

HELAC - PHEGAS : automatic helicity amplitude calculation and parton level generation

Costas G. Papadopoulos

MC4LHC: from parton showers to NNLO, May 7, 2009, CERN

Contributors - Contact Persons

Costas Papadopoulos

costas.papadopoulos@cern.ch

Malgorzata Worek

worek@particle.uni-karlsruhe.de

Alessandro Cafarella

cafarella@inp.demokritos.gr

Web page

<http://www.cern.ch/helac-phegas>

HEP - NCSR Democritos

- Reliable cross section computation and event generation for multiparticle processes, with $\sim 10\text{-}12$ particles in the final state.

- Reliable cross section computation and event generation for multiparticle processes, with $\sim 10\text{-}12$ particles in the final state.
- – **HELAC:** [A.Kanaki and C.G.Papadopoulos, CPC 132 \(2000\) 306, hep-ph/0002082.](#)
Matrix element computation algorithm, based on **Dyson-Schwinger equations**, including: EWK, QCD, fermion masses, reliable arithmetic, running couplings and masses

- Reliable cross section computation and event generation for multiparticle processes, with $\sim 10\text{-}12$ particles in the final state.
- – **HELAC:** [A.Kanaki and C.G.Papadopoulos, CPC 132 \(2000\) 306, hep-ph/0002082.](#)
Matrix element computation algorithm, based on **Dyson-Schwinger equations**, including: EWK, QCD, fermion masses, reliable arithmetic, running couplings and masses
- – **PHEGAS:** [C.G.Papadopoulos, CPC 137 \(2001\) 247, hep-ph/0007335](#)
Monte-Carlo phase space integration/generation based on optimized multichannel approach.

- Reliable cross section computation and event generation for multiparticle processes, with $\sim 10\text{-}12$ particles in the final state.
- – **HELAC:** [A.Kanaki and C.G.Papadopoulos, CPC 132 \(2000\) 306, hep-ph/0002082.](#)
Matrix element computation algorithm, based on **Dyson-Schwinger equations**, including: EWK, QCD, fermion masses, reliable arithmetic, running couplings and masses
- – **PHEGAS:** [C.G.Papadopoulos, CPC 137 \(2001\) 247, hep-ph/0007335](#)
Monte-Carlo phase space integration/generation based on optimized multichannel approach.

[hep-ph/0012004](#) and Tokyo 2001,(CPP2001) Computational particle physics, p. 20-25

[T. Gleisberg, et al. Eur. Phys. J. C 34 \(2004\) 173](#)

- Give the process



- Give the process



- Define the cuts

$$E_i > E_0 \quad \cos \theta_i < c_0 \quad \cos \theta_{ij} < c_1$$

- Give the process



- Define the cuts

$$E_i > E_0 \quad \cos \theta_i < c_0 \quad \cos \theta_{ij} < c_1$$

- The cross section is:

$$\sigma = 1.73(5) \text{ fb at } \sqrt{s} = 500 \text{ GeV}$$

- ... plus any other kinematical distribution !

Old Feynman graphs → computational cost $\sim n!$

HEP - NCSR Democritos

Old Feynman graphs → computational cost $\sim n!$

New Dyson-Schwinger → computational cost $\sim 3^n$

P.Draggiotis, R.H.Kleiss and C.G.Papadopoulos, Phys. Lett. B439 (1998) 157

F. Caravaglios and M. Moretti, Phys. Lett. B358 (1995) 332

F. A. Berends and W. T. Giele, Nucl. Phys. B 306 (1988) 759.

Old Feynman graphs \rightarrow computational cost $\sim n!$

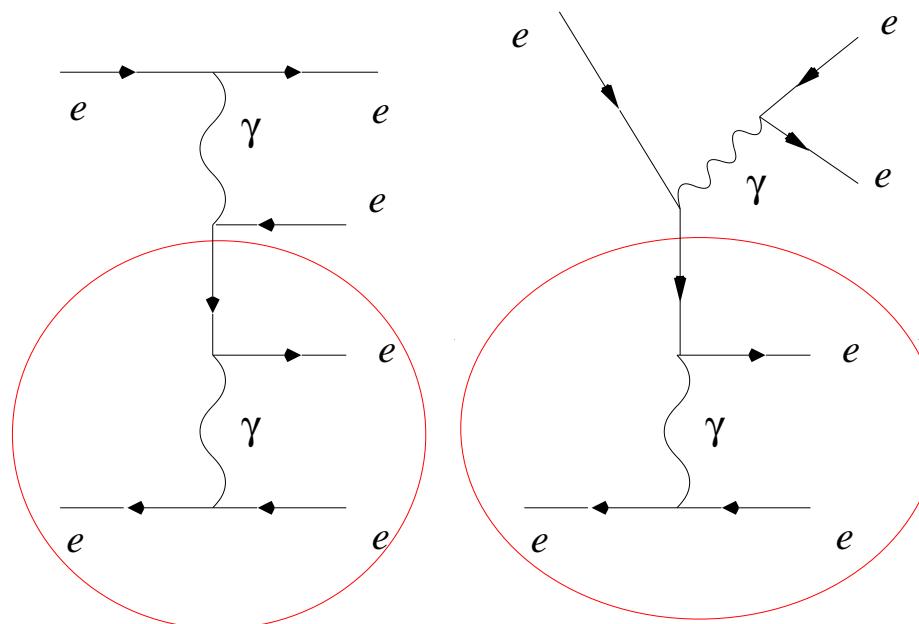
New Dyson-Schwinger \rightarrow computational cost $\sim 3^n$

P.Draggiotis, R.H.Kleiss and C.G.Papadopoulos, Phys. Lett. B439 (1998) 157

F. Caravaglios and M. Moretti, Phys. Lett. B358 (1995) 332

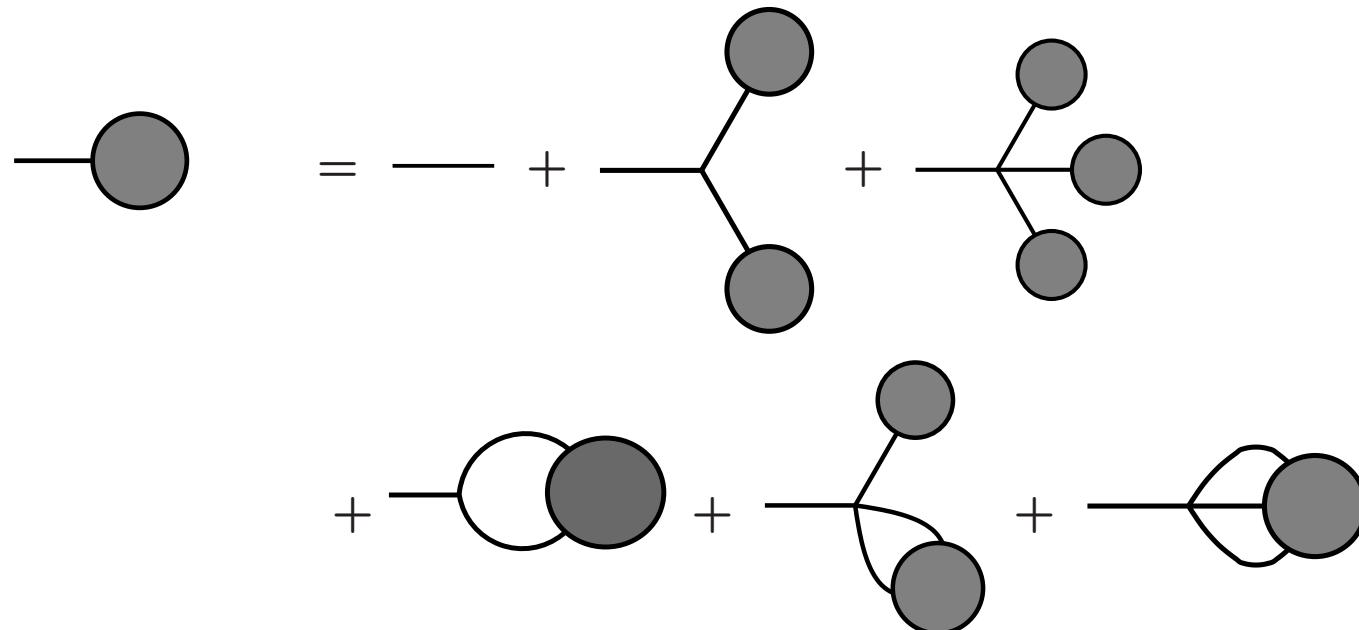
F. A. Berends and W. T. Giele, Nucl. Phys. B 306 (1988) 759.

- Example: $e^- e^+ \rightarrow e^- e^+ e^- e^+$ in QED:

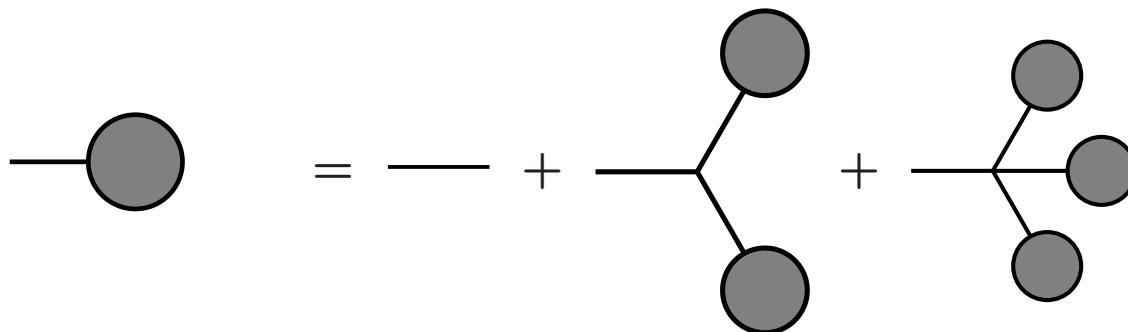


The Dyson-Schwinger recursion

- Imagine a theory with 3- and 4- point vertices and just one field. Then it is straightforward to write an equation that gives the amplitude for $1 \rightarrow n$



The Dyson-Schwinger recursion



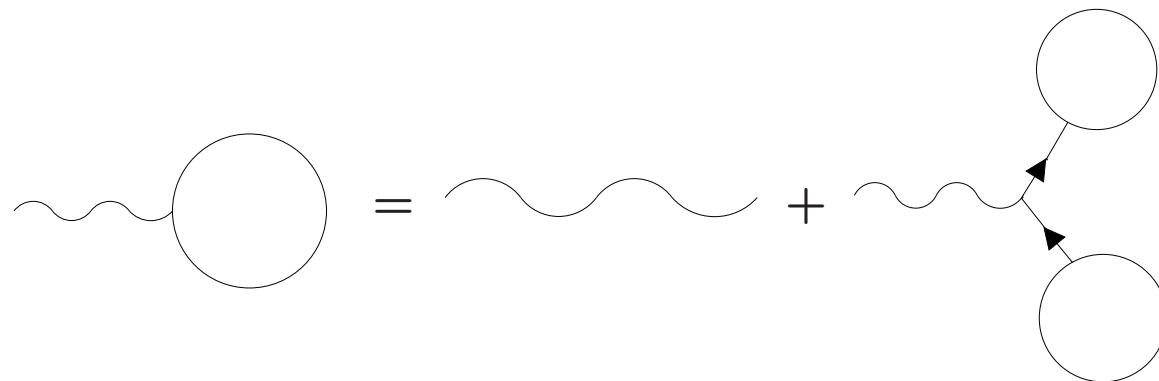
$$a(n) = \delta_{n,1} + \sum \frac{n!}{n_1!n_2!} a(n_1)a(n_2)\delta_{n_1+n_2,n}$$
$$+ \frac{n!}{n_1!n_2!n_3!} \sum a(n_1)a(n_2)a(n_3)\delta_{n_1+n_2+n_3,n}$$

⇒ Systematic approach:

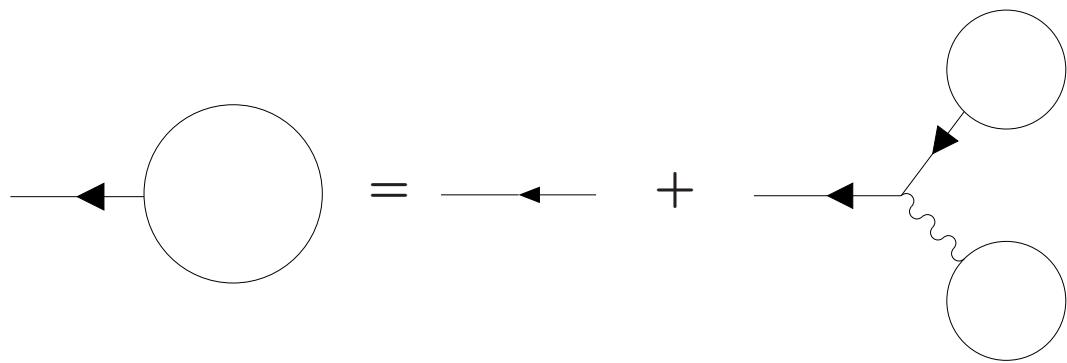
$$b_\mu(P) = \sim\circlearrowleft \quad \psi(P) = \leftarrow\circlearrowleft \quad \bar{\psi}(P) = \rightarrow\circlearrowright$$

⇒ Systematic approach:

$$b_\mu(P) = \text{---} \circlearrowleft \quad \psi(P) = \leftarrow \circlearrowleft \quad \bar{\psi}(P) = \rightarrow \circlearrowright$$



$$b^\mu(P) = \sum_{i=1}^n \delta_{P=p_i} b^\mu(p_i) + \sum_{P=P_1+P_2} (ig) \Pi_\nu^\mu \bar{\psi}(P_2) \gamma^\nu \psi(P_1) \epsilon(P_1, P_2)$$



$$\psi(P) = \sum_{i=1}^n \delta_{P=p_i} \psi(p_i) + \sum_{P=P_1+P_2} (ig) \not{b}(P_2) \frac{(P_1+m)}{P_1^2 - m^2} \psi(P_1) \epsilon(P_1, P_2)$$

- ◆ Let n external particles with momenta $p_i^\mu, i = 1 \dots, n$, and define the momentum P^μ

$$P^\mu = \sum_{i \in I} p_i^\mu , \quad I \subset \{1, \dots, n\}$$

- ◆ Let n external particles with momenta $p_i^\mu, i = 1 \dots, n$, and define the momentum P^μ

$$P^\mu = \sum_{i \in I} p_i^\mu , \quad I \subset \{1, \dots, n\}$$

- ◆ the binary vector $\vec{m} = (m_1, \dots, m_n)$, where its components take the values 0 or 1 :

$$P^\mu = \sum_{i=1}^n m_i p_i^\mu .$$

- ◆ Let n external particles with momenta $p_i^\mu, i = 1 \dots, n$, and define the momentum P^μ

$$P^\mu = \sum_{i \in I} p_i^\mu , \quad I \subset \{1, \dots, n\}$$

- ◆ the binary vector $\vec{m} = (m_1, \dots, m_n)$, where its components take the values 0 or 1 :

$$P^\mu = \sum_{i=1}^n m_i p_i^\mu .$$

- ◆ Moreover this binary vector can be uniquely represented by the integer

$$m = \sum_{i=1}^n 2^{i-1} m_i , \quad 0 \leq m \leq 2^n - 1$$

- ◆ Let n external particles with momenta $p_i^\mu, i = 1 \dots, n$, and define the momentum P^μ

$$P^\mu = \sum_{i \in I} p_i^\mu , \quad I \subset \{1, \dots, n\}$$

- ◆ the binary vector $\vec{m} = (m_1, \dots, m_n)$, where its components take the values 0 or 1 :

$$P^\mu = \sum_{i=1}^n m_i p_i^\mu .$$

- ◆ Moreover this binary vector can be uniquely represented by the integer

$$m = \sum_{i=1}^n 2^{i-1} m_i , \quad 0 \leq m \leq 2^n - 1$$

- ◆ Replace

$$b_\mu(P) \rightarrow b_\mu(m) .$$

- ♣ Convenient ordering of integers in binary representation \Rightarrow level l , defined by

$$l = \sum_{i=1}^n m_i .$$

- ♣ Convenient ordering of integers in binary representation \Rightarrow level l , defined by

$$l = \sum_{i=1}^n m_i .$$

- ♣ external momenta are of level 1

- ♣ Convenient ordering of integers in binary representation \Rightarrow level l , defined by

$$l = \sum_{i=1}^n m_i .$$

- ♣ external momenta are of level 1
- ♣ the total amplitude corresponds to the unique level n integer $2^n - 1$

$$\mathcal{A} = b(1) \cdot b(2^n - 2)$$

- ♣ Convenient ordering of integers in binary representation \Rightarrow level l , defined by

$$l = \sum_{i=1}^n m_i .$$

- ♣ external momenta are of level 1
- ♣ the total amplitude corresponds to the unique level n integer $2^n - 1$

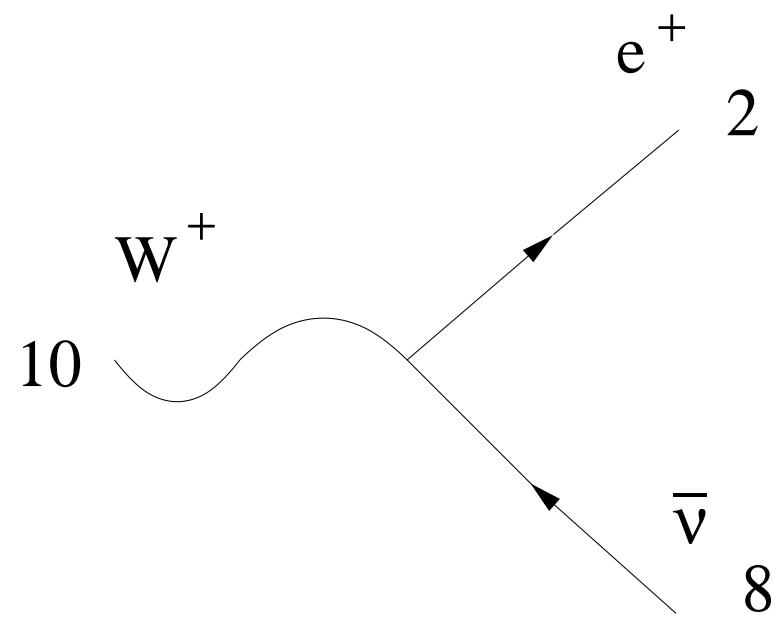
$$\mathcal{A} = b(1) \cdot b(2^n - 2)$$

This ordering dictates the natural path of the computation : starting with level-1 sub-amplitudes, we compute the level-2 ones using the Dyson-Schwinger equations and so on up to level $n - 1$

The solution

$e^-(1) \ e^+(2) \rightarrow e^-(4) \ \bar{\nu}_e(8) \ u(16) \ \bar{d}(32)$

1	10	33	2	-2	8	1
1	12	33	4	-2	8	1
1	48	34	16	-3	32	4
2	26	-4	10	33	16	-3
		...				
2	62	-2	10	33	52	-1
2	62	-2	12	33	50	-1
2	62	-2	58	31	4	-2
2	62	-2	58	32	4	-2
2	62	-2	60	31	2	-2
2	62	-2	60	32	2	-2



HEP - NCSR Democritos

- Dirac algebra simplification: **2-dim vs 4-dim** and chiral representation, including $m_f \neq 0$.
- The sign factor:

$$\epsilon(P_1, P_2) \rightarrow \epsilon(m_1, m_2)$$

we define

$$\epsilon(m_1, m_2) = (-1)^{\chi(m_1, m_2)}$$

$$\chi(m_1, m_2) = \sum_{i=n}^2 \hat{m}_{1i} \left(\sum_{j=1}^{i-1} \hat{m}_{2j} \right)$$

where hatted components are set to 0 if the corresponding external particle is a boson.

- Full EWK theory, both Unitary and Feynman gauges.

