

Light charged Higgs boson scenario in 3HMDs

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Based on A. Akeroyd, S. Moretti, KY, E. Yildirim, arXiv: 1605.05881

KY, arXiv: 1609.04590

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1 doublet is confirmed, then?

- Higgs discovery in $h \rightarrow \tau \tau$: Existence an $SU(2)_L$ doublet scalar.

$$y \overline{L}_L (\Phi) \tau_R$$

$$SU(2)_L \times U(1)_Y \sim (\mathbf{2}, 1/2) \quad (\mathbf{1}, -1)$$

The unique solution is $\Phi \sim (\mathbf{2}, 1/2)$

- Question: How many doublets are there?

$$\Phi_1, \Phi_2, \dots, \Phi_N??$$

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- Question: How many doublets are there?

$$\Phi_1, \Phi_2, \dots, \Phi_N ??$$

- MHDMs automatically satisfy $\rho_{\text{tree}} = 1$.
- MHDMs can reproduce predictions in the minimal Higgs sector.
- MHDMs appear in many new physics beyond the SM.

New Physics & Multi-Doublet Models

New Physics

□ SUSY

□ Composite Higgs models

□ Extended EWGS

□ BSM phenomena

(Neutrino masses, DM,

Baryon asymmetry, Muon $g-2$, ...)



Higgs Sector

At least 2 doublets

Depends on a coset space

ex, $SO(6)/SO(4) \times SO(2) \rightarrow 2HDM$

Multi-doublet structure appears in a low energy effective theory.

Additional doublets are often introduced.

New Physics & Multi-Doublet Models

New Physics

- SUSY
- Composite Higgs models
- Extended EWGS

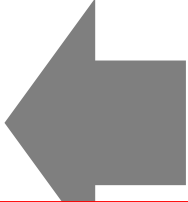
Higgs Sector

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Depends on a coset space

ex, $SO(6)/SO(4) \times SO(2) \rightarrow 2\text{HDM}$

Multi-doublet structure appears in



In this talk, we discuss the phenomenology of **charged Higgs bosons** and its difference between in 2HDMs and 3HDMs.

(Baryon asymmetry, Muon $g-2$, ...)

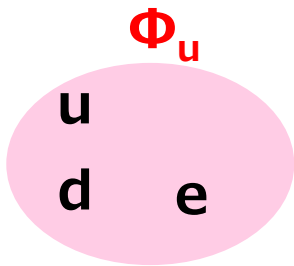
Type of Yukawa interaction in NFC

Glashow, Weinberg (1977); Grossman (1994)

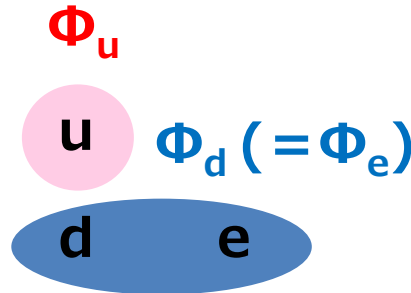
- Simple way to **naturally** avoid the tree level FCNCs is to consider the Lagrangian.

$$-\mathcal{L}_Y = Y_u \bar{Q}_L (i\sigma_2) \Phi_u^* u_R + Y_d \bar{Q}_L \Phi_d d_R + Y_e \bar{L}_L \Phi_e e_R + \text{h.c.}$$

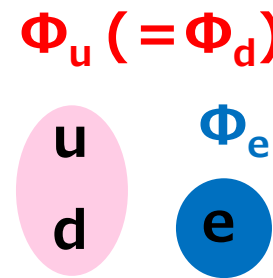
Type-I



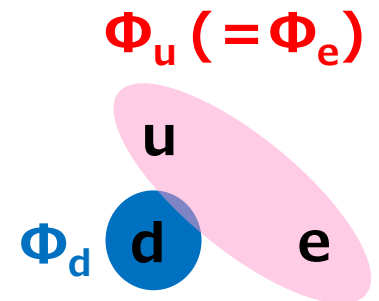
Type-II



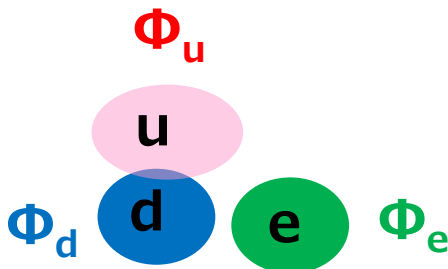
Type-X (Lepton spec.)



Type-Y (Flipped)



Type-Z (Democratic)



Content

- Introduction
- Charged Higgs bosons in 2HDMs and 3HDMs
- Phenomenology
 - Flavour physics
 - Collider physics
- Summary

2HDM/3HDM

2HDM

- 4 types of Yukawa int.
- Higgs basis (one angle)

$$\begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} = \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \Phi \\ \Psi \end{pmatrix} \begin{matrix} \leftarrow v, \text{ NGBs} \\ \leftarrow H^\pm \end{matrix}$$

$$\tan\beta = v_2/v_1$$

- 1 pair of H^\pm
- 2 parameters: $\tan\beta$, mH^\pm

3HDM (See also *Cree & Logan PRD84, 2011*)

- 5 types of Yukawa int.
- Higgs basis (two angles)

$$\begin{pmatrix} \Phi_1 \\ \Phi_2 \\ \Phi_3 \end{pmatrix} = R_{13}(\gamma)R_{12}(\beta) \begin{pmatrix} \Phi \\ \Psi_1 \\ \Psi_2 \end{pmatrix} \begin{matrix} \leftarrow v, \text{ NGBs} \\ \leftarrow \tilde{H}_1^\pm \\ \leftarrow \tilde{H}_2^\pm \end{matrix}$$

R_{ij} : (i-j) rot.

$$\tan\beta = v_2/\sqrt{v_1^2 + v_3^2} \quad \begin{pmatrix} \tilde{H}_1^\pm \\ \tilde{H}_2^\pm \end{pmatrix} = \begin{pmatrix} \cos\theta_C & -\sin\theta_C \\ \sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} H_1^\pm \\ H_2^\pm \end{pmatrix}$$

$$\tan\gamma = v_3/v_1$$

- 2 pairs of H^\pm
- 5(+1) para: β , γ , θ_C , mH_1^\pm , mH_2^\pm (δ_{cp})

Yukawa interactions

- Yukawa Lagrangian is given in the Higgs basis as

$$\mathcal{L}_Y = \bar{Q}_L Y_d \Phi_d d_R + \dots$$

$$= \frac{\sqrt{2}}{v} \bar{Q}_L M_d [\Phi + \underbrace{\sum_a \xi_d^a \Psi_a}] d_R + \dots$$

Charged Higgs couplings

Interaction of charged Higgs bosons

$$-\mathcal{L}_Y = \frac{\sqrt{2}}{v} \sum_a \left[X_a \bar{u} (V_{CKM} m_d P_R) d + Y_a \bar{u} (m_u V_{CKM} P_L) d + Z_a \bar{\nu} (m_e P_R) e \right] H_a^+$$

2HDM

$$X = \xi_d, Y = -\xi_u, Z = \xi_e$$

3HDM

$$\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \begin{pmatrix} c_C & s_C \\ -s_C & c_C \end{pmatrix} \begin{pmatrix} \xi_d^1 \\ \xi_d^2 \end{pmatrix} \quad \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} = - \begin{pmatrix} c_C & s_C \\ -s_C & c_C \end{pmatrix} \begin{pmatrix} \xi_u^1 \\ \xi_u^2 \end{pmatrix} \quad \begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} = \begin{pmatrix} c_C & s_C \\ -s_C & c_C \end{pmatrix} \begin{pmatrix} \xi_e^1 \\ \xi_e^2 \end{pmatrix}$$

	ξ_u	ξ_d	ξ_e
Type-I	$\cot\beta$	$\cot\beta$	$\cot\beta$
Type-II	$\cot\beta$	$-\tan\beta$	$-\tan\beta$
Type-X	$\cot\beta$	$\cot\beta$	$-\tan\beta$
Type-Y	$\cot\beta$	$-\tan\beta$	$\cot\beta$

	ξ_u^1	ξ_d^1	ξ_e^1	ξ_u^2	ξ_d^2	ξ_e^2
Type-I	$\cot\beta$	$\cot\beta$	$\cot\beta$	0	0	0
Type-II	$\cot\beta$	$-\tan\beta$	$-\tan\beta$	0	$-\tan\gamma/\cos\beta$	$-\tan\gamma/\cos\beta$
Type-X	$\cot\beta$	$\cot\beta$	$-\tan\beta$	0	0	$-\tan\gamma/\cos\beta$
Type-Y	$\cot\beta$	$-\tan\beta$	$\cot\beta$	0	$-\tan\gamma/\cos\beta$	0
Type-Z	$\cot\beta$	$-\tan\beta$	$-\tan\beta$	0	$-\tan\gamma/\cos\beta$	$\cot\gamma/\cos\beta$

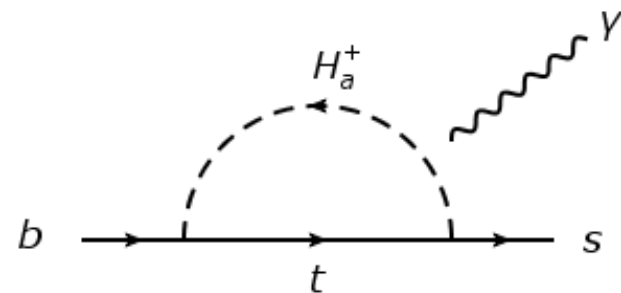
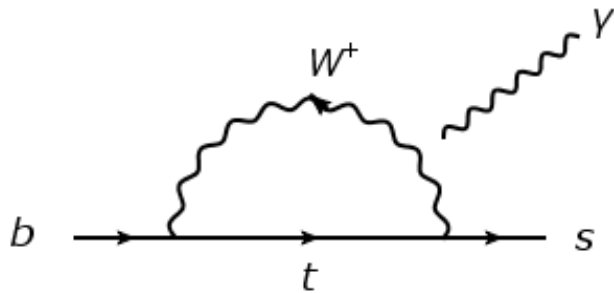
$b \rightarrow s \gamma$ (leading order)

- Effective Lagrangian w/ $m_s = 0$ (integ. out the heavy d.o.f. e.g., W , t , H^\pm)

$$\mathcal{L}_{\text{eff}} \supset \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} C_7(\mu) \mathcal{O}_7(\mu)$$

Dim. 6 dipole operator:

$$\mathcal{O}_7(\mu) = \frac{e}{16\pi^2} \bar{m}_b(\mu) (\bar{s}_L \sigma^{\mu\nu} b_R) F_{\mu\nu}$$



$$C_7(\mu, m_{H_a^\pm}) = C_{7,\text{SM}}(\mu) + \sum_a \left[(X_a Y_a^*) C_{7,XY}(\mu, m_{H_a^\pm}) + |Y_a|^2 C_{7,YY}(\mu, m_{H_a^\pm}) \right]$$

