Cavity simulation

Development in the framework of EUROTRANS: design of a linear proton accelerator driven for radioactive waste transmutation

Modelling of the accelerating cavity and its interaction with the proton beam

- Characteristic equation of a cavity driven by a current generator:

$$\tau \frac{d\tilde{V}_c}{dt} + (1 + j\,y)\,\tilde{V}_c = R_L \tilde{I}_g, \text{ with } \tau = \frac{2Q_L}{\omega_0} \text{ and } y = -\tan\psi \approx 2Q_L \frac{\omega - \omega_0}{\omega_0}$$

- Beam:
  - Represented by a current generator

$$\tau \frac{d\tilde{V}_c}{dt} + (1 + j\,y)\,\tilde{V}_c = R_L (\tilde{I}_g + \tilde{I}_B)$$

  - Represented by the beam loading theorem

$$V_c^+ = V_c - \omega \frac{R_s}{Q}(\xi)\, q_b \cos\left[\phi_c - \phi_s(\xi) - \Delta\phi_b\right]$$

Representation for no-relativist particles: Energy gain and phase slippage

Cavity simulation

- Perturbations:

  - Detuning of the cavity

$$\Delta\omega = \Delta\omega_0 + \Delta\omega_\mu(t) + \Delta\omega_L(t)$$

- Microphonics

$$\Delta\omega_\mu(t) = \sum_i^N \overline{\Delta\omega_i}(t)\sin(\omega_i t + \varphi_i)$$

- Lorentz Forces

$$\frac{d^2\Delta\omega_m}{dt^2} + \frac{\omega_m}{Q_m}\frac{d\Delta\omega_m}{dt} + \omega_m^2\Delta\omega_m = -2\pi k_m \omega_m^2 \left(\frac{V_{cav}}{L_{acc}}\right)^2$$