[A. Denner, Fortsch. Phys. 41, 307 \(1993\).](#)

HELAC

- Construction of the skeleton solution of the Dyson-Schwinger equations. At this stage only integer arithmetic is performed. This is part of the initialization phase.
- Dressing-up the skeleton with momenta, provided by PHEGAS and wave functions, propagators, n -point functions in general.
- Unitary and Feynman gauges implemented. Due to multi-precision arithmetic, tests of gauge invariance can be extended to arbitrary precision.
- All fermions masses can be non-zero.
- All Electroweak and QCD vertices are implemented, including Higgs and would-be Goldstone bosons.

$$V^A = (V^0 + V_z, V^0 - V_z, V_x + iV_y, V_x - iV_y) , \quad A = 1, \dots, 4 .$$

Polarization state-vectors are given by

$$\begin{aligned}\epsilon_-^A &= \left(\frac{-p_T}{\sqrt{2}|\vec{p}|}, \frac{p_T}{\sqrt{2}|\vec{p}|}, \frac{(p_x + ip_y)(|\vec{p}| + p_z)}{\sqrt{2}|\vec{p}|p_T}, \frac{(p_x - ip_y)(-|\vec{p}| + p_z)}{\sqrt{2}|\vec{p}|p_T} \right) \\ \epsilon_+^A &= \left(\frac{p_T}{\sqrt{2}|\vec{p}|}, \frac{-p_T}{\sqrt{2}|\vec{p}|}, \frac{(p_x + ip_y)(|\vec{p}| - p_z)}{\sqrt{2}|\vec{p}|p_T}, \frac{(p_x - ip_y)(-|\vec{p}| - p_z)}{\sqrt{2}|\vec{p}|p_T} \right) \\ \epsilon_0^A &= \left(\frac{|\vec{p}|}{\sqrt{p^2}} + \frac{p_z p_0}{|\vec{p}| \sqrt{p^2}}, \frac{|\vec{p}|}{\sqrt{p^2}} - \frac{p_z p_0}{|\vec{p}| \sqrt{p^2}}, \frac{(p_x + ip_y)p_0}{|\vec{p}| \sqrt{p^2}}, \frac{(p_x - ip_y)p_0}{|\vec{p}| \sqrt{p^2}} \right)\end{aligned}$$

As for the Dirac matrices we are using the chiral representation. The wave functions which describe massive spinors are given by:

$$\begin{aligned}u_+(p) &= \begin{pmatrix} r/c \\ a(p_x + ip_y)/r \\ -mb/r \\ -m(p_x + ip_y)/r \end{pmatrix} & \bar{u}_+(p) &= \begin{pmatrix} mb/r \\ m(p_x - ip_y)/r \\ -r/c \\ -a(p_x - ip_y)/r \end{pmatrix} \\ u_-(p) &= \begin{pmatrix} m(p_x - ip_y)/r \\ -mb/r \\ -a(p_x - ip_y)/r \\ r/c \end{pmatrix} & \bar{u}_-(p) &= \begin{pmatrix} a(p_x + ip_y)/r \\ -r/c \\ -m(p_x + ip_y)/r \\ mb/r \end{pmatrix}\end{aligned}$$

$$v_+(p) = \begin{pmatrix} -m(p_x - ip_y)/r \\ mb/r \\ -a(p_x - ip_y)/r \\ r/c \end{pmatrix}$$

$$v_-(p) = \begin{pmatrix} r/c \\ a(p_x + ip_y)/r \\ mb/r \\ m(p_x + ip_y)/r \end{pmatrix}$$

$$\bar{v}_+(p) = \begin{pmatrix} a(p_x + ip_y)/r \\ -r/c \\ m(p_x + ip_y)/r \\ -mb/r \end{pmatrix}$$

$$\bar{v}_-(p) = \begin{pmatrix} -mb/r \\ -m(p_x - ip_y)/r \\ -r/c \\ -a(p_x - ip_y)/r \end{pmatrix}$$

where:

$$a = p_0 + |\vec{p}|, \quad b = p_z + |\vec{p}|, \quad c = 2|\vec{p}|, \quad r = \sqrt{abc}$$

For a massless particle the spinors are

$$u_R(p) = \begin{pmatrix} \sqrt{p_0 + p_z} \\ (p_x + ip_y)/\sqrt{p_0 + p_z} \\ 0 \\ 0 \end{pmatrix}$$

$$u_L(p) = \begin{pmatrix} 0 \\ 0 \\ -(p_x - ip_y)/\sqrt{p_0 + p_z} \\ \sqrt{p_0 + p_z} \end{pmatrix}$$

$$\bar{u}_R(p) = \begin{pmatrix} 0 \\ 0 \\ -\sqrt{p_0 + p_z} \\ -(p_x - ip_y)/\sqrt{p_0 + p_z} \end{pmatrix}$$

$$\bar{u}_L(p) = \begin{pmatrix} (p_x + ip_y)/\sqrt{p_0 + p_z} \\ -\sqrt{p_0 + p_z} \\ 0 \\ 0 \end{pmatrix}$$

$$V_\mu = \bar{\psi}(P_1)\gamma_\mu (g_R\omega_R + g_L\omega_L) \psi(P_2)$$

turns out to be

$$V^A = \begin{pmatrix} -g_R\psi_1\bar{\psi}_3 - g_L\psi_4\bar{\psi}_2 \\ -g_R\psi_2\bar{\psi}_4 - g_L\psi_3\bar{\psi}_1 \\ -g_R\psi_2\bar{\psi}_3 + g_L\psi_4\bar{\psi}_1 \\ -g_R\psi_1\bar{\psi}_4 + g_L\psi_3\bar{\psi}_2 \end{pmatrix}$$

where $\psi_i(\bar{\psi}_i)$, $i = 1, \dots, 4$ are the components of the spinor $\psi(P_2)$ ($\bar{\psi}(P_1)$) and

$$\omega_L = \frac{1}{2}(1 - \gamma_5), \quad \omega_R = \frac{1}{2}(1 + \gamma_5).$$

On the other hand, the spinor

$$u = (\not{P} + m)\not{\psi}(P_1)\omega_R\psi(P_2)$$

can be reduced to

$$u = \begin{pmatrix} (-b_2p_1 + b_3p_4)\psi_1 + (b_4p_1 - b_1p_4)\psi_2 \\ (b_3p_2 - b_2p_3)\psi_1 + (-b_1p_2 + b_4p_3)\psi_2 \\ m(b_2\psi_1 - b_4\psi_2) \\ m(-b_3\psi_1 + b_1\psi_2) \end{pmatrix}$$

where ψ_i , b_i , p_i , $i = 1, \dots, 4$ are the components of P , $b(P_1)$ and $\psi(P_2)$ respectively.

HELAC - the code

- A two-phase computation
 - phase A: selection and tree-construction (integer arithmetic)
 - phase B: computation (high-precision arithmetic)
- Comparisons: full technical agreement
 - EXCALIBUR $m_f = 0$ comparable speed
 - MADGRAPH $m_f \neq 0$ comparable speed for $n < 7$.
- As expected HELAC shows an exponential (instead of factorial) CPU-time growth.
F.A.Berends, C.G.Papadopoulos and R.Pittau, hep-ph/0002249.
- There is no a priori limitation for the number of particles that HELAC can treat, the only restrictions being that of memory allocation.

Numerical reliability

- Problem due to huge 'gauge' cancellations !
- In order to have a taste of a multi-precision computation we have computed the squared amplitude for the process

$$e^-(p_1)e^+(p_2) \rightarrow e^-(p_3)e^+(p_4)e^-(p_5)e^+(p_6)$$

at two phase space points.

- Phase space point (A) is just a randomly generated one by the phase-space generator RAMBO.
- Phase space point (B)

$$p_1^0/\text{GeV} = 100, \vec{p}_1 + \vec{p}_2 = 0,$$

$$p_3^0/p_1^0 = 0.9, \theta_3 = 0,$$

$$(p_5 + p_6)^2/(p_4 + p_5 + p_6)^2 = 0.1, \theta_4 = \phi_4 = \theta_5 = \phi_5 = 0$$

with $m_e = 0.511 \times 10^{-3}$ GeV.

- Results are provided for these points, by using the real*8 (DP), real*16 (QP) and 34-digit multi-precision version of the code.

David M. Smith, Transactions on Mathematical Software 17 (1991) 273.

(A)

1.539728523150595E-008

1.53972852315058854156763002825013D-08

1.53972852315058854156763002825011853M-8

(B)

1.256276706229023E+023

3.07162601093710915134136924973089D+22

3.07162601093710915127950109241770808M+22

- In the MP-version of the code the precision is defined by the user.

Colour Configuration - EWK \oplus QCD

HEP - NCSR Democritos

Colour Configuration - EWK \oplus QCD

- Ordinary approach $SU(N)$ -type

$$\mathcal{A}^{a_1 \dots a_n} = \sum Tr(T^{a_{\sigma_1}} \dots T^{a_{\sigma_n}}) A(\sigma_1 \dots \sigma_n)$$

Colour Configuration - EWK \oplus QCD

- Ordinary approach $SU(N)$ -type

$$\mathcal{A}^{a_1 \dots a_n} = \sum Tr(T^{a_{\sigma_1}} \dots T^{a_{\sigma_n}}) A(\sigma_1 \dots \sigma_n)$$

$$\mathcal{C}_{ij} = \sum Tr(T^{a_{\sigma_1}} \dots T^{a_{\sigma_n}}) Tr(T^{a_{\sigma'_1}} \dots T^{a_{\sigma'_n}})$$

Colour Configuration - EWK \oplus QCD

- Ordinary approach $SU(N)$ -type

$$\mathcal{A}^{a_1 \dots a_n} = \sum Tr(T^{a_{\sigma_1}} \dots T^{a_{\sigma_n}}) A(\sigma_1 \dots \sigma_n)$$

$$\mathcal{C}_{ij} = \sum Tr(T^{a_{\sigma_1}} \dots T^{a_{\sigma_n}}) Tr(T^{a_{\sigma'_1}} \dots T^{a_{\sigma'_n}})$$

Quarks and gluons treated differently

Colour Configuration - EWK \oplus QCD

HEP - NCSR Democritos

Colour Configuration - EWK \oplus QCD

- New approach $U(N)$ -type

Each color-configuration amplitude is proportional to

$$D_i = \delta_{1,\sigma_i(1)} \delta_{2,\sigma_i(2)} \dots \delta_{n,\sigma_i(n)}$$

where σ_i represents the i -th permutation of the set $1, 2, \dots, n$.

Colour Configuration - EWK \oplus QCD

- New approach $U(N)$ -type

Each color-configuration amplitude is proportional to

$$D_i = \delta_{1,\sigma_i(1)} \delta_{2,\sigma_i(2)} \dots \delta_{n,\sigma_i(n)}$$

where σ_i represents the i -th permutation of the set $1, 2, \dots, n$.

- ★ **quarks** $1 \dots n$
- ★ **antiquarks** $\sigma_i(1 \dots n)$ and
- ★ **gluons** $= q\bar{q}$

Colour Configuration - EWK \oplus QCD

- New approach $U(N)$ -type

Each color-configuration amplitude is proportional to

$$D_i = \delta_{1,\sigma_i(1)} \delta_{2,\sigma_i(2)} \dots \delta_{n,\sigma_i(n)}$$

where σ_i represents the i -th permutation of the set $1, 2, \dots, n$.

- ★ **quarks** $1 \dots n$
- ★ **antiquarks** $\sigma_i(1 \dots n)$ and
- ★ **gluons** $= q\bar{q}$

$$\mathcal{C}_{ij} = \sum D_i D_j = N_c^\alpha , \quad \alpha = \langle \sigma_1, \sigma_2 \rangle$$

Colour Configuration - EWK \oplus QCD

- New approach $U(N)$ -type

Each color-configuration amplitude is proportional to

$$D_i = \delta_{1,\sigma_i(1)} \delta_{2,\sigma_i(2)} \dots \delta_{n,\sigma_i(n)}$$

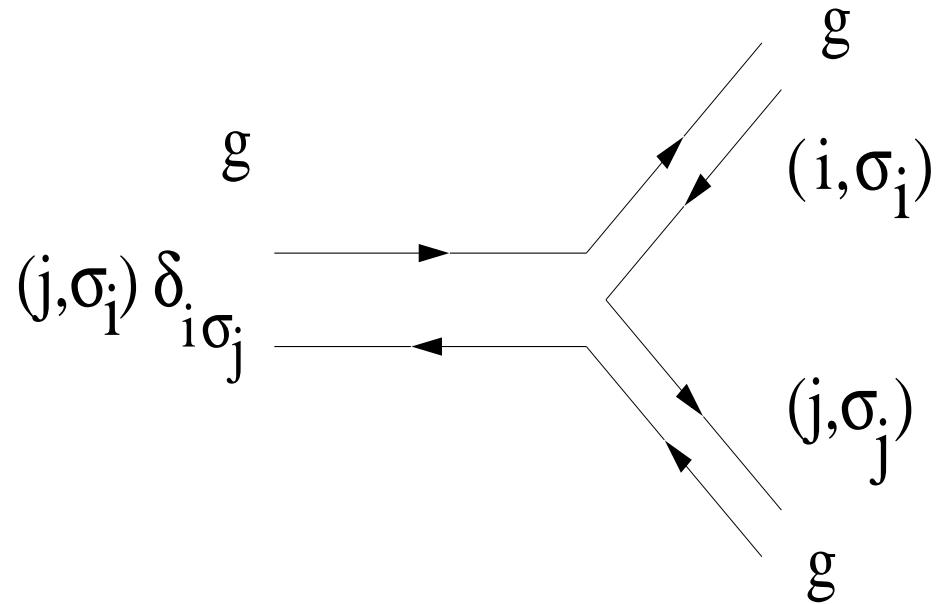
where σ_i represents the i -th permutation of the set $1, 2, \dots, n$.

- ★ **quarks** $1 \dots n$
- ★ **antiquarks** $\sigma_i(1 \dots n)$ and
- ★ **gluons** $= q\bar{q}$

$$\mathcal{C}_{ij} = \sum D_i D_j = N_c^\alpha , \quad \alpha = \langle \sigma_1, \sigma_2 \rangle$$

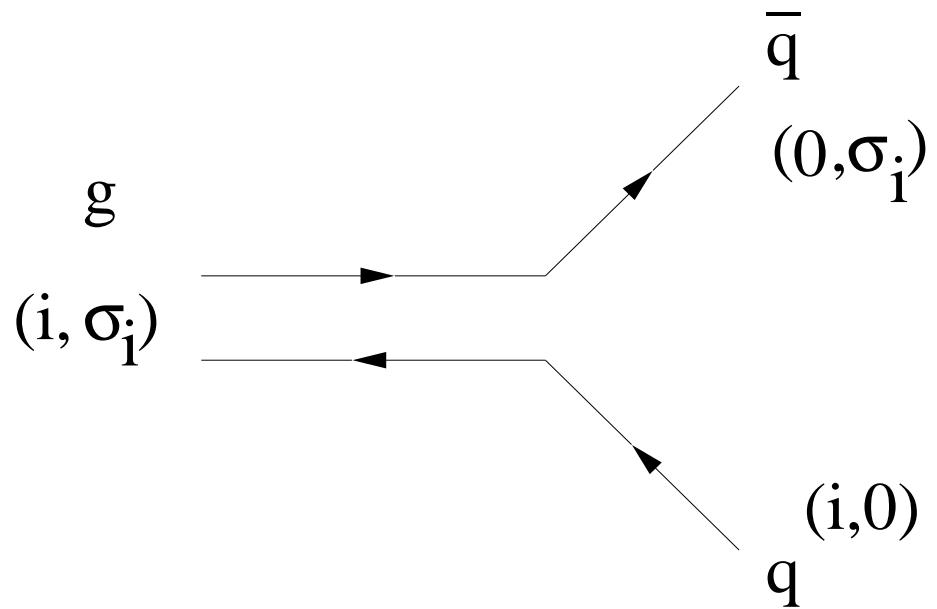
♠ exact color treatment \Rightarrow low color charge

Problem: number of colour connection configurations: $\sim n!$ where n is the number of gluons or $q\bar{q}$ pairs. \Rightarrow Monte-Carlo over continuous colour-space.



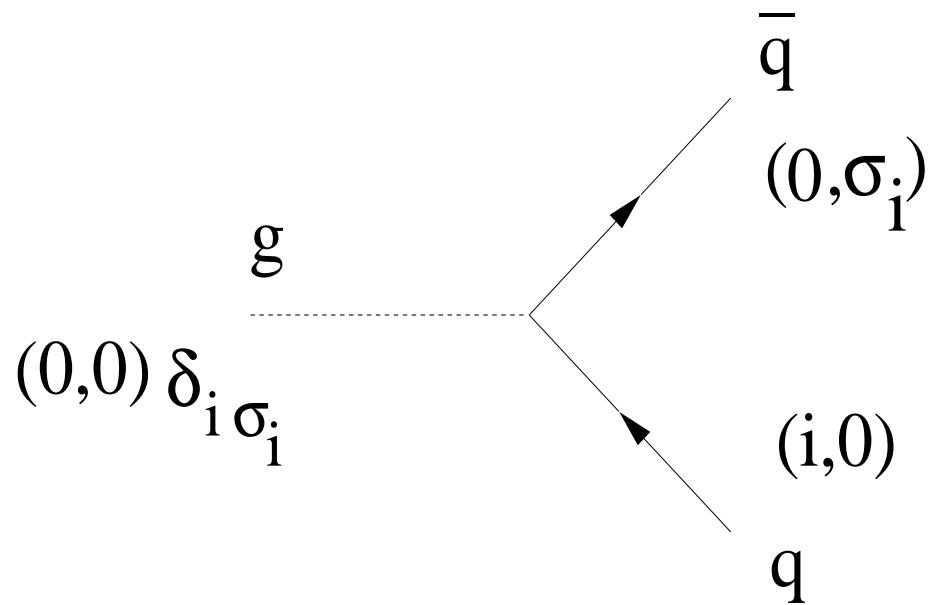
$$\sum f^{abc} t_{AB}^a t_{CD}^b t_{EF}^c = -\frac{i}{4} (\delta_{AD} \delta_{CF} \delta_{EB} - \delta_{AF} \delta_{CB} \delta_{ED})$$

$$\delta_{1\sigma_2} \delta_{2\sigma_3} \delta_{3\sigma_1}$$



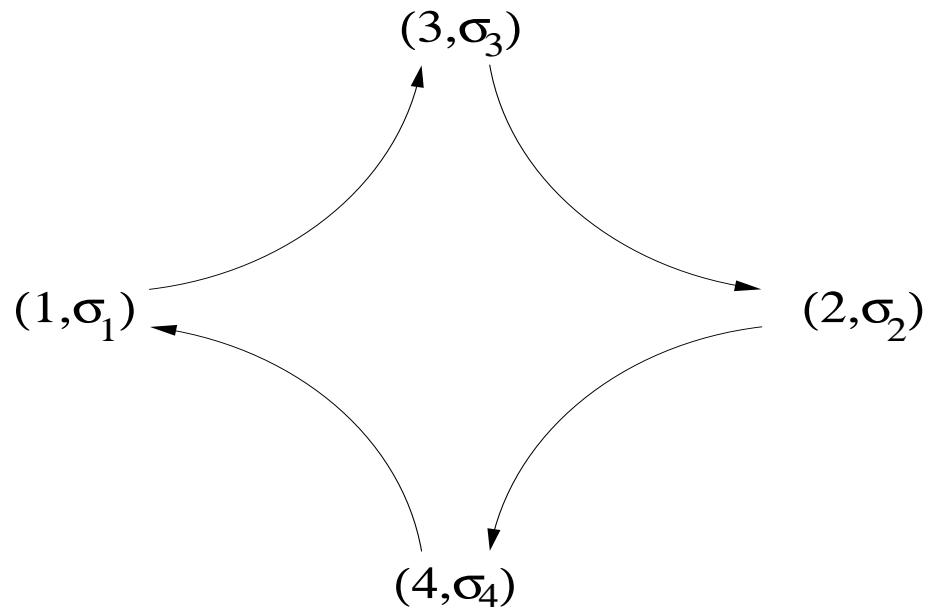
$$\sum t_{AB}^a t_{CD}^b = \frac{1}{2} (\delta_{AD} \delta_{CB} - \frac{1}{N_c} \delta_{AB} \delta_{AC})$$

$$\frac{1}{\sqrt{2}}$$



$$\sum t_{AB}^a t_{CD}^b = \frac{1}{2} (\delta_{AD} \delta_{CB} - \frac{1}{N_c} \delta_{AB} \delta_{AC})$$

$$\frac{1}{\sqrt{2N_c}}$$



$$\delta_{1\sigma_3} \delta_{3\sigma_2} \delta_{2\sigma_4} \delta_{4\sigma_1}$$

$$2g_{12}g_{34} - g_{13}g_{24} - g_{14}g_{23}$$

For an $1 \rightarrow n$ color ordered amplitude the number of 3-vertex

$$\sum_{k=1}^{n-1} (n-k)k$$

For an $1 \rightarrow n$ color ordered amplitude the number of 3-vertex

$$\sum_{k=1}^{n-1} (n-k)k$$

and for the 4-vertex their number is

$$\sum_{k=2}^{n-1} (n-k) \frac{k(k-1)}{2}$$

For an $1 \rightarrow n$ color ordered amplitude the number of 3-vertex

$$\sum_{k=1}^{n-1} (n-k)k$$

and for the 4-vertex their number is

$$\sum_{k=2}^{n-1} (n-k) \frac{k(k-1)}{2}$$

where $n - k$ is the number of k -words in the length n object

$$(1, 2, 3, 4) \rightarrow (1, 2)(2, 3)(3, 4)$$

$$(1, 2, 3, 4) \rightarrow (1, 2, 3)(2, 3, 4)$$

$$\text{numberofterms} = \binom{n+2}{3} + \binom{n+2}{4}$$

$$\text{numberofterms} = \binom{n+2}{3} + \binom{n+2}{4}$$

$gg \rightarrow ng$	2	3	4	5	6	7	8	9
#	5	15	35	70	126	210	330	495
FG-O	3	10	38	154	654	2871	12,925	59,345
FG-U	4	25	220	2,485	34,300	559,405	10,525,900	224,449,225

$$\text{numberofterms} = \binom{n+2}{3} + \binom{n+2}{4}$$

$gg \rightarrow ng$	2	3	4	5	6	7	8	9
#	5	15	35	70	126	210	330	495
FG-O	3	10	38	154	654	2871	12,925	59,345
FG-U	4	25	220	2,485	34,300	559,405	10,525,900	224,449,225

$$a(n) = \frac{1}{2!} \sum \frac{n!}{n_1! n_2!} a(n_1) a(n_2) \delta_{n_1+n_2, n}$$

$$+ \frac{1}{3!} \sum \frac{n!}{n_1! n_2! n_3!} a(n_1) a(n_2) a(n_3) \delta_{n_1+n_2+n_3, n}$$

$$a(1) = 1$$



Quick Connect Profiles

File Edit View Window Help

```

the colour of particles ONE 2 1 0 4 0 3
the colour of particles TWO 1 2 3 0 4 0
for the 8 colour conf. there are      0 subamplitudes
the colour of particles ONE 2 3 0 1 0 4
the colour of particles TWO 1 2 3 0 4 0
for the 9      7 colour conf. there are      30 subamplitudes
 1 2 6 -11   7 1 1 2 35  2 4 -11  3 0 0 0 0 1 1 1
 2 2 6 -11   7 0 1 2 35  2 4 -11  3 0 0 0 0 2 1 1
 3 1 48 31   8 1 1 16 -12  5 32 12  6 0 0 0 0 0 1 0
 4 1 48 32   9 1 1 16 -12  5 32 12  6 0 0 0 0 0 1 0
 5 1 48 35   10 1 1 16 -12  5 32 12  6 0 0 0 0 0 1 2
 6 2 52 -11  11 1 3 48 31  8 4 -11  3 0 0 0 0 1 1 0
 7 2 52 -11  11 0 3 48 31  8 4 -11  3 0 0 0 0 2 1 0
 8 2 52 -11  11 2 3 48 32  9 4 -11  3 0 0 0 0 1 1 0
 9 2 52 -11  11 0 3 48 32  9 4 -11  3 0 0 0 0 2 1 0
10 2 52 -11  11 3 3 48 35 10 4 -11  3 0 0 0 0 1 1 2
11 2 52 -11  11 0 3 48 35 10 4 -11  3 0 0 0 0 2 1 2
12 3 56 11 12 1 3 48 31  8 8 11 4 0 0 0 0 1 1 0
13 3 56 11 12 0 3 48 31  8 8 11 4 0 0 0 0 2 1 0
14 3 56 11 12 2 3 48 32  9 8 11 4 0 0 0 0 1 1 0
15 3 56 11 12 0 3 48 32  9 8 11 4 0 0 0 0 2 1 0
16 3 56 11 12 3 3 48 35 10 8 11 4 0 0 0 0 1 1 2
17 3 56 11 12 0 3 48 35 10 8 11 4 0 0 0 0 2 1 2
18 2 54 -11 13 1 4 2 35 2 52 -11 11 0 0 0 0 1 1 1
19 2 54 -11 13 0 4 2 35 2 52 -11 11 0 0 0 0 2 1 1
20 2 54 -11 13 2 4 48 31  8 6 -11  7 0 0 0 0 1 1 0
21 2 54 -11 13 0 4 48 31  8 6 -11  7 0 0 0 0 2 1 0
22 2 54 -11 13 3 4 48 32  9 6 -11  7 0 0 0 0 1 1 0
23 2 54 -11 13 0 4 48 32  9 6 -11  7 0 0 0 0 2 1 0
24 2 54 -11 13 4 4 48 35 10 6 -11  7 0 0 0 0 1 1 2
25 2 54 -11 13 0 4 48 35 10 6 -11  7 0 0 0 0 2 1 2
26 1 60 35 14 1 2 4 -11 3 56 11 12 0 0 0 0 0 1 1 1
27 1 60 35 14 2 2 52 -11 11 8 11 4 0 0 0 0 0 1 1 1
28 4 62 35 15 1 3 2 35 2 60 35 14 0 0 0 0 0 1 1 1
29 1 62 35 15 2 3 6 -11 7 56 11 12 0 0 0 0 0 1 1 1
30 1 62 35 15 3 3 54 -11 13 8 11 4 0 0 0 0 0 1 1 1
the number of Feynman graphs = 10
the number of Feynman graphs = 10
(feynman.f) the number of Feynman graphs is: 10
the colour of particles ONE 2 3 0 4 0 1
the colour of particles TWO 1 2 3 0 4 0

