Automatizing 1-loop multi-leg calculations for LHC (and ILC) physics

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CERN, May 8, 2009

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- Prelims

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- Prelims
- History

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- Prelims
- History
- Present (and perspectives)

Motivations

- NLO (1-loop) calculations are needed for:
  - computing Backgrounds for New Physics Searches
  - Precision Measurements of fundamental quantities:
    - α_s
    - m_t
    - M_W
- Heavy New Physics states undergo long chain decays
- SM Processes accompanied by multi-jet activity

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$$\Longrightarrow$$

- at LHC multi-leg NLO MCs needed
- at ILC even more

(full set of the EW radiative corrections needed)

Ingredients

- Needed ingredients for an automatic NLO (1-loop) MC generator *(lectures of Kunszt)*:

  V Tensor Reduction

  V (Generalized) Unitarity

  V OPP

  V Rational Parts

  R Treatment of soft and collinear singularities *(lectures of Seymour)*

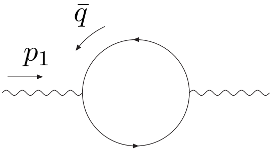
  R Automatic dipole subtraction *(talk of Hoeche)*

  R Matching with Parton Shower *(lectures of Nason)*

In this talk I will concentrate on the four Vs

Tensor Reduction 1

- Passarino-Veltman Reduction (1979) in 3 lines:



$$B^\mu \equiv \int d^n\bar{q}\frac{\bar{q}^\mu}{\bar{D}_0\bar{D}_1} = B_1 p_1^\mu\,, \qquad \bar{D}_i = (\bar{q}+p_i)^2 - m_i^2$$

$$B_1 = \frac{1}{p_1^2}\int d^n\bar{q}\frac{(\bar{q}\cdot p_1)}{\bar{D}_0\bar{D}_1}$$

$$(\bar{q}\cdot p_1) = \frac{1}{2}(\bar{D}_1 - \bar{D}_0 - p_1^2 + m_1^2 - m_0^2)$$

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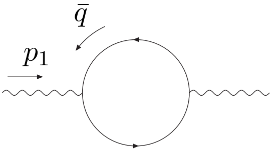

$$
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(\bar{q}\cdot p_1) &= \frac{1}{2}(\bar{D}_1 - \bar{D}_0 - p_1^2 + m_1^2 - m_0^2) \qquad \Rightarrow
\end{aligned}
$$

Tensor Reduction 2

$$\begin{aligned}\mathcal{A}^{1-loop} &= \sum_i d_i \,\mathrm{Box}_i + \sum_i c_i \,\mathrm{Triangle}_i + \sum_i b_i \,\mathrm{Bubble}_i \\ &+ \sum_i a_i \,\mathrm{Tadpole}_i + R\end{aligned}$$

where

$\mathrm{Tadpole}_i = \int d^n\bar{q}\frac{1}{\bar{D}_0}$ $\quad$ $\mathrm{Bubble}_i = \int d^n\bar{q}\frac{1}{\bar{D}_0\bar{D}_1}$

$\mathrm{Triangle}_i = \int d^n\bar{q}\frac{1}{\bar{D}_0\bar{D}_1\bar{D}_2}$ $\quad$ $\mathrm{Box}_i = \int d^n\bar{q}\frac{1}{\bar{D}_0\bar{D}_1\bar{D}_2\bar{D}_3}$

analytic work is necessary

Working at the *integrand* level 1

- The appearance of *would be vanishing* and *spurious* guests:

$$\int d^n\bar{q}\frac{\bar{q}^\mu\cdots\bar{q}^\nu}{\bar{D}_0\bar{D}_1\bar{D}_2\bar{D}_3\bar{D}_4}$$

$$\begin{aligned}\bar{q}^\mu &= \sum_{i=1}^{4}\alpha_i p_i^\mu + \tilde{q}^\mu \\ \alpha_i &= \alpha_i(q\cdot p_j) = \alpha_i(\bar{D}_0,\bar{D}_1,\bar{D}_2,\bar{D}_3,\bar{D}_4)\end{aligned}$$

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$\Rightarrow$ Relations are found among tensors (del Aguila, R. P. 2004).