First order Transfer function: $H^1(s) = \frac{K_0}{\tau s + 1}$,

,

- Amplifier

  Transfer function, Power limitation

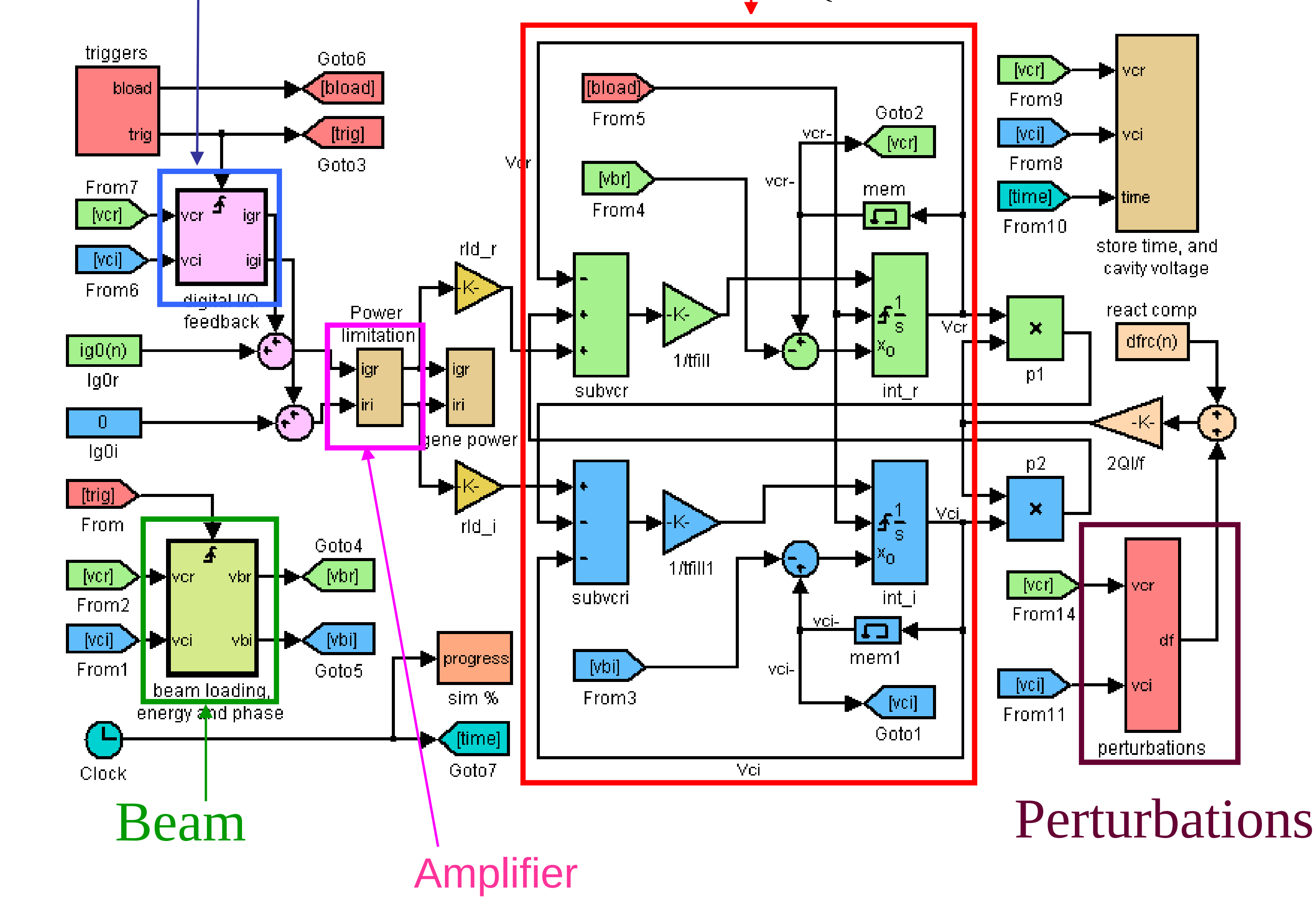
- Feedback control

  PI feedback, delay of digital calculations, ADC-DAC quantification

Implementation architecture for cavity simulation

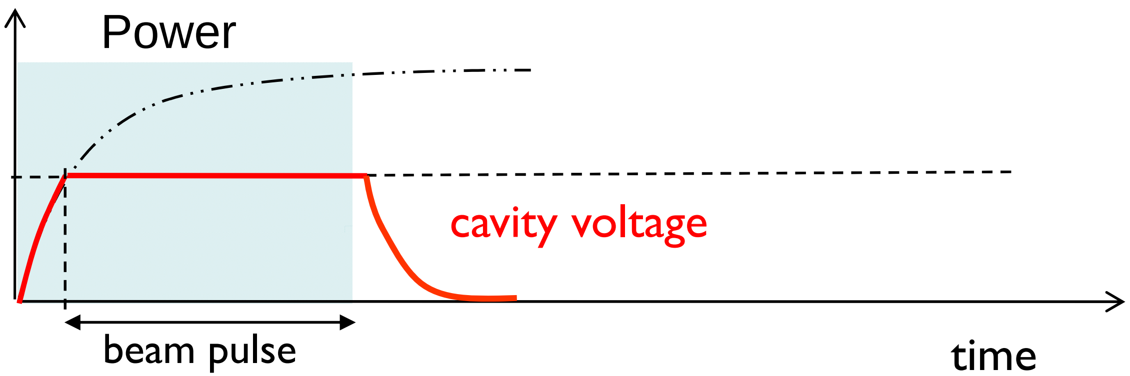
MATLAB/Simulink

$$
\begin{cases}
\tau \dfrac{dV_{cr}}{dt} + V_{cr} - yV_{ci} = R_L I_{gr} \\
\tau \dfrac{dV_{ci}}{dt} + V_{ci} + yV_{cr} = R_L I_{gi}
\end{cases}
$$

