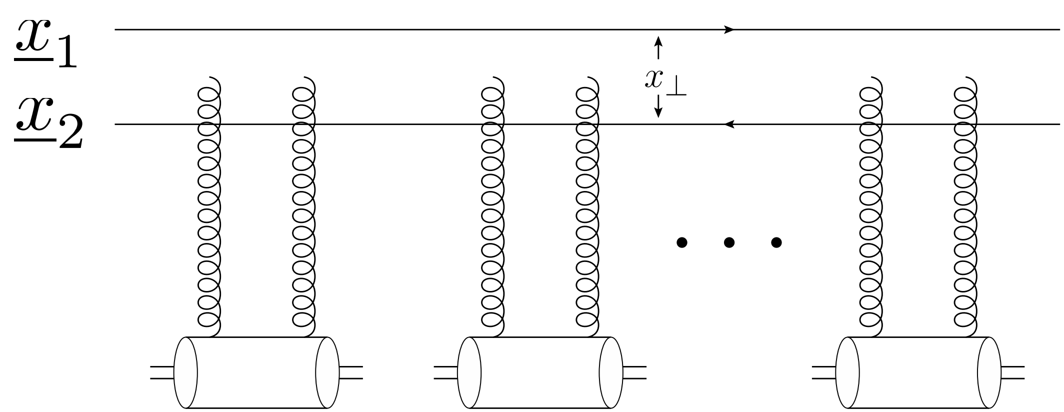
- The eikonal quark propagator is given by the fundamental Wilson line

$$V(\underline{x}) = \mathrm{P}\exp\left[i\,g\int\limits_{-\infty}^{\infty} dx^+\,A^-(x^+, x^- = 0, \underline{x})\right]$$

with the light cone coordinates $$x^\pm = \frac{t \pm z}{\sqrt{2}}$$

- The quark dipole scattering amplitude is

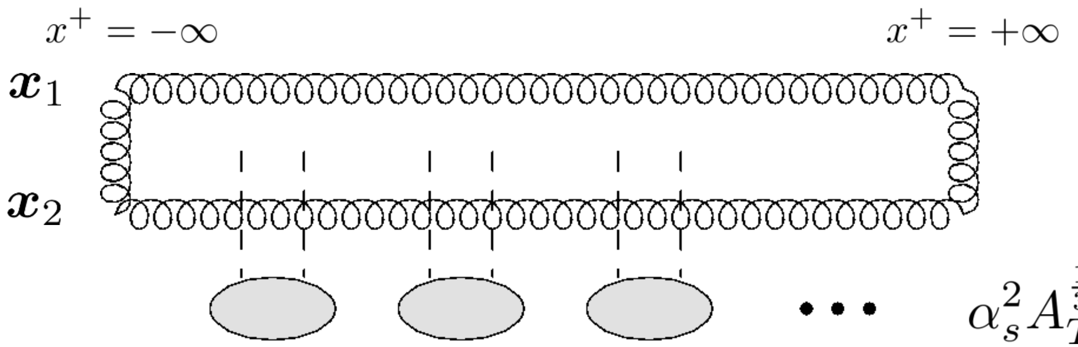
$$N(\underline{x}_1, \underline{x}_2) = 1 - \frac{1}{N_c}\left\langle \mathrm{tr}\left[V(\underline{x}_1)\,V^\dagger(\underline{x}_2)\right]\right\rangle$$



Forward gluon dipole amplitude

- The gluon dipole resums a series of tree level scatterings off many nucleons. The dotted lines represent gluons.

$$S_G(\boldsymbol{x}_1, \boldsymbol{x}_2, y) \equiv \frac{1}{N_c^2 - 1} \left\langle Tr[U_{\boldsymbol{x}_1} U^\dagger_{\boldsymbol{x}_2}] \right\rangle$$



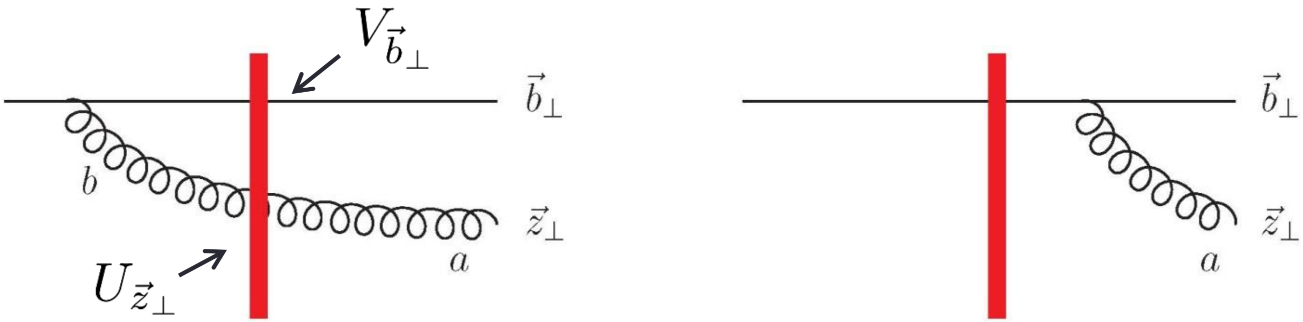
$$\alpha_s^2 A_T^{\frac{1}{3}} \sim 1$$

$$S_G(\boldsymbol{x}_1, \boldsymbol{x}_2, y = 0) = \exp\left[-\frac{1}{4} |\boldsymbol{x}_1 - \boldsymbol{x}_2|^2 Q_{s0}^2 \left(\frac{\boldsymbol{x}_1 + \boldsymbol{x}_2}{2} \right) \ln\left(\frac{1}{|\boldsymbol{x}_1 - \boldsymbol{x}_2| \Lambda} \right) \right]$$

- The forward scattering amplitude is given by.

$$N_G(\boldsymbol{x}_1, \boldsymbol{x}_2, y = 0) = 1 - S_G(\boldsymbol{x}_1, \boldsymbol{x}_2, y = 0)$$

Gluon Production Amplitude for pA Collisions



- High energy scattering between the projectile and the target is an instantaneous interaction (shockwave, red bar) at $x^+ = 0$.
- Gluon emission can happen before or after, not during.
- The projectile interacting with the target results in a power counting of

$$|M|^2 \sim \frac{1}{\alpha_s}(\alpha_s^2 A_P^{\frac{1}{3}})(\alpha_s^2 A_T^{\frac{1}{3}})^N \qquad A_P^{\frac{1}{3}} = 1 \quad \alpha_s^2 A_T^{\frac{1}{3}} \sim 1$$

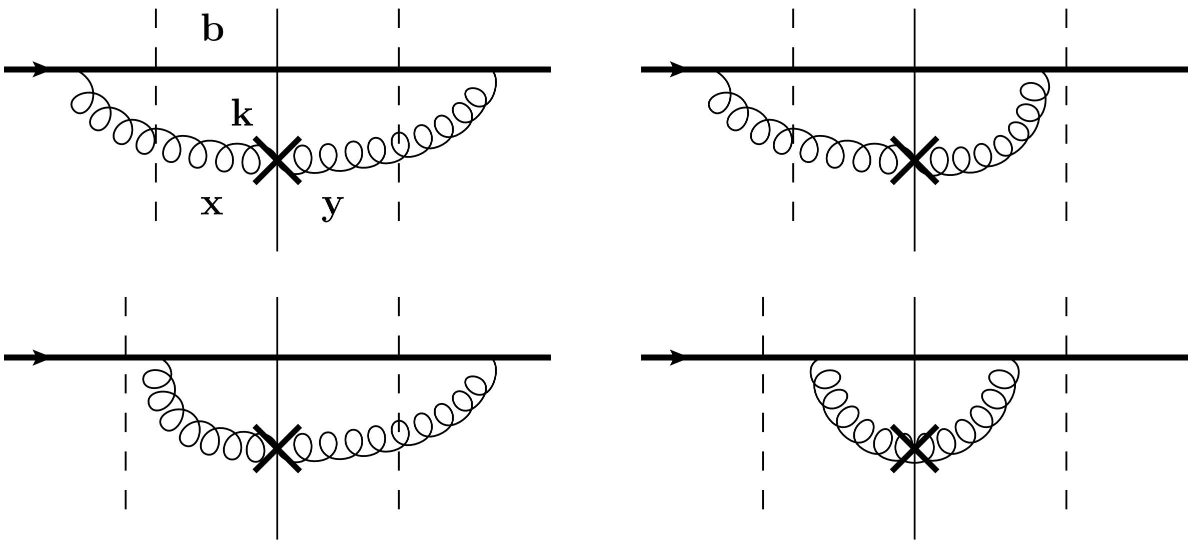
- In total the amplitude is

$$\boxed{M(\vec{z}_\perp, \vec{b}_\perp) = \frac{i\,g}{\pi} \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{b}_\perp)}{|\vec{z}_\perp - \vec{b}_\perp|^2} \left[U_{\vec{z}_\perp}^{ab} - U_{\vec{b}_\perp}^{ab}\right] \left(V_{\vec{b}_\perp} t^b\right)}$$

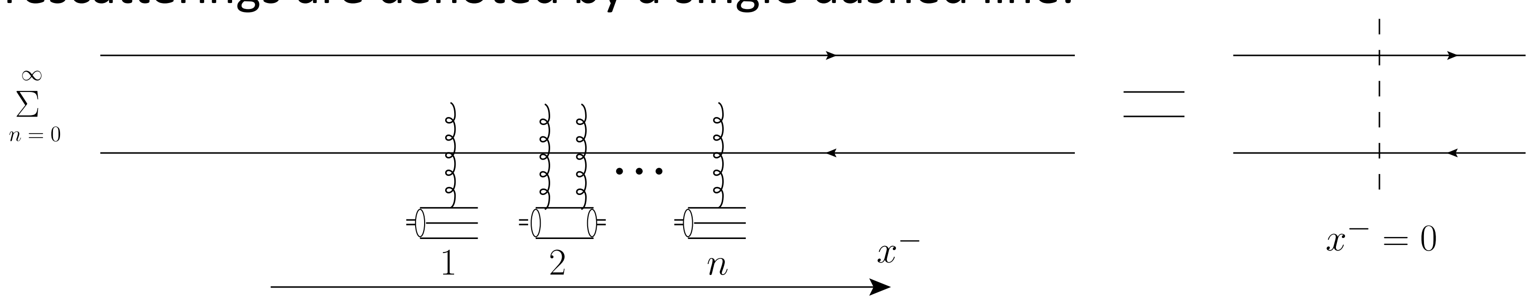
- Used the relation: $\left(t^a V_{\vec{b}_\perp}\right) = \left(V_{\vec{b}_\perp} t^b\right) U_{\vec{b}_\perp}^{ab}$

