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Unitarity of the leptonic mixing matrix

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M.B. Gavela and J. López-Pavón

Thanks also to C. Peña-Garay

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Motivations

- ν masses and mixing → indication of New Physics beyond the Standard Model
- typical explanations (see-saw) involve NP @ *energies* > Λ_{SM}
- NP @ high energy often induces violation of unitarity @ low energy

We relax the assumption of unitarity of the leptonic mixing matrix and we constrain the matrix elements

first only with present oscillation experiments and

then by combining them with other electroweak data

Minimal Unitarity Violation Scheme

$$L=\frac{1}{2}\left(i\overline{\nu}_i\partial\!\!\!/\nu_i-\overline{\nu^c}_i m_{ii}\nu_i\right)-\frac{g}{\sqrt{2}}\left(W^+_\mu\bar{l}_\alpha\gamma^\mu P_L U_{\alpha i}\nu_i\right)-\frac{g}{\cos\theta_W}\left(Z_\mu\overline{\nu}_i\gamma^\mu P_L\nu_i\right)+h.c.+\ldots$$

$$U=\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{-i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{-i\delta} & s_{23}c_{13} \\ s_{23}s_{12}-c_{12}c_{23}s_{13}e^{-i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{-i\delta} & c_{23}c_{13} \end{pmatrix}$$

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Is this Lagrangian well-motivated? … let's go into the flavour basis…

In the flavour basis…

$$\nu_\alpha = N_{\alpha i}\nu_i \qquad N \text{ non-unitary}$$

$$L = \frac{1}{2}\left(i\,\overline{\nu}_\alpha \partial \left(NN^\dagger\right)^{-1}_{\alpha\beta}\nu_\beta - \overline{\nu}^c_\alpha\left[\left(N^{-1}\right)^t m N^{-1}\right]_{\alpha\beta}\nu_\beta + h.c.\right) +$$

$$-\frac{g}{\sqrt{2}}\left(W^+_\mu \overline{l}_\alpha \gamma^\mu P_L \nu_\alpha + h.c.\right) - \frac{g}{\cos\theta_W}\left(Z_\mu \overline{\nu}_\alpha \gamma^\mu P_L \nu_\alpha + h.c.\right) + \ldots$$

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$M_{\alpha\beta} \rightarrow$ diagonalized $\rightarrow$ unitary (bi-unitary) transformation

$K_{\alpha\beta} \rightarrow$ diagonalized and normalized $\rightarrow$ unitary transf. + rescaling

$\downarrow$

source of non-unitarity

In the flavour basis...

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source of non-unitarity

Non-unitarity in the MUV $\leftrightarrow$ non-normalized kinetic terms

Examples

1. The see-saw model

$$L = L_{SM} + i\overline{N}_R \partial\!\!\!/ N_R - Y_\nu \left(\bar{l}_L \tilde{H} N_R + \overline{N}_R \tilde{H}^+ l_L \right) - \frac{1}{2} M \left(\overline{N}_R^c N_R + \overline{N}_R N_R^c \right)$$

Integrate out $N_R \rightarrow$

- 5D operator $\approx \dfrac{d_{\alpha\beta}}{\Lambda} \left(\overline{L}_\alpha^c \tilde{\phi}^* \right) \left(\tilde{\phi}^\dagger L_\beta \right)$
- 6D operator $\approx \dfrac{c_{\alpha\beta}}{\Lambda^2} \left(\overline{L}_\alpha \tilde{\phi} \right) i\partial\!\!\!/ \left(\tilde{\phi}^\dagger L_\beta \right)$

Broncano, Gavela, Jenkins 02

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Broncano, Gavela, Jenkins 02

2. Any theory generating

$$\approx \frac{c_{\alpha\beta}^I}{\Lambda^2} \left(\overline{L}_\alpha \phi\right) iD\!\!\!\!/ \left(\phi^\dagger L_\beta\right) \qquad \approx \frac{c_{\alpha\beta}^{II}}{\Lambda^2} \left(\overline{L}_\alpha \tilde{\phi}\right) iD\!\!\!\!/ \left(\tilde{\phi}^\dagger L_\beta\right)$$

… triplet fermion see-saw..?