```

518,3

20%


[Quick Connect](#) [Profiles](#)
[File](#) [Edit](#) [View](#) [Window](#) [Help](#)

```
sigma= 0.274738D-02 0.171512D-01 23735 93000 93000
-----
sigma= 0.274242D-02 0.170635D-01 23998 94000 94000
-----
sigma= 0.274562D-02 0.169650D-01 24257 95000 95000
-----
sigma= 0.274661D-02 0.168580D-01 24514 96000 96000
-----
sigma= 0.274584D-02 0.167396D-01 24763 97000 97000
-----
sigma= 0.274919D-02 0.166607D-01 25012 98000 98000
-----
sigma= 0.275387D-02 0.165664D-01 25267 99000 99000
-----
sigma= 0.276020D-02 0.165019D-01 25518 100000 100000
```

out of 100000 100001 points have been used
and 25518 points resulted to /= 0 weight

whereas 74483 points to 0 weight

estimator x: 0.276017D-02

estimator y: 0.207463D-08

estimator z: 0.177629D-19

average estimate : 0.276017D-02

+/- 0.455481D-04

variance estimate: 0.207463D-08

+/- 0.133278D-09

lwri: points have used 0.00000000000000E+00

2212 2212 7000.000000000000 7000.000000000000 3 1

% error: 1.650188706631237

<w>/w_max,w_max 3.402995943569451E-03 0.8111008833504001

0.811101E+00 0.472230E+00 0.213615E+00 0.413747E-01 0.113141E-01 0.626923E-02 0.000000E+00

0.000000E+00 0.000000E+00 0.000000E+00

<me>/memax,memax 2.824090411444549E-04 4.886440423606611E-03

iwarning(4) = 46

number of w=1 events 0

number of w>1 events 0

maximum weight used for un:vs 0.00000000000000E+00 0.00000000000000E+00

ONO.199 LOG : INSIDE,UNDER,OVER 0 0 0

NO ENTRIES INSIDE HISTOGRAM

TIME= 2009 5 5 120 20 51 35 13

"output" 1992L, 118814C

1992,2 Bot

Summation/Integration over color

HEP - NCSR Democritos

Summation/Integration over color

$$\mathcal{M}(\{p_i\}_1^n, \{\varepsilon_i\}_1^n, \{a_i\}_1^n) \sim \sum_{P(2, \dots, n)} Tr(t^{a_1} \dots t^{a_n}) \mathcal{A}(\{p_i\}_1^n, \{\varepsilon_i\}_1^n)$$

Summation/Integration over color

$$\mathcal{M}(\{p_i\}_1^n, \{\varepsilon_i\}_1^n, \{a_i\}_1^n) \sim \sum_{P(2, \dots, n)} Tr(t^{a_1} \dots t^{a_n}) \mathcal{A}(\{p_i\}_1^n, \{\varepsilon_i\}_1^n)$$

$$\mathcal{M}(\{p_i\}_1^n, \{\varepsilon_i\}_1^n, \{I_i, J_i\}_1^n) \sim \sum_{P(2, \dots, n)} \delta_{I_1, P(J_1)} \dots \delta_{I_n, P(J_n)} \mathcal{A}(\{p_i\}_1^n, \{\varepsilon_i\}_1^n)$$

Summation/Integration over color

$$\mathcal{M}(\{p_i\}_1^n, \{\varepsilon_i\}_1^n, \{a_i\}_1^n) \sim \sum_{P(2, \dots, n)} Tr(t^{a_1} \dots t^{a_n}) \mathcal{A}(\{p_i\}_1^n, \{\varepsilon_i\}_1^n)$$

$$\mathcal{M}(\{p_i\}_1^n, \{\varepsilon_i\}_1^n, \{I_i, J_i\}_1^n) \sim \sum_{P(2, \dots, n)} \delta_{I_1, P(J_1)} \dots \delta_{I_n, P(J_n)} \mathcal{A}(\{p_i\}_1^n, \{\varepsilon_i\}_1^n)$$

$$\sum_{\{a_i\}_1^n \{\varepsilon_i\}_1^n} |\mathcal{M}(\{p_i\}_1^n, \{\varepsilon_i\}_1^n, \{a_i\}_1^n)|^2 = g^{2n-4} \sum_{\varepsilon} \sum_{ij} \mathcal{A}_i \mathcal{C}_{ij} \mathcal{A}_j^*$$

Summation/Integration over color

$$\mathcal{M}(\{p_i\}_1^n, \{\varepsilon_i\}_1^n, \{a_i\}_1^n) \sim \sum_{P(2, \dots, n)} Tr(t^{a_1} \dots t^{a_n}) \mathcal{A}(\{p_i\}_1^n, \{\varepsilon_i\}_1^n)$$

$$\mathcal{M}(\{p_i\}_1^n, \{\varepsilon_i\}_1^n, \{I_i, J_i\}_1^n) \sim \sum_{P(2, \dots, n)} \delta_{I_1, P(J_1)} \dots \delta_{I_n, P(J_n)} \mathcal{A}(\{p_i\}_1^n, \{\varepsilon_i\}_1^n)$$

$$\sum_{\{a_i\}_1^n \{\varepsilon_i\}_1^n} |\mathcal{M}(\{p_i\}_1^n, \{\varepsilon_i\}_1^n, \{a_i\}_1^n)|^2 = g^{2n-4} \sum_{\varepsilon} \sum_{ij} \mathcal{A}_i \mathcal{C}_{ij} \mathcal{A}_j^*$$

$$\sum_{P(2, \dots, n)} \sim n!$$

Summation/Integration over color

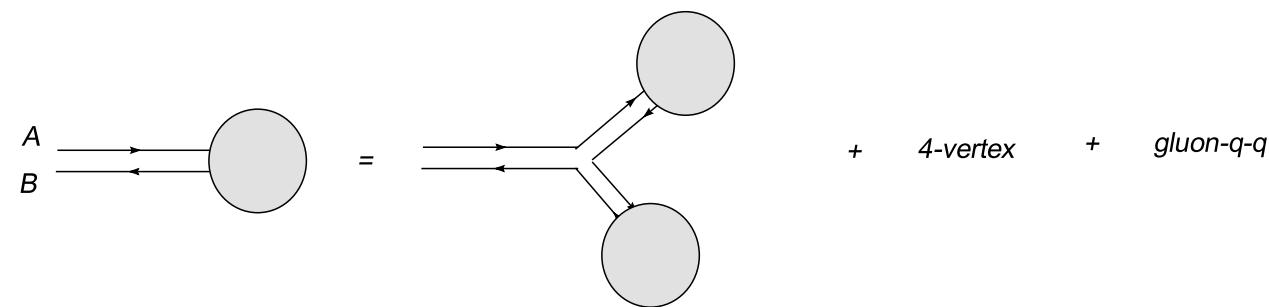
$$\mathcal{M}(\{p_i\}_1^n, \{\varepsilon_i\}_1^n, \{a_i\}_1^n) \sim \sum_{P(2, \dots, n)} Tr(t^{a_1} \dots t^{a_n}) \mathcal{A}(\{p_i\}_1^n, \{\varepsilon_i\}_1^n)$$

$$\mathcal{M}(\{p_i\}_1^n, \{\varepsilon_i\}_1^n, \{I_i, J_i\}_1^n) \sim \sum_{P(2, \dots, n)} \delta_{I_1, P(J_1)} \dots \delta_{I_n, P(J_n)} \mathcal{A}(\{p_i\}_1^n, \{\varepsilon_i\}_1^n)$$

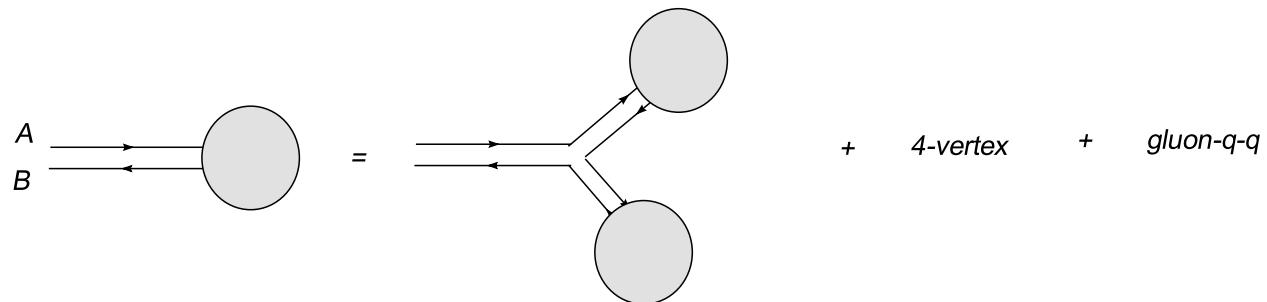
$$\sum_{\{a_i\}_1^n \{\varepsilon_i\}_1^n} |\mathcal{M}(\{p_i\}_1^n, \{\varepsilon_i\}_1^n, \{a_i\}_1^n)|^2 = g^{2n-4} \sum_{\varepsilon} \sum_{ij} \mathcal{A}_i \mathcal{C}_{ij} \mathcal{A}_j^*$$

$$\sum_{P(2, \dots, n)} \sim n!$$

$$\sum_{\{I_i, J_i\}_1^n} \sim 3^n \times 3^n$$



HEP - NCSR Democritos

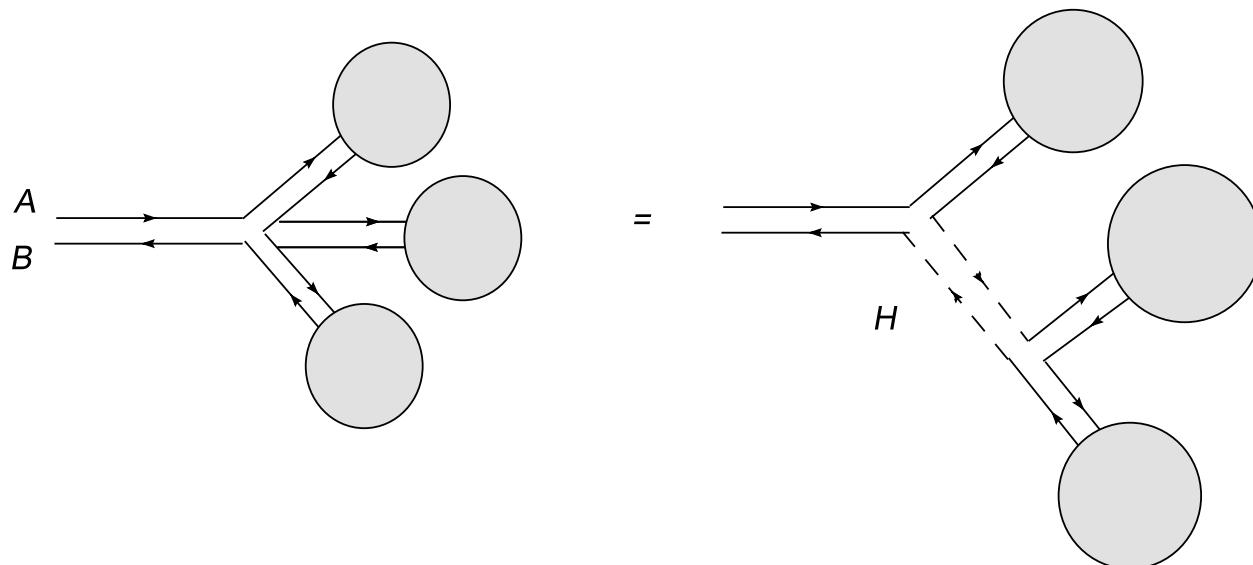


$$\begin{aligned}
 [A^\mu(P);(A,B)] &= \sum_{i=1}^n [\delta(P - p_i) A^\mu(p_i); (A,B)_i] + \\
 \sum [(ig) \Pi_\rho^\mu V^{\rho\nu\lambda}(P, p_1, p_2) A_\nu(p_1) A_\lambda(p_2) \sigma(p_1, p_2); (A,B) = (C,D)_1 \otimes (E,F)_2] \\
 - \sum [(g^2) \Pi_\sigma^\mu G^{\sigma\nu\lambda\rho}(P, p_1, p_2, p_3) A_\nu(p_1) A_\lambda(p_2) A_\rho(p_3) \sigma(p_1, p_2 + p_3); \\
 (A,B) = (C,D)_1 \otimes (E,F)_2 \otimes (G,H)_3] \\
 + \sum_{P=p_1+p_2} [(ig) \Pi_\nu^\mu \bar{\psi}(p_1) \gamma^\nu \psi(p_2) \sigma(p_1, p_2); (A,B) = (0,D)_1 \otimes (C,0)_2]
 \end{aligned}$$

where $A, B, C, D, E, F, G, H = 1, 2, 3$.

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu a}, \quad F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c$$

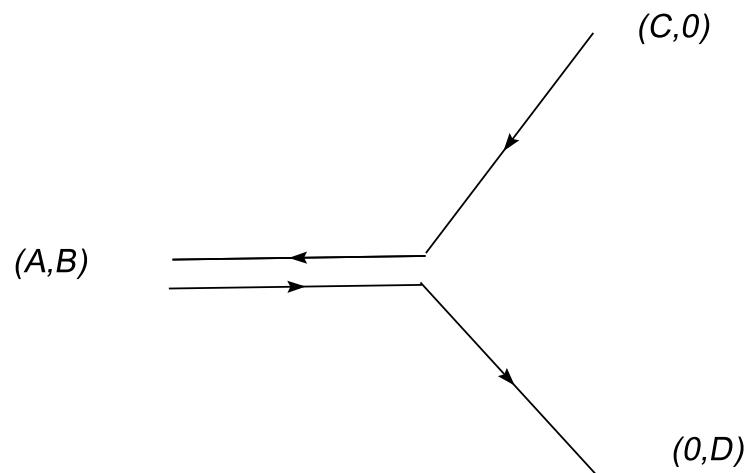
$$\mathcal{L} = -\frac{1}{2} H_{\mu\nu}^a H^{\mu\nu a} + \frac{1}{4} H_{\mu\nu}^a F^{\mu\nu a}.$$



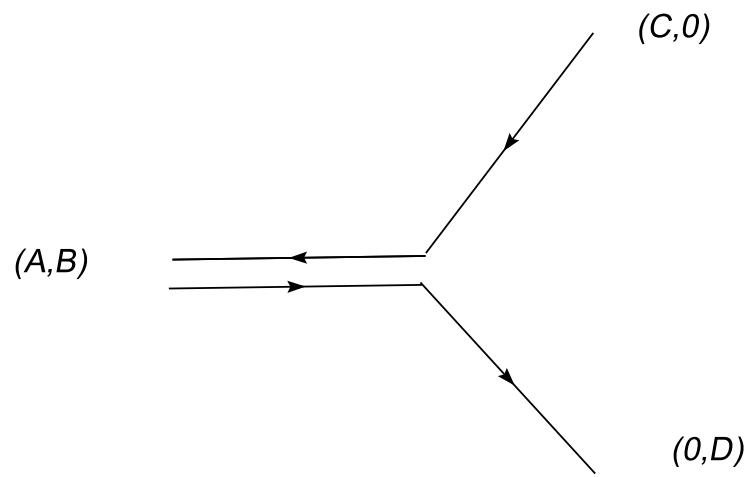
$$\begin{aligned}
[A^\mu(P);(A,B)] &= \sum_{i=1}^n [\delta(P-p_i) A^\mu(p_i);(A,B)_i] + \\
[(ig) \Pi_\rho^\mu V^{\rho\nu\lambda}(P,p_1,p_2) A_\nu(p_1) A_\lambda(p_2) \sigma(p_1,p_2); (A,B) = (C,D)_1 \otimes (E,F)_2] \\
&\quad + \\
[(ig) \Pi_\sigma^\mu (g^{\sigma\lambda} g^{\nu\rho} - g^{\nu\lambda} g^{\sigma\rho}) A_\nu(p_1) H_{\lambda\rho}(p_2) \sigma(p_1,p_2); (A,B) = (C,D)_1 \otimes (E,F)_2] \\
&\quad + \\
[(ig) \Pi_\nu^\mu \bar{\psi}(p_1) \gamma^\nu \psi(p_2) \sigma(p_1,p_2); (A,B) = (0,D)_1 \otimes (C,0)_2]
\end{aligned}$$

and

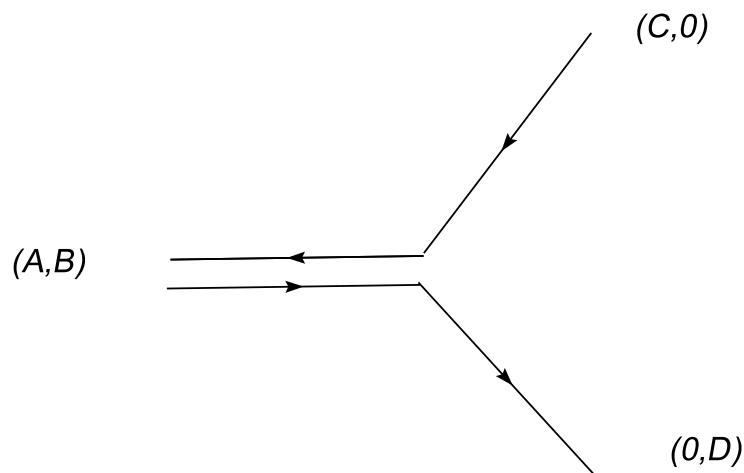
$$\begin{aligned}
[H^{\mu\nu}(P);(A,B)] &= \sum_{P=p_1+p_2} [(ig) (g^{\mu\lambda} g^{\nu\rho} - g^{\nu\lambda} g^{\mu\rho}) A_\lambda(p_1) A_\rho(p_2) \sigma(p_1,p_2); \\
(A,B) &= (C,D)_1 \otimes (E,F)_2].
\end{aligned}$$



HEP - NCSR Democritos

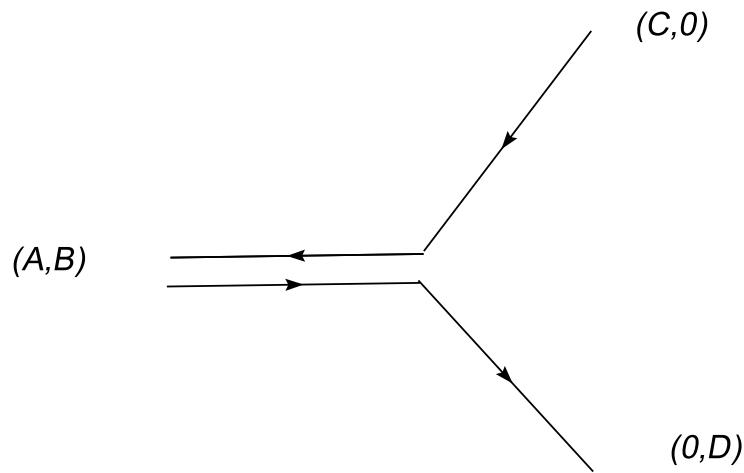


$$(A, B) = (C, 0) \otimes (0, D) = (C, D)_{w=1}, \quad \text{if } C \neq D.$$



$$(A, B) = (C, 0) \otimes (0, D) = (C, D)_{w=1}, \quad \text{if } C \neq D.$$

$$(A, B) = (C, 0) \otimes (0, D) = (1, 1)_{w_1} \oplus (2, 2)_{w_2} \oplus (3, 3)_{w_3}, \quad \text{if } C = D.$$



$$(A, B) = (C, 0) \otimes (0, D) = (C, D)_{w=1}, \quad \text{if } C \neq D.$$

$$(A, B) = (C, 0) \otimes (0, D) = (1, 1)_{w1} \oplus (2, 2)_{w2} \oplus (3, 3)_{w3}, \quad \text{if } C = D.$$

$$(1, 0) \otimes (0, 1) = (1, 1)_{2/3} \oplus (2, 2)_{-1/3} \oplus (3, 3)_{-1/3}$$

$$N_{CC} = \sum_{A=0}^{n_q} \sum_{B=0}^{n_q-1} \sum_{C=0}^{n_q-A-B} \left(\frac{n_q!}{A!B!C!} \right)^2 \delta(n_q = A + B + C)$$

$$N_{CC} = \sum_{A=0}^{n_q} \sum_{B=0}^{n_q-1} \sum_{C=0}^{n_q-A-B} \left(\frac{n_q!}{A!B!C!} \right)^2 \delta(n_q = A + B + C)$$

Process	N_{CC}^{ALL}	N_{CC}	$N_{CC}^F (\%)$
$gg \rightarrow 2g$	6561	639	59.1
$gg \rightarrow 3g$	59049	4653	68.4
$gg \rightarrow 4g$	531441	35169	77.4
$gg \rightarrow 5g$	4782969	272835	85.0
$gg \rightarrow 6g$	43046721	2157759	90.4
$gg \rightarrow 7g$	387420489	17319837	94.0
$gg \rightarrow 8g$	3486784401	140668065	96.4

Process	N_{CC}^{ALL}	N_{CC}	N_{CC}^F (%)
$gg \rightarrow u\bar{u}$	729	93	93.5
$gg \rightarrow g u\bar{u}$	6561	639	91.6
$gg \rightarrow 2g u\bar{u}$	59049	4653	92.6
$gg \rightarrow 3g u\bar{u}$	531441	35169	94.6
$gg \rightarrow 4g u\bar{u}$	4782969	272835	96.4
$gg \rightarrow 5g u\bar{u}$	43046721	2157759	97.8
$gg \rightarrow 6g u\bar{u}$	387420489	17319837	98.6
<hr/>			
$gg \rightarrow c\bar{c}c\bar{c}$	6561	639	99.1
$gg \rightarrow g c\bar{c}c\bar{c}$	59049	4653	98.8
$gg \rightarrow 2g c\bar{c}c\bar{c}$	531441	35169	99.0
$gg \rightarrow 3g c\bar{c}c\bar{c}$	4782969	272835	99.3
$gg \rightarrow 4g c\bar{c}c\bar{c}$	43046721	2157759	99.6

Process	$\sigma_{\text{MC}} \pm \varepsilon$ (nb)	ε (%)
$gg \rightarrow 7g$	$(0.53185 \pm 0.01149) \times 10^{-2}$	2.1
$gg \rightarrow 8g$	$(0.33330 \pm 0.00804) \times 10^{-3}$	2.4
$gg \rightarrow 9g$	$(0.17325 \pm 0.00838) \times 10^{-4}$	4.8
$gg \rightarrow 5gu\bar{u}$	$(0.38044 \pm 0.01096) \times 10^{-3}$	2.8
$gg \rightarrow 3gc\bar{c}c\bar{c}$	$(0.95109 \pm 0.02456) \times 10^{-5}$	2.6
$gg \rightarrow 4gc\bar{c}c\bar{c}$	$(0.81400 \pm 0.02583) \times 10^{-6}$	3.2

Process	$\sigma_{\text{MC}} \pm \varepsilon$ (nb)	ε (%)
$gg \rightarrow Z u \bar{u} gg$	$(0.18948 \pm 0.00344) \times 10^{-3}$	1.8
$gg \rightarrow W^+ \bar{u} d gg$	$(0.62704 \pm 0.01458) \times 10^{-3}$	2.3
$gg \rightarrow ZZ u \bar{u} gg$	$(0.16217 \pm 0.00420) \times 10^{-6}$	2.6
$gg \rightarrow W^+ W^- u \bar{u} gg$	$(0.27526 \pm 0.00752) \times 10^{-5}$	2.7
<hr/>		
$d\bar{d} \rightarrow Z u \bar{u} gg$	$(0.38811 \pm 0.00569) \times 10^{-5}$	1.5
$d\bar{d} \rightarrow W^+ \bar{c} s gg$	$(0.18765 \pm 0.00453) \times 10^{-5}$	2.4
$d\bar{d} \rightarrow ZZ gggg$	$(0.99763 \pm 0.02976) \times 10^{-7}$	2.9
$d\bar{d} \rightarrow W^+ W^- gggg$	$(0.52355 \pm 0.01509) \times 10^{-6}$	2.9

Multi-jet processes

Beyond any colour treatment a summation over different flavours is also needed.

