

# QCD factorisation and flavour symmetries illustrated in $B_{d,s} \rightarrow KK$ decays

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# Two-body nonleptonic $B$ decays

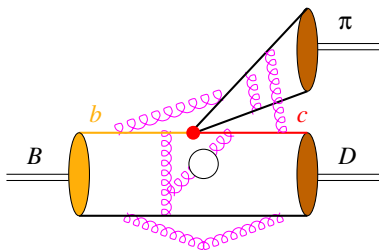
- $B_d$  very much SM-like (Babar, Belle)
- $B_d$  still room for New Physics ? (CDF, D0, LHCb)

Predict SM correlations between  $B_d$  and  $B_s$  decays  
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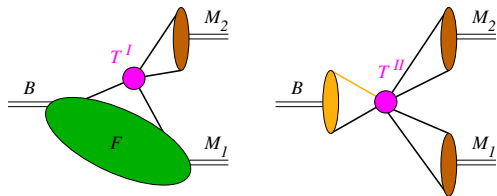
$$\mathcal{A}(B \rightarrow H) = \frac{G_F}{\sqrt{2}} \sum_i \lambda_i C_i(\mu) \langle H | \mathcal{O}_i | B \rangle(\mu)$$

To compute  $\langle H | \mathcal{O}_i | B \rangle$

- (Lattice)
- Light-cone sum rules
- QCD factorisation
- Flavour symmetries

# QCD factorisation

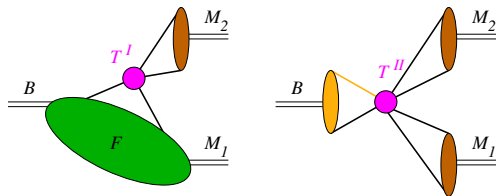
For some classes of decays, in the heavy-quark limit  $m_b \rightarrow \infty$ ,



$$\langle H | \mathcal{O}_i | B \rangle = \left( \begin{array}{c} \text{Long distance} \\ \text{Nonperturbative} \\ \text{Universal} \\ \text{form factors } F \\ \text{distrib amplitude } \phi \end{array} \right) \otimes \left( \begin{array}{c} \text{Short distance} \\ \text{Perturbative} \\ \text{Process dependent} \\ \text{hard-scattering} \\ \text{kernel } T \end{array} \right)$$

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- Good :  $1/m_b, \alpha_s$  expansions with control of short-distance physics
- Bad : Some numerically significant long-distance  $1/m_b$  corrections left out: weak annihilation, spectator-quark interaction

# Flavour symmetries

- Isospin symmetry ( $u \leftrightarrow d$ ) :  $B_d \rightarrow \pi^+\pi^-$ ,  $B_d \rightarrow \pi^0\pi^0$ ,  $B^- \rightarrow \pi^0\pi^-$
- $U$ -spin symmetry ( $d \leftrightarrow s$ ) :  $B_d \rightarrow \pi^+\pi^- \leftrightarrow B_s \rightarrow K^+K^-$
- Good : Global symmetries of QCD, including long- and short-distances
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**Idea** : Combine the two where they are accurate to extract stringent SM correlations between  $B_d$  and  $B_s$  decays

**Illustration** :  $B_d \rightarrow K^0\bar{K}^0$  and  $B_s \rightarrow KK$

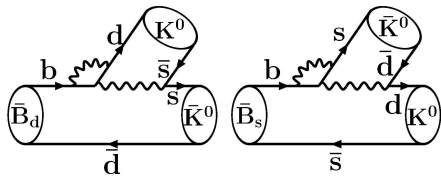
*S. Descotes-Genon, J. Matias and J. Virto  
Phys.Rev.Lett.97:061801,2006*

# $B_q \rightarrow K^0 \bar{K}^0$ : interesting penguin decays

Conventional tree and penguin decomposition

$$\bar{A} \equiv A(\bar{B}_q \rightarrow K^0 \bar{K}^0) = V_{ub} V_{uq}^* T^{q0} + V_{cb} V_{cq}^* P^{q0}$$

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no contribution from  $O_1$  and  $O_2$