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Broncano, Gavela, Jenkins 02

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The effective Lagrangian in the MUV

- 3 light ν
- all unitarity violation from NP @ $E > \Lambda_{SM}$

$$L = \frac{1}{2}\left(i\bar{\nu}_i \partial\!\!\!/ \nu_i - \bar{\nu}^c{}_i m_{ii} \nu_i\right) - \frac{g}{\sqrt{2}}\left(W_\mu^+ \bar{l}_\alpha \gamma^\mu P_L N_{\alpha i} \nu_i\right) - \frac{g}{\cos\theta_W}\left(Z_\mu \bar{\nu}_i \gamma^\mu P_L (N^\dagger N)_{ij} \nu_j\right) + h.c. + \ldots$$

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$\Downarrow$

$$\boxed{\langle \nu_i | \nu_j \rangle = \delta_{ij}}$$

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unchanged

$\Downarrow$

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Non-unitarity effects appear in the interaction:

$$|\nu_\alpha\rangle = \frac{1}{\sqrt{(NN^\dagger)_{\alpha\alpha}}} \sum_i N^*_{\alpha i} |\nu_i\rangle \equiv \sum_i \tilde{N}^*_{\alpha i} |\nu_i\rangle$$

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$$\boxed{\langle \nu_\beta | \nu_\alpha \rangle = \left(\tilde{N}^* \tilde{N}^t\right)_{\alpha\beta} \neq \delta_{\alpha\beta}}$$

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This affects both electroweak decays and oscillation probabilities…

ν oscillations in vacuum

- mass basis $$i\frac{d}{dt}\left|\nu_i\right\rangle = \hat{H}^{free}\left|\nu_i\right\rangle = \sum_j H_{ij}^{free}\left|\nu_j\right\rangle = E_i\left|\nu_i\right\rangle \qquad \left|\nu_i(t)\right\rangle = e^{-iE_i t}\left|\nu_i^0\right\rangle$$

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- flavour basis $$i\frac{d}{dt}\left|\nu_\alpha\right\rangle = \hat{H}^{free}\left|\nu_\alpha\right\rangle = \sum_\beta E_{\alpha\beta}^{free}\left|\nu_\beta\right\rangle$$

with $$E_{\alpha\beta}^{free} = \tilde{N}_{\alpha i}^{*} H_{ij}^{free}\left(\tilde{N}^{*}\right)_{j\beta}^{-1} \neq H_{\alpha\beta}^{free} = \tilde{N}_{\alpha i}^{*} H_{ij}^{free} \tilde{N}_{j\beta}^{t}$$

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$$\boxed{P_{\alpha\beta}(E,L) = \frac{\left|\sum_i N_{\alpha i}^{*} e^{iP_i L} N_{\beta i}\right|^2}{\left(NN^\dagger\right)_{\alpha\alpha}\left(NN^\dagger\right)_{\beta\beta}}}$$

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Zero-distance effect:

$$P_{\alpha\beta}(E,0) = \frac{\left|\left(NN^\dagger\right)_{\alpha\beta}\right|^2}{\left(NN^\dagger\right)_{\alpha\alpha}\left(NN^\dagger\right)_{\beta\beta}}$$

ν oscillations in matter

$$-L^{\text{int}} = \underbrace{\sqrt{2}G_F n_e}_{V_{CC}} \bar{\nu}_e \gamma^0 \nu_e - \underbrace{\frac{1}{\sqrt{2}} G_F n_n}_{V_{NC}} \sum_\alpha \bar{\nu}_\alpha \gamma^0 \nu_\alpha$$

2 families

$$i\frac{d}{dt}\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \left[U^* \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} U^t + \begin{pmatrix} V_{CC} \cancel{- V_{NC}} & 0 \\ 0 & \cancel{-V_{NC}} \end{pmatrix} \right] \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

ν oscillations in matter

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$$i\frac{d}{dt}\begin{pmatrix}\nu_e \\ \nu_\mu\end{pmatrix} = \left[N^*\begin{pmatrix}E_1 & 0 \\ 0 & E_2\end{pmatrix}\left(N^*\right)^{-1} + \begin{pmatrix}\left(V_{CC}-V_{NC}\right)\left(NN^\dagger\right)_{ee} & -V_{NC}\sqrt{\frac{\left(NN^\dagger\right)_{\mu\mu}}{\left(NN^\dagger\right)_{ee}}}\left(NN^\dagger\right)_{\mu e} \\ \left(V_{CC}-V_{NC}\right)\sqrt{\frac{\left(NN^\dagger\right)_{ee}}{\left(NN^\dagger\right)_{\mu\mu}}}\left(NN^\dagger\right)_{e\mu} & -V_{NC}\left(NN^\dagger\right)_{\mu\mu}\end{pmatrix}\right]\begin{pmatrix}\nu_e \\ \nu_\mu\end{pmatrix}$$

