Leptogenesis and LFV
in type I+II seesaw mechanism

Stéphane Lavignac (SPhT Saclay)

- introduction
- reconstruction of the right-handed neutrino spectrum
- implications for leptogenesis
- implications for lepton flavour violation
- conclusions

[based on P. Hosteins, S. L. and C. Savoy, hep-ph/0606078, NPB to appear]

4th "Flavour in the LHC era" meeting
CERN, 9-11 October 2006

Introduction

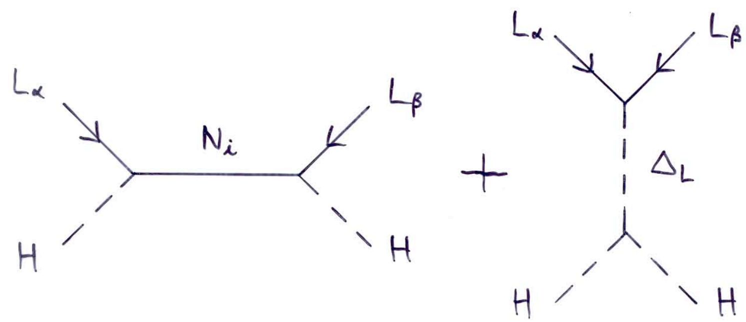
Why consider the type I+II seesaw mechanism?

- Both the type I (heavy right-handed neutrino exchange) and the type II (heavy $SU(2)_L$ triplet exchange) are present in many extensions of the SM, such as left-right symmetric theories and SO(10) GUTs.

- Right-handed neutrinos are suggestive of grand unification. However, SO(10) models with type I seesaw mechanism generally fail to accommodate successful leptogenesis ($Y_\nu \propto Y_u \Rightarrow$ very hierarchical right-handed neutrino masses, with $M_1 \sim 10^5$ GeV).

- More generally, studies of leptogenesis and LFV are usually done in the framework of the type I seesaw mechanism, or assume dominance of one of the two seesaw mechanisms. It is interesting to investigate whether the generic situation where both contributions are comparable in size can lead to qualitatively different results.

Type I+II seesaw mechanism:



Δ_L = SU(2)$_L$ triplet with couplings f_{Lij} to lepton doublets

$$M_\nu \;=\; f_L v_L - \frac{v^2}{v_R} Y^T f_R^{-1} Y \;\equiv\; M_\nu^{II} + M_\nu^I$$

Right-handed neutrino mass matrix: $M_R \;=\; f_R v_R$

$v_R \equiv \langle \Delta_R \rangle$ scale of B-L breaking

Δ_R = SU(2)$_R$ triplet with couplings f_{Rij} to right-handed neutrinos

v_L is small since it is an induced vev: $v_L \;\equiv\; \langle \Delta_L \rangle \sim v^2 v_R / M^2_{\Delta_L}$

In theories with underlying left-right symmetry (such as SO(10) with a $\overline{126}_H$), one has $Y = Y^T$ and $f_L = f_R$ $\Rightarrow$ 2 matrices of couplings Y and f

In a fundamental theory, expect Y to be related to other Yukawa couplings $\Rightarrow$ for phenomenological studies, need to reconstruct the f_{ij} as a function of the Y_{ij} (for a given set of low-energy neutrino parameters)

For n generations, there are 2^n different solutions [Akhmedov, Frigerio]

Reconstruction of the heavy neutrino mass spectrum

The left-right symmetric seesaw formula $M_\nu \;=\; f v_L - \frac{v^2}{v_R} Y f^{-1} Y$

with f, Y complex symmetric matrices (Y invertible), can be rewritten as

$$Z \;=\; \alpha X - \beta X^{-1} \qquad\qquad \alpha \;\equiv\; v_L \;, \quad \beta \;\equiv\; \frac{v^2}{v_R}$$

with $Z = N_Y^{-1} M_\nu (N_Y^{-1})^T,\; X = N_Y^{-1} f (N_Y^{-1})^T,\; N_Y \text{ such that } Y = N_Y N_Y^T$

Z complex symmetric ⇒ can be diagonalized by a complex orthogonal matrix O_Z if its eigenvalues z_i are all distinct:

$$Z \;=\; O_Z \mathrm{Diag}\,(z_1, z_2, z_3) O_Z^T \;, \qquad O_Z O_Z^T \;=\; \mathbf{1}$$

Then X can be diagonalized by the same orthogonal matrix as Z, and its eigenvalues are the solutions of:

$$z_i \;=\; \alpha x_i - \beta x_i^{-1} \quad (i = 1, 2, 3)$$

2 solutions x_i^+, x_i^- for each i ⇒ 2^3 = 8 solutions for X, hence for f:

$$f \;=\; N_Y O_Z \begin{pmatrix} x_1 & 0 & 0 \\ 0 & x_2 & 0 \\ 0 & 0 & x_3 \end{pmatrix} O_Z^T N_Y^T \;, \qquad x_i = x_i^\pm$$

[see Akhmedov, Frigerio for an alternative reconstruction procedure]

Properties of the solutions

We denote the 2 solutions of $z_i = \alpha x_i - \beta x_i^{-1}$ by:

$$x_i^{\pm} \equiv \frac{z_i \pm \sqrt{z_i^2 + 4\alpha\beta}}{2\alpha}$$

(+,+,+) refers to the solution (x_1^+, x_2^+, x_3^+), (+,+,–) to (x_1^+, x_2^+, x_3^-), etc

In the large v_R limit ($4\alpha\beta \ll |z_1|^2$):

$$x_i^+ \simeq \frac{z_i}{\alpha} \quad (\text{"type II branch"}) \qquad\qquad f^{(+,+,+)} \longrightarrow \frac{M_\nu}{v_L}$$

$$x_i^- \simeq -\frac{\beta}{z_i} \quad (\text{"type I branch"}) \qquad\qquad f^{(-,-,-)} \longrightarrow -\frac{v^2}{v_R} Y M_\nu^{-1} Y$$