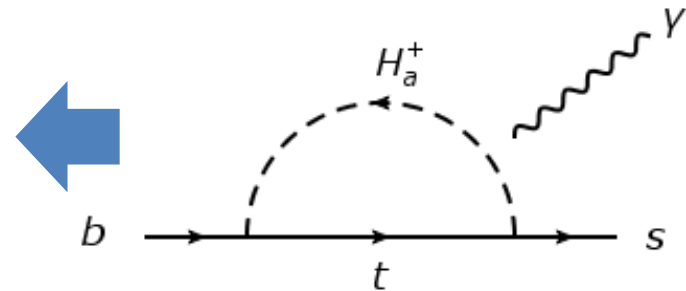
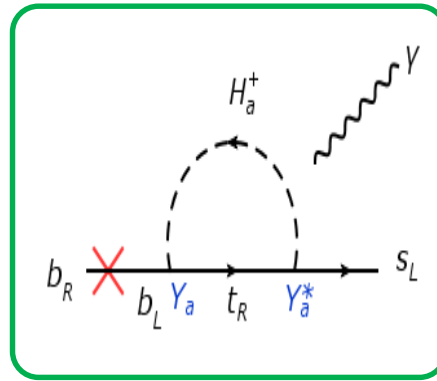
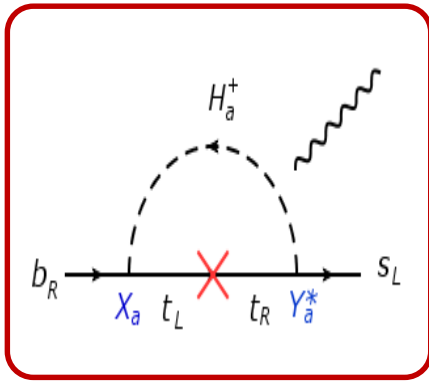
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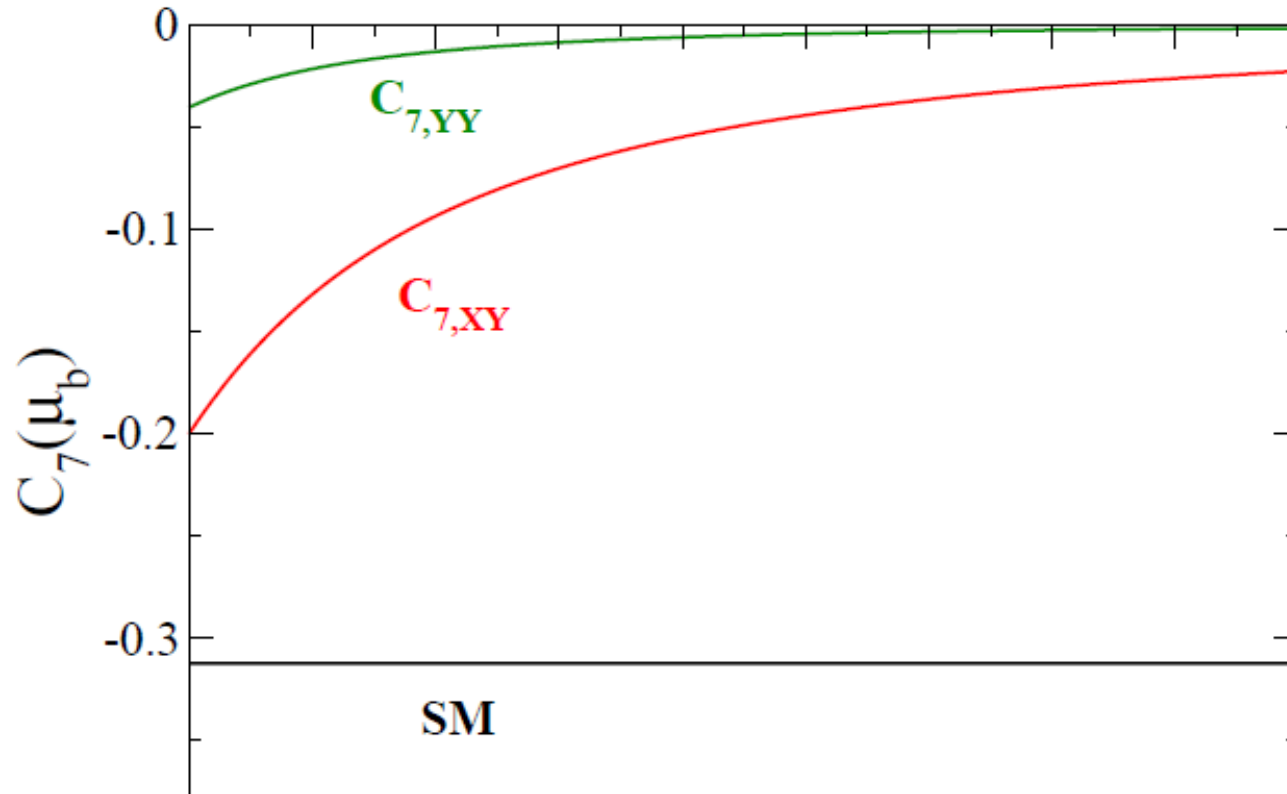
$$\mathcal{O}_7(\mu) = \frac{e}{16\pi^2} \bar{m}_b(\mu) (\bar{s}_L \sigma^{\mu\nu} b_R) F_{\mu\nu}$$



$$C_7(\mu, m_{H_a^\pm}) = C_{7,\text{SM}}(\mu) + \sum_a \left[\underline{(X_a Y_a^*)} C_{7,XY}(\mu, m_{H_a^\pm}) + \underline{|Y_a|^2} C_{7,YY}(\mu, m_{H_a^\pm}) \right]$$

$$\Gamma(b \rightarrow s \gamma) = \frac{G_F^2}{32\pi^4} |V_{ts}^* V_{tb}|^2 \alpha_{\text{em}} m_b^5 |C_7(\mu = \mu_b)|^2$$

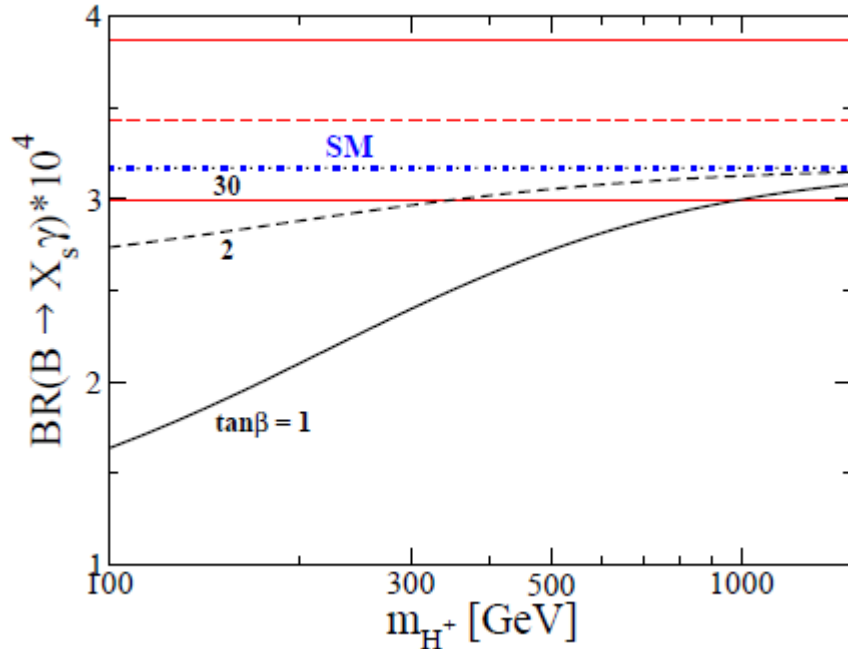
Wilson Coefficient at LO



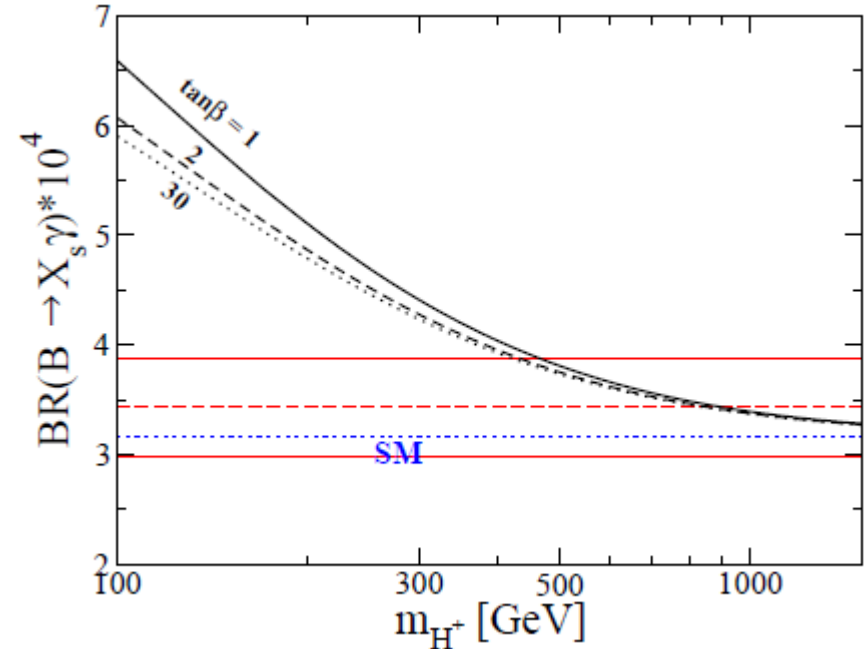
$X Y^* > 0$: Constructive, $X Y^* < 0$: Destructive

$B \rightarrow X_s \gamma$ @NLO (2HDM)

Type-I, X ($XY^* = -\cot^2\beta$)



Type-II, Y ($XY^* = +1$)



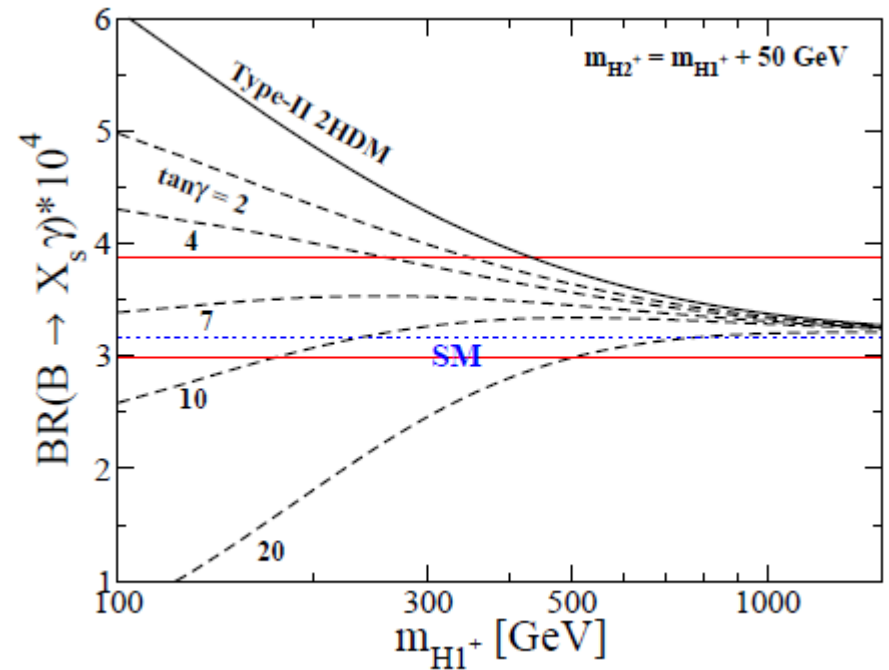
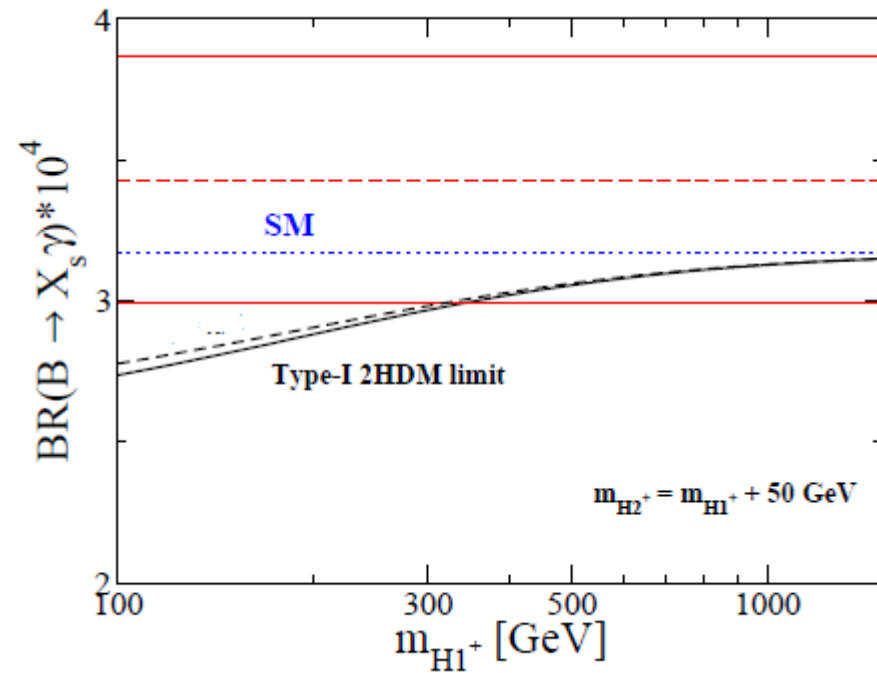
Type-I, -X: Light charged Higgs is strongly constrained only low $\tan\beta$

Type-II, -Y: $m_{H^+} > \sim 450$ GeV (480 GeV@ NNLO, *Misiak, et.al*)

$B \rightarrow X_s \gamma$ @NLO (3HDM, $\tan\beta = 2$, $\theta_c = -\pi/4$)

Type-I, X (no γ dependence)

