Shower Monte Carlo at Next-to-Leading Order

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Outline

Introduction:

- How Showers work
- How NLO works
- The problems of NLO+Shower

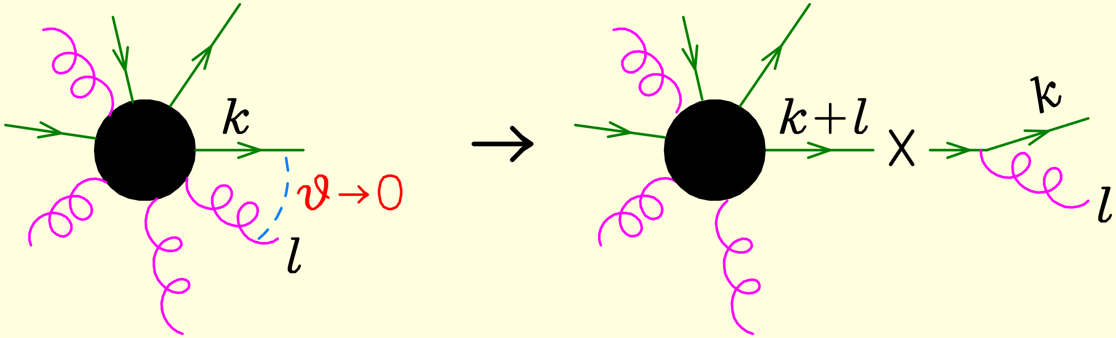
Shower improvements

- NLO and showers: `MC@NLO` and `POWHEG`
- Results:
  - NLO+S vs. NLO vs. LO SMC's ; NLO+S vs. ME+S
  - `POWHEG` vs. `PYTHIA` ME in $2 \to 1$ processes
  - `POWHEG` vs. `MC@NLO`
- `POWHEG` for complex processes: flavour and singularities separation
- NLO+S with angular ordered showers, truncated showers
- Full automation: the `POWHEG BOX`
- Conclusions

Shower basics: Collinear factorization

QCD emissions are enhanced near the collinear limit

Cross sections factorize near collinear limit



$$|M_{n+1}|^2 d\Phi_{n+1} \Longrightarrow |M_n|^2 d\Phi_n \, \frac{\alpha_s}{2\pi} \, \frac{dt}{t} \, P_{q,qg}(z) \, dz \, \frac{d\phi}{2\pi}$$

t : hardness (either virtuality or p_T^2 or $E^2\theta^2$ etc.)

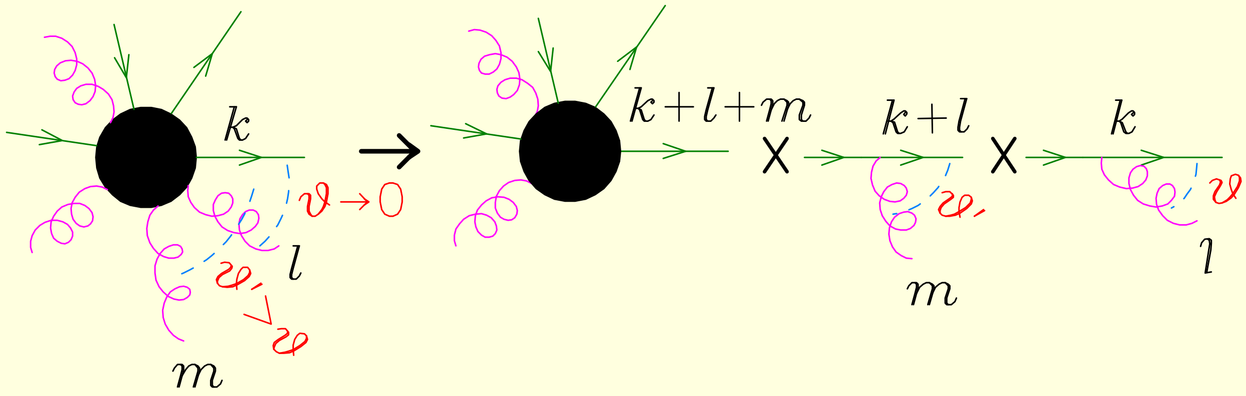
$z = k^0/(k^0 + l^0)$: energy (or $p_{\|}$, or p^+) fraction of quark

$P_{q,qg}(z) = C_F \frac{1+z^2}{1-z}$: Altarelli – Parisi splitting function

(ignore $z \to 1$ IR divergence for now)

If another gluon becomes collinear, iterate the previous formula:

$\theta', \theta \to 0$
with $\theta' > \theta$



$$|M_{n+1}|^2 d\Phi_{n+1} \Longrightarrow |M_{n-1}|^2 d\Phi_{n-1} \times \frac{\alpha_s}{2\pi}\frac{dt'}{t'} P_{q,qg}(z')\,dz' \frac{d\phi'}{2\pi} \times \frac{\alpha_s}{2\pi}\frac{dt}{t} P_{q,qg}(z)\,dz \frac{d\phi}{2\pi}\theta(t'-t)$$

Collinear partons can be described by a factorized integral ordered in t.
For m collinear emissions:

$$\left(\frac{\alpha_s}{2\pi}\right)^m \int_{\theta_{\min}} \frac{d\theta_1}{\theta_1} \int_{\theta_1} \frac{d\theta_2}{\theta_2} \ldots \int_{\theta_{m-1}} \frac{d\theta_m}{\theta_m} \propto \frac{\log^m \frac{1}{\theta^2_{\min}}}{m!} \approx \left(\frac{\alpha_s}{2\pi}\right)^m \frac{\log^m \frac{Q^2}{\Lambda^2}}{m!}$$

where we have taken $\theta_{\min} \approx \Lambda/Q$; (Leading Logs) This is of order 1!

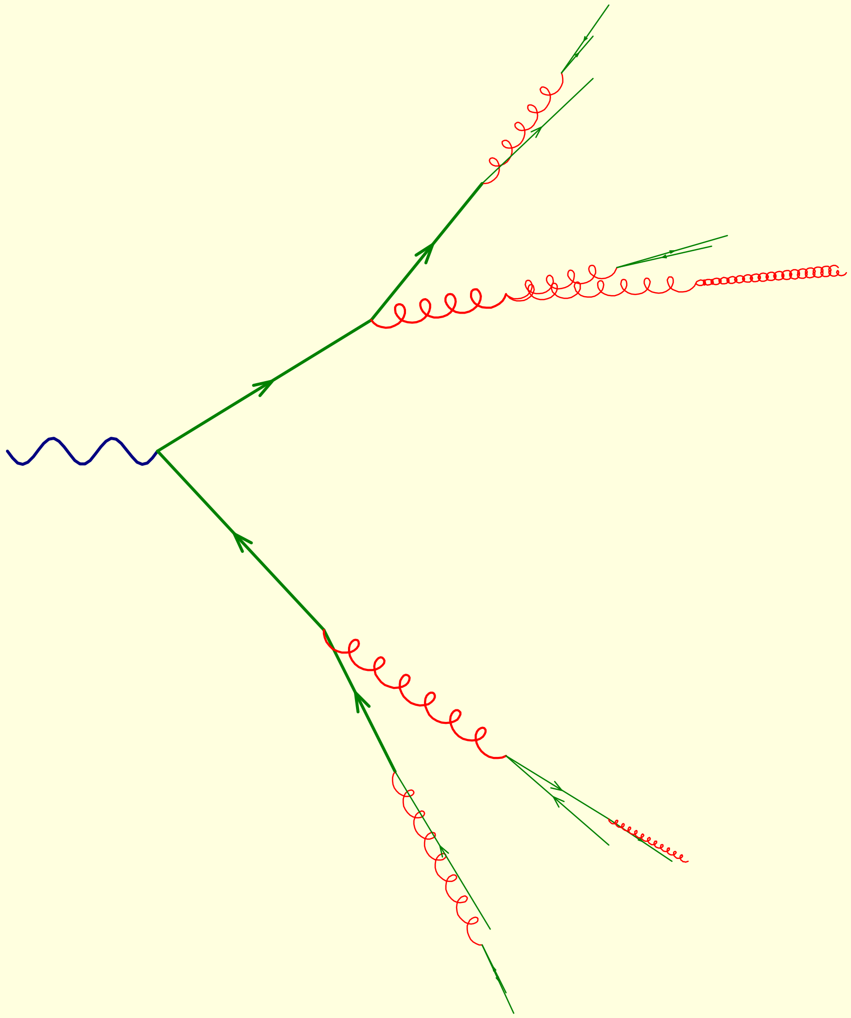
Typical dominant configuration at very high Q^2

Besides $q \to qg$, also $g \to gg$, $g \to q\bar{q}$ come into play.

Typical configurations: intermediate angles of order of geometric average of upstream and downstream angles.

Each angle is $\mathcal{O}(\alpha_s)$ smaller than its upstream angle, and $\mathcal{O}(\alpha_s)$ bigger than its downstream angle.

As relative momenta become smaller α_s becomes bigger, and this picture breaks down.



For a consistent description:
include virtual corrections to same LL approximation

One can show that the effect of virtual corrections is given by

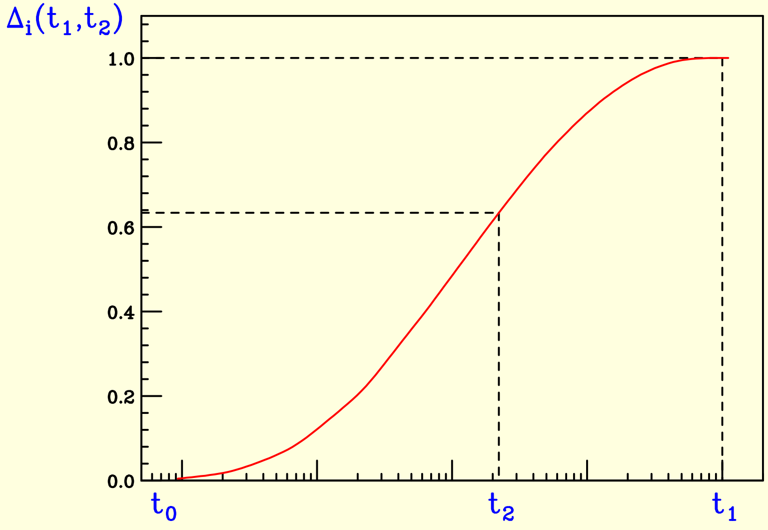
- Let $\alpha(\mu) \Longrightarrow \alpha(t)$ in each vertex, where t is the hardness of the vertex (i.e. hardness of the incoming line)
- For each intermediate line include the factor

$$\Delta_i(t_h, t_l) = \exp\left[-\sum_{(jk)} \int_{t_l}^{t_h} \frac{dt'}{t'} \int dz \, \frac{\alpha_s(t')}{2\pi} P_{i,jk}(z) \right]$$

where t_h is the hardness of the vertex originating the line, and t_l is the hardness of the vertex where the line ends.

Sudakov form factor

$$\Delta_i(t_h, t_l) = \exp\left[-\sum_{(jk)} \int_{t_l}^{t_h} \frac{dt'}{t'} \int dz \, \frac{\alpha_s(t')}{2\pi} \, P_{i,jk}(z) \right]$$



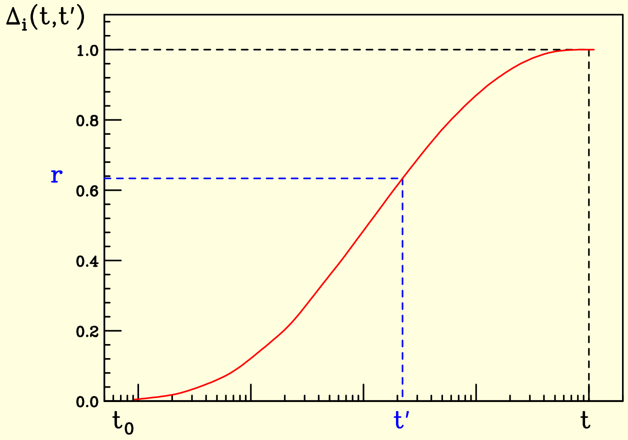
As t_l becomes small the exponent tend to diverge, and $\Delta_i(t_h, t_l)$ approaches 0.
In fact, because of $\alpha_s(t)$, we must stop at $t_0 \gtrsim \Lambda_{\rm QCD}$.

Final Recipe

- Consider all tree graphs.
- Assign ordered hardness parameters t to each vertex.
- Include a factor
$$\frac{\alpha_s(t)}{2\pi} P_{i,jk}(z) \frac{dt}{t} dz \frac{d\phi}{2\pi}$$
at each vertex $i \to jk$.
- Include a factor $\Delta_i(t_1, t_2)$ to each internal line with a parton i, from hardness t_1 to hardness t_2.
- Include a factor $\Delta_i(t, t_0)$ on final lines (t_0: IR cutoff)

Most important: the shower recipe can be easily implemented as a computer code!
Shower Algorithm:

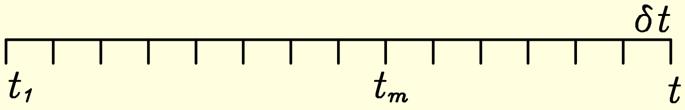
- Generate a uniform random number $0 < r < 1$;
- Solve the equation $\Delta_i(t,t') = r$ for t';
- If $t' < t_0$ stop here (final state line);
- generate z, jk with probability $P_{i,jk}(z)$, and $0 < \phi < 2\pi$ uniformly;
- restart from each branch, with hardness parameter t'.



Probabilistic intepretation: branching probability of line of flavor i

$$dP(t_1,t) = \underbrace{\exp\left[-\sum_{(jk)}\int_t^{t_1}\frac{dt'}{t'}\int dz\,\frac{\alpha_s(t')}{2\pi}P_{i,jk}(z)\right]}_{\Delta(t_1,t)}\frac{\alpha_s(t)}{2\pi}P_{i,jk}(z)\,\frac{dt}{t}\,dz\,\frac{d\phi}{2\pi}$$

break up t_1, t into small subintervals:



$$dP(t_1,t) = \left[\prod_m\left(\underbrace{1-\sum_{(jk)}\frac{\delta t}{t_m}\int dz\,\frac{\alpha_s(t_m)}{2\pi}P_{i,jk}(z)}_{\text{No emission prob. in } t_m,\, t_m+\delta t}\right)\right]\underbrace{\frac{\alpha_s(t)}{2\pi}P_{i,jk}(z)\,\frac{\delta t}{t}\,dz\,\frac{d\phi}{2\pi}}_{\text{emission prob. in } t,\, t+\delta t}$$

So: the probability for the first branching at hardness t is the product of the non-emission probability $\Delta(t_1,\ t)$ in all hardness intervals between t_1 and t, times the emission probability at hardness t.

(more or less) obvious consequences:

- The total branching probability plus the no-branching probability is 1; mathematically

$$\int_{t_0}^{t_1} dP(t_1, t') = \int_{t_0}^{t_1} d\Delta_i(t_1, t') = 1 - \Delta_i(t_1, t_0)$$

- The Sudakov form factor $\Delta_i(t_1, t)$ is the no-branching probability from scale t_1 down to the scale t.

- The branching probability is independent of what happens next (because the total probability of what happens next is 1).

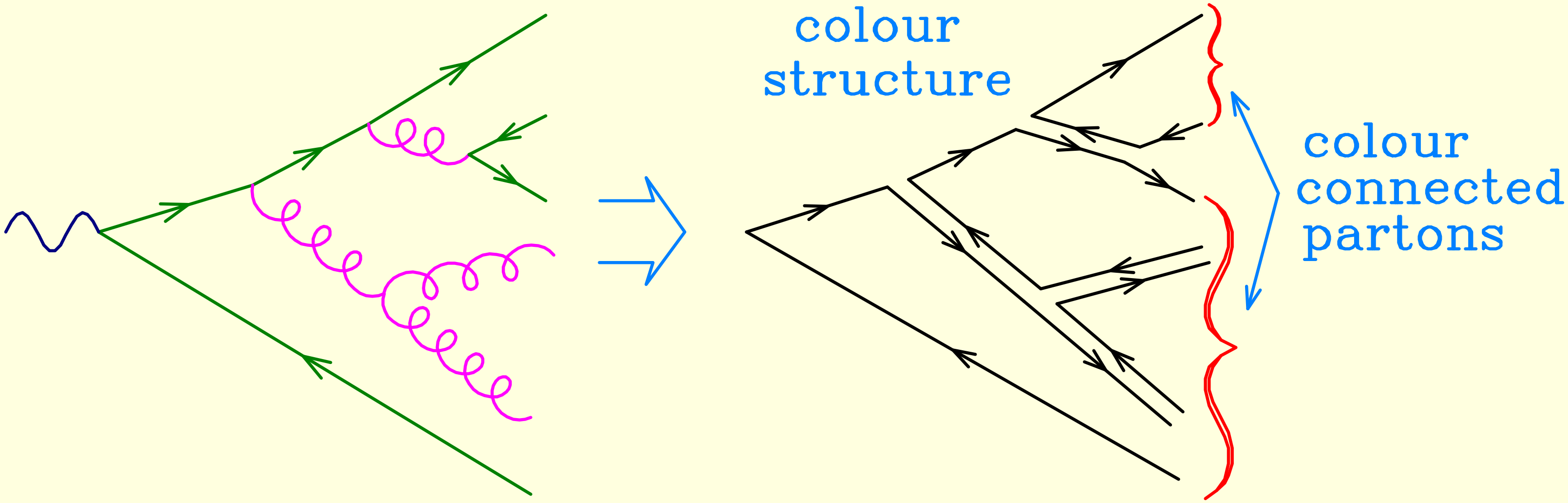
This property is often called unitarity of the shower. It is a consequence of the Kinoshita-Lee-Nauenberg theorem: collinear divergence must cancel in the inclusive cross section.

COLOUR AND HADRONIZATION

SMC's assign colour labels to partons.

Only colour connections are recorded (as in large N limit).

Initial colour assigned according to hard cross section.



Colour assignements are used in the hadronization model.

Most popular models: Lund String Model, Cluster Model.

In all models, color singlect structures are formed out of colour connected partons, and are decayed into hadrons preserving energy and momentum.

NLO Calculations

Scale uncertainty

$$\alpha_s^n(2\mu) \approx \alpha_s^n(\mu)(1 - b_0\alpha_s(\mu)\log(4))^n \approx \alpha_s(\mu)(1 - n\alpha_s(\mu))$$

For $\mu = 100\,\mathrm{GeV}$, $\alpha_s = 0.12$;
uncertainty:

$W+1J$	$W+2J$	$W+3J$
$\pm 12\%$	$\pm 24\%$	$\pm 36\%$

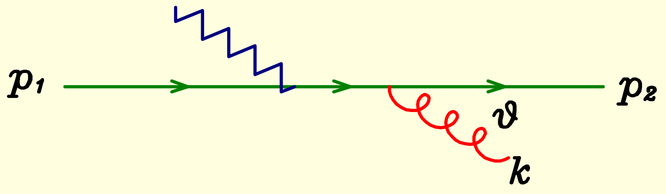
To improve on this, need to go to NLO

Positive experience with NLO calculations at LEP, HERA, Tevatron (we TRUST perturbative QCD after LEP!).
Huge NLO effort towards LHC physics

But: NLO results are cumbersome and unfriendly: typically made up of an n-body (Born+Virtual+Soft and Collinear remnants) and $n+1$ body (real emission) terms, both divergent (finite only when summed up).

Simple example: Z production

"Real" contribution to $q\,\bar{q} \to Z + X$:



$$\frac{C_F}{N_c}\frac{g_Z^2\,g_s^2}{32\pi^2}\frac{1}{S}\left[2(1+y^2)\xi^2+8(1-\xi)\right]\left\{\frac{1}{2}\left(\frac{1}{\xi}\right)_+\left[\left(\frac{1}{1-y}\right)_+ + \left(\frac{1}{1-y}\right)_-\right]\right\}d\xi\,dy\,dY_Z$$

where

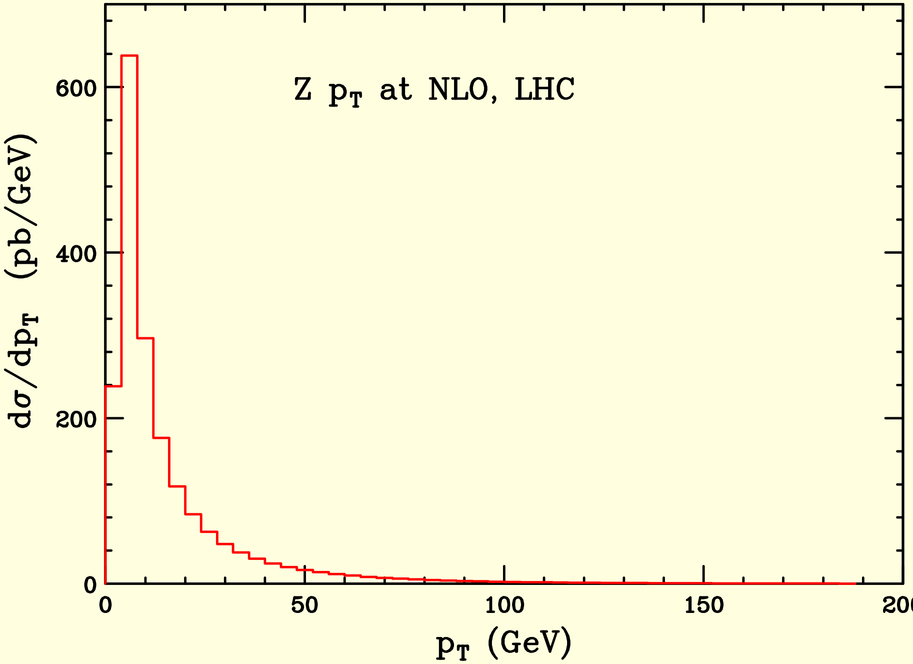
- Y_Z is the Z rapidity
- $y = \cos\theta$, θ being the emission angle of the gluon in the partonic CM
- $\xi = 2k^0/\sqrt{s}$ in the partonic CM ($s = (p_1+p_2)^2$)

$$\left(\frac{1}{\xi}\right)_+ = \lim_{\epsilon\to 0}\left[\frac{1}{\xi+\epsilon} - \log\frac{1}{\epsilon}\,\delta(\xi)\right];\ \int_0^1\left(\frac{1}{\xi}\right)_+ = 0;\ \int_{-1}^1\left(\frac{1}{1\pm y}\right)_+ = 0,$$

Divergent contributions to the cross section for $p_T^Z > 0$ (i.e. $\xi > 0$, $1 \pm y > 0$), compensated by negative divergences (i.e. $\delta(\xi)$, $\delta(1 \pm y)$ terms) at $p_T^Z = 0$, that arise from the virtual corrections.

p_T^Z at NLO:

Negative contributions at $p_T^Z = 0$ compensate the diverging real contributions. For small enough histogram bins the first bin will always turn negative!



A negative bin means: $\mathcal{O}(\alpha_s)$ corrections larger than Born term:
cannot trust perturbation theory!

One should carefully decide the appropriate bin size around the origin.

For more complex processes this becomes a requirement on jet parameters.

So: merging Showers and NLO is not only a problem of overcounting (as with LO matrix elements and showers). Some sort of resummation of the diverging virtual corrections should be carried out, in order to get sensible results in the dangerous regions of collinear and soft emissions.

The key to the solution: the dangerous region is well described by the factorization formula. For example, for $y \to 1$ our cross section becomes

$$\frac{C_F}{N_c}\frac{g_Z^2 g_s^2}{16\pi^2}\frac{1}{S}\left[\frac{x^2+1}{(1-x)_+}\right]\frac{dy}{1-y}\; d\xi\, dy\, dY_W\,, \quad \text{with}\, x = 1-\xi$$

The problem of diverging negative virtual corrections is dealt with and solved in the Shower formalism.

In the following: assume that the hardest SMC radiation is the first one, i.e. that the Shower is ordered in relative p_T. We deal later with the subtle issue of the choice of the ordering variable.

Look back at the cross section for the first emission in a Shower Monte Carlo

$$d\sigma = d\Phi_B\, B(\Phi_B) \left(\underbrace{\Delta_{t_I,t_0}}_{\text{No radiation}} + \sum_{(jk)} \underbrace{\Delta_{t_I,t}\, \frac{\alpha_s(t)}{2\pi} P_{i,jk}(z)\, \frac{dt}{t}\, dz\, \frac{d\phi}{2\pi}}_{\text{radiation}} \right)$$

- t_I is the maximum hardness allowed initially, t_0 is the minimum hardness of emission
- $\Delta_{t_I,t}$ is the no-radiation probability with hardness $> t$

$$\Delta_i(t_I, t) = \exp\left[-\sum_{(jk)} \int_t^{t_I} \frac{dt'}{t'} \int dz\, \frac{\alpha_s(t')}{2\pi} P_{i,jk}(z) \right]$$

Expand the Shower formula at order $\mathcal{O}(\alpha_s)$:

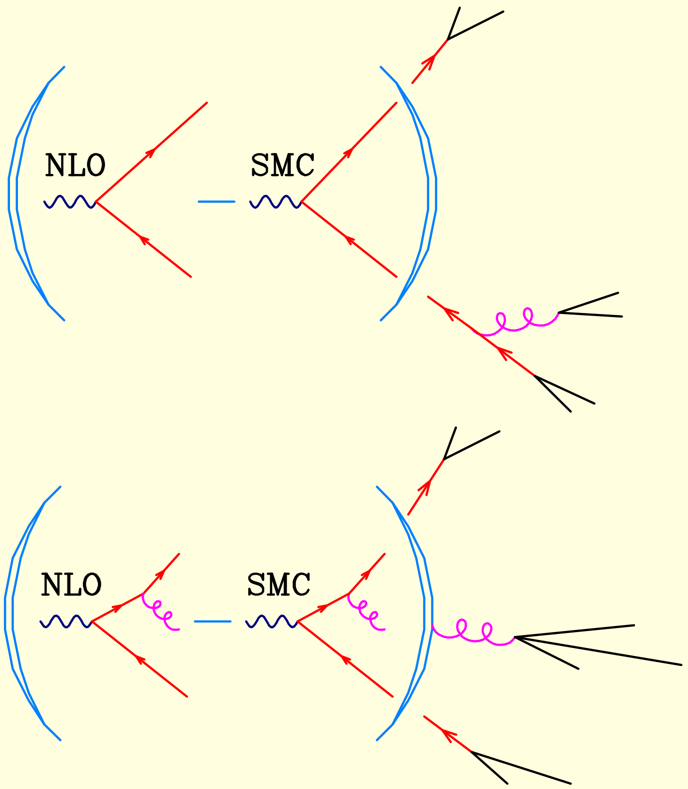
$$d\sigma = d\Phi_B\, B(\Phi_B) \left(1 \underbrace{- \sum_{(jk)} \int_{t_0}^{t_I} \frac{dt'}{t'} \int dz\, \frac{\alpha_s}{2\pi} P_{i,jk}(z)}_{\text{virtual}} + \sum_{(jk)} \underbrace{\frac{\alpha_s}{2\pi} P_{i,jk}(z)\, \frac{dt}{t}\, dz\, \frac{d\phi}{2\pi}}_{\text{real}} \right)$$

As in the NLO calculation, we have a negative divergent contribution for no radiation, and a positive divergent contribution for radiation. The divergence cancels for inclusive cross sections.

So: the SMC has his own approximate NLO virtual and real terms. To get NLO accuracy these terms should be modified to yield the exact NLO.

Notice that SMC algorithms reconstruct from Born kinematics Φ_B and radiation variables t, z, ϕ, the full phase space Φ (momentum reshuffling)

MC@NLO (2002, Frixione+Webber)



Add difference between exact NLO and approximate (MC) NLO.

- Must use MC kinematics
- Difference should be regular (if the MC is OK)
- Difference may be negative

Several collider processes already there: Vector Bosons, Vector Bosons pairs, Higgs, Single Top (also with W), Heavy Quarks, Higgs+W/Z.

How it works (roughly)

The cross section for the hardest event in `MC@NLO` is

$$d\sigma = \underbrace{\bar{B}^{\rm MC}(\Phi_B) d\Phi_B}_{S\ \text{event}} \left[\underbrace{\Delta^{\rm MC}_{t_0} + \Delta^{\rm MC}_{t} \frac{R^{\rm MC}(\Phi)}{B(\Phi_B)} d\Phi^{\rm MC}_r}_{\text{MC shower}} \right] + \left[\underbrace{R(\Phi) - R^{\rm MC}(\Phi)}_{H\ \text{event}} \right] d\Phi$$

$$\bar{B}^{\rm MC}(\Phi_B) = B(\Phi_B) + \underbrace{\left[\underbrace{V(\Phi_B)}_{\text{infinite}} + \underbrace{\int R^{\rm MC}(\Phi)\, d\Phi^{\rm MC}_r}_{\text{infinite}} \right]}_{\text{finite}}$$

Imagine that soft and collinear singularities in $R^{\rm MC}$ are regulated as in V.

The full phase space Φ in parametrized in terms of the Born phase space Φ_B and the radiation variables of the MC $\Phi^{\rm MC}_r$ (typically z, t, ϕ), according to the MC procedure (reshuffling) that yields Φ from Φ_B and $\Phi^{\rm MC}_r$.