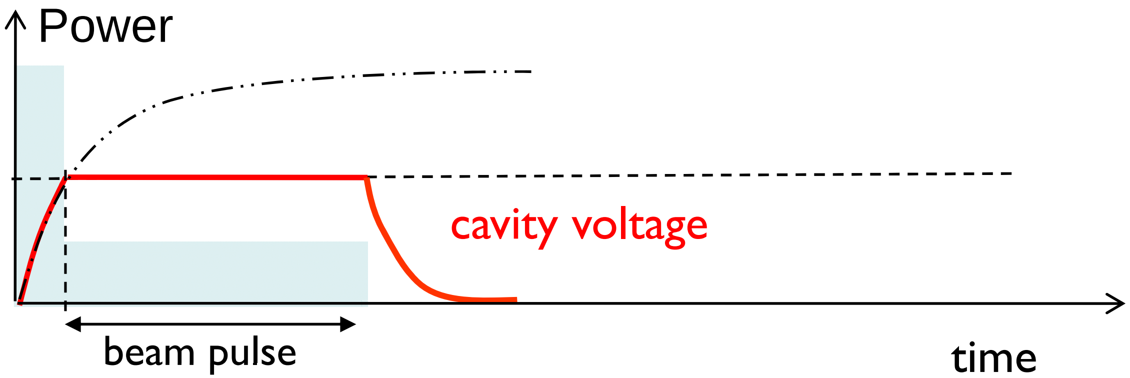


Beam representation

- Beam represented using the beam loading theorem



- Beam represented by a current generator



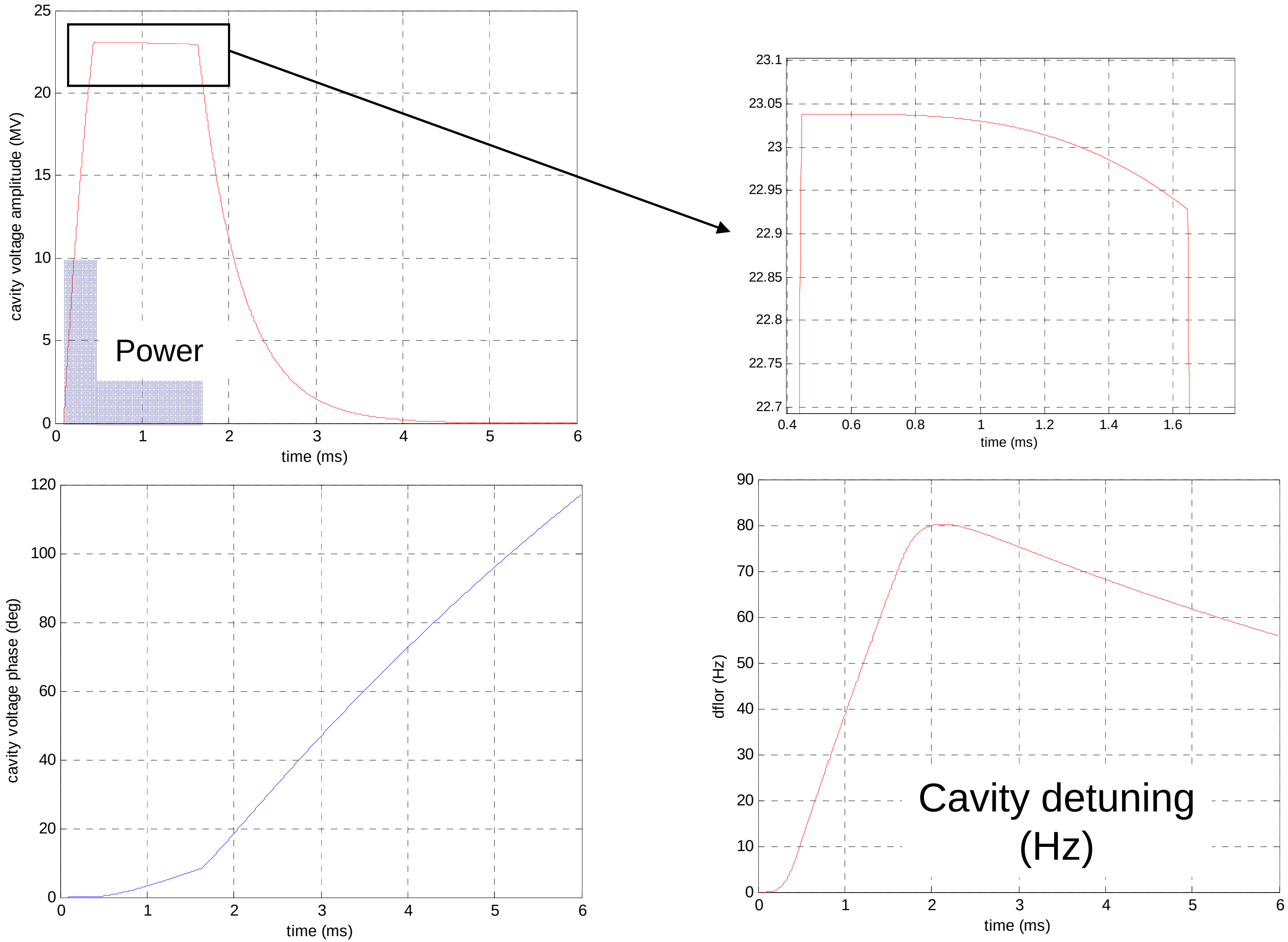
➡ Same method used in the test-stand at Saclay to test HIPPI cavity