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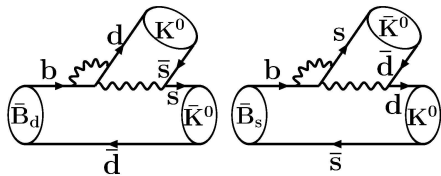


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Difference between tree and penguin  
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$\Rightarrow T^{q0} - P^{q0}$  dominated by short-distance physics  
computed fairly accurately within QCD

$$T^{d0} - P^{d0} = (1.09 \pm 0.43) \cdot 10^{-7} + i(-3.02 \pm 0.97) \cdot 10^{-7} \text{ GeV}$$

$$T^{s0} - P^{s0} = (1.03 \pm 0.41) \cdot 10^{-7} + i(-2.85 \pm 0.93) \cdot 10^{-7} \text{ GeV}$$

# $T - P$ : A sum rule for $B_d \rightarrow K^0 \bar{K}^0$ observables

$$A(t) = \frac{\Gamma(B_d(t) \rightarrow K \bar{K}) - \Gamma(\bar{B}_d(t) \rightarrow K \bar{K})}{\Gamma(B_d(t) \rightarrow K \bar{K}) + \Gamma(\bar{B}_d(t) \rightarrow K \bar{K})} = \frac{A_{dir}^{d0} \cos(\Delta M \cdot t) + A_{mix}^{d0} \sin(\Delta M \cdot t)}{\cosh(\Delta \Gamma_d t/2) - A_{\Delta}^{d0} \sinh(\Delta \Gamma_d t/2)}$$

with CP asymmetries  $\left\{ \begin{array}{l} A_{dir}^{d0} = \frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2}, A_{\Delta}^{d0} + iA_{mix}^{d0} = -\frac{2e^{-i\phi_d} A^* \bar{A}}{|A|^2 + |\bar{A}|^2} \\ |A_{\Delta}^{d0}|^2 + |A_{dir}^{d0}|^2 + |A_{mix}^{d0}|^2 = 1 \end{array} \right.$

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$T^{d0} - P^{d0}$  related to  $B_d \rightarrow K^0 \bar{K}^0$  observables (also true for  $B_s$ )

$$|T^{d0} - P^{d0}|^2 = \frac{BR^{d0} \times 32\pi M_{B_d}^2}{\tau_d \sqrt{M_{B_d}^2 - 4M_K^2}} \times \{x_1 + [x_2 \sin \phi_d - x_3 \cos \phi_d] A_{mix}^{d0} - [x_2 \cos \phi_d + x_3 \sin \phi_d] A_{\Delta}^{d0}\}$$

where  $x_1, x_2, x_3$  depend on CKM factors only,  $\phi_d$   $B$ - $\bar{B}$  mixing angle

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- SM consistency test between  $BR^{d0}$ ,  $|A_{dir}^{d0}|$  and  $A_{mix}^{d0}$  (id. for  $B_s$ )
- SM value of one (say  $|A_{mix}^{d0}|$ ) from the two others ( $BR^{d0}$  and  $A_{dir}^{d0}$ )

## $T - P$ : Hadronic parameters for $B_d \rightarrow K^0 \bar{K}^0$

To extract the hadronic parameters of this decay

- Unknowns  $|T|$ ,  $|P/T|$  and  $\arg(P/T)$
- Observables  $Br = (0.96 \pm 0.26) \cdot 10^{-6}$ ,  $A_{dir}$  (broad range),  $A_{mix}$

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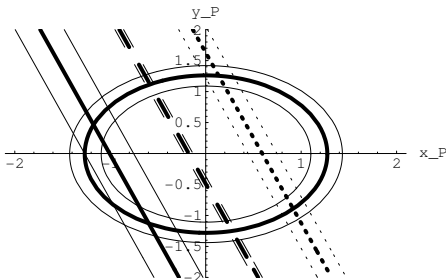
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In the plane  $P = (x_P + iy_P) \cdot 10^{-6}$   
GeV