Up to now the most straightforward way was to count the distinct processes and then multiply with a multiplicity factor, i.e.

process	Flavour
$gg \rightarrow ggg$	1
$q\bar{q} \rightarrow ggg$	8
$qg \rightarrow qgg$	8
$qg \rightarrow qgg$	8
$gg \rightarrow q\bar{q}g$	5
$q\bar{q} \rightarrow q\bar{q}g$	8
$q\bar{q} \rightarrow r\bar{r}g$	32
$qq \rightarrow qqg$	8
$q\bar{r} \rightarrow q\bar{r}g$	24
$qr \rightarrow qrg$	24
$qg \rightarrow qq\bar{q}$	8
$qg \rightarrow qr\bar{r}$	32
$gq \rightarrow qq\bar{q}$	8
$gq \rightarrow qr\bar{r}$	32

initial-state type	distinct processes	multiplicity factor
A (gg)	$C_1(n)$	$\chi(n_0, n_1, \dots, n_f; f)$
B $(q\bar{q})$	$C_2(n)$	$\chi(n_0, n_1, \dots, n_f; f - 1)$
C $(gq \text{ and } qg)$	$C_2(n - 1)$	$\chi(n_0, n_1, \dots, n_f; f - 1)$
D (qq)	$C_2(n - 2)$	$\chi(n_0, n_1, \dots, n_f; f - 1)$
E $(qq' \text{ and } q\bar{q}')$	$C_3(n - 2)$	$\chi(n_0, n_1, \dots, n_f; f - 2)$

In order to clarify what we mean we consider the example of the type A initial state. Each distinct process is defined by an array (n_0, n_1, \dots, n_f) . For instance, in the case of four-jet production we have

$$\begin{aligned}
 (4,0,0,0,0,0) &\quad gg \rightarrow gggg \\
 (2,1,0,0,0,0) &\quad gg \rightarrow ggq\bar{q} \\
 (0,2,0,0,0,0) &\quad gg \rightarrow q\bar{q}q\bar{q} \\
 (0,1,1,0,0,0) &\quad gg \rightarrow q\bar{q}r\bar{r}
 \end{aligned}$$

$$C_1(n) = \sum_{n_0+2n_1+\dots+2n_f=n} \Theta(n_1 \geq n_2 \geq \dots \geq n_f)$$

$$C_2(n) = \sum_{n_0+2n_1+\dots+2n_f=n} \Theta(n_2 \geq n_3 \geq \dots \geq n_f)$$

and

$$C_3(n) = \sum_{n_0+2n_1+\dots+2n_f=n} \Theta(n_3 \geq n_4 \geq \dots \geq n_f)$$

A distinct process, given by the array (n_0, n_1, \dots, n_f) has a multiplicity factor :

$$\chi(n_0, n_1, \dots, n_f; f) = n_f(n_f - 1)\dots(n_f - j + 1)/j!$$

$$\begin{aligned}
 j &= f && \text{if} && \prod_{i=1}^f n_i \neq 0 \\
 j &= f - 1 && \text{if} && \prod_{i=1}^{f-1} n_i \neq 0 \\
 &&&\dots&& \\
 j &= 1 && \text{if} && n_1 \neq 0 \\
 j &= 0 && \text{otherwise} &&
 \end{aligned}$$

Now we can think of a flavour-MC, so the wave function is multiplied by an N_f -dimensional array representing flavour , $\vec{f} = \sqrt{N_f}(f_1, f_2, \dots)$ such that $\langle f_i f_j \rangle = \delta_{ij}$ with a weight proportional to the relevant pdf for initial state flavours

In that case a process like



will actually represent a plethora of processes.

The number of distinct processes is now given by

$$9k + 3 \text{ if } n = 2k \text{ and } 9k + 7 \text{ if } n = 2k + 1$$

# of jets	2	3	4	5	6	7	8	9	10
# of D-processes	12	16	21	24	30	34	39	43	48
# of dist. processes	10	14	28	36	64	78	130	154	241
total # of processes	126	206	621	861	1862	2326	4342	5142	8641

Multi-jet rates

$$p_T \ i > 60 \ GeV, \quad \theta_{ij} > 30^\circ \quad |\eta_i| < 3$$

# jets	3	4	5	6	7	8
$\sigma(nb)$	91.41	6.54	0.458	2.97×10^{-2}	2.21×10^{-3}	2.12×10^{-4}
% Gluon	45.7	39.2	35.7	35.1	33.8	26.6

- quark processes relevant
- $gg \rightarrow ng$ approximation ?
- A new code \Rightarrow JetI
- anybody to tell us how many Feynman graphs in $gg \rightarrow 8g$?
- or $gg \rightarrow 2g3u3\bar{u}$?

- Feynman graphs in $gg \rightarrow 8g$ 10,525,900 !!
- or $gg \rightarrow 2g3u3\bar{u}$ 946,050!

PHEGAS

- Phase space

$$d\Phi_n = (2\pi)^{4-3n} \prod_{i=1}^n \frac{d^3 p_i}{2E_i} \delta \left(\sum E_i - w \right) \delta^3 \left(\sum \vec{p}_i \right)$$

- RAMBO, VEGAS-based nice but completely inefficient!

$$d\sigma_n = \text{FLUX} \times |\mathcal{M}_{2 \rightarrow n}|^2 d\Phi_n$$

need appropriate mappings of peaking structures,
plus optimization!

- Efficiency \Rightarrow to a large number of generators, each one for a specific class of processes.

Multichannel approach

$$\mathcal{I} = \int f(\vec{x}) d\mu(\vec{x}) = \int \frac{f(\vec{x})}{p(\vec{x})} p(\vec{x}) d\mu(\vec{x})$$

$$p(\vec{x}) = \sum_{i=1}^{M_{ch}} \alpha_i p_i(\vec{x}) \quad \sum_{i=1}^{M_{ch}} \alpha_i = 1$$

$$\mathcal{I} \rightarrow \left\langle \frac{f(\vec{x})}{p(\vec{x})} \right\rangle \quad \mathcal{E}^2 N \rightarrow \left\langle \left(\frac{f(\vec{x})}{p(\vec{x})} \right)^2 - \mathcal{I}^2 \right\rangle$$

★ Optimize $\alpha_i \Rightarrow$ Minimize \mathcal{E} ★

R.Kleiss and R.Pittau, Comput. Phys. Commun. 83, 141 (1994).

New Dyson-Schwinger equations: subamplitude is a combination of several peaking structures!

problem unsolved?
QCD antennas

P.D.Draggiotis, A.van Hameren and R.Kleiss, hep-ph/0004047.

New Dyson-Schwinger equations: subamplitude is a combination of several peaking structures!

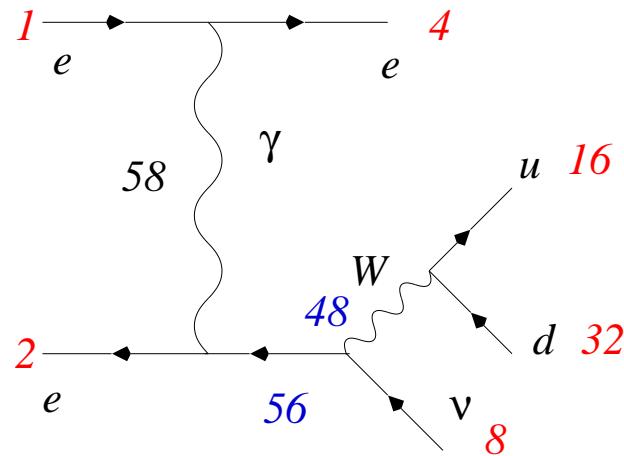
problem unsolved?
QCD antennas

P.D.Draggiotis, A.van Hameren and R.Kleiss, hep-ph/0004047.

Old Feynman graphs: exhibit single peaking structure!

problem solved

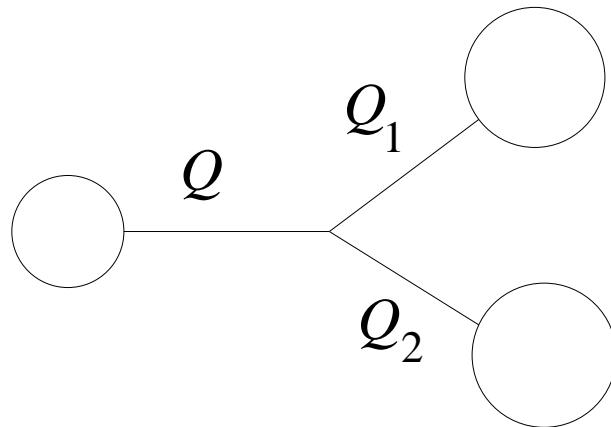
Back to Feynman graphs:



The corresponding intrinsic representation looks like

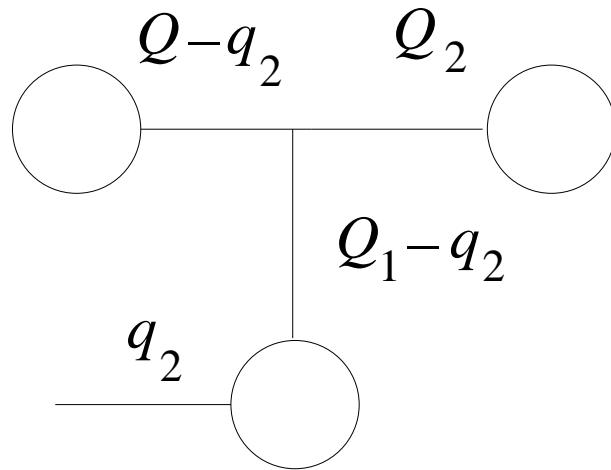
$$\begin{array}{ccccccc} 62 & -2 & 4 & -2 & 58 & 31 \\ 58 & 31 & 2 & -2 & 56 & 2 \\ 56 & 2 & 48 & 33 & 8 & 1 \\ 48 & 33 & 16 & -3 & 32 & 4 \end{array}$$

Time-like momenta $q^2 \geq 0$



$$\begin{aligned} d\Phi_n &= \dots \frac{dQ_1^2}{2\pi} \frac{dQ_2^2}{2\pi} d\Phi_2(Q \rightarrow Q_1, Q_2) \dots \\ &= \dots \frac{dQ_1^2}{2\pi} \frac{dQ_2^2}{2\pi} d\cos\theta \, d\phi \, \frac{\lambda^{1/2}(Q^2, Q_1^2, Q_2^2)}{32\pi^2 \, Q^2} \dots \end{aligned}$$

Space-like momenta



$$\begin{aligned} d\Phi_n &= \dots \frac{dQ_1^2}{2\pi} \frac{dQ_2^2}{2\pi} d\Phi_2(Q \rightarrow Q_1, Q_2) \dots \\ &= \dots \frac{dQ_1^2}{2\pi} \frac{dQ_2^2}{2\pi} dt d\phi \frac{1}{32\pi^2 Q |\vec{q}_2|} \dots \end{aligned}$$

$$t = (Q_1 - q_2)^2 = m_2^2 + Q_1^2 - \frac{E_2}{Q} (Q^2 + Q_1^2 - Q_2^2) + \frac{\lambda^{1/2}}{Q} |\vec{q}_2| \cos \theta$$

- Find limits of $t(Q_1^2, \cos \theta)$:

$$t_{\pm} = m_2^2 + Q_1^2 - \frac{E_2}{Q}(Q^2 + Q_1^2 - Q_2^2) \pm \frac{\lambda^{1/2}}{Q} |\vec{q}_2|$$

In order to find the maximum of t_+ we study the function $\partial t_+ / \partial Q_1^2$ in the region $Q_{1,min}^2 < Q_1^2 < (Q - Q_2)^2$. Since

$$\frac{\partial^2 t_+}{\partial (Q_1^2)^2} = -4Q^2 Q_2^2 \lambda^{-3/2} \frac{|\vec{q}_2|}{Q} \leq 0$$

and

$$\partial t_+ / \partial Q_1^2 |_{Q_1^2 = (Q - Q_2)^2} \rightarrow -\infty$$

we just consider two cases ($|\vec{q}_2| \neq 0$):

1. $\partial t_+ / \partial Q_1^2 |_{Q_1^2 = Q_{1,min}^2} < 0$ in which case
 $t_{max} = t_{+,max} = t_+(Q_1^2 = Q_{1,min}^2)$, and
2. $\partial t_+ / \partial Q_1^2 |_{Q_1^2 = Q_{1,min}^2} > 0$ in which case one can easily derive

$t_{max} = t_+(Q_1^2 = x_-)$ with

$$x_- = Q^2 + Q_2^2 - 2 Q Q_2 \frac{1 - E_2/Q}{\sqrt{\alpha}}, \quad \alpha = \left(1 - \frac{E_2}{Q}\right)^2 - \left(\frac{|\vec{q}_2|}{Q}\right)^2 > 0$$

♠ **Q_1^2 -limits:**

The limits for the Q_1^2 -integration for given t can now be fixed by the condition $|\cos \theta| \leq 1$ or equivalently

$$\Pi(Q_1^2) \leq 0$$

with

$$\Pi(Q_1^2) = \left(t - Q_1^2 - m_2^2 + \frac{E_2}{Q}(Q^2 + Q_1^2 - Q_2^2)\right)^2 - \left(\frac{|\vec{q}_2|}{Q}\right)^2 \lambda$$

If $y_1 \leq y_2$ are the two roots of the polynomial $\Pi(Q_1^2)$ then we have

1. For $a > 0$, $y_- < Q_1^2 < y_+$, with $y_- = \max(y_1, Q_{1,min}^2)$ and $y_+ = \min(y_2, Q_{1,max}^2)$
2. For $a < 0$ we have to satisfy two conditions $Q_1^2 < y_1$ or $y_2 < Q_1^2$ and $Q_{1,min}^2 < Q_1^2 < Q_{1,max}^2$

- At the end we get:

$$d\Phi_n \rightarrow \prod ds_i \ p_i(s_i) \prod dt_j \ p_j(t_j) \prod d\phi_k \prod d\cos\theta_l$$

- $p(x)$ are chosen so that 'singularities' are smoothed out!

- ◆ $(s - m^2)^2 + m^2\Gamma^2$ for massive unstable particles, W^\pm , Z .
- ◆ s^ν for time-like massless propagators, e.g. γ , gluons, fermions.
- ◆ $|t|^\nu$ for space-like massless propagators.

Final states	number of FG	\sqrt{s} (GeV)	Cross section (fb)
$u \bar{d} s \bar{c} \gamma$	90 (74)	200	199.75 (16)
$e^- \bar{\nu}_e \mu^+ \nu_\mu \gamma$	108 (100)	200	29.309 (25)
$\mu^- \bar{\nu}_\mu u \bar{d} \gamma \gamma$	587 (210)	500	1.730 (58)
$\mu^- \bar{\nu}_\mu u \bar{d} c \bar{c}$	209 (102)	500	0.1783 (20)
$\mu^- \bar{\nu}_\mu u \bar{d} c \bar{c} \gamma$	2142 (339)	500	0.02451 (65)

6 (and more) fermion production

- Input parameters and cuts as defined in hep-ph/0206070 and the WG
- DS (Dyson-Schwinger) elements, HC (helicity configurations) and CC (colour connections) determine the matrix element computational cost.
- FG (Feynman graphs) determine the phase-space generation cost. This is drastically reduced by using 'removing channels' techniques.

Top-quark channels

final state	QCD	AMEGIC++ [fb]	HELAC [fb]
$b\bar{b}u\bar{d}d\bar{u}$	yes	32.90(15)	33.05(14)
	yes	49.74(21)	50.20(13)
	no	32.22(34)	32.12(19)
	no	49.42(44)	50.55(26)
$b\bar{b}u\bar{u}gg$	—	11.23(10)	11.136(41)
	—	9.11(13)	8.832(43)
$b\bar{b}gggg$	—	18.82(13)	18.79(11)
	—	24.09(18)	23.80(17)
$b\bar{b}u\bar{d}e^- \bar{\nu}_e$	yes	11.460(36)	11.488(15)
	yes	17.486(66)	17.492(41)
	no	11.312(37)	11.394(18)
	no	17.366(68)	17.353(31)
$b\bar{b}e^+\nu_e e^- \bar{\nu}_e$	—	3.902(31)	3.885(7)
	—	5.954(55)	5.963(11)
$b\bar{b}e^+\nu_e \mu^- \bar{\nu}_\mu$	—	3.847(15)	3.848(7)
	—	5.865(24)	5.868(10)
$b\bar{b}\mu^+\nu_\mu \mu^- \bar{\nu}_\mu$	—	3.808(16)	3.861(19)
	—	5.840(30)	5.839(12)

Vector fusion with Higgs exchange

final state	QCD	AMEGIC++ [fb]	HELAC [fb]
$e^- e^+ u \bar{u} d \bar{d}$	yes	0.6842(85)	0.6858(31)
	yes	1.237(15)	1.265(5)
	no	0.6453(62)	0.6527(35)
	no	1.206(14)	1.2394(75)
$e^- e^+ u \bar{u} e^- e^+$	—	6.06(36)e-03	6.113(87)e-03
	—	6.58(23)e-03	6.614(80)e-03
$e^- e^+ u \bar{u} \mu^- \mu^+$	—	9.24(12)e-03	9.04(11)e-03
	—	9.25(17)e-03	9.145(74)e-03
$\nu_e \bar{\nu}_e u \bar{d} d \bar{u}$	yes	1.15(3)	1.176(6)
	yes	2.36(7)	2.432(12)
	no	1.14(3)	1.134(5)
	no	2.35(7)	2.429(13)
$\nu_e \bar{\nu}_e u \bar{d} e^- \bar{\nu}_e$	—	0.426(11)	0.4309(48)
	—	0.916(30)	0.9121(48)
$\nu_e \bar{\nu}_e u \bar{d} \mu^- \bar{\nu}_\mu$	—	0.425(12)	0.4221(30)
	—	0.878(27)	0.8888(47)

Vector fusion without Higgs exchange

final state	QCD	AMEGIC++ [fb]	HELAC [fb]
$e^- e^+ u \bar{u} d \bar{d}$	yes	0.4838(50)	0.4842(25)
	yes	1.0514(97)	1.0445(51)
	no	0.4502(31)	0.4524(23)
	no	1.0239(79)	1.0227(43)
$e^- e^+ u \bar{u} e^- e^+$	—	3.757(98)e-03	3.577(43)e-03
	—	4.082(56)e-03	4.214(46)e-03
$e^- e^+ u \bar{u} \mu^- \mu^+$	—	5.201(61)e-03	5.119(70)e-03
	—	5.805(67)e-03	5.828(49)e-03
$\nu_e \bar{\nu}_e u \bar{d} d \bar{u}$	yes	0.15007(53)	0.15070(64)
	yes	0.4755(21)	0.4711(24)
	no	0.12828(42)	0.12793(55)
	no	0.4417(19)	0.4398(21)
$\nu_e \bar{\nu}_e u \bar{d} e^- \bar{\nu}_e$	—	0.04546(13)	0.04564(19)
	—	0.16033(63)	0.16011(78)
$\nu_e \bar{\nu}_e u \bar{d} \mu^- \bar{\nu}_\mu$	—	0.0423(12)	0.04180(16)
	—	0.14383(53)	0.14439(65)

Higgs production through Higgsstrahlung

final state	QCD	AMEGIC++ [fb]	HELAC [fb]
$\mu^- \mu^+ \mu^- \bar{\nu}_\mu e^- \bar{\nu}_e$	–	0.03244(27)	0.03210(15)
	–	0.03747(29)	0.03749(32)
$\mu^- \mu^+ u \bar{d} e^- \bar{\nu}_e$	–	0.0924(8)	0.09306(46)
	–	0.1106(22)	0.10901(66)
$\mu^- \mu^+ \mu^- \mu^+ e^- e^+$	–	2.828(67)e-03	2.923(52)e-03
	–	2.731(65)e-03	2.691(42)e-03
$\mu^- \mu^+ u \bar{u} d \bar{d}$	yes	0.2534(24)	0.2540(16)
	yes	0.2634(22)	0.2642(15)
	no	0.2441(23)	0.2471(15)
	no	0.2593(22)	0.2589(14)
$\mu^- \mu^+ u \bar{u} u \bar{u}$	yes	1.125(8)e-02	1.135(22)e-02
	yes	8.767(65)e-03	8.978(58)e-03
	no	7.929(57)e-03	8.078(92)e-03
	no	6.098(35)e-03	6.013(26)e-03