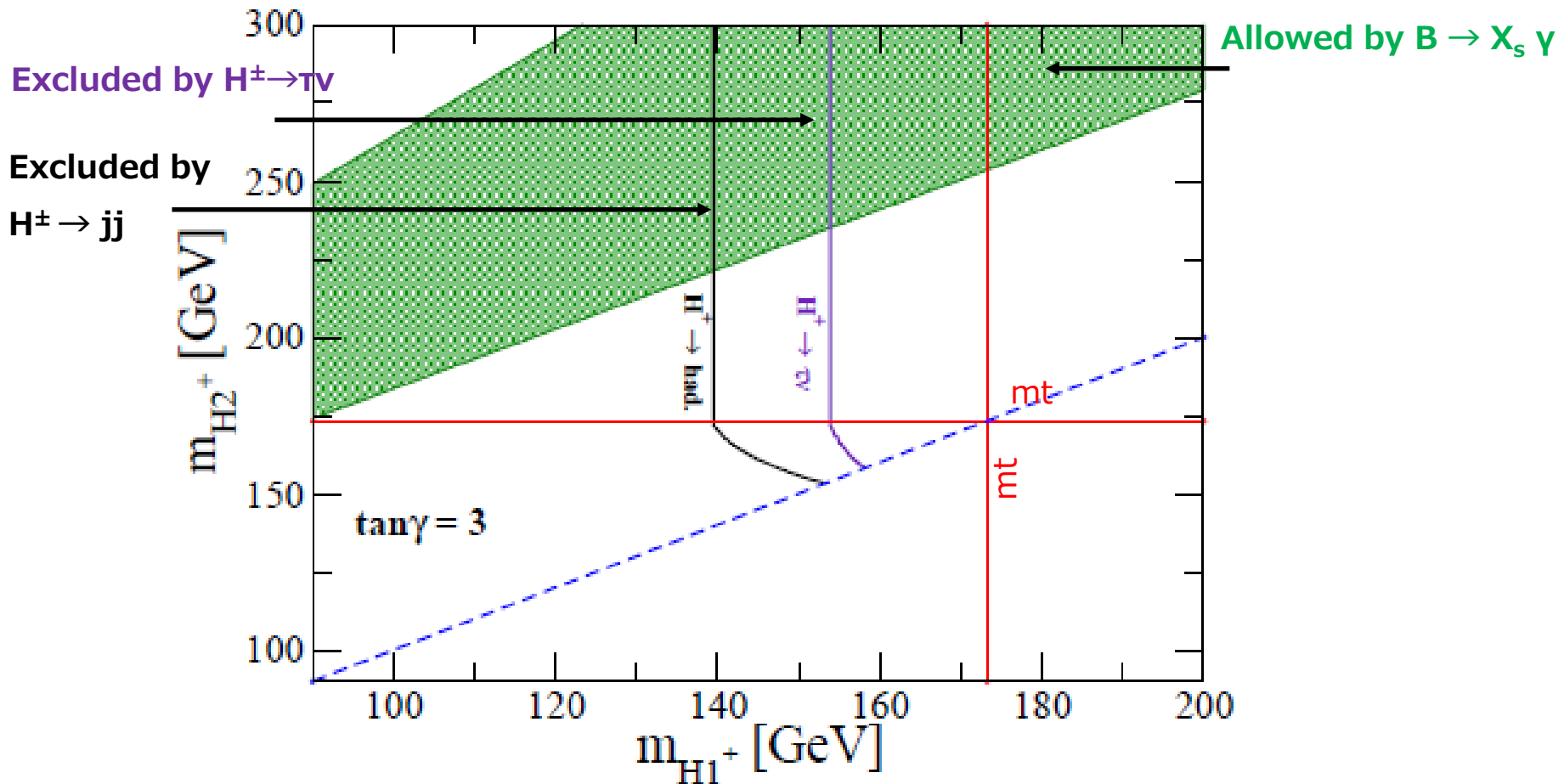
Type-II, Y, Z



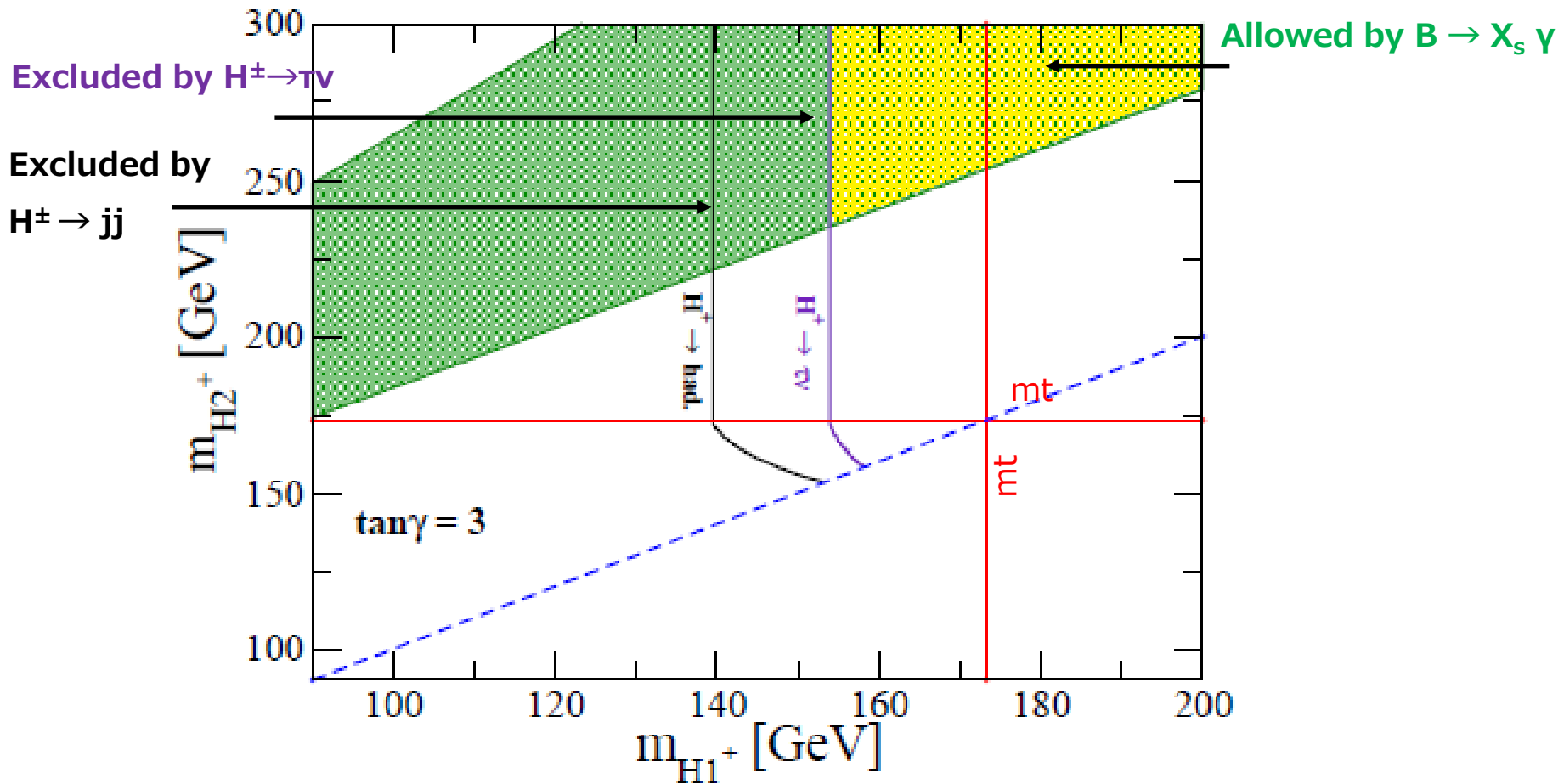
Type-I, X: Almost the same as in the 2HDM

Type-II, Y, Z: Cancellation happens when $\tan\gamma \neq 0$. Light H^\pm is possible.

Allowed regions $(3\text{HDM-}\Upsilon, \tan\beta = 2, \theta_C = -\pi/4)$



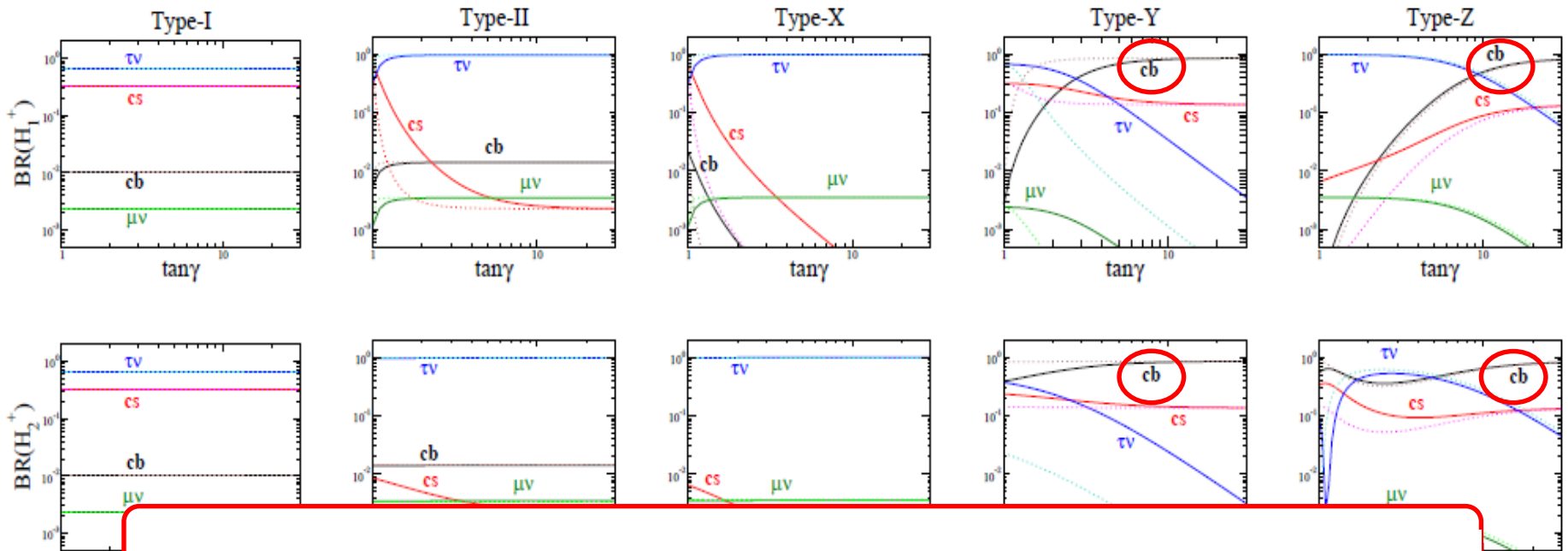
Allowed regions $(3\text{HDM-}\Upsilon, \tan\beta = 2, \theta_C = -\pi/4)$



Decay of charged Higgs bosons

See also for 2HDMs; Aoki, Kanemura, Tsumura, *KY PRD80, 2009*

3HDM, $m_{H_1} = 100$ GeV, $m_{H_2} = 150$ GeV, $\tan\beta = 2$ (5), $\theta_c = -\pi/4$

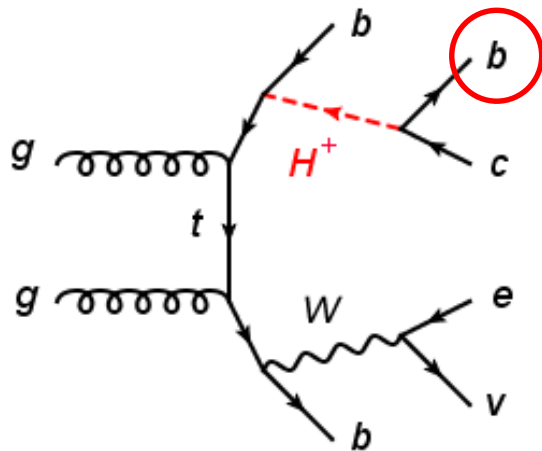


★ In 2HDM-Y, $H^+ \rightarrow cb$ can be dominant, but $b \rightarrow sy$.

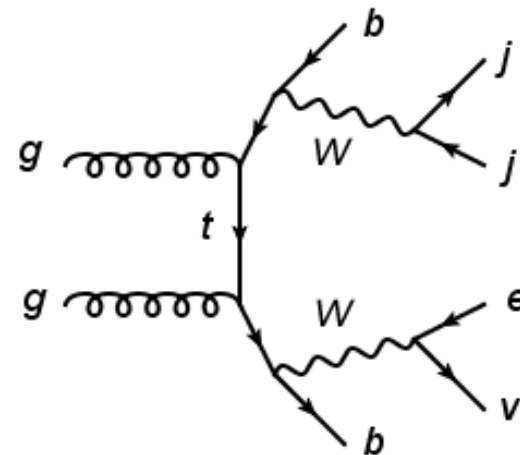
★ $H^+ \rightarrow cb$ can be the smoking gun sig. of the 3HDM!