- $Br + (T - P) \implies$  a circle
- $A_{dir} + (T - P) \implies$  a strip

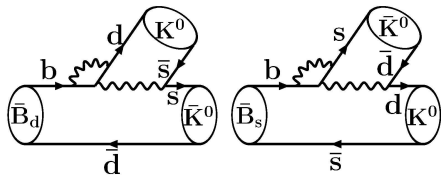
From left to right

$$A_{dir} = -0.17, -0.03, 0.10$$

(QCDF :  $A_{dir} \simeq 0.20$ )

Intersection : hadronic parameters up to a two-fold ambiguity

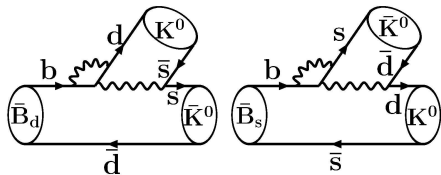
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 $\implies$  Most long-distance effects  
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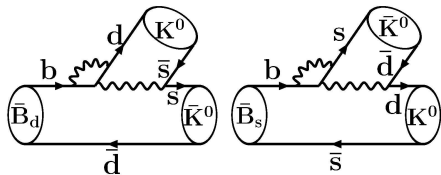


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- Difference in form factors  $f = M_{B_s}^2 F_0^{\bar{B}_s \rightarrow K}(0) / [M_{B_d}^2 F_0^{\bar{B}_d \rightarrow K}(0)]$

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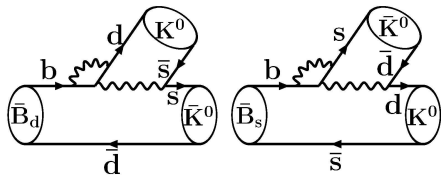


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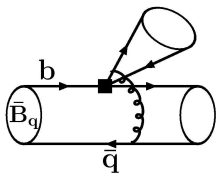
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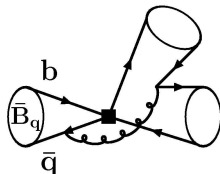
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Hard-spectator scattering  
 ( $B_d$  and  $B_s$  distribution amplitudes)



Weak annihilation  
 (gluon emission off light quark)

# $B_d \rightarrow K^0 \bar{K}^0$ and $B_s \rightarrow K^0 \bar{K}^0$ : QCDF

In QCD factorisation

$$\frac{P^{s0}}{fP^{d0}} = 1 + \frac{A_{KK}^d}{P^{d0}} \left\{ \delta\alpha_4^c - \frac{\delta\alpha_{4EW}^c}{2} + \delta\beta_3^c + 2\delta\beta_4^c - \frac{\delta\beta_{3EW}^c}{2} - \delta\beta_{4EW}^c \right\}$$
$$\frac{T^{s0}}{fT^{d0}} = 1 + \frac{A_{KK}^d}{T^{d0}} \left\{ \delta\alpha_4^u - \frac{\delta\alpha_{4EW}^u}{2} + \delta\beta_3^u + 2\delta\beta_4^u - \frac{\delta\beta_{3EW}^u}{2} - \delta\beta_{4EW}^u \right\}$$

with normalisation  $A_{KK}^q = M_{B_q}^2 F_0^{\bar{B}_q \rightarrow K}(0) f_K G_F / \sqrt{2}$

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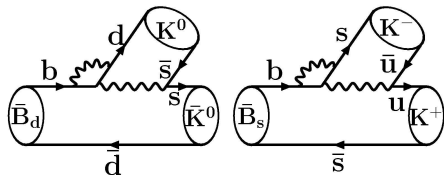
$U$ -spin breaking in very few places

- factorisable ratio  $f = M_{B_s}^2 F_0^{\bar{B}_s \rightarrow K}(0) / [M_{B_d}^2 F_0^{\bar{B}_d \rightarrow K}(0)]$
- $\delta\alpha_i = \alpha_i^P|_{B_s} - \alpha_i^P|_{B_d}$  : hard-spectator scattering
- $\delta\beta_i = \beta_i^P|_{B_s} - \beta_i^P|_{B_d}$  : weak annihilation