The remaining 6 solutions correspond to mixed cases in which M_ν receives significant contributions from both seesaw mechanisms

In the small v_R limit ($|z_3|^2 \ll 4\alpha\beta$):

$$x_i^{\pm} \simeq \pm\sqrt{\beta/\alpha} \qquad\qquad f^{(\pm,\pm,\pm)} \longrightarrow \pm\sqrt{\beta/\alpha}\, Y$$

If Y is hierarchical, $f_i \longrightarrow \sqrt{\beta/\alpha}\, y_i$ holds for all 8 solutions

Application to SO(10) models with two 10's and a $\overline{126}$ in the Higgs sector

$$W \ \ni \ Y^{(1)}_{ij}\, 16_i 16_j 10_1 + Y^{(2)}_{ij}\, 16_i 16_j 10_2 + f_{ij}\, 16_i 16_j \overline{126}$$

If the doublets in the $\overline{126}$ have no vev, then, in the basis of charged lepton mass eigenstates:

$$Yv \ = \ U_q^T \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} U_q \qquad\qquad U_q \ = \ P_u V_{CKM} P_d$$

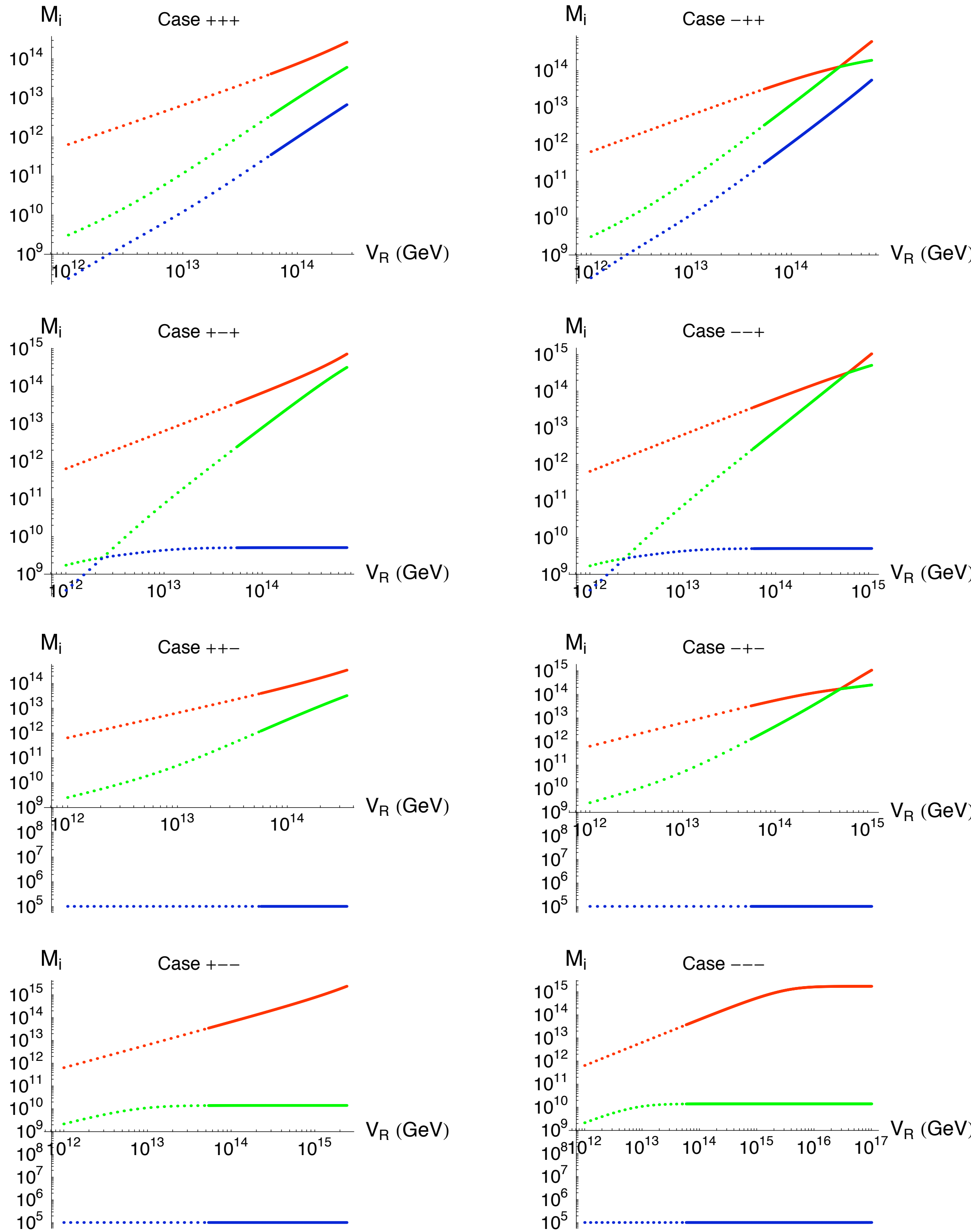
$$M_\nu \ = \ U_l^\star \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} U_l \qquad\qquad U_l \ = \ P_e U_{PMNS} P_\nu$$

For a given choice of the yet unmeasured neutrino parameters (including the Majorana phases contained in P_ν) and of the high energy phases contained in P_u, P_d and P_e, Y and M_ν are known and f can be reconstructed as a function of the B-L breaking scale v_R and of β/α