B : Born cross section; V : exact virtual cross section.

$R^{\rm MC}$: radiation cross section in the MC, typically: $R^{\rm MC} = B\, \frac{1}{t}\, \frac{\alpha}{2\pi} P(z)$

R : exact radiation cross section;

We can check that the $\mathcal{O}(\alpha_s)$ expansion of $d\sigma$ coincides with the exact NLO;

$$d\sigma = \bar{B}^{\rm MC}(\Phi_B)d\Phi_B\left[\Delta^{\rm MC}_{t_0} + \Delta^{\rm MC}_t \frac{R^{\rm MC}(\Phi)}{B(\Phi_B)}d\Phi^{\rm MC}_r\right] + [R(\Phi) - R^{\rm MC}(\Phi)]d\Phi$$

$$\bar{B}^{\rm MC}(\Phi_B) = B(\Phi_B) + \left[V(\Phi_B) + \int R^{\rm MC}(\Phi)\, d\Phi^{\rm MC}_r\right]$$

Expand:

$$d\sigma = \left[B + V + \int R^{\rm MC} d\Phi^{\rm MC}_r\right]d\Phi_B\left[1 - \int \frac{R^{\rm MC}}{B}\, d\Phi^{\rm MC}_r + \frac{R^{\rm MC}}{B}d\Phi^{\rm MC}_r\right] + [R - R^{\rm MC}]d\Phi$$

$$= [B + V]d\Phi_B + B\,d\Phi_B\left[\int \frac{R^{\rm MC}}{B}d\Phi^{\rm MC}_r - \int \frac{R^{\rm MC}}{B}\, d\Phi^{\rm MC}_r + \frac{R^{\rm MC}}{B}d\Phi^{\rm MC}_r\right] + [R - R^{\rm MC}]\, d\Phi$$

$$= [B + V]d\Phi_B + B\,d\Phi_B\left[\frac{R^{\rm MC}}{B}d\Phi^{\rm MC}_r\right] + [R - R^{\rm MC}]\, d\Phi = [B + V]d\Phi_B + R\,d\Phi$$

Recipe for MC@NLO

- Compute totals for S and H events:

$$\sigma_S = \int |\bar{B}^{\rm MC}(\Phi_B)| \, d\Phi_B, \;\; \sigma_H = \int |R - R^{\rm MC}| \, d\Phi$$

- Chose an S or H event with probability proportional to σ_S, σ_H
- For an S event:
  - generate Born kinematics with probability

$$|\bar{B}^{\rm MC}(\Phi_B)| = \left| B(\Phi_B) + \left[V(\Phi_B) + \int R^{\rm MC}(\Phi) \, d\Phi_r^{\rm MC} \right] \right|$$

  - Feed the Born kinematics to the MC for subsequent shower with weight ± 1, same sign as $\bar{B}^{\rm MC}(\Phi_B)$.
- For an H event:
  - generate Radiation kinematics with probability $|R - R^{\rm MC}|$.
  - Feed to the MC (with weight ± 1, same sign as $R - R^{\rm MC}$)

Issues:

- Must use of the MC kinematic mapping $(\Phi_B, \Phi_r^{\mathrm{MC}}) \Rightarrow \Phi$.
- $R - R^{\mathrm{MC}}$ must be non singular: the MC must reproduce exactly the soft and collinear singularities of the radiation matrix element. (Many MC are not accurate in the soft limit)
- The cancellation of divergences in the expression of $\bar{B}^{\mathrm{MC}}$ is taken care of in the framework of the subtraction method (cancellation of divergences under the integral sign) so that the integral in $\bar{B}^{\mathrm{MC}}$ becomes in fact convergent.
- Negative weights in the output (not like standard MC's).

POWHEG

Positive Weight Hardest Emission Generator

Method to generate the hardest emission first, with NLO accuracy, and independently of the SMC (P.N. 2004).

- SMC independent; no need of SMC expert; same calculation can be interfaced to several SMC programs with no extra effort
- SMC inaccuracies in the soft region only affect next-to-hardest emissions; no matching problems
- As the name says, it generates events with positive weight

How it works (roughly)

In words: works like a standard Shower MC for the hardest radiation, with care to maintain higher accuracy.

In a standard MC, the hardest radiation cross section is

$$d\sigma = d\Phi_B\, B(\Phi_B) \left(\underbrace{\Delta_{t_I,t_0}}_{\text{No radiation}} + \underbrace{\Delta_{t_I,t} \frac{\alpha_s(t)}{2\pi} P_{i,jk}(z) \frac{dt}{t} dz \frac{d\phi}{2\pi}}_{\text{radiation}} \right)$$

- t_I is the maximum hardness allowed initially
- $\Delta_{t_I,t}$ in the no-radiation probability with hardness $> t$

SMC algorithm reconstructs from Born kinematics Φ_B and radiation variables t, z, ϕ, the full radiation phase space Φ (momentum reshuffling)

We say that Φ_B is the underlying Born configuration of Φ according to the mapping defined by the MC algorithm

Steps to go NLO:

$$(\Phi_B, t, z, \phi) \Leftrightarrow \Phi \quad \Longrightarrow \quad (\Phi_B, \Phi_r) \Leftrightarrow \Phi,\ d\Phi = d\Phi_B\, d\Phi_r$$

$$B(\Phi_B) \quad \Longrightarrow \quad \bar{B}(\Phi_B) = B(\Phi_B) + \underbrace{\left[\overbrace{V(\Phi_B)}^{\text{INFINITE}} + \overbrace{\int R(\Phi_B, \Phi_r)\, d\Phi_r}^{\text{INFINITE}} \right]}_{\text{FINITE!}}$$

$$\frac{\alpha_s(t)}{2\pi} P_{i,jk}(z)\, \frac{dt}{t}\, dz\, \frac{d\phi}{2\pi} \quad \Longrightarrow \quad \frac{R(\Phi_B, \Phi_r)}{B(\Phi_B)}\, d\Phi_{\text{rad}}$$

POWHEG cross section:

$$d\sigma = d\Phi_B \bar{B}(\Phi_B) \left[\Delta_{t_0} + \Delta_t \frac{R(\Phi)}{B(\Phi_B)} d\Phi_r \right], \qquad \Delta_t = \exp\left[\underbrace{-\int \theta(t_r - t) \frac{R(\Phi_B, \Phi_r)}{B(\Phi_B)} d\Phi_r}_{\text{FINITE because of } \theta \text{ function}} \right]$$

with $t_r = k_T(\Phi_B, \Phi_r)$, the transverse momentum for the radiation.
In the collinear limit, k_t^2 must be of the order of t.

How does it work: $d\sigma = d\Phi_{B}\,\bar{B}(\Phi_{B})\left[\Delta_{t_0} + \Delta_t \frac{R(\Phi)}{B(\Phi_{B})} d\Phi_r\right],$

For small k_T, the factorization theorem yields

$$\frac{R(\Phi)}{B(\Phi_{B})}\, d\Phi_{\rm rad} \approx \frac{\alpha_s(t)}{2\pi} P_{i,jk}(z)\, \frac{dt}{t}\, dz\, \frac{d\phi}{2\pi}$$

and

$$\bar{B} \approx B \times (1 + \mathcal{O}(\alpha_s))$$

Thus: all features of SMC's are preserved at small k_T.
For large k_T, $\Delta \to 1$,

$$d\sigma = \bar{B} \times \frac{R}{B} \approx R \times (1 + \mathcal{O}(\alpha_s)),$$

so large k_t accuracy is preserved. Integrating in $d\Phi_r$ at fixed Φ_{B}

$$\int \delta(\Phi_{B} - \bar{\Phi}_{B})\, d\sigma = \bar{B}(\bar{\Phi}_{B})$$

So NLO accuracy is preserved for inclusive quantities.

Example of mapping $\Phi \Leftrightarrow (\Phi_B, \Phi_r)$: Z pair production

Φ_B variables: choose M_{zz}, Y_{zz} and θ, where

- M_{zz}: invariant mass of the ZZ pair
- Y_{zz}: rapidity of ZZ pair
- θ: go in the (longitudinally) boosted frame where $Y_{zz} = 0$.
  go to the ZZ rest frame with a transverse boost
  In this frame θ is the angle of a Z to the longitudinal direction.

Φ_r variables:

- $x = M_{zz}/s$, (s is the invariant mass of the incoming parton system)
  $x \to 1$ is the soft limit
- y: cosine of the angle of the radiated parton to the beam direction in the partonic CM frame.
- ϕ: radiation azimuth.

Few tricks

Both in `MC@NLO` and `POWHEG`, integrals of the form

$$\bar{B}(\Phi_B) = B(\Phi_B) + \underbrace{\left[\overbrace{V(\Phi_B)}^{\text{INFINITE}} + \overbrace{\int R(\Phi_B, \Phi_r)\, d\Phi_r}^{\text{INFINITE}} \right]}_{\text{FINITE!}}$$

are expressed within the subtraction method as

$$\bar{B}(\Phi_B) = B(\Phi_B) + V_{\rm SV}(\Phi_B) + \int d\Phi_r [R(\Phi_B, \Phi_r) - C(\Phi_B, \Phi_r)]$$

Needs one Φ_r integrations for each Φ point!. To overcome this, we write

$$\tilde{B}(\Phi_B, \Phi_r) = \frac{B(\Phi_B) + V(\Phi_B)}{\int d\Phi_r} + R(\Phi_B, \Phi_r) - C(\Phi_B, \Phi_r)\,, \bar{B}(\Phi_B) = \int \tilde{B}(\Phi_B, \Phi_r) d\Phi_r \,.$$

so that

$$\bar{B}(\Phi_B \Phi) = \int \tilde{B}(\Phi_B \Phi, \Phi_r) d\Phi_r \,.$$

Use standard procedures (SPRING-BASES, Kawabata; MINT, P.N.)
to generate unweighted events for $\tilde{B}(\bar{\Phi}, \Phi_r) d\Phi_r d\bar{\Phi}$,
discard Φ_r (same as integrating over it!).

Radiation in POWHEG: $\Delta(\Phi_{B}, p_T) = \exp\left[-\int \frac{R(\Phi_{B}, \Phi_r)}{B(\Phi_{B})}\theta(k_T(\Phi_{B}, \Phi_r) - p_T)d\Phi_r\right],$

Look for an upper bounding function;

$$\frac{R(\Phi_{B}, \Phi_r)}{B(\Phi_{B})} \leq U(\Phi) = N\frac{\alpha(k_T)}{(1-x)(1-y^2)}$$

Generate x, y according to

$$\exp\left[-\int U(\Phi_{B})\theta(k_T(\Phi_{B}, \Phi_r) - p_T)d\Phi_r\right]$$

accept the event with a probability

$$\frac{R(\Phi_{B}, \Phi_r)}{B(\Phi_{B})U(\Phi_{B})}\,.$$

If the event is rejected generate a new one for smaller p_T, and so on (Veto method)

POWHEG: Interfacing to SMC's

For a p_T ordered SMC, nothing else needs to be done.
Use the standard Les Houches Interface for User's Processes (LHI):
put partonic event generated by POWHEG on the LHI;
Run the SMC in the LHI mode.
The LHI provides a facility to pass the p_T of the event to the SMC (SCALUP).
As far as the hardest emission is concerned, POWHEG can reach:

- NLO accuracy of (integrated) shape variables
- Collinear, double-log, soft (large N_c) accuracy of the Sudakov FF.
  (In fact, corrections that exponentiates are obviously OK)

As far as subsequent (less hard) emissions, the output has the accuracy of the SMC one is using.

Status of POWHEG

Up to now, the following processes have been implemented in POWHEG:

- $hh \to ZZ$ (Ridolfi, P.N., 2006)
- $e^+e^- \to \mathrm{hadrons}$, (Latunde-Dada,Gieseke,Webber, 2006),
  $e^+e^- \to t\bar{t}$, including top decays at NLO (Latunde-Dada,2008),
- $hh \to Q\bar{Q}$ (Frixione, Ridolfi, P.N., 2007)
- $hh \to Z/W$ (Alioli, Oleari, Re, P.N., 2008;)
  (Hamilton,Richardson,Tully, 2008;)
- $hh \to H$ (gluon fusion) (Alioli, Oleari, Re, P.N., 2008; Herwig++)
- $hh \to H$, $hh \to HZ/W$ (Hamilton,Richardson,Tully, 2009;)
- $hh \to t + X$ (single top) NEW (Alioli, Oleari, Re, P.N., 2009)
- $hh \to Z + \mathrm{jet}$, Very preliminary (Alioli, Oleari, Re, P.N., 2009)
- The POWHEG BOX, Very preliminary, (Alioli, Oleari, Re, P.N., 2009)

In practice

MC@NLO: Code and manuals at

http://www.hep.phy.cam.ac.uk/theory/webber/MCatNLO/

1 program for all processes

POWHEG: Codes and manuals in

http://moby.mib.infn.it/~nason/POWHEG

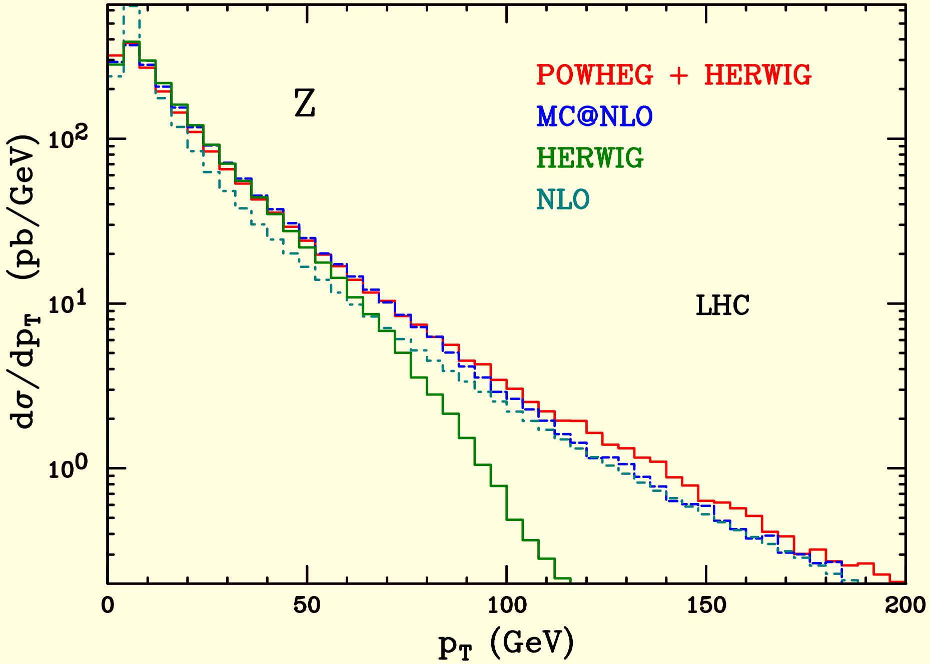
Examples are provide to link POWHEG to HERWIG or PYTHIA, or to generate a Les Houches Event File to be fed later to a SMC for showering.

1 program for each process

In the HERWIG++ code there are few independent implementations of MC@NLO and POWHEG processes

Examples: Z production

HERWIG alone fails ar large p_T;
NLO alone fails at small p_T;
MC@NLO and POWHEG work
in both regions;
Notice:
HERWIG with ME corrections
or any ME program, give the
same NLO shape at large p_T
However: Normalization around
small p_T region is incorrect
(i.e. only LO).



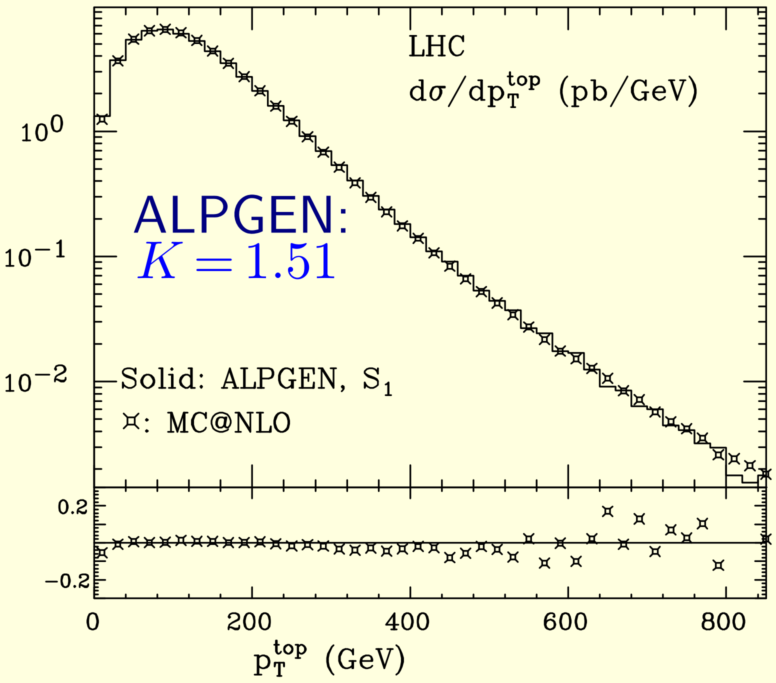
The essence of the improvement with respect to standard shower and ME matched programs is summarized in this plot.
Be careful with the misleading language: Z at LO $\mathcal{O}(1)$, NLO $\mathcal{O}(\alpha_s)$;
At $\mathcal{O}(1)$ there is no Z transverse momentum. Thus, the p_T distribution $p_T > 0$ is of $\mathcal{O}(\alpha_s)$, i.e. has leading order accuracy!

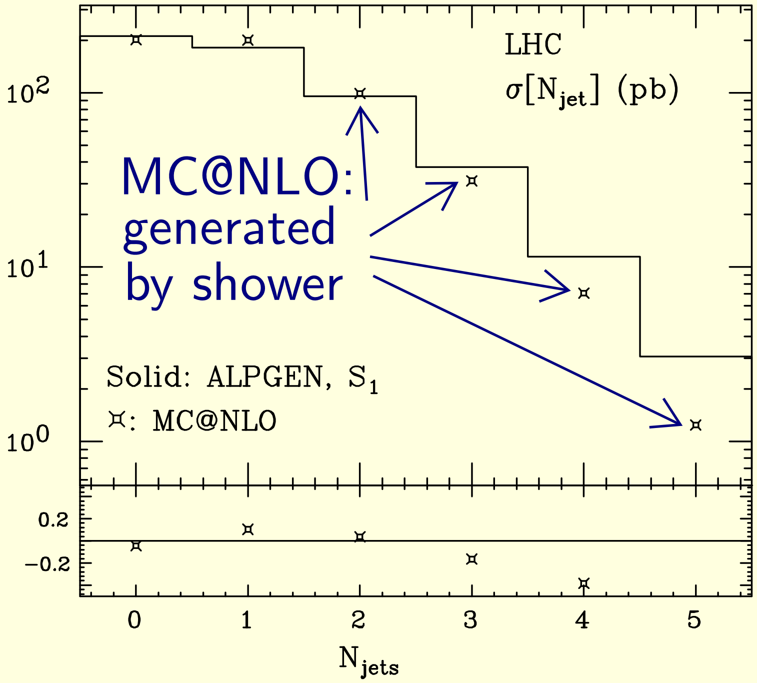
NLO+PS compared with ME programs: ALPGEN and MC@NLO in $t\bar{t}$ production

expect:

- Disadvantage: worse normalization (no NLO)
- Advantage: better high jet multiplicities (exact ME)

(Mangano, Moretti,Piccinini,Treccani, Nov.06)



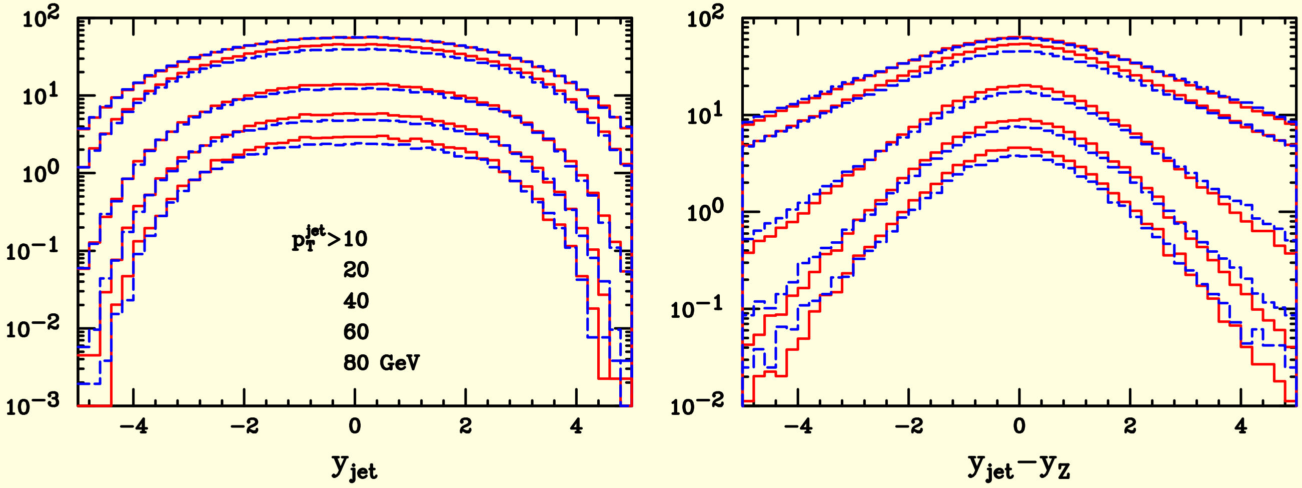


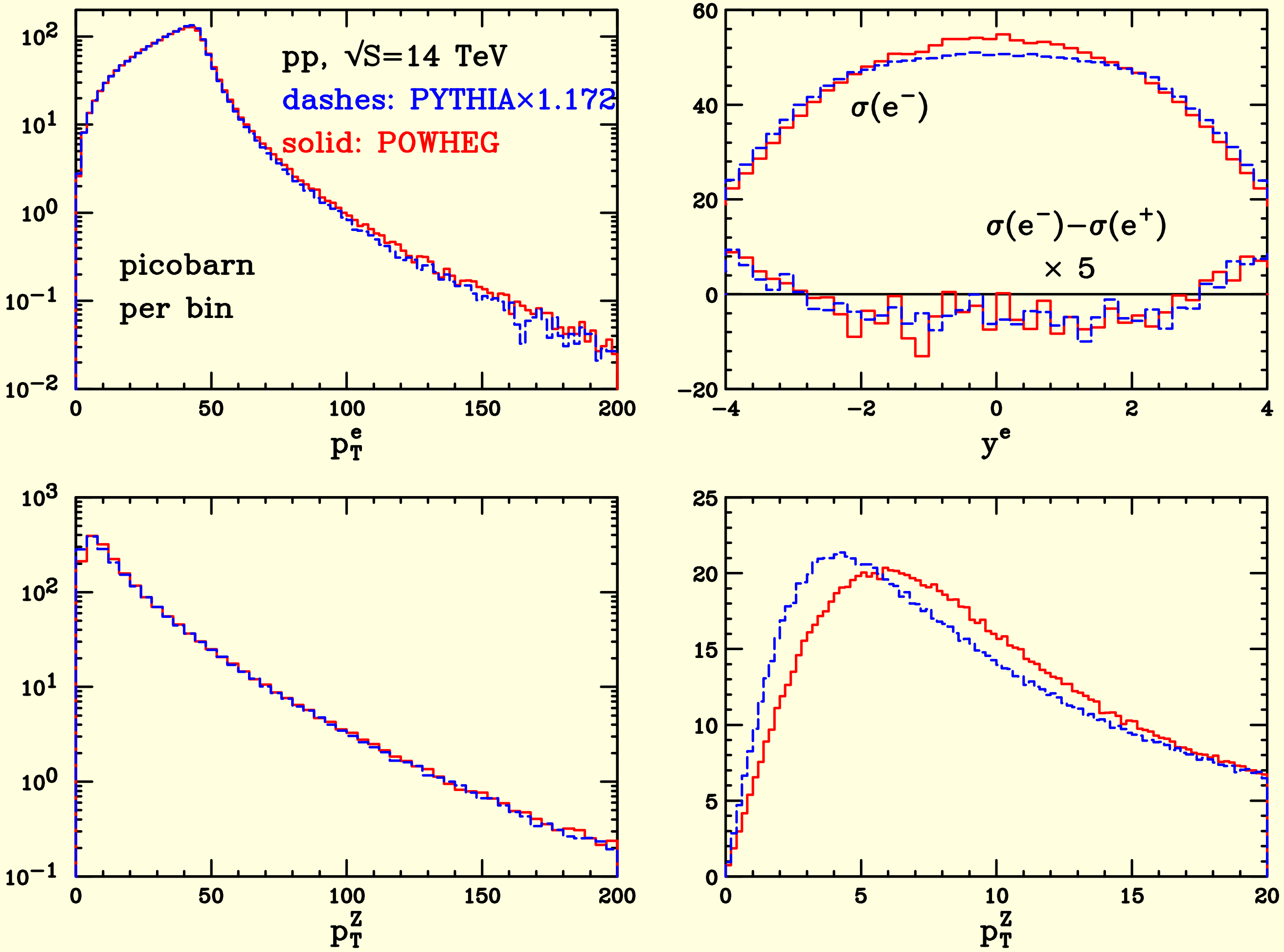
PYTHIA ME vs. POWHEG

For $2 \to 1$ processes (W/Z and Higgs production), PYTHIA ME corrections are very similar to POWHEG; it implements the formula

$$d\sigma = d\Phi_B \underbrace{B(\Phi_B)}_{\bar{B}\text{ in POWHEG}} \left[\Delta_{t_0} + \Delta_t \frac{R(\Phi)}{B(\Phi_B)} d\Phi_r \right], \qquad \Delta_t = \exp\left[- \int \theta(t_r - t) \frac{R(\Phi_B, \Phi_r)}{B(\Phi_B)} d\Phi_r \right]$$