Single gluon production cross section in pA

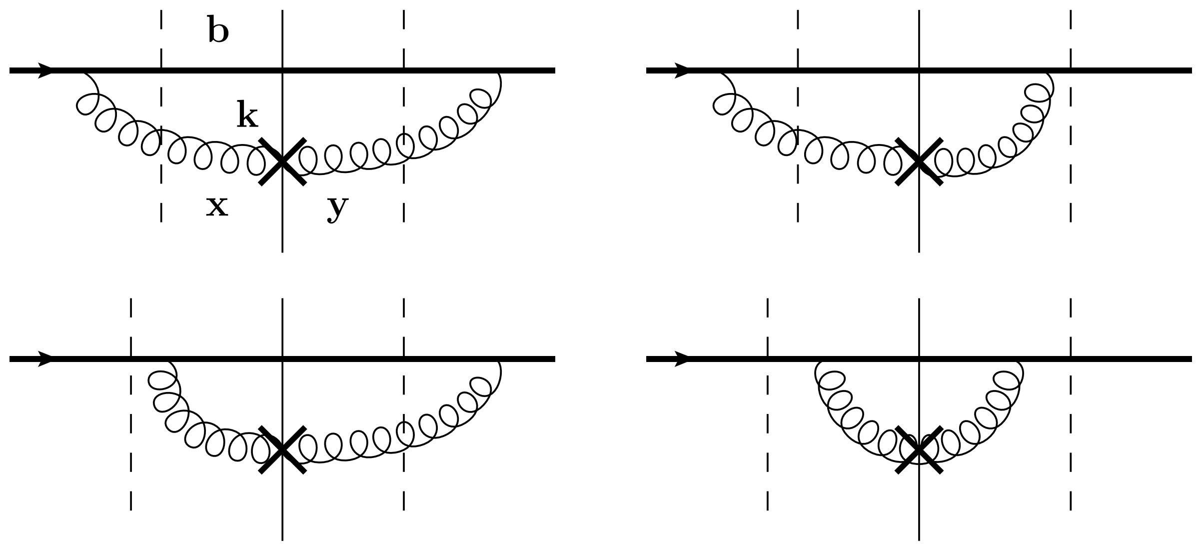
Model the proton by a single quark (can be easily improved upon).
The diagrams are shown below (Yu.K., A. Mueller ’97):



Multiple rescatterings are denoted by a single dashed line:



Single gluon production in pA



The gluon production cross section can be readily written as (U = Wilson line in **adjoint** representation, represents gluon interactions with the target)

$$\left\langle \frac{d\sigma^{pA_2}}{d^2k\,dy\,d^2b} \right\rangle = \frac{\alpha_s\,C_F}{4\,\pi^4} \int d^2x\,d^2y\,e^{-i\,\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}\,\frac{\mathbf{x}-\mathbf{b}}{|\mathbf{x}-\mathbf{b}|^2}\cdot\frac{\mathbf{y}-\mathbf{b}}{|\mathbf{y}-\mathbf{b}|^2}$$

$$\times\,\left\langle \frac{1}{N_c^2-1}\,Tr[U_{\mathbf{x}}U_{\mathbf{y}}^\dagger] - \frac{1}{N_c^2-1}\,Tr[U_{\mathbf{x}}U_{\mathbf{b}}^\dagger] - \frac{1}{N_c^2-1}\,Tr[U_{\mathbf{b}}U_{\mathbf{y}}^\dagger] + 1 \right\rangle$$

Power Counting

- The resulting cross section is

$$\frac{d\sigma}{d^2k_T\,dy} = \frac{\alpha_s\,C_F}{\underbrace{4\,\pi^4}} \int d^2z\,d^2z'\,d^2b\,e^{-i\vec{k}_\perp\cdot(\vec{z}_\perp-\vec{z}\,'_\perp)}\,\frac{\vec{z}_\perp-\vec{b}_\perp}{|\vec{z}_\perp-\vec{b}_\perp|^2}\cdot\frac{\vec{z}\,'_\perp-\vec{b}_\perp}{|\vec{z}\,'_\perp-\vec{b}_\perp|^2}$$
$$\times\,\underbrace{\left[S_G(\vec{z}_\perp,\vec{z}\,'_\perp)-S_G(\vec{b}_\perp,\vec{z}\,'_\perp)-S_G(\vec{z}_\perp,\vec{b}_\perp)+1\right]}$$

- Overall power counting:
  - Classical field (squared): $\dfrac{1}{\alpha_s}$
  - Single nucleon in the projectile: $\alpha_s^2 A_P^{\frac{1}{3}}$
  - Interactions in the target: $(\alpha_s^2 A_T^{\frac{1}{3}})^N \sim 1$
- In total: $\dfrac{1}{\alpha_s}(\alpha_s^2 A_P^{\frac{1}{3}}) = \alpha_s$ as for a proton $A_P = 1$

Classical Gluon Production in Heavy-Light Ion Collisions

Heavy-Light Collision Case

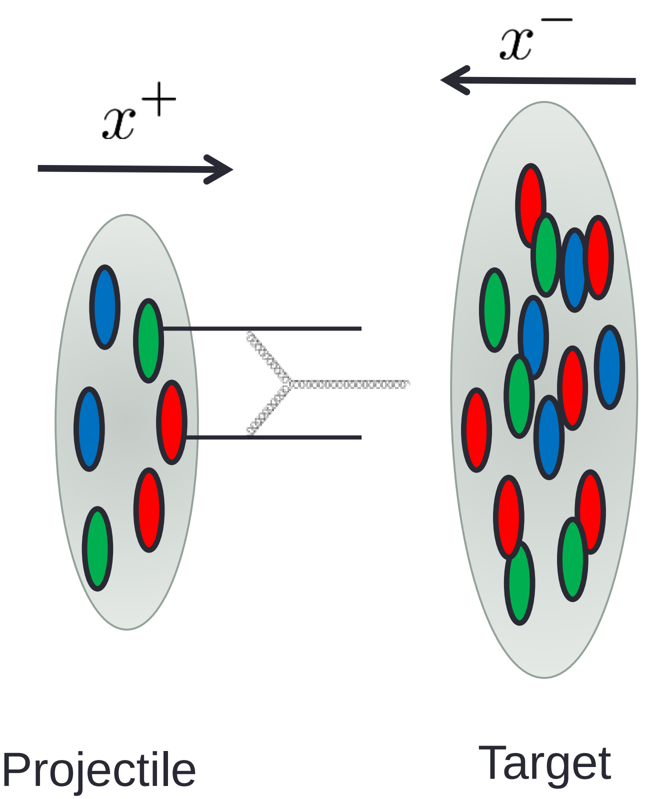
- Target nucleus has same power counting as before.
- Projectile has many nucleons, but not too many such that

$$A_P \gg 1$$

$$\alpha_s \ll \alpha_s^2 A_P^{\frac{1}{3}} \lesssim 1 \qquad \alpha_s^2 A_T^{\frac{1}{3}} \sim 1$$

- Two nucleons from projectile.
- Power counting for the cross section:

$$\sim \frac{1}{\alpha_s}(\alpha_s^2 A_P^{\frac{1}{3}})^2(\alpha_s^2 A_T^{\frac{1}{3}})^N \sim \alpha_s^3 A_P^{1/3}$$



Types of Diagrams

- Diagrams have two quarks from the projectile and are order g^6.
- Huge number of diagrams.
- Diagrams can be separated into three classes:

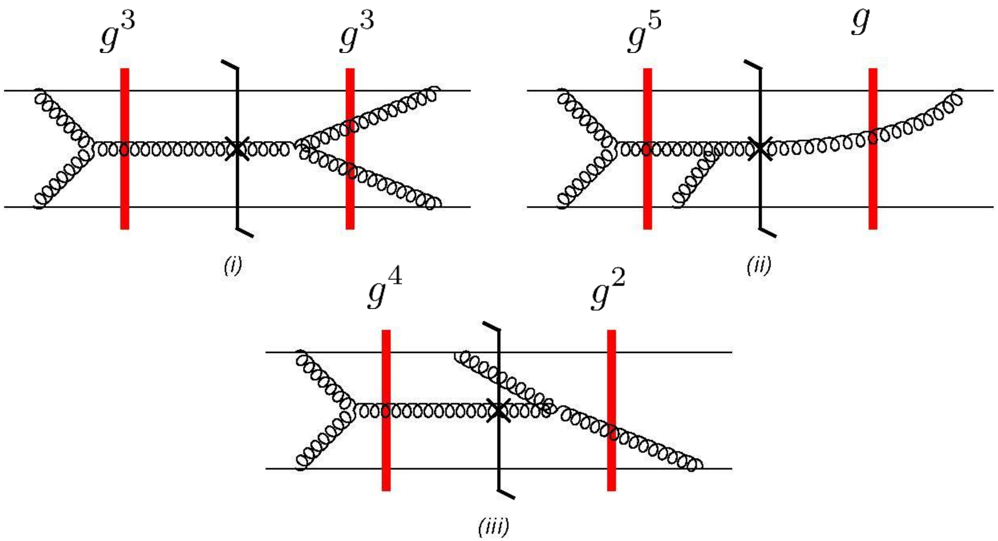
i) Square of order-g^3 amplitudes

ii) Interference between order-g^5 and order-g amplitudes

iii) Interference between order-g^4 and order-g^2 amplitudes

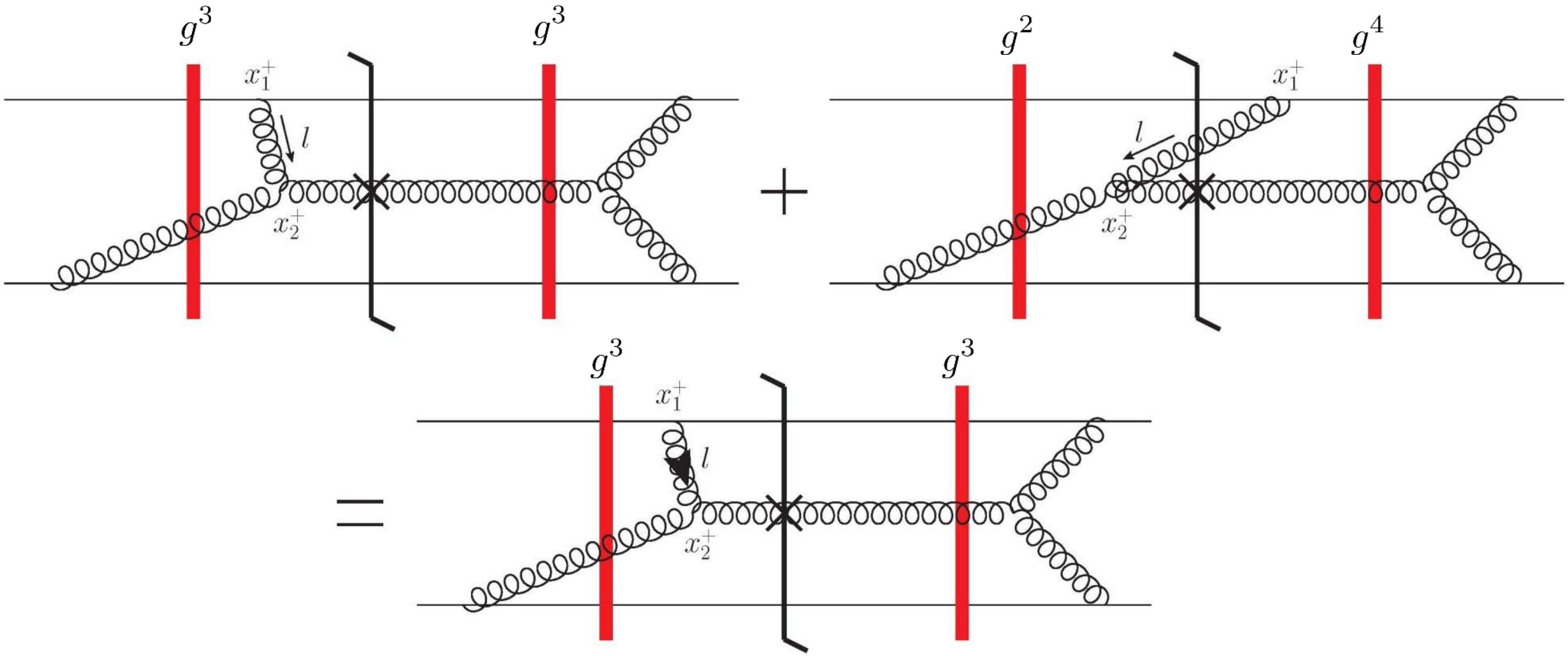
- These can be combined together in various ways to reduce the number of diagrams.
- Light-cone gauge,

$$\eta_\mu \, A^\mu = A^+ = 0$$



$$\frac{-i\,D_{\mu\nu}(l)}{l^2 + i\,\epsilon} \qquad \text{where} \qquad D_{\mu\nu}(l) = g_{\mu\nu} - \frac{1}{\eta \cdot l}(\eta_\mu\, l_\nu + \eta_\nu\, l_\mu)$$

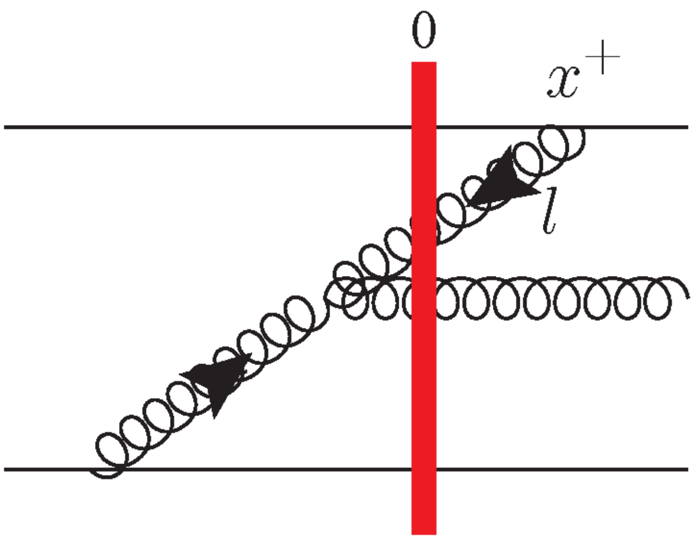
Retarded Green Function

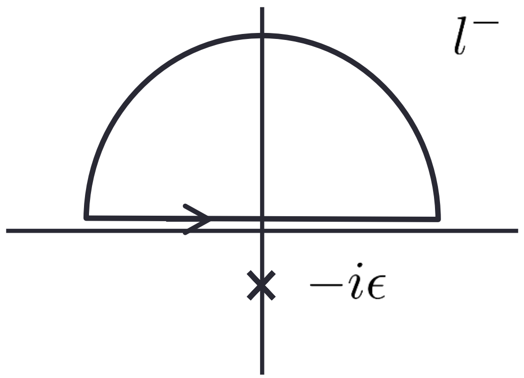


- Adding the top two diagrams turns the propagator into a retarded propagator, represented by the arrow.

$$\frac{-i\,D_{\mu\nu}(l)}{l^2 + i\,\epsilon} + 2\pi\,\theta(-l^+)\,\delta(l^2)\,D_{\mu\nu}(l) = \frac{-i\,D_{\mu\nu}(l)}{l^2 + i\,\epsilon\,l^+}$$

Backwards Propagators



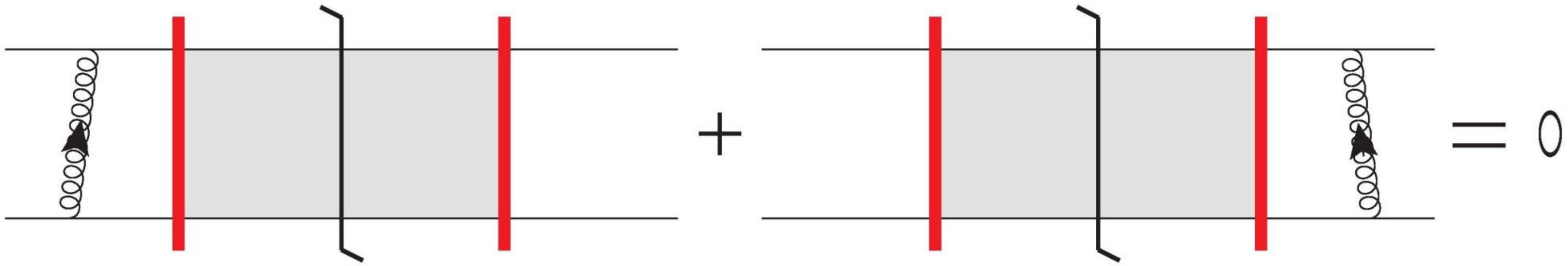


- l^- momentum component must flow forward in x^+ "time":

$$\propto \int dl^- \frac{\Theta(x^+)}{2l^+l^- - l_\perp^2 + i\,\epsilon\, l^+} e^{il^- x^+} = 0$$

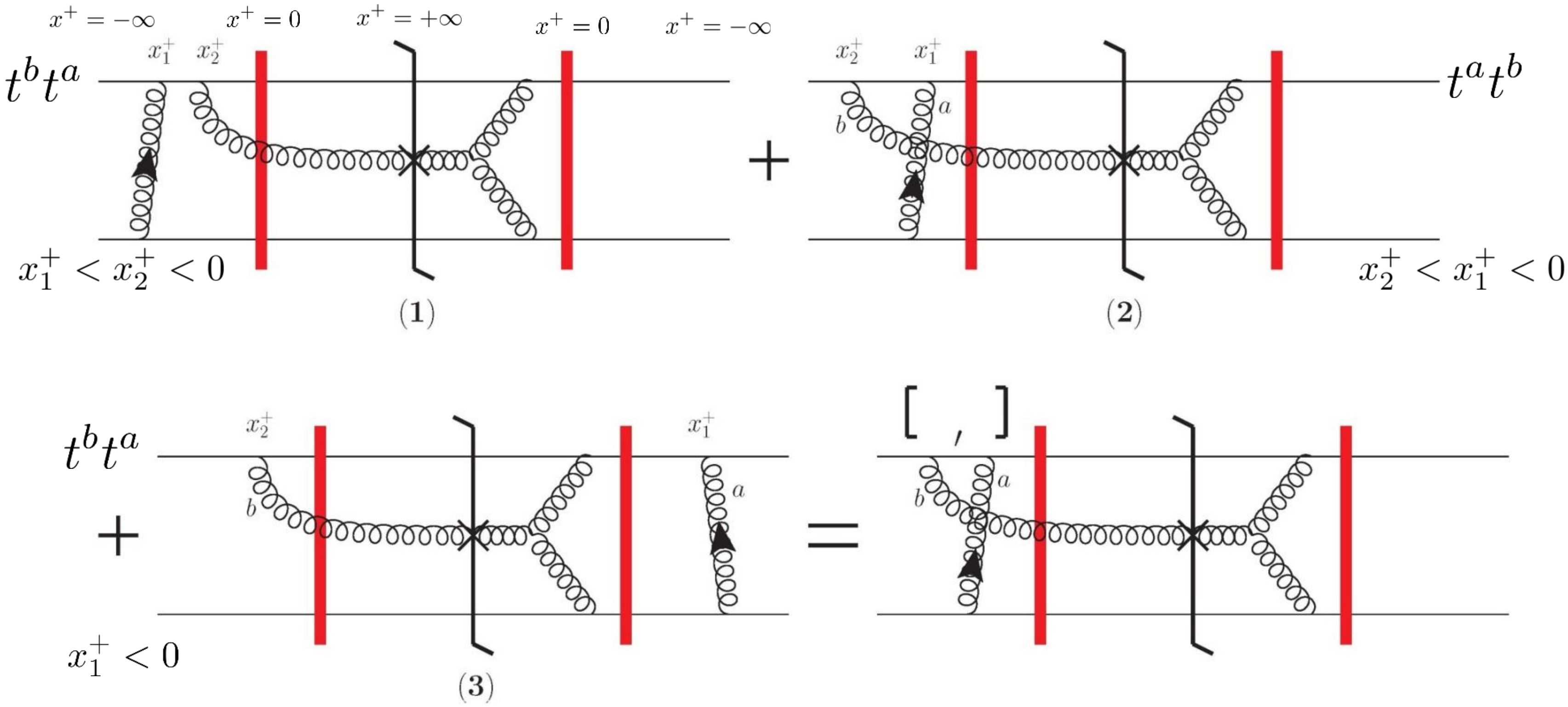
- Flowing backwards into the shock wave is not allowed.

