What BFKL has to say about three-jet production, four-jet production and average rapidity ratios at the LHC

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LHC Forward Physics meeting, 15-16 March 2016, CERN

Outline

- Motivation
- 3-jet production within the BFKL framework
- New observables relevant to 3-jet production
- 4-jet production within the BFKL framework
- New observables relevant to 4-jet production
- Multi-jet production within the BFKL framework
- Average rapidity ratios with the **BFKLex** MC
- Conclusions and outlook

BFKL phenomenology

- LHC has produced and will further produce an abundance of data
- This is the best time to investigate the applicability of the BFKL resummation program within the context of a hadron collider
- In the last years: the big hit from the theory/experimental side was the study of Mueller-Navelet jets (dijets). We only touch here one subfield for which BFKL is relevant, small-x physics/forward physics is much richer of course: Diffraction, Saturation, DPI etc.

CMS-FSQ-12-002

CERN-PH-EP/2015-309
2016/01/26

Azimuthal decorrelation of jets widely separated in rapidity in pp collisions at $\sqrt{s} = 7\,\mathrm{TeV}$

The CMS Collaboration*

... Therein, in Conclusions it reads:

The observed sensitivity to the implementation of the colour-coherence effects in the DGLAP MC generators and the reasonable data-theory agreement shown by the NLL BFKL analytical calculations at large Δy, may be considered as indications that the kinematical domain of the present study lies in between the regions described by the DGLAP and BFKL approaches. Possible manifestations of BFKL signatures are expected to be more pronounced at increasing collision energies.

BFKL phenomenology

- It seems that searching for BFKL signals in a hadron collider is more fruitful when we focus on azimuthal angle dependencies than the usual “growth with energy” behaviour.
- We should exploit more the azimuthal decorrelations by considering more exclusive quantities (3- and 4-jets which introduce also a pT dependence coming from the extra jets)
- We shouldn't leave unexplored any rapidity patterns within the Multi-Regge kinematics.

BFKL obviously prefers a factorization between transverse and longitudinal (rapidity) degrees of freedom. If indeed this is the case then we do learn something new about high energy QCD.

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There are other ways we can explore this territory.

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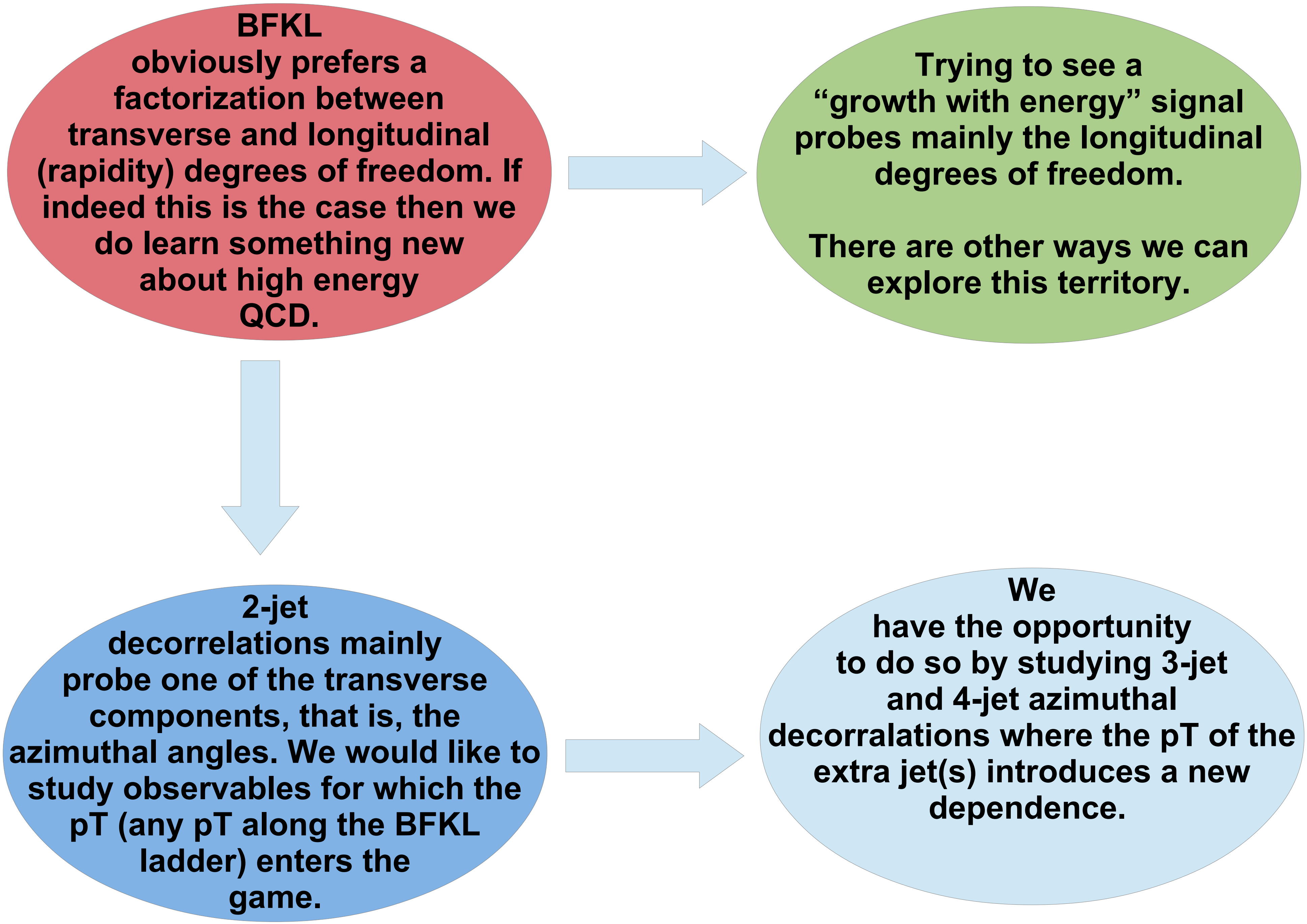
2-jet decorrelations mainly probe one of the transverse components, that is, the azimuthal angles. We would like to study observables for which the pT (any pT along the BFKL ladder) enters the game.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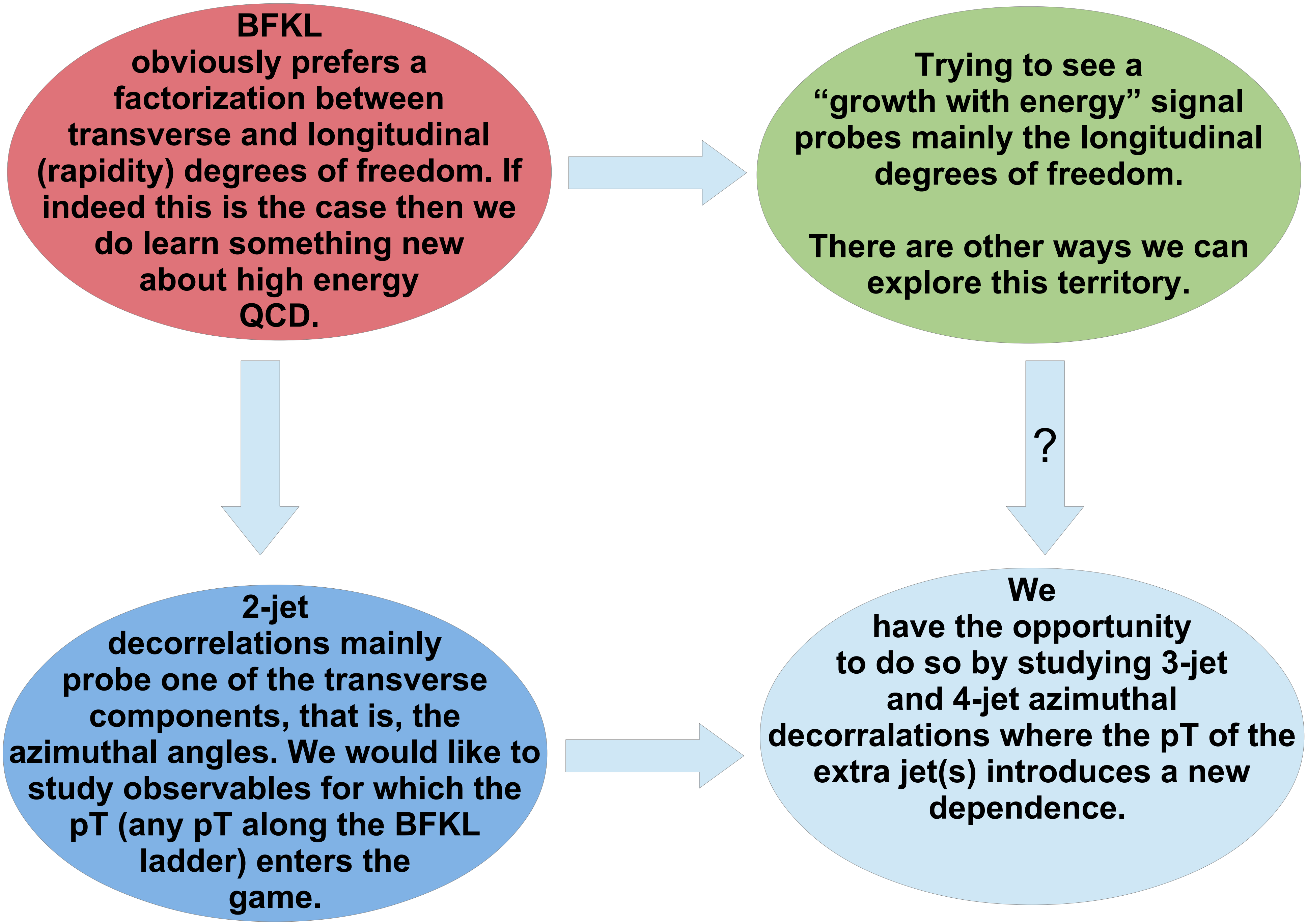
We have the opportunity to do so by studying 3-jet and 4-jet azimuthal decorralations where the pT of the extra jet(s) introduces a new dependence.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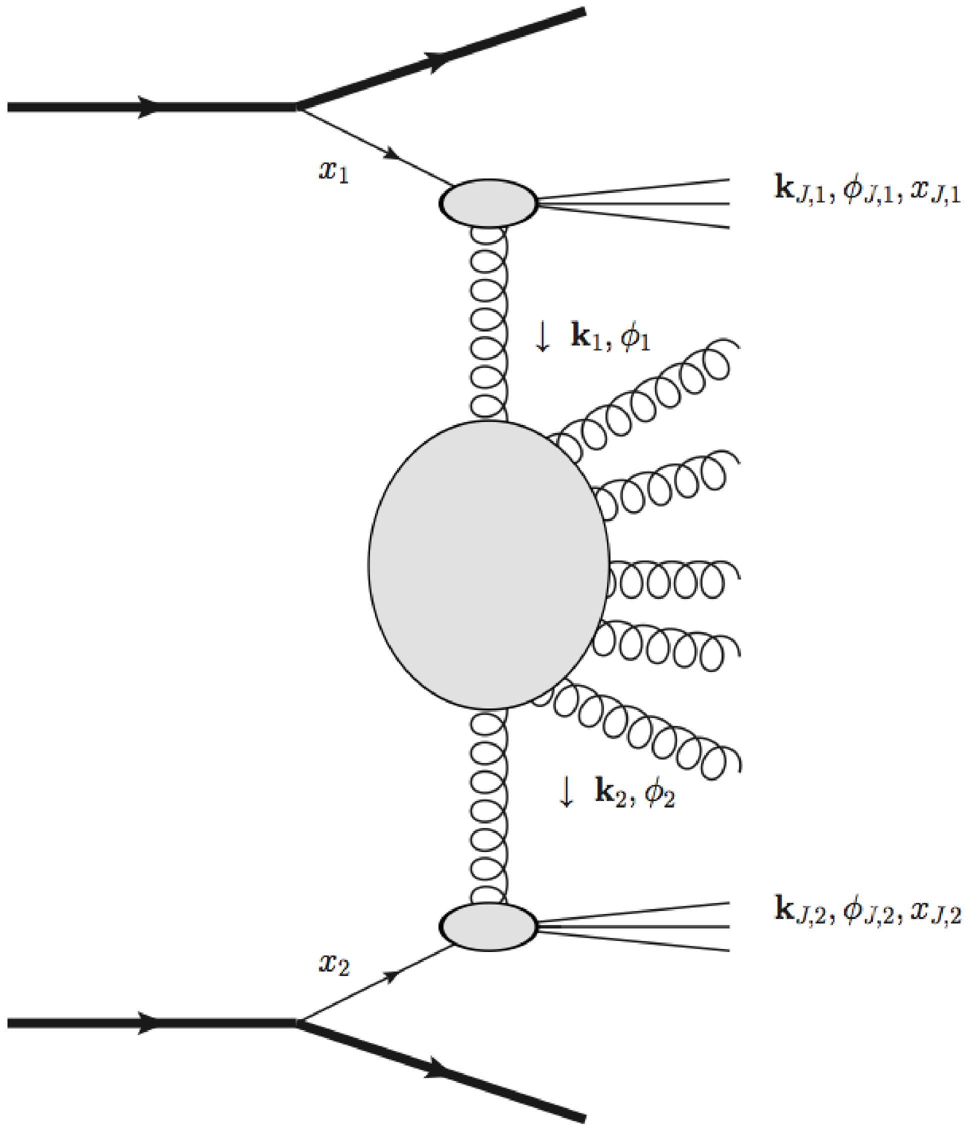
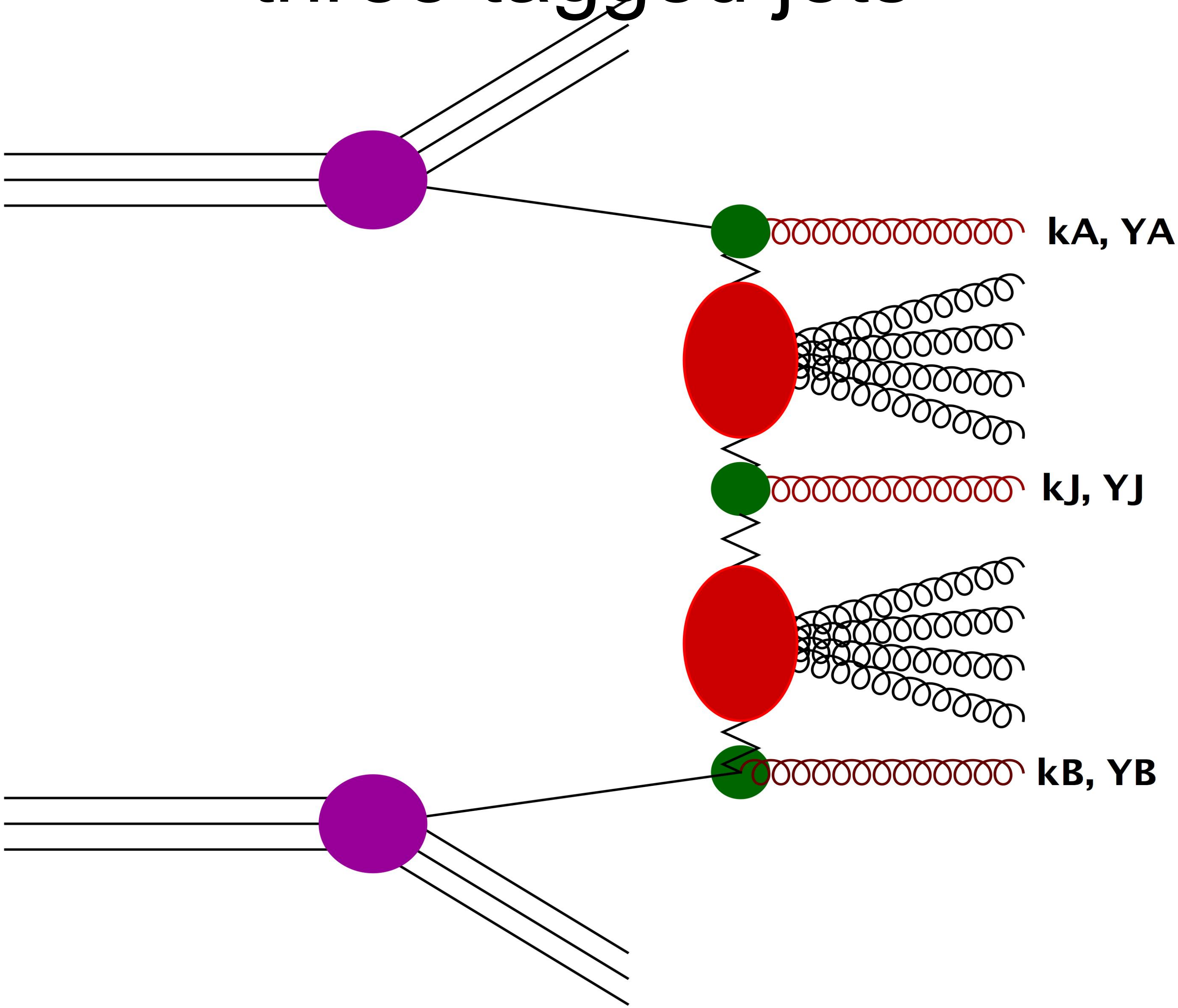
Mueller-Navelet jets