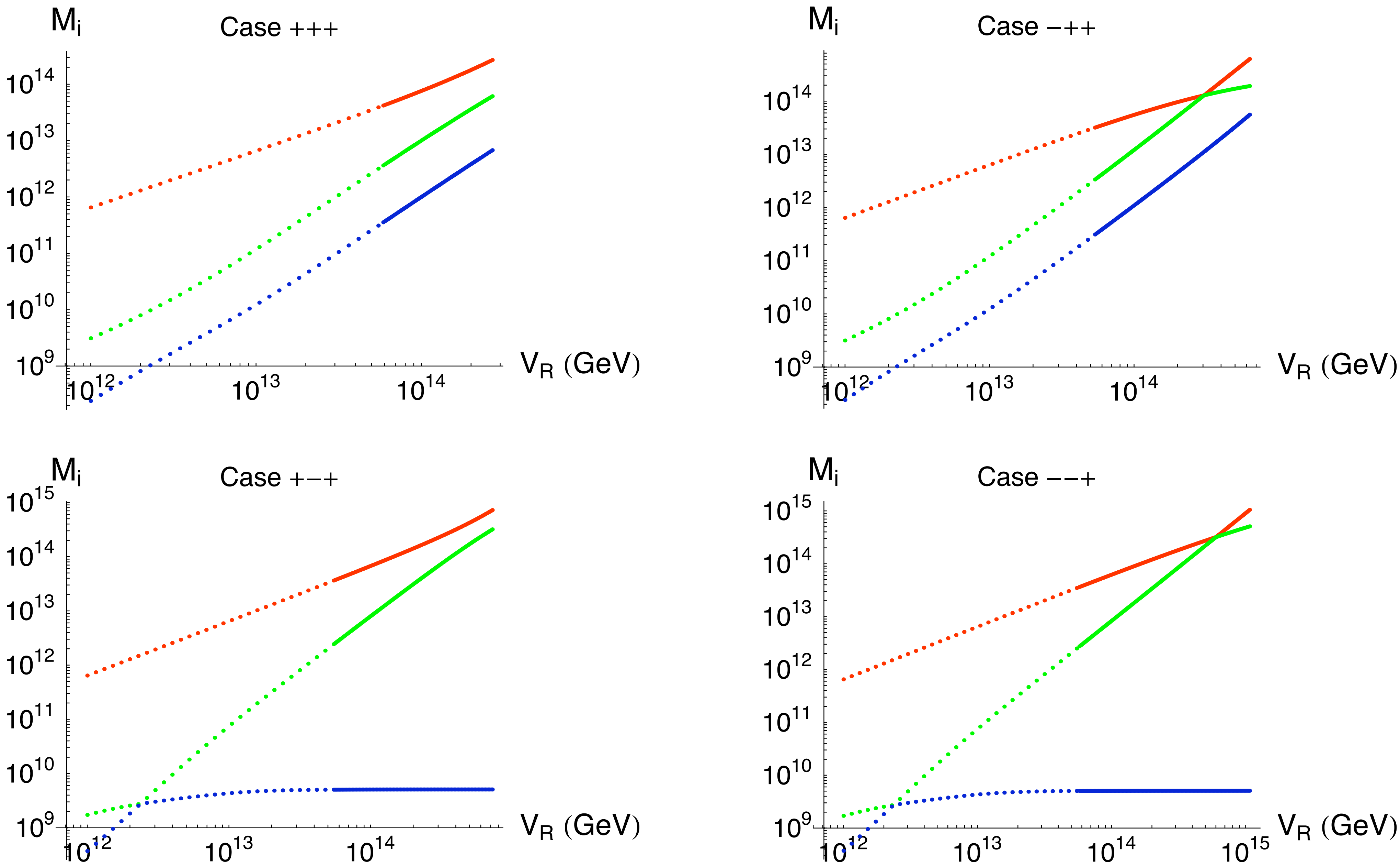
$\beta/\alpha = v^2/v_L v_R \sim (M_{\Delta_L}/v_R)^2$ depends on details of the model. Assume $\beta/\alpha = 1$.

Perturbativity constraint: require that the f_{ij} remain perturbative up to the Landau pole of the SO(10) gauge coupling (~ 2 x 10^{17} GeV)