Cancellations



- Shaded region represents any late-time interaction.
- Moving the retarded gluon propagator across the cut gives rise to a minus sign.
- The sum of the diagrams is zero.

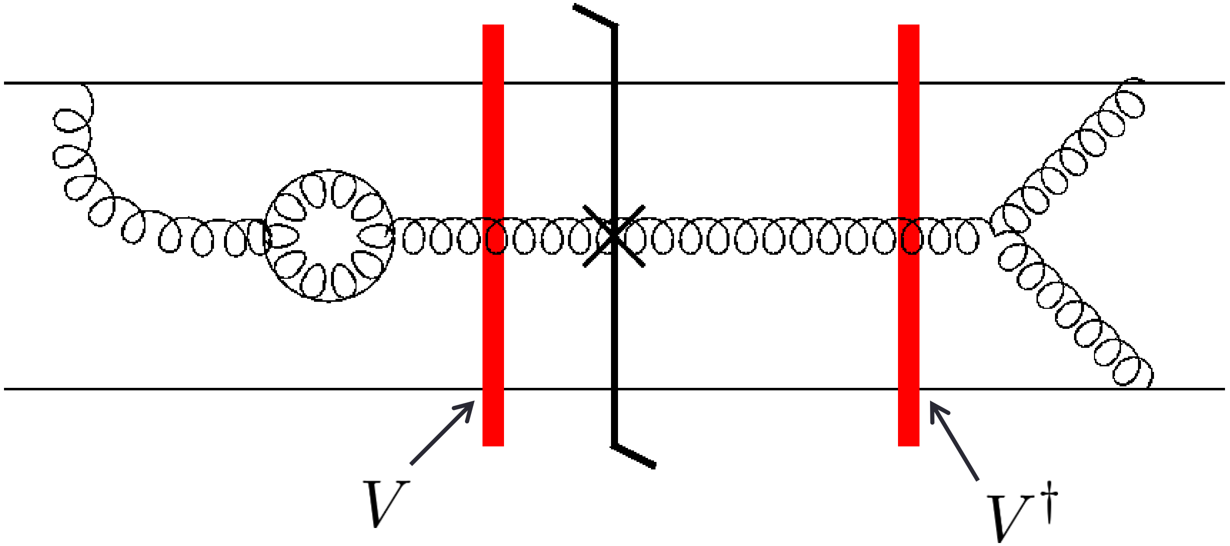
Commutators



- Using the cancellation shown previously diagrams (1), (2), and (3) can be combined into a single diagram, diagram (2), with the color factor on the quark line replaced by a commutator, denoted by the square brackets:

$$t^a t^b \rightarrow [t^a, t^b]$$

No Quantum Contributions

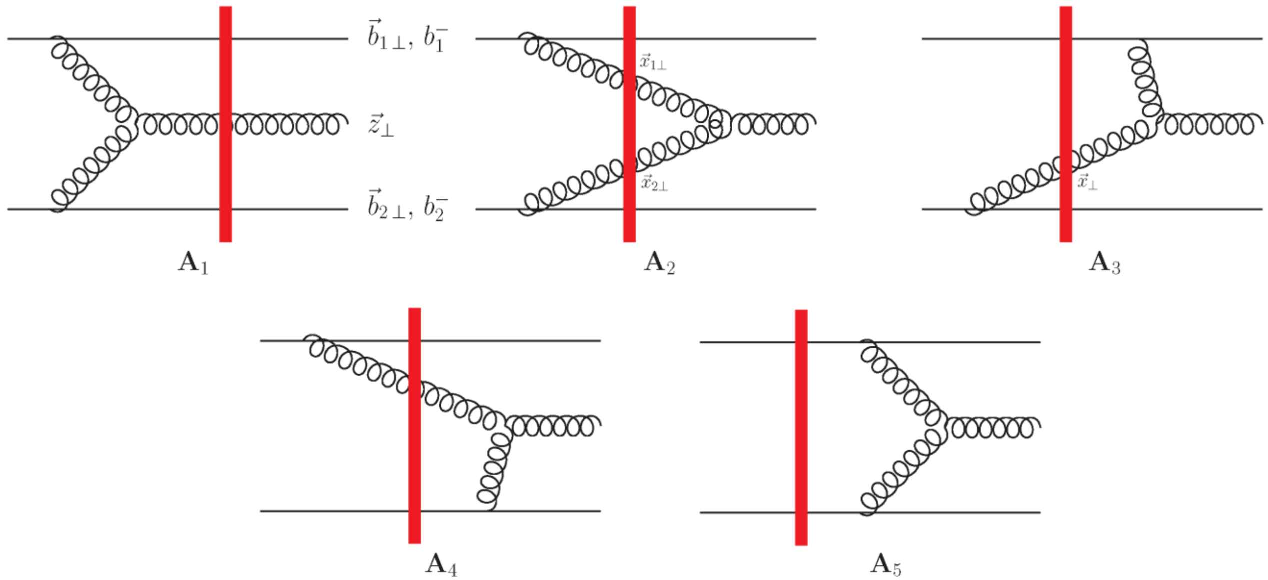


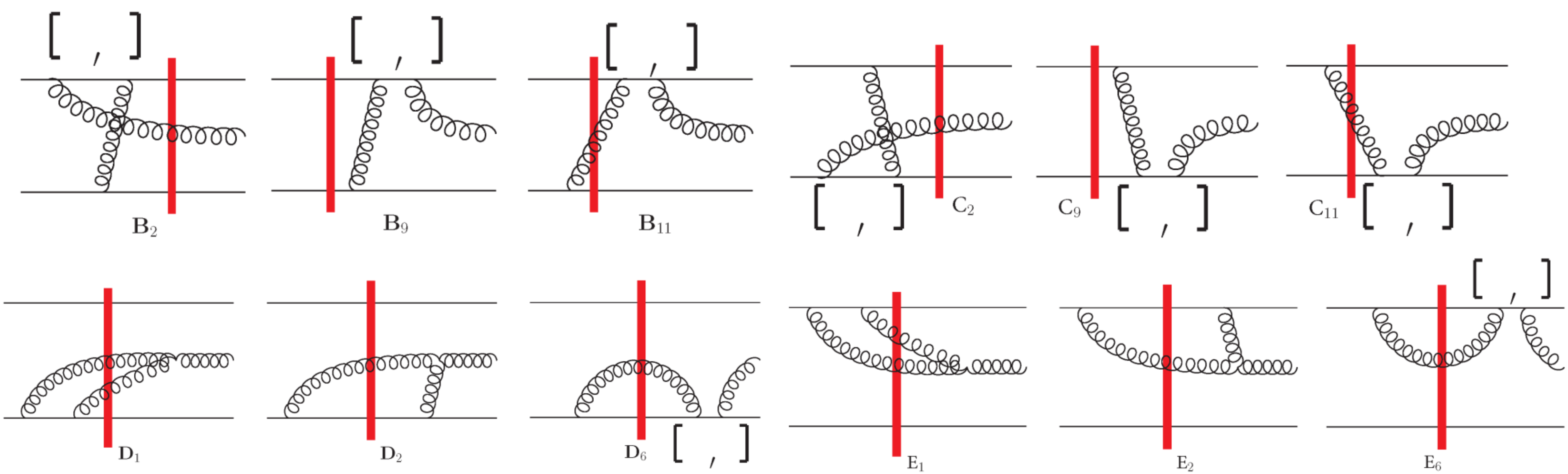
- Quantum corrections go away at this order.
- Left with classical fields.
- Zero due to color averaging of quark two.

$$tr[t^a V^\dagger V] = tr[t^a] = 0$$

Final Diagrams

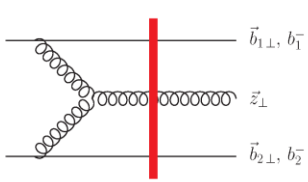
Using these tricks the number of diagrams are reduced to a manageable amount. Here’s a subset of the remaining graphs:



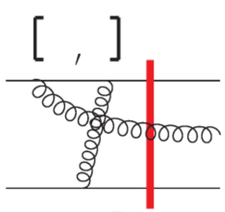


Results: Amplitude – A, B, and C graphs

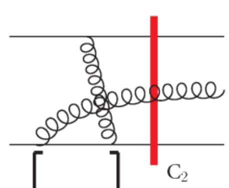
$$
\begin{aligned}
&\sum_i A_i + \sum_i B_i + \sum_i C_i \\
&= -\frac{g^3}{4\,\pi^4}\int d^2x_1\, d^2x_2\, \delta[(\vec z_\perp - \vec x_{1\perp})\times(\vec z_\perp - \vec x_{2\perp})] \Bigg[\frac{\vec\epsilon_\perp^{\,\lambda *}\cdot(\vec x_{2\perp}-\vec x_{1\perp})}{|\vec x_{2\perp}-\vec x_{1\perp}|^2}\, \frac{\vec x_{1\perp}-\vec b_{1\perp}}{|\vec x_{1\perp}-\vec b_{1\perp}|^2}\cdot\frac{\vec x_{2\perp}-\vec b_{2\perp}}{|\vec x_{2\perp}-\vec b_{2\perp}|^2} - \frac{\vec\epsilon_\perp^{\,\lambda *}\cdot(\vec x_{1\perp}-\vec b_{1\perp})}{|\vec x_{1\perp}-\vec b_{1\perp}|^2}\, \frac{\vec z_\perp-\vec x_{1\perp}}{|\vec z_\perp-\vec x_{1\perp}|^2}\cdot\frac{\vec x_{2\perp}-\vec b_{2\perp}}{|\vec x_{2\perp}-\vec b_{2\perp}|^2} \\
&\quad - \frac{\vec\epsilon_\perp^{\,\lambda *}\cdot(\vec x_{2\perp}-\vec b_{2\perp})}{|\vec x_{2\perp}-\vec b_{2\perp}|^2}\, \frac{\vec x_{1\perp}-\vec b_{1\perp}}{|\vec x_{1\perp}-\vec b_{1\perp}|^2}\cdot\frac{\vec z_\perp-\vec x_{2\perp}}{|\vec z_\perp-\vec x_{2\perp}|^2}\Bigg]\, f^{abc}\left[U^{bd}_{\vec x_{1\perp}} - U^{bd}_{\vec b_{1\perp}}\right]\left[U^{ce}_{\vec x_{2\perp}} - U^{ce}_{\vec b_{2\perp}}\right]\left(V_{\vec b_{1\perp}}\, t^d\right)_1 \left(V_{\vec b_{2\perp}}\, t^e\right)_2 \\
&\quad + \frac{i\,g^3}{4\,\pi^3}\, f^{abc}\left(V_{\vec b_{1\perp}}\, t^d\right)_1 \left(V_{\vec b_{2\perp}}\, t^e\right)_2 \int d^2x \Bigg[U^{bd}_{\vec b_{1\perp}}\left(U^{ce}_{\vec x_\perp} - U^{ce}_{\vec b_{2\perp}}\right)\Bigg(\frac{\vec\epsilon_\perp^{\,\lambda *}\cdot(\vec z_\perp-\vec x_\perp)}{|\vec z_\perp-\vec x_\perp|^2}\, \frac{\vec x_\perp-\vec b_{1\perp}}{|\vec x_\perp-\vec b_{1\perp}|^2}\cdot\frac{\vec x_\perp-\vec b_{2\perp}}{|\vec x_\perp-\vec b_{2\perp}|^2} \\
&\quad - \frac{\vec\epsilon_\perp^{\,\lambda *}\cdot(\vec z_\perp-\vec b_{1\perp})}{|\vec z_\perp-\vec b_{1\perp}|^2}\, \frac{\vec z_\perp-\vec x_\perp}{|\vec z_\perp-\vec x_\perp|^2}\cdot\frac{\vec x_\perp-\vec b_{2\perp}}{|\vec x_\perp-\vec b_{2\perp}|^2} - \frac{\vec\epsilon_\perp^{\,\lambda *}\cdot(\vec z_\perp-\vec b_{1\perp})}{|\vec z_\perp-\vec b_{1\perp}|^2}\, \frac{\vec x_\perp-\vec b_{1\perp}}{|\vec x_\perp-\vec b_{1\perp}|^2}\cdot\frac{\vec x_\perp-\vec b_{2\perp}}{|\vec x_\perp-\vec b_{2\perp}|^2}\Bigg) \\
&\quad - \left(U^{bd}_{\vec x_\perp} - U^{bd}_{\vec b_{1\perp}}\right) U^{ce}_{\vec b_{2\perp}} \Bigg(\frac{\vec\epsilon_\perp^{\,\lambda *}\cdot(\vec z_\perp-\vec x_\perp)}{|\vec z_\perp-\vec x_\perp|^2}\, \frac{\vec x_\perp-\vec b_{1\perp}}{|\vec x_\perp-\vec b_{1\perp}|^2}\cdot\frac{\vec x_\perp-\vec b_{2\perp}}{|\vec x_\perp-\vec b_{2\perp}|^2} - \frac{\vec\epsilon_\perp^{\,\lambda *}\cdot(\vec z_\perp-\vec b_{2\perp})}{|\vec z_\perp-\vec b_{2\perp}|^2}\, \frac{\vec z_\perp-\vec x_\perp}{|\vec z_\perp-\vec x_\perp|^2}\cdot\frac{\vec x_\perp-\vec b_{1\perp}}{|\vec x_\perp-\vec b_{1\perp}|^2} \\
&\quad - \frac{\vec\epsilon_\perp^{\,\lambda *}\cdot(\vec z_\perp-\vec b_{2\perp})}{|\vec z_\perp-\vec b_{2\perp}|^2}\, \frac{\vec x_\perp-\vec b_{1\perp}}{|\vec x_\perp-\vec b_{1\perp}|^2}\cdot\frac{\vec x_\perp-\vec b_{2\perp}}{|\vec x_\perp-\vec b_{2\perp}|^2}\Bigg)\Bigg] \\
&\quad - \frac{i\,g^3}{4\,\pi^2}\, f^{abc}\left(V_{\vec b_{1\perp}}\, t^d\right)_1 \left(V_{\vec b_{2\perp}}\, t^e\right)_2 \Bigg[\left(U^{bd}_{\vec z_\perp} - U^{bd}_{\vec b_{1\perp}}\right) U^{ce}_{\vec b_{2\perp}}\, \frac{\vec\epsilon_\perp^{\,\lambda *}\cdot(\vec z_\perp-\vec b_{1\perp})}{|\vec z_\perp-\vec b_{1\perp}|^2}\, \ln\frac{1}{|\vec z_\perp-\vec b_{2\perp}|\,\Lambda} - U^{bd}_{\vec b_{1\perp}}\left(U^{ce}_{\vec z_\perp} - U^{ce}_{\vec b_{2\perp}}\right) \frac{\vec\epsilon_\perp^{\,\lambda *}\cdot(\vec z_\perp-\vec b_{2\perp})}{|\vec z_\perp-\vec b_{2\perp}|^2}\, \ln\frac{1}{|\vec z_\perp-\vec b_{1\perp}|\,\Lambda}\Bigg] \\
&\quad - \frac{i\,g^3}{4\,\pi^3}\int d^2x\, \left[U^{ab}_{\vec x_\perp} - U^{ab}_{\vec z_\perp}\right] f^{bde} \left(V_{\vec b_{1\perp}}\, t^d\right)_1 \left(V_{\vec b_{2\perp}}\, t^e\right)_2\, \frac{\vec\epsilon_\perp^{\,\lambda *}\cdot(\vec z_\perp-\vec x_\perp)}{|\vec z_\perp-\vec x_\perp|^2}\, \frac{\vec x_\perp-\vec b_{1\perp}}{|\vec x_\perp-\vec b_{1\perp}|^2}\cdot\frac{\vec x_\perp-\vec b_{2\perp}}{|\vec x_\perp-\vec b_{2\perp}|^2}\, \mathrm{Sign}(b_2^- - b_1^-)
\end{aligned}
$$



A₁



B₂



C₂

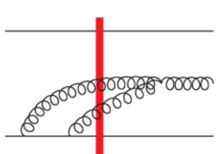
$$\vec x_\perp \times \vec y_\perp = x_1\, y_2 - x_2\, y_1$$