1. non-diagonal elements

2. NC effects do not disappear

N elements from oscillations: e-row

Only disappearance exps $\rightarrow$ informations only on $|N_{\alpha i}|^2$

CHOOZ: $\Delta_{12}\approx 0$

$$\Delta_{ij} = \Delta m_{ij}^2 L / 2E$$

$$\hat{P}(\bar{\nu}_e \to \bar{\nu}_e) \cong \left(|N_{e1}|^2 + |N_{e2}|^2\right)^2 + |N_{e3}|^4 + 2\left(|N_{e1}|^2 + |N_{e2}|^2\right)|N_{e3}|^2 \cos(\Delta_{23})$$

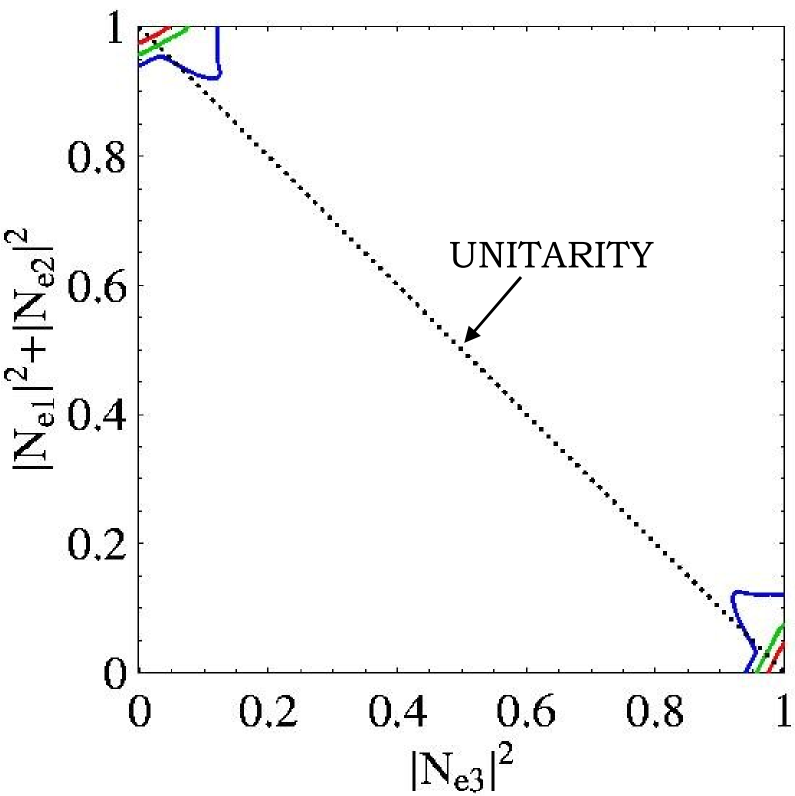
K2K $(\nu_\mu \rightarrow \nu_\mu)$: Δ_{23}