Backgrounds to Higgsstrahlung

final state	QCD	AMEGIC++ [fb]	HELAC [fb]
$\mu^- \mu^+ \mu^- \bar{\nu}_\mu e^- \bar{\nu}_e$	–	0.01845(14)	0.01843(13)
	–	0.03054(23)	0.03092(19)
$\mu^- \mu^+ u \bar{d} e^- \bar{\nu}_e$	–	0.05284(57)	0.05209(33)
	–	0.08911(53)	0.08925(48)
$\mu^- \mu^+ \mu^- \mu^+ e^- e^+$	–	2.204(52)e-03	2.346(49)e-03
	–	2.280(66)e-03	2.277(62)e-03
$\mu^- \mu^+ u \bar{u} d \bar{d}$	yes	0.1412(10)	0.1404(11)
	yes	0.2092(12)	0.2075(13)
	no	0.1358(20)	0.1341(12)
	no	0.2040(12)	0.2015(11)
$\mu^- \mu^+ u \bar{u} u \bar{u}$	yes	5.937(24)e-03	5.937(25)e-03
	yes	6.134(29)e-03	6.108(27)e-03
	no	2.722(10)e-03	2.710(11)e-03
	no	3.290(12)e-03	3.303(12)e-03

Triple Higgs coupling

final state	QCD	AMEGIC++ [fb]	HELAC [fb]
$\mu^- \mu^+ b\bar{b}b\bar{b}$	yes	2.560(26)e-02	2.583(26)e-02
	yes	3.096(60)e-02	3.019(43)e-02
	no	1.711(55)e-02	1.666(28)e-02
	no	2.34(12)e-02	2.36(10)e-02

Backgrounds to triple Higgs coupling

final state	QCD	AMEGIC++ [fb]	HELAC [fb]
$\mu^- \mu^+ b\bar{b}b\bar{b}$	yes	7.002(32)e-03	7.044(22)e-03
	yes	6.308(24)e-03	6.364(21)e-03
	no	2.955(11)e-03	2.972(12)e-03
	no	3.704(15)e-03	3.695(13)e-03

FS	CC	DS	FG	HC	$\sigma(\text{fb})$	T
$\mu^- \mu^+ b\bar{b}b\bar{b}$	2	256+256	1158	24	0.00827(18)	
$b\bar{b}b\bar{b}u\bar{d}\mu^- \nu_\mu$	2	938+938	23116	18	0.001576(17)	
$u\bar{d}e^- \bar{\nu}_e \gamma$	1	119	108	6	10.87(18)	

Input parameters: Working Group

$$g g \rightarrow b \bar{b} b \bar{b} W^- W^+$$

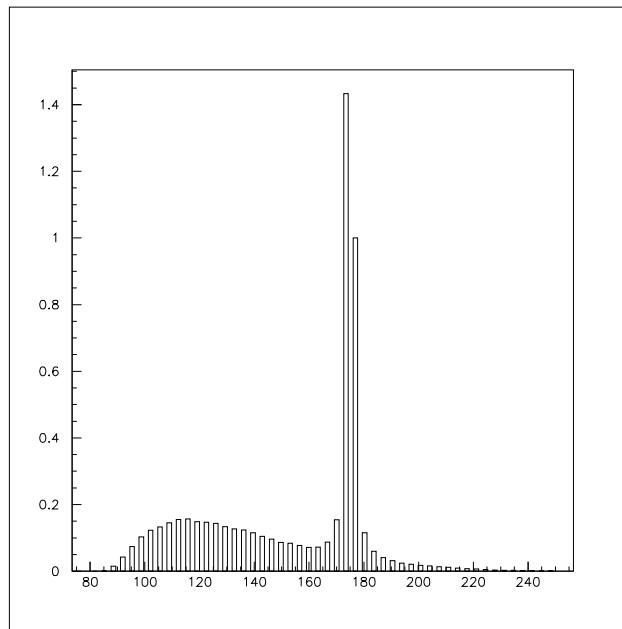
- challenging process, from a computational point of view
- a nice example to demonstrate the ability of PHEGAS/HELAC to deal with QCD processes.
- background of $t\bar{t}H$ production

MC points $w > 0$	result (fb)	error (fb)	efficiency (%)	efficiency $w > 0$ (%)
99442	4.716	0.024	3.3	33

- energy $\sqrt{s} = 500$ GeV and to 1×10^6 MC points.
- Feynman graphs for this process is 960, with $4!$ colour configurations, without taking into account electroweak contributions from Z and γ intermediate states.

Parameters used are $g_{QCD} = 1$, $m_{top} = 175$ GeV and $\Gamma_{top} = 1.5$ GeV.
Moreover the following set of cuts has been applied:

$$M_{q,q'} > 20\text{GeV}, \quad E_q > 20\text{GeV}, \quad |\cos \theta(q, \text{beam})| < 0.9,$$



$$p\ p \rightarrow t\ \bar{t}\ b\ \bar{b}\ b\ \bar{b}$$

- Another challenging process, from a computational point of view
- A nice example to demonstrate the ability of PHEGAS/HELAC to deal with QCD processes in a **realistic setup**.
- A background of $t\bar{t}HH$ production, which seems interesting in a high-luminosity LHC version for HHH coupling.
- Feynman graphs for this process is 1454 (gg), with $5!$ colour configurations.
- A two-phase implementation has been set up.
- Structure functions and α_s from PDFLIB, CTEQ-4L (LO).
- Kinematical decays of $t \rightarrow bW^+$ has been implemented
- Cuts: $p_T^b > 20\text{GeV}$, $|\eta_b| < 2.5$, $\Delta R > 0.5$

The result is 1.053 ± 0.073 (fb) @ LHC

- $e^- e^+ \rightarrow e^- e^+ \mu^- \mu^+$

σ_{tot} (in nb)

$\sqrt{s}(\text{GeV})$	BDK	NEXTCALIBUR
20	98.9 ± 0.6	99.20 ± 0.98
35	131.4 ± 2.2	131.03 ± 0.88
50	154.4 ± 0.9	152.33 ± 0.83
100	205.9 ± 1.2	204.17 ± 1.73
200	—	263.50 ± 1.31
200 (all)	—	265.58 ± 1.44

- $e^- e^+ \rightarrow e^- e^+ e^- e^+$

σ_{tot} (in nb $\times 10^7$)

\sqrt{s} (GeV)	BDK	NEXTCALIBUR
20	$0.920 \pm .011$	$0.905 \pm .011$
35	$1.070 \pm .015$	$1.079 \pm .014$
50	$1.233 \pm .018$	$1.214 \pm .016$
100	$1.459 \pm .025$	$1.485 \pm .020$
200	—	$1.776 \pm .019$
200 (all)	—	$1.787 \pm .030$

Current Status

- Single process mode: all SM processes. Only limitation memory and CPU cost ! to be judged by the user. Experience with as many as 10 particles in the final state.
- Summation over processes mode: all SM processes with fl_{ini} and fl_{fin} flavors for 'jets'. Only limitation memory and CPU cost ! to be judged by the user. Parallelism !
- Complete generation for pp and $p\bar{p}$ collisions, including all sub-processes. We do not exclude any processes!
- Interfacing with Pythia, including CKKW-like reweighting and use of UPVETO à la MLM.
- Extra version with HG^n and $H\gamma^n$ couplings

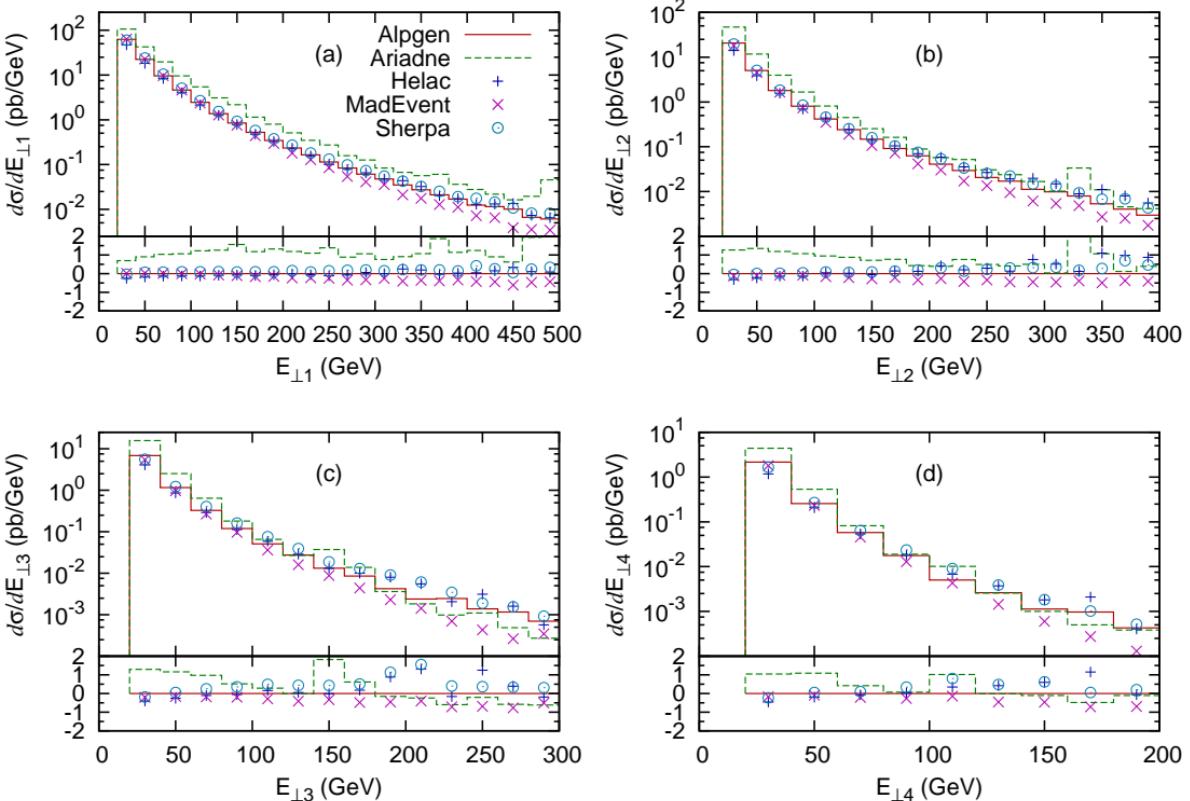


Figure 12: Inclusive E_T spectra of the leading 4 jets at the LHC (pb/GeV). In all cases the full line gives the ALPGEN results, the dashed line gives the ARIADNE result and the “+”, “x” and “o” points give the HELAC, MADEVENT and SHERPA results respectively.

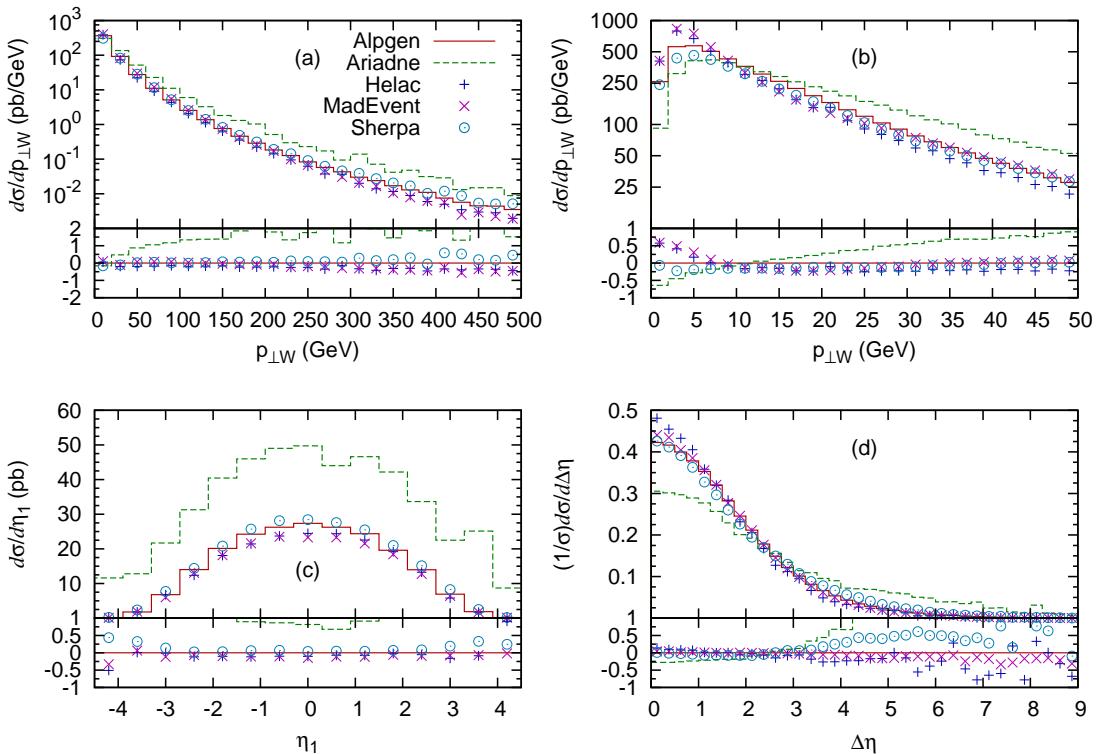


Figure 14: (a) and (b) p_T spectrum of W^+ bosons at the LHC (pb/GeV). (c) η spectrum of the leading jet, for $p_T^{\text{jet}_1} > 100$ GeV; absolute normalization (pb). (d) Pseudo-rapidity separation between the W^+ and the leading jet, $|\eta(W^+) - \eta(\text{jet}_1)|$, for $p_T^{\text{jet}_1} > 40$ GeV, normalized to unit area. Lines and points are as in fig. 12.

Code Structure

Files: *.f

- `main_mc.f` main file
- `intpar.f` integer arithmetic
- `master.f` master file for DS solution
- `pan1.f` non-dressed vertices and amplitude calculation
- `pan2.f` dressed vertices

plus several tool-files

Files: *.h

- common_int.h
common/helac_int/n,io(20),ifl(20)

Files: *.sh to allow for several useful interventions

- ktreweight_yes.sh
cp ktreweight_yes.h
ktreweight.h touch main_mc.f

subdirectory Summation_processes: scripts plus .lhe file

subdirectory pythia-interface: interface, upveto, etc

README : mainly for the single-process mode

Summation_processes/README: summation and scripting

On line demo - I

- Define your input in user.inp. Let say $W + 1\text{jet}$.
- Type 'sh run.sh user.inp'
- That's it !

On line demo - II

- Special try for $gg \rightarrow b\bar{b}e^+\nu_e\mu^-\bar{\nu}_\mu q\bar{q}$ to be called $t\bar{t} + 2\text{jets}$
- Please be patient for 2 minutes:
Colour connections = 22
Feynman Graphs = 384
- Study all EW corrections, interferences, subprocesses, etc

OPP REDUCTION - INTRO

G. Ossola., C. G. Papadopoulos and R. Pittau, Nucl. Phys. B **763**, 147 (2007) – arXiv:hep-ph/0609007

and JHEP **0707** (2007) 085 – arXiv:0704.1271 [hep-ph]

R. K. Ellis, W. T. Giele and Z. Kunszt, JHEP **0803**, 003 (2008)

Any m -point one-loop amplitude can be written, **before integration**, as

$$A(\bar{q}) = \frac{N(q)}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}}$$

A bar denotes objects living in $n = 4 + \epsilon$ dimensions

$$\bar{D}_i = (\bar{q} + p_i)^2 - m_i^2$$

$$\bar{q}^2 = q^2 + \tilde{q}^2$$

$$\bar{D}_i = D_i + \tilde{q}^2$$

External momenta p_i are 4-dimensional objects

THE OLD “MASTER” FORMULA

$$\begin{aligned}\int A &= \sum_{i_0 < i_1 < i_2 < i_3}^{m-1} d(i_0 i_1 i_2 i_3) D_0(i_0 i_1 i_2 i_3) \\ &+ \sum_{i_0 < i_1 < i_2}^{m-1} c(i_0 i_1 i_2) C_0(i_0 i_1 i_2) \\ &+ \sum_{i_0 < i_1}^{m-1} b(i_0 i_1) B_0(i_0 i_1) \\ &+ \sum_{i_0}^{m-1} a(i_0) A_0(i_0) \\ &+ \text{rational terms}\end{aligned}$$

THE OLD “MASTER” FORMULA

$$A(\bar{q}) = \frac{N(q)}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}}$$

$$N(q) \rightarrow q^{\mu_1} \dots q^{\mu_m} \rightarrow g^{\mu_1 \mu_2} p_i^{\mu_3} \dots$$

G. Passarino and M. J. G. Veltman, “One Loop Corrections For E+ E- Annihilation Into Mu+ Mu- In The Weinberg Model,” Nucl. Phys. B **160** (1979) 151.

THE OLD “MASTER” FORMULA

$$T_{\mu_1 \dots \mu_P}^N(p_1, \dots, p_{N-1}, m_0, \dots, m_{N-1}) = \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int d^D q \frac{q_{\mu_1} \cdots q_{\mu_P}}{D_0 D_1 \cdots D_{N-1}}$$

$$T_{\mu_1 \dots \mu_P}^N(p_1, \dots, p_{N-1}, m_0, \dots, m_{N-1}) = \sum_{i_1, \dots, i_P=0}^{N-1} T_{i_1 \dots i_P}^N p_{i_1 \mu_1} \cdots p_{i_P \mu_P}.$$

$$D_\mu = \sum_{i=1}^3 p_{i\mu} D_i,$$

$$D_{\mu\nu} = g_{\mu\nu} D_{00} + \sum_{i,j=1}^3 p_{i\mu} p_{j\nu} D_{ij},$$

$$D_{\mu\nu\rho} = \sum_{i=1}^3 (g_{\mu\nu} p_{i\rho} + g_{\nu\rho} p_{i\mu} + g_{\mu\rho} p_{i\nu}) D_{00i} + \sum_{i,j,k=1}^3 p_{i\mu} p_{j\nu} p_{k\rho} D_{ijk},$$

$$\begin{aligned} D_{\mu\nu\rho\sigma} &= (g_{\mu\nu} g_{\rho\sigma} + g_{\mu\rho} g_{\nu\sigma} + g_{\mu\sigma} g_{\nu\rho}) D_{0000} \\ &\quad + \sum_{i,j=1}^3 (g_{\mu\nu} p_{i\rho} p_{j\sigma} + g_{\nu\rho} p_{i\mu} p_{j\sigma} + g_{\mu\rho} p_{i\nu} p_{j\sigma} \\ &\quad \quad + g_{\mu\sigma} p_{i\nu} p_{j\rho} + g_{\nu\sigma} p_{i\mu} p_{j\rho} + g_{\rho\sigma} p_{i\mu} p_{j\nu}) D_{00ij} \\ &\quad + \sum_{i,j,k,l=1}^3 p_{i\mu} p_{j\nu} p_{k\rho} p_{l\sigma} D_{ijkl}. \end{aligned}$$

THE OLD “MASTER” FORMULA

$$A(\bar{q}) = \frac{N(q)}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}}$$

$$N(q) \rightarrow q^{\mu_1} \dots q^{\mu_m} \rightarrow g^{\mu_1 \mu_2} p_i^{\mu_3} \dots$$

G. Passarino and M. J. G. Veltman, “One Loop Corrections For E+ E- Annihilation Into Mu+ Mu- In The Weinberg Model,” Nucl. Phys. B **160** (1979) 151.

W. L. van Neerven and J. A. M. Vermaseren, “Large Loop Integrals,” Phys. Lett. B **137**, 241 (1984)

The derivation of the reduction formula starts as in ref. [1] with the Schouten identity which is a relation between five Levi-Civita tensors:

$$\epsilon^{p_1 p_2 p_3 p_4} Q_\mu = \epsilon^{\mu p_2 p_3 p_4} Q \cdot p_1 + \epsilon^{p_1 \mu p_3 p_4} Q \cdot p_2 + \epsilon^{p_1 p_2 \mu p_4} Q \cdot p_3 + \epsilon^{p_1 p_2 p_3 \mu} Q \cdot p_4 . \quad (6)$$

THE OLD “MASTER” FORMULA

$$A(\bar{q}) = \frac{N(q)}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}}$$

$$N(q) \rightarrow q^{\mu_1} \dots q^{\mu_m} \rightarrow g^{\mu_1 \mu_2} p_i^{\mu_3} \dots$$

G. Passarino and M. J. G. Veltman, “One Loop Corrections For E+ E- Annihilation Into Mu+ Mu- In The Weinberg Model,” Nucl. Phys. B **160** (1979) 151.

W. L. van Neerven and J. A. M. Vermaseren, “Large Loop Integrals,” Phys. Lett. B **137**, 241 (1984)

The derivation of the reduction formula starts as in ref. [1] with the Schouten identity which is a relation between five Levi-Civita tensors:

$$\epsilon^{p_1 p_2 p_3 p_4} Q_\mu = \epsilon^{\mu p_2 p_3 p_4} Q \cdot p_1 + \epsilon^{p_1 \mu p_3 p_4} Q \cdot p_2 + \epsilon^{p_1 p_2 \mu p_4} Q \cdot p_3 + \epsilon^{p_1 p_2 p_3 \mu} Q \cdot p_4 . \quad (6)$$

which yields the final formula for the scalar one-loop five-point function:

$$\begin{aligned} E_{01234}(w^2 - 4\Delta_4 m_0^2) &= D_{1234} [2\Delta_4 - w \cdot (v_1 + v_2 + v_3 + v_4)] \\ &+ D_{0234} v_1 \cdot w + D_{0134} v_2 \cdot w + D_{0124} v_3 \cdot w + D_{0123} v_4 \cdot w . \end{aligned} \quad (19)$$

THE OLD “MASTER” FORMULA

$$A(\bar{q}) = \frac{N(q)}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}}$$

$$N(q) \rightarrow q^{\mu_1} \dots q^{\mu_m} \rightarrow g^{\mu_1 \mu_2} p_i^{\mu_3} \dots$$

G. Passarino and M. J. G. Veltman, “One Loop Corrections For E+ E- Annihilation Into Mu+ Mu- In The Weinberg Model,” Nucl. Phys. B **160** (1979) 151.

W. L. van Neerven and J. A. M. Vermaseren, “Large Loop Integrals,” Phys. Lett. B **137**, 241 (1984)

The derivation of the reduction formula starts as in ref. [1] with the Schouten identity which is a relation between five Levi-Civita tensors:

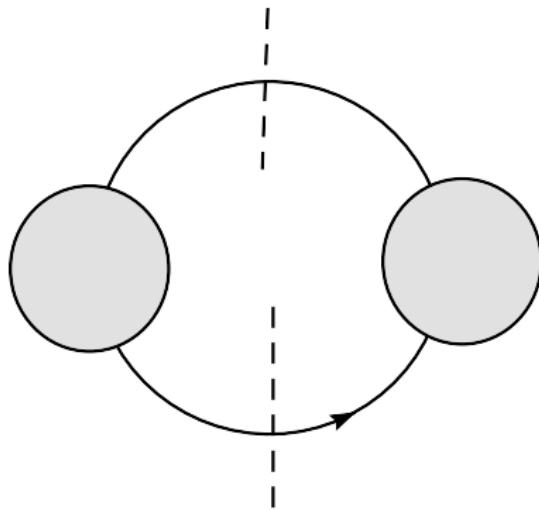
$$\epsilon^{p_1 p_2 p_3 p_4} Q_\mu = \epsilon^{\mu p_2 p_3 p_4} Q \cdot p_1 + \epsilon^{p_1 \mu p_3 p_4} Q \cdot p_2 + \epsilon^{p_1 p_2 \mu p_4} Q \cdot p_3 + \epsilon^{p_1 p_2 p_3 \mu} Q \cdot p_4 . \quad (6)$$

which yields the final formula for the scalar one-loop five-point function:

$$\begin{aligned} E_{01234}(w^2 - 4\Delta_4 m_0^2) &= D_{1234} [2\Delta_4 - w \cdot (v_1 + v_2 + v_3 + v_4)] \\ &+ D_{0234} v_1 \cdot w + D_{0134} v_2 \cdot w + D_{0124} v_3 \cdot w + D_{0123} v_4 \cdot w . \end{aligned} \quad (19)$$

This method is completely different from the one used in ref. [3].