$$\Lambda = \text{IR cutoff}$$

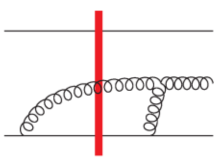
Results: Amplitude – D graphs

$$
\begin{aligned}
\sum_i D_i = & -\frac{g^3}{8\,\pi^4}\int d^2x_1\, d^2x_2\, \delta[(\vec{z}_\perp - \vec{x}_{1\perp})\times(\vec{z}_\perp - \vec{x}_{2\perp})] \Bigg[\frac{\vec{\epsilon}_\perp^{\,\lambda *}\cdot(\vec{x}_{2\perp}-\vec{x}_{1\perp})}{|\vec{x}_{2\perp}-\vec{x}_{1\perp}|^2}\, \frac{\vec{x}_{1\perp}-\vec{b}_{2\perp}}{|\vec{x}_{1\perp}-\vec{b}_{2\perp}|^2}\cdot\frac{\vec{x}_{2\perp}-\vec{b}_{2\perp}}{|\vec{x}_{2\perp}-\vec{b}_{2\perp}|^2} \\
& - \frac{\vec{\epsilon}_\perp^{\,\lambda *}\cdot(\vec{x}_{1\perp}-\vec{b}_{2\perp})}{|\vec{x}_{1\perp}-\vec{b}_{2\perp}|^2}\, \frac{\vec{z}_\perp-\vec{x}_{1\perp}}{|\vec{z}_\perp-\vec{x}_{1\perp}|^2}\cdot\frac{\vec{x}_{2\perp}-\vec{b}_{2\perp}}{|\vec{x}_{2\perp}-\vec{b}_{2\perp}|^2} + \frac{\vec{\epsilon}_\perp^{\,\lambda *}\cdot(\vec{x}_{2\perp}-\vec{b}_{2\perp})}{|\vec{x}_{2\perp}-\vec{b}_{2\perp}|^2}\, \frac{\vec{x}_{1\perp}-\vec{b}_{2\perp}}{|\vec{x}_{1\perp}-\vec{b}_{2\perp}|^2}\cdot\frac{\vec{z}_\perp-\vec{x}_{2\perp}}{|\vec{z}_\perp-\vec{x}_{2\perp}|^2}\Bigg] \\
& \times\, f^{abc}\left[U^{bd}_{\vec{x}_{1\perp}} - U^{bd}_{\vec{b}_{2\perp}}\right]\left[U^{ce}_{\vec{x}_{2\perp}} - U^{ce}_{\vec{b}_{2\perp}}\right]\left(V_{\vec{b}_{1\perp}}\right)_1\left(V_{\vec{b}_{2\perp}}\, t^e\, t^d\right)_2 \\
& + \frac{i\,g^3}{4\,\pi^3}\int d^2x\, f^{abc}\, U^{bd}_{\vec{b}_{2\perp}}\left[U^{ce}_{\vec{x}_\perp} - U^{ce}_{\vec{b}_{2\perp}}\right]\left(V_{\vec{b}_{1\perp}}\right)_1\left(V_{\vec{b}_{2\perp}}\, t^e\, t^d\right)_2 \Bigg(\frac{\vec{\epsilon}_\perp^{\,\lambda *}\cdot(\vec{z}_\perp-\vec{x}_\perp)}{|\vec{z}_\perp-\vec{x}_\perp|^2}\, \frac{1}{|\vec{x}_\perp-\vec{b}_{2\perp}|^2} \\
& - \frac{\vec{\epsilon}_\perp^{\,\lambda *}\cdot(\vec{z}_\perp-\vec{b}_{2\perp})}{|\vec{z}_\perp-\vec{b}_{2\perp}|^2}\, \frac{\vec{z}_\perp-\vec{x}_\perp}{|\vec{z}_\perp-\vec{x}_\perp|^2}\cdot\frac{\vec{x}_\perp-\vec{b}_{2\perp}}{|\vec{x}_\perp-\vec{b}_{2\perp}|^2} - \frac{\vec{\epsilon}_\perp^{\,\lambda *}\cdot(\vec{z}_\perp-\vec{b}_{2\perp})}{|\vec{z}_\perp-\vec{b}_{2\perp}|^2}\, \frac{1}{|\vec{x}_\perp-\vec{b}_{2\perp}|^2}\Bigg) \\
& + \frac{i\,g^3}{4\,\pi^2}\, f^{abc}\, U^{bd}_{\vec{b}_{2\perp}}\left[U^{ce}_{\vec{z}_\perp} - U^{ce}_{\vec{b}_{2\perp}}\right]\left(V_{\vec{b}_{1\perp}}\right)_1\left(V_{\vec{b}_{2\perp}}\, t^e\, t^d\right)_2\, \frac{\vec{\epsilon}_\perp^{\,\lambda *}\cdot(\vec{z}_\perp-\vec{b}_{2\perp})}{|\vec{z}_\perp-\vec{b}_{2\perp}|^2}\, \ln\frac{1}{|\vec{z}_\perp-\vec{b}_{2\perp}|\,\Lambda}
\end{aligned}
$$

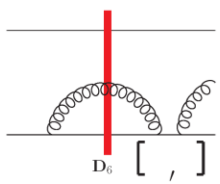
- To get the E graph results switch quark 1 with quark 2 ($1 \leftrightarrow 2$)



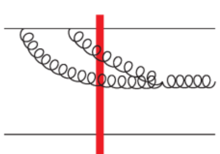
D_1



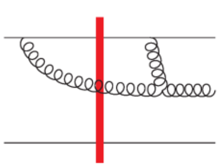
D_2



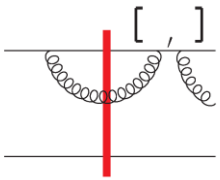
D_6



E_1



E_2



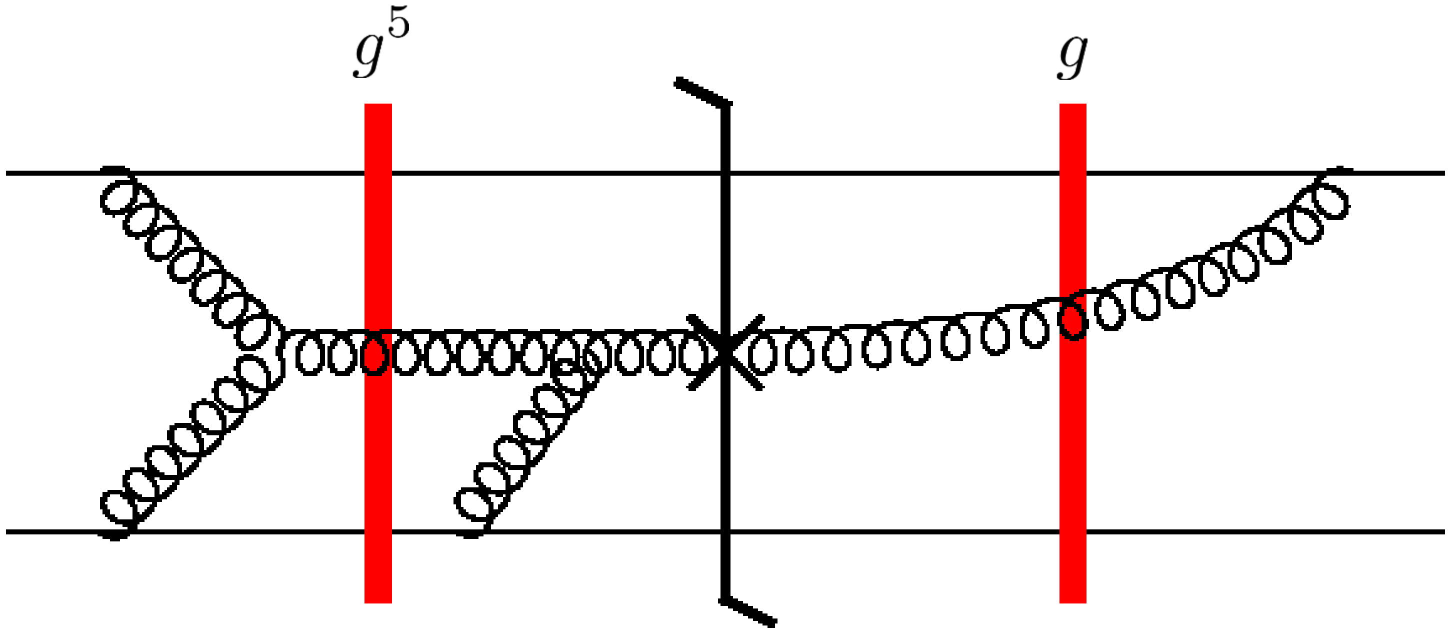
E_6

Compare this with the dilute projectile (pA) amplitude!

$$\boxed{M(\vec{z}_\perp, \vec{b}_\perp) = \frac{i\, g}{\pi} \, \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{b}_\perp)}{|\vec{z}_\perp - \vec{b}_\perp|^2} \, \left[U^{ab}_{\vec{z}_\perp} - U^{ab}_{\vec{b}_\perp} \right] \, \left(V_{\vec{b}_\perp} \, t^b \right)}$$

Conclusions and Outlook

Outlook



- Need to calculate g^5 amplitude to get the single inclusive gluon production cross-section.
- Work in progress. So far seems much harder for some graphs.

Conclusions

- Analytic calculation of the first saturation correction in the projectile (while keeping all-orders in the target) is very hard.

- It has to be done: again, without it cannot find quantum corrections to do phenomenology and to test thermalization scenarios.

- Ulrich (and hydro) are way ahead of me and are far in the future…

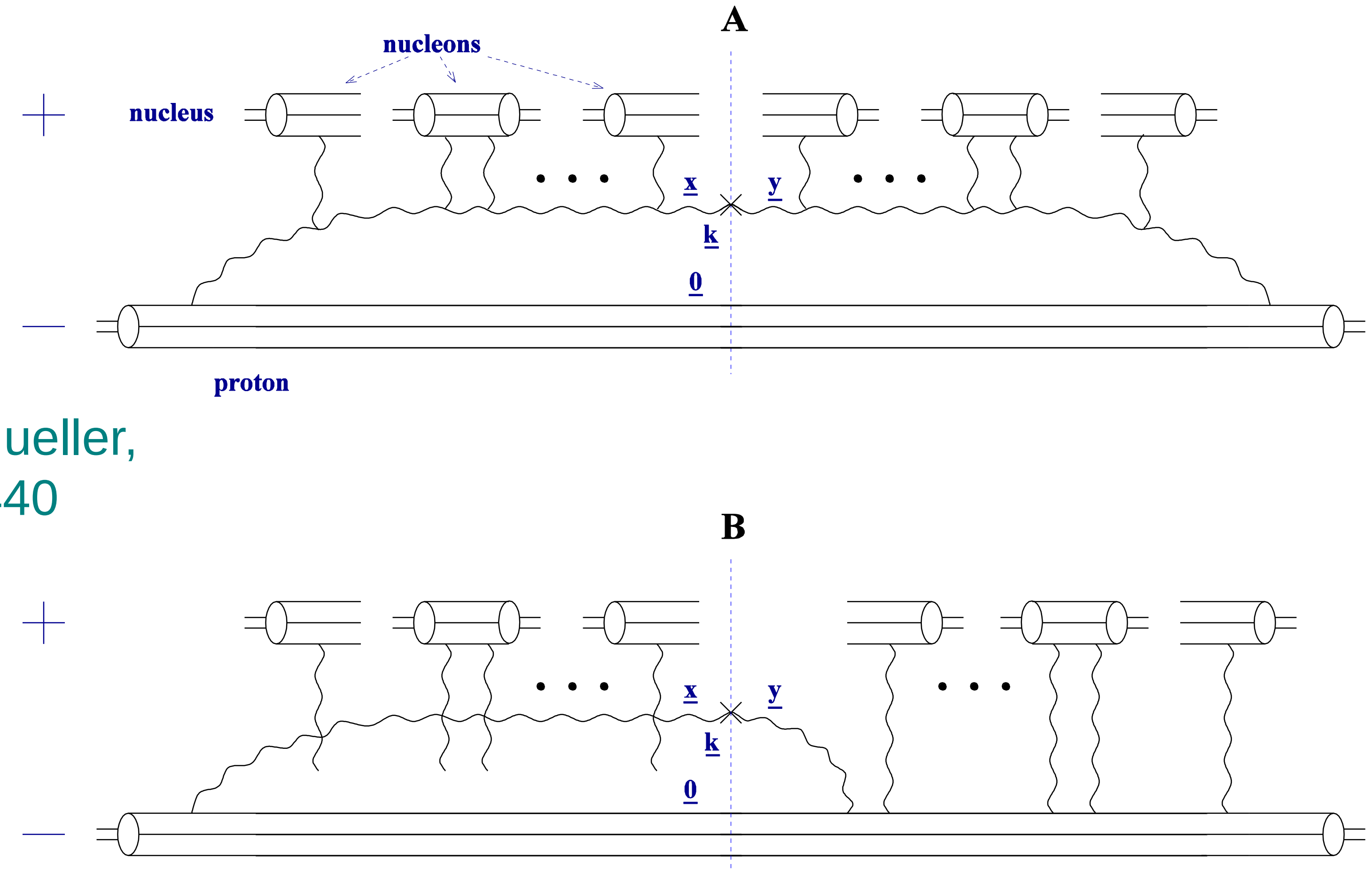
- Happy Birthday!!!

