Leptogenesis and LFV in type I+II seesaw mechanism

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Introduction

Why consider the type I+II seesaw mechanism?

• Both the type I (heavy right-handed neutrino exchange) and the type II (heavy SU(2)$_L$ triplet exchange) are present in many extensions of the SM, such as left-right symmetric theories and SO(10) GUTs.

• Right-handed neutrinos are suggestive of grand unification. However, SO(10) models with type I seesaw mechanism generally fail to accommodate successful leptogenesis ($Y_V \propto Y_u \Rightarrow$ very hierarchical right-handed neutrino masses, with $M_1 \sim 10^5$ GeV).

• More generally, studies of leptogenesis and LFV are usually done in the framework of the type I seesaw mechanism, or assume dominance of one of the two seesaw mechanisms. It is interesting to investigate whether the generic situation where both contributions are comparable in size can lead to qualitatively different results.
Type I+II seesaw mechanism:

\[ \Delta_L = SU(2)_L \text{ triplet with} \]
couplings \( f_{Lij} \) to lepton doublets

\[
M_\nu = f_L v_L - \frac{v^2}{v_R} Y^T f_R^{-1} Y \equiv M_\nu^{II} + M_\nu^I
\]

Right-handed neutrino mass matrix:

\[ v_R \equiv \langle \Delta_R \rangle \text{ scale of B-L breaking} \]

\[ \Delta_R = SU(2)_R \text{ triplet with couplings } f_{Rij} \text{ to right-handed neutrinos} \]

\[ v_L \text{ is small since it is an induced vev:} \]

\[ v_L \equiv \langle \Delta_L \rangle \sim \frac{v^2 v_R}{M_\Delta^2} \]

In theories with underlying left-right symmetry (such as SO(10) with a \( \overline{126}_H \)),
one has \( Y = Y^T \) and \( f_L = f_R \) \( \Rightarrow \) 2 matrices of couplings \( Y \) and \( f \)

In a fundamental theory, expect \( Y \) to be related to other Yukawa couplings

\( \Rightarrow \) for phenomenological studies, need to reconstruct the \( f_{ij} \) as a function

of the \( Y_{ij} \) (for a given set of low-energy neutrino parameters)

For \( n \) generations, there are \( 2^n \) different solutions [Akhmedov, Frigerio]
Reconstruction of the heavy neutrino mass spectrum

The left-right symmetric seesaw formula \( M_\nu = f v_L - \frac{v^2}{v_R} Y f^{-1} Y \)

with \( f, Y \) complex symmetric matrices (\( Y \) invertible), can be rewritten as

\[
Z = \alpha X - \beta X^{-1} \quad \alpha \equiv v_L, \quad \beta \equiv \frac{v^2}{v_R}
\]

with \( Z = N_Y^{-1} M_\nu (N_Y^{-1})^T, \quad X = N_Y^{-1} f (N_Y^{-1})^T, \quad N_Y \) such that \( Y = N_Y N_Y^T \)

\( Z \) complex symmetric \( \Rightarrow \) can be diagonalized by a complex orthogonal matrix \( O_Z \) if its eigenvalues \( z_i \) are all distinct:

\[
Z = O_Z \text{Diag} \left(z_1, z_2, z_3\right) O_Z^T, \quad O_Z O_Z^T = 1
\]

Then \( X \) can be diagonalized by the same orthogonal matrix as \( Z \), and its eigenvalues are the solutions of:

\[
z_i = \alpha x_i - \beta x_i^{-1} \quad (i = 1, 2, 3)
\]

2 solutions \( x_i^+, x_i^- \) for each \( i \) \( \Rightarrow \) \( 2^3 = 8 \) solutions for \( X \), hence for \( f \):

\[
f = N_Y O_Z \begin{pmatrix} x_1 & 0 & 0 \\ 0 & x_2 & 0 \\ 0 & 0 & x_3 \end{pmatrix} O_Z^T N_Y^T, \quad x_i = x_i^\pm
\]

[see Akhmedov, Frigerio for an alternative reconstruction procedure]
Properties of the solutions

We denote the 2 solutions of \( z_i = \alpha x_i - \beta x_i^{-1} \) by:

\[
x_i^\pm \equiv \frac{z_i \pm \sqrt{z_i^2 + 4\alpha\beta}}{2\alpha}
\]

(+,+,+) refers to the solution \((x_1^+, x_2^+, x_3^+)\), (+,+,−) to \((x_1^+, x_2^+, x_3^-)\), etc.