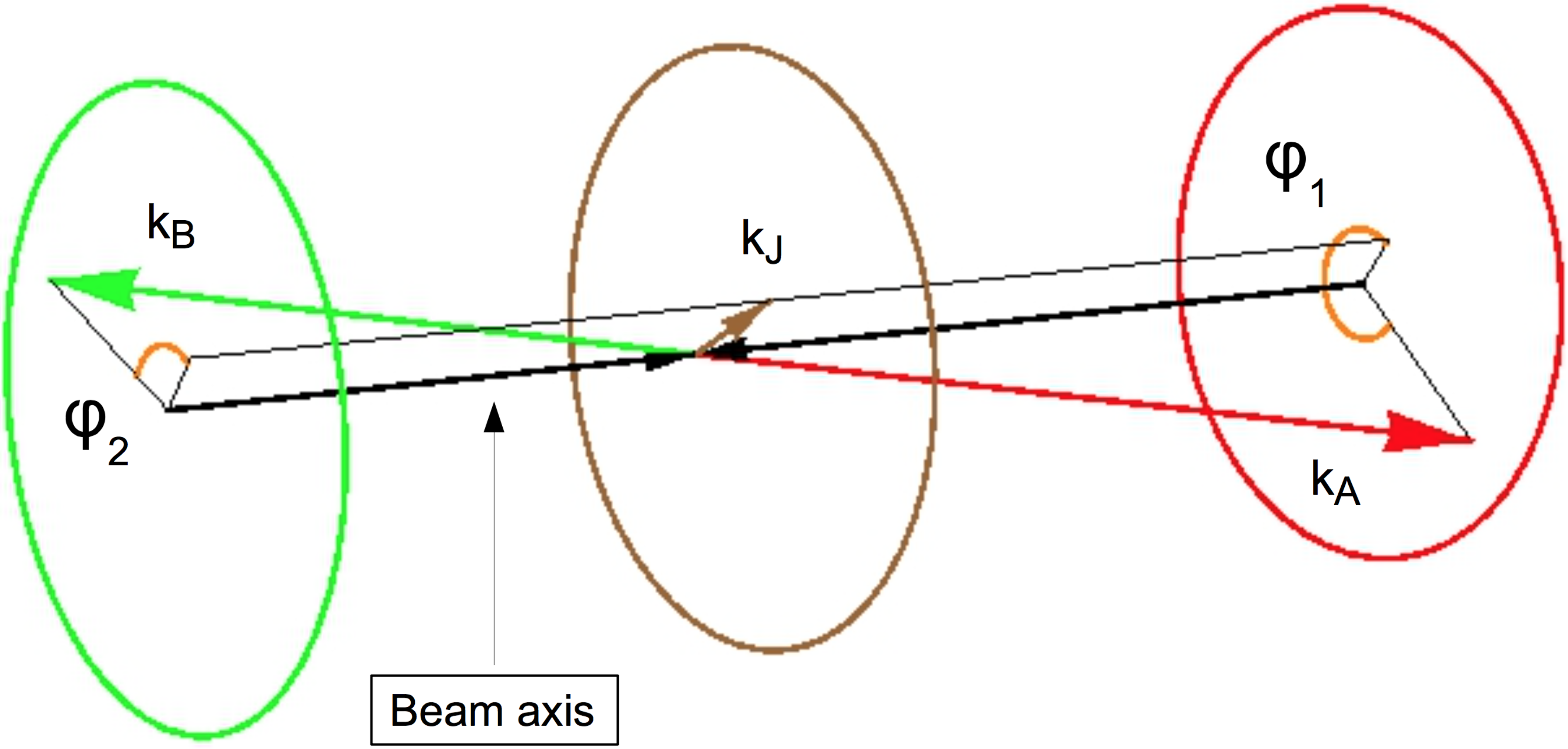


Colferai, Schwennsen, Szymanowski, Wallon 2010

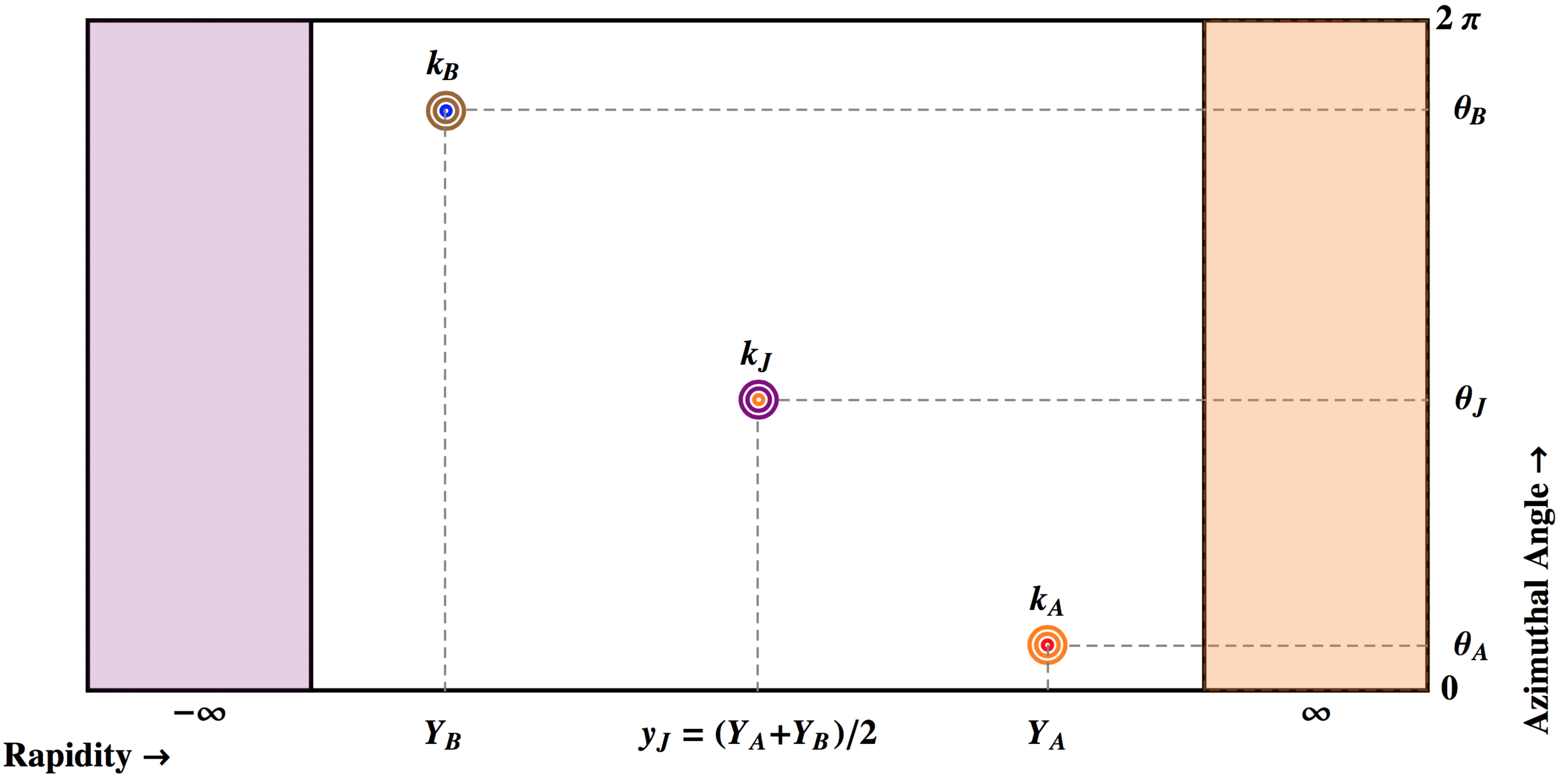
Now, let us move to events with three tagged jets