Example of simulation

SPL cavity
β=0.92
Eacc=24MV/m
R/Q=500Ω
QL=1.1.10^6
Pulse=1.2ms

- Beam represented by a current generator

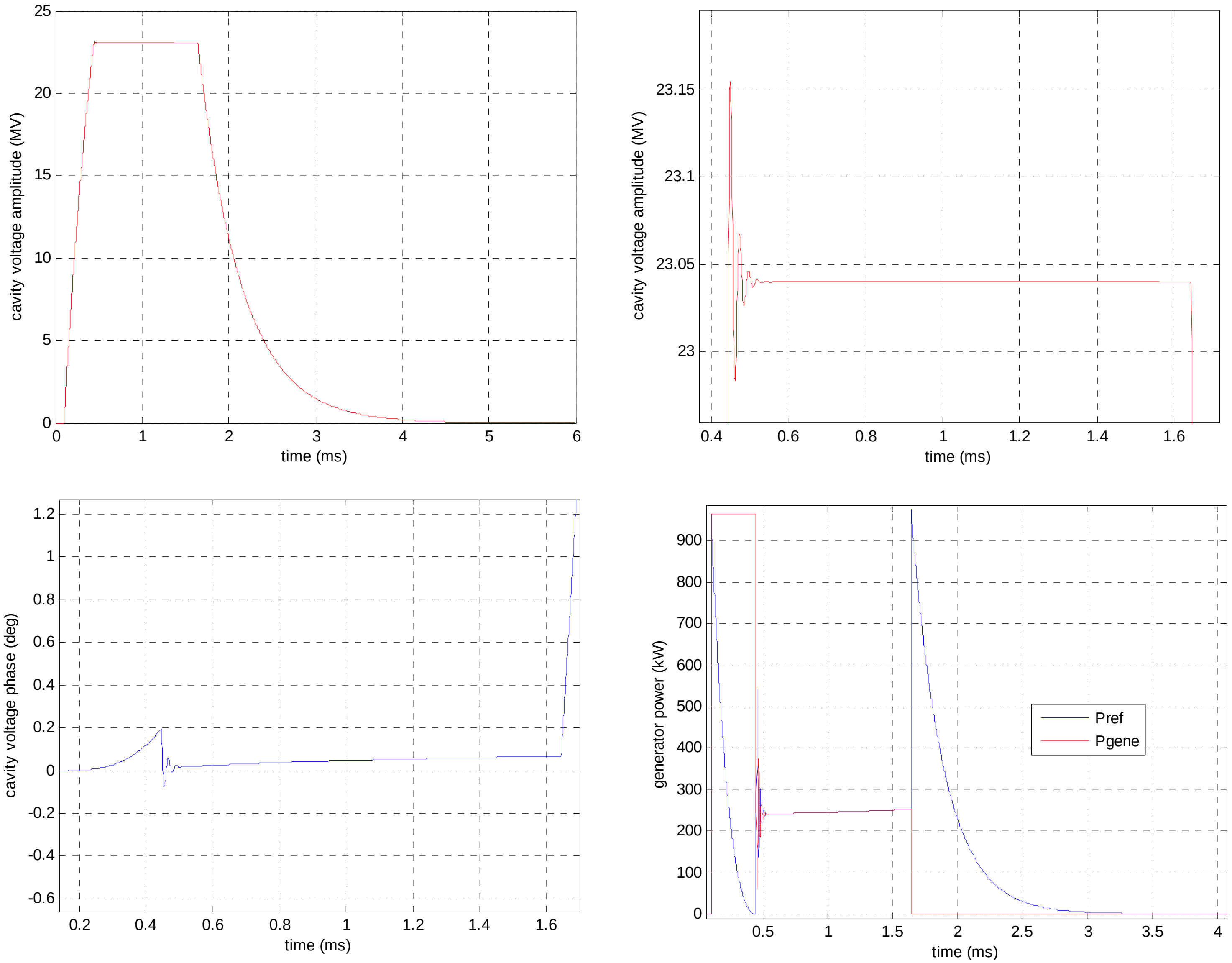
<u>Open Loop</u> Static Lorentz detuning factor K_L=-1 Hz/(MV/m)² and τ_L=10ms