In the large \( v_R \) limit \((4\alpha\beta \ll |z|^2)\):

\[
x_i^+ \simeq \frac{z_i}{\alpha} \quad \text{("type II branch")}
\]

\[
f^{(+,+,+)} \rightarrow \frac{M_\nu}{v_L}
\]

\[
x_i^- \simeq -\frac{\beta}{z_i} \quad \text{("type I branch")}
\]

\[
f^{(-,-,-)} \rightarrow -\frac{v^2}{v_R} Y M_\nu^{-1} Y
\]

The remaining 6 solutions correspond to mixed cases in which \( M_\nu \) receives significant contributions from both seesaw mechanisms.

In the small \( v_R \) limit \(|z_i|^2 \ll 4\alpha\beta\):

\[
x_i^\pm \simeq \pm \sqrt{\beta/\alpha}
\]

\[
f^{(\pm,\pm,\pm)} \rightarrow \pm \sqrt{\beta/\alpha} Y
\]

If \( Y \) is hierarchical, \( f_i \rightarrow \sqrt{\beta/\alpha} y_i \) holds for all 8 solutions.
Application to SO(10) models with two 10’s and a $\overline{126}$ in the Higgs sector

$$W \ni Y_{ij}^{(1)}_{16i16j101} + Y_{ij}^{(2)}_{16i16j102} + f_{ij}16i16j\overline{126}$$

If the doublets in the $\overline{126}$ have no vev, then, in the basis of charged lepton mass eigenstates:

$$Y_{\nu} = U_q^T \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} U_q$$

$$U_q = P_uV_{CKM}P_d$$

$$M_{\nu} = U_l^* \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} U_l$$

$$U_l = P_eU_{PMNS}P_{\nu}$$

For a given choice of the yet unmeasured neutrino parameters (including the Majorana phases contained in $P_{\nu}$) and of the high energy phases contained in $P_u, P_d$ and $P_e, Y$ and $M_{\nu}$ are known and $f$ can be reconstructed as a function of the B-L breaking scale $v_R$ and of $\beta/\alpha$.

$$\beta/\alpha = v^2/v_{LVR} \sim (M_{\Delta L}/v_R)^2$$
depends on details of the model. Assume $\beta/\alpha = 1$.

Perturbativity constraint: require that the $f_{ij}$ remain perturbative up to the Landau pole of the SO(10) gauge coupling ($\sim 2 \times 10^{17}$ GeV)

$$\Rightarrow$$

constrains $\beta/\alpha \leq O(1)$ and restricts the range of $v_R$. 
Case +++

Case --+

Case ++

Case --
Fine-tuning greater than 10% in the (3, 3) entry of the light neutrino mass matrix.

\[ f_3 \leq 1. \] Dotted lines indicate a fine-tuning greater than 10% in the (3, 3) entry of the light neutrino mass matrix.
Features of the right-handed neutrino spectrum

– at large $v_R$, the solutions ($+,+,+$) and ($-, -, -$) tend to the type II (triplet exchange) and type I (heavy neutrino exchange) cases:

\begin{align*}
(+, +, +): & \quad M_1 : M_2 : M_3 \sim m_1 : m_2 : m_3 \\
(-, -, -): & \quad M_1 : M_2 : M_3 \sim m_u^2 : m_c^2 : m_t^2
\end{align*}

– at small $v_R$, the type I and type II contribution compensate for each other in such a way that

\[ M_1 : M_2 : M_3 \sim m_u : m_c : m_t \]

– 4 solutions are characterized by $M_1 \approx 10^5 \text{GeV}$

– 2 solutions are characterized by $M_1 \approx 5 \times 10^9 \text{GeV}$

Mixing angles

\[
f = U_f \begin{pmatrix} f_1 & 0 & 0 \\ 0 & f_2 & 0 \\ 0 & 0 & f_3 \end{pmatrix} U_f^T \implies U_f^T \begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix}
\]

Dirac couplings in the basis of RHN mass eigenstates

- 2 solutions have RHN mixing angles very close to the CKM angles

- in the other 6 solutions, the mixing angles are close to the CKM angles at small $v_R$, then take larger values at large $v_R$
Leptogenesis

\[ M_1 \ll M_2, M_3, M_{\Delta L} \Rightarrow Y_B \text{ determined by the decays of } N_1 \]

The triplet contributes to \( \epsilon_{N_1} \) through the vertex diagram:

\[ \Rightarrow \quad \epsilon_{N_i} = \epsilon^I_{N_i} + \epsilon^{II}_{N_i} \quad \text{[Hambye, Senjanovic]} \]