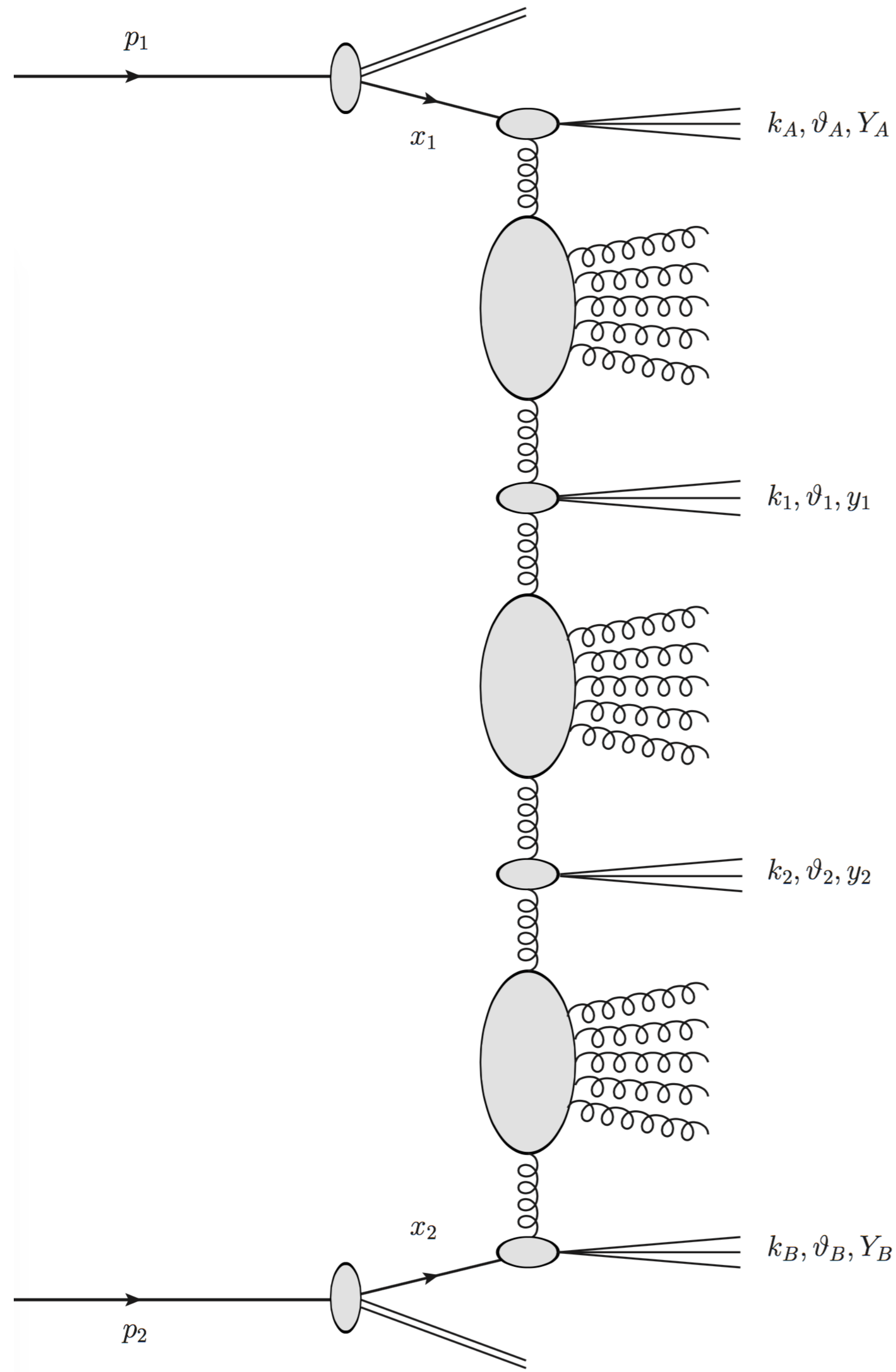
An event with three tagged jets



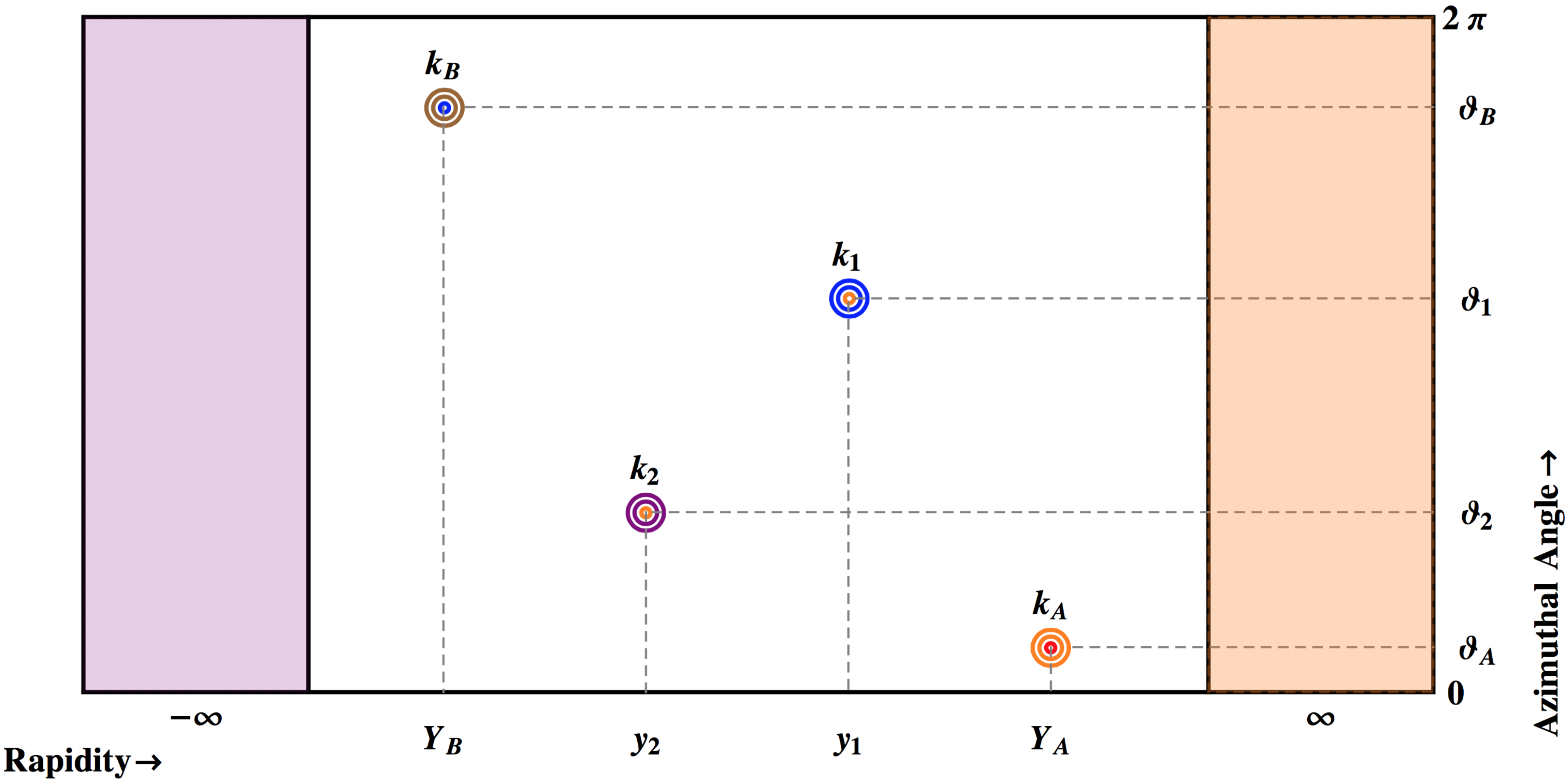
A primitive lego-plot (3-jets)



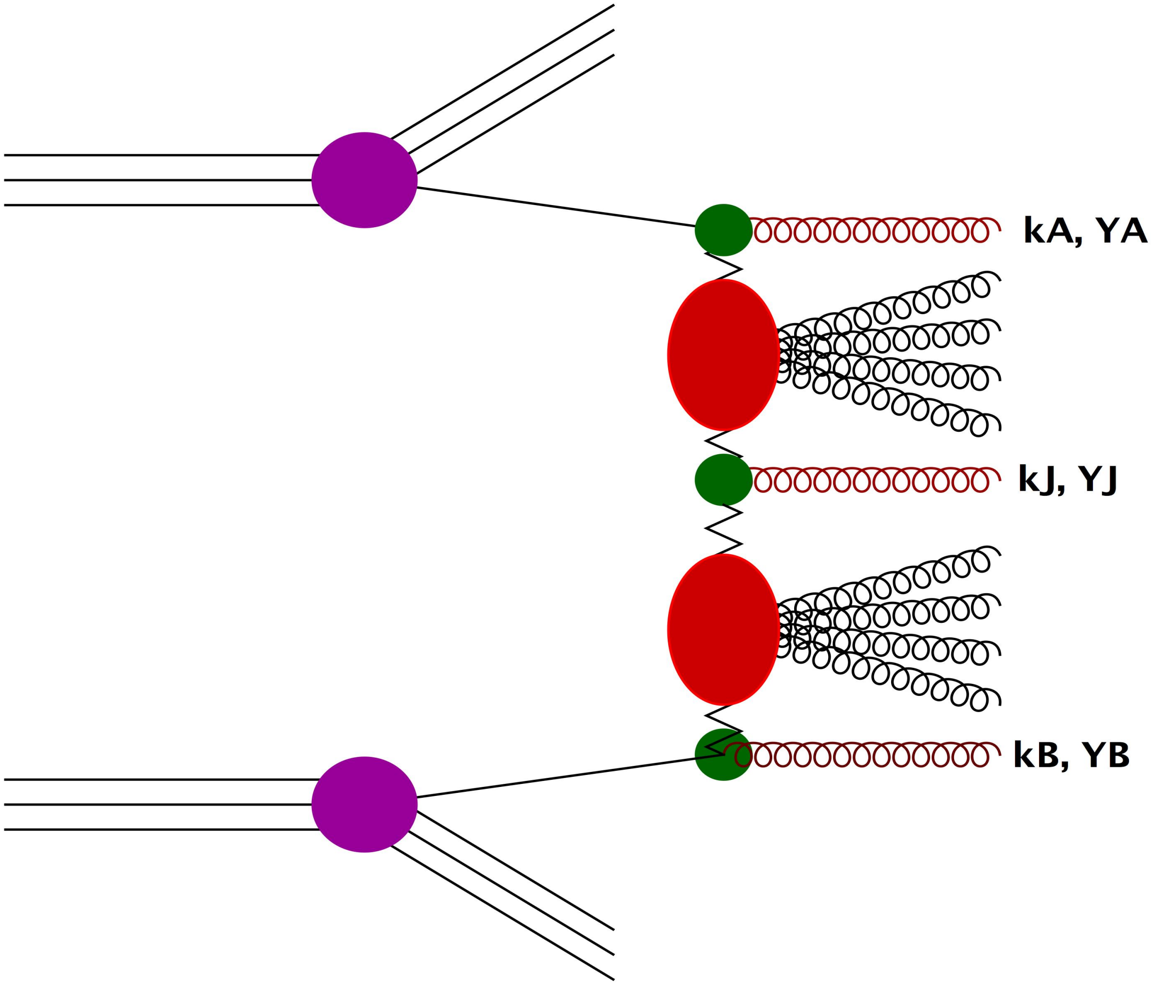
An event with four tagged jets



A primitive lego-plot (4-jets)



3-jets partonic cross section



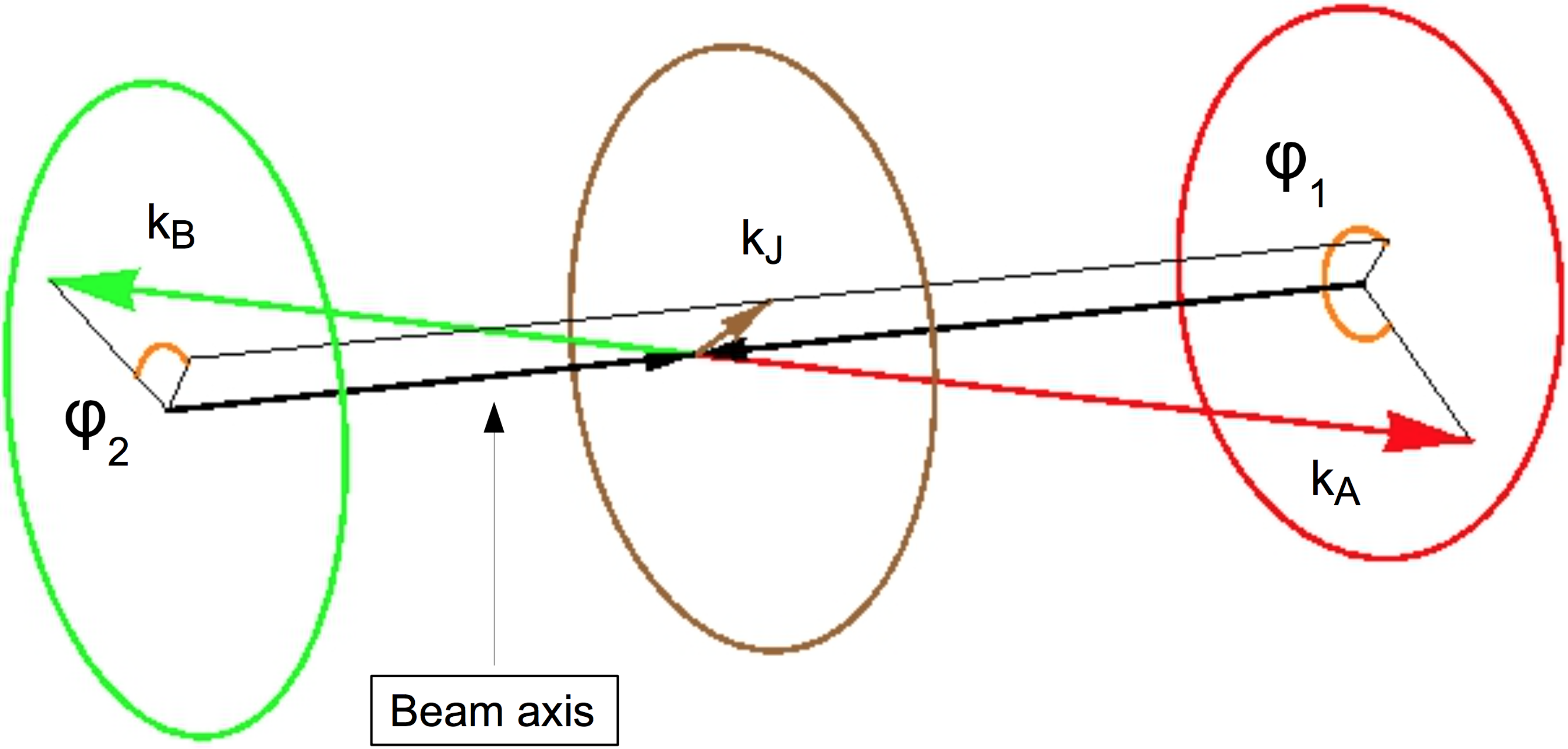
Assuming that
$Y_A > y_J > Y_B$ and also that k_A and k_B are fixed we can write for the differential cross section:

$$\frac{d^3\sigma^{3-\mathrm{jet}}}{d^2\vec{k}_J dy_J} = \frac{\bar{\alpha}_s}{\pi k_J^2} \int d^2\vec{p}_A \int d^2\vec{p}_B \, \delta^{(2)}\left(\vec{p}_A + \vec{k}_J - \vec{p}_B\right) \times \varphi\left(\vec{k}_A, \vec{p}_A, Y_A - y_J\right) \varphi\left(\vec{p}_B, \vec{k}_B, y_J - Y_B\right)$$