$\Rightarrow$ What about $\displaystyle\int d^n\bar{q}\frac{\bar{q}^\mu\cdots\bar{q}^\nu}{\bar{D}_0\bar{D}_1\bar{D}_2\bar{D}_3}$?

Working at the *integrand* level 2

$$\bar{q}^\mu = \sum_{i=1}^{3} \alpha_i p_i^\mu + \alpha_0 \epsilon^{\mu p_1 p_2 p_3} + \tilde{q}^\mu$$

$$\alpha_0 = \frac{\epsilon(q p_1 p_2 p_3)}{\Delta(123)}$$

with

$$\int d^n \bar{q} \frac{\epsilon(q p_1 p_2 p_3)}{\bar{D}_0 \bar{D}_1 \bar{D}_2 \bar{D}_3} = 0$$

- Furthermore

$$\bar{q}^\mu \bar{q}^\nu \sim \alpha_0 + \alpha_0^2 + \cdots$$

$$\alpha_0^2 = \alpha_0^2(\bar{D}_0, \bar{D}_1, \bar{D}_2, \bar{D}_3, q^2 = \bar{D}_0 + m_0^2 - \tilde{q}^2)$$

In the meanwhile· · ·

· · · on the other side of the ocean· · ·

Cutting · · ·

- Double cuts ⇔ gluing 2 tree-level amplitudes

(Bern, Dixon, Dunbar, Kosower 1994)

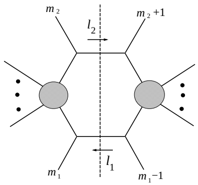


Fig. 5

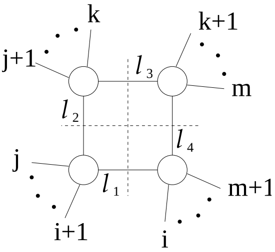
- Different double cuts are applied to disentangle 1-loop scalar functions *by looking at the analytic structure of the result*

- R is reconstructed by looking at collinear and infrared limits

$\cdots$ and more cutting

- Quadruple cuts $\Leftrightarrow$ gluing 4 tree-level amplitudes

(Britto, Cachazo, Feng 2004)



- q integration frozen $\Rightarrow$ *coefficient* d_i *of the box extracted*

The OPP Method (in a nutshell) 1

(Ossola, Papadopoulos, R. P. 2006)

$$\mathcal{A} = \sum_{I \subset \{0,1,\ldots,m-1\}} \int \frac{\mu^{4-d} d^d \bar{q}}{(2\pi)^d} \frac{\bar{N}_I(\bar{q})}{\prod_{i \in I} \bar{D}_i(\bar{q})}$$

$$\begin{aligned} \mathcal{A} &= \sum_i d_i \ \mathrm{Box}_i + \sum_i c_i \ \mathrm{Triangle}_i + \sum_i b_i \ \mathrm{Bubble}_i \\ &+ \sum_i a_i \ \mathrm{Tadpole}_i + R \end{aligned}$$

$$\bar{N}_I(\bar{q}) = N(q) + \tilde{N}(q, \tilde{q}, \epsilon)$$

The OPP Method (in a nutshell) 2

The function to be sampled numerically to extract the coefficients

$$
\begin{aligned}
N(q) &= \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3)\right] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i \\
&+ \sum_{i_0<i_1<i_2}^{m-1} \left[c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2)\right] \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i \\
&+ \sum_{i_0<i_1}^{m-1} \left[b(i_0 i_1) + \tilde{b}(q; i_0 i_1)\right] \prod_{i \neq i_0, i_1}^{m-1} D_i \\
&+ \sum_{i_0}^{m-1} \left[a(i_0) + \tilde{a}(q; i_0)\right] \prod_{i \neq i_0}^{m-1} D_i \\
&+ \tilde{P}(q) \prod_{i}^{m-1} D_i
\end{aligned}
$$

What about R $(= R_1 + R_2)$?