The total CP asymmetry can be written [Hambye, Senjanovic; Antusch, King]:

\[ \epsilon_{N_1} = \epsilon^I_{N_1} + \epsilon^{II}_{N_1} = \frac{3}{8\pi} \sum_{k,l} \text{Im} \left[ Y_{1k} Y_{1l} (M_\nu)^*_{kl} \right] \frac{M_1}{v^2} \]

It depends on the reconstructed \( f_{ij} \) couplings through \( M_1 \) and \( U_f \) (since \( Y \) is expressed in the basis of RHN mass eigenstates). The high-energy phases enter this expression directly (in \( Y \) and \( M_\nu \)) and indirectly (via \( U_f \)).

Since the triplet is heavy, the dilution of the generated lepton asymmetry mainly depends on the effective mass parameter:

\[ \tilde{m}_1 = \frac{(YY^\dagger)_{11} v^2}{M_1} \]

which also depends on \( M_1 \) and \( U_f \).
CP asymmetry versus the B-L breaking scale

Hierarchical light neutrino mass spectrum with $m_1 = 10^{-3}$ eV (and $\sin^2 \theta_{13} = 0.009$, $\delta = 0$) – various choices of the Majorana and high-energy phases

3 different behaviours among the 8 solutions for $f$

1) solutions $(\pm, \pm, -)$ [ $M_1 \approx 10^5$ GeV]

Leptogenesis fails to produce the observed baryon asymmetry for these solutions ($|\epsilon_{N1}| \leq$ few $10^{-11}$ $\Rightarrow$ no improvement wrt the type I case)

$N_2$ decays [Di Bari; Vives] do not seem to help, at least for our choice of parameters: $N_2$ decays produce an asymmetry mainly in tau leptons, but the asymmetry is rather small and the wash-out of the tau flavour is strong ($\epsilon_2 \sim 10^{-7}$, $\tilde{m}_1^\tau \sim 10^{-2}$).
2) solutions (±,+,+)

$\epsilon_{N1}$ can reach large values, even if the only input phase is the CKM phase (left panel), but the wash-out tends to be strong (typically $\tilde{m}_1 \sim 10^{-2} \text{eV}$)

This can be compensated for by large values of $\epsilon_{N1}$, but at the price of a larger $M_1$ ($|\epsilon_{N1}| \geq 10^{-5}$ for $M_1 \geq 10^{11} \text{GeV}$) ⇒ conflict with gravitino overproduction in the supersymmetric case
3) solutions (±,−,+) \[ M_1 \approx 5 \times 10^9 \text{ GeV} \]

The peaks in the left and right panels are due to a level crossing \( M_1 = M_2 \); there resonant leptogenesis is possible (but strong cancellation between the type I and type II contributions to \( M_v \))

\( \epsilon_{N1} \) can reach the few \( 10^{-7} \) level \( (10^{-6} \) in some cases), but the wash-out tends to be too important \( (\tilde{m}_1 \sim 10^{-2} \text{ eV}) \)

However, different values of the input parameters (light neutrino mass parameters; phases) or the inclusion of corrections leading to realistic charged fermion masses could improve the situation
Lepton flavour violation

We can estimate the RHN + triplet contribution [Borzumati, Masiero; Rossi] to flavour violation in the slepton sector by:

$$(m^2_L)_{ij} \simeq -\frac{3m_0^2 + A_0^2}{8\pi^2} C_{ij} , \quad (m^2_{\tilde{e} R})_{ij} \simeq 0 , \quad A^e_{ij} \simeq -\frac{3}{8\pi^2} A_0 y_e C_{ij}$$

where the $C_{ij}$'s encapsulate the dependence on the seesaw parameters:

$$C_{ij} \equiv \sum_k Y^*_{ki} Y_{kj} \ln \left(\frac{M_U}{M_k}\right) + 3 (f f^\dagger)_{ij} \ln \left(\frac{M_U}{M_{\Delta_L}}\right)$$

$M_U =$ scale where universality among soft terms is assumed (we take $M_U = 10^{17}$ GeV and $M_{\Delta_L} = \nu_R$)

Experimental upper limits on the LFV decays $l_i \to l_j \gamma$ can be turned into upper bounds on the $C_{ij}$'s as a function of the supersymmetric mass parameters and of $\tan \beta$ [S.L., Masina, Savoy]:

![Graphs showing $C_{32}$ and $C_{21}$ as functions of $m_{32}$ and $m_{21}$, with upper limits on BR($\tau \to \mu \gamma$) and BR($\mu \to e \gamma$) indicated.](image-url)
We can then compare the predicted Cij’s for a given solution f with the “experimental” upper bounds |C_{23}| \leq 10 (τ→μγ) and |C_{12}| \leq 0.1 (μ→eγ) [taking tanβ = 10 and m_0, M_{1/2} \leq O(1 \text{ TeV})]:

![Graph showing coefficients C_{ij} as a function of v_R for solutions (+, +, +) and (−, −, −) in the case of a hierarchical light neutrino mass spectrum with m_1 = 10^{-3} \text{ eV}, \beta = \alpha, and no CP violation beyond the CKM phase. The green [light grey] curve corresponds to |C_{23}|, and the blue [black] curve to |C_{12}|. The horizontal lines indicate the “experimental” constraints |C_{23}| < 10 and |C_{12}| < 0.1 (see text).]

Due to the small CKM angles (V_L = U_q), the type II contribution always dominates in the C_{ij}’s, except in the large v_R region of solutions (−,−,−) [type I limit] and (+,−,−)

The predictions lie significantly below the experimental bounds, except in the large v_R region where, depending on the supersymmetric parameters, μ→eγ can exceed its present upper limit
Conclusions

- The possibilities to account for the observed neutrino data in the left-right symmetric seesaw mechanism is much richer than in the cases of type I and type II dominance, with interesting implications for leptogenesis and LFV.

- In particular, the mixed solutions where both seesaw mechanisms give a significant contribution to neutrino masses provide new opportunities for successful leptogenesis in SO(10) GUTs.

- A detailed study of leptogenesis in realistic SO(10) models (including a correct description of charged fermion masses and taking into account the wash-out of the lepton asymmetry) is in progress.
Back-up slides
Figure 3: Effect of $\beta \neq \alpha$ on the right-handed neutrino masses. The input parameters are the same as in Fig. 1, except $\beta/\alpha = 0.01$.

Figure 4: Effect of high-energy phases on the right-handed neutrino masses. The input parameters are the same as in Fig. 1, except $\Phi_2 = \pi/4$ (left panel), $\Phi_1 = \pi/4$ (right panel).
Figure 5: Effect of the light neutrino mass hierarchy on the right-handed neutrino masses. The input parameters are the same as in Fig. 1, except that the light neutrino mass hierarchy is inverted, with $m_3 = 10^{-3}$ eV and opposite CP parities for $m_1$ and $m_2$.

<table>
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<tr>
<th>parameter / lepton flavour</th>
<th>$\alpha = e$</th>
<th>$\alpha = \mu$</th>
<th>$\alpha = \tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon_1^\nu$</td>
<td>$2.7 \times 10^{-13}$</td>
<td>$-6.0 \times 10^{-12}$</td>
<td>$-1.5 \times 10^{-11}$</td>
</tr>
<tr>
<td>$\epsilon_2^\nu$</td>
<td>$5.6 \times 10^{-11}$</td>
<td>$-1.5 \times 10^{-9}$</td>
<td>$1.4 \times 10^{-7}$</td>
</tr>
<tr>
<td>$\epsilon_3^\nu$</td>
<td>$-1.8 \times 10^{-14}$</td>
<td>$5.0 \times 10^{-13}$</td>
<td>$-4.5 \times 10^{-11}$</td>
</tr>
<tr>
<td>$m_1^\nu$</td>
<td>$3.3 \times 10^{-3}$ eV</td>
<td>$1.6 \times 10^{-2}$ eV</td>
<td>$2.2 \times 10^{-2}$ eV</td>
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<tr>
<td>$\tilde{m}_2^\nu$</td>
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<td>$1.1 \times 10^{-2}$ eV</td>
<td>$3.5 \times 10^{-2}$ eV</td>
</tr>
<tr>
<td>$m_3^\nu$</td>
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<td>$1.1 \times 10^{-5}$ eV</td>
<td>$9.4 \times 10^{-3}$ eV</td>
</tr>
</tbody>
</table>

Table 1: Parameters that control flavour effects in leptogenesis in the type I case (large $v_R$ limit of solution $(-,-,-)$), in the case of a hierarchical light neutrino mass spectrum with $m_1 = 10^{-3}$ eV, $\Phi_2^\nu = \pi/4$ and all other CP-violating phases but the CKM phase set to zero.