Backup Slides

Gluon Production in pA: McLerran-Venugopalan model

The diagrams one has to resum are shown here: they resum powers of

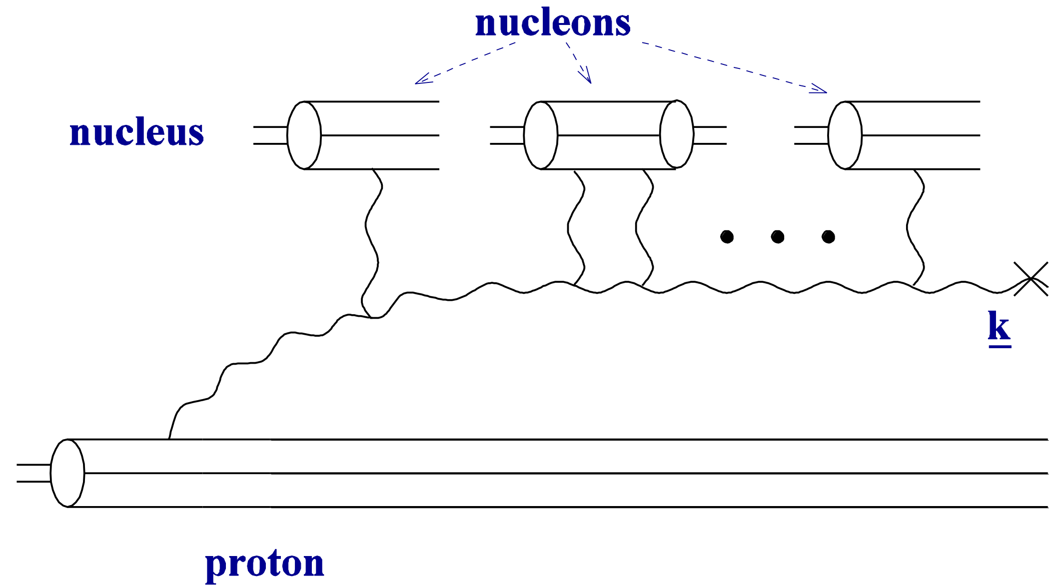
$$\alpha_s^2 \, A^{1/3}$$



Yu. K., A.H. Mueller, hep-ph/9802440

Gluon Production in pA: McLerran-Venugopalan model

Classical gluon production: we need to resum only the multiple rescatterings of the gluon on nucleons. Here's one of the graphs considered.



Yu. K., A.H. Mueller, '98

Resulting inclusive gluon production cross section is given by

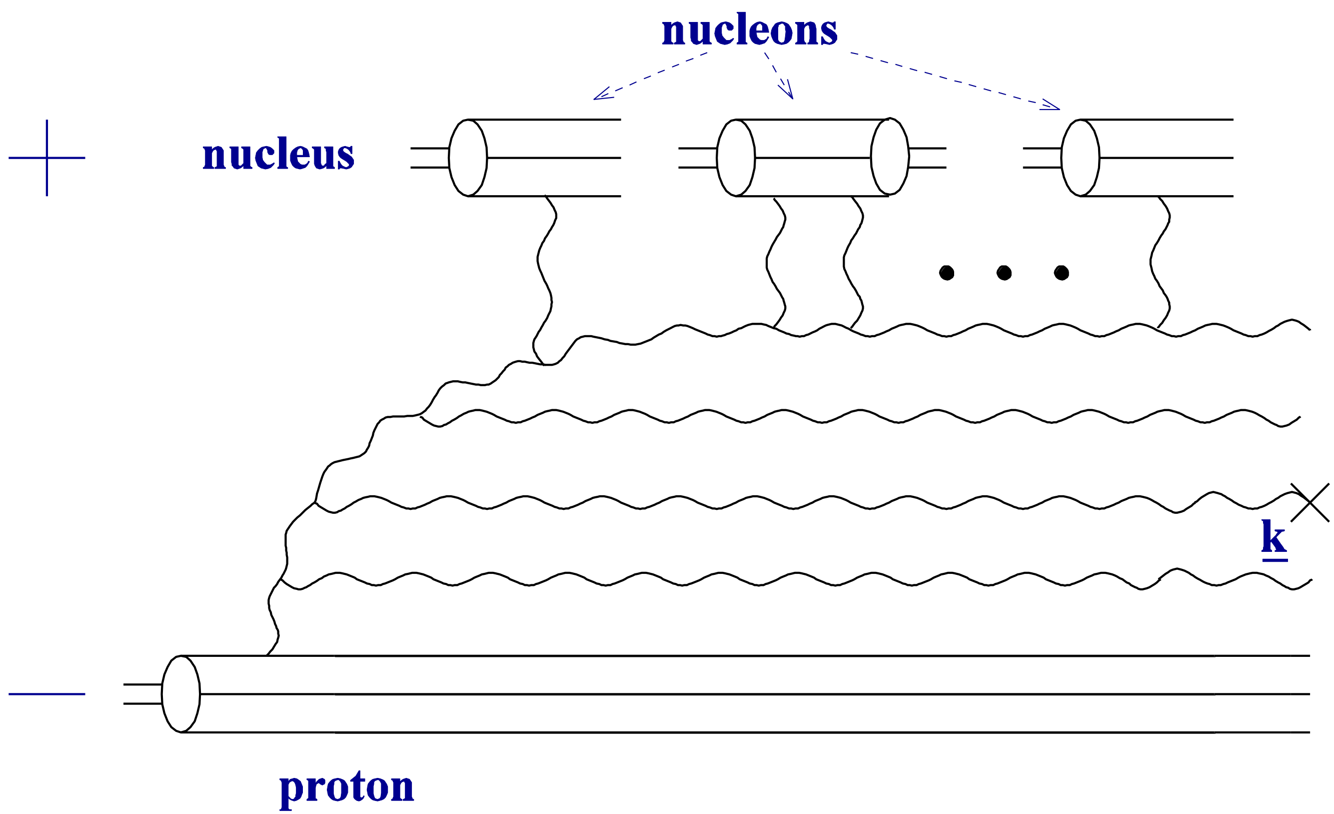
$$\frac{d\sigma}{d^2k\,dy} = \int d^2b\,\frac{d^2x\,d^2y}{(2\,\pi)^2}\,e^{i\,\underline{k}\cdot(\underline{x}-\underline{y})}\,\frac{\alpha_s\,C_F}{\pi^2}\,\frac{\underline{x}\cdot\underline{y}}{\underline{x}_\perp^2\,\underline{y}_\perp^2}\,\left[N_G(\underline{x}) + N_G(\underline{y}) - N_G(\underline{x}-\underline{y})\right]$$

proton+gluon wave function

With the gluon-gluon dipole-nucleus forward scattering amplitude

$$N_G(x, Y=0) = 1 - e^{-x^2 Q_s^2/4}$$

Including Quantum Evolution



To understand the energy dependence of particle production in pA one needs to include quantum evolution resumming graphs like this one. It resums powers of $\alpha \ln 1/x = \alpha Y$.

(Yu. K., K. Tuchin, ' 01)

Gluon Production in pA

Amazingly the gluon production cross section reduces to a k_T-factorization expression (Yu. K., Tuchin, ' 01; cf. Gribov, Levin, Ryskin '83):

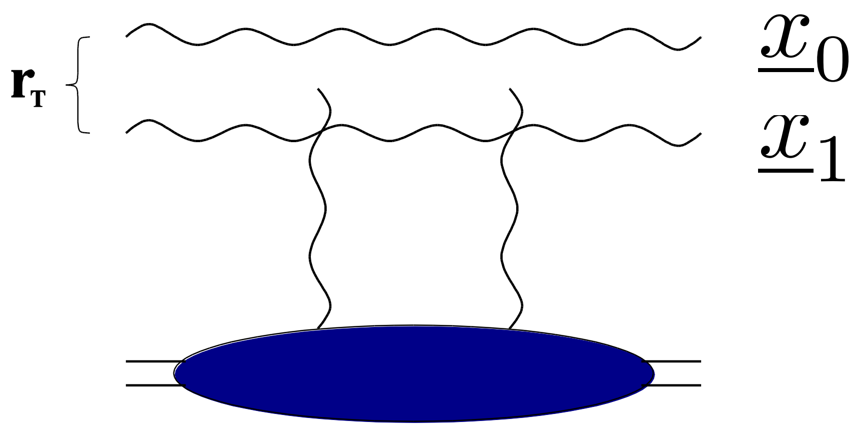
$$\frac{d\sigma^{pA}}{d^2k\,dy} = \frac{2\,\alpha_s}{C_F}\,\frac{1}{k_T^2}\int d^2q\,\phi^p(\underline{q}, Y-y)\,\phi^A(\underline{k}-\underline{q}, y)$$

with the proton and nucleus unintegrated gluon distributions defined by

$$\phi^{p,A}(\underline{k}, y) = \frac{C_F}{\alpha_s\,(2\,\pi)^3}\int d^2b\,d^2x_\perp\,e^{-i\,\underline{k}\cdot\underline{x}}\,\nabla_x^2\,N_G^{p,A}(\underline{x}, \underline{b}, y)$$

with $N_G^{p,A}$ the scattering amplitude of a GG dipole on p or A. (Includes multiple rescatterings and small-x evolution.)