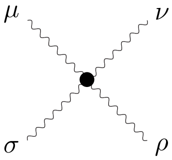
- The origin of R_1 (2006):

$$\frac{1}{\bar{D}_i} = \frac{1}{D_i}\left(1 - \frac{\tilde{q}^2}{\bar{D}_i}\right)$$

- The origin of R_2 (2008):

$$R_2 = \int \frac{\mu^{4-d} d^d \bar{q}}{(2\pi)^d} \quad \frac{\tilde{N}(q, \tilde{q}, \epsilon)}{\bar{D}_0 \cdots \bar{D}_{m-1}}$$

Computable with effective tree-level Feynman Rules (For QCD: Draggiotis, Garzelli, Papadopoulos, R. P. 2009). E.g.



$$= \frac{ie^4}{12\pi^2}\left(g_{\mu\nu}g_{\rho\sigma} + g_{\mu\rho}g_{\nu\sigma} + g_{\mu\sigma}g_{\nu\rho}\right)$$

A classic example

$$\int \frac{\mu^{4-d} d^d \bar{q}}{(2\pi)^d} \quad \frac{1}{\bar{D}_0 \bar{D}_1 \bar{D}_2 \bar{D}_3 \bar{D}_4 \bar{D}_5 \bar{D}_6} \qquad \Rightarrow \qquad N(q) = 1$$

$$d(0123) = \frac{1}{2} \left(\frac{1}{D_4(q^+) D_5(q^+) D_6(q^+)} + \frac{1}{D_4(q^-) D_5(q^-) D_6(q^-)} \right)$$

$$D_i(q^{\pm}) = 0 \qquad i = 0, 1, 2, 3$$

Sampling $N(q)$: Gluing or Opening?

- Gluing tree-level amplitudes using OPP numerical subtraction:
  - `Rockets` (Giele, Zanderighi, Ellis, Kunszt, Melnikov 2008)
  - `BlackHat` (Berger, Bern, Dixon, Febres Cordero, Forde, Ita, Kosower, Maitre 2008)
  - `C++` (Lazopoulos 2008)

- OPP $+$ Opening the loop to use tree-level like Recursion Rels:
  - `CutTools+Helac` (van Hameren, Papadopoulos, R. P. 2009)

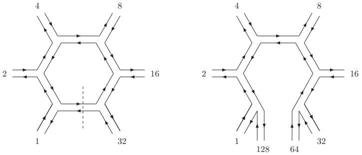
$\Rightarrow$ *the virtual parts (1 point) of all processes contained in the so called 2007 Les Houches wish list have been computed*

An example of Opening

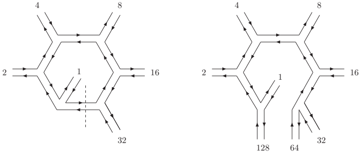
- After the one-particle Opening one has to deal with an n + 2 tree-order matrix element *the same Feynman rules apply*

- As for the color

Planar diagram:



Non planar diagram:



Conclusions

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- Bottleneck problems (Virtual Corrections and Rational Terms) have been solved
- Building an automatic multi-leg NLO (1-loop) generator is feasible
- The final form of it is still to be determined (re-weighting of tree level results, standardization of the interface with real radiation + Parton Shower)
- Please contribute with any idea to the SM and NLO Multi-leg WG in Les-Houches 2009
  `http://wwwlapp.in2p3.fr/conferences/LesHouches`

Perspectives

- Working at the *integrand level* pays off

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- What about 2 loops?

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- Working at the *integrand level* pays off
- What about 2 loops?
- Think simply on <u>new</u> unsolved problems