Reconstruction of a missing particle with vertex information

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Decays with a missing particle

- Generally considered as not fully reconstructible
- Few well known exceptions:
  - momentum of the decaying particle and all but one of the decay products are known
  - detector hermeticity: missing energy measured precisely
  - collinear approximation ($H^0 \rightarrow \tau^+\tau^-$)
  - $k$-factor in semileptonic B decays
- But: 4-momentum of missing particle can be reconstructed with additional topological information
Example decay: \( B_s^0 \rightarrow D_s^- \ell^+ \nu \)

Six unknown variables: \( P_B^i, P_\nu^i, i = x, y \) and \( z \)

Four equations

\[
\sqrt{m_B^2 + \vec{P}_B^2} = \sqrt{m_{(D_s \ell)}^2 + \vec{P}_{(D_s \ell)}^2} + |\vec{P}_\nu|
\]

\[
\vec{P}_B = \vec{P}_{(D_s \ell)} + \vec{P}_\nu
\]
Event topology

$\text{Bs} \rightarrow \text{Ds} \, \mu \, \nu$

$\text{P}_{\text{Ds}}$ $\text{P}_{\text{Ds} \mu}$ $\text{P}_{\text{Bs}}$

$\text{Ds}$ $\mu$ $\nu$

$\text{Bs}$ $\text{P}_{\text{v}}$
New system of equations

\[ \sqrt{m_B^2 + \vec{P}_B^2} = \sqrt{m_{(D_s\ell)}^2 + \vec{P}_{(D_s\ell)}^2 + |\vec{P}_\nu|} \]

\[ |\vec{P}_B| = P_{(D_s\ell)}^\parallel + P_{\nu}^\parallel \]

\[ P_{\nu}^\perp = -P_{(D_s\ell)}^\perp \]
Solution

\[ P_ν^∥ = -a \pm \sqrt{r} \]

where

\[
a = \frac{\left( m_B^2 - m^2 - 2 \cdot P^2_⊥ \right) \cdot P^∥}{2 \cdot (P^∥ - E^2)}
\]

\[
r = \frac{\left( m_B^2 - m^2 - 2 \cdot P^2_⊥ \right)^2 \cdot E^2}{4 \cdot (P^∥ - E^2)^2} + \frac{E^2 \cdot P^2_⊥}{P^2^∥ - E^2}
\]

Here we use the following notations: \( P_⊥ = P^⊥_{(D_sℓ)} \), \( P^∥ = P^∥_{(D_sℓ)} \), \( E = E_{(D_sℓ)} \), \( m = m_{(D_sℓ)} \)
Event generation

Decay channel: $B_s^0 \rightarrow D_s^- \mu^+ \nu_\mu$, $D_s^- \rightarrow \phi \pi^-$, $\phi \rightarrow K^+ K^-$

MC generator: PYTHIA V6.227, $E_{CM}=14$ TeV.

Kinematics and resolutions

- Hadrons: $p_T \geq 1$ GeV/$c$, muon: $p_T \geq 3$ GeV/$c$.
- Momentum uncertainty:
  - pseudorapidity: $\sigma_\eta = 5.8 \times 10^{-4}$,
  - $\phi$: $\sigma_\phi = 0.58$ mrad,
  - transverse momentum: $\sigma(1/p_T) = 0.013(\text{GeV}/c)^{-1}$.
- The primary vertex: $\sigma_{x,y} = 20 \mu m$, the secondary vertex: $\sigma_{||} = 70 \mu m$ in flight direction of the $B_s^0$ and $\sigma_\perp = 10 \mu m$ in the perpendicular direction.
The most important ingredient in the measurement of the $B_s^0$ oscillation frequency is the proper time $c\tau$:

$$c\tau = \frac{L_{xy} m(B_s^0)}{p_T(B_s^0)}$$

$$c\tau = \frac{L_{xy} m(B_s^0)}{p_T(D_s\ell)} \times k$$

$$k = \frac{p_T(D_s\ell)}{p_T(B_s^0)}$$
$k$-factor distribution
Momentum resolution

\[ p_t \text{-resolution} \]

\[ m(D_s^I) \text{ [GeV/c}^2\text{]} \]

- K-factor
- \( v \) reco \( (\sigma_t = 10\mu m) \)
- \( v \) reco \( (\sigma_t = 30\mu m) \)