Starting point...
THEN:

A idea would be to integrate over all angles after using the projections on the two azimuthal angle differences between the central jet and k_A and k_B respectively

Back to the basic picture



Integrate over all angles after using projections

$$\frac{d^3\sigma^{3-\mathrm{jet}}}{d^2\vec{k}_J dy_J} = \frac{\bar{\alpha}_s}{\pi k_J^2} \int d^2\vec{p}_A \int d^2\vec{p}_B\, \delta^{(2)}\left(\vec{p}_A + \vec{k}_J - \vec{p}_B\right) \times \varphi\left(\vec{k}_A, \vec{p}_A, Y_A - y_J\right) \varphi\left(\vec{p}_B, \vec{k}_B, y_J - Y_B\right)$$

$$\int_0^{2\pi} d\theta_A \int_0^{2\pi} d\theta_B \int_0^{2\pi} d\theta_J \cos\left(M\left(\theta_A - \theta_J - \pi\right)\right) \cos\left(N\left(\theta_J - \theta_B - \pi\right)\right) \frac{d^3\sigma^{3-\mathrm{jet}}}{d^2\vec{k}_J dy_J}$$
$$= \bar{\alpha}_s \sum_{L=0}^{N} \binom{N}{L} \left(k_J^2\right)^{\frac{L-1}{2}} \int_0^\infty dp^2 \left(p^2\right)^{\frac{N-L}{2}} \int_0^{2\pi} d\theta \frac{(-1)^{M+N} \cos\left(M\theta\right) \cos\left((N-L)\theta\right)}{\sqrt{\left(p^2 + k_J^2 + 2\sqrt{p^2 k_J^2}\cos\theta\right)^N}}$$
$$\times\, \phi_M\left(p_A^2, p^2, Y_A - y_J\right) \phi_N\left(p^2 + k_J^2 + 2\sqrt{p^2 k_J^2}\cos\theta, p_B^2, y_J - Y_B\right)$$

Integrate over all angles after using projections

$$\int_0^{2\pi} d\theta_A \int_0^{2\pi} d\theta_B \int_0^{2\pi} d\theta_J \cos\left(M\left(\theta_A - \theta_J - \pi\right)\right)$$
$$\cos\left(N\left(\theta_J - \theta_B - \pi\right)\right) \frac{d^3\sigma^{3-\mathrm{jet}}}{d^2\vec{k}_J dy_J}$$
$$= \bar{\alpha}_s \sum_{L=0}^{N} \binom{N}{L} \left(k_J^2\right)^{\frac{L-1}{2}} \int_0^\infty dp^2 \left(p^2\right)^{\frac{N-L}{2}}$$
$$\int_0^{2\pi} d\theta \frac{(-1)^{M+N} \cos\left(M\theta\right) \cos\left((N-L)\theta\right)}{\sqrt{\left(p^2 + k_J^2 + 2\sqrt{p^2 k_J^2}\cos\theta\right)^N}}$$
$$\times\, \phi_M\left(p_A^2, p^2, Y_A - y_J\right) \phi_N\left(p^2 + k_J^2 + 2\sqrt{p^2 k_J^2}\cos\theta, p_B^2, y_J - Y_B\right)$$

$$\left\langle \cos\left(M\left(\theta_A - \theta_J - \pi\right)\right) \cos\left(N\left(\theta_J - \theta_B - \pi\right)\right)\right\rangle$$
$$= \frac{\int_0^{2\pi} d\theta_A d\theta_B d\theta_J \cos\left(M\left(\theta_A - \theta_J - \pi\right)\right) \cos\left(N\left(\theta_J - \theta_B - \pi\right)\right) \frac{d^3\sigma^{3-\mathrm{jet}}}{d^2\vec{k}_J dy_J}}{\int_0^{2\pi} d\theta_A d\theta_B d\theta_J \frac{d^3\sigma^{3-\mathrm{jet}}}{d^2\vec{k}_J dy_J}}$$

... so that you can define new observables:

$$\mathcal{R}_{P,Q}^{M,N} = \frac{\langle \cos\left(M\left(\theta_A - \theta_J - \pi\right)\right)\cos\left(N\left(\theta_J - \theta_B - \pi\right)\right)\rangle}{\langle \cos\left(P\left(\theta_A - \theta_J - \pi\right)\right)\cos\left(Q\left(\theta_J - \theta_B - \pi\right)\right)\rangle}$$

How would an experimentalist measure this*?

$$\mathcal{R}_{P,Q}^{M,N} = \frac{\langle \cos\left(M\left(\theta_A - \theta_J - \pi\right)\right)\cos\left(N\left(\theta_J - \theta_B - \pi\right)\right)\rangle}{\langle \cos\left(P\left(\theta_A - \theta_J - \pi\right)\right)\cos\left(Q\left(\theta_J - \theta_B - \pi\right)\right)\rangle}$$

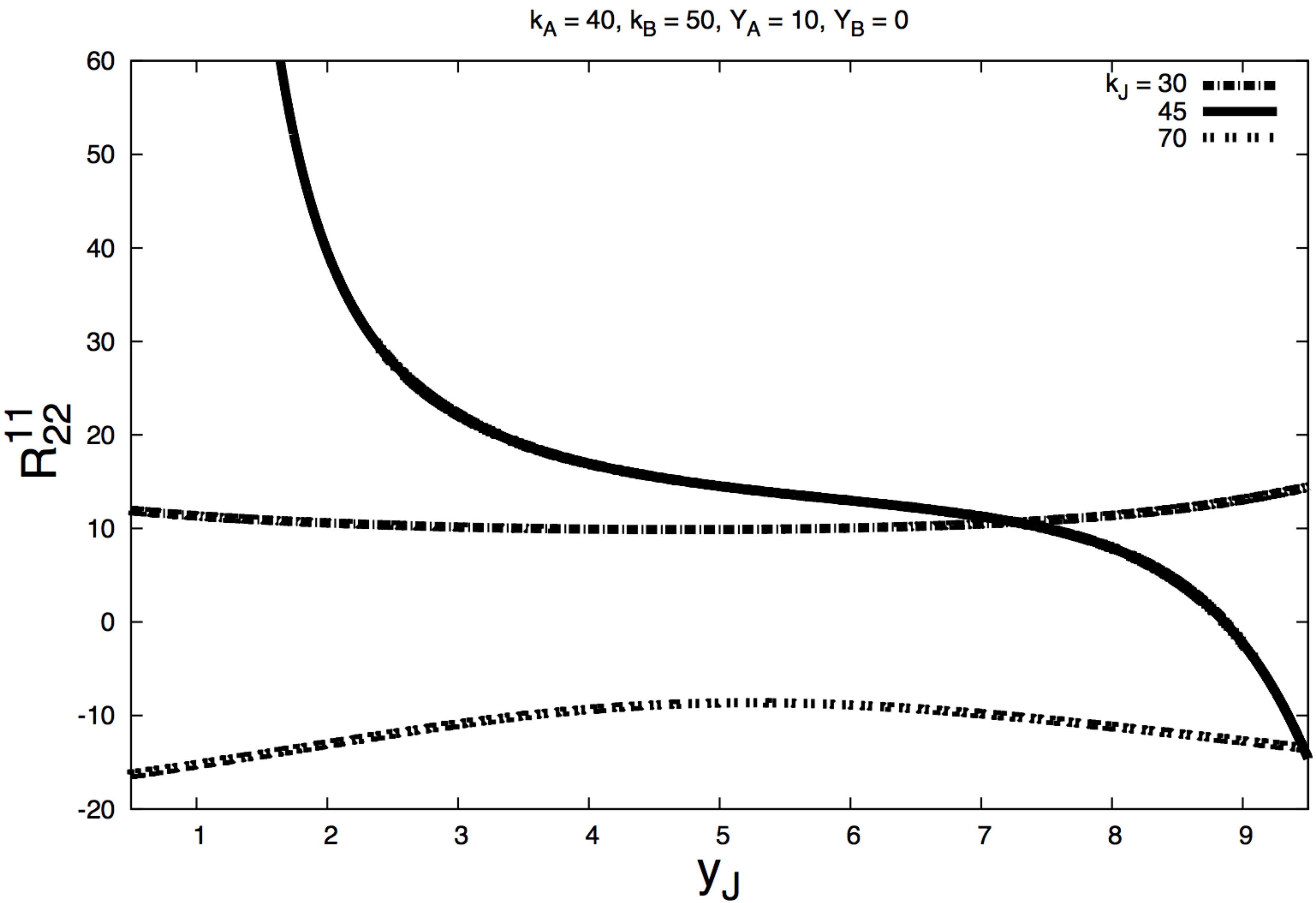
* Coming from a theorist, this would appear to be more of a cooking recipe, apologies to our experimental colleagues in advance for any naivety here.

How would an experimentalist measure this?

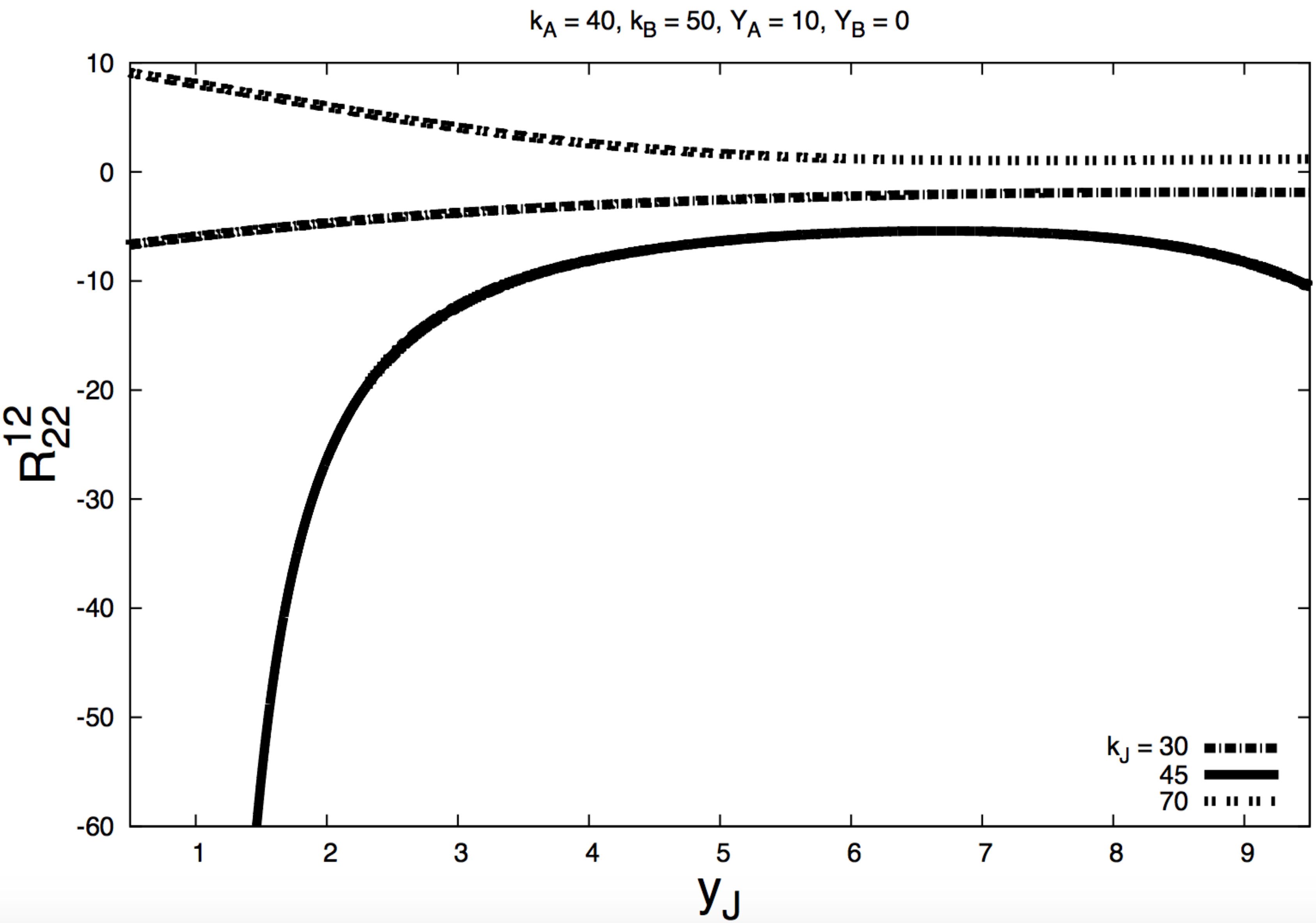
1. For 7 and 8 TeV energies, just pick up the data that were used for dijet studies.
2. From these data, isolate those events that have in addition a very central jet.
3. Choose integers M, N, P, Q (for a first study they can be 1, 2 or 3).
4. For each event, measure the azimuthal angle difference between the forward-central (θ_A-θ_J-π) and the backward-central (θ_J-θ_B-π) jets and calculate the quantity below. Finally compute the average.