Example of simulation

Closed Loop

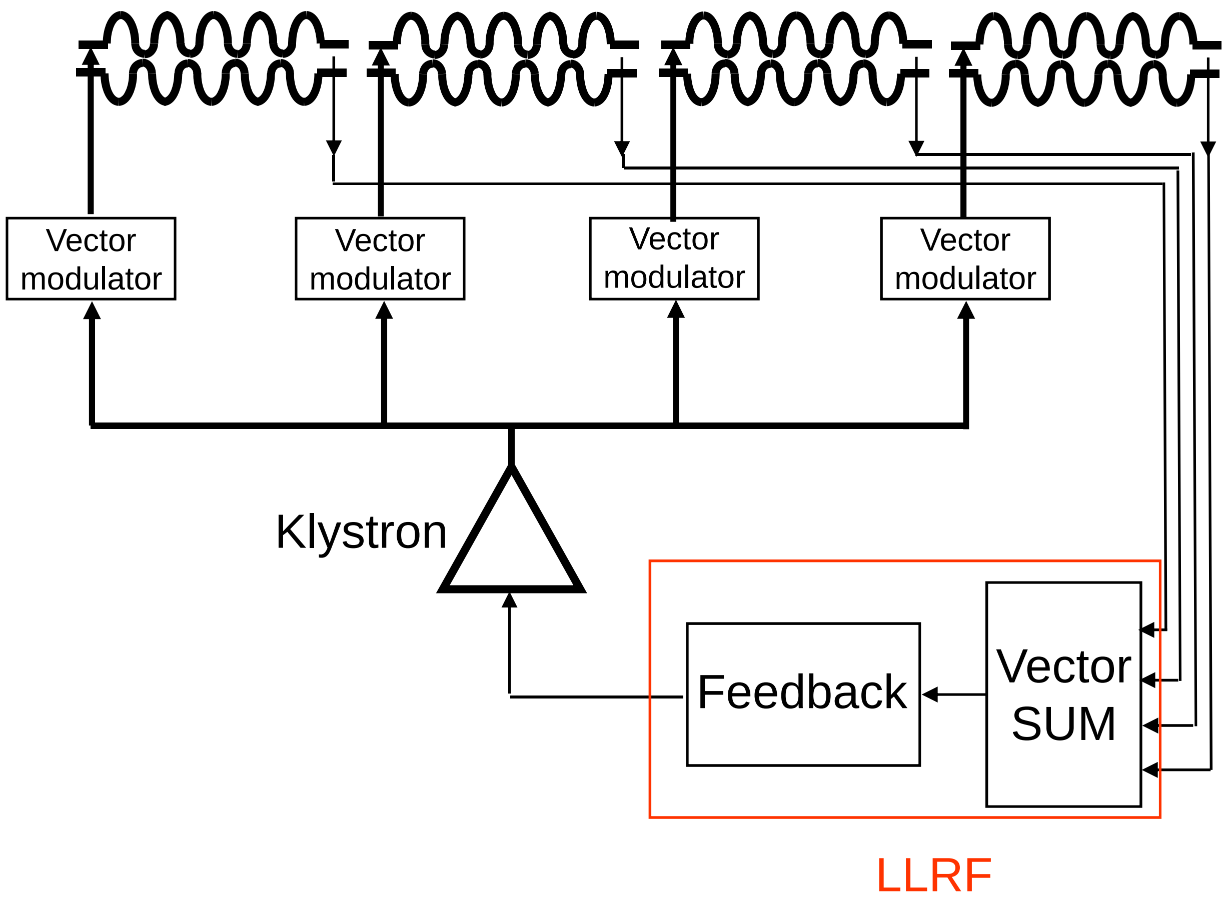
Gain P=100
Gain I=1/2^6
Delay=5μs



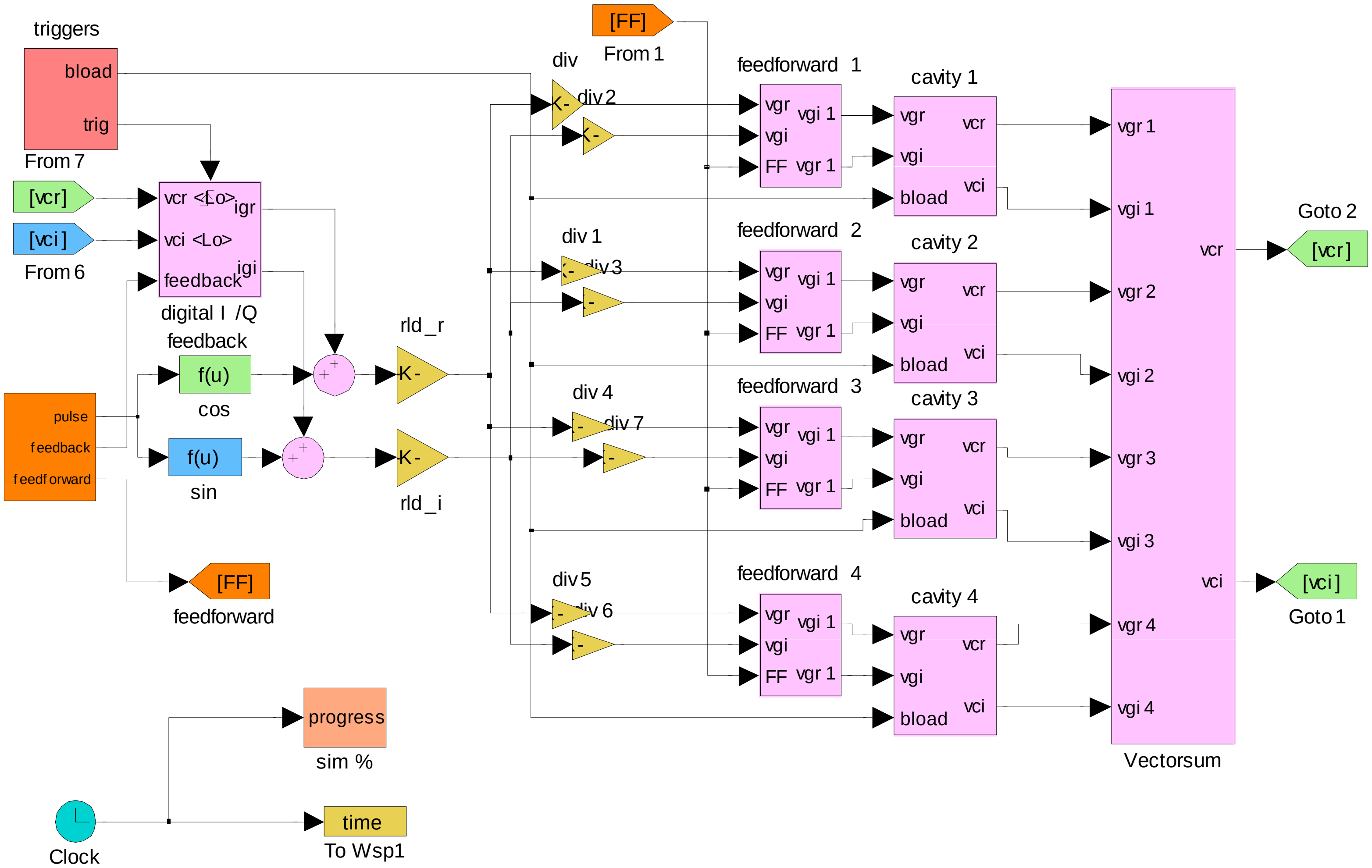
Power margin needed for the feedback

Possible Layout for SPL

- System with 1 klystron per 4 cavities:



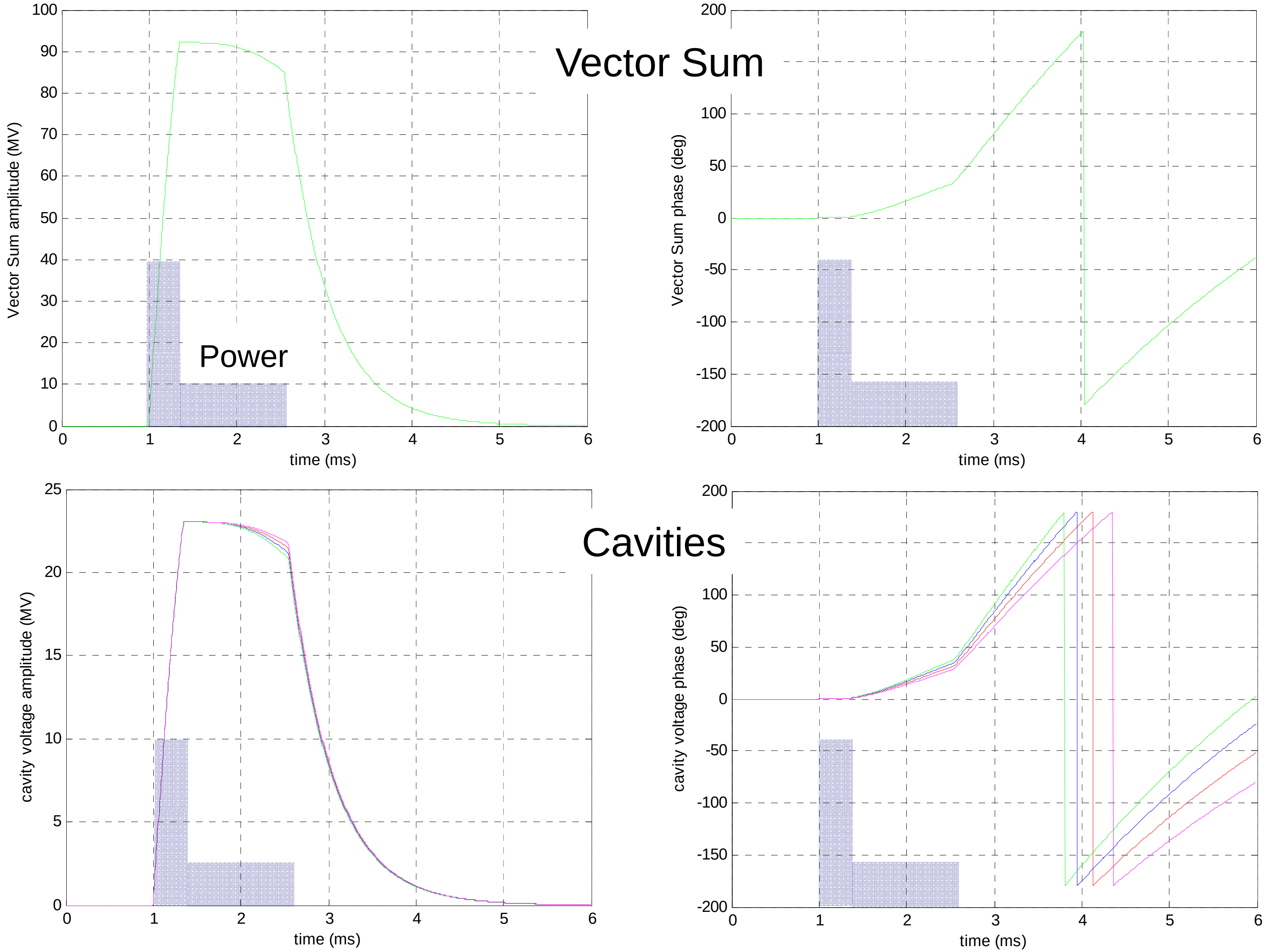
System architecture



- Feedforward: Vector modulator used as a feedforward system
- Simulations don’t take into account active Lorentz-force detuning compensation system

System simulation

<u>Open Loop</u> Static Lorentz detuning factor cavity 1 K_L= -2 Hz/(MV/m)² and τ_L=5ms
cavity 2 K_L= -2.2 Hz/(MV/m)² and τ_L=5ms
cavity 3 K_L= -2.4 Hz/(MV/m)² and τ_L=5ms
cavity 4 K_L= -1.8 Hz/(MV/m)² and τ_L=5ms