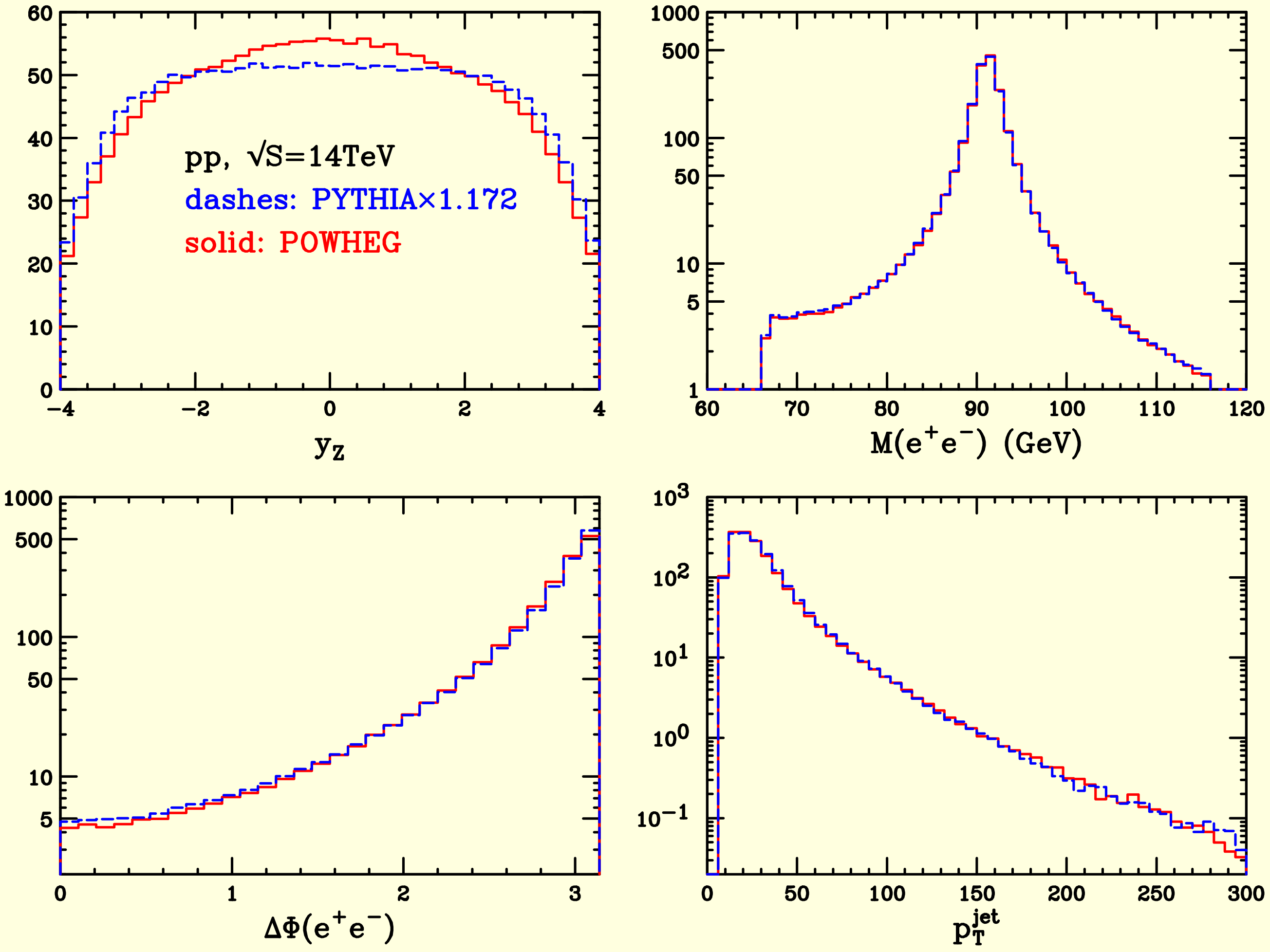
Dashes: PYTHIA X 1.172, Solid: POWHEG





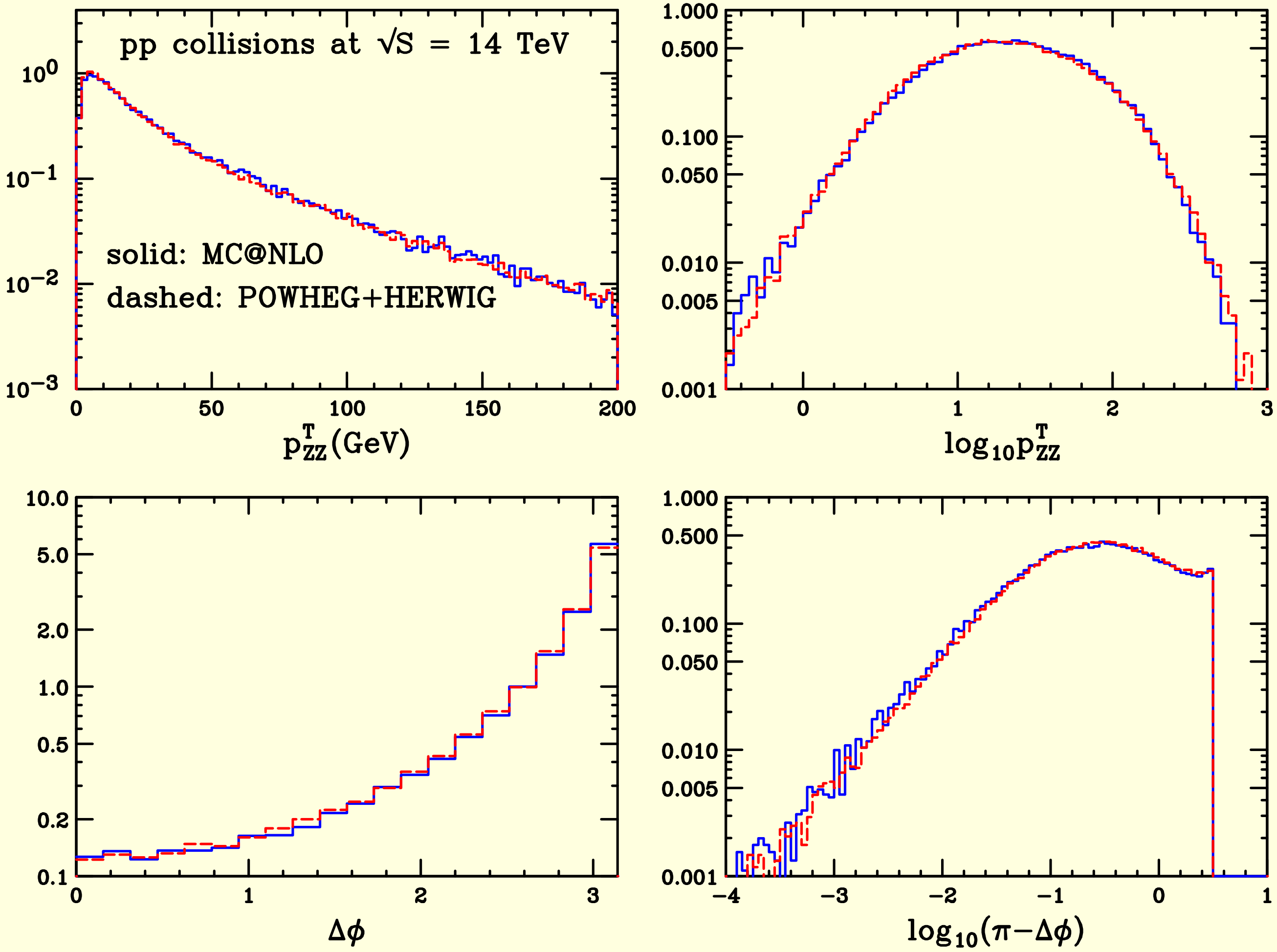
pp, √S=14 TeV
dashes: PYTHIA×1.172
solid: POWHEG
picobarn
per bin
p_T^e
$\sigma(e^-)$
$\sigma(e^-)-\sigma(e^+)$
× 5
y^e
p_T^Z
p_T^Z

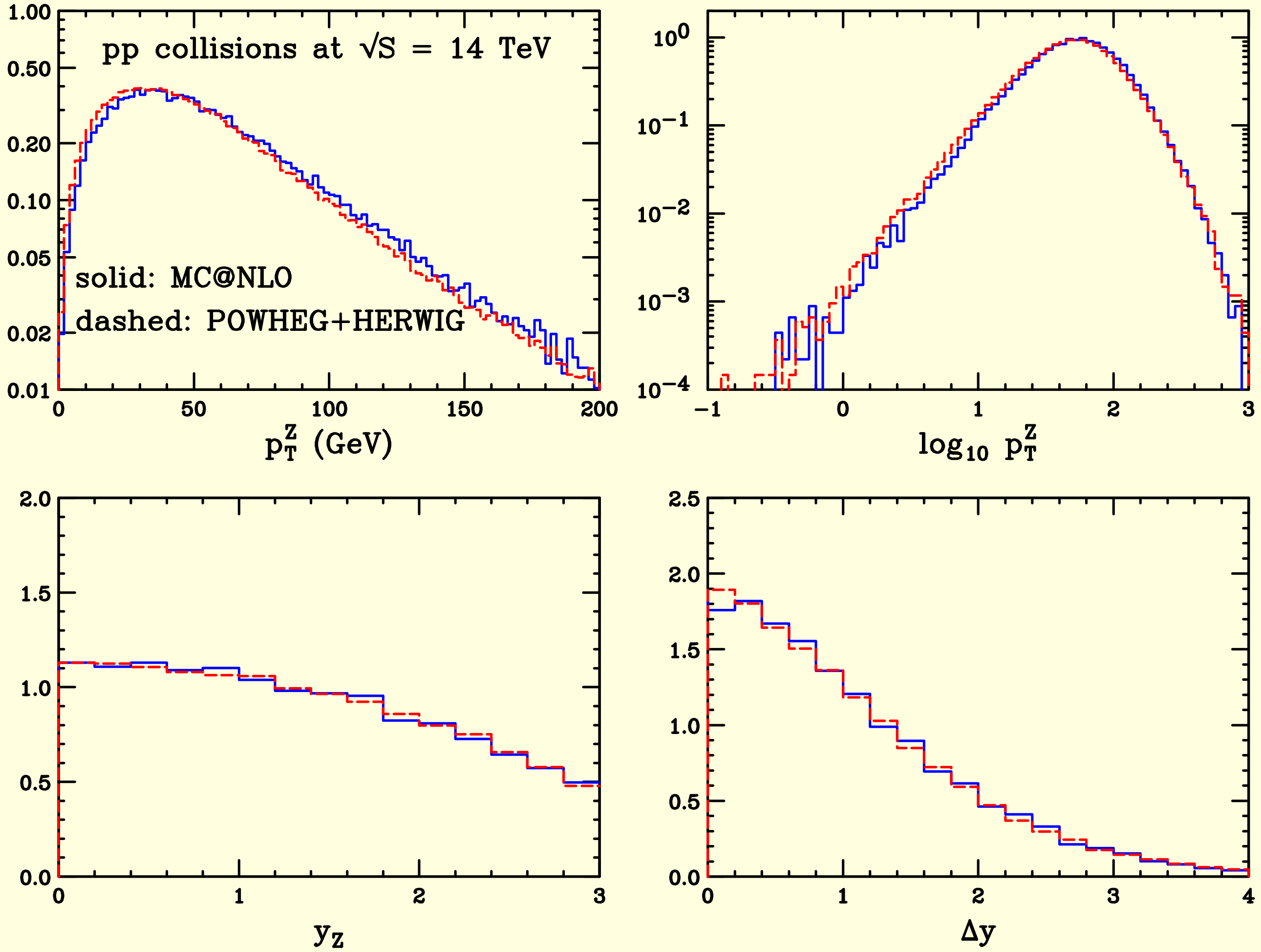
Different shape in y_Z distribution understood as NLO effect

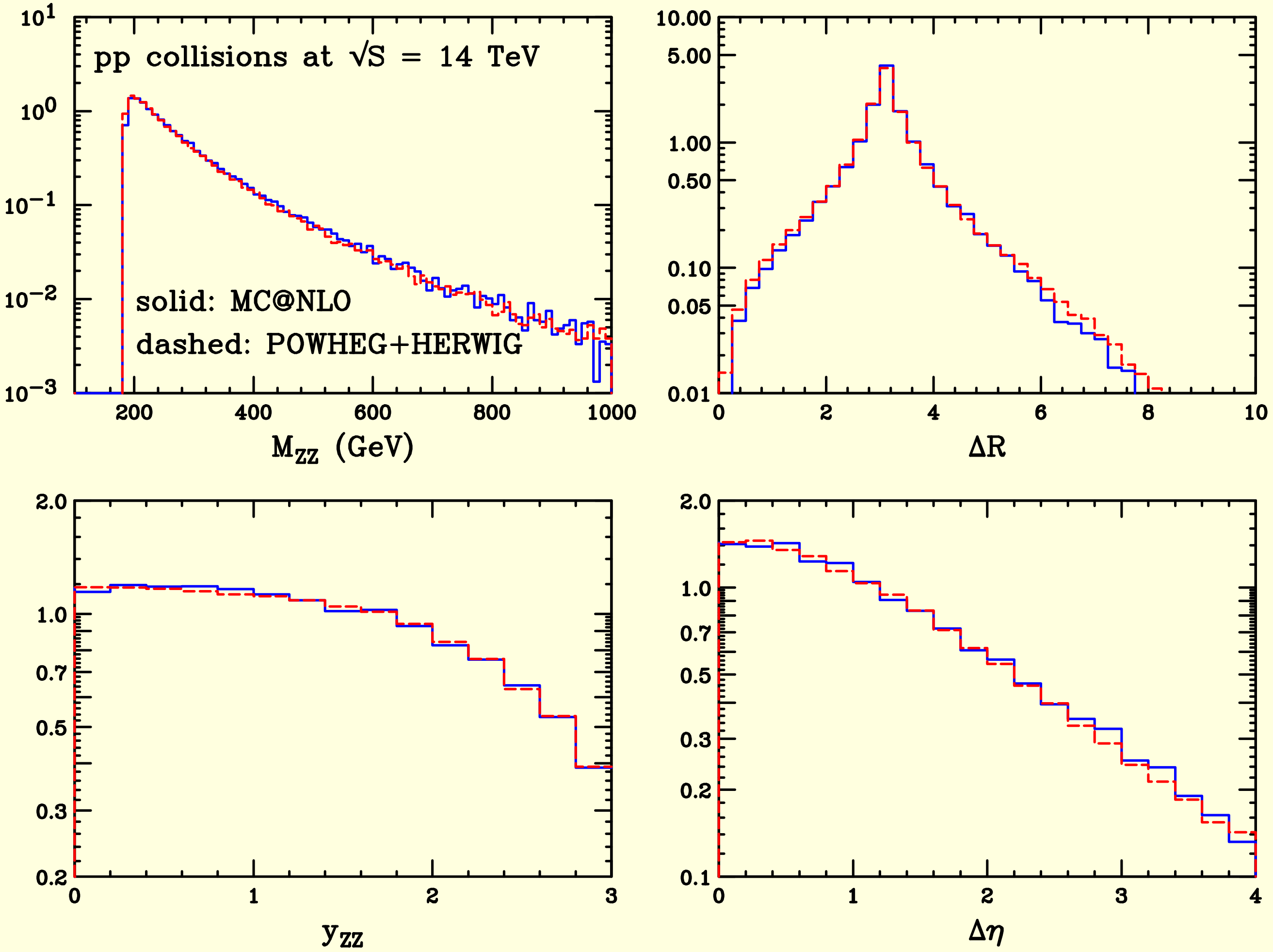


Comparisons of POWHEG+HERWIG vs. MC@NLO

Z pair production

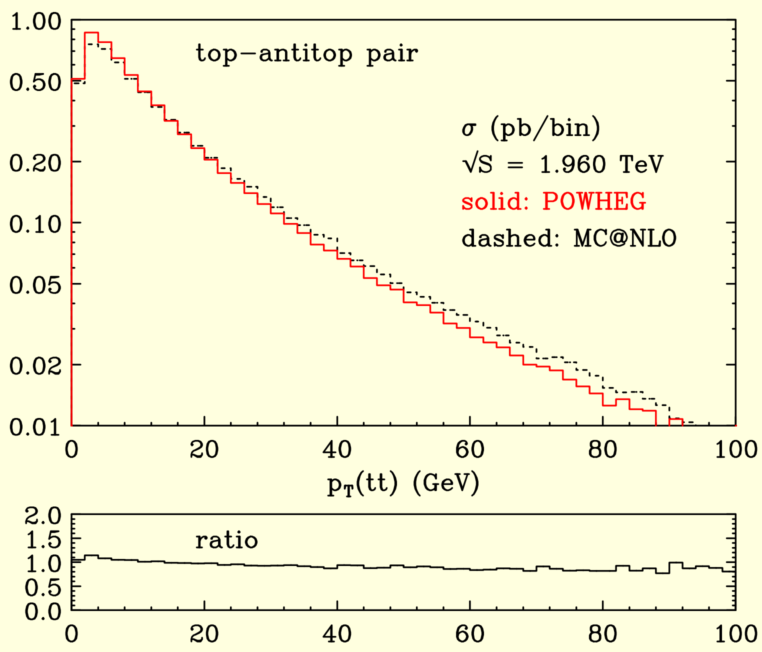


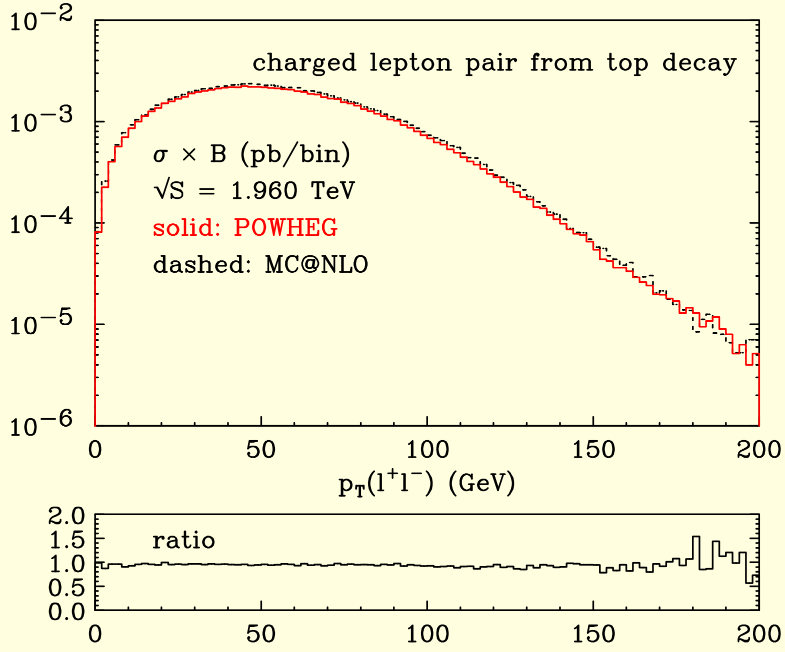

pp collisions at √S = 14 TeV
solid: MC@NLO
dashed: POWHEG+HERWIG
p_T^Z (GeV)
$\log_{10} p_T^Z$
y_Z
Δy



Remarkable agreement for most quantities;

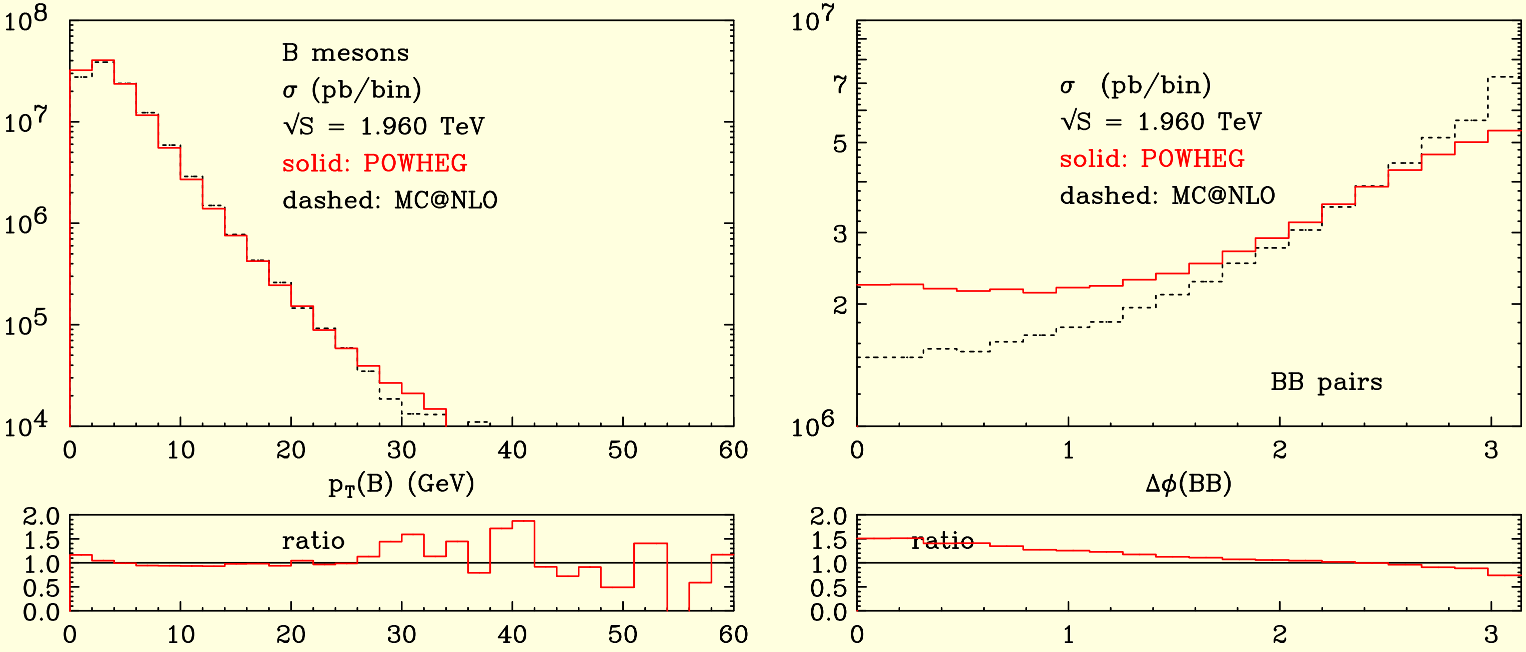
POWHEG and MC@NLO comparison: Top pair production





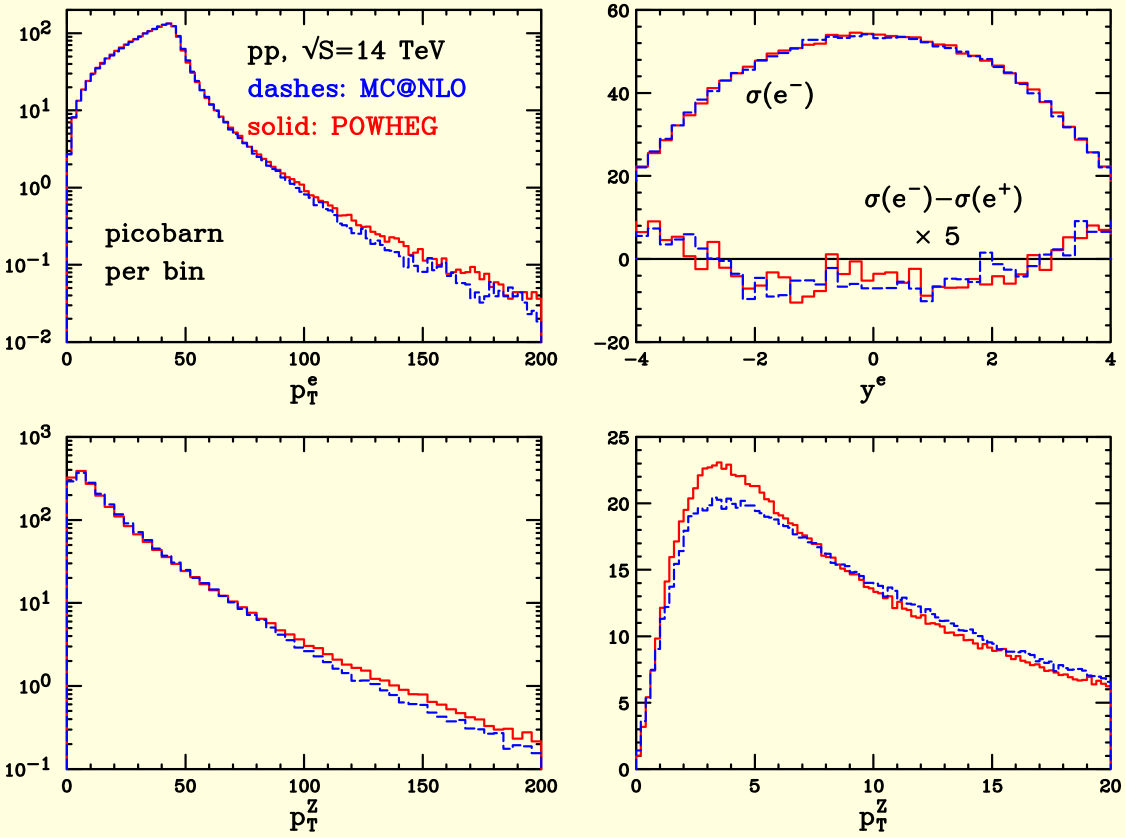
Good agreement for all observable considered
(differences can be ascribed to different treatment of higher order terms)

Bottom pair production



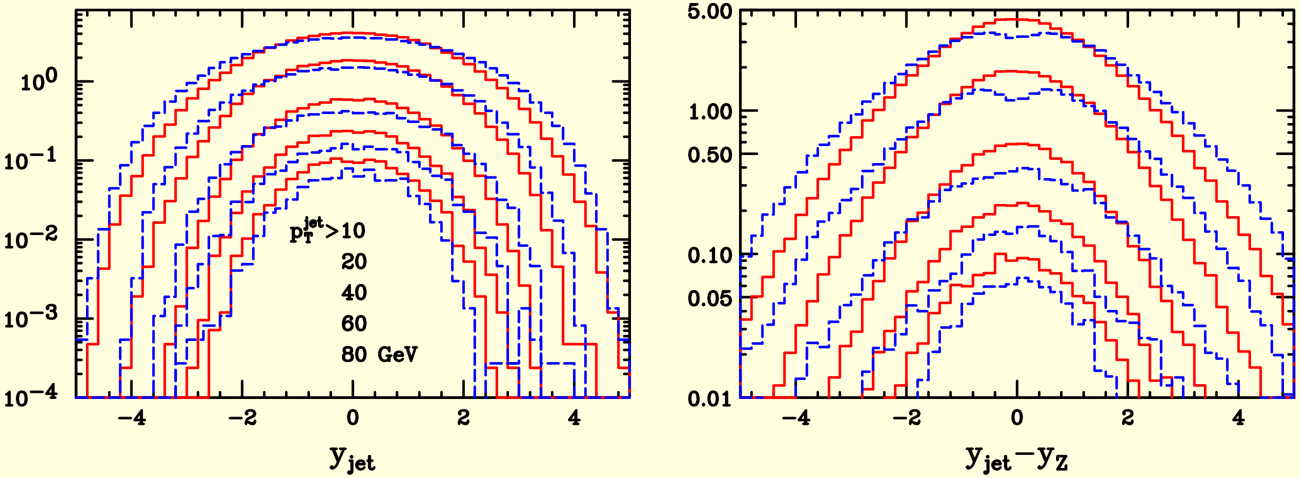
- Very good agreement For large scales (ZZ, $t\bar{t}$ production)
- Differences at small scales ($b\bar{b}$ at the Tevatron)
- POWHEG more reliable in extreme cases like $b\bar{b}, c\bar{c}$ at LHC (yields positive results, MC@NLO has problems with negative weights)

Z production: POWHEG+HERWIG vs. MC@NLO

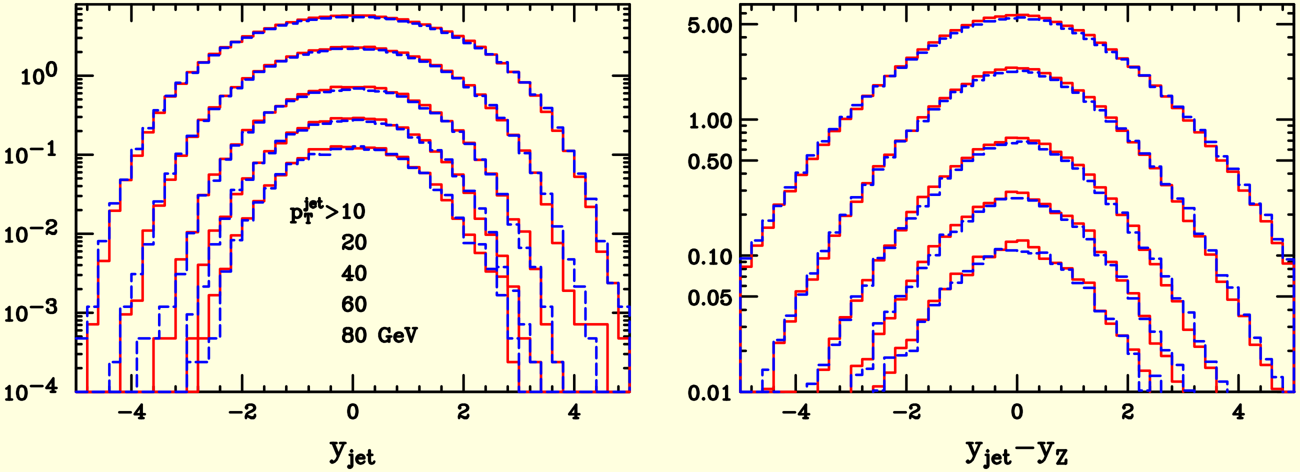


Small differences in high and low p_T region

Z production: rapidity of hardest jet (TEVATRON)