Collider signatures

Signal



Background



Akeroyd, Moretti, Hernandez-Sanchez, PRD85 (2012)

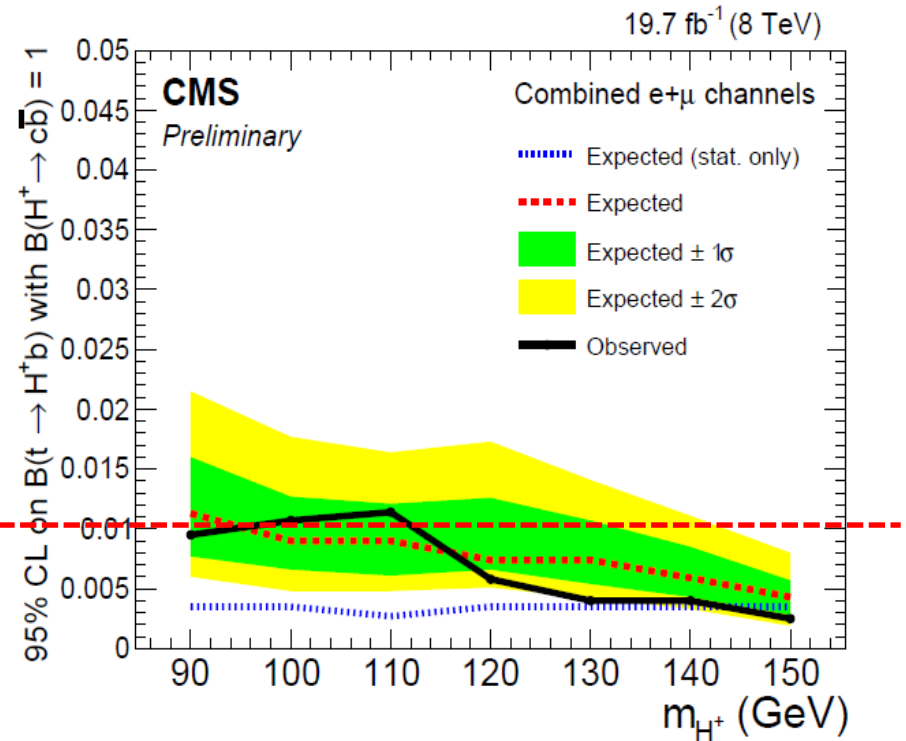
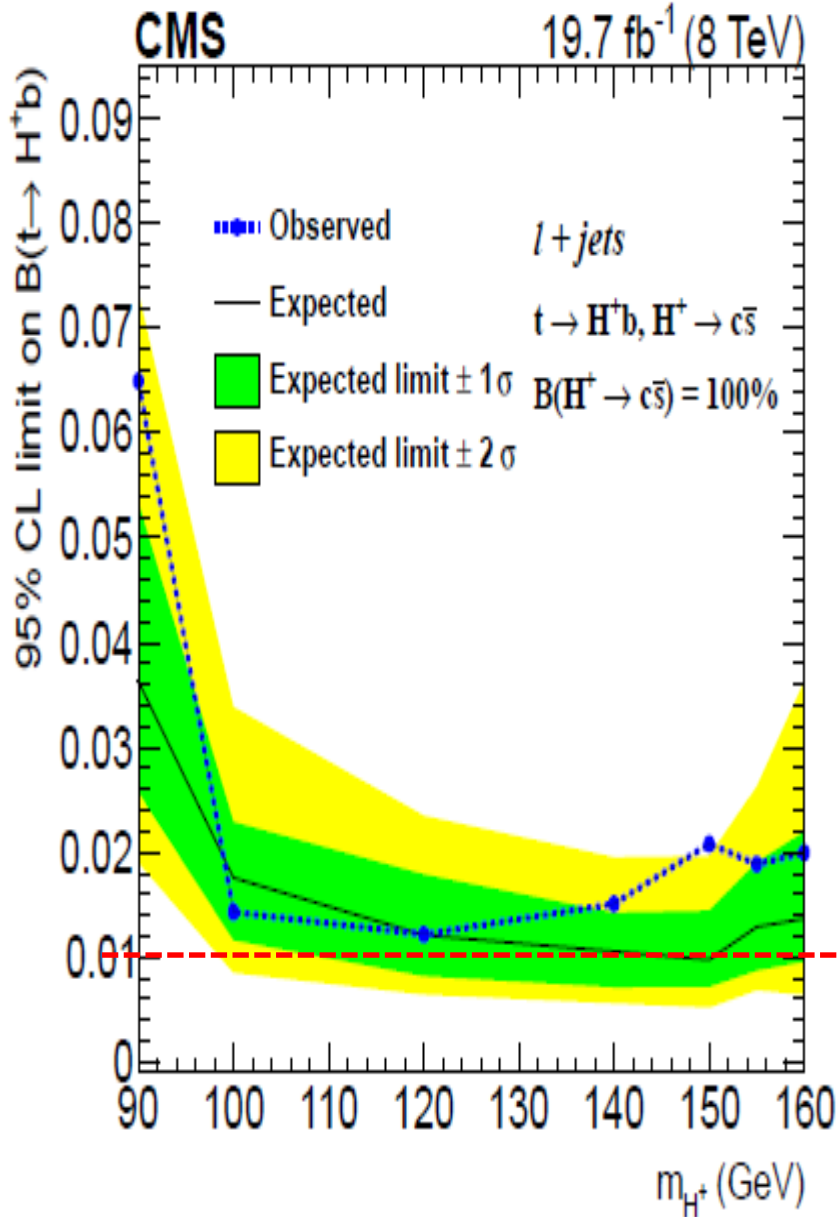
$$\frac{[S/\sqrt{B}]_{b\text{tag}}}{[S/\sqrt{B}]_{\text{btag}}} \sim \frac{\epsilon_b \sqrt{2}}{\sqrt{(\epsilon_j + \epsilon_c)}} \sim 2.13.$$

ϵ_b (b tagging eff.)= 0.5

ϵ_c (c miss tagging rate)= 0.1

ϵ_j (j miss tagging rate)= 0.01

★ 3rd b-quark tagging is important to reduce the BG!

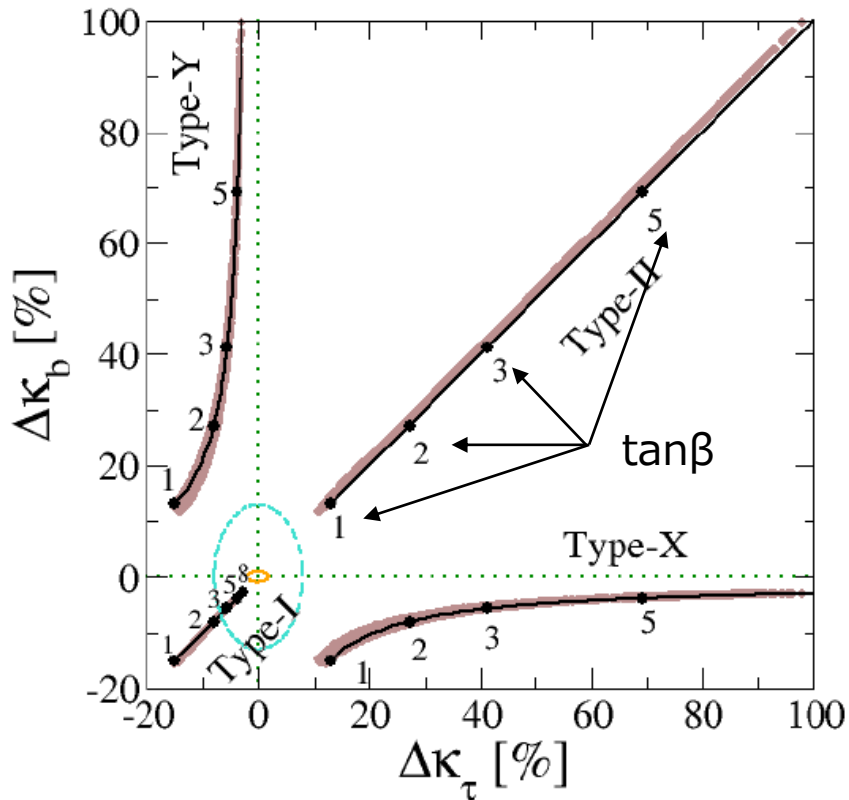


h(125) couplings ($\Delta\kappa_x = g_{hXX}^{3\text{HDM}}/g_{hXX}^{\text{SM}} - 1$)

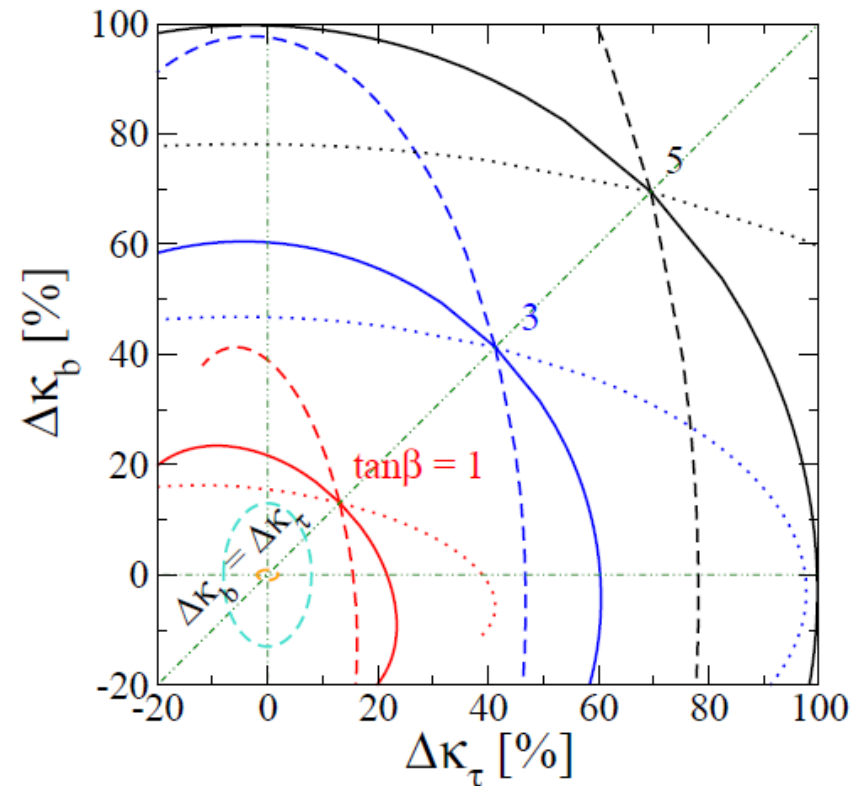
Under $\Delta\kappa_V = -1\%$, $\Delta\kappa_t < 0$

$\tan\gamma = 1$ (solid)
 $\tan\gamma = 2$ (dash)
 $\tan\gamma = 1/2$ (dot)
 Others are scanned

2HDM



3HDM (Type-Z)