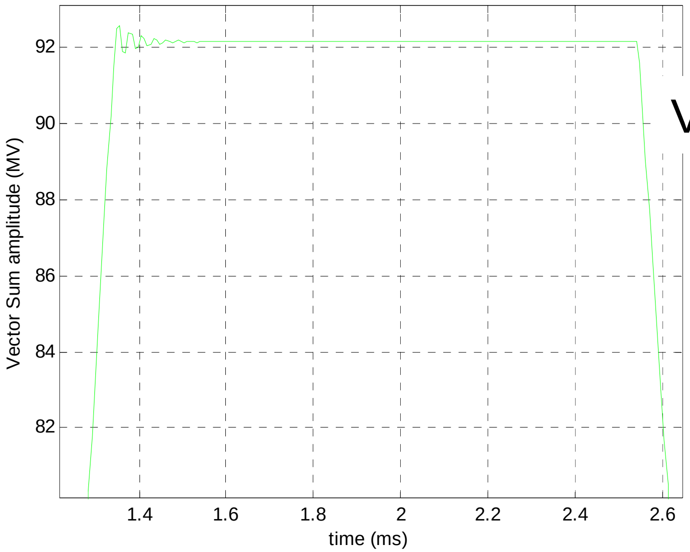


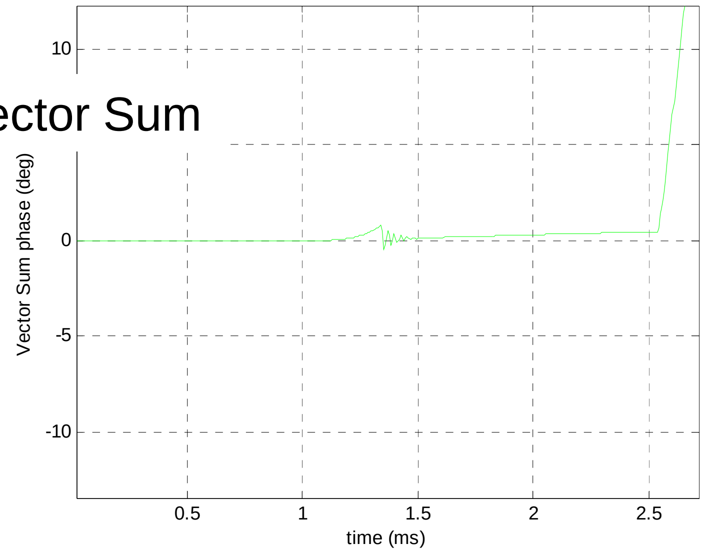
System simulation

Closed Loop

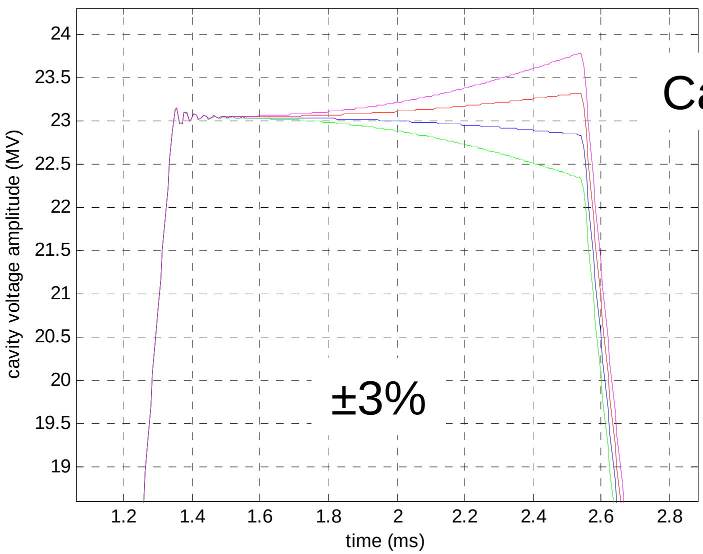
Gain P=100
Gain I=1/2^6
Delay=5µs

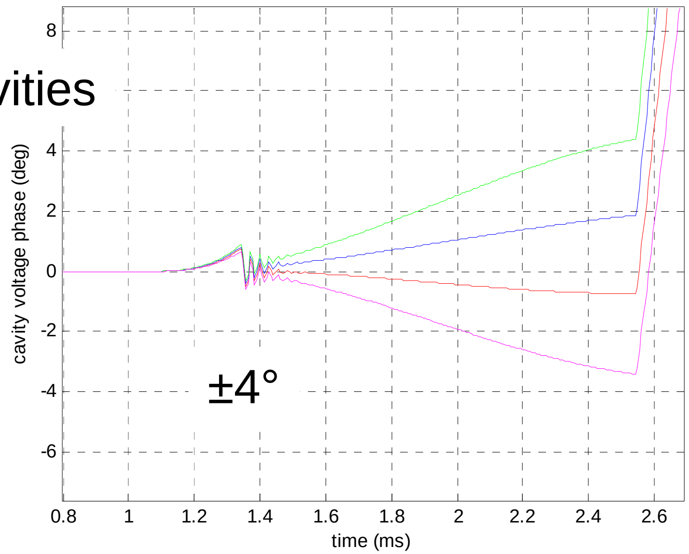
Vector Sum





Cavities



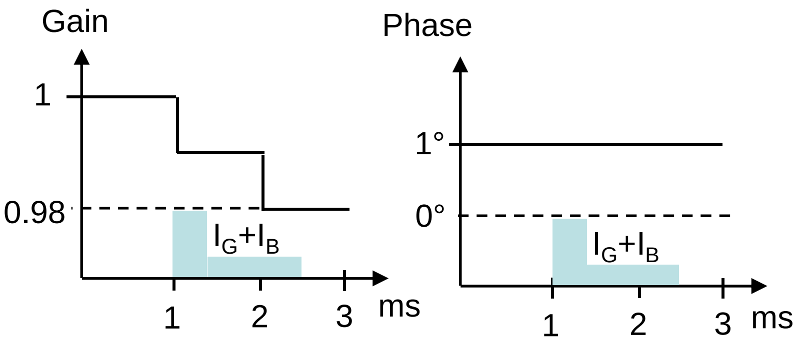


Example of simulation

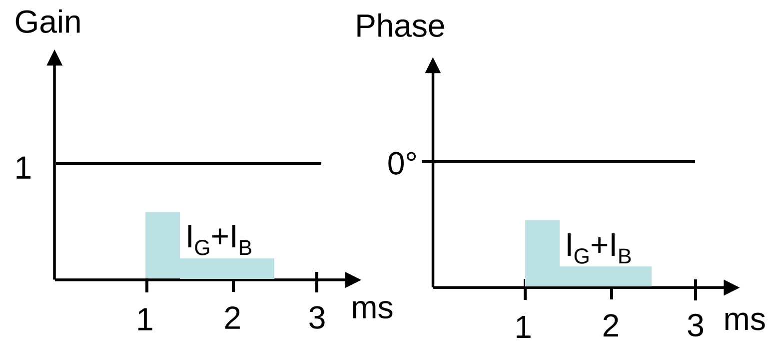
Active feedforward with Vector Modulator:

Phase of $I_{forward}$: 2°/ms
Amplitude of $I_{forward}$ 3%/ms

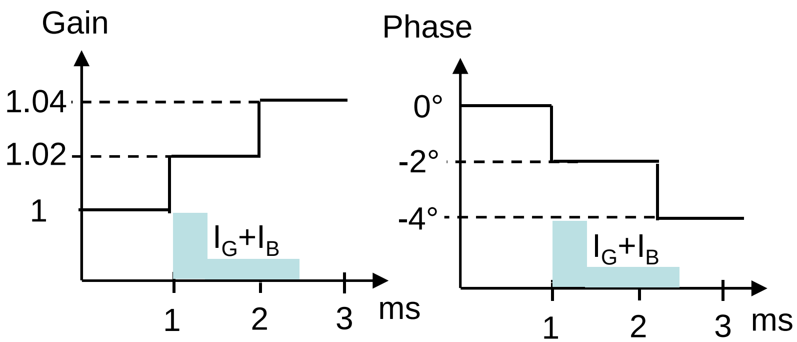
Cavity 1



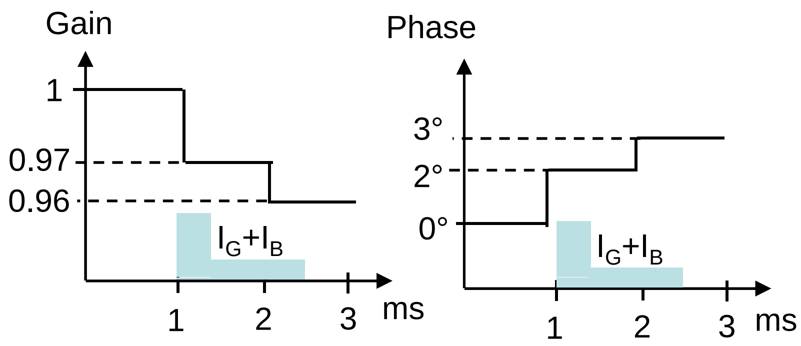
Cavity 2



Cavity 3

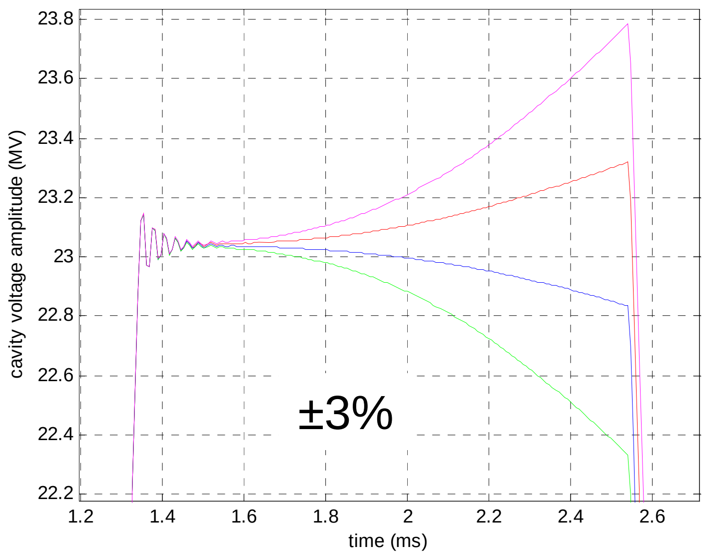


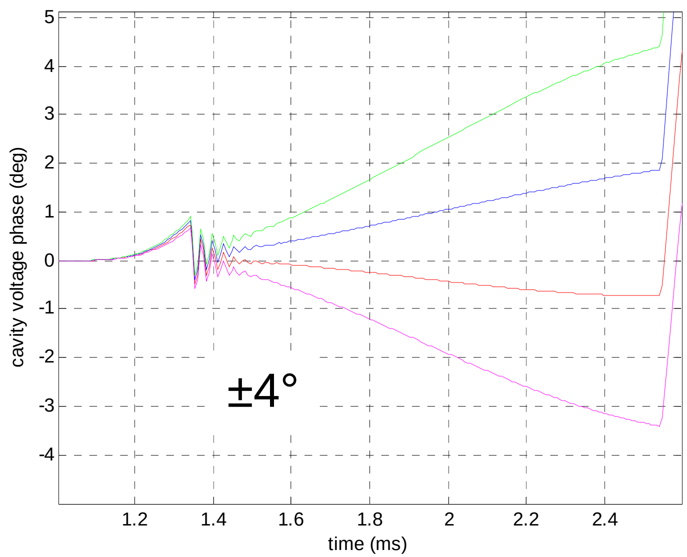
Cavity 4



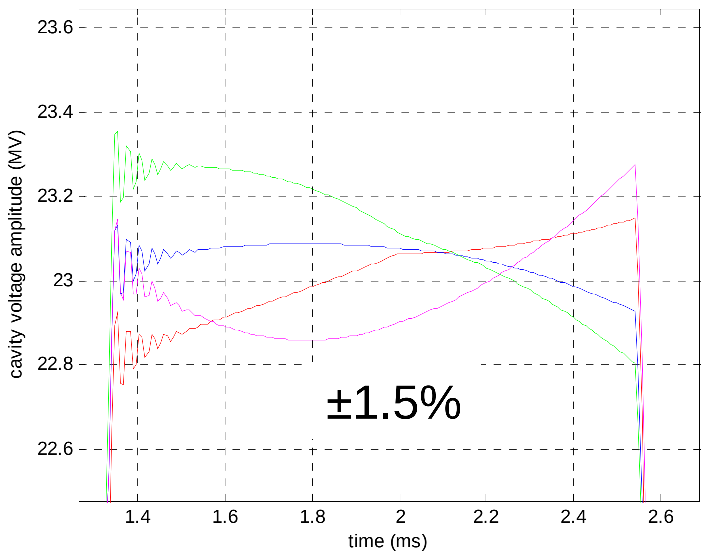
Example of simulation

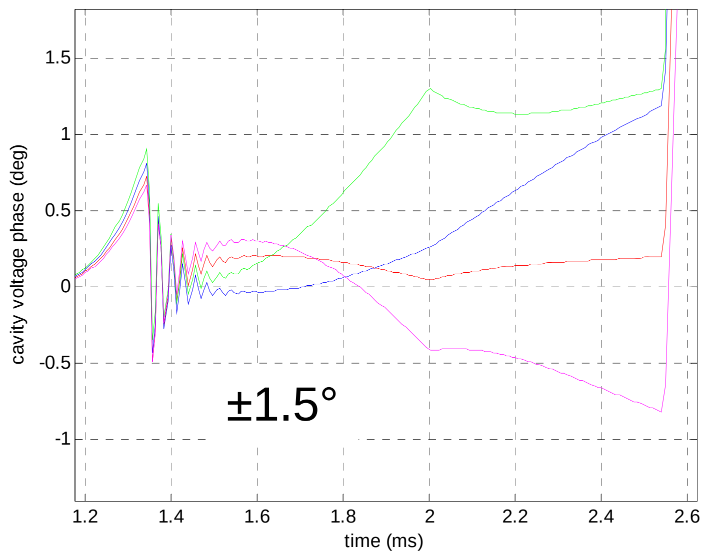
Without feedforward





With feedforward





Summary

➡ Development of tools to define LLRF system

Needs:

- Cavity and perturbation parameters

  Spread in cavity voltage, coupling, Lorentz detuning coefficients, microphonics ….

- Vector modulator parameters

- To take into account Lorentz tuning system which reduce the effect of Lorentz forces

  Measurements at HIPPI tests

- Beam parameters (Input energy, gain energy, Beam phase)

  Use of the beam loading theorem in the simulation to get errors in energy and phase for every bunch at the cavity output