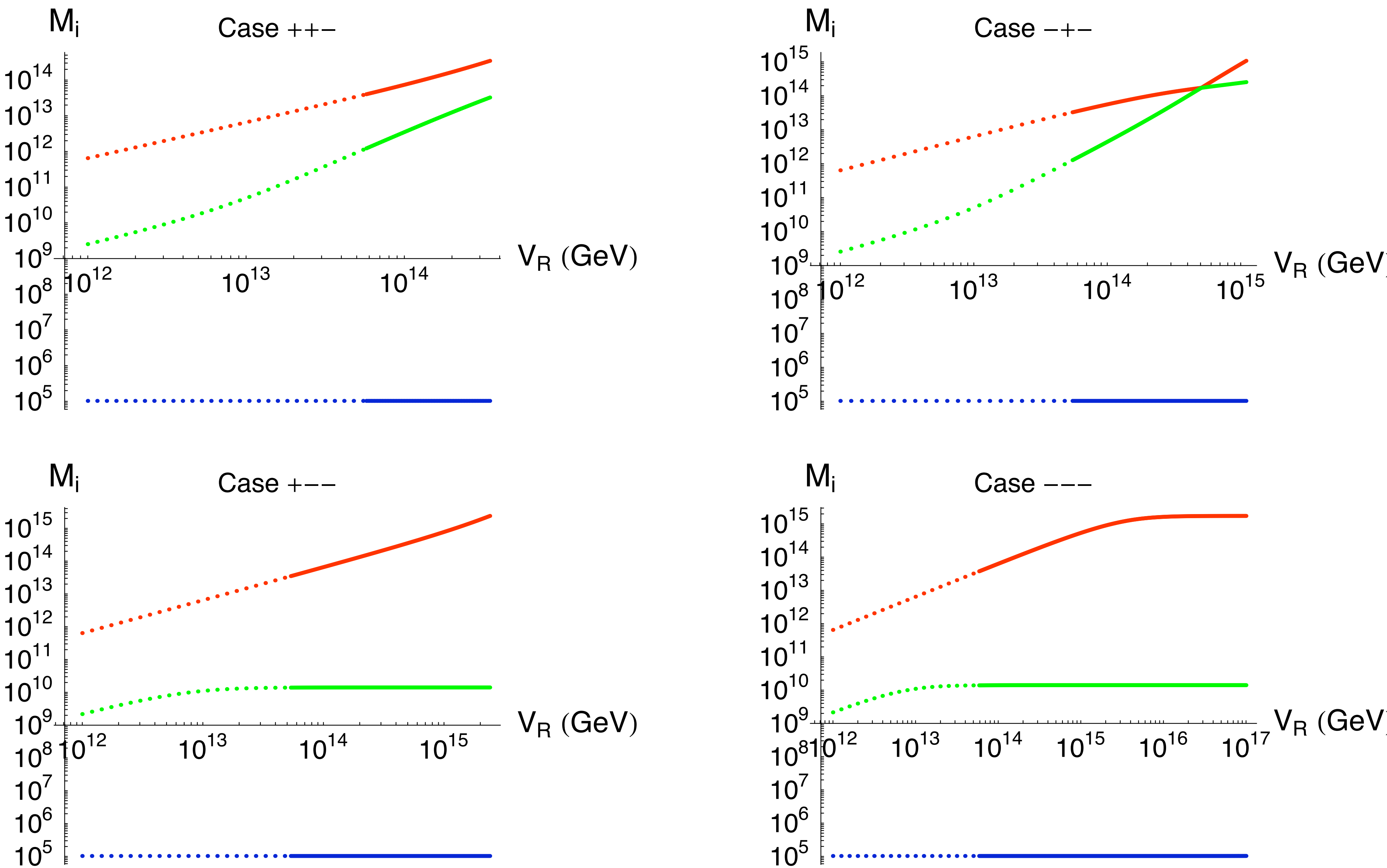
⇒ constrains $\beta/\alpha \leq O(1)$ and restricts the range of v_R



Case +++
Case −++
Case +−+
Case −−+
Case ++−
Case −+−
Case +−−
Case −−−
M_i
V_R (GeV)
10^{9} 10^{10} 10^{11} 10^{12} 10^{13} 10^{14} 10^{15}
10^{5} 10^{6} 10^{7} 10^{8}
10^{12} 10^{13} 10^{14} 10^{15} 10^{16} 10^{17}



M_i
Case +++
V_R (GeV)
Case −++
Case +−+
Case −−+



M_i
Case ++−
Case −+−
Case +−−
Case −−−
V_R (GeV)

Features of the right-handed neutrino spectrum

– at large v_R, the solutions (+,+,+) and (–,–,–) tend to the type II (triplet exchange) and type I (heavy neutrino exchange) cases:

$$(+,+,+): \qquad M_1 : M_2 : M_3 \sim m_1 : m_2 : m_3$$

$$(-,-,-): \qquad M_1 : M_2 : M_3 \sim m_u^2 : m_c^2 : m_t^2$$

– at small v_R, the type I and type II contribution compensate for each other in such a way that $M_1 : M_2 : M_3 \sim m_u : m_c : m_t$