$$\mathcal{R}^{M,N}_{P,Q} = \frac{\langle \cos\left(M\left(\theta_A - \theta_J - \pi\right)\right) \cos\left(N\left(\theta_J - \theta_B - \pi\right)\right)\rangle}{\langle \cos\left(P\left(\theta_A - \theta_J - \pi\right)\right) \cos\left(Q\left(\theta_J - \theta_B - \pi\right)\right)\rangle}$$

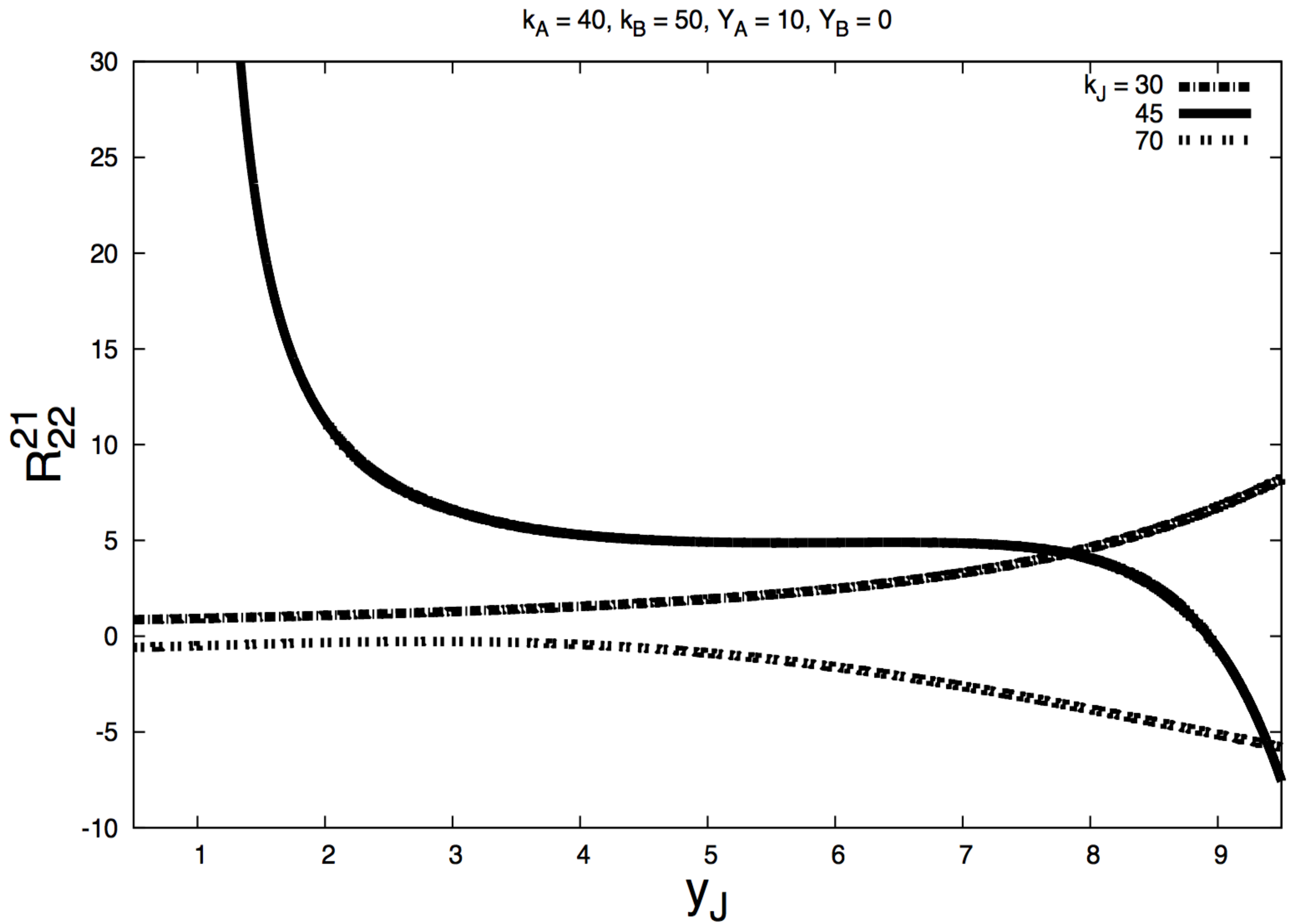
Partonic behaviour of R^{11}_{22}



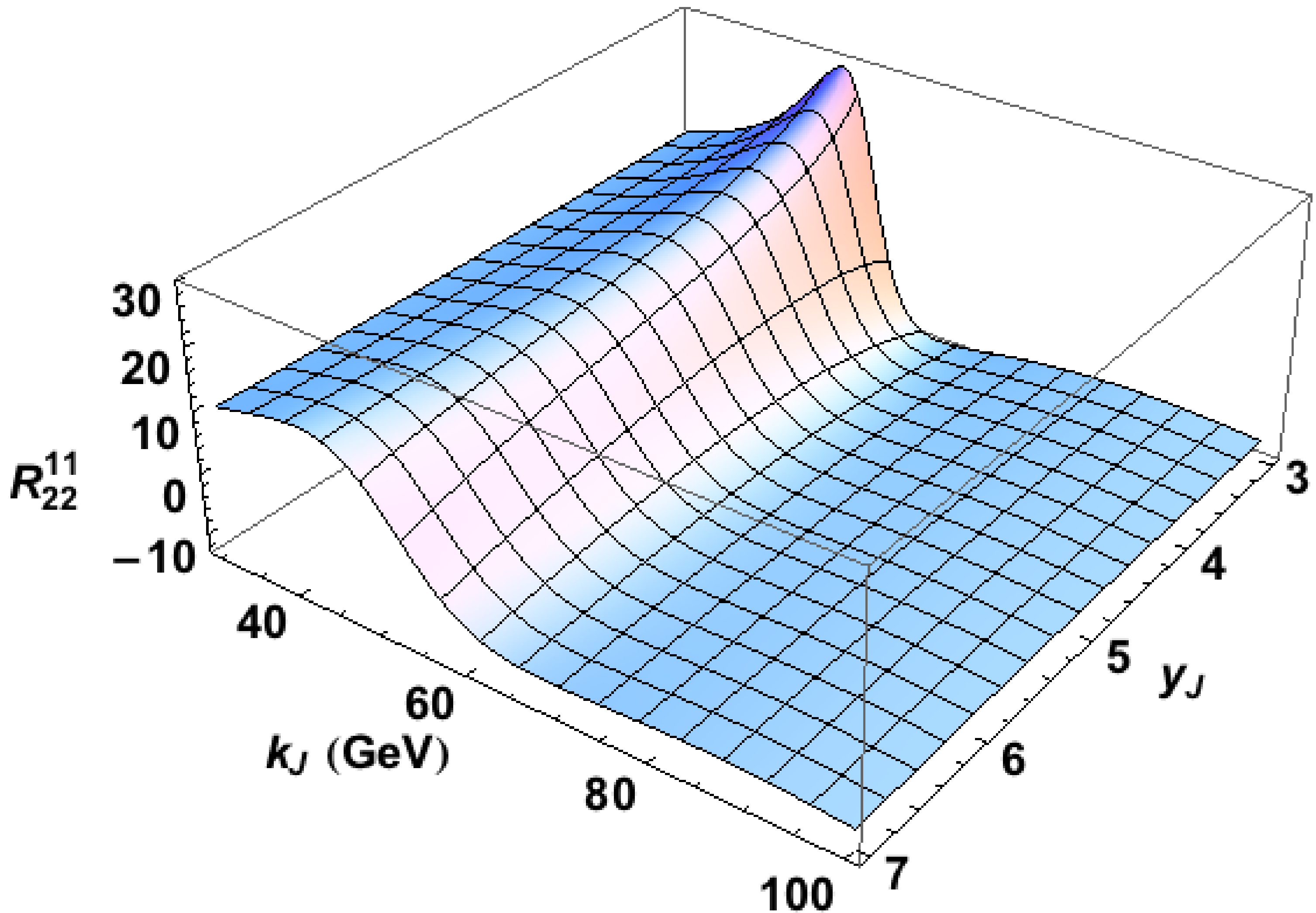
Partonic behaviour of R^{12}_{22}



Partonic behaviour of R^{21}_{22}

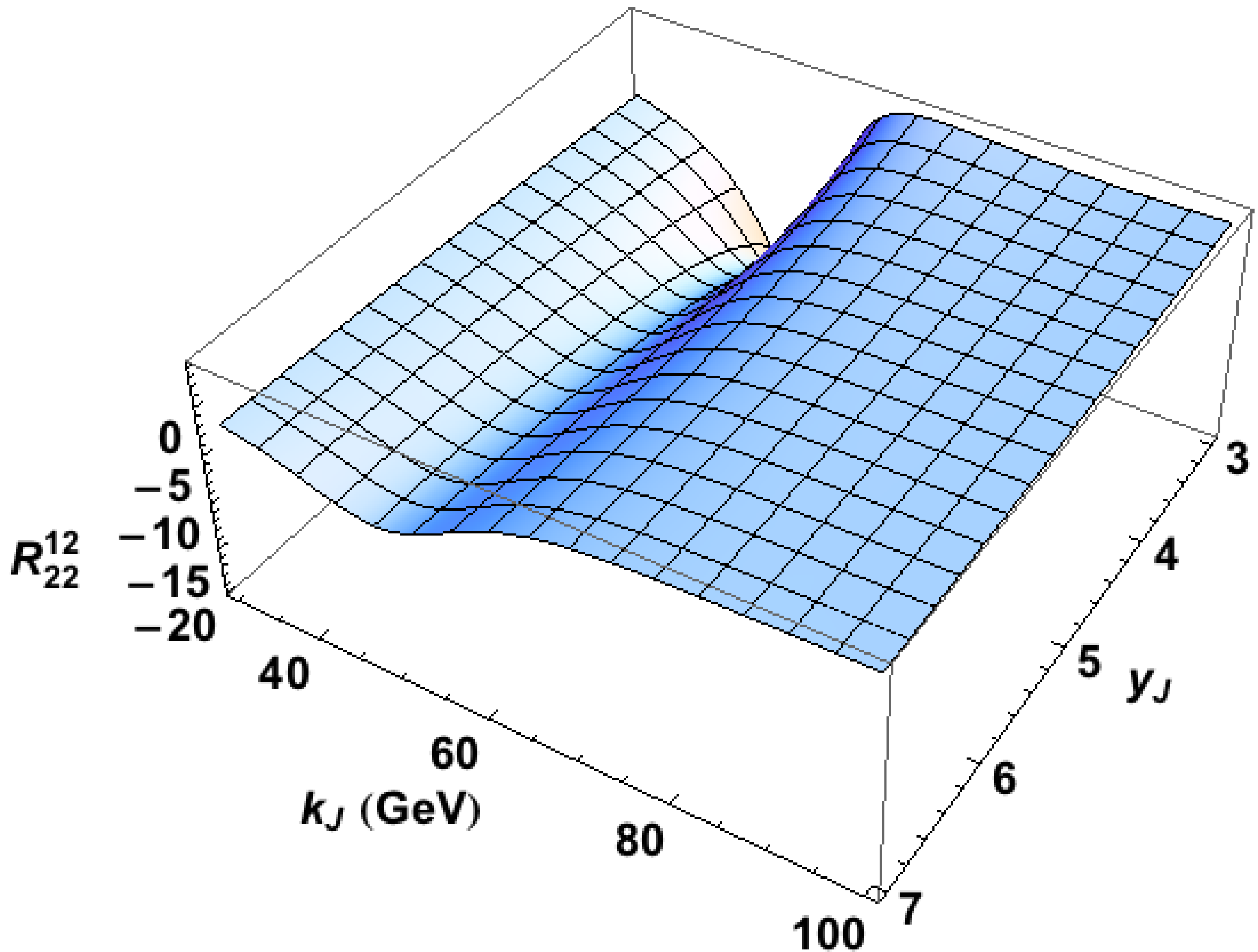


3D plot for (partonic) R^{11}_{22}



k_A = 40 GeV, k_B = 50 GeV, Y_A = 10, Y_B = 0

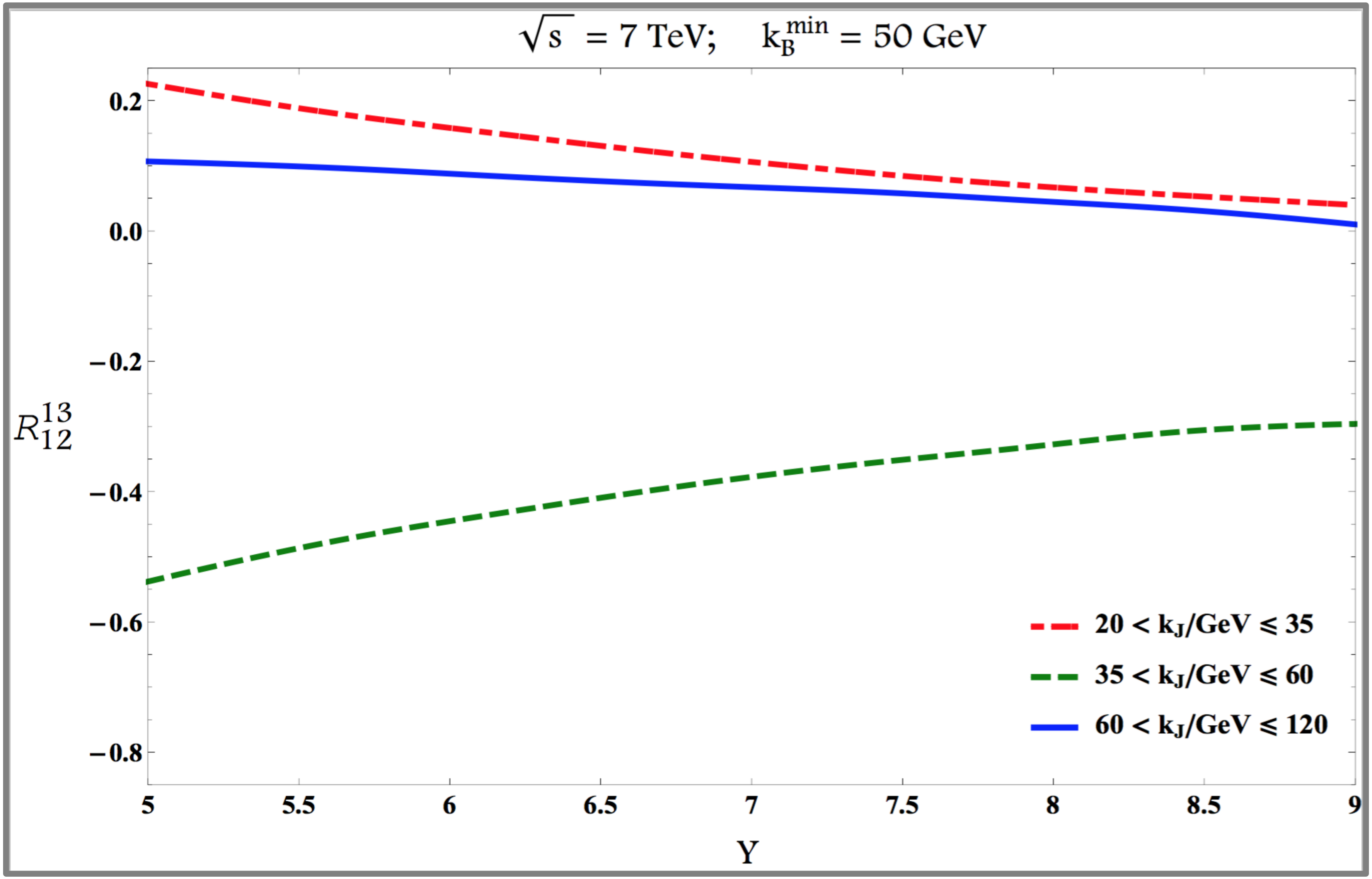
3D plot for (partonic) R^{12}_{22}



k_A = 40 GeV, k_B = 50 GeV, Y_A = 10, Y_B = 0

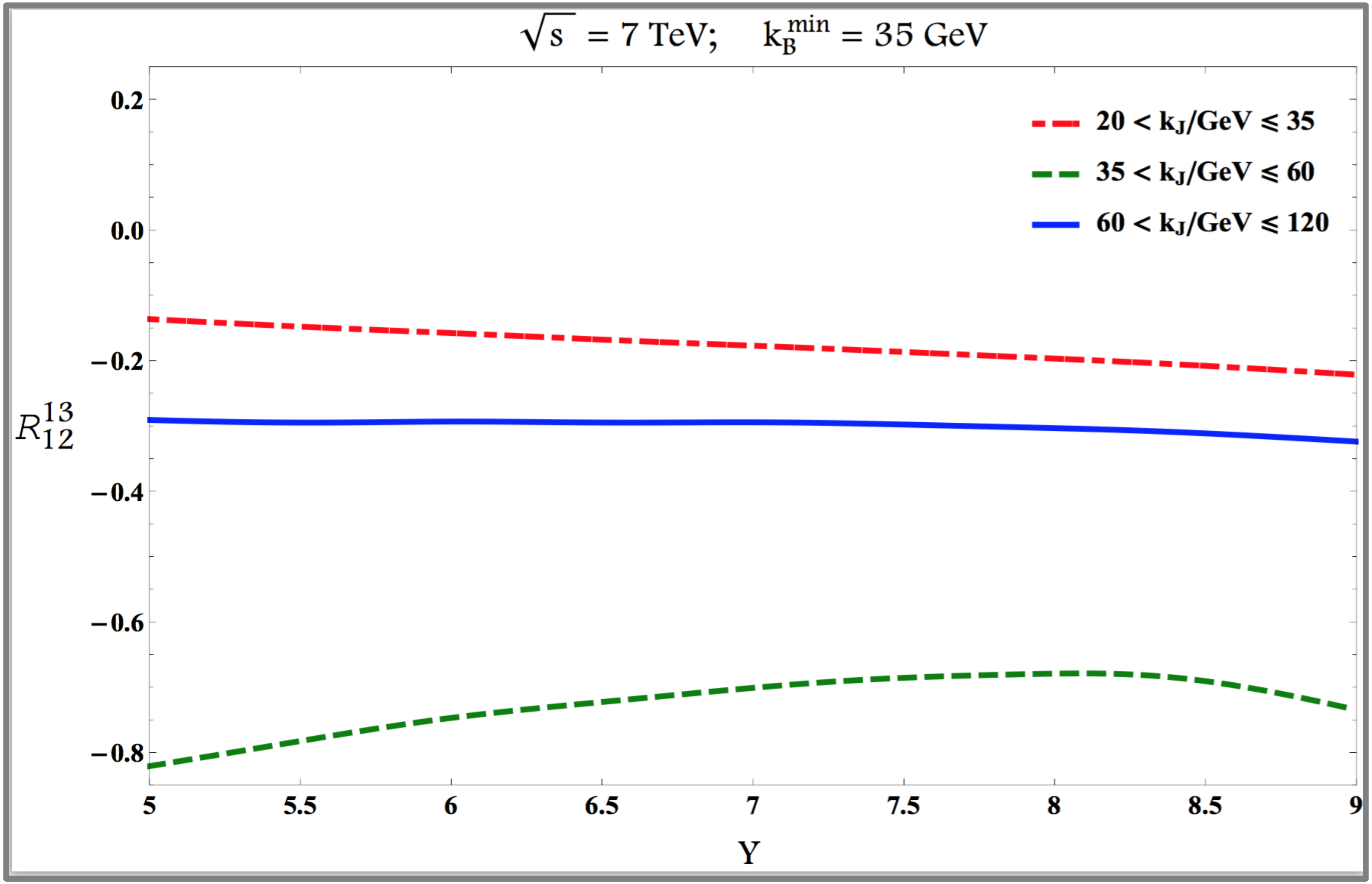