$\implies$  Very small differences in agreement with  $U$ -spin arguments

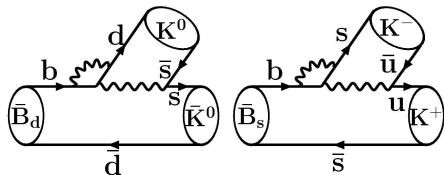
**Reliable QCDF bounds :**  $\left| \frac{P^{s0}}{fP^{d0}} - 1 \right| \leq 3\%$  and  $\left| \frac{T^{s0}}{fT^{d0}} - 1 \right| \leq 3\%$

$B_d \rightarrow K^0 \bar{K}^0$  and  $B_s \rightarrow K^+ K^-$  : Flavour sym.



$U$ -spin and isospin  
 $B_s \rightarrow K^+ K^-$  penguin related to  $P^{d0}$   
 $\implies$  Most long-distance effects  
 are the same

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Only a few places where nature of spectator quark matters

- Factorisable ratio  $f$
- $\delta\alpha$  : Hard-spectator scattering
- $\delta\beta$  : Annihilation with gluon emission from light quark in  $B_{d,s}$  meson
- Electroweak corrections

# $B_d \rightarrow K^0 \bar{K}^0$ and $B_s \rightarrow K^+ K^-$ : QCDF

Penguin relation confirmed by QCDF : only EW terms + small  $\delta\alpha, \delta\beta$

$$\frac{P^{s\pm}}{fP^{d0}} = 1 + \frac{A_{KK}^d}{P^{d0}} \times \left\{ \frac{3}{2}(\alpha_{4EW}^c + \beta_{4EW}^c) + \delta\alpha_4^c + \delta\alpha_{4EW}^c + \delta\beta_3^c + 2\delta\beta_4^c - \frac{1}{2}(\delta\beta_{3EW}^c - \delta\beta_{4EW}^c) \right\}$$

Reliable QCDF bound :  $\left| \frac{P^{s\pm}}{fP^{d0}} - 1 \right| \leq 2\%$

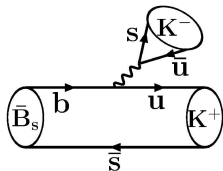


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No such simple relation for the tree part  
Some related contributions but  
 $O_1$  tree contribution  
to  $B_s \rightarrow K^+ K^-$  unmatched

- QCDF estimate of  $O_1$  term in  $T^{s\pm}$ :  $\left| \frac{T^{s\pm}}{A_{KK}^s \bar{\alpha}_1} - 1 - \frac{T^{d0}}{A_{KK}^d \bar{\alpha}_1} \right| \leq 4\%$
- Cabibbo suppressed in  $B_s \rightarrow K^+ K^-$

# Hadronic parameters for $B_s \rightarrow K^+ K^-$

Take same form factors as QCDF, and CKM factors  $\lambda_p^{(q)} = V_{pb} V_{pq}^*$

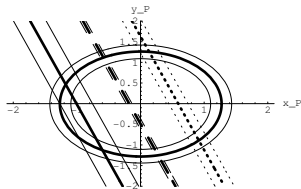
$$\begin{aligned} \lambda_u^{(d)} &= 0.0038 \cdot e^{-i\gamma} & \lambda_u^{(s)} &= 0.00088 \cdot e^{-i\gamma} \\ \lambda_c^{(d)} &= -0.0094 & \lambda_c^{(s)} &= 0.04 \end{aligned} \quad \text{and } \gamma = 62^\circ$$

$B_d \rightarrow K^0 \bar{K}^0$  :  $Br, A_{dir}, T - P \implies$  Hadronic parameters

$\implies$  Hadronic parameters for  $B_s \rightarrow K^+ K^-$  from bounds stretched to 5%

$A_{dir}^{d0}$	$ T^{s\pm}  \times 10^6$	$ P^{s\pm}/T^{s\pm} $	$\arg(P^{s\pm}/T^{s\pm})$
-0.2	$12.7 \pm 2.8$	$0.09 \pm 0.03$	$(45 \pm 33)^\circ$
-0.1	$12.1 \pm 2.7$	$0.10 \pm 0.03$	$(78 \pm 27)^\circ$
0	$11.5 \pm 2.6$	$0.10 \pm 0.03$	$(105 \pm 15)^\circ$
0.1	$11.1 \pm 2.6$	$0.11 \pm 0.03$	$(137 \pm 27)^\circ$
0.2	$10.8 \pm 2.6$	$0.11 \pm 0.03$	$(180 \pm 10)^\circ$