1. Degeneracy

$$|N_{e1}|^2 + |N_{e2}|^2 \leftrightarrow |N_{e3}|^2$$

2. $|N_{e1}|^2$, $|N_{e2}|^2$

cannot be disentangled



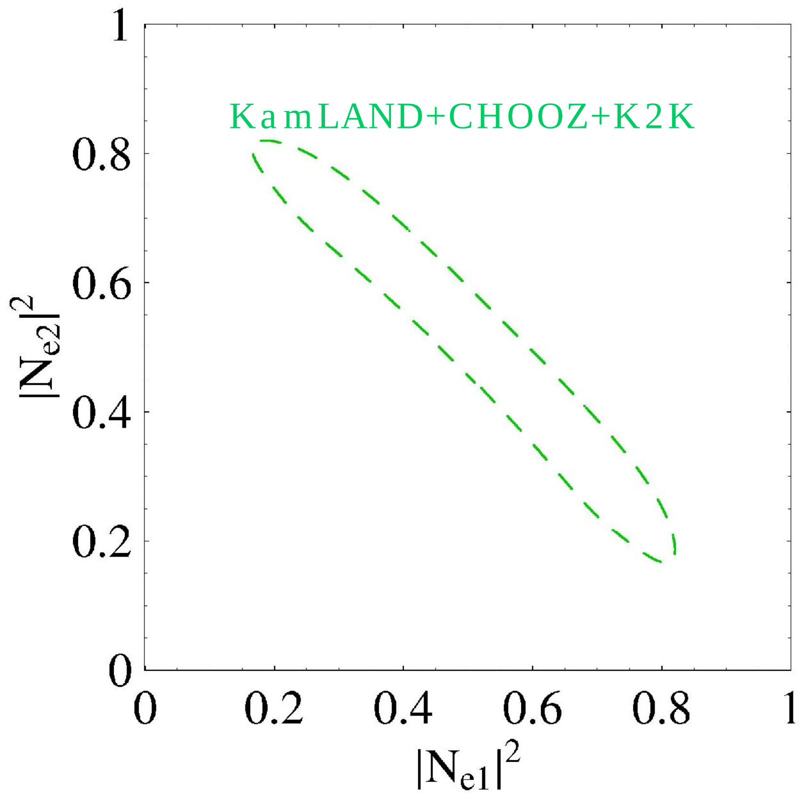
N elements from oscillations: e-row

KamLAND: $\Delta_{23}>>1$

$$\hat{P}(\bar{\nu}_e \to \bar{\nu}_e) \cong |N_{e1}|^4 + |N_{e2}|^4 + |N_{e3}|^4 + 2|N_{e1}|^2|N_{e2}|^2 \cos(\Delta_{12})$$

$$\begin{cases} |N_{e1}|^2 + |N_{e2}|^2 \approx 1 \\ |N_{e3}|^2 \approx 0 \end{cases}$$

→ first degeneracy solved



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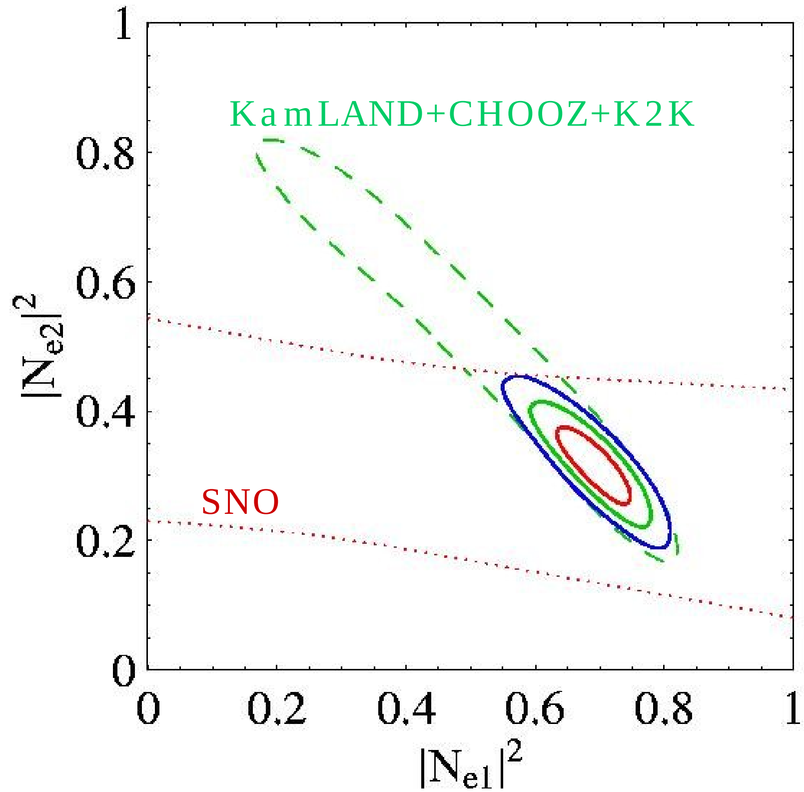
$$\begin{cases} |N_{e1}|^2 + |N_{e2}|^2 \approx 1 \\ |N_{e3}|^2 \approx 0 \end{cases}$$