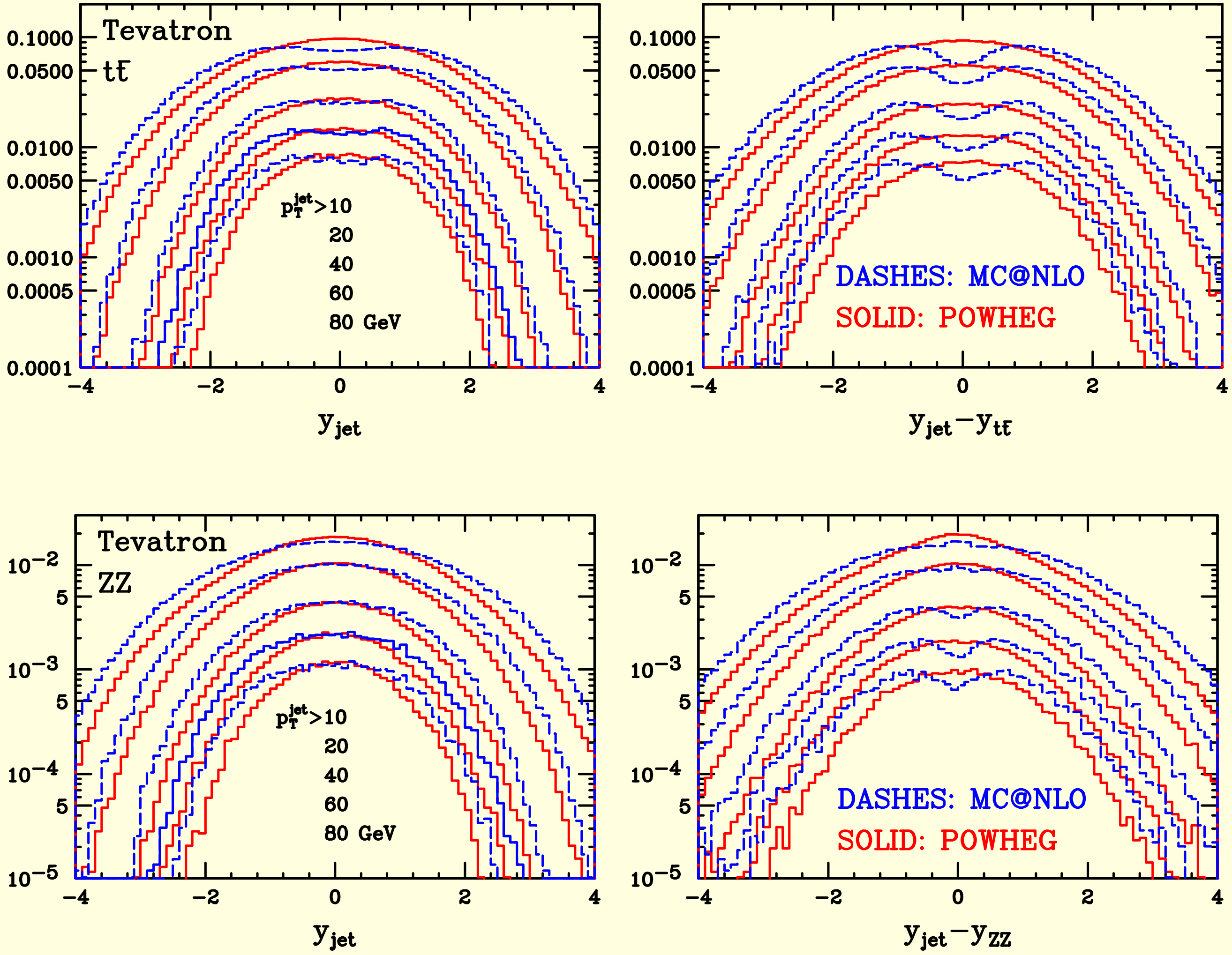


POWHEG+HERWIG
MC@NLO



POWHEG+PYTHIA
PYTHIA

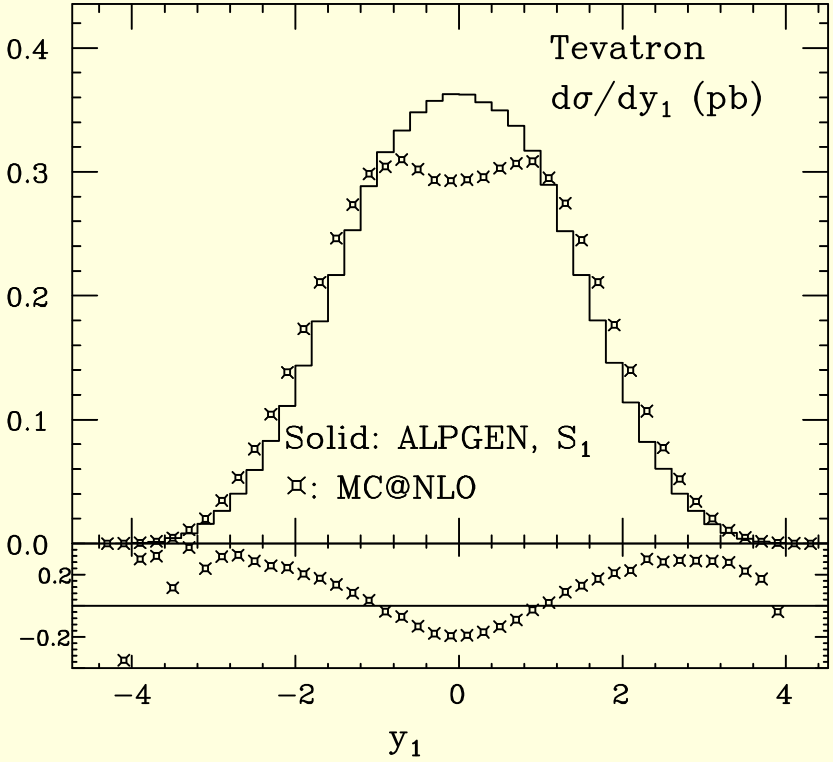
Dip in central region in MC@NLO also in $t\bar{t}$ and ZZ

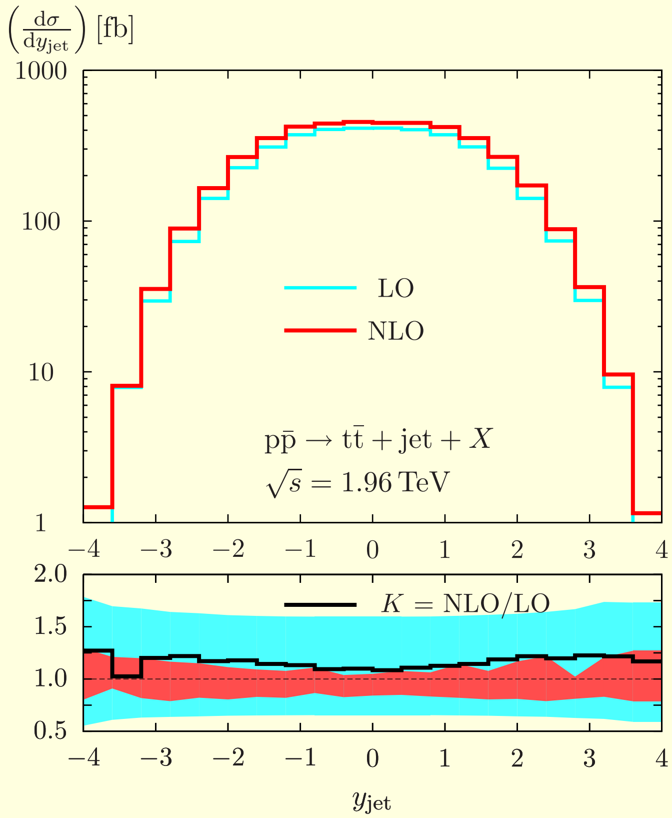


POWHEG+HERWIG
MC@NLO

POWHEG+HERWIG
MC@NLO

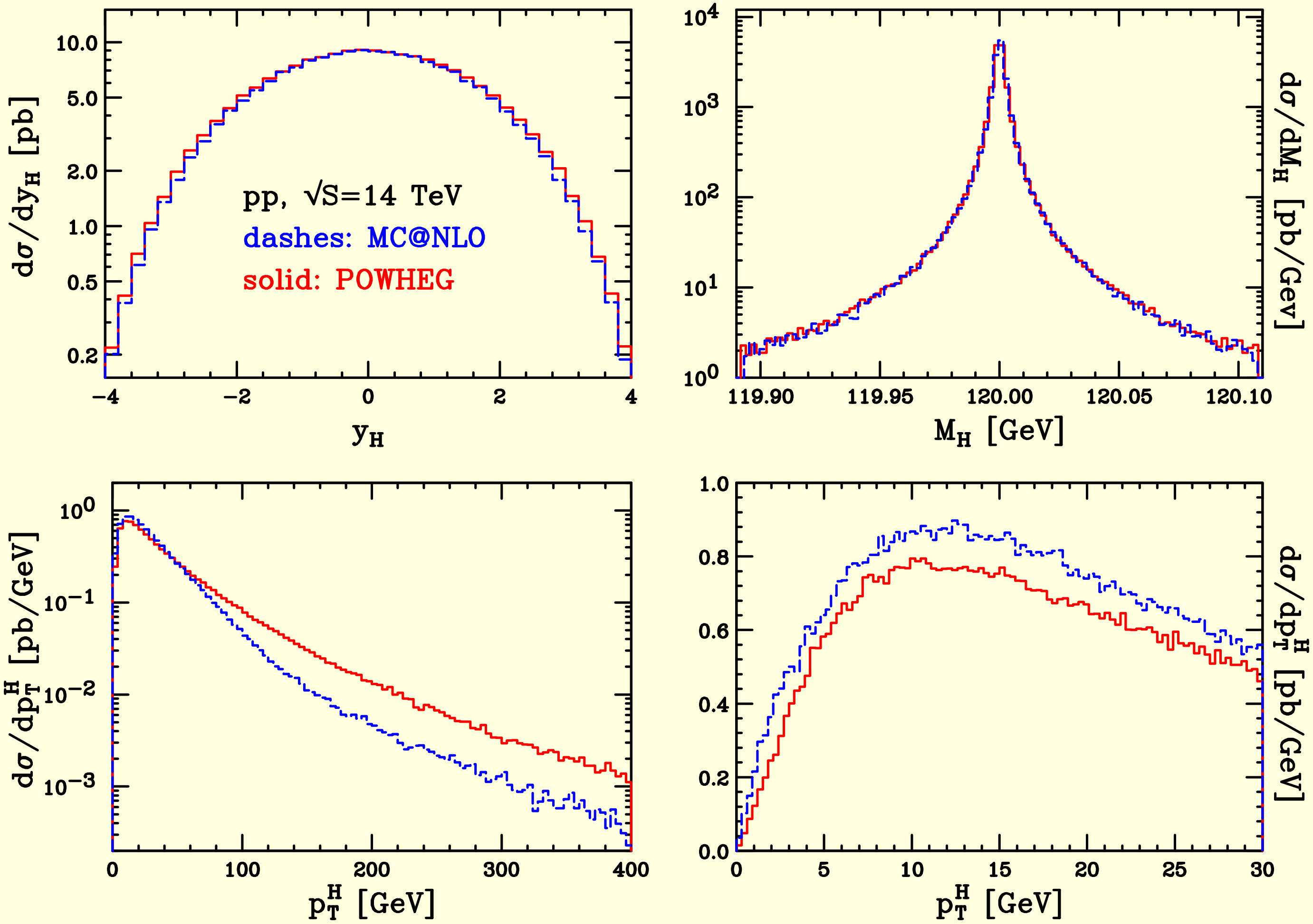
ALPGEN and $t\bar{t}+\mathrm{jet}$ at NLO vs. MC@NLO



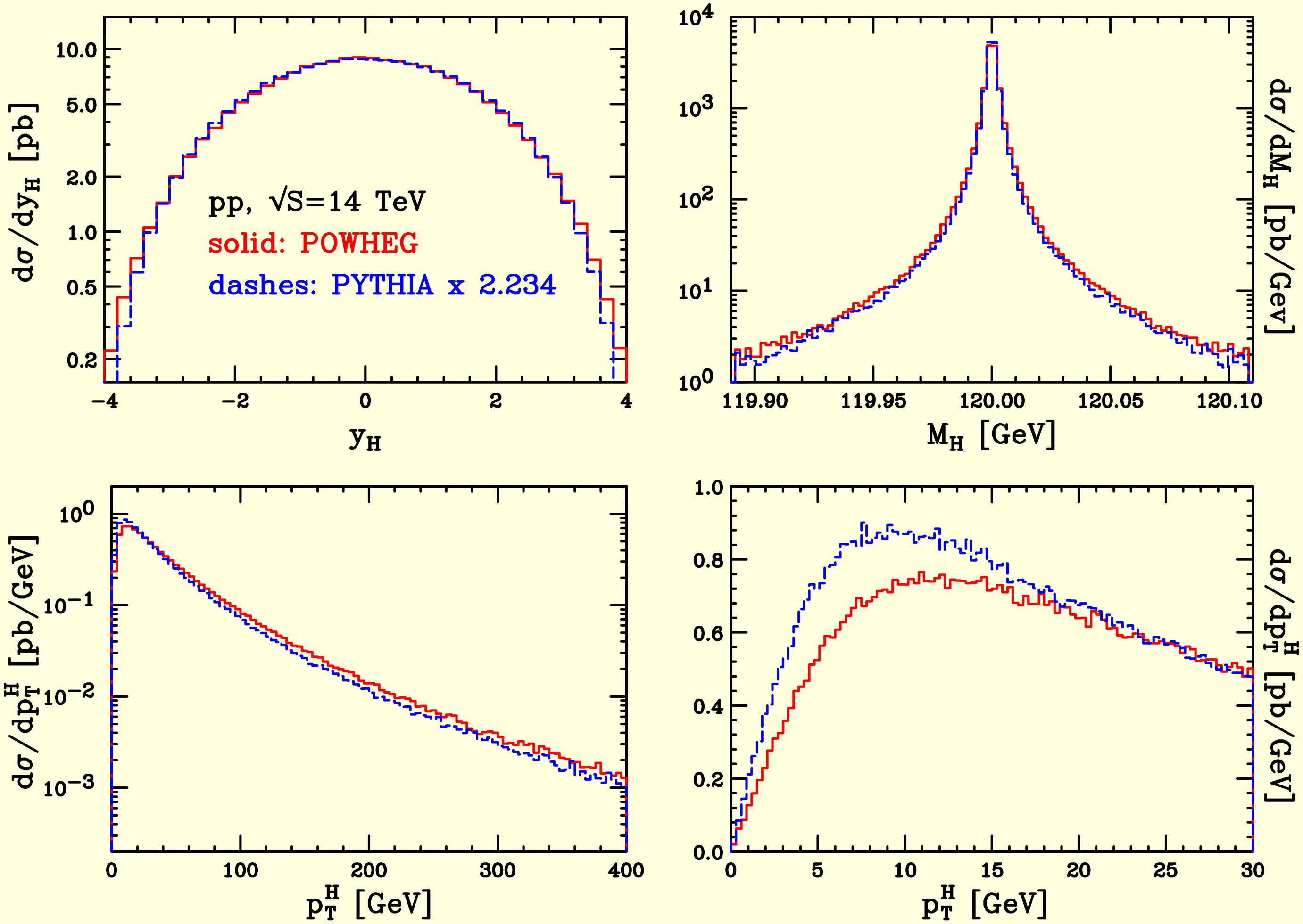


POWHEG distribution as in ALPGEN (Mangano,Moretti,Piccinini,Treccani,Nov.06) and in $t\bar{t}+\mathrm{jet}$ at NLO (Dittmaier, Uwer, Weinzierl) : no dip present.

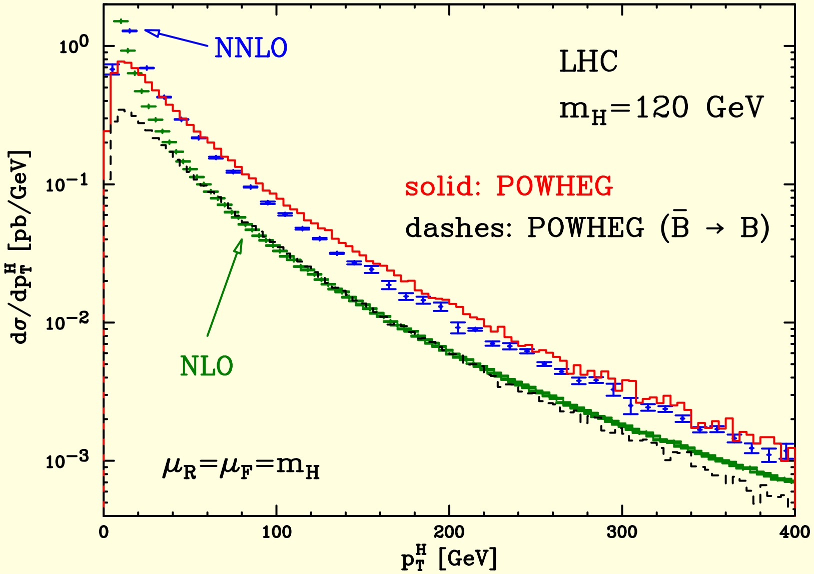
Higgs boson via gluon fusion at LHC

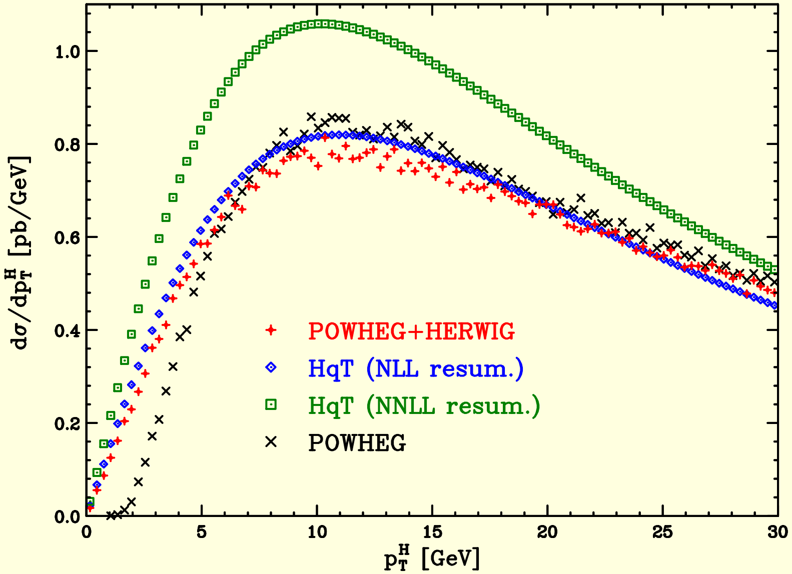


Higgs boson via gluon fusion at LHC



POWHEG vs. NNLO vs. NNLL



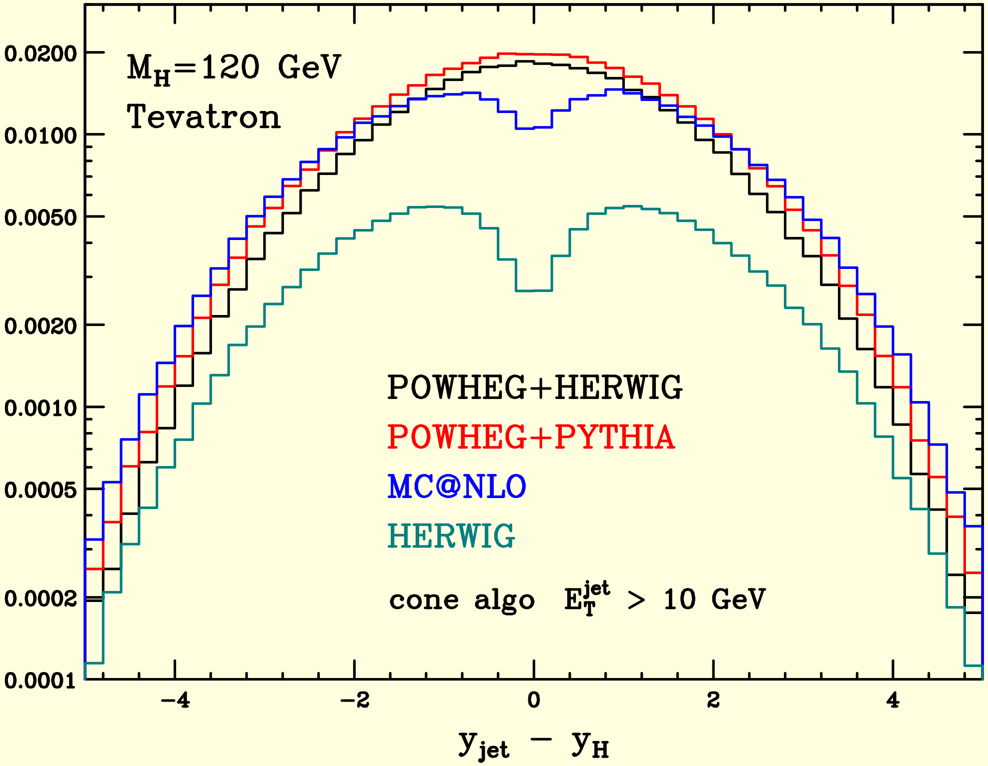


$$d\sigma = \bar{B}(\Phi_B)\, d\Phi_B \left\{ \Delta(\Phi_B, p_T^{\min}) + \Delta(\Phi_B, p_T) \frac{R(\Phi_B, \Phi_r)}{B(\Phi_B)} d\Phi_r \right\}$$

$$\approx \frac{\bar{B}(\Phi_B)}{B(\Phi_B)} R(\Phi_B, \Phi_r)\, d\Phi_r = \{1 + \mathcal{O}(\alpha_s)\} R(\Phi)\, d\Phi$$

Better agreement with NNLO this way.

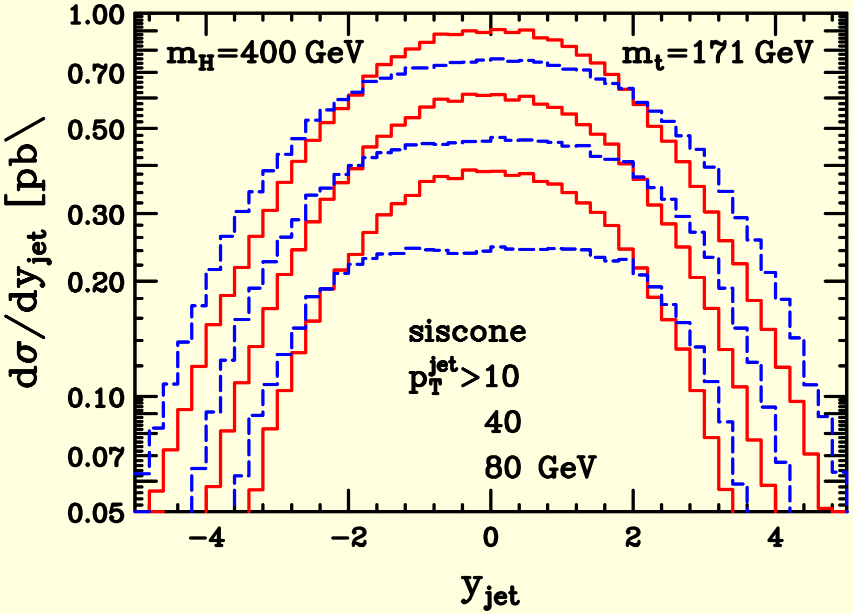
Jet rapidity in h production

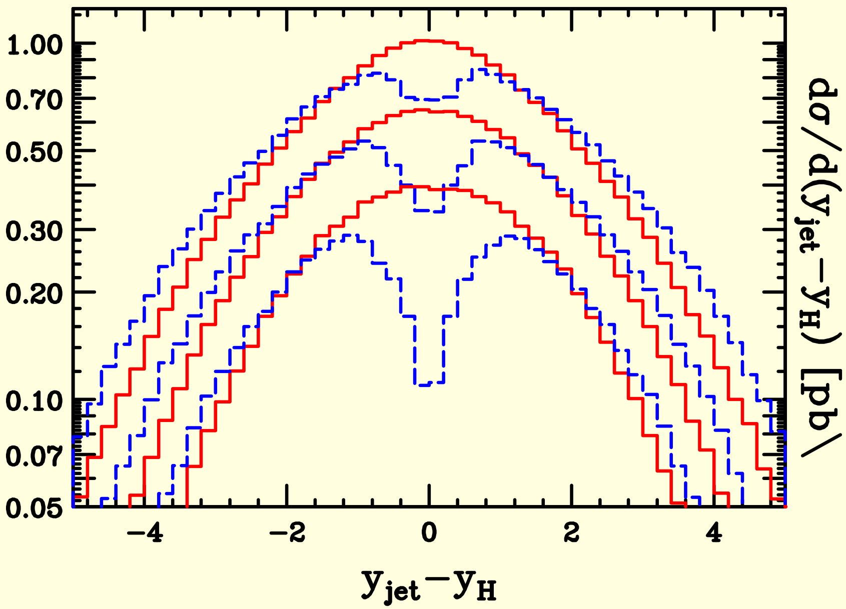


Dip in MC@NLO inerithed from even deeper dip in HERWIG

(MC@NLO tries to fill dead regions in HERWIG, a mismatch remains).

Gets worse for larger E_T cuts:





Questions:

Why MC@NLO has a dip in the hardest jet rapidity?

Why POWHEG has no dip? Is that because of the hardest p_T spectrum?

Hard p_T spectrum in POWHEG

We understand the cause; we keep it because yields results closer to NNLO;
There is enough flexibility to get rid of it, if one wants!
Go back to the POWHEG cross section:

$$d\sigma = \bar{B}(\Phi_{B})\left[\Delta_{t_0} + \Delta_t \frac{R(\Phi)}{B(\Phi_{B})} d\Phi_r\right], \qquad \Delta_t = \exp\left[-\int \theta(t_r - t)\frac{R(\Phi_{B}, \Phi_r)}{B(\Phi_{B})} d\Phi_r\right]$$

Break $R = R_s + R_f$, with R_f finite in collinear and soft limit, define

$$d\sigma' = \bar{B}^s(\Phi_{B})\left[\Delta^s_{t_0} + \Delta^s_t \frac{R_s(\Phi)}{B(\Phi_{B})} d\Phi_r\right] + R_f(\Phi) d\Phi$$

with:

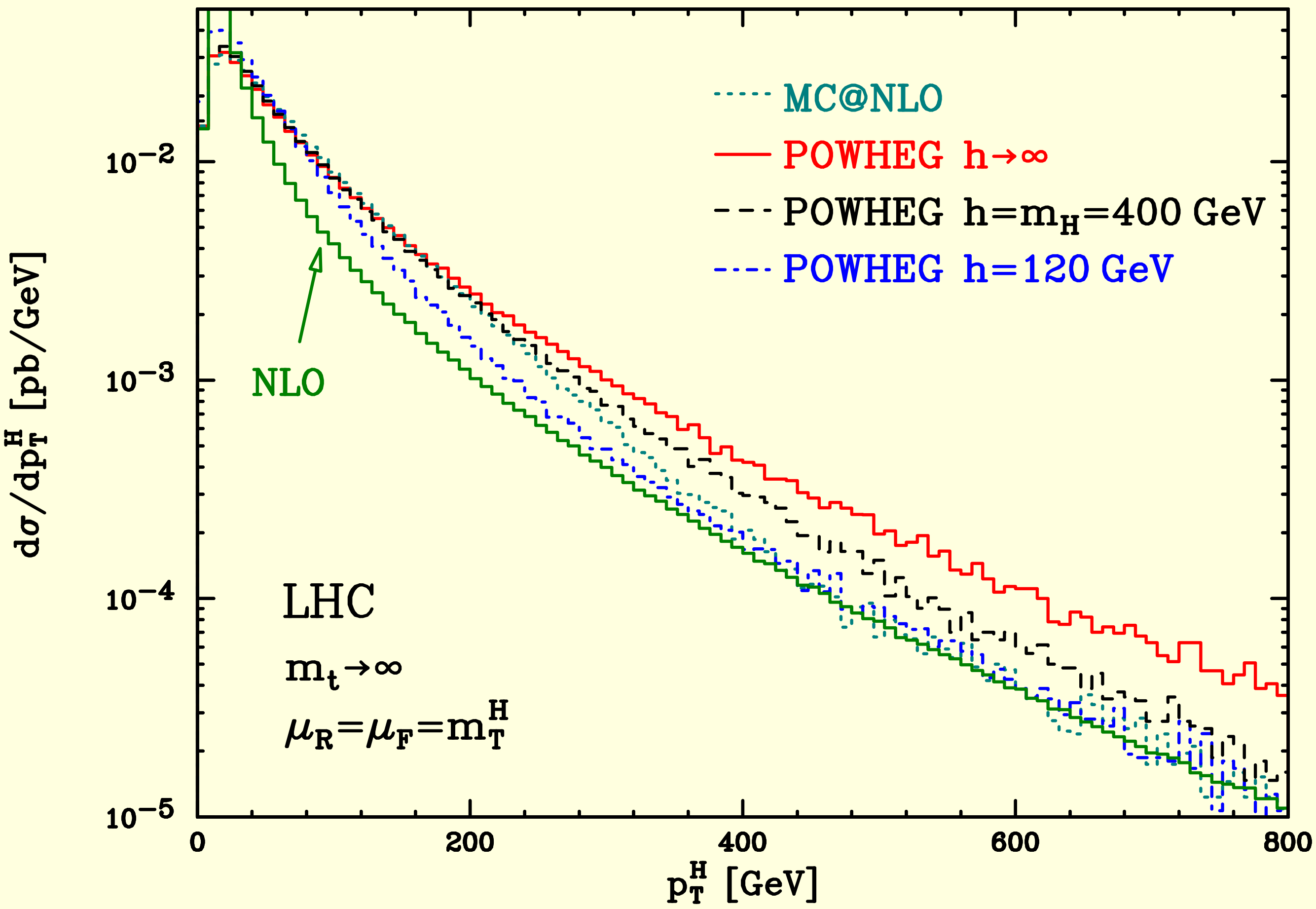
$$\Delta^s_t = \exp\left[-\int \theta(t_r - t)\frac{R^s(\Phi_{B}, \Phi_r)}{B(\Phi_{B})} d\Phi_r\right].$$

Easy to prove that: $d\sigma'$ is equivalent to $d\sigma$.
In other words, the part of the real cross section that is treated with the Shower technique can be varied.