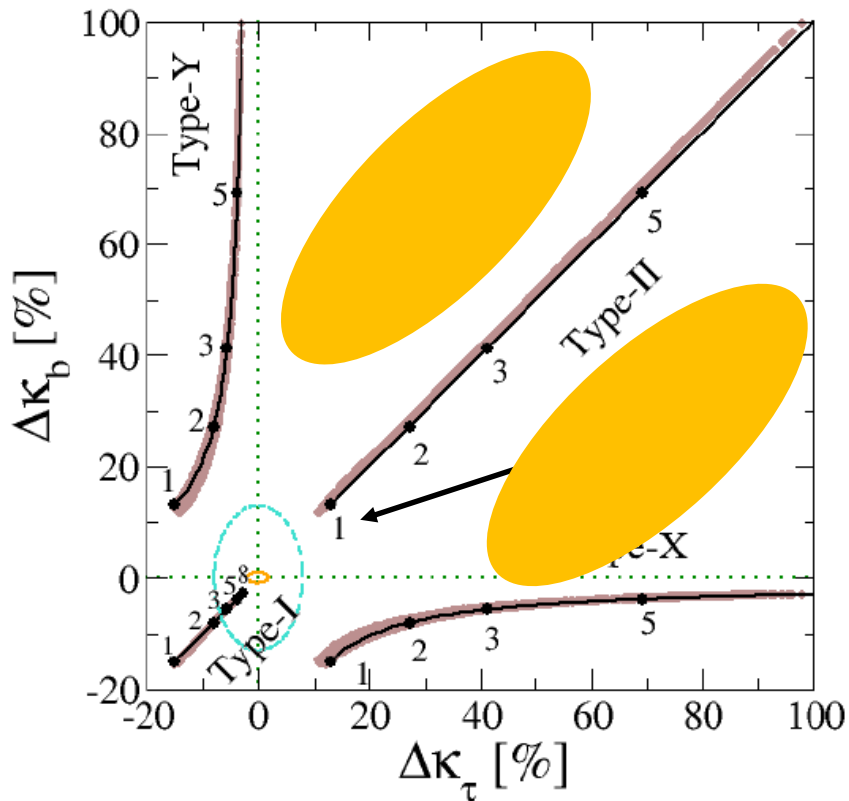


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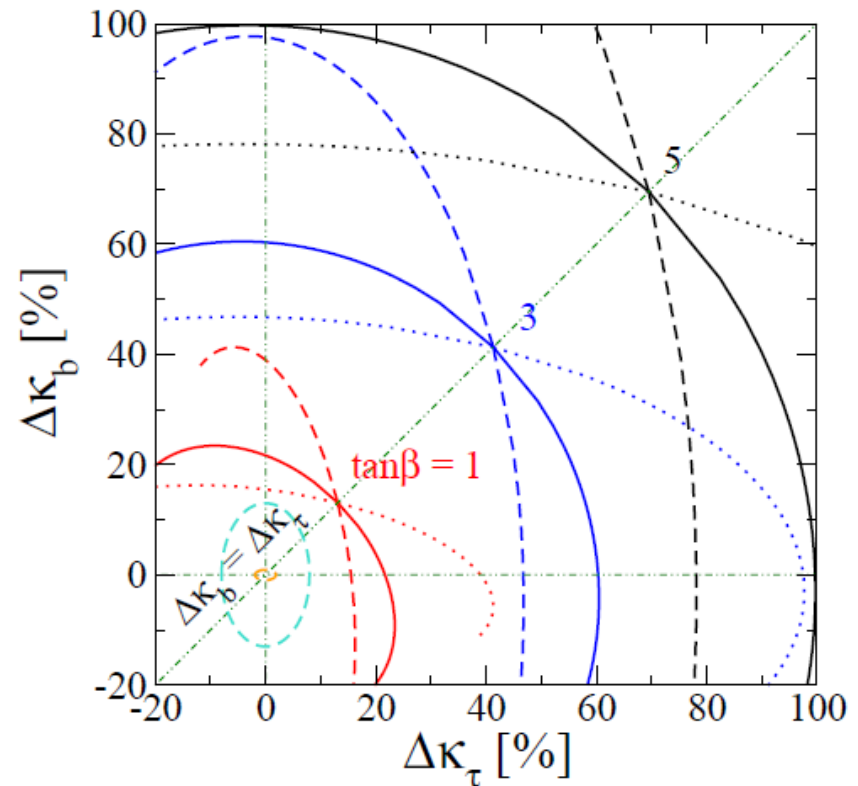
Under $\Delta\kappa_V = -1\%$, $\Delta\kappa_t < 0$

$\tan\beta = 1$ (solid)
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 Others are scanned

2HDM



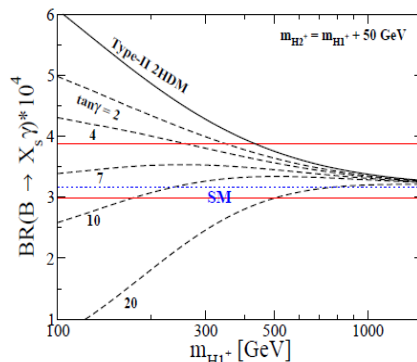
3HDM (Type-Z)



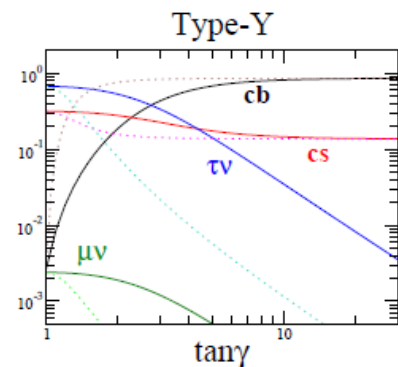
Summary

- The Higgs sector could have the multi-doublet structure like “flavour” .
- There are critical differences between the 2HDMs and the 3HDMs ($N \geq 3$).

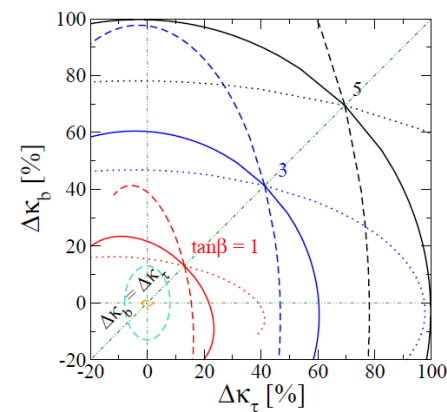
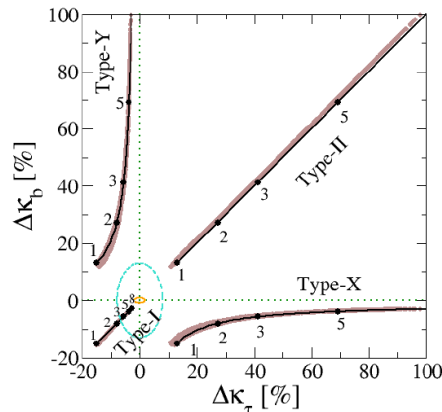
★ Cancellation in $B \rightarrow X_s \gamma$



★ $H^{\pm} \rightarrow cb$ can be smoking gun.



★ Deviation in $h(125)$ couplings



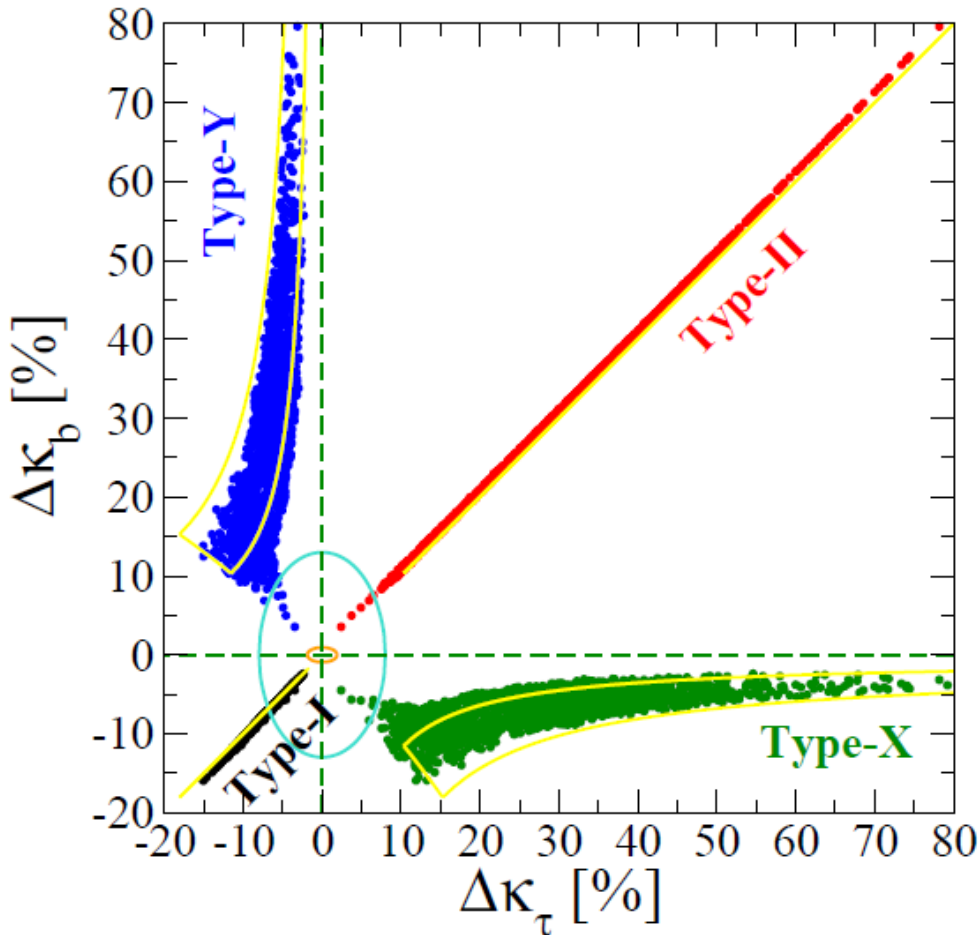
Higgs basis in 3HDMs

$$\begin{pmatrix} \Phi_1 \\ \Phi_2 \\ \Phi_3 \end{pmatrix} = R \begin{pmatrix} \Phi \\ \Psi_2 \\ \Psi_3 \end{pmatrix} \quad R = = \begin{pmatrix} \cos \gamma & 0 & -\sin \gamma \\ 0 & 1 & 0 \\ \sin \gamma & 0 & \cos \gamma \end{pmatrix} \begin{pmatrix} \cos \beta & -\sin \beta & 0 \\ \sin \beta & \cos \beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
 = \begin{pmatrix} \cos \beta \cos \gamma & -\sin \beta \cos \gamma & -\sin \gamma \\ \sin \beta & \cos \beta & 0 \\ \cos \beta \sin \gamma & -\sin \beta \sin \gamma & \cos \gamma \end{pmatrix}, \quad \xi_f^a = R_{fa}/R_{f1}$$

	Φ_u	Φ_d	Φ_e	ξ_u^1	ξ_d^1	ξ_e^1	ξ_u^2	ξ_d^2	ξ_e^2
Type-I	Φ_2	Φ_2	Φ_2	$\cot \beta$	$\cot \beta$	$\cot \beta$	0	0	0
Type-II	Φ_2	Φ_1	Φ_1	$\cot \beta$	$-\tan \beta$	$-\tan \beta$	0	$-\tan \gamma / \cos \beta$	$-\tan \gamma / \cos \beta$
Type-X	Φ_2	Φ_2	Φ_1	$\cot \beta$	$\cot \beta$	$-\tan \beta$	0	0	$-\tan \gamma / \cos \beta$
Type-Y	Φ_2	Φ_1	Φ_2	$\cot \beta$	$-\tan \beta$	$\cot \beta$	0	$-\tan \gamma / \cos \beta$	0
Type-Z	Φ_2	Φ_1	Φ_3	$\cot \beta$	$-\tan \beta$	$-\tan \beta$	0	$-\tan \gamma / \cos \beta$	$\cot \gamma / \cos \beta$

$\Delta\kappa_\tau$ VS $\Delta\kappa_b$ in 2HDMs (1-loop)

$$\Delta\kappa_V = (-1 \pm 0.4)\%, \Delta\kappa_t < 0$$



Parameter scan:

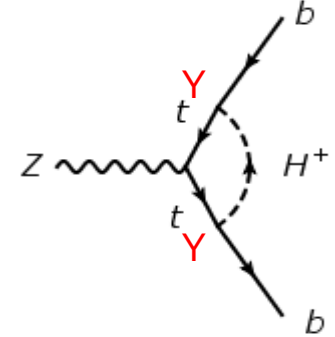
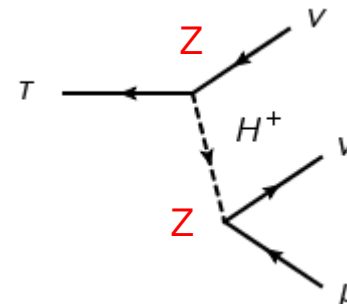
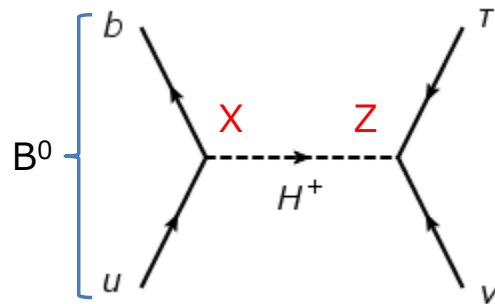
- $\tan\beta > 1$
- $m_\phi > 300$ GeV
- $\sin(\beta-\alpha) < 1$
- $|\lambda_{h\phi\phi}| > 0$
- $\Lambda_{\text{cutoff}} > 3$ TeV