Now introduce PDF's and running of the strong coupling to get theoretical predictions on a hadronic level

$$k_A^{\min} = 35 \text{ GeV}, k_B^{\min} = 50 \text{ GeV}, k_A^{\max} = k_B^{\max} = 60 \text{ GeV (asymmetric)}$$



Y is the rapidity difference between the most forward/backward jet

$$k_A^{\min} = 35 \text{ GeV}, \; k_B^{\min} = 35 \text{ GeV}, \; k_A^{\max} = k_B^{\max} = 60 \text{ GeV (symmetric)}$$



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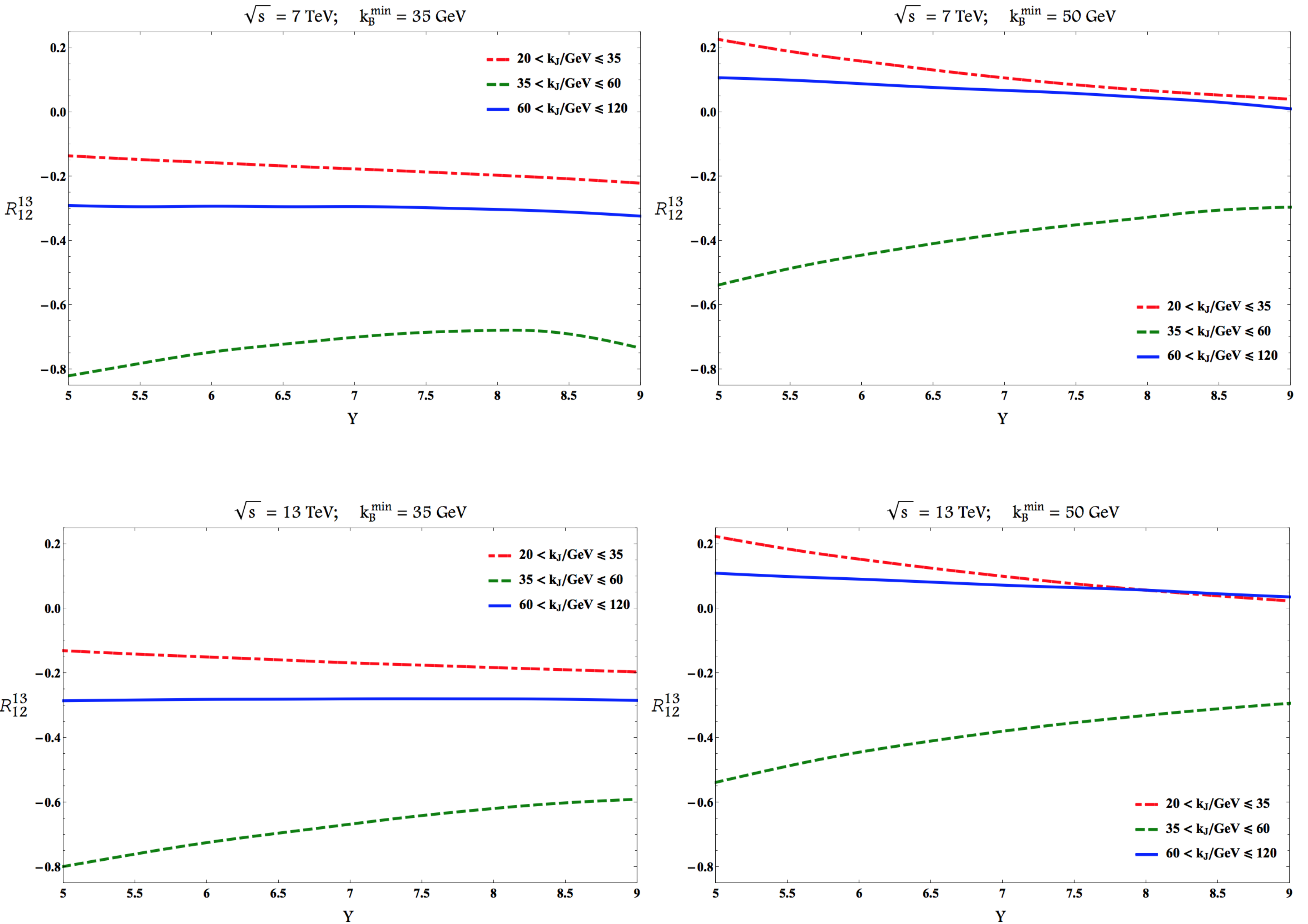


$\sqrt{s}$ = 7 TeV; k_B^{min} = 35 GeV
$\sqrt{s}$ = 7 TeV; k_B^{min} = 50 GeV
$\sqrt{s}$ = 13 TeV; k_B^{min} = 35 GeV
$\sqrt{s}$ = 13 TeV; k_B^{min} = 50 GeV
$20 < k_J/\text{GeV} \leq 35$
$35 < k_J/\text{GeV} \leq 60$
$60 < k_J/\text{GeV} \leq 120$
R_{12}^{13}
Y
0.2
0.0
−0.2
−0.4
−0.6
−0.8
5
5.5
6
6.5
7
7.5
8
8.5
9