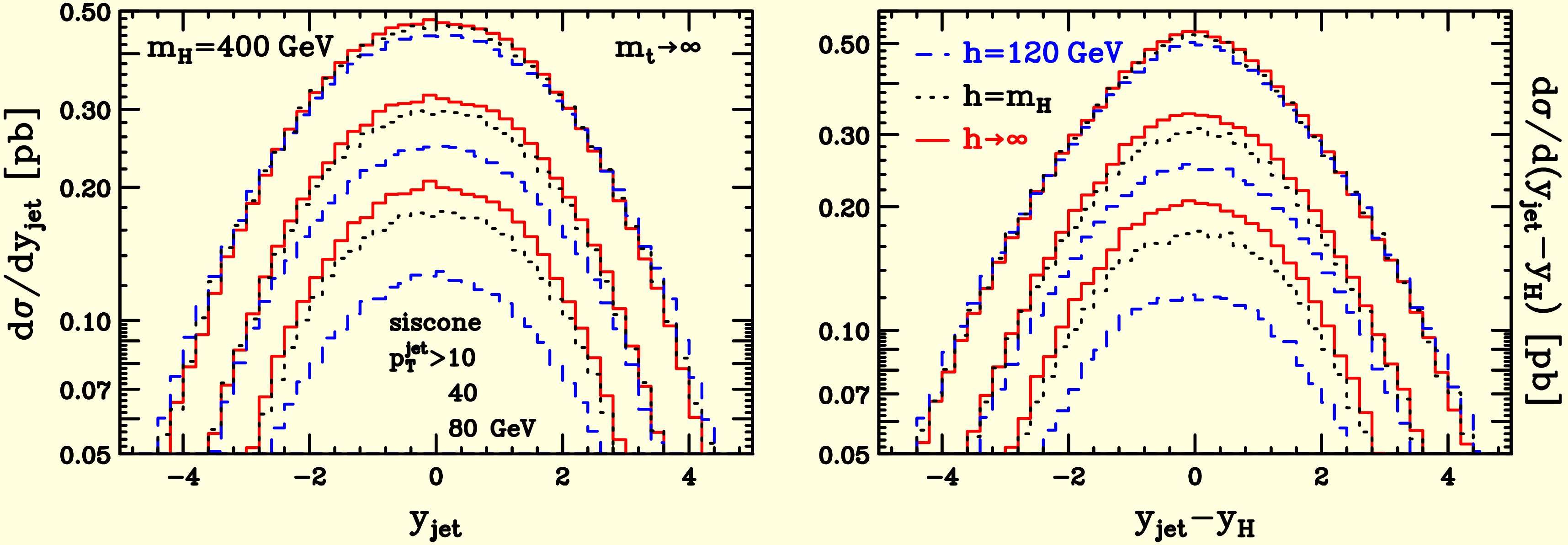
$$R_s = R\frac{h^2}{k_T^2 + h^2}$$

$$R_f = R\frac{k_T^2}{k_T^2 + h^2}$$

Agrees with NLO at high p_T.



No new features (dips and the like) arise in the other distributions:



So: high k_T cross section and dips are unrelated issues.

Why is there a dip in MC@NLO?

Write the `MC@NLO` hardest jet cross section in the `POWHEG` language;
Hardest emission (P.N., 2004) can be written as

$$d\sigma = \underbrace{\bar{B}^{\mathrm{HW}}(\Phi_B)d\Phi_B}_{S\ \mathrm{event}}\left[\underbrace{\Delta_{t_0}^{\mathrm{HW}} + \Delta_t^{\mathrm{HW}}\frac{R^{\mathrm{HW}}(\Phi)}{B(\Phi_B)}d\Phi_r^{\mathrm{HW}}}_{\mathrm{HERWIG\ shower}}\right] + \left[\underbrace{R(\Phi) - R^{\mathrm{HW}}(\Phi)}_{H\ \mathrm{event}}\right]d\Phi$$

$$\bar{B}^{\mathrm{HW}}(\Phi_B) = B(\Phi_B) + \underbrace{\left[\underbrace{V(\Phi_B)}_{\mathrm{infinite}} + \underbrace{\int R^{\mathrm{HW}}(\Phi_B,\Phi_r)\,d\Phi_r}_{\mathrm{infinite}}\right]}_{\mathrm{finite}}$$

(Imagine that soft and collinear singularities in R^{HW} are regulated as in V!).
Like `POWHEG` with $R_s = R^{\mathrm{HW}}$. But now $R_f = R - R^{\mathrm{HW}}$ can be negative.
This formula illustrates why MC@NLO and POWHEG are equivalent at NLO.
But differences can arise at NNLO ...

For large k_T:

$$d\sigma = \left[\frac{\bar{B}^{\mathrm{HW}}(\Phi_{B})}{B(\Phi_{B})} R^{\mathrm{HW}}(\Phi) + R(\Phi) - R^{\mathrm{HW}}(\Phi) \right] d\Phi_{B}\, d\Phi_r^{\mathrm{HW}}$$

$$= \underbrace{R(\Phi) d\Phi}_{\text{no dip}} + \underbrace{\left(\frac{\bar{B}^{\mathrm{HW}}(\Phi_{B})}{B(\Phi_{B})} - 1 \right)}_{\mathcal{O}(\alpha_s),\ \text{but large for Higgs}} \underbrace{R^{\mathrm{HW}}(\Phi)}_{\text{Pure Herwig dip}} d\Phi$$

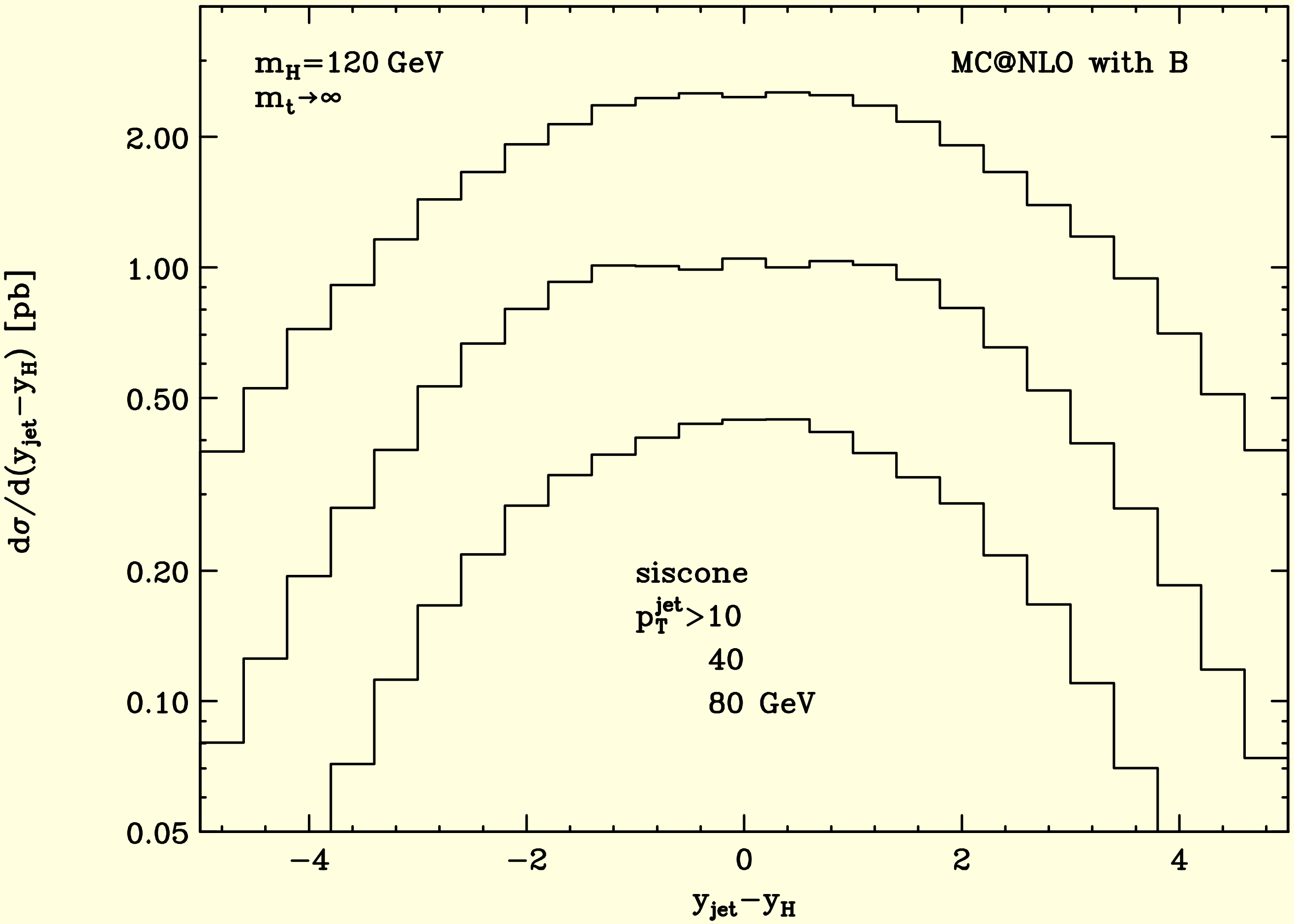
So: a contribution with a dip is added to the exact NLO result;

The contribution is $\mathcal{O}(\alpha_s R)$, i.e. NNLO!

Can we test this hypothesis? Replace $\bar{B}^{\mathrm{HW}}(\Phi_n) \Rightarrow B(\Phi_n)$ in MC@NLO!

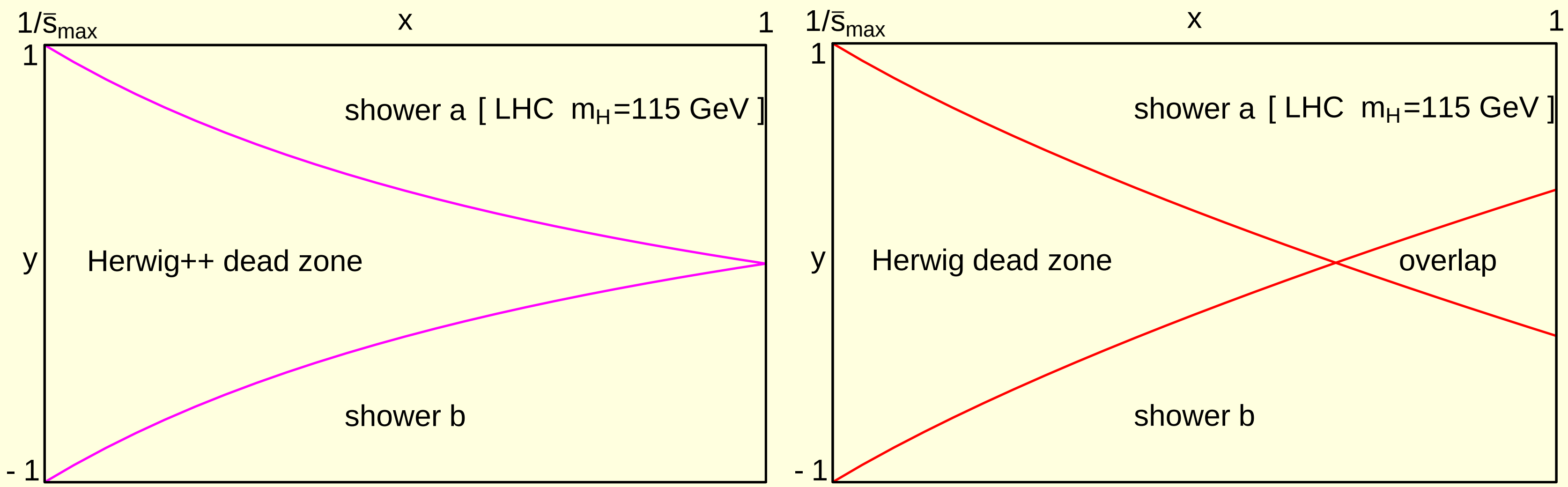
the dip should disappear ...

MC@NLO with $B^{\rm HW}$ replaced by B

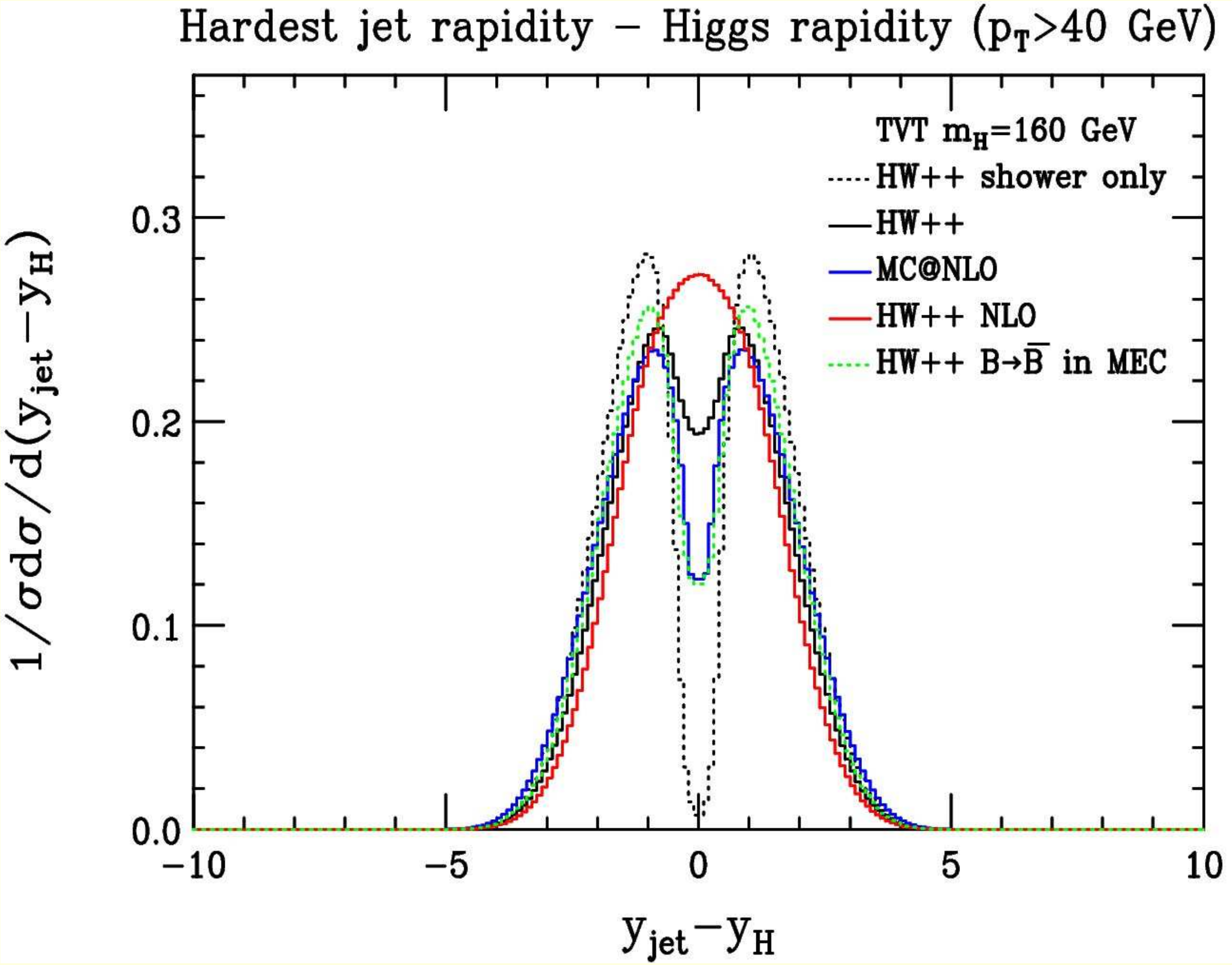


No visible dip is present! (on the right track, more studies needed cd Does...)

Detailed study of the problem also by Hamilton,Richardson,Tully, 2009

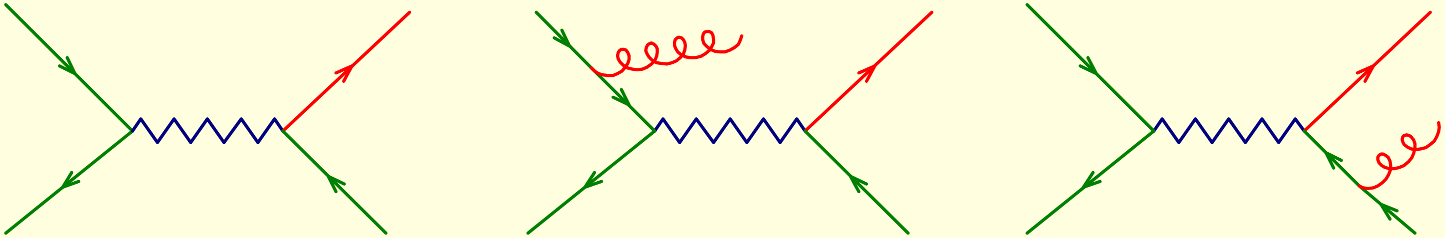


Both HERWIG and HERWIG++ have a dead radiation region corresponding to central rapidity and high energy


Hardest jet rapidity – Higgs rapidity (p_T>40 GeV)
TVT m_H=160 GeV
HW++ shower only
HW++
MC@NLO
HW++ NLO
HW++ B→$\overline{B}$ in MEC
$1/\sigma d\sigma/d(y_{jet}-y_H)$
0.0
0.1
0.2
0.3
−10
−5
0
5
10
$y_{jet}-y_H$

Single Top

Both initial state and final state radiation is present;



Born Initial state radiation final state radiation

The separation of the different singular regions is based upon the general formulation of `POWHEG` given in Frixione, Oleari, P.N. 2007

Flavour and singularities separation

There are several allowed flavour structures in the n body process.
A flavour structure is a flavour assignment to the incoming and outgoing partons. B and V contributions are labelled by the flavour structure index f_b.

There are several allowed flavour structures in the $n+1$ body process.
Thus R is labelled by a flavour structure index f_r.
Each component R_{f_r} has several singularity regions. We thus write

$$R = \sum_{\alpha_r} R^{\alpha_r}$$

where each R^{α_r} has a specific flavour structure, and is singular in only one singular region. In FKS one writes

$$R^{\alpha_r} = R \times \mathcal{S}_{\alpha_r}, \qquad \sum_{\alpha_r} S_{\alpha_r} = 1$$

The S factors in the FKS formalism are defined as

$$\mathcal{S}_i = \frac{1}{N d_i}, \; \mathcal{S}_{ij} = \frac{1}{N d_{ij}} h\left(\frac{E_i}{E_i + E_j}\right),$$

where N is define so that $\sum_{\alpha_r} S_{\alpha_r} = 1$,

$$d_i = \left(\sqrt{s} E_i/2\right)^a (1 - \cos^2\theta_i)^b, \;\; d_{ij} = (E_i E_j)^a (1 - \cos\theta_{ij})^b,$$

$$\lim_{z \to 0} h(z) = 1, \;\; \lim_{z \to 1} h(z) = 0, \;\; h(z) + h(1 - z) = 1.$$

For example:

$$h(z) = \frac{(1 - z)^c}{z^c + (1 - z)^c}$$

So, the $\mathcal{S}_i$ factors single out the region where parton i is collinear to either initial state line, or is soft, while S_{ij} single out the region where parton i is collinear to parton j or is soft.

The underlying Born

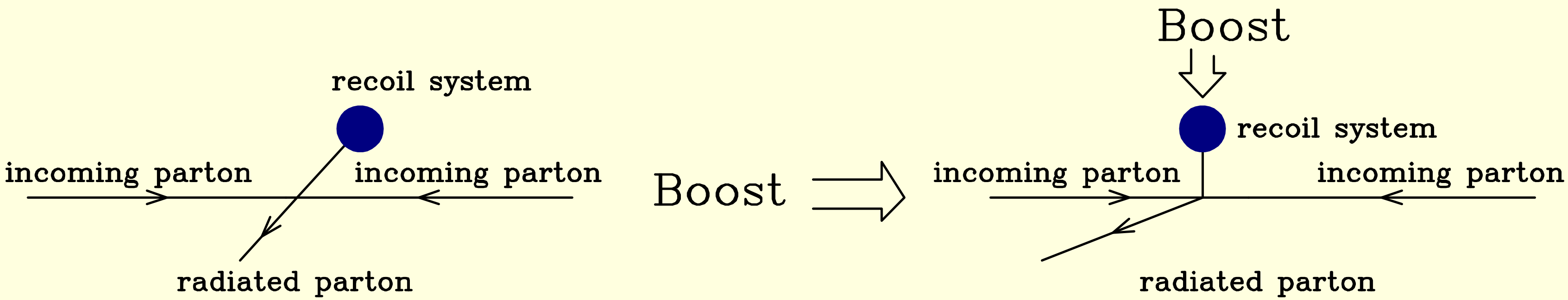
This is a basic concept in the POWHEG formalism;
To each region α_r we associate an underlying Born flavour configuration f_b, obtained as follows:

- If the singular region is associated to a parton becoming soft, then the parton must be a gluon, and it is simply removed to get the underlying Born configuration
- If the region is associated to two parton becoming collinear, then, in order for the region to be singular, the two partons must come from the splitting of another parton. The two partons are removed, and are replaced by the single parent parton with the appropriate flavour

Notice that in a shower Monte Carlo one first generates the Born process (i.e. the underlying Born configuration) and then lets one initial or final line undergo collinear splitting. Here we look at each singular region of the real matrix element, and ask from which underlyng Born process it could have been produced via a shower.

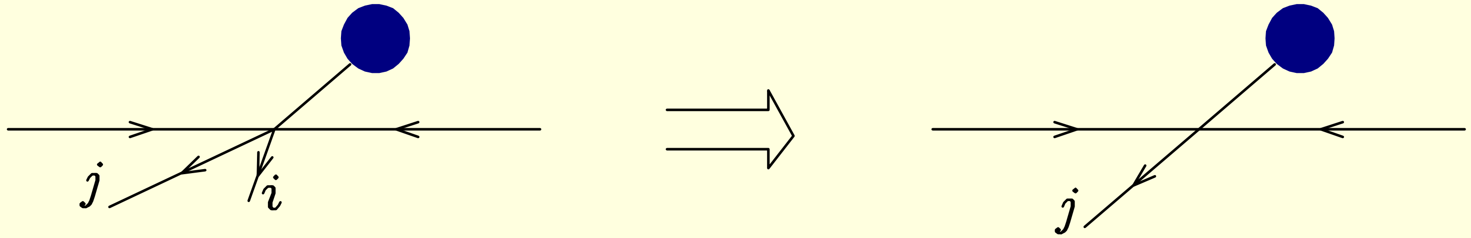
The underlying Born kinematics

To each kinematic configuration for the full radiation phase space Φ, one associates an underlying Born kinematics Φ_B and a set of radiation variables $\Phi_r = (y, z, \phi)$. For initial state radiation Φ_B is obtained by going with a longitudinal boost to the frame where the system recoiling against radiation has zero longitudinal momentum. In this frame one boosts the recoil system in the transverse direction, so that its transverse momentum becomes zero



The radiation variables are $y = \cos\theta$, θ being the angle between the radiated parton and the positive rapidity incoming parton, $\xi = 2E/\sqrt{s}$, where E is the energy of the radiated parton, and ϕ is its azimuth.

For final state radiation, the splitting partons are merged by summing their 3-momenta in the partonic CM frame. The 3-momentum is scaled, and the recoil system is boosted so that momentum and energy are conserved.



The radiation variables are $y = \cos\theta$, θ being the angle between the radiated partons, $\xi = 2E_i/\sqrt{s}$, ϕ is the azimuth of the ij plane relative to $\vec{k}_i + \vec{k}_j$. (This differs from FKS kinematics , where ϕ is relative to $\vec{k}_j$).

The $\bar{B}$ function carries a flavour structure index, and is given by

$$\bar{B}^{f_b}(\Phi_B) = [B(\Phi_B) + V(\Phi_B)]_{f_b} + \sum_{\alpha_r \in \{\alpha_r | f_b\}} [d\Phi_r \, R(\Phi)]_{\alpha_r}$$

The R_{α_r} appearing here have singularities regulated by $+$ prescriptions in the FKS framework.
we have

- $\{\alpha_r | f_b\}$ is the set of all singular regions having the underlying Born configuration with flavour structure f_b.
- $[...]_{\alpha_r}$ means that everything inside is relative to the α_r singular term: thus R is R_{α_r}, the parametrization (Φ_B, Φ_r) is the one appropriate to the α_r singular region

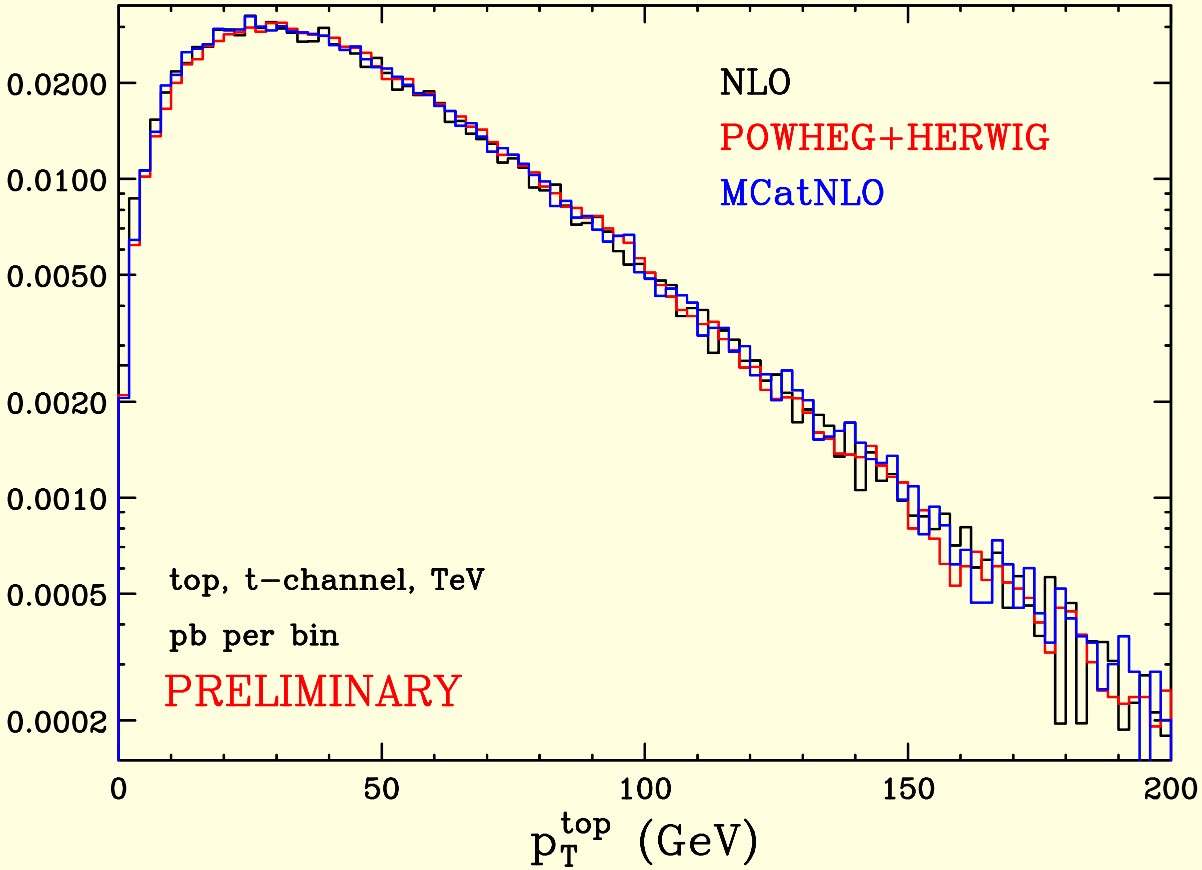
Sudakov FF also carries an f_b index:

$$\Delta^{f_b}(\Phi_n, p_T) = \exp\left\{ - \sum_{\alpha_r \in \{\alpha_r | f_b\}} \int \frac{[d\Phi_r\, R(\Phi_n, \Phi_r)\theta(k_T - p_T)]_{\alpha_r}}{B^{f_b}(\Phi_n)} \right\}$$