Other flavour constraints

□ $B \rightarrow TV$

□ $T \rightarrow \mu\nu\nu$

□ $Z \rightarrow bb$



$$|Z| < 40 (m_{H^+}/100 \text{ GeV})$$

$$|X*Z| < 1080 (m_{H^+}/100 \text{ GeV})^2$$

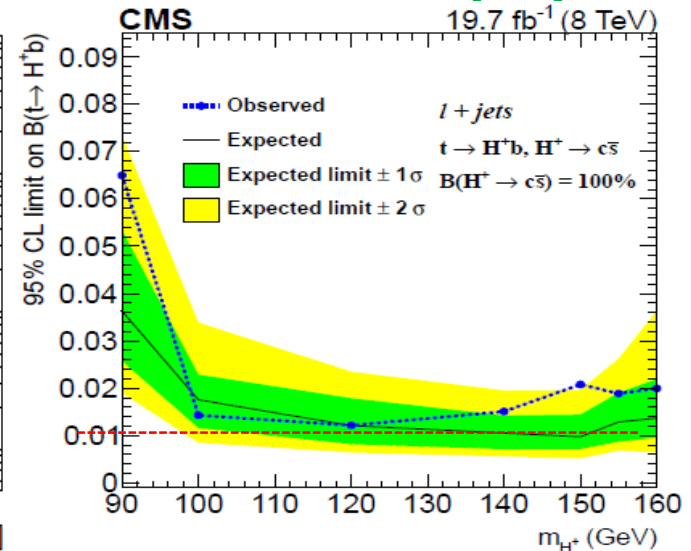
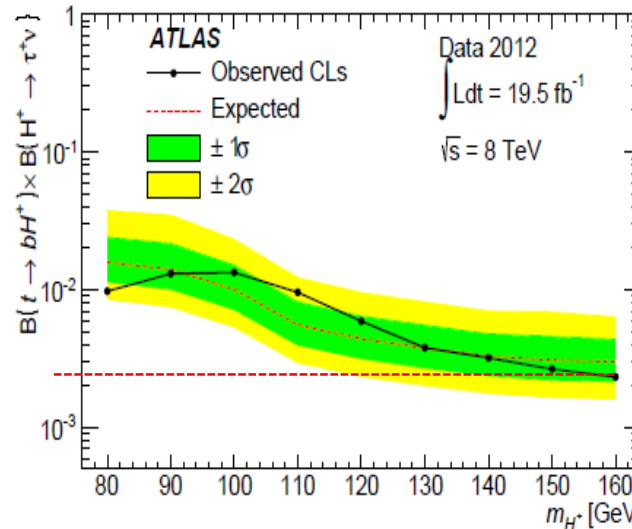
$$|Y| < 0.72 + 0.34 (m_{H^+}/100 \text{ GeV})$$

Cree, Logan, PRD84 (2011)

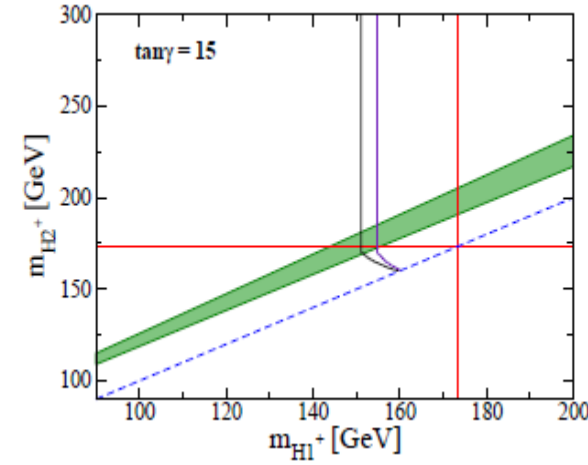
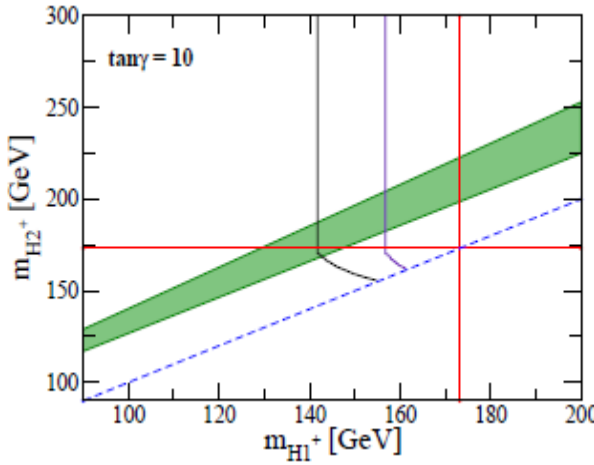
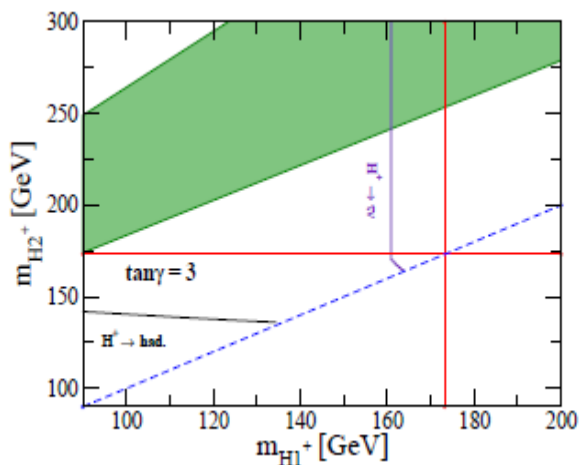
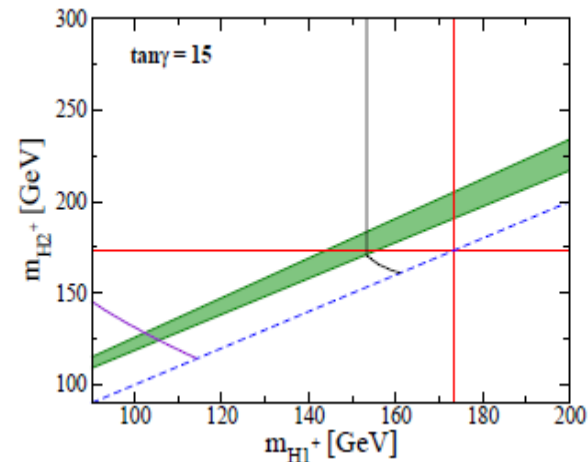
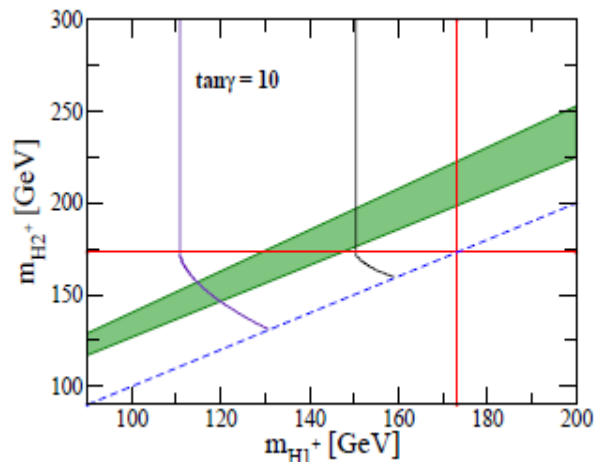
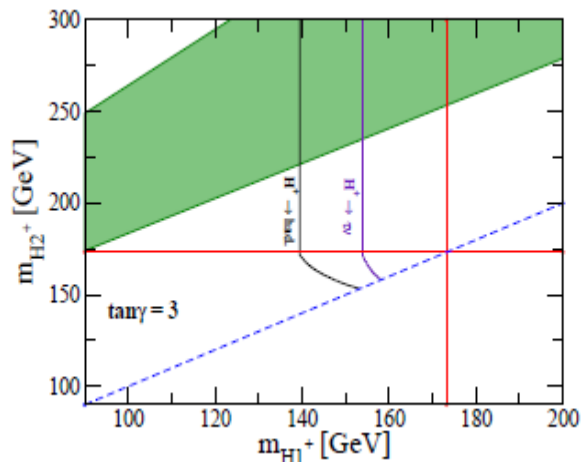
□ $t \rightarrow H^\pm b$ (LHC)

1412.6663 [ATLAS]

1510.04252 [CMS]