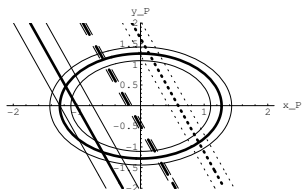
# Two-fold degeneracy in $(T^{d0}, P^{d0})$



Two solutions for  $(T^{s\pm}, P^{s\pm})$  :

- Similar  $|T^{s\pm}|$  and  $|P^{s\pm}/T^{s\pm}|$
- Solution used :  $20^\circ \leq \arg(P^{s\pm}/T^{s\pm}) \leq 180^\circ$
- 2nd sol :  $-150^\circ \leq \arg(P^{s\pm}/T^{s\pm}) \leq 20^\circ$

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HFAG on  $B_d \rightarrow \pi^+\pi^-$  data

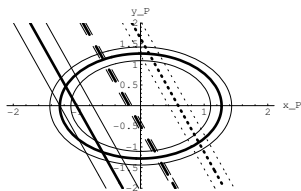
$$BR = (5.0 \pm 0.4) \times 10^{-6}$$

$$A_{dir} = -0.33 \pm 0.11$$

$$A_{mix} = 0.49 \pm 0.12$$

$$\left. \begin{array}{l} BR = (5.0 \pm 0.4) \times 10^{-6} \\ A_{dir} = -0.33 \pm 0.11 \\ A_{mix} = 0.49 \pm 0.12 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} |T_{\pi\pi}^{d\pm}| = (5.48 \pm 0.42) \times 10^{-6} \\ |P_{\pi\pi}^{d\pm}/T_{\pi\pi}^{d\pm}| = 0.13 \pm 0.05 \\ \arg(P_{\pi\pi}^{d\pm}/T_{\pi\pi}^{d\pm}) = (131 \pm 18)^\circ \end{array} \right.$$

# Two-fold degeneracy in $(T^{d0}, P^{d0})$



Two solutions for  $(T^{s\pm}, P^{s\pm})$  :

- Similar  $|T^{s\pm}|$  and  $|P^{s\pm}/T^{s\pm}|$
- Solution used :  $20^\circ \leq \arg(P^{s\pm}/T^{s\pm}) \leq 180^\circ$
- 2nd sol :  $-150^\circ \leq \arg(P^{s\pm}/T^{s\pm}) \leq 20^\circ$

HFAG on  $B_d \rightarrow \pi^+\pi^-$  data

$$\left. \begin{aligned} BR &= (5.0 \pm 0.4) \times 10^{-6} \\ A_{dir} &= -0.33 \pm 0.11 \\ A_{mix} &= 0.49 \pm 0.12 \end{aligned} \right\} \Rightarrow \left\{ \begin{aligned} |T_{\pi\pi}^{d\pm}| &= (5.48 \pm 0.42) \times 10^{-6} \\ |P_{\pi\pi}^{d\pm}/T_{\pi\pi}^{d\pm}| &= 0.13 \pm 0.05 \\ \arg(P_{\pi\pi}^{d\pm}/T_{\pi\pi}^{d\pm}) &= (131 \pm 18)^\circ \end{aligned} \right.$$

Approximate  $U$ -spin  $B_d \rightarrow \pi^+\pi^-/B_s \rightarrow K^+K^-$

- Discard 2nd sol:  $\arg(P^{s\pm}/T^{s\pm})$  should be positive
- Favours sol used with  $A_{dir}^{d0} > 0$

# Observables in $B_s \rightarrow K^+ K^-$

$(P^{s\pm}, T^{s\pm})$  yields  $U$ -spin breaking between  $\bar{B}_s \rightarrow K^+ K^-$  and  $\bar{B}_d \rightarrow \pi^+ \pi^-$

$$\xi = \left| \frac{P^{s\pm}}{T^{s\pm}} \frac{T_{\pi\pi}^{d\pm}}{P_{\pi\pi}^{d\pm}} \right| = 0.8 \pm 0.4$$