– 4 solutions are characterized by $M_1 \approx 10^5 \,\mathrm{GeV}$

– 2 solutions are characterized by $M_1 \approx 5 \times 10^9 \,\mathrm{GeV}$

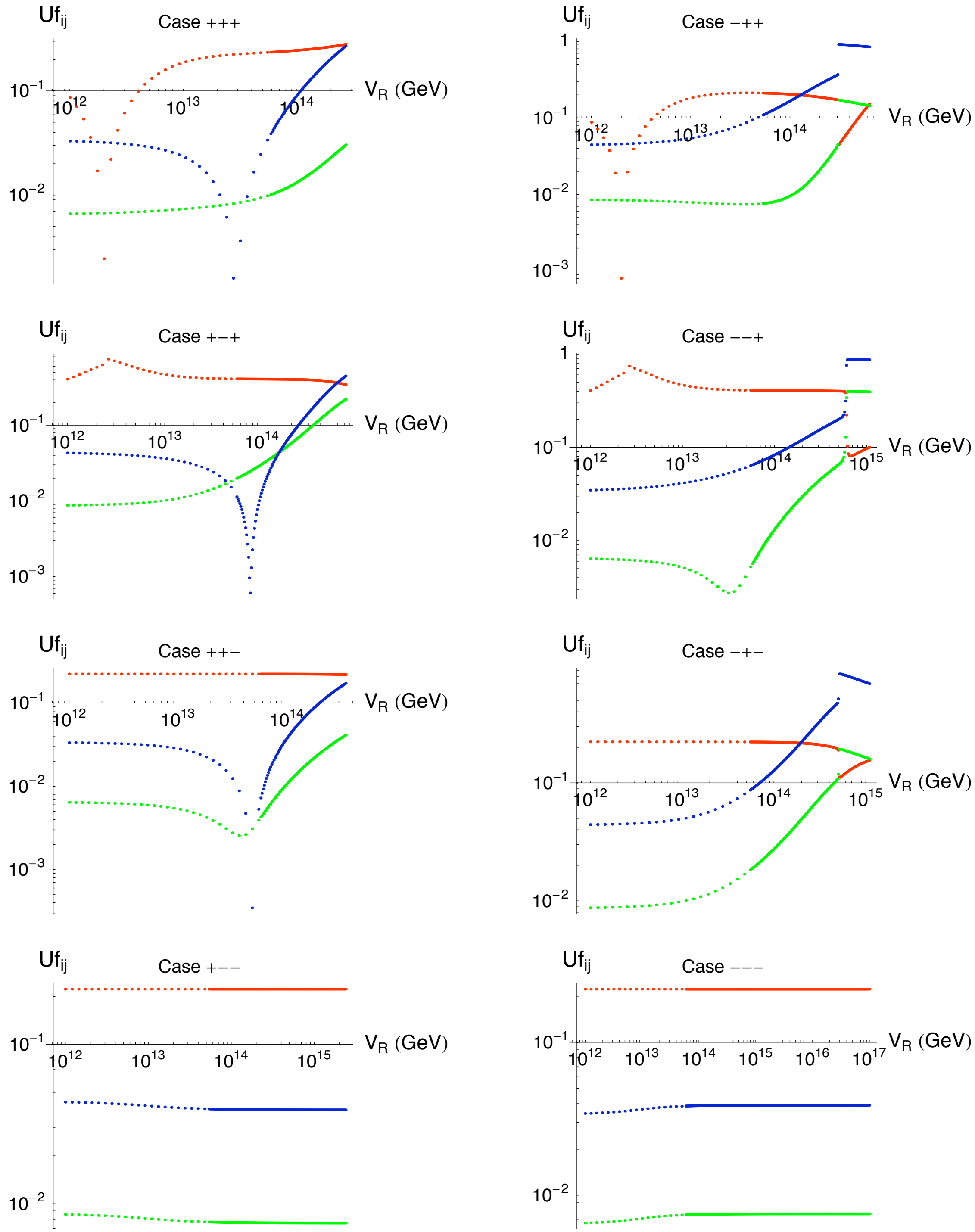
Mixing angles

$$f = U_f \begin{pmatrix} f_1 & 0 & 0 \\ 0 & f_2 & 0 \\ 0 & 0 & f_3 \end{pmatrix} U_f^T \quad \Longrightarrow \quad U_f^\dagger Y$$

Dirac couplings in the basis of RHN mass eigenstates

- 2 solutions have RHN mixing angles very close to the CKM angles

- in the other 6 solutions, the mixing angles are close to the CKM angles at small v_R, then take larger values at large v_R

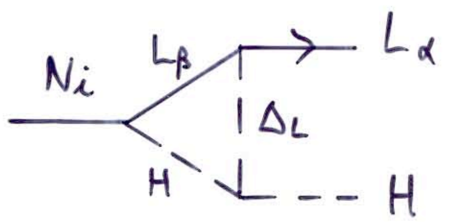


Uf_{ij}
Case +++
V_R (GeV)
10^{-1}
10^{-2}
10^{12}
10^{13}
10^{14}
Uf_{ij}
Case −++
V_R (GeV)
1
10^{-1}
10^{-2}
10^{-3}
10^{12}
10^{13}
10^{14}
Uf_{ij}
Case +−+
V_R (GeV)
10^{-1}
10^{-2}
10^{-3}
10^{12}
10^{13}
10^{14}
Uf_{ij}
Case −−+
V_R (GeV)
1
10^{-1}
10^{-2}
10^{12}
10^{13}
10^{14}
10^{15}
Uf_{ij}
Case ++−
V_R (GeV)
10^{-1}
10^{-2}
10^{-3}
10^{12}
10^{13}
10^{14}
Uf_{ij}
Case −+−
V_R (GeV)
10^{-1}
10^{-2}
10^{12}
10^{13}
10^{14}
10^{15}
Uf_{ij}
Case +−−
V_R (GeV)
10^{-1}
10^{-2}
10^{12}
10^{13}
10^{14}
10^{15}
Uf_{ij}
Case −−−
V_R (GeV)
10^{-1}
10^{-2}
10^{12}
10^{13}
10^{14}
10^{15}
10^{16}
10^{17}

Leptogenesis

$M_1 \ll M_2, M_3, M_{\Delta_L}$ ⇒ Y_B determined by the decays of N_1

The triplet contributes to ϵ_{N_1} through the vertex diagram:



$$\Rightarrow \quad \epsilon_{N_i} \;=\; \epsilon^{I}_{N_i} + \epsilon^{II}_{N_i}$$ [Hambye, Senjanovic]

The total CP asymmetry can be written [Hambye, Senjanovic; Antusch, King]:

$$\epsilon_{N_1} \;=\; \epsilon^{I}_{N_1} + \epsilon^{II}_{N_1} \;=\; \frac{3}{8\pi}\, \frac{\sum_{k,l} \mathrm{Im}\left[Y_{1k} Y_{1l} \left(M_\nu\right)^{\star}_{kl}\right]}{(YY^\dagger)_{11}}\, \frac{M_1}{v^2}$$

It depends on the reconstructed f_{ij} couplings through M_1 and U_f (since Y is expressed in the basis of RHN mass eigenstates). The high-energy phases enter this expression directly (in Y and M_ν) and indirectly (via U_f)

Since the triplet is heavy, the dilution of the generated lepton asymmetry mainly depends on the effective mass parameter:

$$\tilde{m}_1 \;=\; \frac{(YY^\dagger)_{11}\, v^2}{M_1}$$

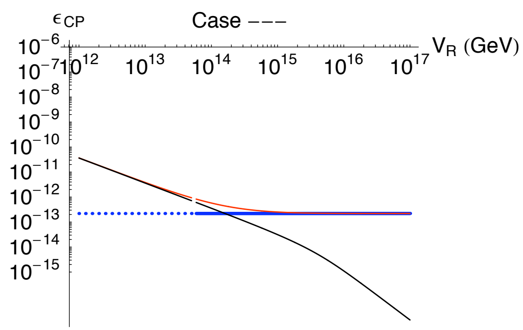
which also depends on M_1 and U_f

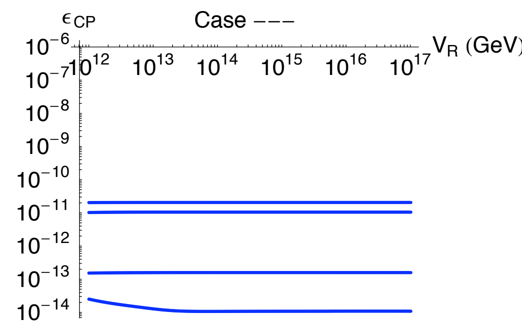
CP asymmetry versus the B-L breaking scale