UNITARITY



Started in 90's, mainly QCD, amplitude level (analytical results)

Z. Bern, L. J. Dixon, D. C. Dunbar and D. A. Kosower,

[arXiv:hep-ph/9403226].

Gluing tree amplitudes plus colinear limits → extract coefficients

UNITARITY

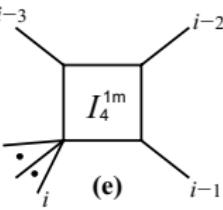
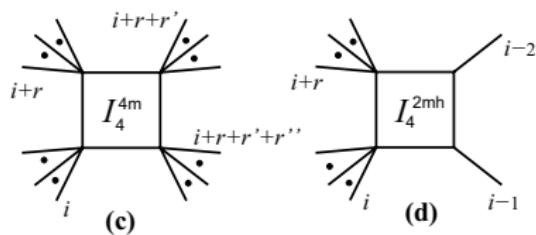
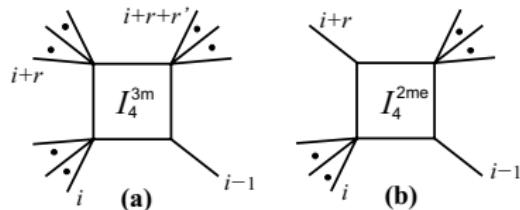
$$\begin{aligned}\mathcal{C} * \int A &= \sum_{i_0 < i_1 < i_2 < i_3}^{m-1} d(i_0 i_1 i_2 i_3) \mathcal{C} * D_0(i_0 i_1 i_2 i_3) \\ &+ \sum_{i_0 < i_1 < i_2}^{m-1} c(i_0 i_1 i_2) \mathcal{C} * C_0(i_0 i_1 i_2) \\ &+ \sum_{i_0 < i_1}^{m-1} b(i_0 i_1) \mathcal{C} * B_0(i_0 i_1)\end{aligned}$$

UNITARITY

	Integral	Unique Function
a	$I_4^{0m}(s, t)$	$\ln(-s) \ln(-t)$
b	$I_3^{1m}(s)$	$\ln(-s)^2$
c	$I_3^{1m}(t)$	$\ln(-t)^2$
d	$I_2(s)$	$\ln(-s)$
e	$I_2(t)$	$\ln(-t)$

Table 1: The set of integral functions that may appear in a cut-constructible massless four-point amplitude, together with the independent logarithms.

UNITARITY



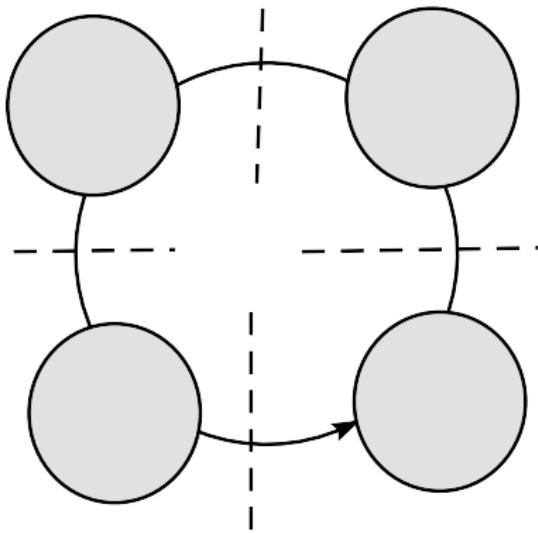
UNITARITY

	Integral	Unique Function
a	$I_{4:r,r';i}^{3m}$	$\ln(-t_i^{[r]}) \ln(-t_{i+r+r'}^{[n-r-r'-1]})$
b	$I_{4:r;i}^{2m\,e}$	$\ln(-t_i^{[r]}) \ln(-t_{i+r+1}^{[n-r-2]})$
c	$I_{4:r,r',r'';i}^{4m}$	$\ln(-t_i^{[r]}) \ln(-t_{i+r+r'}^{[r'']})$
d	$I_{4:r;i}^{2m\,h}$	$\ln(-t_i^{[r]}) \ln(-t_{i+r}^{[n-r-1]})$
e	$I_{4;i}^{1m}$	$\ln(-t_i^{[r]}) \ln(-t_i^{[r+1]})$
f	$I_{3:r,r';i}^{3m}$	$\ln(-t_i^{[r]}) \ln(-t_{i+r}^{[r']})$

Table 2: Following the ordering shown and taking large $t_i^{[r]}$ makes the proof of uniqueness of the cuts straightforward.

QUADRUPLE CUTS

R. Britto, F. Cachazo and B. Feng, [arXiv:hep-th/0412103].
Quadruple cut with complex momenta $\rightarrow d(i_0 i_1 i_2 i_3)$



OPP “MASTER” FORMULA - I

General expression for the 4-dim $N(q)$ at the integrand level in terms of D_i

$$\begin{aligned} N(q) &= \sum_{i_0 < i_1 < i_2 < i_3}^{m-1} \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3) \right] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i \\ &+ \sum_{i_0 < i_1 < i_2}^{m-1} [c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2)] \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i \\ &+ \sum_{i_0 < i_1}^{m-1} \left[b(i_0 i_1) + \tilde{b}(q; i_0 i_1) \right] \prod_{i \neq i_0, i_1}^{m-1} D_i \\ &+ \sum_{i_0}^{m-1} [a(i_0) + \tilde{a}(q; i_0)] \prod_{i \neq i_0}^{m-1} D_i \end{aligned}$$

OPP “MASTER” FORMULA - II

$$N(q) = \sum_{i_0 < i_1 < i_2 < i_3}^{m-1} [d(i_0 i_1 i_2 i_3)] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i + \sum_{i_0 < i_1 < i_2}^{m-1} [c(i_0 i_1 i_2)] \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i \\ + \sum_{i_0 < i_1}^{m-1} [b(i_0 i_1)] \prod_{i \neq i_0, i_1}^{m-1} D_i + \sum_{i_0}^{m-1} [a(i_0)] \prod_{i \neq i_0}^{m-1} D_i$$

- The quantities $d(i_0 i_1 i_2 i_3)$ are the coefficients of 4-point functions with denominators labeled by i_0 , i_1 , i_2 , and i_3 .
- $c(i_0 i_1 i_2)$, $b(i_0 i_1)$, $a(i_0)$ are the coefficients of all possible 3-point, 2-point and 1-point functions, respectively.

OPP “MASTER” FORMULA - II

$$\begin{aligned} N(q) &= \sum_{i_0 < i_1 < i_2 < i_3}^{m-1} [d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3)] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i + \sum_{i_0 < i_1 < i_2}^{m-1} [c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2)] \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i \\ &+ \sum_{i_0 < i_1}^{m-1} [b(i_0 i_1) + \tilde{b}(q; i_0 i_1)] \prod_{i \neq i_0, i_1}^{m-1} D_i + \sum_{i_0}^{m-1} [a(i_0) + \tilde{a}(q; i_0)] \prod_{i \neq i_0}^{m-1} D_i \end{aligned}$$

The quantities \tilde{d} , \tilde{c} , \tilde{b} , \tilde{a} are the “spurious” terms

- They still depend on q (integration momentum)
- They should vanish upon integration

What is the explicit expression of the spurious term?

SPURIOUS TERMS - I

Following F. del Aguila and R. Pittau, arXiv:hep-ph/0404120

- Express any q in $N(q)$ as

$$q^\mu = -p_0^\mu + \sum_{i=1}^4 G_i \ell_i^\mu, \quad \ell_i^2 = 0$$

$$\begin{aligned} k_1 &= \ell_1 + \alpha_1 \ell_2, \quad k_2 = \ell_2 + \alpha_2 \ell_1, \quad k_i = p_i - p_0 \\ \ell_3^\mu &= \langle \ell_1 | \gamma^\mu | \ell_2 \rangle, \quad \ell_4^\mu = \langle \ell_2 | \gamma^\mu | \ell_1 \rangle \end{aligned}$$

- The coefficients G_i either reconstruct denominators D_i
 - They give rise to d, c, b, a coefficients

SPURIOUS TERMS - I

Following F. del Aguila and R. Pittau, arXiv:hep-ph/0404120

- Express any q in $N(q)$ as

$$q^\mu = -p_0^\mu + \sum_{i=1}^4 G_i \ell_i^\mu, \quad \ell_i^2 = 0$$

$$\begin{aligned} k_1 &= \ell_1 + \alpha_1 \ell_2, \quad k_2 = \ell_2 + \alpha_2 \ell_1, \quad k_i = p_i - p_0 \\ \ell_3^\mu &= \langle \ell_1 | \gamma^\mu | \ell_2 \rangle, \quad \ell_4^\mu = \langle \ell_2 | \gamma^\mu | \ell_1 \rangle \end{aligned}$$

- The coefficients G_i either reconstruct denominators D_i or vanish upon integration

- They give rise to d, c, b, a coefficients
- They form the spurious $\tilde{d}, \tilde{c}, \tilde{b}, \tilde{a}$ coefficients

SPURIOUS TERMS - II

- $\tilde{d}(q)$ term (only 1)

$$\tilde{d}(q) = \tilde{d} T(q),$$

where \tilde{d} is a constant (does not depend on q)

$$T(q) \equiv Tr[(\not{q} + \not{p}_0)\not{\ell}_1\not{\ell}_2\not{k}_3\not{\gamma}_5]$$

SPURIOUS TERMS - II

- $\tilde{d}(q)$ term (only 1)

$$\tilde{d}(q) = \tilde{d} T(q),$$

where \tilde{d} is a constant (does not depend on q)

$$T(q) \equiv \text{Tr}[(q + p_0)\ell_1\ell_2k_3\gamma_5]$$

- $\tilde{c}(q)$ terms (they are 6)

$$\tilde{c}(q) = \sum_{j=1}^{j_{max}} \left\{ \tilde{c}_{1j} [(q + p_0) \cdot \ell_3]^j + \tilde{c}_{2j} [(q + p_0) \cdot \ell_4]^j \right\}$$

In the renormalizable gauge, $j_{max} = 3$

SPURIOUS TERMS - II

- $\tilde{d}(q)$ term (only 1)

$$\tilde{d}(q) = \tilde{d} T(q),$$

where \tilde{d} is a constant (does not depend on q)

$$T(q) \equiv \text{Tr}[(q + p_0)\ell_1\ell_2k_3\gamma_5]$$

- $\tilde{c}(q)$ terms (they are 6)

$$\tilde{c}(q) = \sum_{j=1}^{j_{max}} \left\{ \tilde{c}_{1j} [(q + p_0) \cdot \ell_3]^j + \tilde{c}_{2j} [(q + p_0) \cdot \ell_4]^j \right\}$$

In the renormalizable gauge, $j_{max} = 3$

- $\tilde{b}(q)$ and $\tilde{a}(q)$ give rise to 8 and 4 terms, respectively

A SIMPLE EXAMPLE

$$\int \frac{1}{D_0 D_1 D_2 D_3 D_4}$$

A SIMPLE EXAMPLE

$$\int \frac{1}{D_0 D_1 D_2 D_3 D_4}$$

$$1 = \sum \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3) \right] D_{i_4}$$

A SIMPLE EXAMPLE

$$\int \frac{1}{D_0 D_1 D_2 D_3 D_4}$$

$$1 = \sum \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3) \right] D_{i_4}$$

$$\int \frac{1}{D_0 D_1 D_2 D_3 D_4} \sum \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3) \right] D_{i_4}$$

A SIMPLE EXAMPLE

$$\int \frac{1}{D_0 D_1 D_2 D_3 D_4}$$

$$1 = \sum \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3) \right] D_{i_4}$$

$$\int \frac{1}{D_0 D_1 D_2 D_3 D_4} \sum \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3) \right] D_{i_4}$$

$$\int \frac{1}{D_0 D_1 D_2 D_3 D_4} = \sum d(i_0 i_1 i_2 i_3) D_0(i_0 i_1 i_2 i_3)$$

A SIMPLE EXAMPLE

$$\int \frac{1}{D_0 D_1 D_2 D_3 D_4}$$

$$1 = \sum \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3) \right] D_{i_4}$$

$$\int \frac{1}{D_0 D_1 D_2 D_3 D_4} \sum \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3) \right] D_{i_4}$$

$$\int \frac{1}{D_0 D_1 D_2 D_3 D_4} = \sum d(i_0 i_1 i_2 i_3) D_0(i_0 i_1 i_2 i_3)$$

$$d(i_0 i_1 i_2 i_3) = \frac{1}{2} \left(\frac{1}{D_{i_4}(q^+)} + \frac{1}{D_{i_4}(q^-)} \right)$$

A SIMPLE EXAMPLE

$$\int \frac{1}{D_0 D_1 D_2 D_3 D_4}$$

$$1 = \sum \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3) \right] D_{i_4}$$

$$\int \frac{1}{D_0 D_1 D_2 D_3 D_4} \sum \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3) \right] D_{i_4}$$

$$\int \frac{1}{D_0 D_1 D_2 D_3 D_4} = \sum d(i_0 i_1 i_2 i_3) D_0(i_0 i_1 i_2 i_3)$$

$$d(i_0 i_1 i_2 i_3) = \frac{1}{2} \left(\frac{1}{D_{i_4}(q^+)} + \frac{1}{D_{i_4}(q^-)} \right)$$

- Melrose, Nuovo Cim. 40 (1965) 181
- G. Källén, J.Toll, J. Math. Phys. 6, 299 (1965)

A NEXT TO SIMPLE EXAMPLE

$$\int \frac{1}{D_0 D_1 D_2 D_3 \dots D_{m-1}}$$

A NEXT TO SIMPLE EXAMPLE

$$\int \frac{1}{D_0 D_1 D_2 D_3 \dots D_{m-1}}$$
$$1 = \sum \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3) \right] D_{i_4} D_{i_5} \dots D_{i_{m-1}}$$

A NEXT TO SIMPLE EXAMPLE

$$\int \frac{1}{D_0 D_1 D_2 D_3 \dots D_{m-1}}$$

$$1 = \sum \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3) \right] D_{i_4} D_{i_5} \dots D_{i_{m-1}}$$

$$\int \frac{1}{D_0 D_1 D_2 D_3 \dots D_{m-1}} \sum \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3) \right] D_{i_4} D_{i_5} \dots D_{i_{m-1}}$$

A NEXT TO SIMPLE EXAMPLE

$$\int \frac{1}{D_0 D_1 D_2 D_3 \dots D_{m-1}}$$

$$1 = \sum \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3) \right] D_{i_4} D_{i_5} \dots D_{i_{m-1}}$$

$$\int \frac{1}{D_0 D_1 D_2 D_3 \dots D_{m-1}} \sum \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3) \right] D_{i_4} D_{i_5} \dots D_{i_{m-1}}$$

$$\int \frac{1}{D_0 D_1 D_2 D_3 \dots D_{m-1}} = \sum d(i_0 i_1 i_2 i_3) D_0(i_0 i_1 i_2 i_3)$$

A NEXT TO SIMPLE EXAMPLE

$$\int \frac{1}{D_0 D_1 D_2 D_3 \dots D_{m-1}}$$

$$1 = \sum \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3) \right] D_{i_4} D_{i_5} \dots D_{i_{m-1}}$$

$$\int \frac{1}{D_0 D_1 D_2 D_3 \dots D_{m-1}} \sum \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3) \right] D_{i_4} D_{i_5} \dots D_{i_{m-1}}$$

$$\int \frac{1}{D_0 D_1 D_2 D_3 \dots D_{m-1}} = \sum d(i_0 i_1 i_2 i_3) D_0(i_0 i_1 i_2 i_3)$$

$$d(i_0 i_1 i_2 i_3) = \frac{1}{2} \left(\prod_{j \neq i_0, i_1, i_2, i_3} \frac{1}{D_j(q^+)} + \prod_{j \neq i_0, i_1, i_2, i_3} \frac{1}{D_j(q^-)} \right)$$

A NEXT TO SIMPLE EXAMPLE

$$\int \frac{1}{D_0 D_1 D_2 D_3 \dots D_{m-1}}$$

$$1 = \sum \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3) \right] D_{i_4} D_{i_5} \dots D_{i_{m-1}}$$

$$\int \frac{1}{D_0 D_1 D_2 D_3 \dots D_{m-1}} \sum \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3) \right] D_{i_4} D_{i_5} \dots D_{i_{m-1}}$$

$$\int \frac{1}{D_0 D_1 D_2 D_3 \dots D_{m-1}} = \sum d(i_0 i_1 i_2 i_3) D_0(i_0 i_1 i_2 i_3)$$

$$d(i_0 i_1 i_2 i_3) = \frac{1}{2} \left(\prod_{j \neq i_0, i_1, i_2, i_3} \frac{1}{D_j(q^+)} + \prod_{j \neq i_0, i_1, i_2, i_3} \frac{1}{D_j(q^-)} \right)$$

A NEXT TO SIMPLE EXAMPLE

$$\begin{vmatrix} T_0^5 & -T_0^4(0) & -T_0^4(1) & -T_0^4(2) & -T_0^4(3) & -T_0^4(4) \\ 1 & Y_{00} & Y_{01} & Y_{02} & Y_{03} & Y_{04} \\ 1 & Y_{10} & Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ 1 & Y_{20} & Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ 1 & Y_{30} & Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ 1 & Y_{40} & Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{vmatrix} = 0,$$

GENERAL STRATEGY

Now we know the form of the spurious terms:

$$\begin{aligned} N(q) &= \sum_{i_0 < i_1 < i_2 < i_3}^{m-1} [d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3)] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i + \sum_{i_0 < i_1 < i_2}^{m-1} [c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2)] \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i \\ &+ \sum_{i_0 < i_1}^{m-1} [b(i_0 i_1) + \tilde{b}(q; i_0 i_1)] \prod_{i \neq i_0, i_1}^{m-1} D_i + \sum_{i_0}^{m-1} [a(i_0) + \tilde{a}(q; i_0)] \prod_{i \neq i_0}^{m-1} D_i \end{aligned}$$

Our calculation is now reduced to an algebraic problem

GENERAL STRATEGY

Now we know the form of the spurious terms:

$$\begin{aligned} N(q) &= \sum_{i_0 < i_1 < i_2 < i_3}^{m-1} [d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3)] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i + \sum_{i_0 < i_1 < i_2}^{m-1} [c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2)] \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i \\ &+ \sum_{i_0 < i_1}^{m-1} [b(i_0 i_1) + \tilde{b}(q; i_0 i_1)] \prod_{i \neq i_0, i_1}^{m-1} D_i + \sum_{i_0}^{m-1} [a(i_0) + \tilde{a}(q; i_0)] \prod_{i \neq i_0}^{m-1} D_i \end{aligned}$$

Our calculation is now reduced to an **algebraic problem**

Extract all the coefficients by evaluating $N(q)$ for a set of values of the integration momentum q

GENERAL STRATEGY

Now we know the form of the spurious terms:

$$\begin{aligned} N(q) &= \sum_{i_0 < i_1 < i_2 < i_3}^{m-1} [d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3)] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i + \sum_{i_0 < i_1 < i_2}^{m-1} [c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2)] \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i \\ &+ \sum_{i_0 < i_1}^{m-1} [b(i_0 i_1) + \tilde{b}(q; i_0 i_1)] \prod_{i \neq i_0, i_1}^{m-1} D_i + \sum_{i_0}^{m-1} [a(i_0) + \tilde{a}(q; i_0)] \prod_{i \neq i_0}^{m-1} D_i \end{aligned}$$

Our calculation is now reduced to an **algebraic problem**

Extract all the coefficients by evaluating $N(q)$ for a set of values of the integration momentum q

There is a very good set of such points: **Use values of q for which a set of denominators D_i vanish** → The system becomes “triangular”: solve first for 4-point functions, then 3-point functions and so on

EXAMPLE

$$\begin{aligned}N(q) &= d + \tilde{d}(q) + \sum_{i=0}^3 [c(i) + \tilde{c}(q; i)] D_i + \sum_{i_0 < i_1}^3 [b(i_0 i_1) + \tilde{b}(q; i_0 i_1)] D_{i_0} D_{i_1} \\&+ \sum_{i_0=0}^3 [a(i_0) + \tilde{a}(q; i_0)] D_{i \neq i_0} D_{j \neq i_0} D_{k \neq i_0}\end{aligned}$$

We look for a q of the form $q^\mu = -p_0^\mu + x_i \ell_i^\mu$ such that

$$D_0 = D_1 = D_2 = D_3 = 0$$

→ we get a system of equations in x_i that has two solutions q_0^\pm

EXAMPLE

$$N(q) = d + \tilde{d}(q)$$

Our “master formula” for $q = q_0^\pm$ is:

$$N(q_0^\pm) = [d + \tilde{d} T(q_0^\pm)]$$

→ solve to extract the coefficients d and \tilde{d}

EXAMPLE

$$\begin{aligned} N(q) - d - \tilde{d}(q) &= \sum_{i=0}^3 [c(i) + \tilde{c}(q; i)] D_i + \sum_{i_0 < i_1}^3 [b(i_0 i_1) + \tilde{b}(q; i_0 i_1)] D_{i_0} D_{i_1} \\ &+ \sum_{i_0=0}^3 [a(i_0) + \tilde{a}(q; i_0)] D_{i \neq i_0} D_{j \neq i_0} D_{k \neq i_0} \end{aligned}$$

Then we can move to the extraction of **c coefficients** using

$$N'(q) = N(q) - d - \tilde{d} T(q)$$

and setting to zero three denominators (ex: $D_1 = 0$, $D_2 = 0$, $D_3 = 0$)

EXAMPLE

$$N(q) - \textcolor{blue}{d} - \tilde{d}(q) = [\textcolor{blue}{c}(0) + \tilde{c}(q; 0)] D_0$$

We have infinite values of q for which

$$\textcolor{violet}{D}_1 = \textcolor{violet}{D}_2 = \textcolor{violet}{D}_3 = 0 \quad \text{and} \quad \textcolor{violet}{D}_0 \neq 0$$

→ Here we need 7 of them to determine $\textcolor{blue}{c}(0)$ and $\tilde{c}(q; 0)$

RATIONAL TERMS - I

- Let's go back to the integrand

$$A(\bar{q}) = \frac{N(q)}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}}$$