# Observables in $B_s \rightarrow K^+ K^-$

$(P^{s\pm}, T^{s\pm})$  yields  $U$ -spin breaking between  $\bar{B}_s \rightarrow K^+ K^-$  and  $\bar{B}_d \rightarrow \pi^+ \pi^-$

$$\xi = \left| \frac{P^{s\pm}}{T^{s\pm}} \frac{T_{\pi\pi}^{d\pm}}{P_{\pi\pi}^{d\pm}} \right| = 0.8 \pm 0.4$$

$A_{dir}^{d0}$	$BR^{s\pm} \times 10^6$	$A_{dir}^{s\pm} \times 10^2$	$A_{mix}^{s\pm} \times 10^2$
-0.2	$21.9 \pm 7.9 \pm 4.3$	$24.3 \pm 18.4$	$24.7 \pm 15.5$
-0.1	$19.6 \pm 7.3 \pm 4.2$	$35.7 \pm 14.4$	$7.7 \pm 15.7$
0	$17.8 \pm 6.0 \pm 3.7$	$37.0 \pm 12.3$	$-9.3 \pm 10.6$
0.1	$16.4 \pm 5.7 \pm 3.3$	$29.7 \pm 19.9$	$-26.3 \pm 15.6$
0.2	$15.4 \pm 5.6 \pm 3.1$	$6.8 \pm 28.9$	$-40.2 \pm 14.6$

# Observables in $B_s \rightarrow K^+ K^-$

$(P^{s\pm}, T^{s\pm})$  yields  $U$ -spin breaking between  $\bar{B}_s \rightarrow K^+ K^-$  and  $\bar{B}_d \rightarrow \pi^+ \pi^-$

$$\xi = \left| \frac{P^{s\pm}}{T^{s\pm}} \frac{T_{\pi\pi}^{d\pm}}{P_{\pi\pi}^{d\pm}} \right| = 0.8 \pm 0.4$$

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- $U$ -spin on  $B_d \rightarrow \pi^+ \pi^-$ :  $A_{mix}^{s\pm} < 0$  and  $\arg \frac{P^{s\pm}}{T^{s\pm}} \simeq 130^\circ \implies A_{dir}^{d0} \geq 0$
- QCDF alone:  $A_{dir}^{d0} \simeq 20\%$
- Babar:  $A_{dir}^{d0} = -0.40 \pm 0.41 \pm 0.06$  [hep-ex/0608036]



# Observables in $B_s \rightarrow K^+ K^-$

$(P^{s\pm}, T^{s\pm})$  yields  $U$ -spin breaking between  $\bar{B}_s \rightarrow K^+ K^-$  and  $\bar{B}_d \rightarrow \pi^+ \pi^-$

$$\xi = \left| \frac{P^{s\pm}}{T^{s\pm}} \frac{T_{\pi\pi}^{d\pm}}{P_{\pi\pi}^{d\pm}} \right| = 0.8 \pm 0.4$$

$A_{dir}^{d0}$	$BR^{s\pm} \times 10^6$	$A_{dir}^{s\pm} \times 10^2$	$A_{mix}^{s\pm} \times 10^2$
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CDF measurement [Beauty 2006]:  $BR^{s\pm} = (24.4 \pm 1.4 \pm 4.6) \times 10^{-6}$

# Observables in $B_s \rightarrow K^0 \bar{K}^0$

$B_d \rightarrow K^0 \bar{K}^0$  :  $Br, A_{dir}, T - P \implies$  Hadronic parameters

$\implies$  Hadronic parameters for  $B_s \rightarrow K^0 \bar{K}^0$  from bounds stretched to 5%