Hierarchical light neutrino mass spectrum with $m_1 = 10^{-3}$ eV (and $\sin^2\theta_{13} = 0.009$, $\delta = 0$) – various choices of the Majorana and high-energy phases

3 different behaviours among the 8 solutions for f

1) solutions (±,±,–) [$M_1 \approx 10^5\,\mathrm{GeV}$]

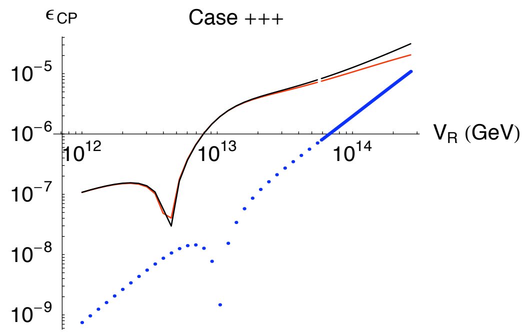


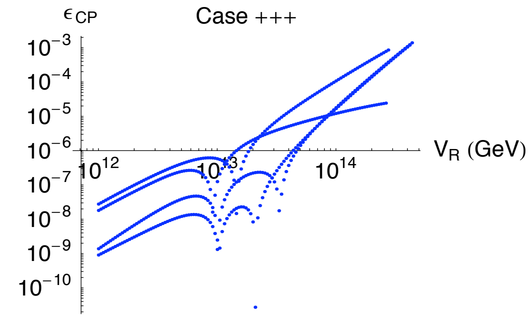


Leptogenesis fails to produce the observed baryon asymmetry for these solutions ($|\epsilon_{N_1}| \leq$ few 10^{-11} ⇒ no improvement wrt the type I case)

N_2 decays [Di Bari;Vives] do not seem to help, at least for our choice of parameters: N_2 decays produce an asymmetry mainly in tau leptons, but the asymmetry is rather small and the wash-out of the tau flavour is strong ($\epsilon_2^\tau \sim 10^{-7},\ \tilde{m}_1^\tau \sim 10^{-2}$).

2) solutions (±,+,+)

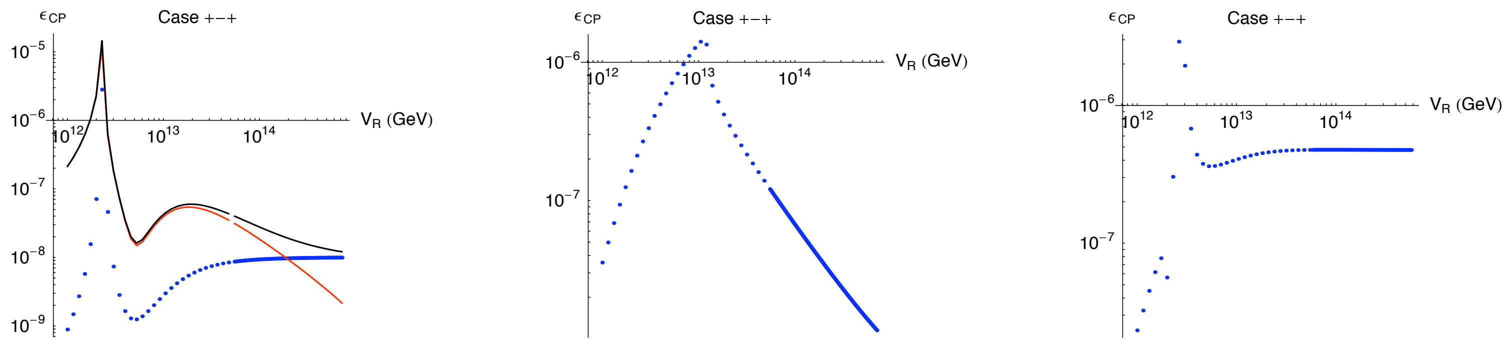




ϵ_{N1} can reach large values, even if the only input phase is the CKM phase (left panel), but the wash-out tends to be strong (typically $\tilde{m}_1 \sim 10^{-2}\,\mathrm{eV}$)

This can be compensated for by large values of ϵ_{N1}, but at the price of a larger M_1 ($|\epsilon_{N1}| \geq 10^{-5}$ for $M1 \geq 10^{11}$ GeV) ⇒ conflict with gravitino overproduction in the supersymmetric case

3) solutions (±,–,+) [$M_1 \approx 5 \times 10^9\,\mathrm{GeV}$]



The peaks in the left and right panels are due to a level crossing $M_1 = M_2$; there resonant leptogenesis is possible (but strong cancellation between the type I and type II contributions to M_ν)

ϵ_{N_1} can reach the few 10^{-7} level (10^{-6} in some cases), but the wash-out tends to be too important ($\tilde{m}_1 \sim 10^{-2}\,\mathrm{eV}$)

However, different values of the input parameters (light neutrino mass parameters; phases) or the inclusion of corrections leading to realistic charged fermion masses could improve the situation

Lepton flavour violation

We can estimate the RHN + triplet contribution [Borzumati, Masiero; Rossi] to flavour violation in the slepton sector by:

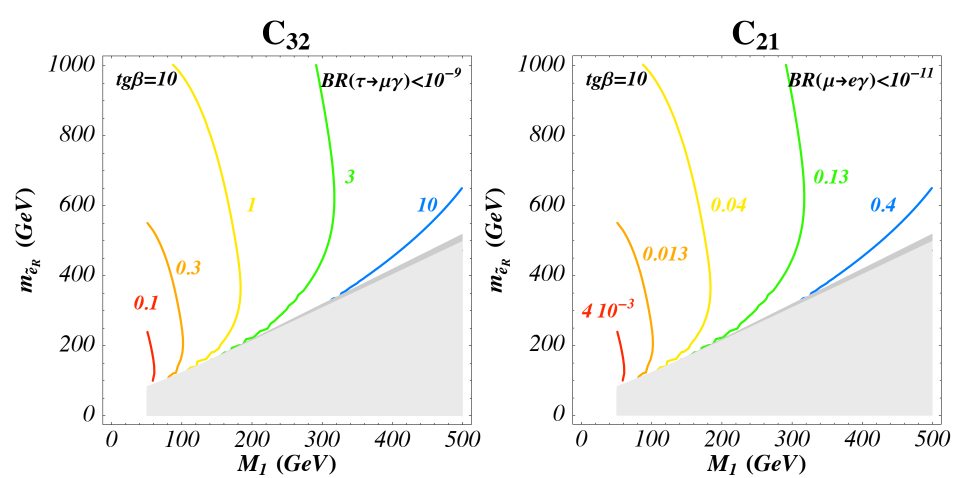
$$(m^2_{\tilde{L}})_{ij} \simeq -\frac{3m_0^2 + A_0^2}{8\pi^2}\, C_{ij}\,, \qquad (m^2_{\tilde{e}_R})_{ij} \simeq 0\,, \qquad A^e_{ij} \simeq -\frac{3}{8\pi^2}\, A_0 y_{e_i} C_{ij}$$

where the C_{ij}'s encapsulate the dependence on the seesaw parameters:

$$C_{ij} \equiv \sum_k Y^\star_{ki} Y_{kj} \ln\left(\frac{M_U}{M_k}\right) + 3\,(f f^\dagger)_{ij} \ln\left(\frac{M_U}{M_{\Delta_L}}\right)$$

M_U = scale where universality among soft terms is assumed (we take $M_U = 10^{17}$ GeV and $M_{\Delta_L} = v_R$)

Experimental upper limits on the LFV decays $l_i \to l_j \gamma$ can be turned into upper bounds on the C_{ij}'s as a function of the supersymmetric mass parameters and of $\tan\beta$ [S.L., Masina, Savoy]:



We can then compare the predicted Cij's for a given solution f with the "experimental" upper bounds $|C_{23}| \leq 10$ ($\tau \to \mu\gamma$) and $|C_{12}| \leq 0.1$ ($\mu \to e\gamma$) [taking $\tan\beta = 10$ and $m_0, M_{1/2} \leq O(1\text{ TeV})$]:

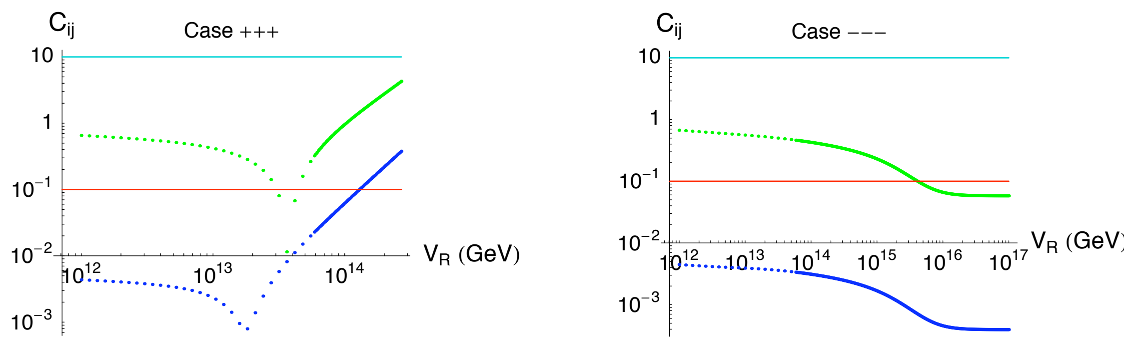


Figure 9: Coefficients C_{12} and C_{23} as a function of v_R for the solutions $(+,+,+)$ and $(-,-,-)$ in the case of a hierarchical light neutrino mass spectrum with $m_1 = 10^{-3}$ eV, $\beta = \alpha$, and no CP violation beyond the CKM phase. The green [light grey] curve corresponds to $|C_{23}|$, and the blue [black] curve to $|C_{12}|$. The horizontal lines indicate the "experimental" constraints $|C_{23}| < 10$ and $|C_{12}| < 0.1$ (see text).

Due to the small CKM angles ($V_L = U_q$), the type II contribution always dominates in the C_{ij}'s, except in the large v_R region of solutions (–,–,–) [type I limit] and (+,–,–)

The predictions lie significantly below the experimental bounds, except in the large v_R region where, depending on the supersymmetric parameters, $\mu \to e\gamma$ can exceed its present upper limit

Conclusions

- The possibilities to account for the observed neutrino data in the left-right symmetric seesaw mechanism is much richer than in the cases of type I and type II dominance, with interesting implications for leptogenesis and LFV
- In particular, the mixed solutions where both seesaw mechanisms give a significant contribution to neutrino masses provide new opportunities for successful leptogenesis in SO(10) GUTs
- A detailed study of leptogenesis in realistic SO(10) models (including a correct description of charged fermion masses and taking into account the wash-out of the lepton asymmetry) is in progress