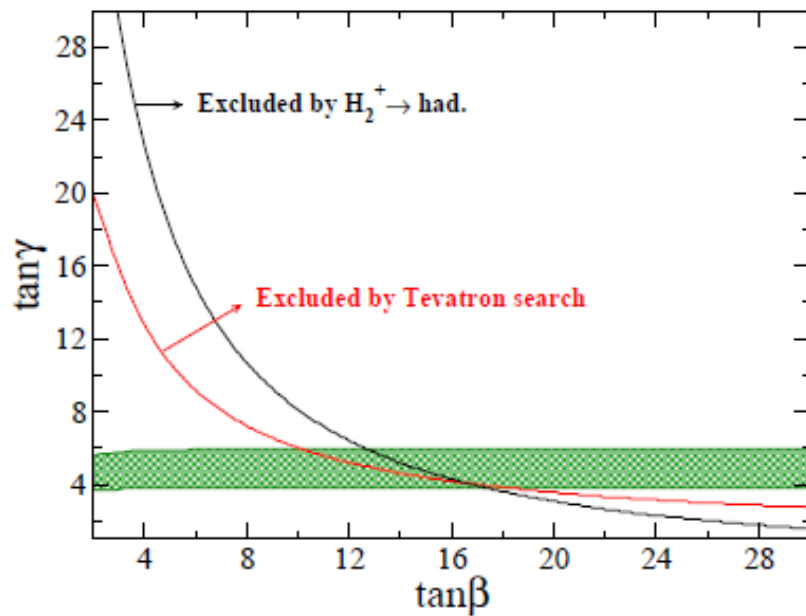


Combined Analysis

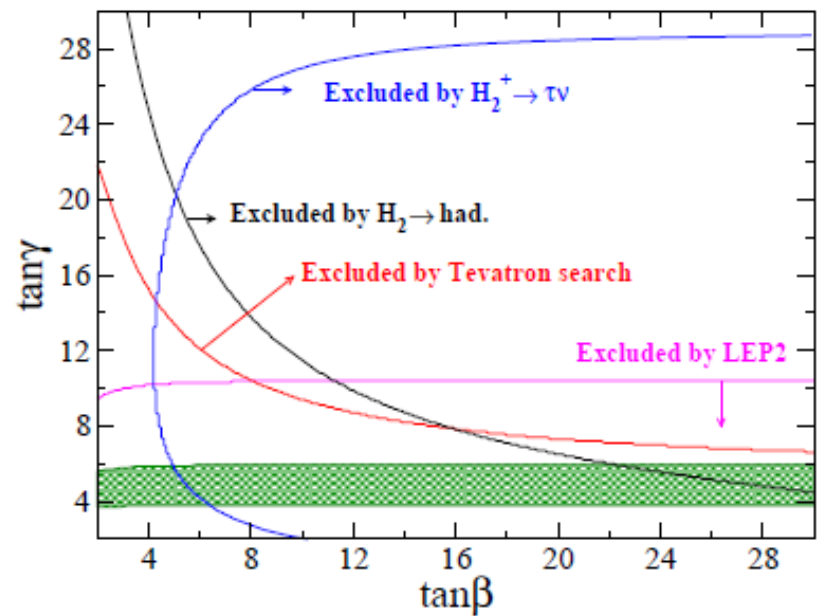


Scenario for $m_{H_{1+}} \sim m_W$

Type-Y 3HDM

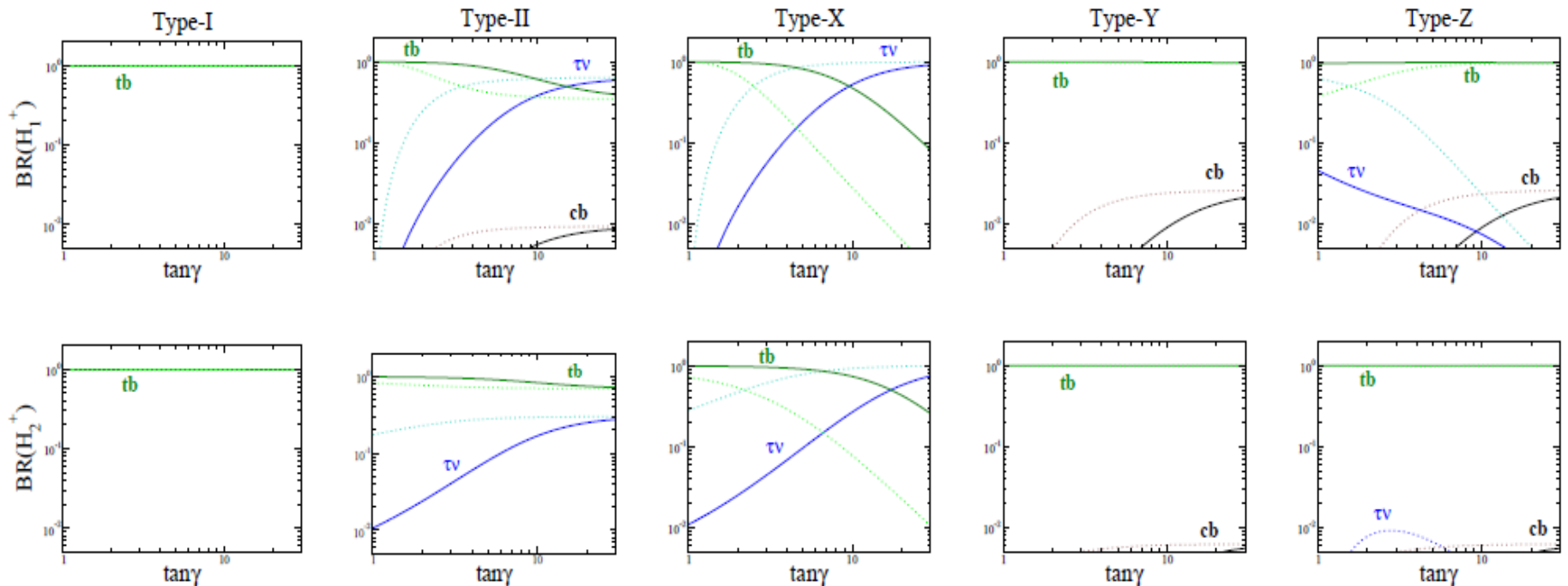


Type-Z 3HDM



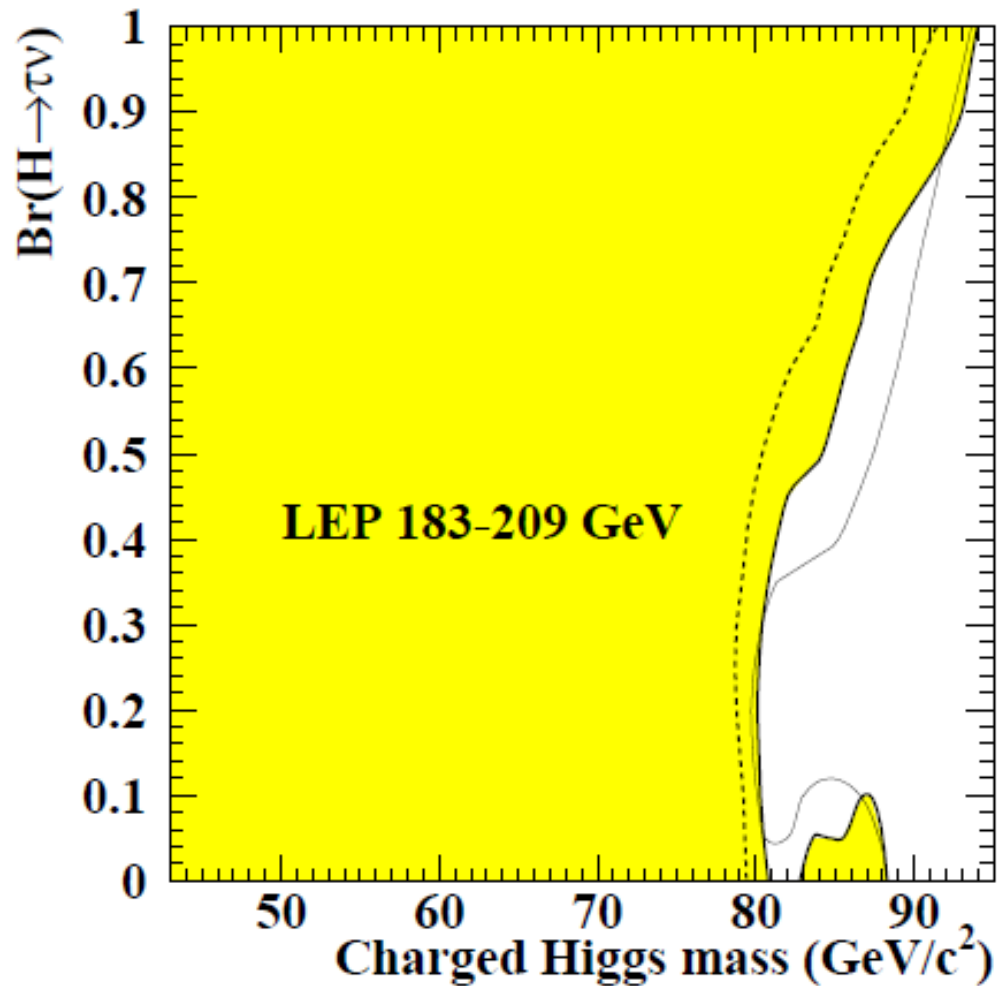
Charged Higgs decays

3HDM, $m_{H_1} = 200$ GeV, $m_{H_2} = 250$ GeV, $\tan\beta = 2$ (5), $\theta_c = -\pi/4$



Type-I, Y, Z: tb , Type-II, X : tv @ large $\tan\gamma$

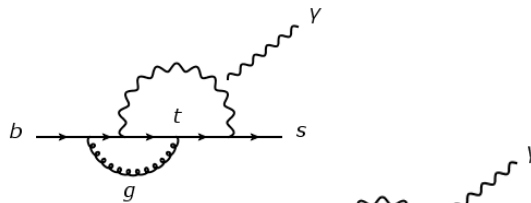
Scenario for $m_{H_{1+}} \sim m_W$



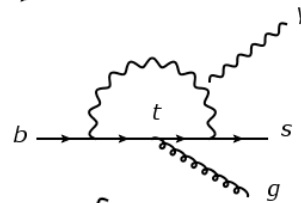
B → X_s γ @NLO

□ To compare the exp. value of BR(B → X_s γ) , we need to take into account the three factors

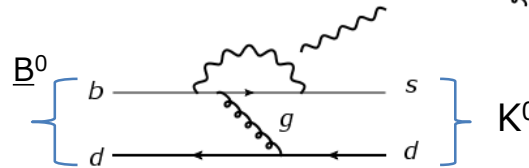
1. Gluon-loop



2. Gluon-emission



3. Non-pert. effects



[NLO calculation]

Ciuchini, Franco, Martinelli, Reina, Silvestrini, PLB334, 137 (1994), [hep-ph/9406239].

Ciuchini, Degrassi, Gambino, Giudice, NPB527, 21 (1998), [hep-ph/9710335].

Borzumati, Greub, PRD58, 074004 (1998), [hep-ph/9802391].

Kagan, Neubert, EPJC7, 5 (1999), [hep-ph/9805303].

Gambino, Misiak, NPB611, 338 (2001), [hep-ph/0104034].

[NNLO calculation]

Hermann, Misiak, Steinhauser, JHEP1211, 036 (2012), [arXiv:1208.2788 [hep-ph]].

Misiak et.al,

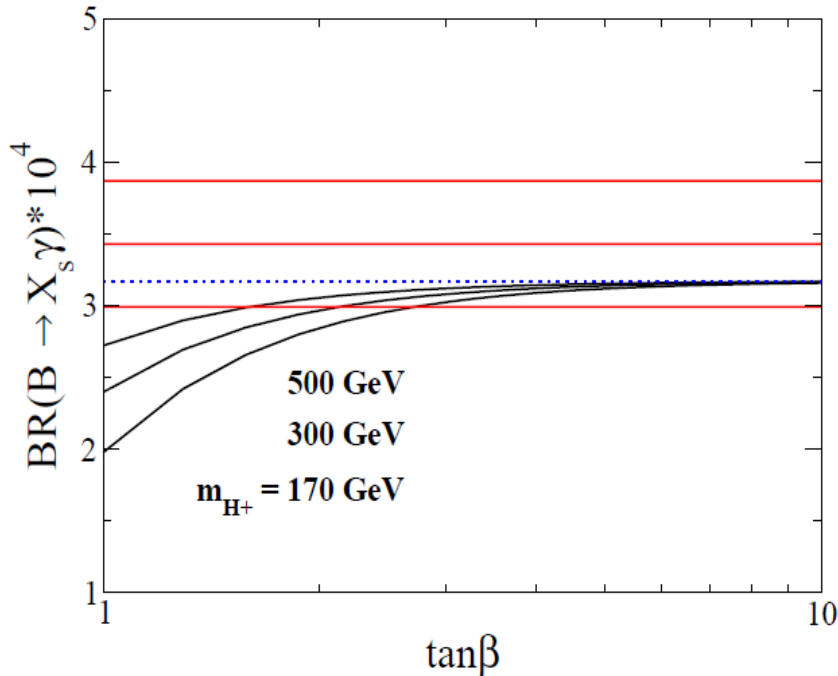
PRL114, 221801 (2015), [arXiv:1503.01789 [hep-ph]].

$$\Gamma(B \rightarrow X_s \gamma)_{\text{NLO}} = \Gamma_{b \rightarrow s \gamma}^{\text{NLO}} + \Gamma_{b \rightarrow s \gamma g} + \Gamma_{\text{non-pert.}}$$

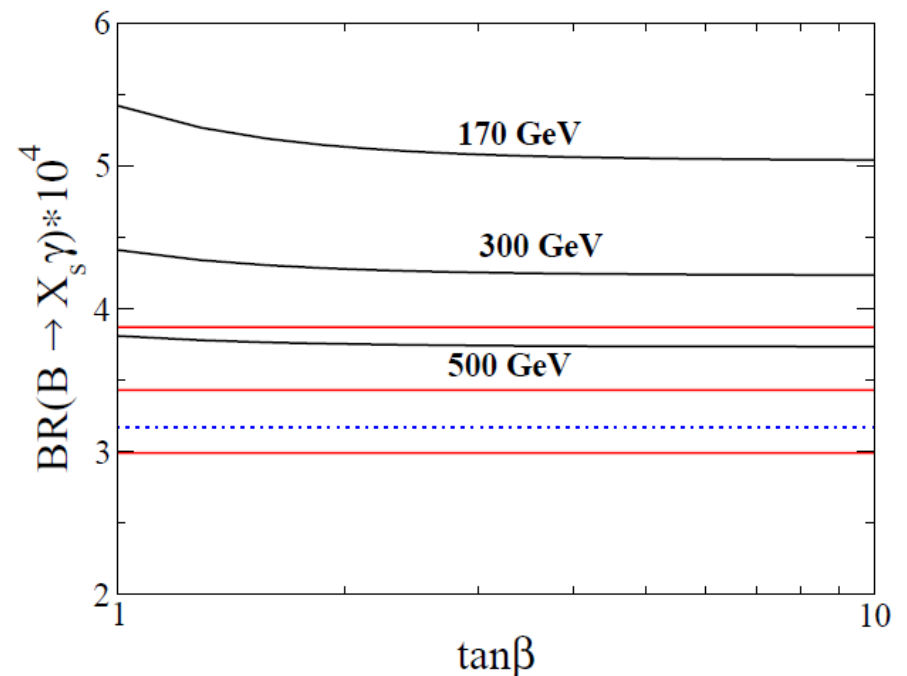
$$\text{BR}(B \rightarrow X_s \gamma) = \frac{\Gamma(B \rightarrow X_s \gamma)}{\Gamma(B \rightarrow X_c \ell \nu)} \text{BR}(B \rightarrow X_c \ell \nu)$$

$B \rightarrow X_s \gamma$ (2HDM)