Gluon dipole

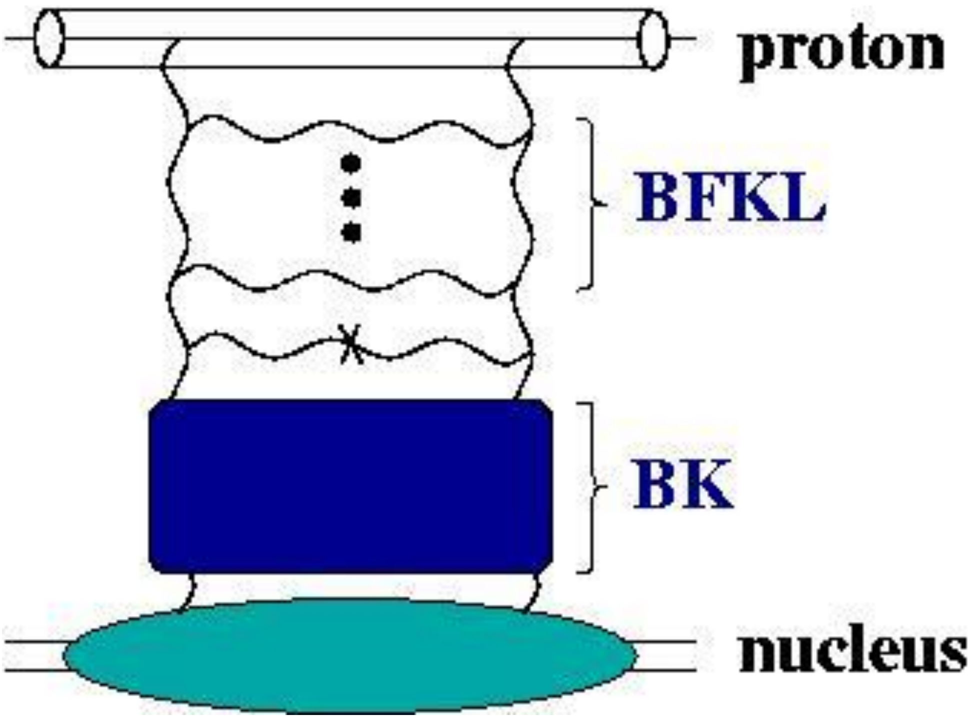


- The gluon dipole amplitude is

$$N_G(x_0, x_1, Y) = 2\,N(x_0, x_1, Y) - [N(x_0, x_1, Y)]^2$$

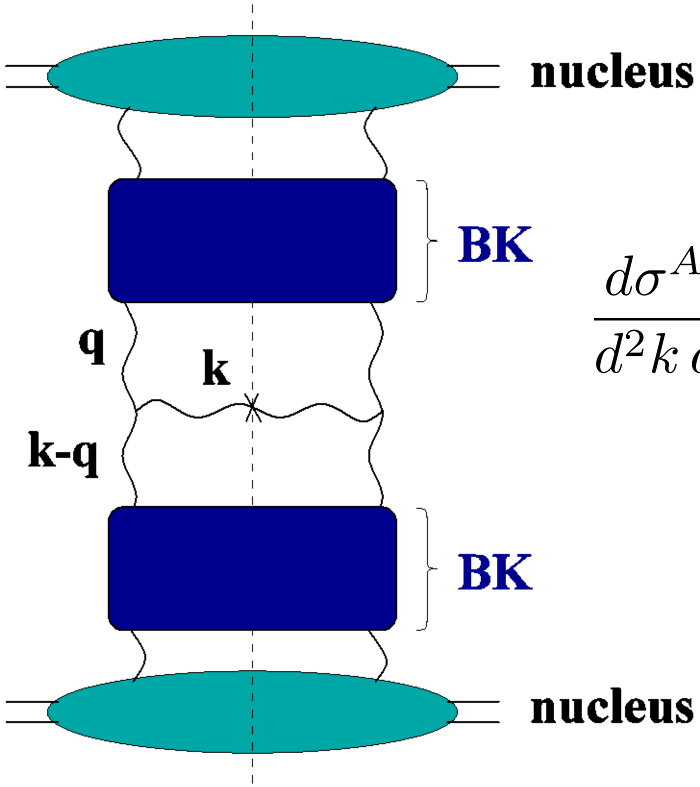
Gluon Production in pA

A simplified diagrammatic way of thinking about this result is



CGC Gluon Production in AA

This is an unsolved problem. Albacete and Dumitru ‘10, following Kharzeev, Levin, and Nardi ‘01, approximate the full unknown solution by

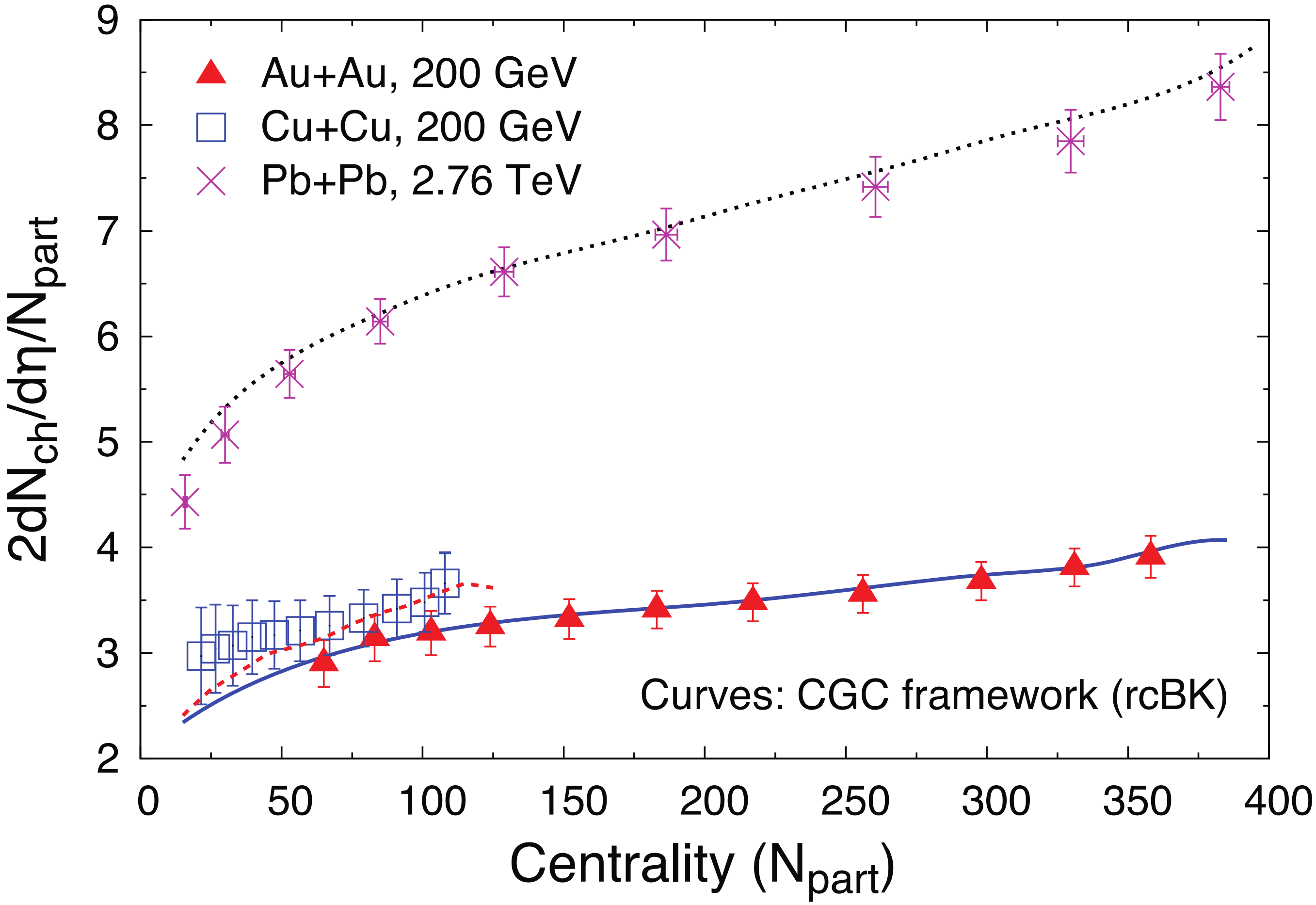


$$\frac{d\sigma^{AA}}{d^2k\,dy} = \frac{2\,\alpha_s}{C_F}\,\frac{1}{k_T^2}\int d^2q\,\phi^A(\underline{q}, Y-y)\,\phi^A(\underline{k}-\underline{q}, y)$$

Gluon production in AA

- While the p_T-dependence is modified by the quark-gluon plasma, the hope is that the rapidity dependence is not
- This is approximately true due to causality: different rapidity regions are causally disconnected from each other.
- Also, ideal hydrodynamics preserves entropy = number of degrees of freedom = number of particles.

Multiplicity with centrality at RHIC and LHC



Albacete and Dumitru ‘10