Back-up slides

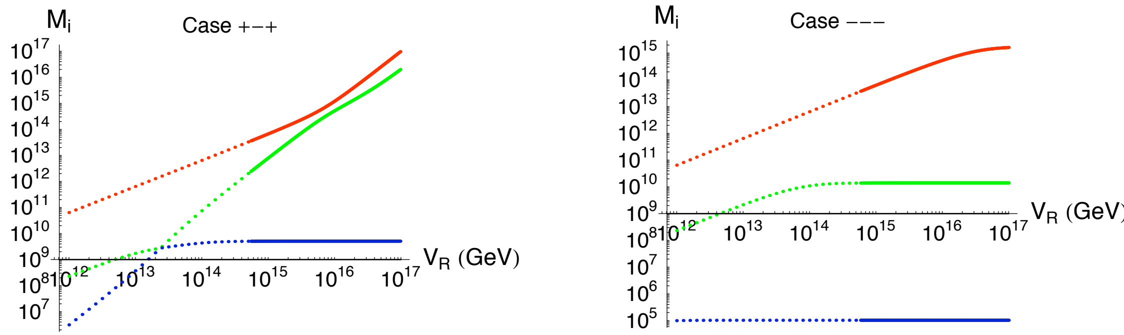


Figure 3: Effect of $\beta \neq \alpha$ on the right-handed neutrino masses. The input parameters are the same as in Fig. 1, except $\beta/\alpha = 0.01$.

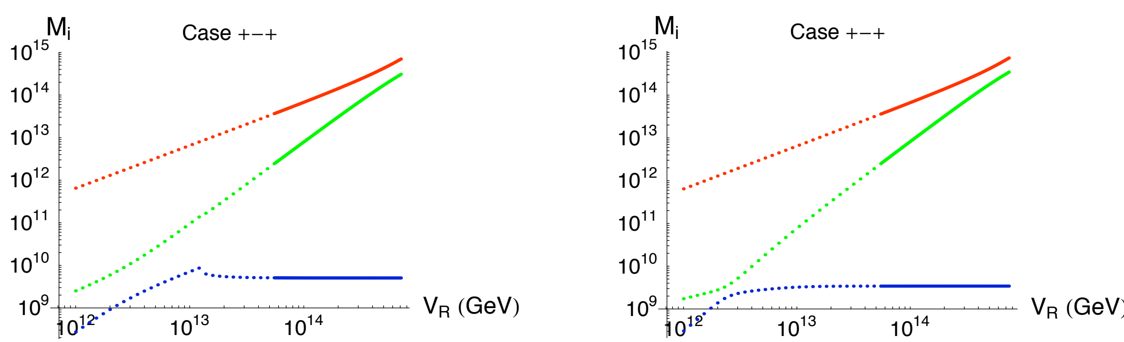


Figure 4: Effect of high-energy phases on the right-handed neutrino masses. The input parameters are the same as in Fig. 1, except $\Phi_2^u = \pi/4$ (left panel), $\Phi_1^d = \pi/4$ (right panel).

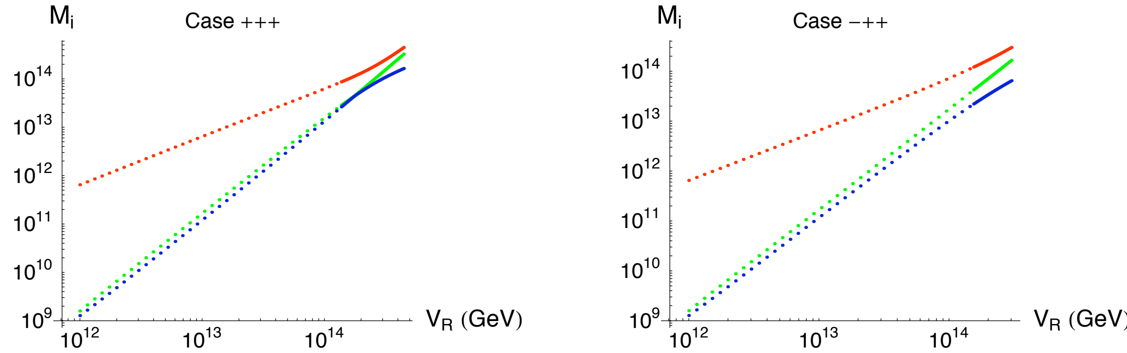


Figure 5: Effect of the light neutrino mass hierarchy on the right-handed neutrino masses. The input parameters are the same as in Fig. 1, except that the light neutrino mass hierarchy is inverted, with $m_3 = 10^{-3}$ eV and opposite CP parities for m_1 and m_2.

parameter / lepton flavour	$\alpha = e$	$\alpha = \mu$	$\alpha = \tau$
ϵ_1^α	2.7×10^{-13}	-6.0×10^{-12}	-1.5×10^{-11}
ϵ_2^α	5.6×10^{-11}	-1.5×10^{-9}	1.4×10^{-7}
ϵ_3^α	-1.8×10^{-14}	5.0×10^{-13}	-4.5×10^{-11}
$\tilde{m}_1^\alpha$	3.3×10^{-3} eV	1.6×10^{-2} eV	2.2×10^{-2} eV
$\tilde{m}_2^\alpha$	5.9×10^{-4} eV	1.1×10^{-2} eV	3.5×10^{-2} eV
$\tilde{m}_3^\alpha$	4.0×10^{-7} eV	1.1×10^{-5} eV	9.4×10^{-3} eV

Table 1: Parameters that control flavour effects in leptogenesis in the type I case (large v_R limit of solution $(-,-,-)$), in the case of a hierarchical light neutrino mass spectrum with $m_1 = 10^{-3}$ eV, $\Phi_2^\nu = \pi/4$ and all other CP-violating phases but the CKM phase set to zero.