$\rightarrow$ first degeneracy solved

SNO:

$$\hat{P}(\nu_e \to \nu_e) \cong 0.1|N_{e1}|^2 + 0.9|N_{e2}|^2$$

$\rightarrow$ all $|N_{ei}|^2$ determined



N elements from oscillations: μ-row

Atmospheric + K2K: $\Delta_{12}\approx 0$

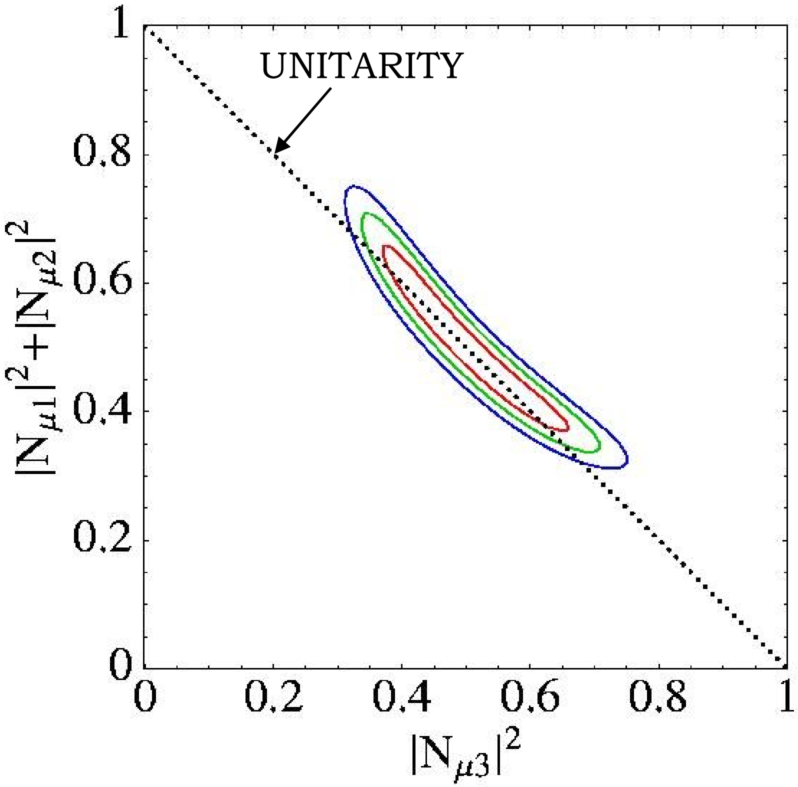
$$\hat{P}(\nu_\mu \to \nu_\mu) \cong \left(|N_{\mu 1}|^2 + |N_{\mu 2}|^2\right)^2 + |N_{\mu 3}|^4 + 2\left(|N_{\mu 1}|^2 + |N_{\mu 2}|^2\right)|N_{\mu 3}|^2 \cos(\Delta_{23})$$

1. Degeneracy

   $$|N_{\mu 1}|^2 + |N_{\mu 2}|^2 \leftrightarrow |N_{\mu 3}|^2$$

2. $|N_{\mu 1}|^2$, $|N_{\mu 2}|^2$

   cannot be disentangled



N elements from oscillations only

with unitarity
OSCILLATIONS

$$|U| = \begin{pmatrix} 0.79-0.88 & 0.47-0.61 & <0.20 \\ 0.19-0.52 & 0.42-0.73 & 0.58-0.82 \\ 0.20-0.53 & 0.44-0.74 & 0.56-0.81 \end{pmatrix}$$

González-García 04

without unitarity
OSCILLATIONS
3σ

$$|N| = \begin{pmatrix} \multicolumn{2}{c}{\begin{array}{cc} 0.75-0.89 & 0.45-0.66 \end{array}} & <0.34 \\ \multicolumn{2}{c}{\left[\left(|N_{\mu 1}|^2 + |N_{\mu 2}|^2\right)^{1/2} = 0.57-0.86\right]} & 0.57-0.86 \\ ? \quad ? & & ? \end{pmatrix}$$

...adding near detectors...