$A_{dir}^{d0}$	$BR^{s0} \times 10^6$	$A_{dir}^{s0} \times 10^2$	$A_{mix}^{s0} \times 10^2$
-0.2	$18.4 \pm 6.5 \pm 3.6$	$0.8 \pm 0.3$	$-0.3 \pm 0.8$
-0.1	$18.2 \pm 6.4 \pm 3.6$	$0.4 \pm 0.3$	$-0.7 \pm 0.7$
0	$18.1 \pm 6.3 \pm 3.6$	$0 \pm 0.3$	$-0.8 \pm 0.7$
0.1	$18.2 \pm 6.4 \pm 3.6$	$-0.4 \pm 0.3$	$-0.7 \pm 0.7$
0.2	$18.4 \pm 6.5 \pm 3.6$	$-0.8 \pm 0.3$	$-0.3 \pm 0.8$

- Very small asymmetries, but BR very stable within SM
- All constraints derived from SM relation between  $b \rightarrow d$  and  $b \rightarrow s$  and should be upset by New Physics

Improve our understanding of  $B$ -decays by combining two theoretical tools

## QCD factorisation and flavour symmetries

- Stringent SM correlations between  $B_d$  and  $B_s$  sector (altered by NP)
- Flavour symmetry helps to deal with  $1/m_b$ -suppressed but numerically significant contributions
- QCD factorisation assesses more precisely some poorly known  $SU(3)$ -breaking effects

# Conclusion

Illustration with  $B_d \rightarrow K^0 \bar{K}^0$ , which can be related to  $B_s \rightarrow K \bar{K}$

- $T^{d0} - P^{d0}$  accurately known in QCDF and related to observables
- $Br(B_d \rightarrow K^0 \bar{K}^0)$  (measured) and  $A_{dir}^{d0}$  (loose range, expected  $\geq 0$ ) enough to fix tree and penguin
- Large and correlated asymmetries in  $B_d \rightarrow K^0 \bar{K}^0$  and  $B_s \rightarrow K^+ K^-$
- Improved determination of  $U$ -spin ratios
- Clean SM predictions (improvable with  $f$  and  $A_{dir}^{d0}$ )

$$Br(B_s \rightarrow K^+ K^-) = (20 \pm 8 \pm 4 \pm 2) \cdot 10^{-6} \quad (\text{OK with CDF})$$

$$Br(B_s \rightarrow K^0 \bar{K}^0) = (18 \pm 6 \pm 4 \pm 2) \cdot 10^{-6}$$

# Conclusion

Illustration with  $B_d \rightarrow K^0 \bar{K}^0$ , which can be related to  $B_s \rightarrow K \bar{K}$

- $T^{d0} - P^{d0}$  accurately known in QCDF and related to observables
- $Br(B_d \rightarrow K^0 \bar{K}^0)$  (measured) and  $A_{dir}^{d0}$  (loose range, expected  $\geq 0$ ) enough to fix tree and penguin
- Large and correlated asymmetries in  $B_d \rightarrow K^0 \bar{K}^0$  and  $B_s \rightarrow K^+ K^-$
- Improved determination of  $U$ -spin ratios
- Clean SM predictions (improvable with  $f$  and  $A_{dir}^{d0}$ )

$$Br(B_s \rightarrow K^+ K^-) = (20 \pm 8 \pm 4 \pm 2) \cdot 10^{-6} \quad (\text{OK with CDF})$$

$$Br(B_s \rightarrow K^0 \bar{K}^0) = (18 \pm 6 \pm 4 \pm 2) \cdot 10^{-6}$$

Experiment : More accurate  $A_{dir}^{d0}$  ? More  $B_s$  observables ?

Theory : Improved  $f = \frac{M_{B_s}^2 F_0^{\bar{B}_s \rightarrow K}(0)}{[M_{B_d}^2 F_0^{\bar{B}_d \rightarrow K}(0)]}$  ? Application to other modes ?