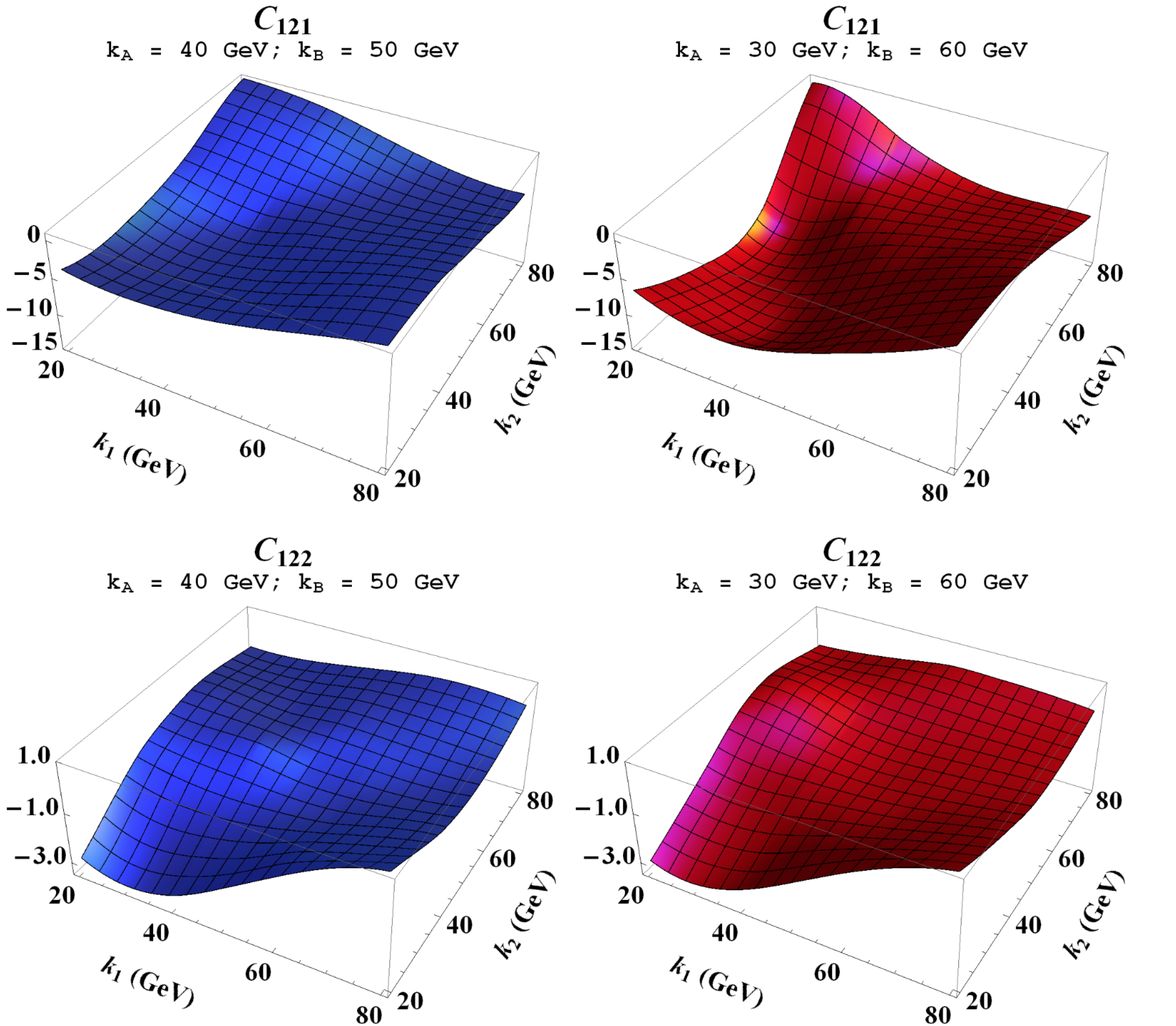
Now what about the 4-jets?

Easy to imagine the generalization:

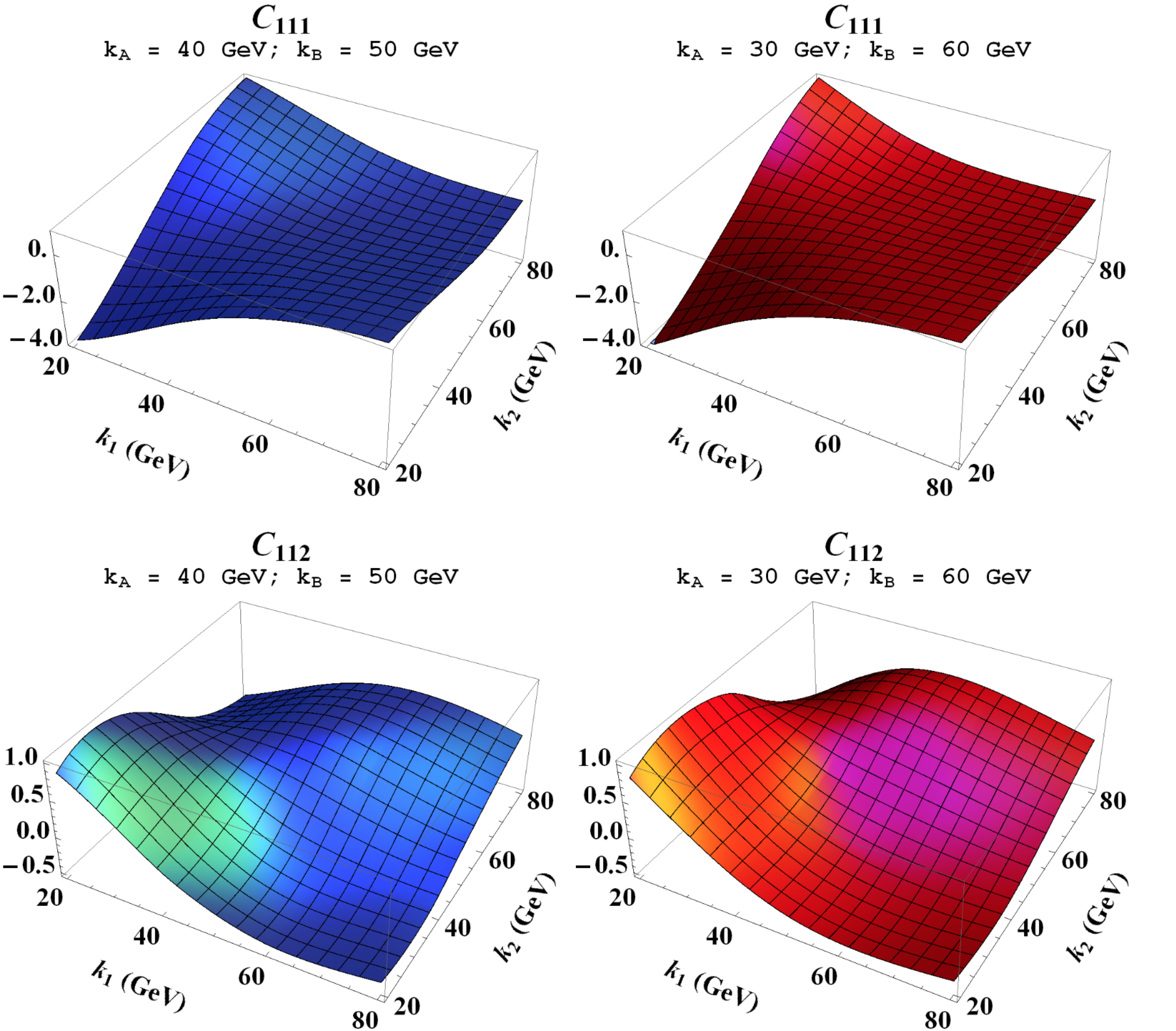
$$\mathcal{R}^{MNL}_{PQR} = \frac{\langle \cos(M(\vartheta_A - \vartheta_1 - \pi)) \cos(N(\vartheta_1 - \vartheta_2 - \pi)) \cos(L(\vartheta_2 - \vartheta_B - \pi)) \rangle}{\langle \cos(P(\vartheta_A - \vartheta_1 - \pi)) \cos(Q(\vartheta_1 - \vartheta_2 - \pi)) \cos(R(\vartheta_2 - \vartheta_B - \pi)) \rangle}$$

At partonic level, we see interesting patterns similar to the oscillation modes of a two-dimensional membrane.

Clearly, this is something we need to investigate further, it is just mentioned here only to intrigue you.



C121
kA = 40 GeV; kB = 50 GeV
0
−5
−10
−15
20
40
60
80
k1 (GeV)
20
40
60
80
k2 (GeV)
C121
kA = 30 GeV; kB = 60 GeV
0
−5
−10
−15
20
40
60
80
k1 (GeV)
20
40
60
80
k2 (GeV)
C122
kA = 40 GeV; kB = 50 GeV
1.0
−1.0
−3.0
20
40
60
80
k1 (GeV)
20
40
60
80
k2 (GeV)
C122
kA = 30 GeV; kB = 60 GeV
1.0
−1.0
−3.0
20
40
60
80
k1 (GeV)
20
40
60
80
k2 (GeV)



C_{111}
k_A = 40 GeV; k_B = 50 GeV
C_{111}
k_A = 30 GeV; k_B = 60 GeV
C_{112}
k_A = 40 GeV; k_B = 50 GeV
C_{112}
k_A = 30 GeV; k_B = 60 GeV
0.
−2.0
−4.0
1.0
0.5
0.0
−0.5
20
40
60
80
k_1 (GeV)
k_2 (GeV)

Enough with the transverse degrees of freedom*, what can we study that is directly connected to rapidities?

Let us assume multi-jet events (anything above three jets in the final state)

*One should be careful here, we do not imply that there is such thing as completely decoupled transverse degrees of freedom. When we study azimuthal decorralations we still have an explicit dependence on the rapidity parameters.

The high-energy radiation pattern from the **BFKLex*** Monte Carlo

***This is an implementation of the iterative solution of the BFKL equation as a Monte Carlo code.**

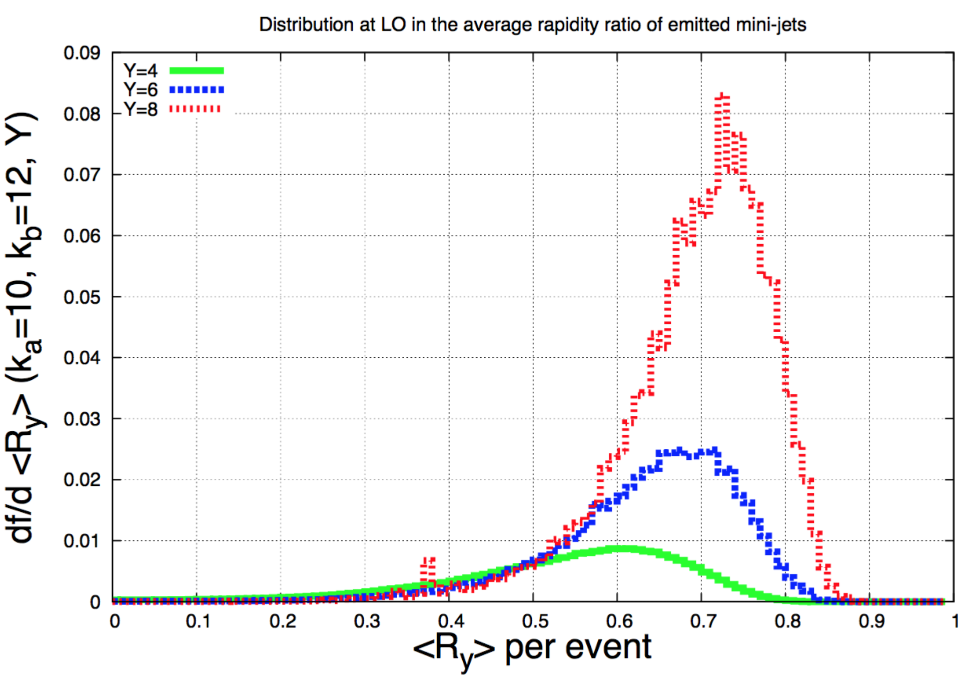
Present status:

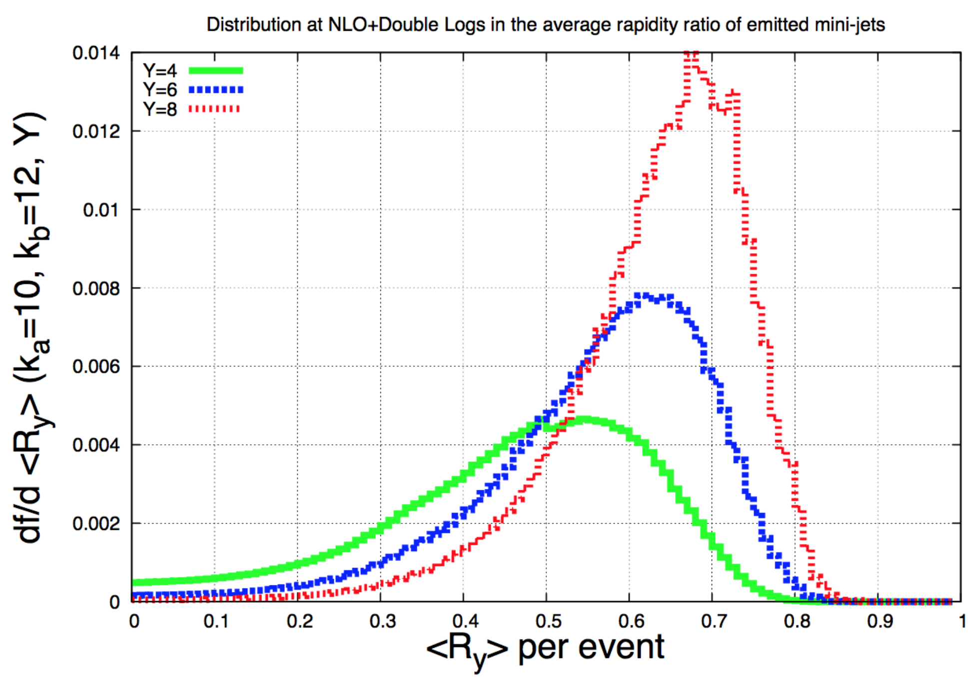
- NLO BFKL, collinearly improved
- Interfaced with PDF's and FastJet