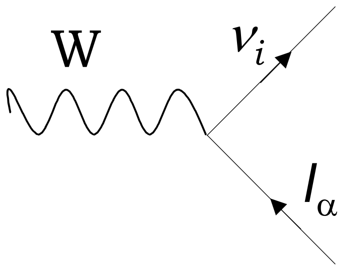
Test of zero-distance effect: $P_{\alpha\beta}(E,0) = \left|\left(NN^\dagger\right)_{\alpha\beta}\right|^2 \neq \delta_{\alpha\beta}$

- MINOS: $(NN^\dagger)_{\mu\mu} = 1 \pm 0.05$
- BUGEY: $(NN^\dagger)_{ee} = 1 \pm 0.04$
- NOMAD: $(NN^\dagger)_{\mu\tau} < 0.09$ $(NN^\dagger)_{e\tau} < 0.013$
- KARMEN: $(NN^\dagger)_{\mu e} < 0.05$

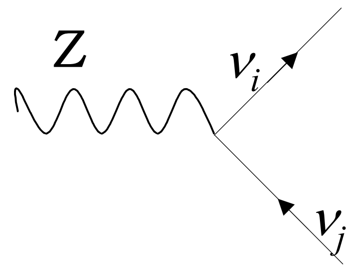
$$|N| = \begin{pmatrix} 0.75-0.89 & 0.45-0.66 & < 0.27 \\ 0.00-0.69 & 0.22-0.81 & 0.57-0.85 \\ ? & ? & ? \end{pmatrix}$$

$\rightarrow$ also all $|N_{\mu i}|^2$ determined

Electroweak decays



$$\approx N_{\alpha i}$$

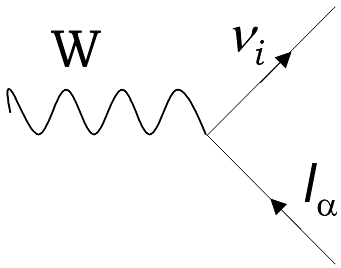


$$\approx \left(N^\dagger N\right)_{ij}$$

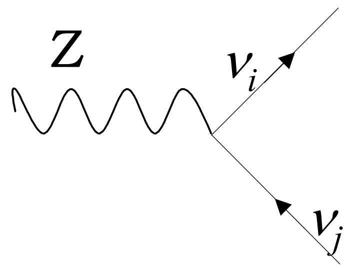
$$\Gamma = \Gamma_{SM} \sum_i \left|N_{\alpha i}\right|^2 = \Gamma_{SM} \left(NN^\dagger\right)_{\alpha\alpha}$$

$$\Gamma = \Gamma_{SM} \sum_{ij} \left|\left(N^\dagger N\right)_{ij}\right|^2$$

Electroweak decays



$$\approx N_{\alpha i}$$



$$\approx \left(N^\dagger N\right)_{ij}$$

$$\Gamma = \Gamma_{SM} \sum_i \left|N_{\alpha i}\right|^2 = \Gamma_{SM} \left(N N^\dagger\right)_{\alpha\alpha}$$

$$\Gamma = \Gamma_{SM} \sum_{ij} \left|\left(N^\dagger N\right)_{ij}\right|^2$$

COMMENT

With decays we can only constrain *($NN^\dagger$)* and *($N^\dagger N$)*,
we cannot extract the matrix elements

→ *we need oscillations!*

Different from quark sector…

$(NN^\dagger)$ from decays

- W decays $\rightarrow \dfrac{\left(NN^\dagger\right)_{\alpha\alpha}}{\sqrt{\left(NN^\dagger\right)_{ee}}\sqrt{\left(NN^\dagger\right)_{\mu\mu}}}$
- Invisible Z $\rightarrow \dfrac{\sum_{\alpha\beta}\left(NN^\dagger\right)_{\alpha\beta}}{\sqrt{\left(NN^\dagger\right)_{ee}}\sqrt{\left(NN^\dagger\right)_{\mu\mu}}}$
- Universality tests $\rightarrow \dfrac{\left(NN^\dagger\right)_{\alpha\alpha}}{\left(NN^\dagger\right)_{\beta\beta}}$