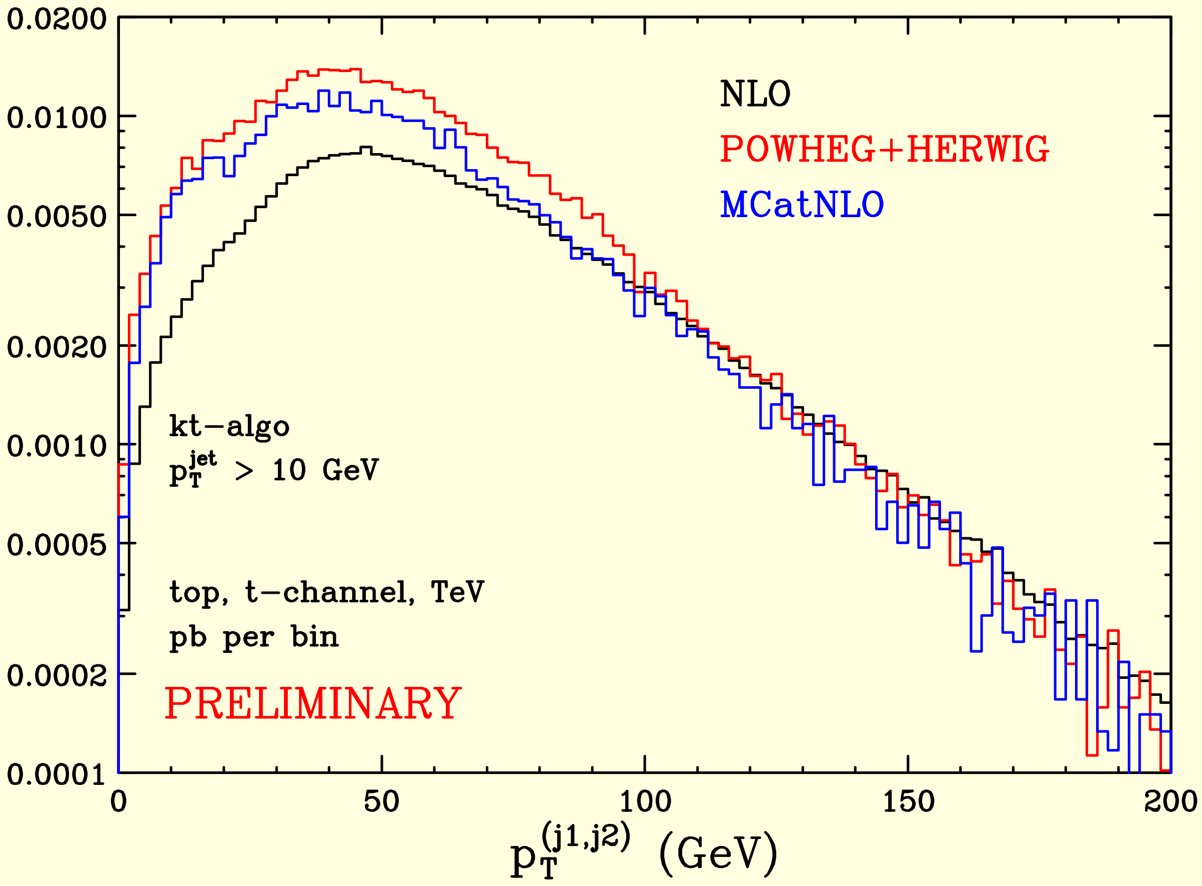
or

$$\Delta^{f_b}(\Phi_n, p_T) = \prod_{\alpha_r \in \{\alpha_r | f_b\}} \exp\left\{ - \sum \int \frac{[d\Phi_r\, R(\Phi_n, \Phi_r)\theta(k_T - p_T)]_{\alpha_r}}{B^{f_b}(\Phi_n)} \right\}$$

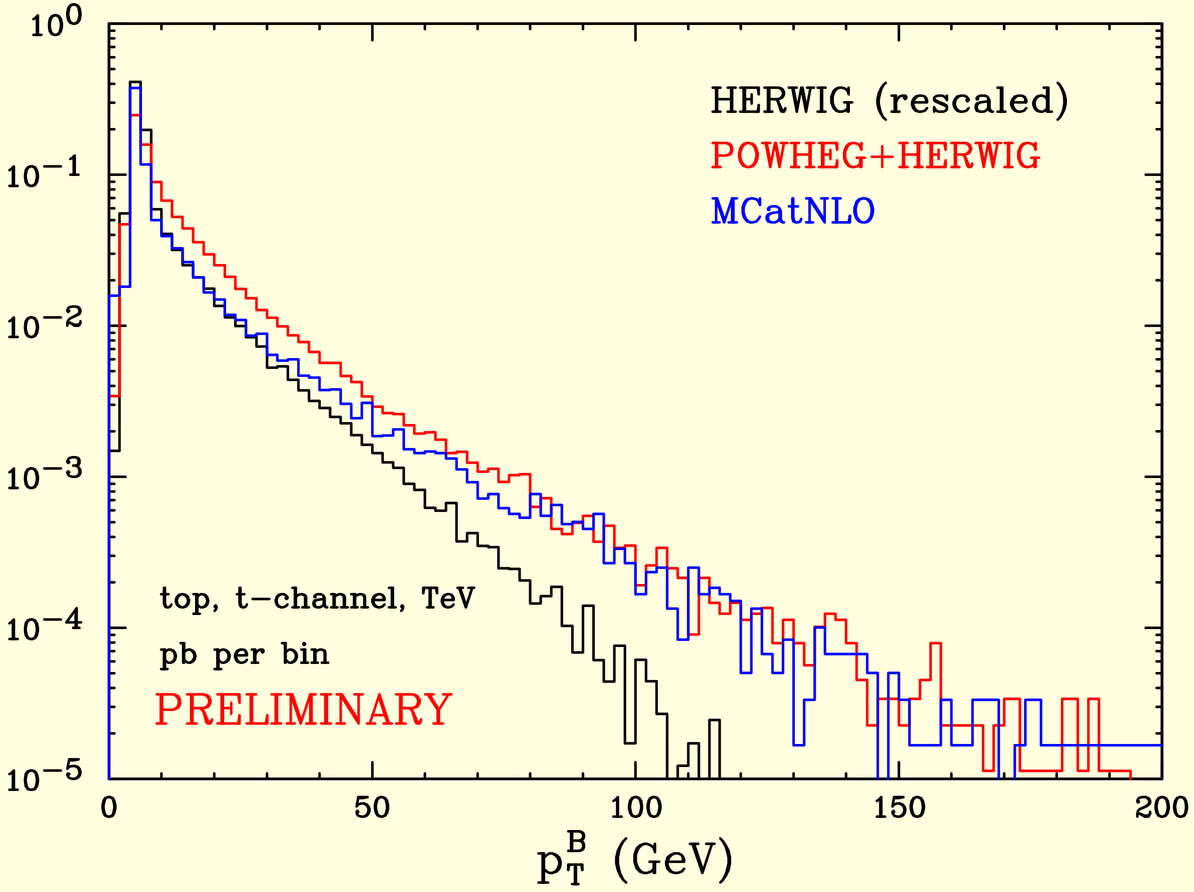
The Sudakov form factor is a product of elementary Sudakov form factors associated with each radiation region. Technically, one generates radiation by generating a k_T with each elementary form factor, and choosing the one with the largest k_T at the end.



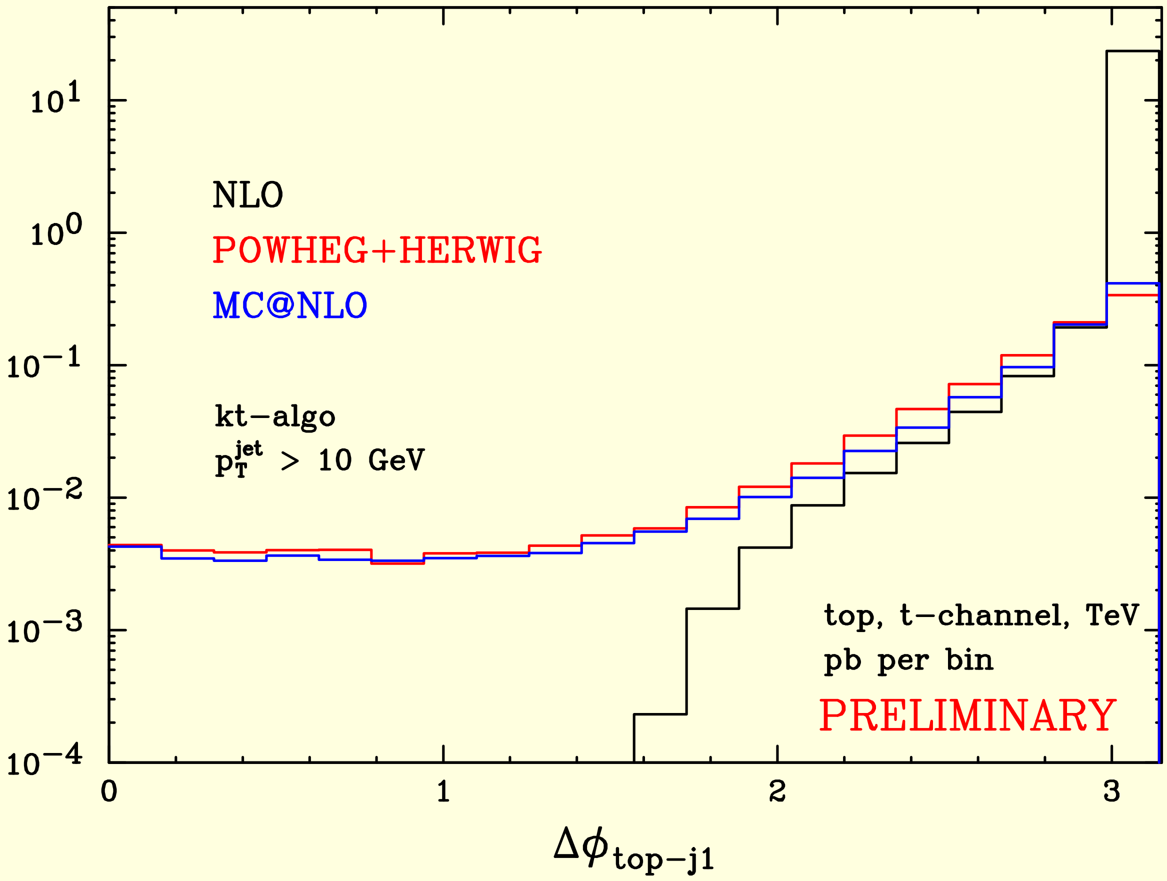
0.0200
0.0100
0.0050
0.0020
0.0010
0.0005
0.0002
0
50
100
150
200
p_T^{top} (GeV)
NLO
POWHEG+HERWIG
MCatNLO
top, t−channel, TeV
pb per bin
PRELIMINARY



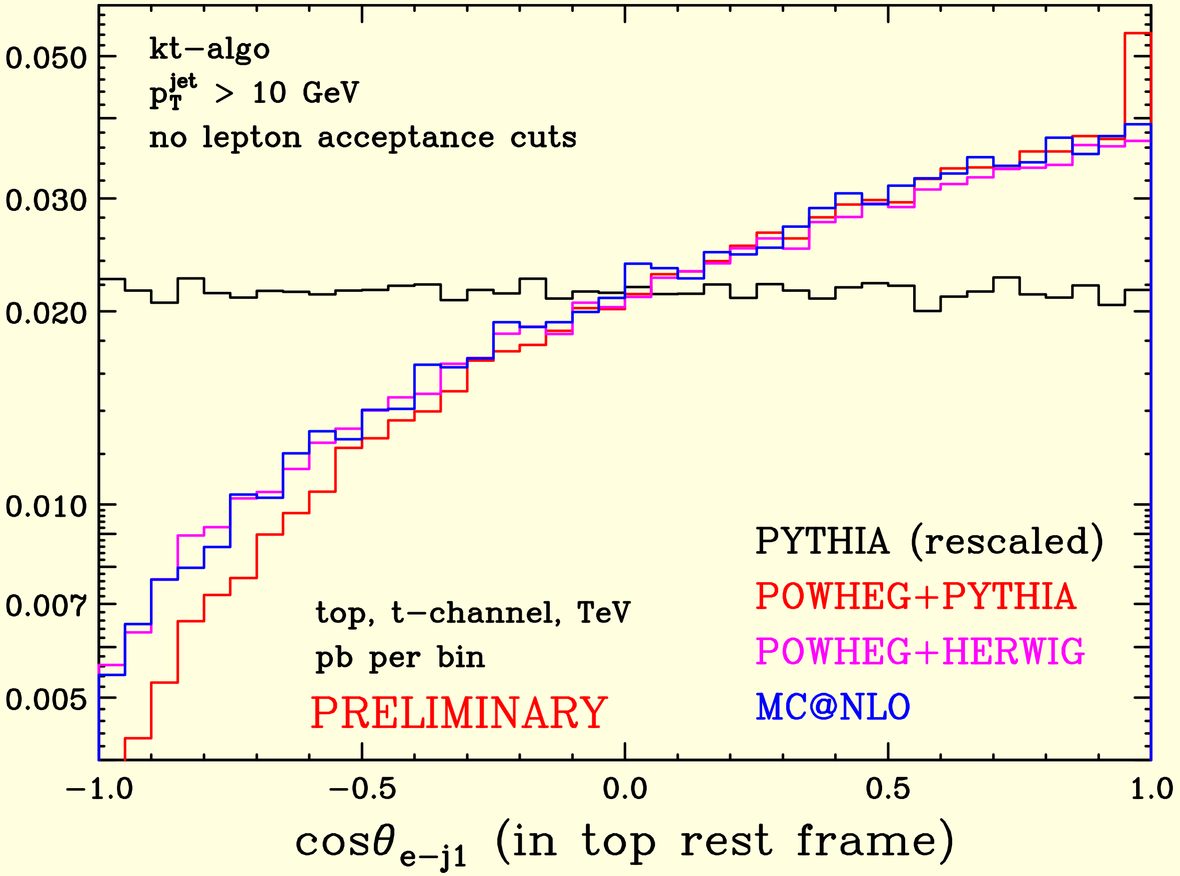
NLO
POWHEG+HERWIG
MCatNLO
kt−algo
$p_T^{jet} > 10$ GeV
top, t−channel, TeV
pb per bin
PRELIMINARY
0.0200
0.0100
0.0050
0.0020
0.0010
0.0005
0.0002
0.0001
0
50
100
150
200
$p_T^{(j1,j2)}$ (GeV)



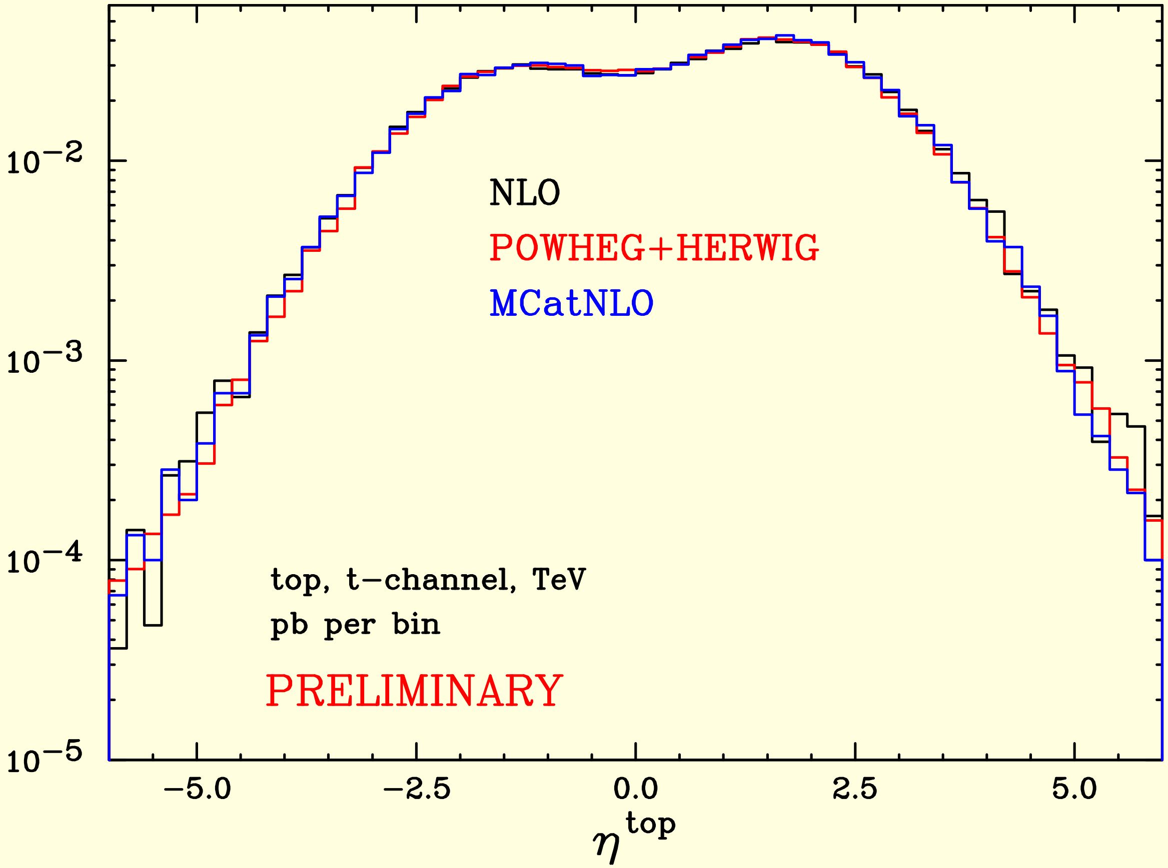
HERWIG (rescaled)
POWHEG+HERWIG
MCatNLO
top, t−channel, TeV
pb per bin
PRELIMINARY
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0
50
100
150
200
p_T^B (GeV)



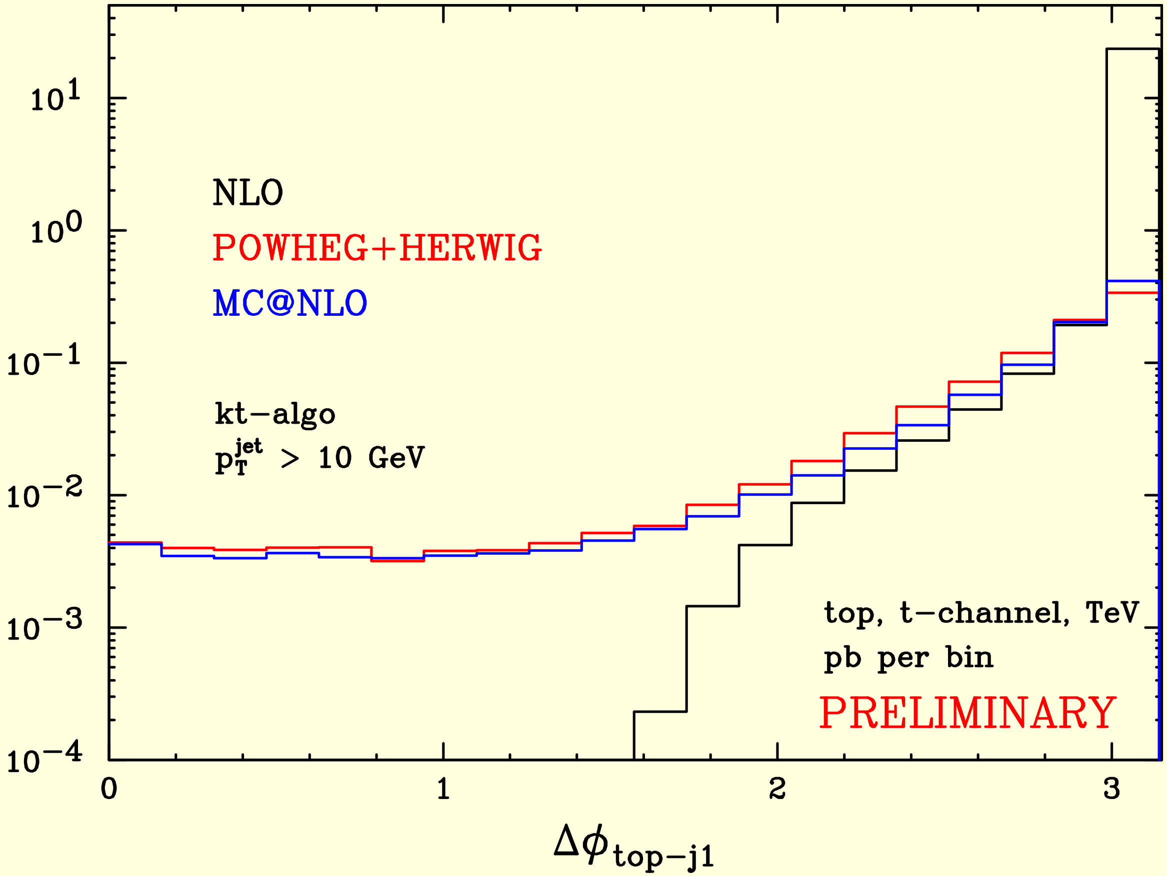
NLO
POWHEG+HERWIG
MC@NLO
kt−algo
p_T^{jet} > 10 GeV
top, t−channel, TeV
pb per bin
PRELIMINARY
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0
1
2
3
$\Delta\phi_{top-j1}$



kt−algo
p_T^{jet} > 10 GeV
no lepton acceptance cuts
0.050
0.030
0.020
0.010
0.007
0.005
PYTHIA (rescaled)
POWHEG+PYTHIA
POWHEG+HERWIG
MC@NLO
top, t−channel, TeV
pb per bin
PRELIMINARY
−1.0
−0.5
0.0
0.5
1.0
$\cos\theta_{e-j1}$ (in top rest frame)



10^{-2}
10^{-3}
10^{-4}
10^{-5}
NLO
POWHEG+HERWIG
MCatNLO
top, t−channel, TeV
pb per bin
PRELIMINARY
−5.0
−2.5
0.0
2.5
5.0
η^{top}



NLO
POWHEG+HERWIG
MC@NLO
kt−algo
p_T^{jet} > 10 GeV
top, t−channel, TeV
pb per bin
PRELIMINARY
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0
1
2
3
$\Delta\phi_{top-j1}$

Double logs and angular ordered Showers

We have discussed MC+NLO assuming that the hardest radiation is the first one. This is the case only in dipole shower programs (`ARIADNE`, newer `PYTHIA` versions). In virtuality ordered (old `PYTHIA`) or angular ordered showers the hardest event may not be the first.

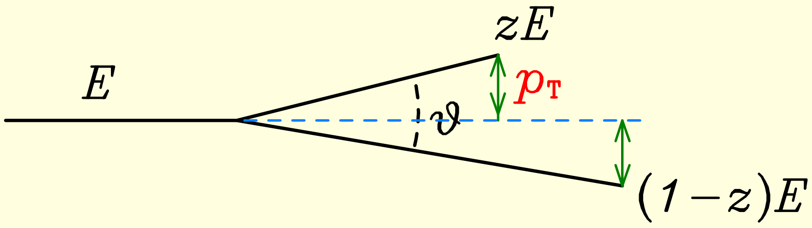
Accuracy of SMC's

Soft divergences and double log region

$z \to 1$ $(z \to 0)$ region problematic: for $z \to 1$: $P_{qq}, P_{gg} \propto \frac{1}{1-z}$

Choice of hardness variable makes a difference

$$\begin{aligned} \text{virtuality:} \quad t &\equiv E^2 z(1-z) \overbrace{\theta^2}^{1-\cos\theta} \\ p_T^2: \quad t &\equiv E^2 z^2 (1-z)^2\, \theta^2 \\ \text{angle:} \quad t &\equiv E^2\, \theta^2 \end{aligned}$$

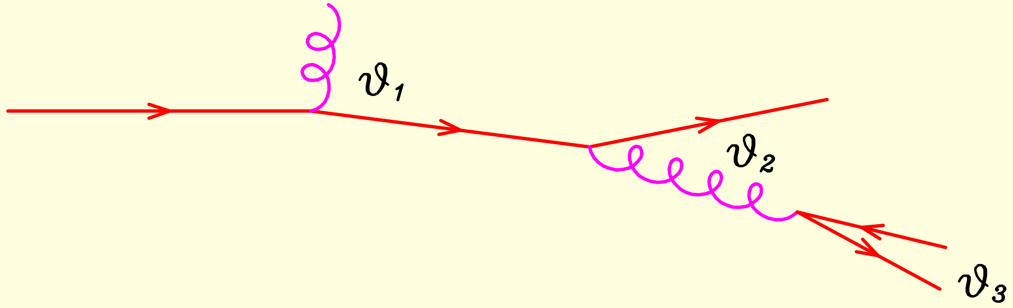


$$\underbrace{\int \frac{dt}{t} \int_{\sqrt{t}/E}^{1-\sqrt{t}/E} \frac{dz}{1-z}}_{\text{virtuality}: z(1-z) > t/E^2} \approx \frac{\log^2 \frac{t}{E^2}}{4}; \underbrace{\int \frac{dt}{t} \int_{t/E^2}^{1-t/E^2} \frac{dz}{1-z}}_{p_T^2: z^2(1-z)^2 > t/E} \approx \frac{\log^2 \frac{t}{E^2}}{2}; \underbrace{\int \frac{dt}{t} \int_0^1 \frac{dz}{1-z}}_{\text{angle}} \approx \log t \log \Lambda$$

Sizeable difference in double log structure!

Angular ordering is the correct choice (Mueller 1981)

$\frac{d\theta}{\theta}\frac{\alpha_s(p_T^2)}{2\pi}P(z)\mathrm{dz}$



$\theta_1 > \theta_2 > \theta_3 \ldots$

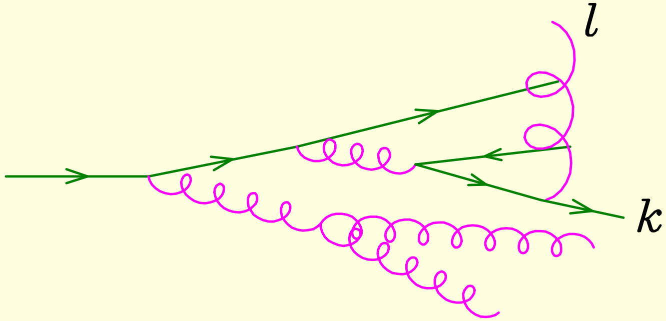
$p_T^2 = E^2 z^2 (1-z)^2\, \theta^2$

$\alpha_s(p_T^2)$ for a correct treatment of charge renormalization in soft region.

$$\Delta_i(t,t') = \exp\left[-\int_{t'}^{t}\frac{dt}{t}\int_{\sqrt{\frac{t_0}{t}}}^{1-\sqrt{\frac{t_0}{t}}} dz\frac{\alpha_s(p_T^2)}{2\pi}\sum_{(jk)} P_{i,jk}(z)\right]$$

$$\approx \exp\left[-\frac{c_i}{4\pi b_0}\left\{\log\frac{t}{\Lambda^2}\log\frac{\log\frac{t}{\Lambda^2}}{\log\frac{t_0}{\Lambda^2}} - \log\frac{t}{t_0}\right\}_{t'}^{t}\right] \qquad (c_q = C_F, c_g = 2C_A)$$

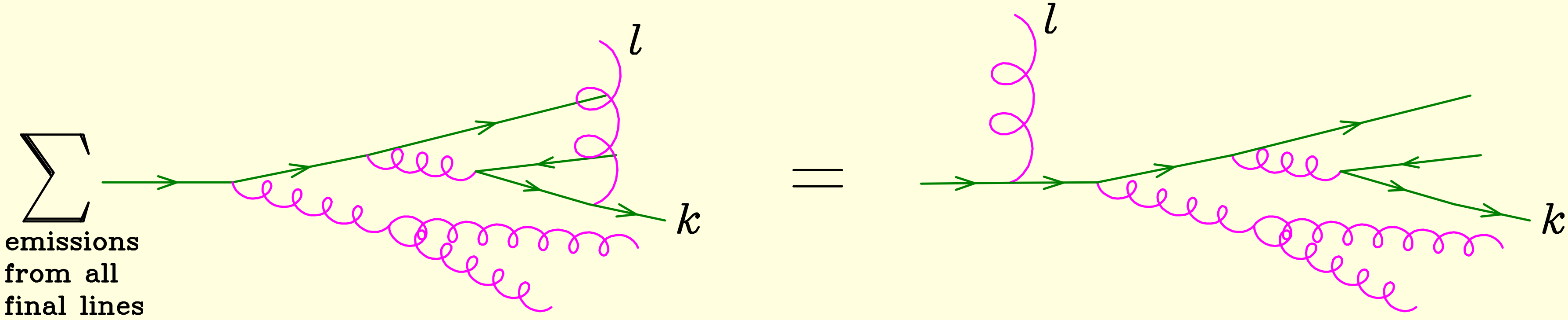
Sudakov damping stronger than any power of t.



With virtuality ordering:

Soft emissions give small virtuality.
At end of shower, large amount of unrestricted (all angles) soft radiation

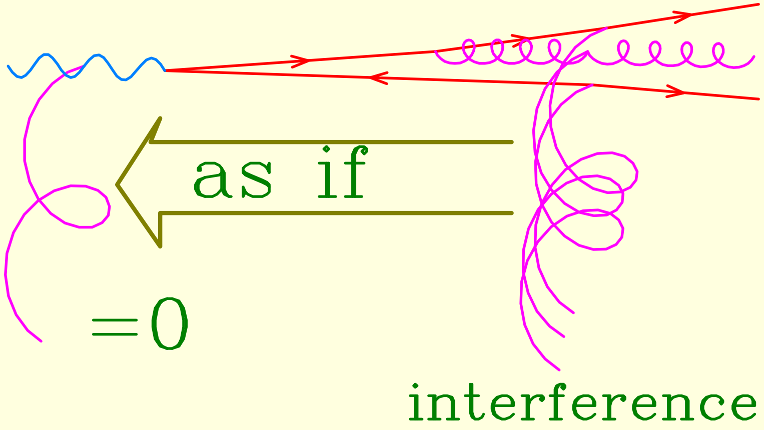
But soft gluons emitted at large angles from final state partons add coherently!



large angle, high energy: already ordered in angle
large angle, small energy: should be reordered by angle;

Thus: order in angle

Look at the example:



Angular ordering accounts

for soft gluon interference.

Intensity for photon jets $= 0$

Intensity for gluon jets $= C_A$

instead of $2C_F + C_A$

Consistent with a boosted jet pair, in the case of a photon jet.
In angular ordered SMC large angle soft emission is generated first.
Hardest emission (i.e. highest p_T) happens later.

Is it important?