- Insert the expression for $N(q) \rightarrow$ we know all the coefficients

$$N(q) = \sum_{i_0 < i_1 < i_2 < i_3}^{m-1} \left[d + \tilde{d}(q) \right] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i + \sum_{i_0 < i_1 < i_2}^{m-1} [c + \tilde{c}(q)] \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i + \cdots$$

- Finally rewrite all denominators using

$$\frac{D_i}{\bar{D}_i} = \bar{Z}_i, \quad \text{with} \quad \bar{Z}_i \equiv \left(1 - \frac{\tilde{q}^2}{\bar{D}_i} \right)$$

RATIONAL TERMS - I

$$\begin{aligned}
 A(\bar{q}) &= \sum_{i_0 < i_1 < i_2 < i_3}^{m-1} \frac{d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3)}{\bar{D}_{i_0} \bar{D}_{i_1} \bar{D}_{i_2} \bar{D}_{i_3}} \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} \bar{Z}_i \\
 &+ \sum_{i_0 < i_1 < i_2}^{m-1} \frac{c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2)}{\bar{D}_{i_0} \bar{D}_{i_1} \bar{D}_{i_2}} \prod_{i \neq i_0, i_1, i_2}^{m-1} \bar{Z}_i \\
 &+ \sum_{i_0 < i_1}^{m-1} \frac{b(i_0 i_1) + \tilde{b}(q; i_0 i_1)}{\bar{D}_{i_0} \bar{D}_{i_1}} \prod_{i \neq i_0, i_1}^{m-1} \bar{Z}_i \\
 &+ \sum_{i_0}^{m-1} \frac{a(i_0) + \tilde{a}(q; i_0)}{\bar{D}_{i_0}} \prod_{i \neq i_0}^{m-1} \bar{Z}_i
 \end{aligned}$$

The rational part is produced, after integrating over $d^n q$, by the \tilde{q}^2 dependence in \bar{Z}_i

$$\bar{Z}_i \equiv \left(1 - \frac{\tilde{q}^2}{\bar{D}_i} \right)$$

RATIONAL TERMS - I

The “Extra Integrals” are of the form

$$I_{s;\mu_1 \cdots \mu_r}^{(n;2\ell)} \equiv \int d^n q \tilde{q}^{2\ell} \frac{q_{\mu_1} \cdots q_{\mu_r}}{\bar{D}(k_0) \cdots \bar{D}(k_s)},$$

where

$$\bar{D}(k_i) \equiv (\bar{q} + k_i)^2 - m_i^2, k_i = p_i - p_0$$

These integrals:

- **have dimensionality** $\mathcal{D} = 2(1 + \ell - s) + r$
- **contribute only when** $\mathcal{D} \geq 0$, **otherwise are of** $\mathcal{O}(\epsilon)$

RATIONAL TERMS - II

Expand in D-dimensions ?

$$\bar{D}_i = D_i + \tilde{q}^2$$

RATIONAL TERMS - II

Expand in D-dimensions ?

$$\begin{aligned} N(q) &= \sum_{i_0 < i_1 < i_2 < i_3}^{m-1} \left[d(i_0 i_1 i_2 i_3; \tilde{q}^2) + \tilde{d}(q; i_0 i_1 i_2 i_3; \tilde{q}^2) \right] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} \bar{D}_i \\ &+ \sum_{i_0 < i_1 < i_2}^{m-1} \left[c(i_0 i_1 i_2; \tilde{q}^2) + \tilde{c}(q; i_0 i_1 i_2; \tilde{q}^2) \right] \prod_{i \neq i_0, i_1, i_2}^{m-1} \bar{D}_i \\ &+ \sum_{i_0 < i_1}^{m-1} \left[b(i_0 i_1; \tilde{q}^2) + \tilde{b}(q; i_0 i_1; \tilde{q}^2) \right] \prod_{i \neq i_0, i_1}^{m-1} \bar{D}_i \\ &+ \sum_{i_0}^{m-1} \left[a(i_0; \tilde{q}^2) + \tilde{a}(q; i_0; \tilde{q}^2) \right] \prod_{i \neq i_0}^{m-1} \bar{D}_i + \tilde{P}(q) \prod_i^{m-1} \bar{D}_i \end{aligned}$$

RATIONAL TERMS - II

Expand in D-dimensions ?

$$\begin{aligned} N(q) &= \sum_{i_0 < i_1 < i_2 < i_3}^{m-1} \left[d(i_0 i_1 i_2 i_3; \tilde{q}^2) + \tilde{d}(q; i_0 i_1 i_2 i_3; \tilde{q}^2) \right] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} \bar{D}_i \\ &+ \sum_{i_0 < i_1 < i_2}^{m-1} \left[c(i_0 i_1 i_2; \tilde{q}^2) + \tilde{c}(q; i_0 i_1 i_2; \tilde{q}^2) \right] \prod_{i \neq i_0, i_1, i_2}^{m-1} \bar{D}_i \\ &+ \sum_{i_0 < i_1}^{m-1} \left[b(i_0 i_1; \tilde{q}^2) + \tilde{b}(q; i_0 i_1; \tilde{q}^2) \right] \prod_{i \neq i_0, i_1}^{m-1} \bar{D}_i \\ &+ \sum_{i_0}^{m-1} \left[a(i_0; \tilde{q}^2) + \tilde{a}(q; i_0; \tilde{q}^2) \right] \prod_{i \neq i_0}^{m-1} \bar{D}_i + \tilde{P}(q) \prod_i^{m-1} \bar{D}_i \end{aligned}$$

$$m_i^2 \rightarrow m_i^2 - \tilde{q}^2$$

RATIONAL TERMS - II

Polynomial dependence on \tilde{q}^2

$$b(ij; \tilde{q}^2) = b(ij) + \tilde{q}^2 b^{(2)}(ij), \quad c(ijk; \tilde{q}^2) = c(ijk) + \tilde{q}^2 c^{(2)}(ijk).$$

RATIONAL TERMS - II

Polynomial dependence on \tilde{q}^2

$$b(ij; \tilde{q}^2) = b(ij) + \tilde{q}^2 b^{(2)}(ij), \quad c(ijk; \tilde{q}^2) = c(ijk) + \tilde{q}^2 c^{(2)}(ijk).$$

$$\int d^n \bar{q} \frac{\tilde{q}^2}{\bar{D}_i \bar{D}_j} = -\frac{i\pi^2}{2} \left[m_i^2 + m_j^2 - \frac{(p_i - p_j)^2}{3} \right] + \mathcal{O}(\epsilon),$$

$$\int d^n \bar{q} \frac{\tilde{q}^2}{\bar{D}_i \bar{D}_j \bar{D}_k} = -\frac{i\pi^2}{2} + \mathcal{O}(\epsilon), \quad \int d^n \bar{q} \frac{\tilde{q}^4}{\bar{D}_i \bar{D}_j \bar{D}_k \bar{D}_l} = -\frac{i\pi^2}{6} + \mathcal{O}(\epsilon).$$

RATIONAL TERMS - II

Furthermore, by defining

$$\mathcal{D}^{(m)}(q, \tilde{q}^2) \equiv \sum_{i_0 < i_1 < i_2 < i_3}^{m-1} \left[d(i_0 i_1 i_2 i_3; \tilde{q}^2) + \tilde{d}(q; i_0 i_1 i_2 i_3; \tilde{q}^2) \right] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} \bar{D}_i,$$

the following expansion holds

$$\mathcal{D}^{(m)}(q, \tilde{q}^2) = \sum_{j=2}^m \tilde{q}^{(2j-4)} d^{(2j-4)}(q),$$

where the last coefficient is independent on q

$$d^{(2m-4)}(q) = d^{(2m-4)}.$$

RATIONAL TERMS - II

In practice, once the 4-dimensional coefficients have been determined, one can redo the fits for different values of \tilde{q}^2 , in order to determine $b^{(2)}(ij)$, $c^{(2)}(ijk)$ and $d^{(2m-4)}$.

$$\begin{aligned} R_1 &= -\frac{i}{96\pi^2}d^{(2m-4)} - \frac{i}{32\pi^2} \sum_{i_0 < i_1 < i_2}^{m-1} c^{(2)}(i_0 i_1 i_2) \\ &\quad - \frac{i}{32\pi^2} \sum_{i_0 < i_1}^{m-1} b^{(2)}(i_0 i_1) \left(m_{i_0}^2 + m_{i_1}^2 - \frac{(p_{i_0} - p_{i_1})^2}{3} \right). \end{aligned}$$

G. Ossola, C. G. Papadopoulos and R. Pittau, arXiv:0802.1876 [hep-ph]

RATIONAL TERMS - R_2

A different source of Rational Terms, called R_2 , can also be generated from the ϵ -dimensional part of $N(q)$

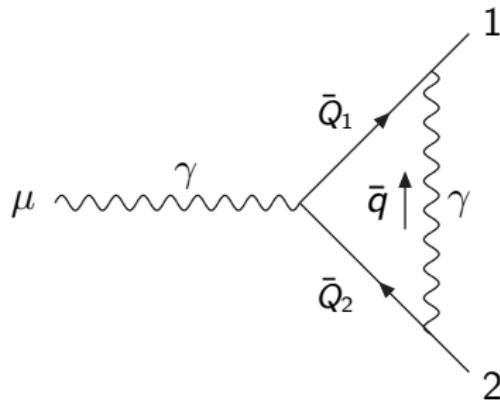
$$\bar{N}(\bar{q}) = N(q) + \tilde{N}(\tilde{q}^2, \epsilon; q)$$

$$R_2 \equiv \frac{1}{(2\pi)^4} \int d^n \bar{q} \frac{\tilde{N}(\tilde{q}^2, \epsilon; q)}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}} \equiv \frac{1}{(2\pi)^4} \int d^n \bar{q} \mathcal{R}_2$$

$$\begin{aligned}\bar{q} &= q + \tilde{q}, \\ \bar{\gamma}_{\bar{\mu}} &= \gamma_\mu + \tilde{\gamma}_{\tilde{\mu}}, \\ \bar{g}^{\bar{\mu}\bar{\nu}} &= g^{\mu\nu} + \tilde{g}^{\tilde{\mu}\tilde{\nu}}.\end{aligned}$$

New vertices/particles or GKM-approach

RATIONAL TERMS - R_2



$$\begin{aligned}\bar{Q}_1 &= \bar{q} + p_1 = Q_1 + \tilde{q} \\ \bar{Q}_2 &= \bar{q} + p_2 = Q_2 + \tilde{q}\end{aligned}$$

$$\bar{D}_0 = \bar{q}^2$$

$$\bar{D}_1 = (\bar{q} + p_1)^2$$

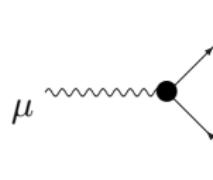
$$\bar{D}_2 = (\bar{q} + p_2)^2$$

$$\begin{aligned}\bar{N}(\bar{q}) &\equiv e^3 \left\{ \bar{\gamma}_{\bar{\beta}} (\bar{Q}_1 + m_e) \gamma_\mu (\bar{Q}_2 + m_e) \bar{\gamma}^{\bar{\beta}} \right\} \\ &= e^3 \left\{ \gamma_\beta (Q_1 + m_e) \gamma_\mu (Q_2 + m_e) \gamma^\beta \right. \\ &\quad \left. - \epsilon (Q_1 - m_e) \gamma_\mu (Q_2 - m_e) + \epsilon \tilde{q}^2 \gamma_\mu - \tilde{q}^2 \gamma_\beta \gamma_\mu \gamma^\beta \right\},\end{aligned}$$

RATIONAL TERMS - R_2

$$\begin{aligned}\int d^n \bar{q} \frac{\tilde{q}^2}{\bar{D}_0 \bar{D}_1 \bar{D}_2} &= -\frac{i\pi^2}{2} + \mathcal{O}(\epsilon), \\ \int d^n \bar{q} \frac{q_\mu q_\nu}{\bar{D}_0 \bar{D}_1 \bar{D}_2} &= -\frac{i\pi^2}{2\epsilon} g_{\mu\nu} + \mathcal{O}(1),\end{aligned}$$

$$R_2 = -\frac{ie^3}{8\pi^2} \gamma_\mu + \mathcal{O}(\epsilon),$$


$$= -\frac{ie^3}{8\pi^2} \gamma_\mu$$

RATIONAL TERMS - R_2

Rational counterterms

$$\mu \overset{p}{\rightsquigarrow} \bullet \sim \nu = -\frac{ie^2}{8\pi^2} g_{\mu\nu} (2m_e^2 - p^2/3)$$

$$\overset{p}{\rightarrow} \bullet \rightarrow = \frac{ie^2}{16\pi^2} (-p + 2m_e)$$

$$\begin{array}{c} \mu \quad \nu \\ \swarrow \quad \searrow \\ \sigma \quad \rho \end{array} = \frac{ie^4}{12\pi^2} (g_{\mu\nu}g_{\rho\sigma} + g_{\mu\rho}g_{\nu\sigma} + g_{\mu\sigma}g_{\nu\rho})$$

SUMMARY

Calculate $N(q)$

SUMMARY

Calculate $N(q)$

- We do not need to repeat this for all Feynman diagrams. We can group them and solve for (sub)amplitudes directly

SUMMARY

Calculate $N(q)$

- We do not need to repeat this for all Feynman diagrams. We can group them and solve for (sub)amplitudes directly
- Calculate $N(q)$ numerically via recursion relations

SUMMARY

Calculate $N(q)$

- We do not need to repeat this for all Feynman diagrams. We can group them and solve for (sub)amplitudes directly
- Calculate $N(q)$ numerically via recursion relations
- Just specify external momenta, polarization vectors and masses and proceed with the reduction!

SUMMARY

Calculate $N(q)$

- We do not need to repeat this for all Feynman diagrams. We can group them and solve for (sub)amplitudes directly
- Calculate $N(q)$ numerically via recursion relations
- Just specify external momenta, polarization vectors and masses and proceed with the reduction!

Compute all coefficients

SUMMARY

Calculate $N(q)$

- We do not need to repeat this for all Feynman diagrams. We can group them and solve for (sub)amplitudes directly
- Calculate $N(q)$ numerically via recursion relations
- Just specify external momenta, polarization vectors and masses and proceed with the reduction!

Compute all coefficients

- by evaluating $N(q)$ at certain values of integration momentum

SUMMARY

Calculate $N(q)$

- We do not need to repeat this for all Feynman diagrams. We can group them and solve for (sub)amplitudes directly
- Calculate $N(q)$ numerically via recursion relations
- Just specify external momenta, polarization vectors and masses and proceed with the reduction!

Compute all coefficients

- by evaluating $N(q)$ at certain values of integration momentum

Evaluate scalar integrals

SUMMARY

Calculate $N(q)$

- We do not need to repeat this for all Feynman diagrams. We can group them and solve for (sub)amplitudes directly
- Calculate $N(q)$ numerically via recursion relations
- Just specify external momenta, polarization vectors and masses and proceed with the reduction!

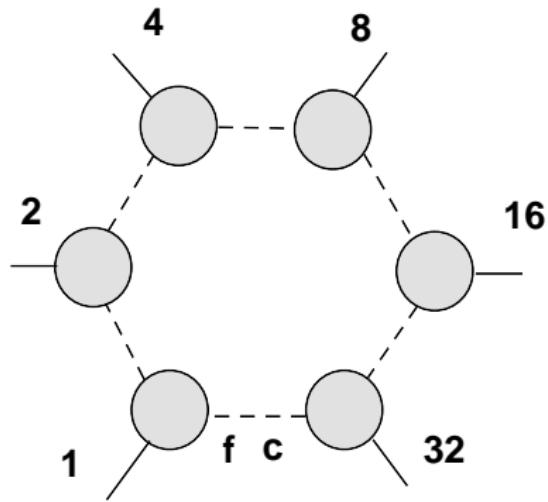
Compute all coefficients

- by evaluating $N(q)$ at certain values of integration momentum

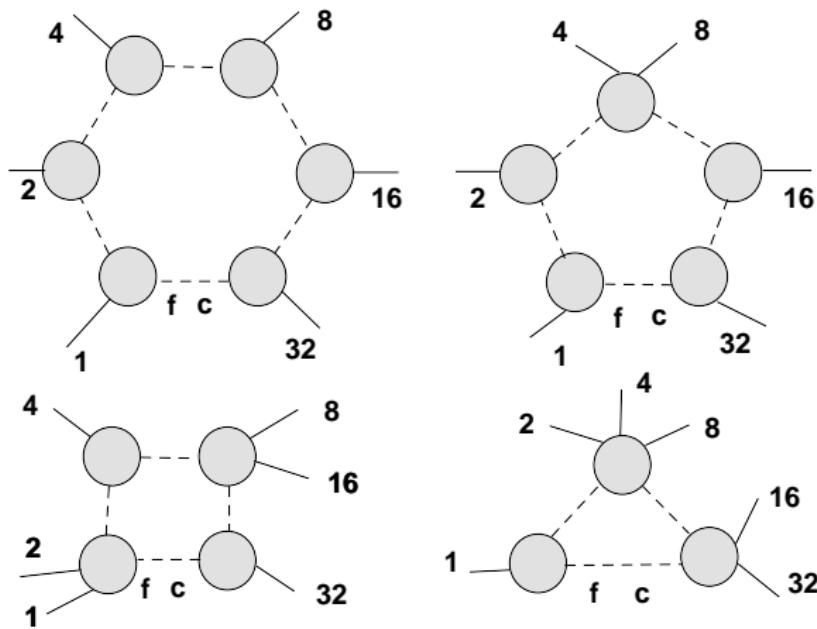
Evaluate scalar integrals

- massive integrals → FF [G. J. van Oldenborgh]
- massless+massive integrals → OneLOop [A. van Hameren]

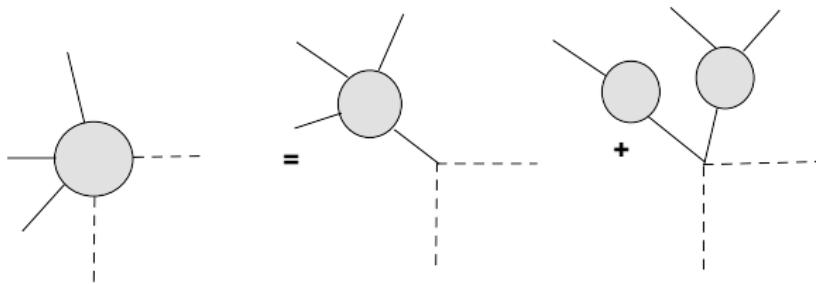
HELAC 1-LOOP



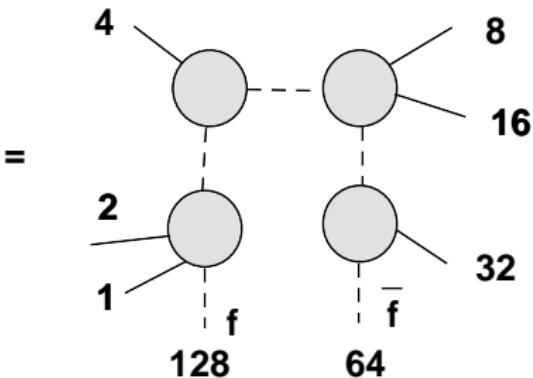
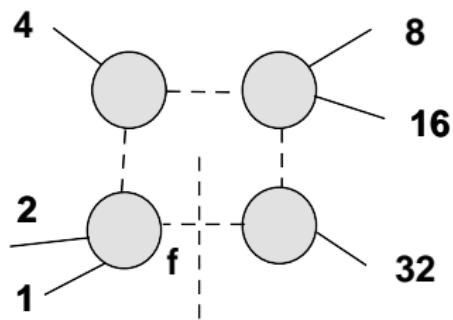
HELAC 1-LOOP



HELAC 1-LOOP



HELAC 1-LOOP



HELAC COLOR TREATMENT

$$\mathcal{M}_{j_2, \dots, j_k}^{a_1, i_2, \dots, i_k} t_{i_1 j_1}^{a_1} \rightarrow \mathcal{M}_{j_1, j_2, \dots, j_k}^{i_1, i_2, \dots, i_k}$$

$$\mathcal{M}_{j_1, j_2, \dots, j_k}^{i_1, i_2, \dots, i_k} = \sum_{\sigma} \delta_{i_{\sigma_1}, j_1} \delta_{i_{\sigma_2}, j_2} \dots \delta_{i_{\sigma_k}, j_k} A_{\sigma}$$

$$\sum_{\{i\}, \{j\}} |\mathcal{M}_{j_1, j_2, \dots, j_k}^{i_1, i_2, \dots, i_k}|^2$$

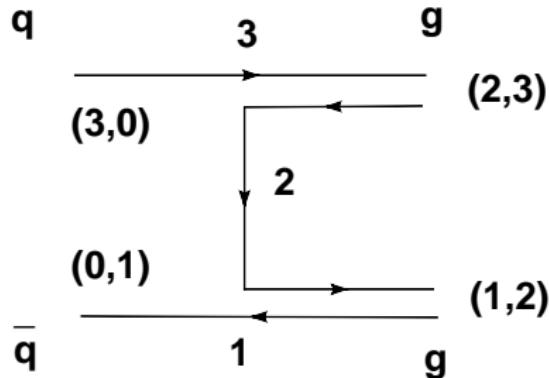
$$\sum_{\sigma, \sigma'} A_{\sigma}^* \mathcal{C}_{\sigma, \sigma'} A_{\sigma'}$$

$$\mathcal{C}_{\sigma, \sigma'} \equiv \sum_{\{i\}, \{j\}} \delta_{i_{\sigma_1}, j_1} \delta_{i_{\sigma_2}, j_2} \dots \delta_{i_{\sigma_k}, j_k} \delta_{i_{\sigma'_1}, j_1} \delta_{i_{\sigma'_2}, j_2} \dots \delta_{i_{\sigma'_k}, j_k}$$

HELAC COLOR TREATMENT

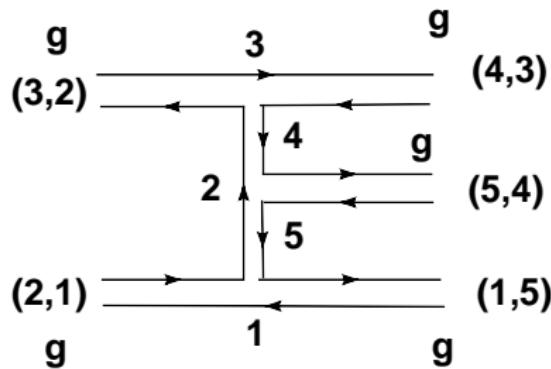
$$(x_1, y_1) \dots (x_n, y_n)$$

where y_i take the values $\{1, 2, \dots, n_l\}$ if i is a gluon or an outgoing quark (incoming anti-quark) otherwise $y_i = 0$, whereas x_i take the values $\{\sigma_1, \sigma_2, \dots, \sigma_{n_l}\}$ if i is a gluon or an incoming quark (outgoing anti-quark) otherwise $x_i = 0$. So for instance for a $q\bar{q} \rightarrow gg$ process, $n_l = 3$ and a possible color connection is given by $(3,0)(0,1)(1,2)(2,3)$



HELAC COLOR TREATMENT

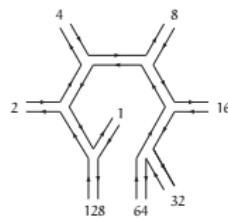
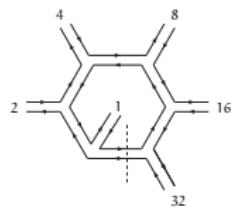
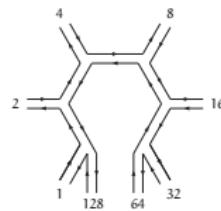
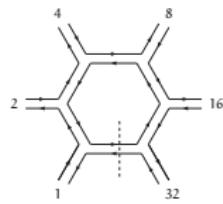
whereas for $gg \rightarrow ggg$, $n_l = 5$ and a possible color connection is given by
 $(2,1)(3,2)(4,3)(5,4)(1,5)$



$$\mathcal{C}_{\sigma, \sigma'} = N_c^{m(\sigma, \sigma')}$$

where $m(\sigma, \sigma')$ count the number of common cycles of the two permutations.