Infos on $(NN^\dagger)_{\alpha\alpha}$

$(NN^\dagger)$ from decays

- W decays $\rightarrow \dfrac{(NN^\dagger)_{\alpha\alpha}}{\sqrt{(NN^\dagger)_{ee}}\sqrt{(NN^\dagger)_{\mu\mu}}}$
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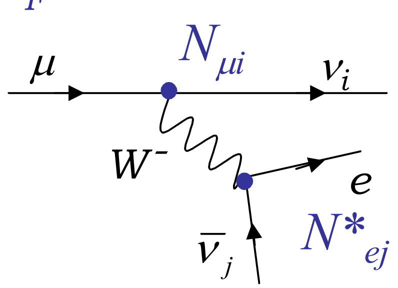
Infos on $(NN^\dagger)_{\alpha\alpha}$

G_F is measured in μ-decay

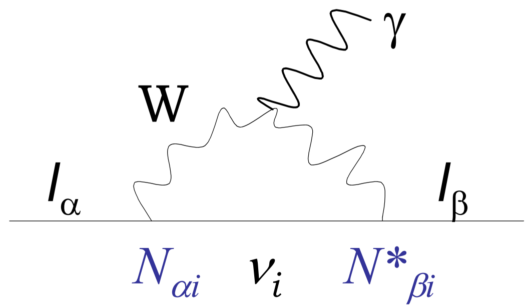


$$\Gamma = \frac{G_F^2 m_\mu^5}{192\pi^3}(NN^\dagger)_{ee}(NN^\dagger)_{\mu\mu}$$

$$G_F^2 = \frac{G_{F,\exp}^2}{(NN^\dagger)_{ee}(NN^\dagger)_{\mu\mu}}$$

$(NN^\dagger)$ from decays

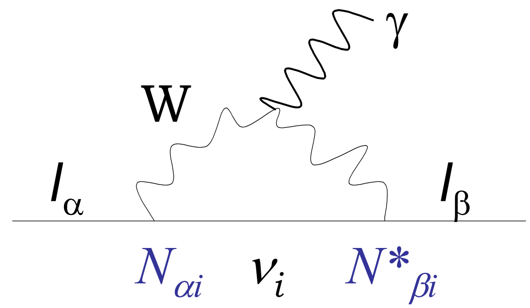
- Rare leptons decays



Infos on $(NN^\dagger)_{\alpha\beta}$

$(NN^\dagger)$ from decays

- Rare leptons decays



Infos on $(NN^\dagger)_{\alpha\beta}$

SM $\rightarrow$ GIM suppression: $$Br(l_\alpha \rightarrow l_\beta \gamma) = \frac{3\alpha}{32\pi}\left|\sum_{i=2,3} U^*_{\alpha i} U_{\beta i} \frac{\Delta m^2_{1i}}{M^2_W}\right| < 10^{-54}$$

Now $\rightarrow$ no suppression:
$\rightarrow$ constant term leading

$$\rightarrow \frac{\left|\left(NN^\dagger\right)_{\alpha\beta}\right|^2}{\left(NN^\dagger\right)_{\alpha\alpha}\left(NN^\dagger\right)_{\beta\beta}}$$

$(NN^\dagger)$ from decays

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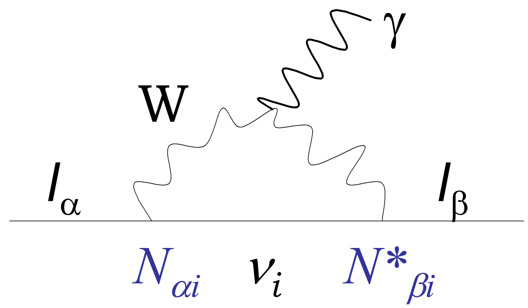

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$$\to \frac{\left|\left(NN^\dagger\right)_{\alpha\beta}\right|^2}{\left(NN^\dagger\right)_{\alpha\alpha}\left(NN^\dagger\right)_{\beta\beta}}$$

- μ-e conversion in nuclei
- $\mu \to e^+ e^- e$

$(NN^\dagger)$ and $(N^\dagger N)$ from decays

$$\left|NN^\dagger\right| \approx \begin{pmatrix} 1.002 \pm 0.005 & < 7.2\cdot 10^{-5} & < 1.6\cdot 10^{-2} \\ < 7.2\cdot 10^{-5} & 1.003 \pm 0.005 & < 1.3\cdot 10^{-2} \\ < 1.6\cdot 10^{-2} & < 1.3\cdot 10^{-2} & 1.003 \pm 0.005 \end{pmatrix}$$