The proposal of a new rapidity-related observable

$$\langle \mathcal{R}_y \rangle = \frac{1}{N+1} \sum_{i=1}^{N+1} \frac{y_i}{y_{i-1}}$$

$$y_0 = y_a,\ y_{N+1} = y_b = 0 \text{ and } y_{i-1} > y_i$$



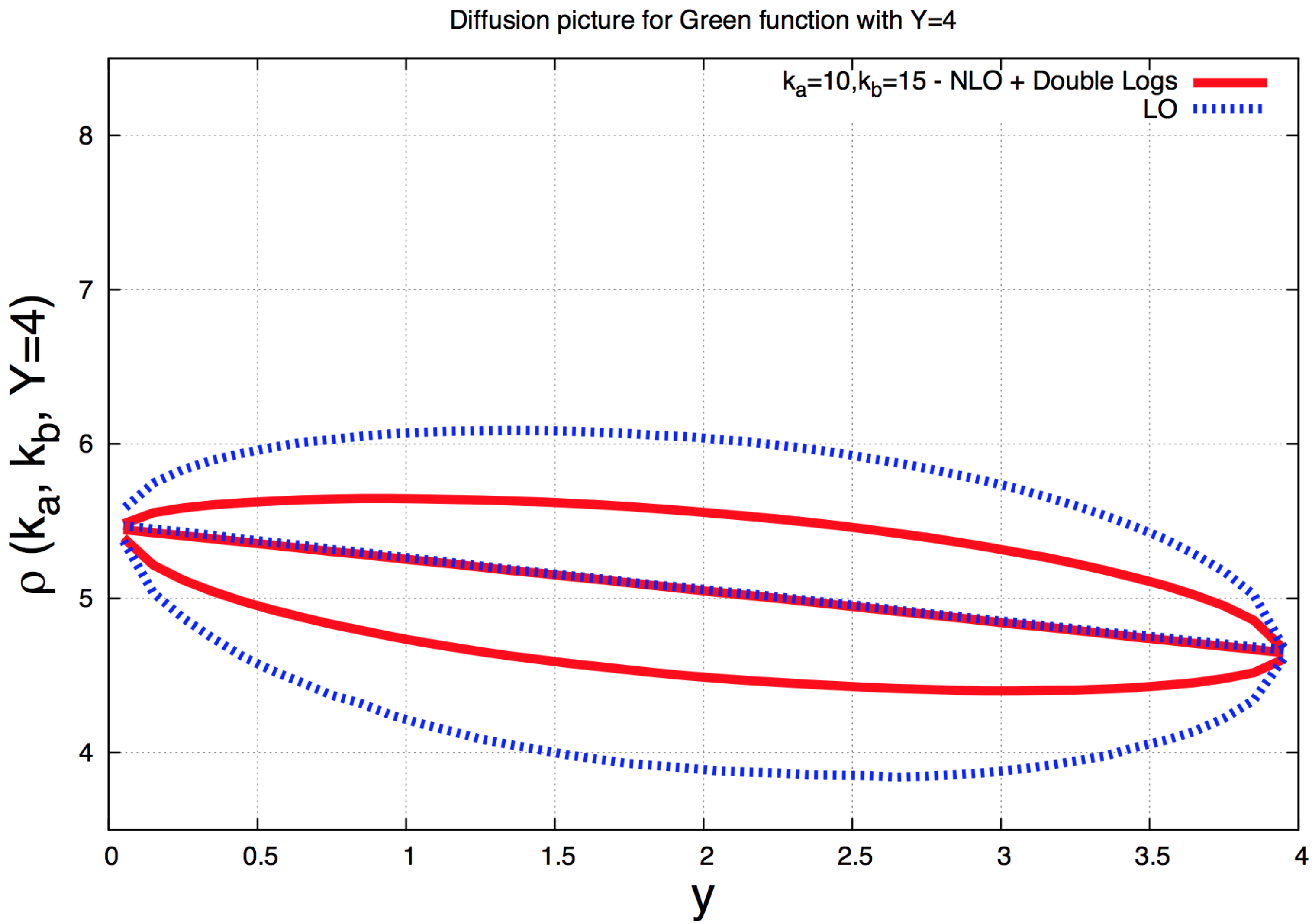


Conclusions & Outlook

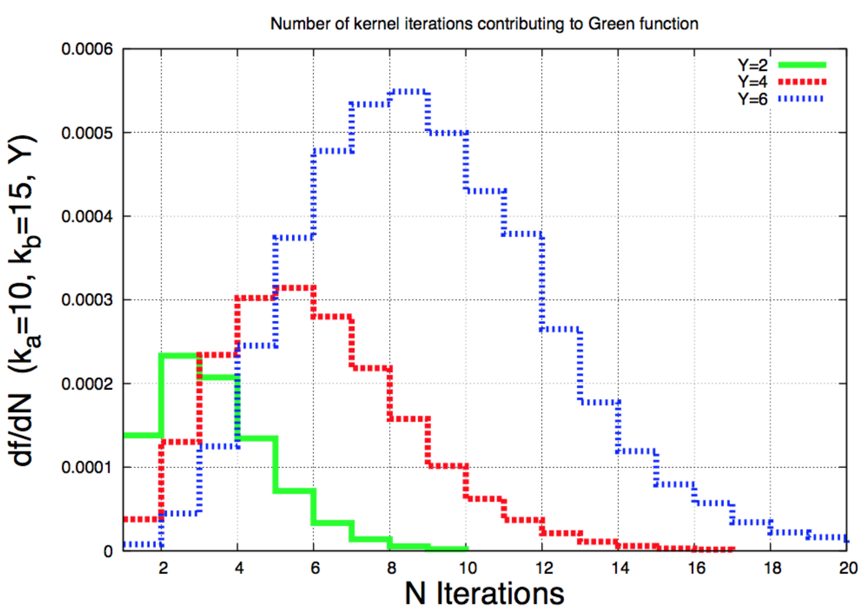
- We use events with three and four tagged jets to propose new observables with a distinct signal of BFKL dynamics.
- We use ratios of correlation functions to minimize the influence of higher order corrections.
- We also propose the study of the average rapidity ratio between subsequent jets in multi-jet events.
- We need to compare against experimental data to see whether these new BFKL probes deliver results that outline the window of applicability of the BFKL framework at the LHC.
- Any new input from the experimental side would be extremely valuable

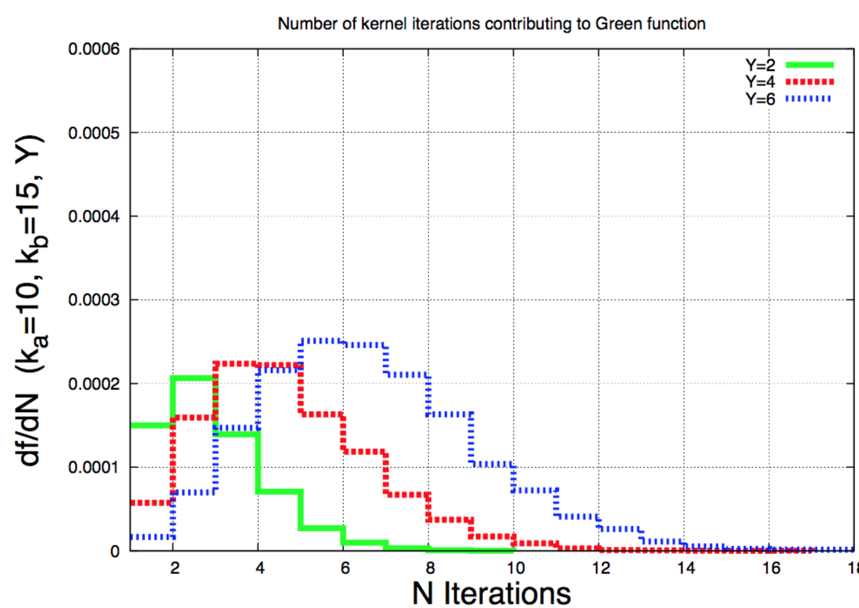
Backup slides

BFKLex Monte Carlo interesting facts: Diffusion



BFKLex Monte Carlo interesting facts: “multiplicity”





1. Integrate over the angle difference of k_A and k_B and also over the angle of the central jet

$$\frac{d^3\sigma^{3-\text{jet}}}{d^2\vec{k}_J dy_J} = \frac{\bar{\alpha}_s}{\pi k_J^2} \int d^2\vec{p}_A \int d^2\vec{p}_B\, \delta^{(2)}\left(\vec{p}_A + \vec{k}_J - \vec{p}_B\right) \times \varphi\left(\vec{k}_A, \vec{p}_A, Y_A - y_J\right) \varphi\left(\vec{p}_B, \vec{k}_B, y_J - Y_B\right)$$

$$\int_0^{2\pi} d\Delta\phi \cos\left(M\Delta\phi\right) \int_0^{2\pi} d\theta_J \frac{d^3\sigma^{3-\text{jet}}}{d^2\vec{k}_J dy_J}$$

$$= \bar{\alpha}_s \sum_{L=0}^{M} \int_0^{\infty} dp^2 \int_0^{2\pi} d\theta \frac{(-1)^M \binom{M}{L} \left(k_J^2\right)^{\frac{L-1}{2}} \left(p^2\right)^{\frac{M-L}{2}} \cos\left(L\,\theta\right)}{\sqrt{\left(p^2 + k_J^2 + 2\sqrt{p^2 k_J^2}\cos\theta\right)^M}}$$

$$\times \phi_M\left(p_A^2, p^2, Y_A - y_J\right) \phi_M\left(p^2 + k_J^2 + 2\sqrt{p^2 k_J^2}\cos\theta, p_B^2, y_J - Y_B\right)$$

$$\Delta\phi \equiv \theta_A - \theta_B - \pi$$

1. Integrate over the angle difference of k_A and k_B and also over the angle of the central jet

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$$\times \phi_M\left(p_A^2, p^2, Y_A - y_J\right) \phi_M\left(p^2 + k_J^2 + 2\sqrt{p^2 k_J^2}\cos\theta, p_B^2, y_J - Y_B\right)$$

where:

$$\phi_n\left(p_A^2, p_B^2, Y\right) = 2\int_0^{\infty} d\nu \cos\left(\nu \ln \frac{p_A^2}{p_B^2}\right) \frac{e^{\bar{\alpha}_s \chi_{|n|}(\nu) Y}}{\pi\sqrt{p_A^2 p_B^2}},$$

$$\chi_n\left(\nu\right) = 2\psi(1) - \psi\left(\frac{1+n}{2} + i\nu\right) - \psi\left(\frac{1+n}{2} - i\nu\right)$$

1. Integrate over the angle difference of k_A and k_B and also over the angle of the central jet

$$\int_0^{2\pi} d\Delta\phi \cos(M\Delta\phi) \int_0^{2\pi} d\theta_J \frac{d^3\sigma^{3-\text{jet}}}{d^2\vec{k}_J dy_J}$$

$$= \bar{\alpha}_s \sum_{L=0}^{M} \int_0^{\infty} dp^2 \int_0^{2\pi} d\theta \frac{(-1)^M \binom{M}{L} \left(k_J^2\right)^{\frac{L-1}{2}} \left(p^2\right)^{\frac{M-L}{2}} \cos(L\,\theta)}{\sqrt{\left(p^2 + k_J^2 + 2\sqrt{p^2 k_J^2}\cos\theta\right)^M}}$$

$$\times \phi_M\left(p_A^2, p^2, Y_A - y_J\right) \phi_M\left(p^2 + k_J^2 + 2\sqrt{p^2 k_J^2}\cos\theta, p_B^2, y_J - Y_B\right)$$

$$\langle \cos(M(\theta_A - \theta_B - \pi))\rangle = \frac{\int_0^{2\pi} d\Delta\phi \cos(M\Delta\phi) \int_0^{2\pi} d\theta_J \frac{d^3\sigma^{3-\text{jet}}}{d^2\vec{k}_J dy_J}}{\int_0^{2\pi} d\Delta\phi \int_0^{2\pi} d\theta_J \frac{d^3\sigma^{3-\text{jet}}}{d^2\vec{k}_J dy_J}}$$

$$\mathcal{R}_N^M = \frac{\langle \cos(M(\theta_A - \theta_B - \pi))\rangle}{\langle \cos(N(\theta_A - \theta_B - \pi))\rangle}$$

1. ...and then plot for different k_J