- Eccessive multiplicity growth in virtuality ordered MC
- Angular ordered MC's (`HERWIG`) agree with mutliplicity data in e^+e^- annihilation
- Agreement of `PYTHIA` with multiplicity data was achieved by superimposing an angular ordered veto over the virtuality ordered shower. This amounts to take the interference as being totally distructive. No major differences between `PYTHIA` and `HERWIG` were seen because of this reason.

MC@NLO and HERWIG

In this case

$$d\sigma = \underbrace{\bar{B}^{\rm MC}(\Phi_B) d\Phi_B}_{S \text{ event}} \left[\underbrace{\Delta^{\rm MC}_{t_0} + \Delta^{\rm MC}_{t} \frac{R^{\rm MC}(\Phi)}{B(\Phi_B)} d\Phi^{\rm MC}_r}_{\rm MC\ shower} \right] + \left[\underbrace{R(\Phi) - R^{\rm MC}(\Phi)}_{H \text{ event}} \right] d\Phi .$$

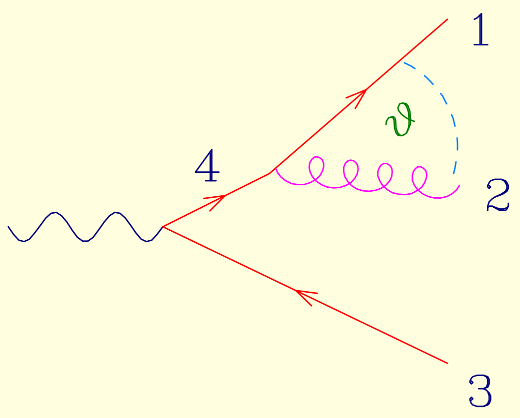
The S events should already be treated correctly by the MC; the net effect of the shower development in HERWIG is to generate the hardest radiation according to the above formula (P.N. 2004).
There are however $2 \to 2$ processes in HERWIG that may need a truncated shower to consistently treat colour connections.

The H event may need truncated shower, as any ME result interfaced to the angular ordered shower; however, being not singular, the region that needs the truncated shower is power suppressed by the phase space.

Interfacing POWHEG with angular ordered SMC's

- Generate event with harderst emission
- Generate all subsequent emissions with a p_T veto equal to the hardest emission p_T
- Pair up the partons that are nearest in p_T
- Generate an angular ordered shower associated with the paired parton, stopping at the angle of the paired partons:
  Truncated shower, (P.N., 2004)
- Generate all subsequent (vetoed) showers

Example of truncated shower: e^+e^-



Nearby partons: 1,2

Truncated shower: 1,2 pair,

from maximum angle to θ

1 and 2 shower from θ to cutoff

3 showers from maximum to cutoff

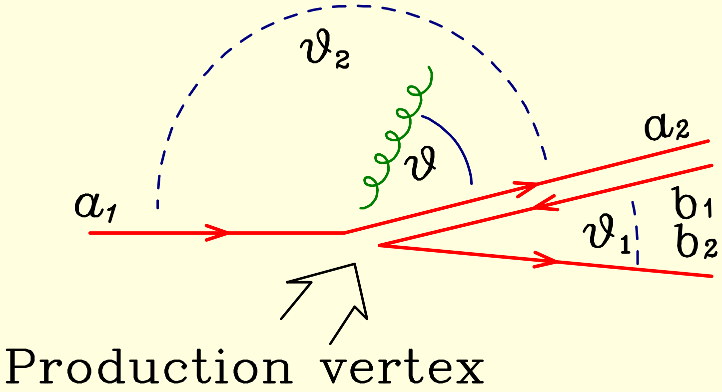
The truncated shower reintroduces coherent soft radiation from 1,2 at angles larger than θ (Angular ordered SMC's generate those earlier).

Truncated shower are generally needed for interfacing ME calculations with angular ordered MC;
They are not a specific problem of `POWHEG`.
They are now implemented in `HERWIG++`

Issue of truncated showers

Truncated shower are generally needed in angular ordered SMC's

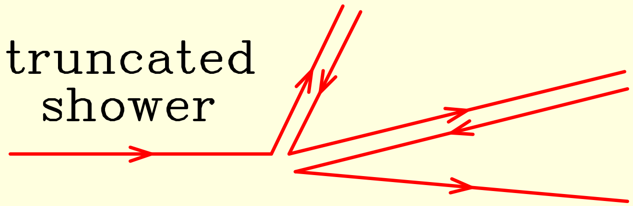
- Every time the shower is initiated by a relatively complex matrix element a truncated shower is needed
- CKKW mocks the effect of truncated shower with a trick (but it misses the correct colour flow)

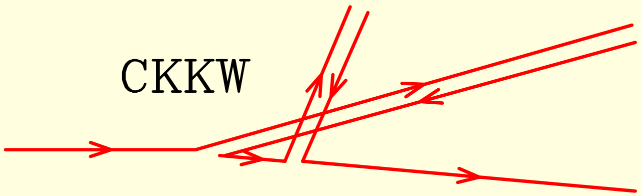


Consider $e^+e^- \to q\,\bar{q}\,g$.
Assume θ_1 small. Consider gluon emission with angle $\theta \gg \theta_1$, $\theta \ll \theta_2$.
Coherence requires that the emission strength is C_F (gluon and quark coherently)

In HERWIG: initial angle for gluon radiation is θ_1 or θ_2 with a 50% probability. Thus (in the above region) strength is $C_A/2 \approx C_F$ (but only in the average!!)

In CKKW: radiation from gluon restricted to $\theta < \theta_1$, quark radiates with angle up to θ_2. Thus only the quark radiates in the above region, with strength C_F. However, the colour connection is incorrect! Large colour gap in CKKW!



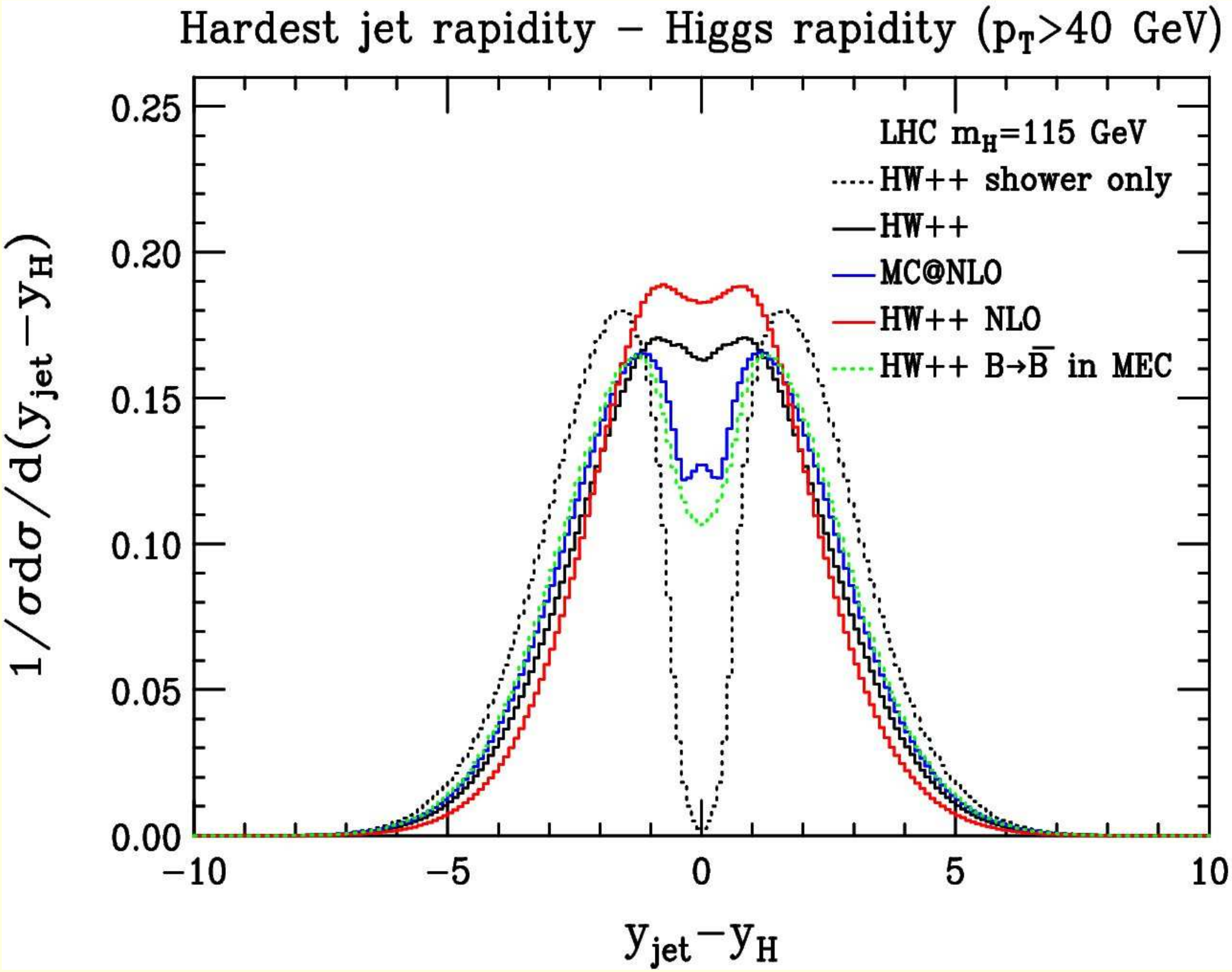


So: coherent showers are always needed when doing ME-Shower matching with angular ordered showers.

Implementation of truncated showers: HERWIG++

Method: once the POWHEG radiation is generated, if it does not fall in the HERWIG dead zone, find the born level kinematics and the HERWIG shower parameters θ_0, z_0 and ϕ_0 that would have generated the same kinematic configuration. Then run the shower on that Born kinematics, veto on its p_T, stop it when an angle greater than θ_0 is generated. At this stage, replace the splitting variables with θ_0, z, ϕ.

Very small dip in rapidity observed for LHC energies; is it related to the different behaviour adopted in the dead region?


Hardest jet rapidity – Higgs rapidity (pT>40 GeV)
LHC mH=115 GeV
HW++ shower only
HW++
MC@NLO
HW++ NLO
HW++ B→B̄ in MEC
1/σdσ/d(yjet−yH)
yjet−yH
0.25
0.20
0.15
0.10
0.05
0.00
−10
−5
0
5
10

Towards automation: the POWHEG BOX

The MIB (Milano-Bicocca) group (Alioli, Oleari, Re, P.N.) is working on an automatic implementation of POWHEG for generic NLO processes.

This framework is being tested in the process $hh \to Z + 1\mathrm{jet}$.

The POWHEG BOX

Build a computer code framework, such that, given the Born cross section, the finite part of the virtual corrections, and the real graph cross section, one builds immediately a POWHEG generator. More precisely, the user must supply:

- The Born phase space
- The lists of Born and Real processes (i.e. $u\,\bar{s} \to W^+ c\,\bar{c}$, etc.)
- The Born squared amplitudes $\mathcal{B} = |\mathcal{M}|^2$, $\mathcal{B}_{ij}$, $\mathcal{B}_{j,\mu_j,\mu'_j}$, for all relevant partonic processes; $\mathcal{B}_{ij}$ is the colour ordered Born amplitude squared, $\mathcal{B}_{j,\mu\nu}$ is the spin correlated amplitude, where j runs over all external gluons in the amplitude. All these amplitudes are common ingredient of an NLO calculation.
- The Real squared amplitude, for all relevant partonic processes. This may also be obtained by interfacing the program to MADGRAPH.
- The finite part of the virtual amplitude contribution, for all relevant partonic processes.

Strategy

Use the FKS framework according to the general formulation of POWHEG given in (Frixione, Oleari, P.N. 2007), hiding all FKS implementation details.
In other words, we use FKS, but the user needs not to understand it.
(Attempts to use the popular Catani-Seymour method
have turned out to be too cumbersome).
It includes:

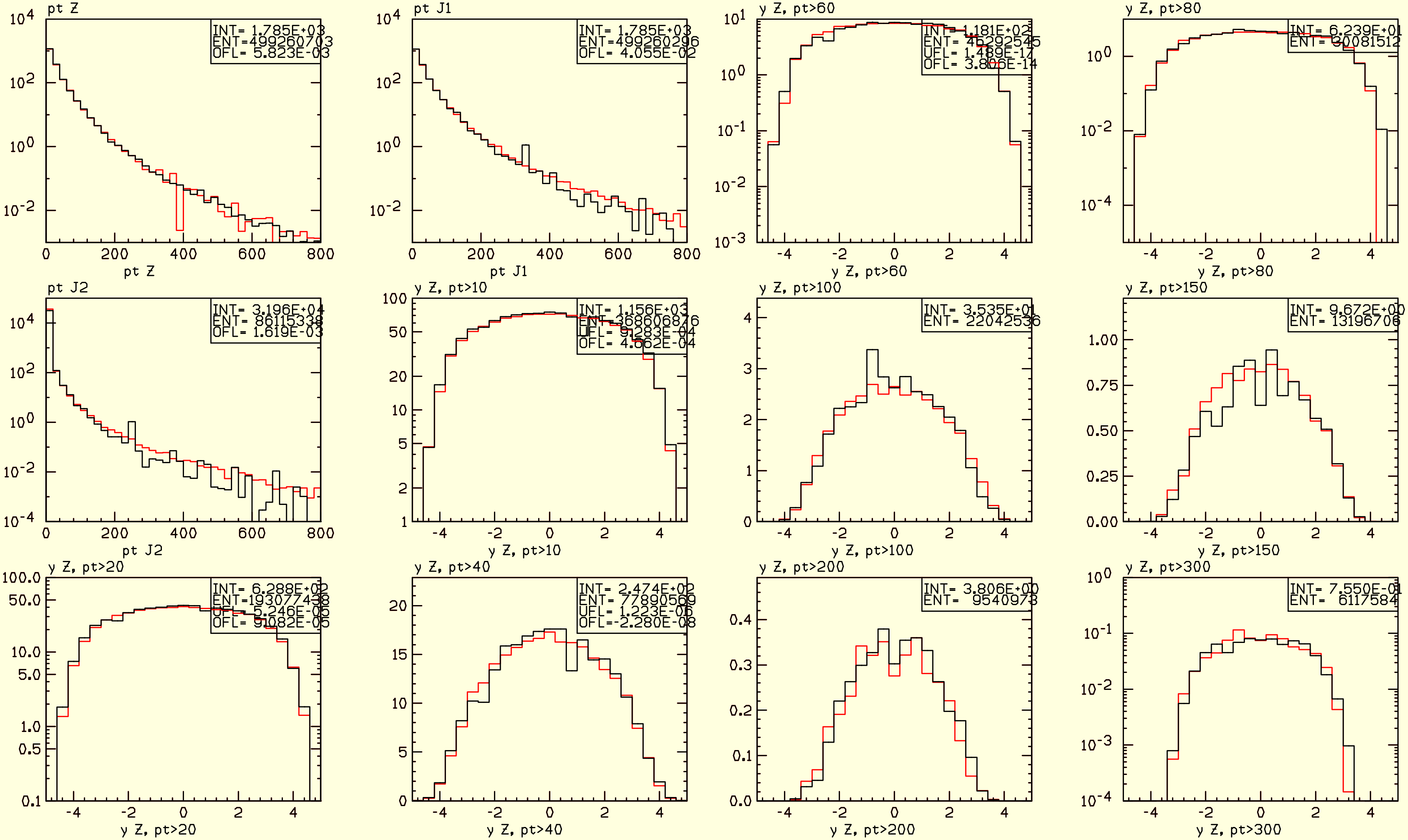
- The phase space for ISR and FSR, according to FNO2006.
- The combinatorics, the calculation of all R_α, the soft and coll. limits
- The calculation of $\tilde{B}$
- The calculation of the upper bounds for the generation of radiation
- The generation of radiation
- Writing the event to the Les Houches interface

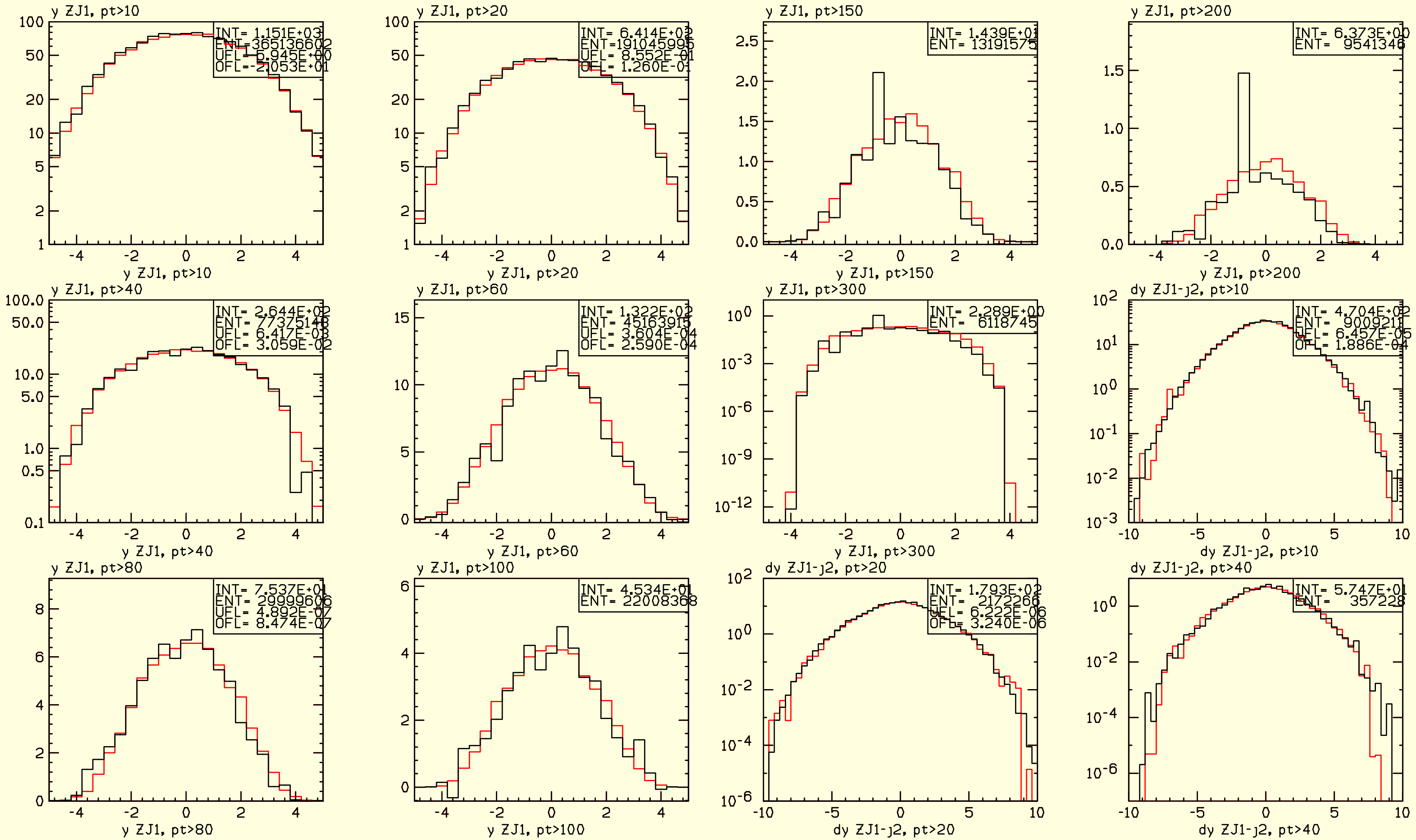
It works! Lots of testing needed now ...
Byproduct: generic NLO implementation using the FKS method

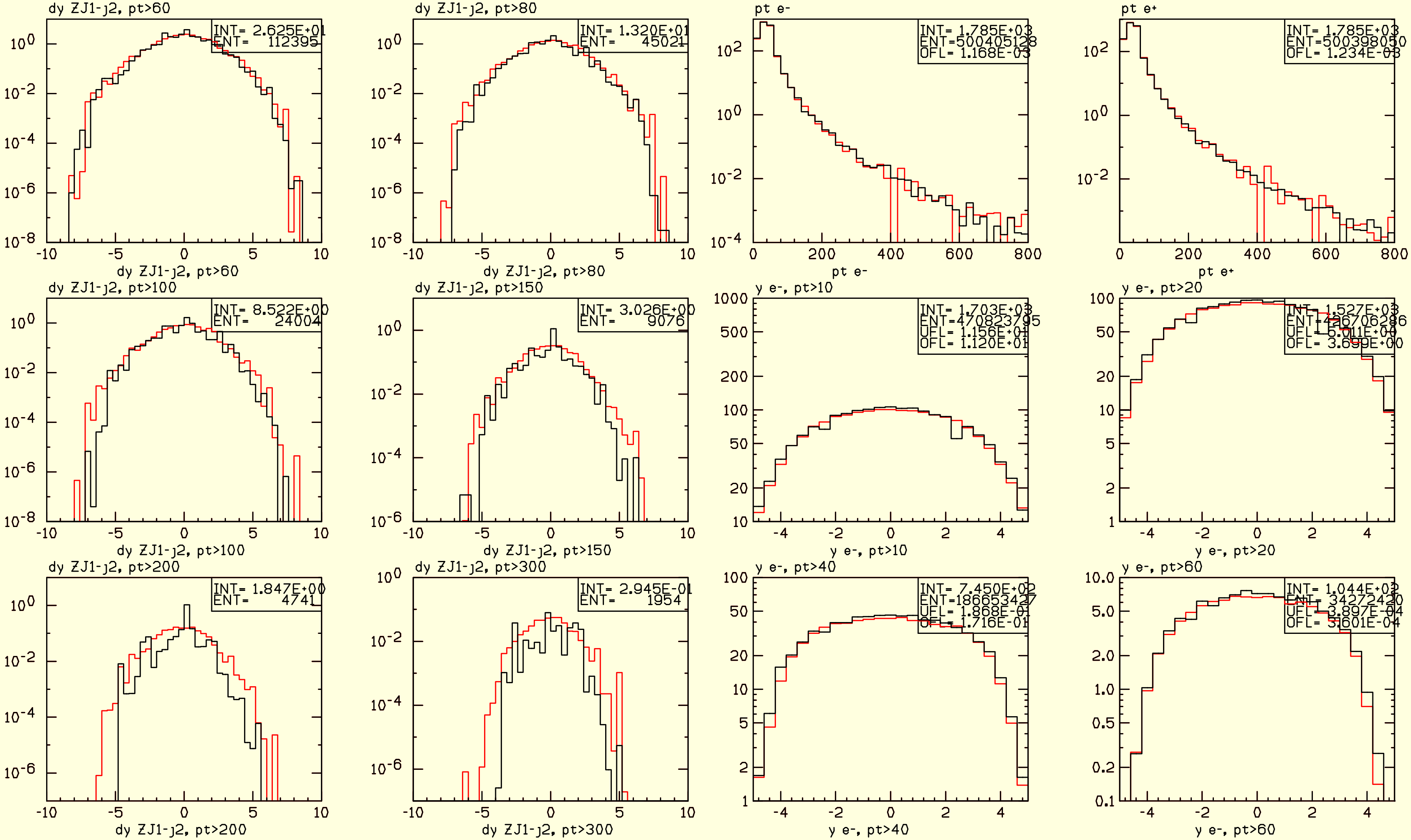
Case study: $Z + \mathrm{jet}$ production

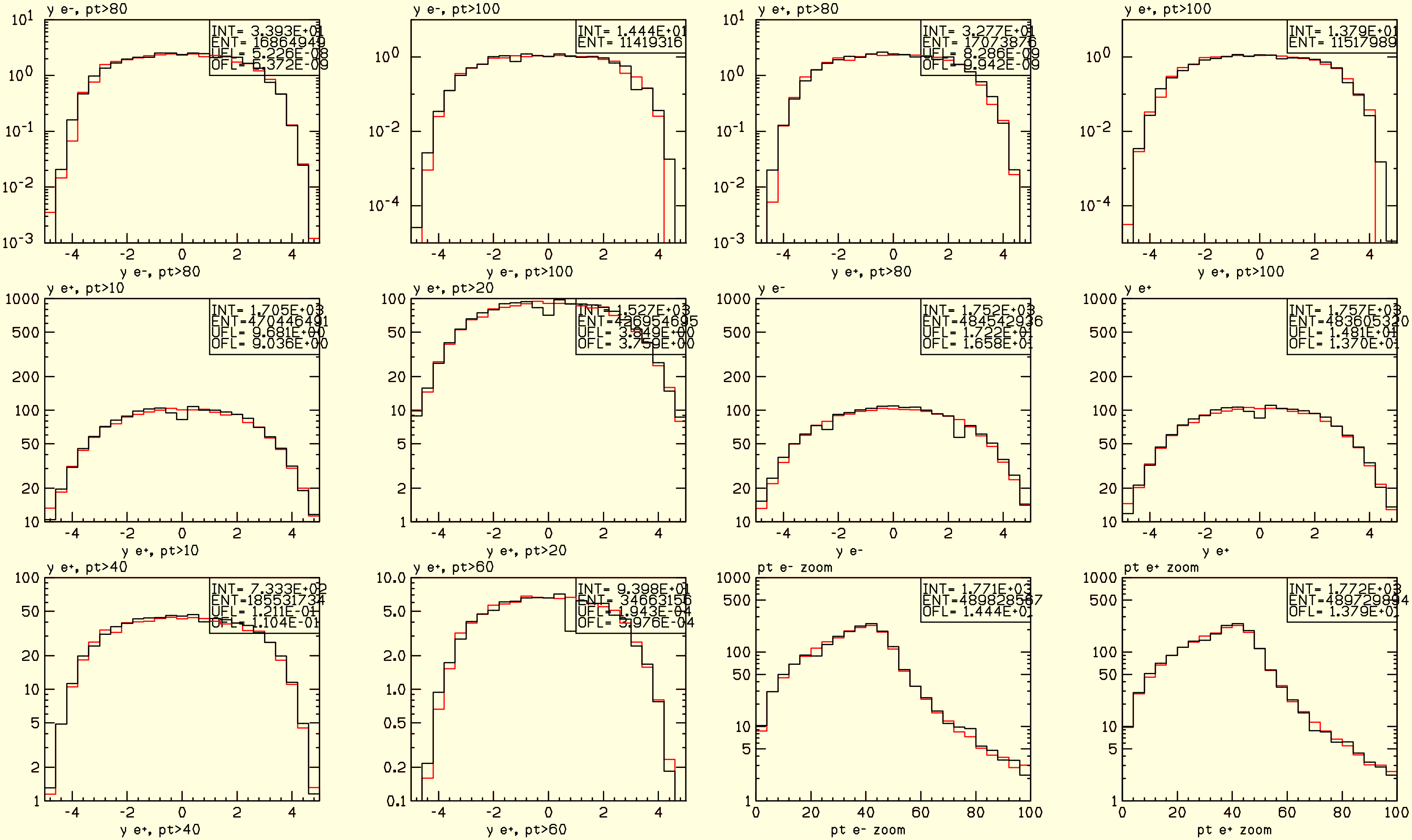
Get virtual matrix elements from `MCFM`;
Compare first NLO predictions obtained with `MCFM` and the `POWHEG BOX`