Type-I, X



Type-II, Y

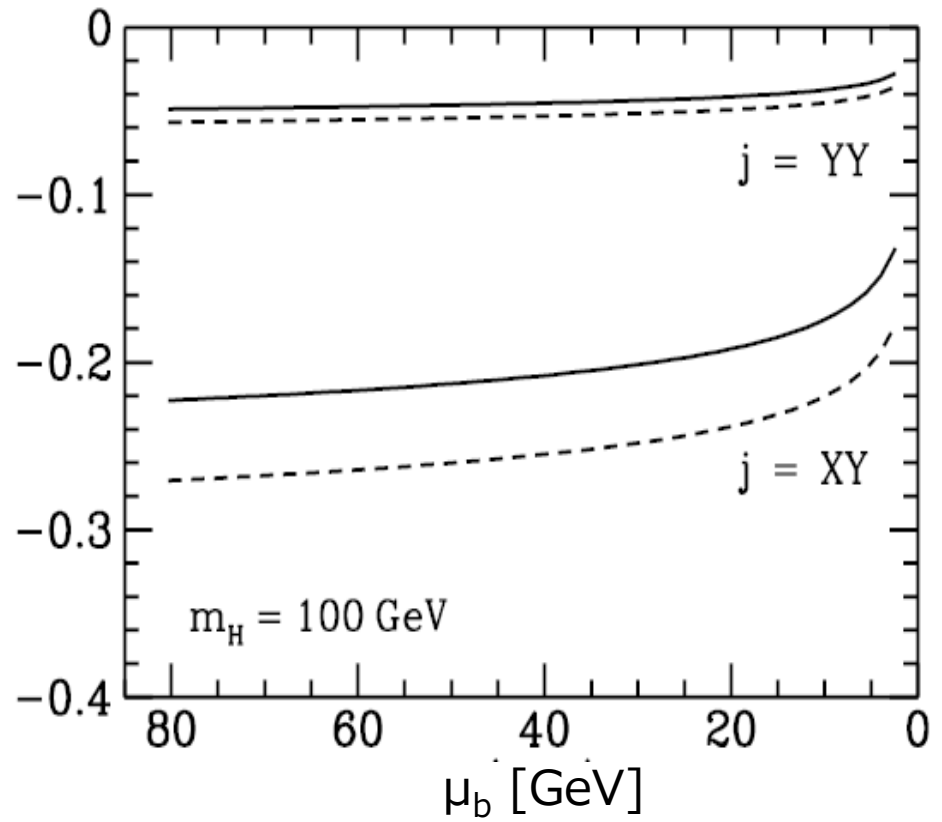
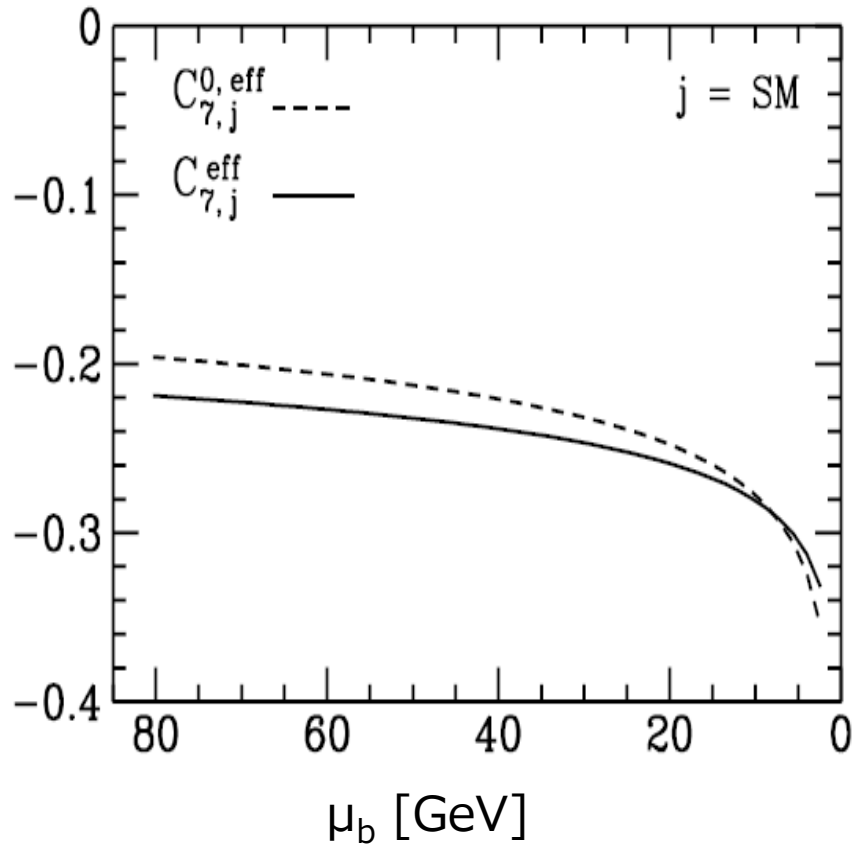


Type-I, -X : $\tan\beta > \sim 3$ で $m_{H^+} < m_t$ が許される。

Type-II, -Y : less sensitivity to $\tan\beta$

LO \rightarrow NLO

Borzumati, Greub, PRD58, 074004 (1998)



Higgs potential of 2HDM

- Higgs potential with softly-broken Z_2 symmetry and CP-conservation

$$V = m_1^2 |\Phi_1|^2 + m_2^2 |\Phi_2|^2 - m_3^2 [\Phi_1^\dagger \Phi_2 + \text{h.c.}] \\ + \frac{1}{2} \lambda_1 |\Phi_1|^4 + \frac{1}{2} \lambda_2 |\Phi_2|^4 + \lambda_3 |\Phi_1|^2 |\Phi_2|^2 + \lambda_4 |\Phi_1^\dagger \Phi_2|^2 + \frac{1}{2} \lambda_5 [(\Phi_1^\dagger \Phi_2)^2 + \text{h.c.}]$$

- 8 parameters

v (=246 GeV), m_h (=125 GeV),

m_{H^\pm} , m_A , m_{H^\pm} , $\sin(\beta-\alpha)$, $\tan\beta$, and M^2

$$M^2 = m_3^2 / (\sin\beta \cos\beta)$$

- Mass parameters [$\sin(\beta-\alpha) \sim 1$]

$$m_h^2 \sim \lambda v^2, m_\Phi^2 \sim M^2 + \lambda' v^2$$

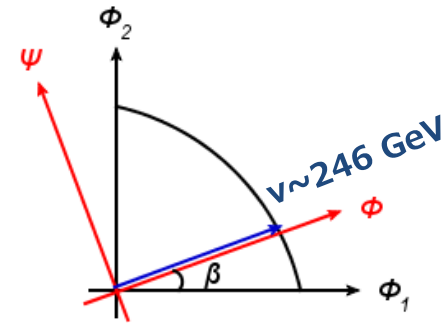
$\Phi = H^\pm, A, H$

Two Higgs Doublet Model (2HDM)

□ The Higgs basis

$$\begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} = \begin{pmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} \Phi \\ \Psi \end{pmatrix}$$

$$\tan \beta = v_2/v_1$$



$$\Phi = \left[\begin{array}{c} \text{NG boson} \\ G^+ \\ \frac{1}{\sqrt{2}}(h'_1 + v + iG^0) \end{array} \right]$$

$$\Psi = \left[\begin{array}{c} \text{Charged Higgs} \\ H^+ \\ \frac{1}{\sqrt{2}}(h'_2 + iA) \end{array} \right]$$

CP-even Higgs

CP-odd Higgs

$$\begin{pmatrix} h'_1 \\ h'_2 \end{pmatrix} \begin{pmatrix} \cos(\beta - \alpha) & \sin(\beta - \alpha) \\ -\sin(\beta - \alpha) & \cos(\beta - \alpha) \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix} \text{ SM-like Higgs with 125 GeV}$$

Gauge/Yukawa interactions

★ **Alignment limit: $(R_H)_{11} \rightarrow 1$**
 hVV and hff couplings \rightarrow SM

$$\mathcal{L}_Y = \bar{Q}_L Y_d \Phi_d d_R + \dots = \frac{\sqrt{2}}{v} \bar{Q}_L M_d [\Phi + \sum_a \xi_d^a \Psi_a] d_R + \dots$$

$h \text{---} \begin{array}{l} \text{V} \\ \text{---} \\ \text{V} \end{array} = (\text{SM}) \times (R_H)_{11}$
 $h \text{---} \begin{array}{l} \text{f} \\ \text{---} \\ \bar{\text{f}} \end{array} = (\text{SM}) \times [(R_H)_{11} + \sum_a \xi_f^a (R_H)_{a1}]$

Gauge/Yukawa interactions

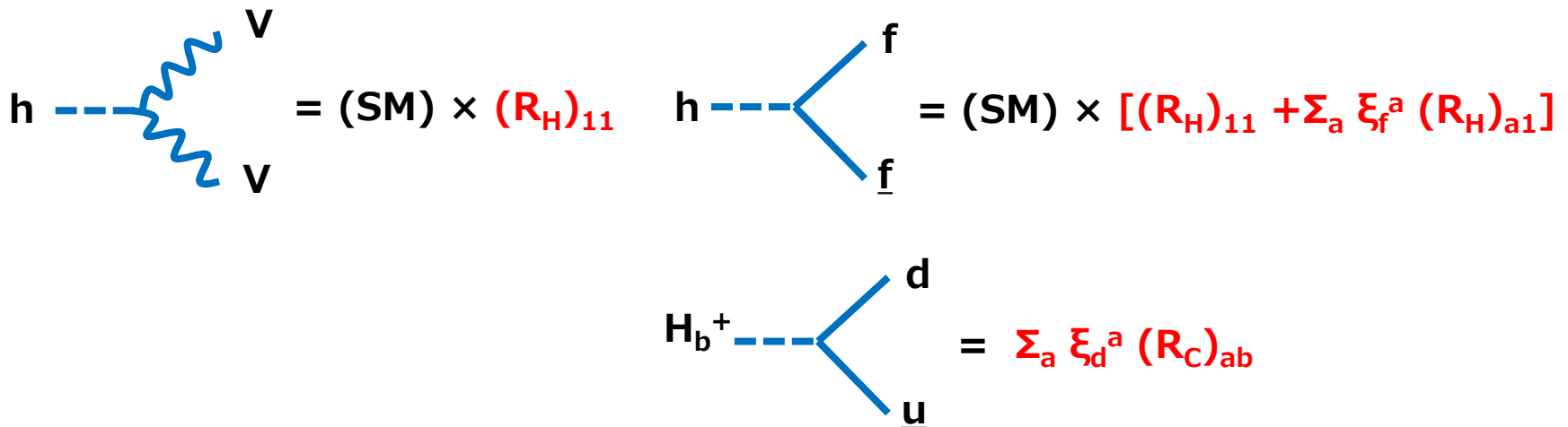
□ Gauge interaction

$$\mathcal{L}_{\text{kin}} = |D_\mu \Phi|^2 + \sum_a |D_\mu \Psi_a|^2$$

□ Yukawa interaction

$$\xi_f^a = R_{fa}/R_{f1}$$

$$\mathcal{L}_Y = \bar{Q}_L Y_d \Phi_d d_R + \dots = \frac{\sqrt{2}}{v} \bar{Q}_L M_d [\Phi + \sum_a \xi_d^a \Psi_a] d_R + \dots$$



Higgs basis

Weak basis

Higgs basis

$$\begin{pmatrix} \Phi_1 \\ \Phi_2 \\ \vdots \\ \Phi_N \end{pmatrix} = R \begin{pmatrix} \Phi \\ \Psi_2 \\ \vdots \\ \Psi_N \end{pmatrix}$$

$$\Phi = \begin{pmatrix} G^+ \\ \frac{v + \tilde{H}_1 + iG^0}{\sqrt{2}} \end{pmatrix} \quad G^+, G^0: \text{NG boson}$$

$$\Psi_a = \begin{pmatrix} \tilde{H}_a^+ \\ \frac{\tilde{H}_a + i\tilde{A}_a}{\sqrt{2}} \end{pmatrix} \quad a = 2, \dots, N$$

$$R = R_{12}(\beta_1)R_{13}(\beta_2)\cdots R_{1N}(\beta_{N-1}) \quad R_{ij} : \text{rotation for the } (i,j) \text{ plane.}$$

$$\begin{pmatrix} \tilde{A}_2 \\ \tilde{A}_3 \\ \vdots \\ \tilde{A}_N \end{pmatrix} = R_A \begin{pmatrix} A_2 \\ A_3 \\ \vdots \\ A_N \end{pmatrix} \quad \begin{pmatrix} \tilde{H}_2^\pm \\ \tilde{H}_3^\pm \\ \vdots \\ \tilde{H}_N^\pm \end{pmatrix} = R_C \begin{pmatrix} H_2^\pm \\ H_3^\pm \\ \vdots \\ H_N^\pm \end{pmatrix} \quad \begin{pmatrix} \tilde{H}_1 \\ \tilde{H}_2 \\ \vdots \\ \tilde{H}_N \end{pmatrix} = R_H \begin{pmatrix} h \\ H_2 \\ \vdots \\ H_N \end{pmatrix}$$

125 GeV