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Proper time resolution I

\textit{k}-factor method

\begin{align*}
\text{Const 1} & & 2652 \pm 29.8 \\
\text{Mean 1} & & 0.01909 \pm 0.00096 \\
\text{Sigma 1} & & 0.1003 \pm 0.0015 \\
\text{Const 2} & & 730.8 \pm 20.8 \\
\text{Mean 2} & & -0.01576 \pm 0.00291 \\
\text{Sigma 2} & & 0.3381 \pm 0.0043
\end{align*}

neutrino reconstruction method

\begin{align*}
\text{Const 1} & & 2273 \pm 38.9 \\
\text{Mean 1} & & -0.00161 \pm 0.00094 \\
\text{Sigma 1} & & 0.07717 \pm 0.00154 \\
\text{Const 2} & & 655.7 \pm 36.6 \\
\text{Mean 2} & & 0.004666 \pm 0.002116 \\
\text{Sigma 2} & & 0.1928 \pm 0.0033
\end{align*}

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The distributions are fitted with two Gaussian, the average width $\sigma$:

$$
\sigma^2 = \frac{N_n^2 \sigma_n^2 + N_w^2 \sigma_w^2}{N_n^2 + N_w^2},
$$

here $\sigma_n$ ($\sigma_w$) and $N_n$ ($N_w$) are the width and normalization of the narrow (wide) Gaussian.

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_n$</th>
<th>$\sigma_w$</th>
<th>$\sigma$</th>
<th>$N_n$</th>
<th>$N_w$</th>
</tr>
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<tbody>
<tr>
<td>$k$-factor</td>
<td>100 fs</td>
<td>338 fs</td>
<td>132 fs</td>
<td>2700</td>
<td>730</td>
</tr>
<tr>
<td>$\nu$-reco</td>
<td>77 fs</td>
<td>193 fs</td>
<td>91 fs</td>
<td>2300</td>
<td>660</td>
</tr>
</tbody>
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Amplitude method

- Candidates split into two samples:
  - the same flavor ($P_{unmix}$) and
  - the opposite flavor ($P_{mix}$) at production and decay.
- Proper decay time of the $B_s^0$ mesons is reconstructed.
- Two samples are used to define the time-dependent asymmetry:
  \[
  a(t) = \frac{P_{unmix} - P_{mix}}{P_{unmix} + P_{mix}} \propto A \times D \times \cos(\Delta m_s t)
  \]
  where $D$ is a global dilution factor accounting for background, miss-tagging and proper-time resolution and $A$ is the amplitude.
Amplitude fit

- In the fit the oscillation frequency $\Delta m_s$ is fixed, leaving the amplitude $A$ as a free parameter.
- A scan over $\Delta m_s$ is performed starting from zero.
- If $\Delta m_s$ is consistent with the true one, the $A \approx 1$, else $A \approx 0$.
- The error on the $A$ is calculated according to

$$\sigma_A = \frac{1}{1 - 2W} \times \sqrt{\frac{2}{S + B} \times \frac{S + B}{S}} \times e^{\frac{\Delta m_s^2 \sigma_t^2}{2}}$$

where $W$ is the mistagging probability, $S$ the number of signal, $B$ the number of background events, and $\sigma_t$ the proper time resolution.
Assumptions

- Number of signal events: **45000**
- Signal to Background ratio: **1:1**
- Mistagging probability: **40%**
- Simulated oscillation frequency: $\Delta m_s = 17.25 \text{ ps}^{-1}$
Results

\textit{k}-factor method \hspace{2cm} \textit{neutrino reconstruction method}

![Graphs showing amplitude vs. \(\Delta m_s [\text{ps}^{-1}]\) for the \(k\)-factor method and the neutrino reconstruction method.]

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S/B for the 2 methods

- Smaller number of signal and background events due to negative radicand \( r \) (factor 2)
- More background events: second (wrong) solution of quadratic equation for signal and background (factor 3)

- \( k \)-factor method: 1:1
- Neutrino reconstruction method: 1:3.

But the sensitivity of \( \nu \)-reconstruction method is at higher values of \( \Delta m_s \) due to better proper time resolution!
Sensitivity of the method

sensitivity vs $\sigma_{xy}$

sensitivity vs $\sigma_{\perp}$
Conclusion

- Missing particles can be reconstructed using vertex information.
- Example: $B_s^0\bar{B}_s^0$ oscillations with semileptonic $B_s^0$ decays.
- The sensitivity of proposed method is higher than of conventional $k$-factor method except if the vertex resolution is too bad.
- Proposed method can be used in some other cases where the known topology of a decay compensate for the incompleteness of kinematical information.
Other examples

- Life time measurements in the semileptonic $B$ decays
  - $\tau$ reconstruction in $\tau \rightarrow 3h^\pm + \nu_\tau$ decays
    - $H^0 \rightarrow \tau^+ \mu^- (?)$
    - $H^0 \rightarrow \tau^+ \tau^- (???)$
    - $B^0_s \rightarrow \tau^+ \tau^- (???)$
  - $B^0_s \rightarrow \mu^+ \mu^- + \gamma (?)$
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