HELAC COLOR TREATMENT - 1 LOOP



HELAC R2 TERMS

$$\frac{p}{\mu_1, a_1 \text{---} \overset{\overbrace{\text{00000}}}{\bullet} \text{---} \mu_2, a_2} = \frac{ig^2 N_{col}}{48\pi^2} \delta_{a_1 a_2} \left[\frac{p^2}{2} g_{\mu_1 \mu_2} + \lambda_{HV} \left(g_{\mu_1 \mu_2} p^2 - p_{\mu_1} p_{\mu_2} \right) + \frac{N_f}{N_{col}} (p^2 - 6 m_q^2) g_{\mu_1 \mu_2} \right]$$

$$\frac{p_1 \quad p_2 \quad \mu_2, a_2}{\overbrace{\text{00000}} \text{---} \overset{\overbrace{\text{00000}}}{\bullet} \text{---} \mu_3, a_3} = -\frac{g^3 N_{col}}{48\pi^2} \left(\frac{7}{4} + \lambda_{HV} + 2 \frac{N_f}{N_{col}} \right) f^{a_1 a_2 a_3} V_{\mu_1 \mu_2 \mu_3}(p_1, p_2, p_3)$$

$$\begin{aligned} & \frac{\mu_1, a_1 \quad \mu_2, a_2}{\text{00000} \text{---} \overset{\overbrace{\text{00000}}}{\bullet} \text{---} \mu_3, a_3} \\ &= -\frac{ig^4 N_{col}}{96\pi^2} \sum_{P(234)} \left\{ \left[\frac{\delta_{a_1 a_2} \delta_{a_3 a_4} + \delta_{a_1 a_3} \delta_{a_4 a_2} + \delta_{a_1 a_4} \delta_{a_2 a_3}}{N_{col}} \right. \right. \\ & \quad + 4 \operatorname{Tr}(t^{a_1} t^{a_3} t^{a_2} t^{a_4} + t^{a_1} t^{a_4} t^{a_2} t^{a_3}) (3 + \lambda_{HV}) \\ & \quad \left. \left. - \operatorname{Tr}(\{t^{a_1} t^{a_2}\} \{t^{a_3} t^{a_4}\}) (5 + 2\lambda_{HV}) \right] g_{\mu_1 \mu_2} g_{\mu_3 \mu_4} \right. \\ & \quad \left. + 12 \frac{N_f}{N_{col}} \operatorname{Tr}(t^{a_1} t^{a_2} t^{a_3} t^{a_4}) \left(\frac{5}{3} g_{\mu_1 \mu_3} g_{\mu_2 \mu_4} - g_{\mu_1 \mu_2} g_{\mu_3 \mu_4} - g_{\mu_2 \mu_3} g_{\mu_1 \mu_4} \right) \right\} \end{aligned}$$

$$\frac{p}{l \text{---} \bullet \text{---} k} = \frac{ig^2}{16\pi^2} \frac{N_{col}^2 - 1}{2N_{col}} \delta_{kl} (-\not{p} + 2m_q) \lambda_{HV}$$

$$\frac{k}{\mu, a \text{---} \overset{\overbrace{\text{00000}}}{\bullet} \text{---} l} = \frac{ig^3}{16\pi^2} \frac{N_{col}^2 - 1}{2N_{col}} t_{kl}^a \gamma_\mu (1 + \lambda_{HV})$$

HELAC R2 TERMS

$$= -\frac{g^2}{16\pi^2} \frac{N_{col}^2 - 1}{2N_{col}} \delta_{kl} \gamma_\mu (v + a\gamma_5) (1 + \lambda_{HV})$$

$$= -\frac{g^2}{8\pi^2} \frac{N_{col}^2 - 1}{2N_{col}} \delta_{kl} (c + d\gamma_5) (1 + \lambda_{HV})$$

$$= a \frac{ig^2}{12\pi^2} \delta_{a_1 a_2} \epsilon_{\mu\alpha_1\alpha_2\beta} (p_1 - p_2)^\beta$$

$$= c \frac{g^2}{8\pi^2} \delta_{a_1 a_2} g_{\alpha_1 \alpha_2} m_q$$

$$= -\frac{ig^2}{24\pi^2} \delta_{a_1 a_2} (v_1 v_2 + a_1 a_2) (g_{\mu_1 \mu_2} g_{\alpha_1 \alpha_2} + g_{\mu_1 \alpha_1} g_{\mu_2 \alpha_2} + g_{\mu_1 \alpha_2} g_{\mu_2 \alpha_1})$$

$$= \frac{ig^2}{8\pi^2} \delta_{a_1 a_2} (c_1 c_2 - d_1 d_2) g_{\alpha_1 \alpha_2}$$

$$= -\frac{g^3}{24\pi^2} \{v Tr(t^{a_1} \{t^{a_2} t^{a_3}\}) (g_{\mu\alpha_1} g_{\alpha_2\alpha_3} + g_{\mu\alpha_2} g_{\alpha_1\alpha_3} + g_{\mu\alpha_3} g_{\alpha_1\alpha_2}) - i9a [Tr(t^{a_1} t^{a_2} t^{a_3}) - Tr(t^{a_1} t^{a_3} t^{a_2})] \epsilon_{\mu\alpha_1\alpha_2\alpha_3}\}$$

HELAC 1-LOOP

INFO =====																		
INFO COLOR 1 out of 6																		
INFO	1	12	35	7	1	1	4	-4	3	8	4	4	0	0	0	0	1	2
INFO	1	48	35	8	1	1	16	-8	5	32	8	6	0	0	0	0	1	2
INFO	2	14	-3	9	1	1	12	35	7	2	-3	2	0	0	0	0	1	2
INFO	2	14	-3	9	0	1	12	35	7	2	-3	2	0	0	0	0	1	2
INFO	2	28	-8	10	1	1	12	35	7	16	-8	5	0	0	0	0	1	2
INFO	2	28	-8	10	0	1	12	35	7	16	-8	5	0	0	0	0	2	1
INFO	3	44	8	11	1	1	12	35	7	32	8	6	0	0	0	0	1	2
INFO	3	44	8	11	0	1	12	35	7	32	8	6	0	0	0	0	2	1
INFO	2	50	-3	12	1	1	48	35	8	2	-3	2	0	0	0	0	1	2
INFO	2	50	-3	12	0	1	48	35	8	2	-3	2	0	0	0	0	2	1
INFO	2	52	-4	13	1	1	48	35	8	4	-4	3	0	0	0	0	1	2
INFO	2	52	-4	13	0	1	48	35	8	4	-4	3	0	0	0	0	2	1
INFO	3	56	4	14	1	1	48	35	8	8	4	4	0	0	0	0	1	2
INFO	3	56	4	14	0	1	48	35	8	8	4	4	0	0	0	0	2	1
INFO	1	60	35	15	1	4	4	-4	3	56	4	14	0	0	0	0	1	2
INFO	1	60	35	15	2	4	16	-8	5	44	8	11	0	0	0	0	1	2
INFO	1	60	35	15	3	4	28	-8	10	32	8	6	0	0	0	0	1	2
INFO	1	60	35	15	4	4	52	-4	13	8	4	4	0	0	0	0	1	2
INFO	2	62	-3	16	1	3	12	35	7	50	-3	12	0	0	0	0	1	2
INFO	2	62	-3	16	0	3	12	35	7	50	-3	12	0	0	0	0	2	1
INFO	2	62	-3	16	2	3	48	35	8	14	-3	9	0	0	0	0	1	2
INFO	2	62	-3	16	0	3	48	35	8	14	-3	9	0	0	0	0	2	1
INFO	2	62	-3	16	3	3	60	35	15	2	-3	2	0	0	0	0	1	2
INFO	2	62	-3	16	0	3	60	35	15	2	-3	2	0	0	0	0	2	1
INFO =====																		
INFO COLOR 2 out of 6																		
INFO	1	12	35	7	1	1	4	-4	3	8	4	4	0	0	0	0	1	1
INFO	1	48	35	8	1	1	16	-8	5	32	8	6	0	0	0	0	1	1

HELAC 1-LOOP

papadopo@aiolos:/tmp - Shell - Konsole

```
INFO =====
INFO COLOR 4 out of 6
INFO number of nums 143
INFO NUM 1 of 143 10
INFO 3 96 8 9 1 1 64 35 7 32 8 6 0 0 0 0 1 1 2
INFO 3 96 8 9 0 1 64 35 7 32 8 6 0 0 0 0 2 1 2
INFO 1 112 35 10 1 1 16 -8 5 96 8 9 0 0 0 0 0 1 1 1
INFO 3 120 4 11 1 1 112 35 10 8 4 4 0 0 0 0 1 1 1
INFO 3 120 4 11 0 1 112 35 10 8 4 4 0 0 0 0 2 1 1
INFO 1 124 35 12 1 1 4 -4 3 120 4 11 0 0 0 0 0 1 1 1
INFO 2 126 -3 13 1 1 124 35 12 2 -3 2 0 0 0 0 1 1 1
INFO 2 126 -3 13 0 1 124 35 12 2 -3 2 0 0 0 0 2 1 1
INFO 2 254 -3 14 1 1 128 35 8 126 -3 13 0 0 0 0 1 1 2
INFO 2 254 -3 14 0 1 128 35 8 126 -3 13 0 0 0 0 2 1 2
INFO 6 32 16 8 4 2 1 35 8 35 4 35 -3 0 0 0 0 3 1
INFOYY 1
INFO NUM 2 of 143 10
INFO 3 96 8 9 1 1 64 35 7 32 8 6 0 0 0 0 1 1 1
INFO 3 96 8 9 0 1 64 35 7 32 8 6 0 0 0 0 2 1 1
INFO 1 112 35 10 1 1 16 -8 5 96 8 9 0 0 0 0 0 1 1 1
INFO 3 120 4 11 1 1 112 35 10 8 4 4 0 0 0 0 1 1 1
INFO 3 120 4 11 0 1 112 35 10 8 4 4 0 0 0 0 2 1 1
INFO 1 124 35 12 1 1 4 -4 3 120 4 11 0 0 0 0 0 1 1 2
INFO 2 126 -3 13 1 1 124 35 12 2 -3 2 0 0 0 0 1 1 2
INFO 2 126 -3 13 0 1 124 35 12 2 -3 2 0 0 0 0 2 1 2
INFO 2 254 -3 14 1 1 128 35 8 126 -3 13 0 0 0 0 1 1 1
INFO 2 254 -3 14 0 1 128 35 8 126 -3 13 0 0 0 0 2 1 1
INFO 6 32 16 8 4 2 1 35 8 35 4 35 -3 0 0 0 0 3 1
INFOYY 1
INFO NUM 3 of 143 10
INFO 2 80 -8 0 1 1 64 35 7 16 -8 5 0 0 0 0 1 1 2
```

HELAC 1-LOOP

INFO NUM 127 of 143 15																		
INFO	1	48	35	9	1	1	16	-8	5	32	8	6	0	0	0	0	1	1
INFO	3	112	3	10	1	1	48	35	9	64	3	7	0	0	0	0	1	1
INFO	3	112	3	10	0	1	48	35	9	64	3	7	0	0	0	0	2	1
INFO	1	12	35	11	1	1	4	-4	3	8	4	4	0	0	0	0	1	1
INFO	1	240	35	12	1	1	128	-3	8	112	3	10	0	0	0	0	-1	1
INFO	2	242	-3	13	1	1	240	35	12	2	-3	2	0	0	0	0	1	1
INFO	2	242	-3	13	0	1	240	35	12	2	-3	2	0	0	0	0	2	1
INFO	3	248	4	14	1	1	240	35	12	8	4	4	0	0	0	0	1	1
INFO	3	248	4	14	0	1	240	35	12	8	4	4	0	0	0	0	2	1
INFO	1	252	35	15	1	2	4	-4	3	248	4	14	0	0	0	0	1	1
INFO	4	252	35	15	2	2	12	35	11	240	35	12	0	0	0	0	1	1
INFO	2	254	-3	16	1	2	12	35	11	242	-3	13	0	0	0	0	1	1
INFO	2	254	-3	16	0	2	12	35	11	242	-3	13	0	0	0	0	2	1
INFO	2	254	-3	16	2	2	252	35	15	2	-3	2	0	0	0	0	1	1
INFO	2	254	-3	16	0	2	252	35	15	2	-3	2	0	0	0	0	2	1
INFO	2	48	15	3	3	0	0	0	0	0	0	0	0	0	0	0	2	5
INFOYY	5																	
INFO NUM 128 of 143 11																		
INFO	1	12	35	7	1	1	4	-4	3	8	4	4	0	0	0	0	1	1
INFO	1	48	35	8	1	1	16	-8	5	32	8	6	0	0	0	0	1	1
INFO	2	28	-8	9	1	1	12	35	7	16	-8	5	0	0	0	0	1	1
INFO	2	28	-8	9	0	1	12	35	7	16	-8	5	0	0	0	0	2	1
INFO	3	56	4	10	1	1	48	35	8	8	4	4	0	0	0	0	1	1
INFO	3	56	4	10	0	1	48	35	8	8	4	4	0	0	0	0	2	1
INFO	1	60	35	11	1	3	4	-4	3	56	4	10	0	0	0	0	1	1
INFO	4	60	35	11	2	3	12	35	7	48	35	8	0	0	0	0	1	1
INFO	1	60	35	11	3	3	28	-8	9	32	8	6	0	0	0	0	1	1
INFO	25	62	-3	12	1	1	60	35	11	2	-3	2	0	0	0	0	1	1
INFO	25	62	-3	12	0	1	60	35	11	2	-3	2	0	0	0	0	2	1
INFO	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
INFOYY	1																	
INFO NUM 129 of 143 12																		
INFO	23	12	35	7	1	1	4	-4	3	8	4	4	0	0	0	0	1	1

HELAC RESULTS

$pp \rightarrow t\bar{t}bb$			
$u\bar{u} \rightarrow t\bar{t}bb$			
	ϵ^{-2}	ϵ^{-1}	ϵ^0
HELAC-1L	-2.347908989000179E-07	-2.082520105681483E-07	3.909384299635230E-07
$I(\epsilon)$	-2.347908989000243E-07	-2.082520105665445E-07	
$gg \rightarrow t\bar{t}bb$			
HELAC-1L	-1.435108168334016E-06	-2.085070773763073E-06	3.616343483497464E-06
$I(\epsilon)$	-1.435108168334035E-06	-2.085070773651439E-06	

	p_x	p_y	p_z	E
$u(g)$	0	0	250	250
$\bar{u}(g)$	0	0	-250	250
t	12.99421901255723	-9.591511769543683	75.05543670827210	190.1845561691092
\bar{t}	53.73271578143694	-0.2854146459513714	17.68101382654795	182.9642163285034
b	-41.57664370692741	3.895531135098977	-91.94931862397770	100.9874727883170
\bar{b}	-25.15029108706678	5.981395280396083	-0.7871319108423604	25.86375471407044

HELAC RESULTS

$pp \rightarrow VVbb$ and $pp \rightarrow VV + 2 \text{ jets}$			
$u\bar{u} \rightarrow W^+W^-b\bar{b}$			
	ϵ^{-2}	ϵ^{-1}	ϵ^0
HELAC-1L	-2.493916939359002E-07	-4.885901774740355E-07	1.592538533368835E-07
$I(\epsilon)$	-2.493916939359001E-07	-4.885901774752593E-07	
$gg \rightarrow W^+W^-b\bar{b}$			
HELAC-1L	-2.686310592221201E-07	-6.078682316434646E-07	-2.431624440346638E-07
$I(\epsilon)$	-2.686310592221206E-07	-6.078682340168020E-07	

	p_x	p_y	p_z	E
$u(g)$	0	0	250	250
$\bar{u}(g)$	0	0	-250	250
W^+	22.40377113462118	-16.53704884550758	129.4056091248114	154.8819879118765
W^-	92.64238702192333	-0.4920930146078141	30.48443210132545	126.4095336206695
b	-71.68369328357026	6.716416578342183	-158.5329205583824	174.1159068988160
\bar{b}	-43.36246487297426	10.31272528177322	-1.357120667754454	44.59257156863792

HELAC RESULTS

$pp \rightarrow V + 3 \text{ jets}$			
$u\bar{d} \rightarrow W^+ ggg$			
	ϵ^{-2}	ϵ^{-1}	ϵ^0
HELAC-1L	-1.995636628164684E-05	-5.935610843551600E-05	-5.323285370666314E-05
$I(\epsilon)$	-1.995636628164686E-05	-5.935610843566534E-05	
$u\bar{u} \rightarrow Z ggg$			
HELAC-1L	-7.148261887172997E-06	-2.142170009323704E-05	-1.906378375774021E-05
$I(\epsilon)$	-7.148261887172976E-06	-2.142170009540120E-05	

	p_x	p_y	p_z	E
u	0	0	250	250
\bar{d}	0	0	-250	250
W^+	23.90724239064912	-17.64681636854432	138.0897548661186	162.5391101447744
g	98.85942812363483	-0.5251163702879512	32.53017998659339	104.0753327455388
g	-76.49423931754684	7.167141557113385	-169.1717405928078	185.8004692730082
g	-46.27243119673712	11.00479118171890	-1.448194259904179	47.58508783667868

HELAC RESULTS

$pp \rightarrow t\bar{t} + 2 \text{ jets}$			
$u\bar{u} \rightarrow t\bar{t}gg$			
	ϵ^{-2}	ϵ^{-1}	ϵ^0
HELAC-1L	-6.127108113312741E-05	-1.874963444741646E-04	-3.305349683690902E-04
$I(\epsilon)$	-6.127108113312702E-05	-1.874963445081074E-04	
$gg \rightarrow t\bar{t}gg$			
HELAC-1L	-3.838786514961561E-04	-9.761168899507888E-04	-5.225385984750410E-04
$I(\epsilon)$	-3.838786514961539E-04	-9.761168898436521E-04	

	p_x	p_y	p_z	E
$u(g)$	0	0	250	250
$\bar{u}(g)$	0	0	-250	250
t	12.99421901255723	-9.591511769543683	75.05543670827210	190.1845561691092
\bar{t}	53.73271578143694	-0.2854146459513714	17.68101382654795	182.9642163285034
g	-41.57664370692741	3.895531135098977	-91.94931862397770	100.9874727883170
g	-25.15029108706678	5.981395280396083	-0.7871319108423604	25.86375471407044

HELAC RESULTS

$pp \rightarrow bbbb$			
$u\bar{u} \rightarrow bbbb$			
	ϵ^{-2}	ϵ^{-1}	ϵ^0
HELAC-1L	-9.205269484951069E-08	-2.404679886692200E-07	-2.553568662778129E-07
$I(\epsilon)$	-9.205269484951025E-08	-2.404679886707971E-07	
$gg \rightarrow bbbb$			
HELAC-1L	-2.318436429821683E-05	-6.958360737366907E-05	-7.564212339279291E-05
$I(\epsilon)$	-2.318436429821662E-05	-6.958360737341511E-05	

	p_x	p_y	p_z	E
$u(g)$	0	0	250	250
$\bar{u}(g)$	0	0	-250	250
b	24.97040523056789	-18.43157602837212	144.2306511496888	147.5321146846735
\bar{b}	103.2557390255471	-0.5484684659584054	33.97680766420219	108.7035966213640
b	-79.89596300367462	7.485866671764871	-176.6948628845280	194.0630765341365
\bar{b}	-48.33018125244035	11.49417782256567	-1.512595929362970	49.70121215982584

OUTLOOK

OPP

OUTLOOK

OPP

- changes the computational approach at one loop

OUTLOOK

OPP

- changes the computational approach at one loop
- Numerical but still algebraic: speed and precision not a problem

OUTLOOK

OPP

- changes the computational approach at one loop
- Numerical but still algebraic: speed and precision not a problem

Current

OUTLOOK

OPP

- changes the computational approach at one loop
- Numerical but still algebraic: speed and precision not a problem

Current

- Automatize through Dyson-Schwinger equations Full one-loop amplitudes (color, rational)

OUTLOOK

OPP

- changes the computational approach at one loop
- Numerical but still algebraic: speed and precision not a problem

Current

- Automatize through Dyson-Schwinger equations Full one-loop amplitudes (color, rational)
- Several calculations

OUTLOOK

OPP

- changes the computational approach at one loop
- Numerical but still algebraic: speed and precision not a problem

Current

- Automatize through Dyson-Schwinger equations Full one-loop amplitudes (color, rational)
- Several calculations

Future

OUTLOOK

OPP

- changes the computational approach at one loop
- Numerical but still algebraic: speed and precision not a problem

Current

- Automatize through Dyson-Schwinger equations Full one-loop amplitudes (color, rational)
- Several calculations

Future

- Automatize the real contributions (dipoles)

OUTLOOK

OPP

- changes the computational approach at one loop
- Numerical but still algebraic: speed and precision not a problem

Current

- Automatize through Dyson-Schwinger equations Full one-loop amplitudes (color, rational)
- Several calculations

Future

- Automatize the real contributions (dipoles)

A generic NLO calculator *ante portas*

TOOLS 2009 ?

BlackHat

C.F. Berger, Z. Bern, L.J. Dixon, F. Febres Cordero, D. Forde, H. Ita, D.A. Kosower, D. Maitre, arXiv:0803.4180 [hep-ph]

Rocket

W. T. Giele and G. Zanderighi, arXiv:0805.2152 [hep-ph]

CutTools

G. Ossola, C. G. Papadopoulos and R. Pittau, [arXiv:0711.3596 [hep-ph]].

HELAC-1LOOP

A. van Hameren, C. G. Papadopoulos and R. Pittau [arXiv:0903.4665 [hep-ph]].