Experimentally

$(NN^\dagger)$ and $(N^\dagger N)$ from decays

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Experimentally

$$N = HV \qquad NN^\dagger = H^2 = 1 + \varepsilon \quad \text{with } \varepsilon = \varepsilon^\dagger$$

$$N^\dagger N = 1 + V^\dagger \varepsilon V = 1 + \varepsilon'$$

$$|\varepsilon'_{ij}| \le \sqrt{\sum_{\alpha\beta} |\varepsilon_{\alpha\beta}|^2} \approx 0.03$$

$$|N^\dagger N| \approx \begin{pmatrix} 1.00 \pm 0.032 & < 0.032 & < 0.032 \\ < 0.032 & 1.00 \pm 0.032 & < 0.032 \\ < 0.032 & < 0.032 & 1.00 \pm 0.032 \end{pmatrix}$$

Estimation
(the most conservative)

$\rightarrow$ *N* is unitary at % level

N elements from oscillations & decays

with unitarity
OSCILLATIONS

$$|U| = \begin{pmatrix} 0.79-0.88 & 0.47-0.61 & <0.20 \\ 0.19-0.52 & 0.42-0.73 & 0.58-0.82 \\ 0.20-0.53 & 0.44-0.74 & 0.56-0.81 \end{pmatrix}$$

González-García 04

without unitarity
OSCILLATIONS
+DECAYS
3σ

$$|N| = \begin{pmatrix} 0.76-0.89 & 0.45-0.65 & <0.20 \\ 0.19-0.54 & 0.42-0.73 & 0.57-0.82 \\ 0.13-0.56 & 0.36-0.75 & 0.54-0.82 \end{pmatrix}$$

In the future…

MEASUREMENT OF MATRIX ELEMENTS

- $|N_{e3}|^2$, μ-row $\rightarrow$ MINOS, T2K, Superbeams, NUFACTs…
- τ-row $\rightarrow$ high energies: NUFACTs
- phases $\rightarrow$ *appearance* experiments: NUFACTs, β-beams

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TESTS OF UNITARITY

Rare leptons decays

PRESENT

- $\mu \rightarrow e\gamma$ $\quad \left(NN^\dagger\right)_{e\mu} < 7.2 \cdot 10^{-5}$
- $\tau \rightarrow e\gamma$ $\quad \left(NN^\dagger\right)_{e\tau} < 0.016$
- $\tau \rightarrow \mu\gamma$ $\quad \left(NN^\dagger\right)_{\mu\tau} < 0.013$

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Rare leptons decays

	PRESENT	FUTURE
• $\mu \to e\gamma$	$\left(NN^\dagger\right)_{e\mu} < 7.2 \cdot 10^{-5}$	$\sim 10^{-6}$ MEG $\sim 10^{-7}$ NUFACT
• $\tau \to e\gamma$	$\left(NN^\dagger\right)_{e\tau} < 0.016$	
• $\tau \to \mu\gamma$	$\left(NN^\dagger\right)_{\mu\tau} < 0.013$	

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ZERO-DISTANCE EFFECT

40Kt Iron calorimeter @ NUFACT

- $\nu_e \rightarrow \nu_\mu$ $\left(NN^\dagger\right)_{e\mu} < 2.3 \cdot 10^{-4}$

4Kt OPERA-like @ NUFACT

- $\nu_e \rightarrow \nu_\tau$ $\left(NN^\dagger\right)_{e\tau} < 2.9 \cdot 10^{-3}$
- $\nu_\mu \rightarrow \nu_\tau$ $\left(NN^\dagger\right)_{\mu\tau} < 2.6 \cdot 10^{-3}$

In the future...

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- $|N_{e3}|^2$, μ-row $\rightarrow$ MINOS, T2K, Superbeams, NUFACTs...
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Conclusions

If we don't assume unitarity for the leptonic mixing matrix

- Present oscillation experiments alone can only measure the e-row and part of the μ-row
- EW decays probes unitarity at % level
- Combining oscillations and EW decays we obtain values for the leptonic mixing matrix comparable with the ones obtained with the unitary analysis

Future experiments will:

- improve the present measurements on the e- and μ-rows
- give informations on the τ-row and on phases (appearance exps)
- test unitarity by constraining the zero-distance effect with a near detector
- discriminate among different NP scenarios