# Backup

# Comparing QCDF and our approach

Main uncertainties from long-distance (IR-divergent) terms

- QCD factorisation : Source of substantial errors to model

Observable	QCDF default set	QCDF S4
$BR^{s0} \times 10^6$	$24.7^{+2.5+13.7+2.6+25.6}_{-2.4-9.2-2.9-9.8}$	38.3
$A_{dir}^{s0} \times 10^2$	$0.9^{+0.2+0.2+0.1+0.2}_{-0.2-0.2-0.1-0.3}$	0.6
$BR^{s\pm} \times 10^6$	$22.7^{+3.5+12.7+2.0+24.1}_{-3.2-8.4-2.0-9.1}$	36.1
$A_{dir}^{s\pm} \times 10^2$	$4.0^{+1.0+2.0+0.5+10.4}_{-1.0-2.3-0.5-11.3}$	-4.7

*Beneke and Neubert, Nucl.Phys.B675:333-415,2003*

- Our approach : Extracted from other flavour-related decays

$A_{dir}^{d0}$	$BR^{s0} \times 10^6$	$A_{dir}^{s0} \times 10^2$	$BR^{s\pm} \times 10^6$	$A_{dir}^{s\pm} \times 10^2$
-0.2	$18.4 \pm 6.5 \pm 3.6$	$0.8 \pm 0.3$	$21.9 \pm 7.9 \pm 4.3$	$24.3 \pm 18.4$
-0.1	$18.2 \pm 6.4 \pm 3.6$	$0.4 \pm 0.3$	$19.6 \pm 7.3 \pm 4.2$	$35.7 \pm 14.4$
0	$18.1 \pm 6.3 \pm 3.6$	$0 \pm 0.3$	$17.8 \pm 6.0 \pm 3.7$	$37.0 \pm 12.3$
0.1	$18.2 \pm 6.4 \pm 3.6$	$-0.4 \pm 0.3$	$16.4 \pm 5.7 \pm 3.3$	$29.7 \pm 19.9$
0.2	$18.4 \pm 6.5 \pm 3.6$	$-0.8 \pm 0.3$	$15.4 \pm 5.6 \pm 3.1$	$6.8 \pm 28.9$

$$A_{dir}^{d0} = -0.40 \pm 0.41 \pm 0.06 \text{ [BaBar]} \quad BR^{s\pm} = (24.4 \pm 1.4 \pm 4.6) \times 10^{-6} \text{ [CDF]}$$

# Comparing flavour symmetries and our approach

Quantitative statement about  $U$ -spin breaking

- Flavour symmetries : guesstimated fudge factors

$$\xi = \left| \frac{P^{s\pm}}{T^{s\pm}} \times \frac{T_{\pi\pi}^{d\pm}}{P_{\pi\pi}^{d\pm}} \right| = 1.0 \pm 0.2 \quad (\text{assumed})$$

$$R_c = \left| \frac{T^{s\pm}}{T_{\pi\pi}^{d\pm}} \right| = 1.76 \pm 0.17 \quad (\text{sum rule})$$

$$4.2 \cdot 10^{-6} \leq BR^{s\pm} \leq 61.9 \cdot 10^{-6}$$

*S. Baek, D. London, J. Matias, J. Virto, JHEP 0602:027,2006*

- Our approach : Estimate through QCDF analysis of  $U$ -spin relations

$$\xi = 0.8 \pm 0.4 \quad (\text{computed})$$

$$R_c = 2.0 \pm 0.8 \quad (\text{computed})$$

$$BR^{s\pm} = (20 \pm 8 \pm 4 \pm 2) \cdot 10^{-6}$$



# Dependence on $\gamma$

- Almost no dependence of predictions for CP asymmetries in  $B_s \rightarrow K \bar{K}^0$  and mixed asymmetry in  $B_s \rightarrow K^+ K^-$
- A limited sensitivity of order 10% for the other observables, e.g. if we take  $A_{dir}^{d0} = 0$

Observable	$\gamma = 62^\circ$	$\gamma = 68^\circ$
$BR^{s0} \times 10^6$	$18.1 \pm 6.3 \pm 3.6$	$17.0 \pm 5.9 \pm 3.6$
$BR^{s\pm} \times 10^6$	$17.8 \pm 6.0 \pm 3.6$	$17.1 \pm 5.8 \pm 3.6$
$A_{dir}^{s\pm} \times 10^2$	$37.0 \pm 12.3$	$40.5 \pm 12.5$

⇒ Currently under investigation