EDM Ring, Five Quads
lattice, orbit and spin tracking

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EDM ring with two magnets
B= 2T, n=1, $\alpha_p=1$, footprint 5x10 meter$^2$

- Lattice optimized with MAD using a Montecarlo method
- At n=1 the ring is unstable to first order, but stable to second order: good enough for a ring operating at quasi fixed energy
  - $\alpha_p=1$ difficult to achieve. It freezes the size of the ring. Also, it produces a high dispersion in half of the ring
Five Fat Quads
Sector Bends

Alternating gradient
Bends to reduce
dispersion
FFQ parameters (1)

\[ p_c = 1.5 \ [\text{GeV}] \ (\text{deuterons}) \]
\[ \rho = 2.5 \ [\text{m}] \]
\[ \gamma = 1.3414 \]
\[ \beta_s = 0.6665 \]
\[ B_\rho = 5.003 \ [\text{Tm}] \]
\[ B_0 = 2.00 \ [\text{T}] \]
FFQ parameters (2)

\[ L_d = \pi \rho (\pi^2/6 - 1) = 5.0653 \text{ [m]}, \text{ drift length} \]
\[ n = 1, \quad \text{bend field index} \]
\[ K_{1B} = -n/\rho^2 \quad [1/\text{m}^2], \text{ bend quad strength} \]
\[ L_Q = 0.32 \quad \text{[m]}, \quad \text{quad length} \]
\[ L_H = 0.08 \quad \text{[m]}, \quad \text{quad-to-bend} \]
FFQ parameters(3)

- \( K_{1Q_1} = 0.166512 \ [1/m^2] \)
- \( K_{1Q_2} = 1.282920 \)
- \( K_{1Q_3} = -1.564187 \)
- \( G_1 = K_{1Q_1}B\rho = 0.833136 \ [T/m] \)
- \( G_2 = K_{1Q_2}B\rho = 6.41904 \)
- \( G_3 = K_{1Q_3}B\rho = -7.82635 \)
**FFQ parameters(4)**

- Total length: $25.838564 \text{ [m]}$
- $\nu_x = 0.637806$, horizontal beta tune
- $\nu_y = 1.182998$, vertical beta tune
- $\alpha_p = 1.00001$, momentum compaction
- $\beta_{X,\text{max}} = 16.8168 \text{ [m]}$
- $\beta_{y,\text{max}} = 36.5071 \text{ [m]}$
- $\gamma_{tr} = 0.999997$, transition
- $D_{x,\text{max}} = 20.6318$, dispersion
Lattice (MAD)

EDM – Five Fat Quads
EDM base line
Tracking (1)

Track polarized deuterons (magnetic), or, equivalent protons with energy close to an Intrinsic spin resonance

\[ G_\gamma = n + \nu_y \]

To check limits of spin stability. Cross the resonance by slow acceleration
FFQ tracking (2)

1 + G\gamma = 1.183; \quad G = g/2 - 1
G(p) = 1.79285, \quad G(d) = -0.14301
pc(p) = 1.3764 GeV: G\gamma(p) = [3].183
pc(d) = 1.4983 GeV: \gamma(d) = 1.2796
\quad G\gamma(d) = 0.183

search for spin resonance at n +/- 0.183

Accelerate in a RF cavity with V = 5 KV

Code Spink based on the BMT equation, modified to add edm
Tracking code *Spink*

Phase space propagation (thick elements)

\[ r_i^{(n+1)} = \sum_{j=1,6} R_{ij} r_j^{(n)} + \sum_{j=1,6} \sum_{k=1,6} T_{ijk} r_j^{(n)} r_k^{(n)} \]

\[ r_6^{(N+1)} = r_6^{(N)} + eV\left(\sin(hr_5^{(N)}) - \sin(\phi_s)\right) \]

Thomas-BMT equation for spin + EDM

\[
\frac{d\vec{S}}{dt} = \frac{e}{\gamma m} \vec{S} \times \vec{F}
\]

\[
\vec{F} = (1 + G\gamma) \vec{B}_\perp + (1 + G) \vec{B}_\parallel + \eta\vec{\beta} \times \vec{B}, \quad \vec{E} = 0
\]
Spink

Rotation of a vector $S$ by an angle $\mu$ around an axis $b$
(thin elements)

$$\frac{dS}{d\mu} = S \times b, \text{or } S = RS$$

$$R = I \cos \mu + W (1 - \cos \mu) + A \sin \mu$$

$$W = \begin{pmatrix} b_1^2 & b_1 b_2 & b_1 b_3 \\ b_2 b_1 & b_2^2 & b_2 b_3 \\ b_3 b_1 & b_3 b_2 & b_3^2 \end{pmatrix}, \quad A = \begin{pmatrix} 0 & b_3 & -b_2 \\ -b_3 & 0 & b_1 \\ b_2 & -b_1 & 0 \end{pmatrix}$$

$$b, \mu = f(B, G, \eta)$$
Spink

- MAD ➔ mad_read ➔ symplectification ➔ Sy_lattice descriptor (maps, twiss, errors) Guaranteed the conservation of invariants –say, Courant-Snyder- for several million of turns. Fast

- MakePop ➔ particle population in 6 D. Random Gaussian

- MPI directives ➔ parallelization (trivial) For many particle tracking

- Sy + Pop + MPI ➔ Spink

- runs on Linux Cluster
Transverse phase space
(to check symplecticity after many turns)
Longitudinal Phase Space

Orbit tracking in the EDM ring

protons, $\varepsilon=10^{-3}$, $q/p=10^{-3}$, $\delta\gamma\delta\eta=10^{-7}$

sy.osc=125 turns
Track spin - resonance
Spin tracking in the EDM ring

Cross spin resonance at $G\gamma = 3.183$

Various speeds of crossing
(for Froissart-Stora)

Represented is the vertical component of spin vs. spin tune $G\gamma$
Various long. phases -cΔt

Spin tracking in the EDM ring

Track particles at various longitudinal position in the bunch

They cross the resonance differently, producing some depolarization
Spin resonance- vary: -cΔt
Spin tracking in the EDM ring

protons, ε=10^{-6}, dp/p=0

Some more of the same......
Various –cΔt (2)

Central part of the bunch. Particles up to 4 m away from the synchronous particle behave coherently.

Particles within a 1 m long bunch behave coherently.
Vertical component of the spin. Injection with spin longitudinal. Tilt of 10^{-3} rad of quadrupole Q2. Vertical motion suppressed. $10^6$ turns