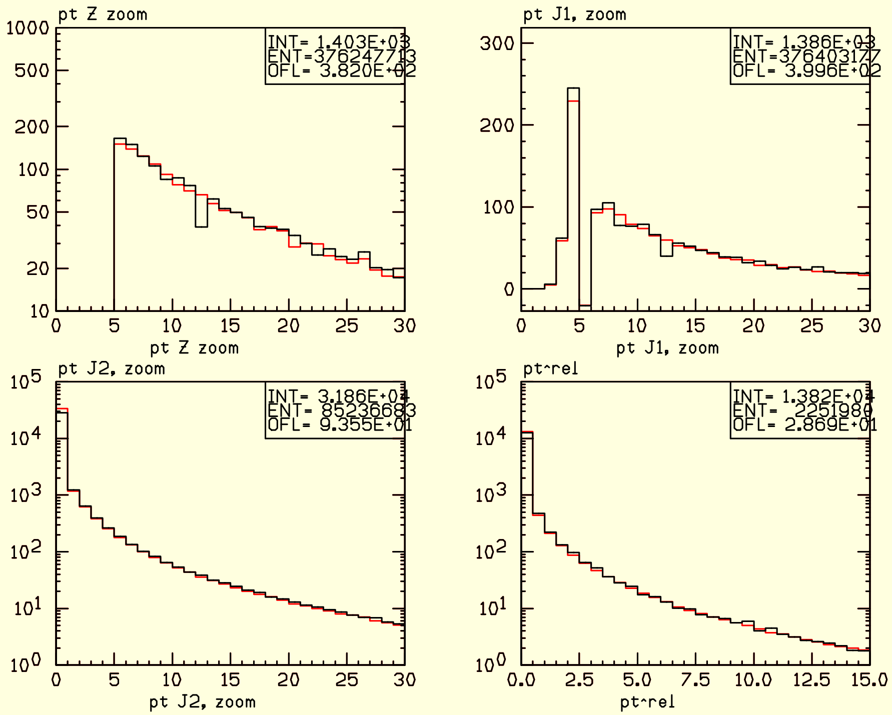
Virtual corrections are the same, but subtraction terms, soft and collinear remnants are all different; non trivial test of setup;


pt Z
INT= 1.785E+03
ENT=499260703
OFL= 5.823E-03
pt J1
INT= 1.785E+03
ENT=499260296
OFL= 4.055E-02
y Z, pt>60
INT= 1.181E+02
y Z, pt>80
INT= 6.239E+01
pt J2
INT= 3.196E+04
ENT= 86115338
OFL= 1.619E-03
y Z, pt>10
INT= 1.156E+03
OFL= 4.662E-04
y Z, pt>100
INT= 3.535E+01
ENT= 22042536
y Z, pt>150
INT= 9.672E+00
ENT= 13196706
y Z, pt>20
INT= 6.288E+02
ENT=193077438
y Z, pt>40
INT= 2.474E+02
ENT= 77890569
UFL= 1.223E-06
OFL=-2.280E-08
y Z, pt>200
INT= 3.806E+00
ENT= 9540973
y Z, pt>300
INT= 7.550E-01
ENT= 6117584


y ZJ1, pt>10
INT= 1.151E+03
ENT=365136602
UFL= 6.945E+00
OFL= 2.053E+01
y ZJ1, pt>20
INT= 6.414E+02
ENT=191045995
UFL= 8.552E-01
OFL= 1.260E-01
y ZJ1, pt>150
INT= 1.439E+01
ENT= 13191575
y ZJ1, pt>200
INT= 6.373E+00
ENT= 9541346
y ZJ1, pt>40
INT= 2.644E+02
ENT= 77375148
UFL= 6.417E-03
OFL= 3.059E-02
y ZJ1, pt>60
INT= 1.322E+02
ENT= 45163915
UFL= 3.604E-04
OFL= 2.590E-04
y ZJ1, pt>300
INT= 2.289E+00
ENT= 6118745
dy ZJ1-j2, pt>10
INT= 4.704E+02
ENT= 9009211
UFL= 6.457E-05
OFL= 1.886E-04
y ZJ1, pt>80
INT= 7.537E+01
ENT= 29999606
UFL= 4.892E-07
OFL= 8.474E-07
y ZJ1, pt>100
INT= 4.534E+01
ENT= 22008368
dy ZJ1-j2, pt>20
INT= 1.793E+02
ENT= 2172266
UFL= 6.222E-06
OFL= 3.240E-06
dy ZJ1-j2, pt>40
INT= 5.747E+01
ENT= 357228

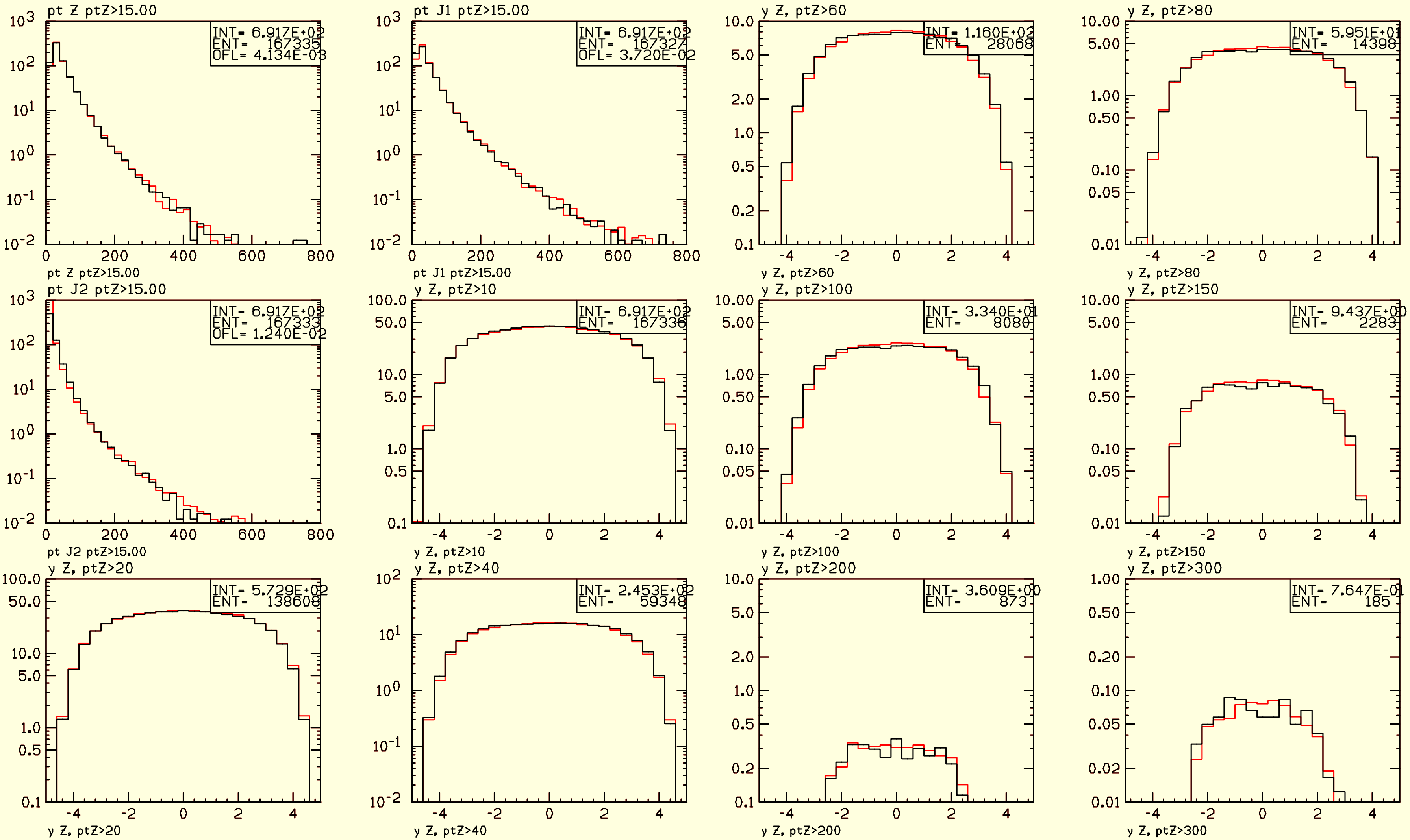

dy ZJ1-j2, pt>60
INT= 2.625E+01
ENT= 112395
dy ZJ1-j2, pt>80
INT= 1.320E+01
ENT= 45021
pt e-
INT= 1.785E+03
ENT=500405128
OFL= 1.168E-03
pt e+
INT= 1.785E+03
ENT=500398050
OFL= 1.234E-03
dy ZJ1-j2, pt>100
INT= 8.522E+00
ENT= 24004
dy ZJ1-j2, pt>150
INT= 3.026E+00
ENT= 9076
y e-, pt>10
INT= 1.703E+03
ENT=470823795
UFL= 1.156E+01
OFL= 1.120E+01
y e-, pt>20
INT= 1.527E+03
ENT=436706286
UFL= 5.011E+00
OFL= 3.699E+00
dy ZJ1-j2, pt>200
INT= 1.847E+00
ENT= 4741
dy ZJ1-j2, pt>300
INT= 2.945E-01
ENT= 1954
y e-, pt>40
INT= 7.450E+02
ENT=186653427
UFL= 1.868E-01
OFL= 1.716E-01
y e-, pt>60
INT= 1.044E+02
ENT= 34272420
UFL= 3.897E-04
OFL= 3.601E-04

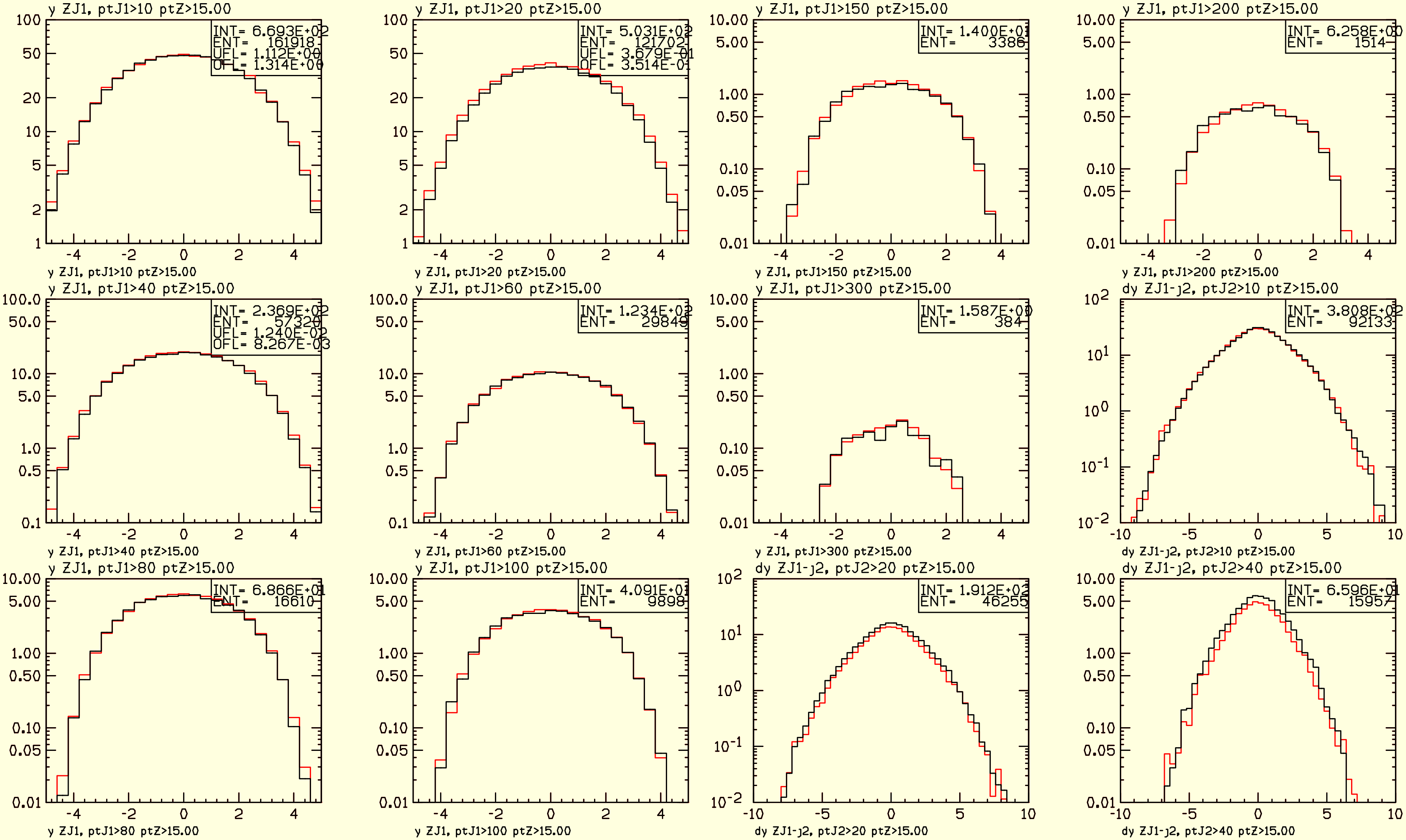

y e-, pt>80
INT= 3.393E+01
ENT= 16864949
y e-, pt>100
INT= 1.444E+01
ENT= 11419316
y e+, pt>80
INT= 3.277E+01
ENT= 17073876
y e+, pt>100
INT= 1.379E+01
ENT= 11517989
y e+, pt>10
INT= 1.705E+03
ENT=470446491
UFL= 9.681E+00
OFL= 9.036E+00
y e+, pt>20
INT= 1.527E+03
y e-
INT= 1.752E+03
ENT=484542936
UFL= 1.722E+01
OFL= 1.658E+01
y e+
INT= 1.757E+03
ENT=483605320
UFL= 1.481E+01
OFL= 1.370E+01
y e+, pt>40
INT= 7.333E+02
ENT=185531734
UFL= 1.211E-01
y e+, pt>60
INT= 9.398E+01
ENT= 34663156
pt e- zoom
INT= 1.771E+03
ENT=489828567
OFL= 1.444E+01
pt e+ zoom
INT= 1.772E+03
ENT=489729894
OFL= 1.379E+01

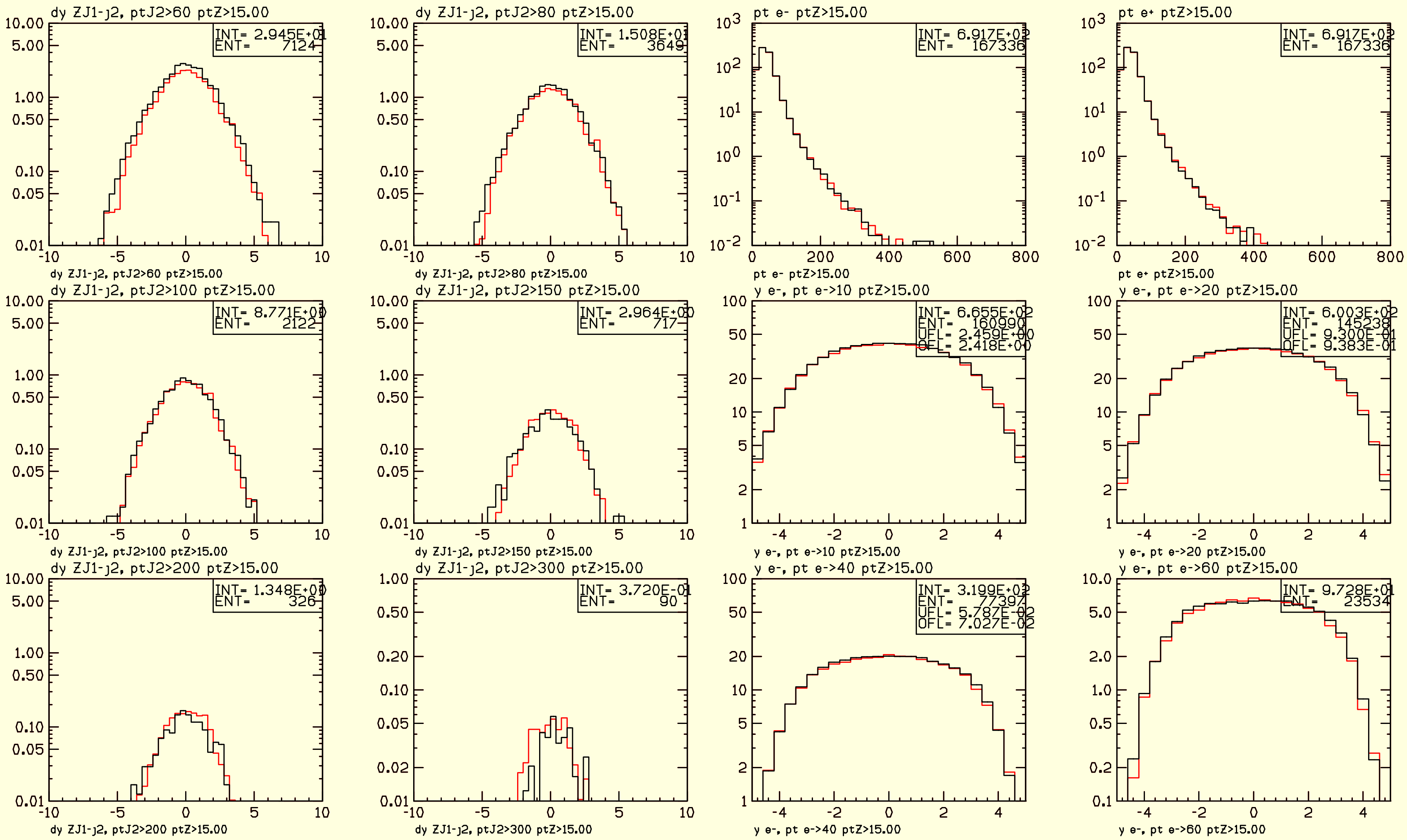


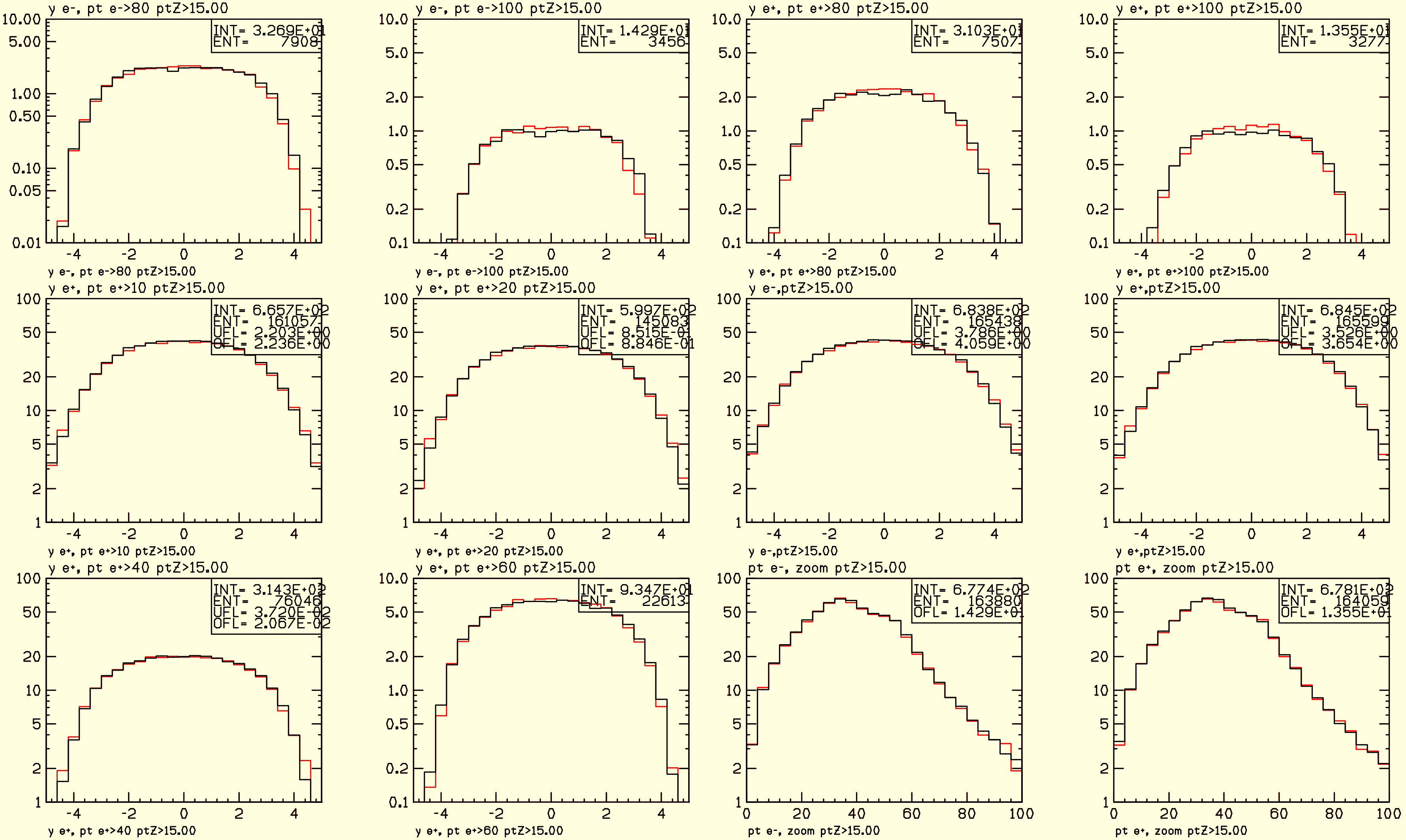
Everything seems to work ...

Now compare POWHEG+HERWIG with NLO (red)

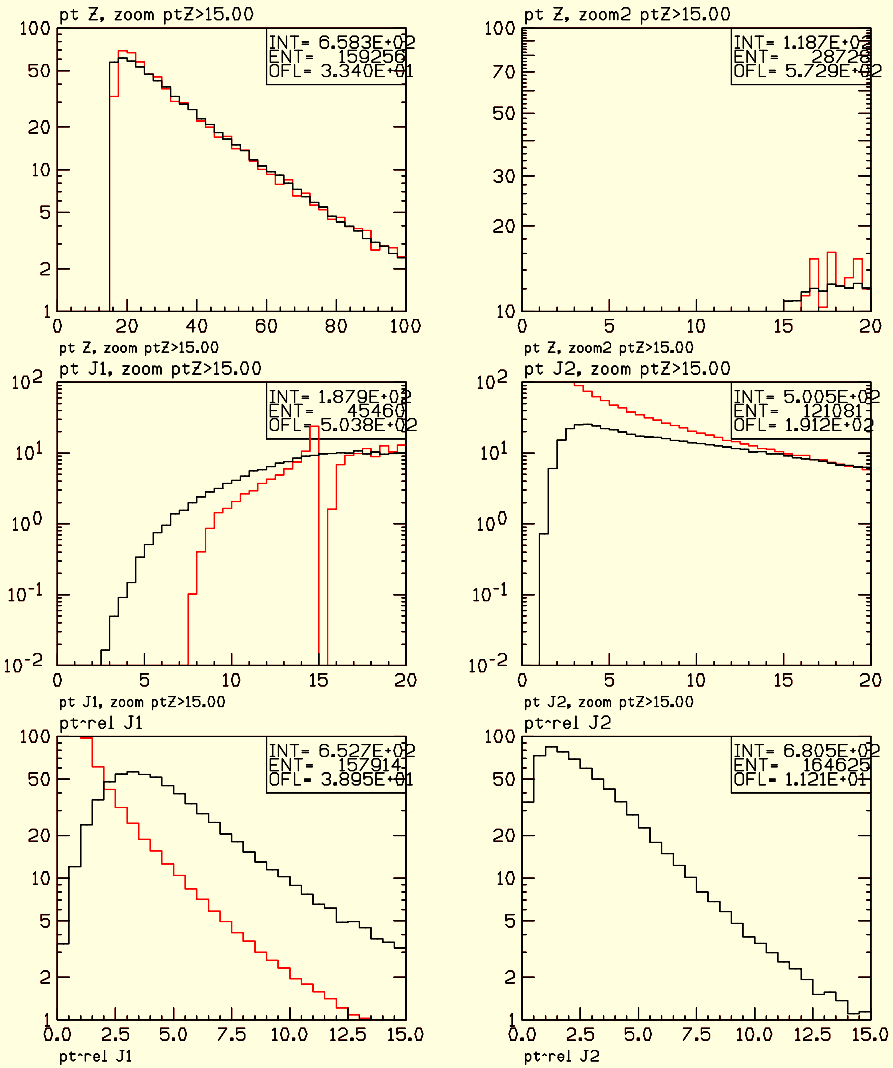



y ZJ1, ptJ1>10 ptZ>15.00
INT= 6.693E+02
ENT= 161918
UFL= 1.112E+00
OFL= 1.314E+00
y ZJ1, ptJ1>20 ptZ>15.00
INT= 5.031E+02
ENT= 121702
UFL= 3.679E-01
OFL= 3.514E-01
y ZJ1, ptJ1>150 ptZ>15.00
INT= 1.400E+01
ENT= 3386
y ZJ1, ptJ1>200 ptZ>15.00
INT= 6.258E+00
ENT= 1514
y ZJ1, ptJ1>40 ptZ>15.00
INT= 2.369E+02
ENT= 57320
UFL= 1.240E-02
OFL= 8.267E-03
y ZJ1, ptJ1>60 ptZ>15.00
INT= 1.234E+02
ENT= 29849
y ZJ1, ptJ1>300 ptZ>15.00
INT= 1.587E+00
ENT= 384
dy ZJ1-j2, ptJ2>10 ptZ>15.00
INT= 3.808E+02
ENT= 92133
y ZJ1, ptJ1>80 ptZ>15.00
INT= 6.866E+01
ENT= 16610
y ZJ1, ptJ1>100 ptZ>15.00
INT= 4.091E+01
ENT= 9898
dy ZJ1-j2, ptJ2>20 ptZ>15.00
INT= 1.912E+02
ENT= 46255
dy ZJ1-j2, ptJ2>40 ptZ>15.00
INT= 6.596E+01
ENT= 15957


dy ZJ1-j2, ptJ2>60 ptZ>15.00
INT= 2.945E+01
ENT= 7124
dy ZJ1-j2, ptJ2>80 ptZ>15.00
INT= 1.508E+01
ENT= 3649
pt e- ptZ>15.00
INT= 6.917E+02
ENT= 167336
pt e+ ptZ>15.00
INT= 6.917E+02
ENT= 167336
dy ZJ1-j2, ptJ2>100 ptZ>15.00
INT= 8.771E+00
ENT= 2122
dy ZJ1-j2, ptJ2>150 ptZ>15.00
INT= 2.964E+00
ENT= 717
y e-, pt e->10 ptZ>15.00
INT= 6.655E+02
ENT= 160990
UFL= 2.459E+00
OFL= 2.418E+00
y e-, pt e->20 ptZ>15.00
INT= 6.003E+02
ENT= 145238
UFL= 9.300E-01
OFL= 9.383E-01
dy ZJ1-j2, ptJ2>200 ptZ>15.00
INT= 1.348E+00
ENT= 326
dy ZJ1-j2, ptJ2>300 ptZ>15.00
INT= 3.720E-01
ENT= 90
y e-, pt e->40 ptZ>15.00
INT= 3.199E+02
ENT= 77397
UFL= 5.787E-02
OFL= 7.027E-02
y e-, pt e->60 ptZ>15.00
INT= 9.728E+01
ENT= 23534


y e-, pt e->80 ptZ>15.00
INT= 3.269E+01
ENT= 7908
y e-, pt e->100 ptZ>15.00
INT= 1.429E+01
ENT= 3456
y e+, pt e+>80 ptZ>15.00
INT= 3.103E+01
ENT= 7507
y e+, pt e+>100 ptZ>15.00
INT= 1.355E+01
ENT= 3277
y e+, pt e+>10 ptZ>15.00
INT= 6.657E+02
ENT= 161057
UFL= 2.203E+00
OFL= 2.236E+00
y e+, pt e+>20 ptZ>15.00
INT= 5.997E+02
ENT= 145083
UFL= 8.515E-01
OFL= 8.846E-01
y e-,ptZ>15.00
INT= 6.838E+02
ENT= 165438
UFL= 3.786E+00
OFL= 4.059E+00
y e+,ptZ>15.00
INT= 6.845E+02
ENT= 165599
UFL= 3.526E+00
OFL= 3.654E+00
y e+, pt e+>40 ptZ>15.00
INT= 3.143E+02
ENT= 76046
UFL= 3.720E-02
OFL= 2.067E-02
y e+, pt e+>60 ptZ>15.00
INT= 9.347E+01
ENT= 22613
pt e-, zoom ptZ>15.00
INT= 6.774E+02
ENT= 163880
OFL= 1.429E+01
pt e+, zoom ptZ>15.00
INT= 6.781E+02
ENT= 164059
OFL= 1.355E+01

Distributions sensitive to more than two jet show noticeably different. All others in agreement with NLO



Conclusions

- NLO accuracy with Shower MC has become a reality in recent years.
- Two methods have provided usable implementations for collider physics `MC@NLO` and `POWHEG`.
- Agreement and differences among the two method are relatively well understood
- A path to full automation of POWHEG implementations of arbitrary NLO calculation is open
- Even so, we are just at the beginning: many interesting problems remain to be addressed, and it is likely that further theoretical progress is still possible in this framework

Caveats in POWHEG

Born zeros

- Singularities in B
- Zeros in B

Both cause problems, but they are easily fixed.
For example, zeros in B: further separate

$$R_{\alpha_r} = \frac{k_T^2}{k_T^2 + B} R_{\alpha_r} + \frac{B}{k_T^2 + B} R_{\alpha_r}$$

The first term in non-singular (can be generated directly without Sudakov), while in the second term the zero in B cancels in the Sudakov exponent.

Accuracy of the Sudakov Form Factor

POWHEG's Sudakov FF has the form (with $c \approx 1$)

$$\Delta_t = \exp\left[-\int_t^{Q^2} \frac{dk_T^2}{k_T^2}\frac{\alpha_S(c\,k_T^2)}{\pi}\left\{A\log\frac{M^2}{k_T^2} + B\right\}\right]$$

We know that the NLL Sudakov form factor has the form

$$\Delta_t^{\mathrm{NLL}} = \exp\left[-\int_t^{Q^2} \frac{dk_T^2}{k_T^2}\frac{\alpha_S(k_T^2)}{\pi}\left\{\left(A_1 + A_2\frac{\alpha_s(k_T^2)}{\pi}\right)\log\frac{M^2}{k_T^2} + B\right\}\right]$$

provided the colour structure of the process is sufficiently simple
($\leqslant 3$ coloured legs). Can use this to fix c in POWHEG's Sudakov FF.
(Suggested in (Catani, Webber, Marchesini, 1991) for HERWIG)
$\geqslant 4$ coloured legs: exponentiation only holds in LL,
or LL + (NLL large N_c) if planar colour structures are suitably separated
Summarizing:
POWHEG Sudakov is: always LL accurate,
NLL accurate for $\leqslant 3$ coloured legs, NLL accurate in leading N_c in all cases.