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"Flavour in the era of LHC"
EDM and g-2 miniworkshop

A plan of comprehensive investigation of systematic errors and spin coherence time for the deuteron resonance EDM experiment.

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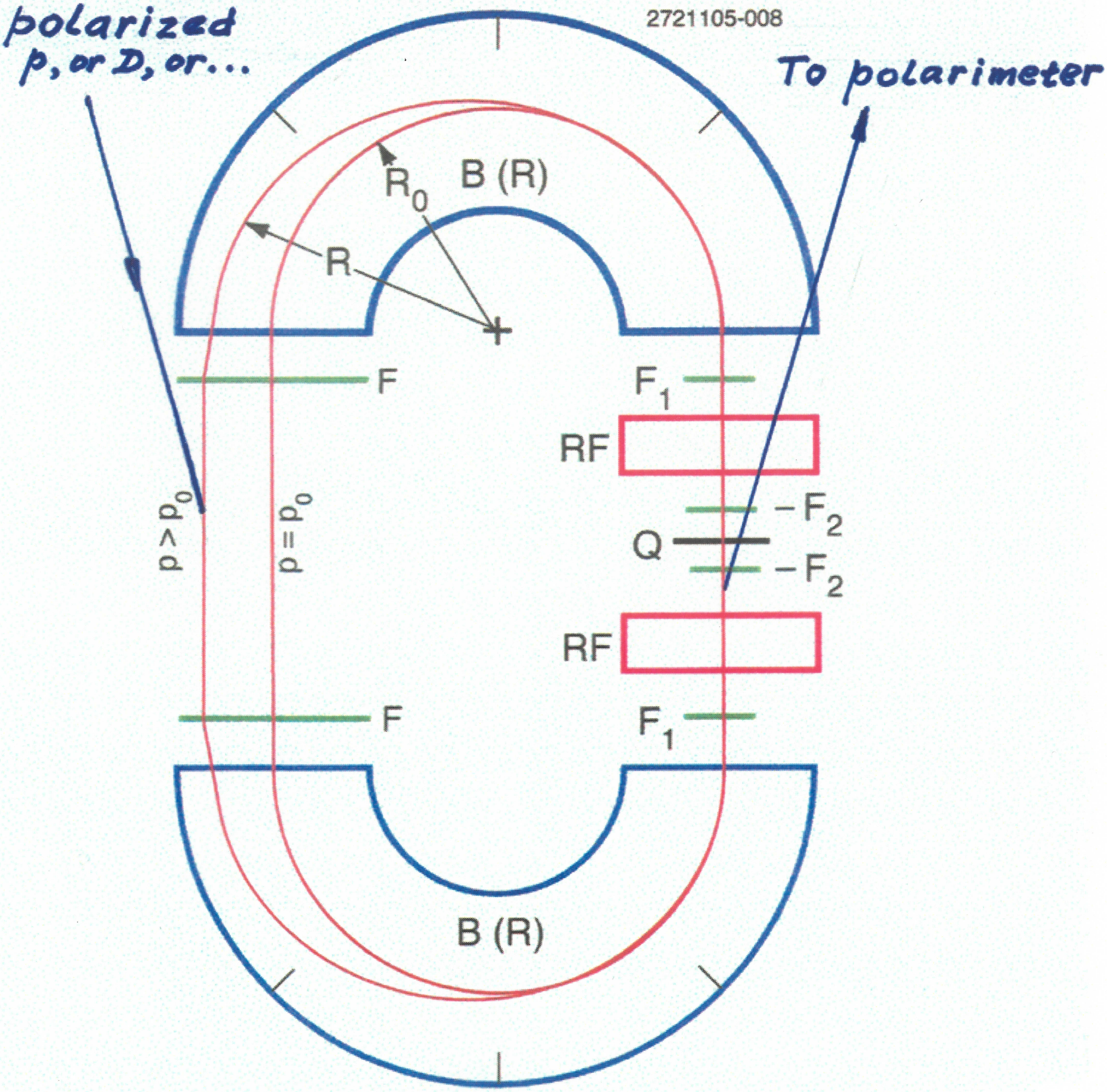


Fig. 1

Resonance EDM ring.

$\delta d_D \sim 10^{-29}$ e·cm

$\delta d_p \sim \delta d_{He\text{-}3} \sim 10^{-28}$ e.cm

The final choice of parameters needs comprehensive investigations

Will be not changed:

$mc^2 = 1875.6134$ MeV

$a = (g-2)/2 = -0.1429878$

$\alpha_p \equiv \langle D(s)/R(s) \rangle = 1$

$D(s) = 0$ in one of big straight sections

Still under investigations:

$p = 1 \div 1.5$ GeV/c

$R \sim 2.5$ m , $B \sim 2$ T

$n \equiv -R(\partial B/\partial R)/B = 1$ or $n < 1$

$f_c \sim 8$ MHz

$f_a = a\gamma f_c \sim 1.5$ MHz

$f_{RF1} = h_1 f_c \sim 400$ MHz

$f_{RF2} = h_2 f_c \pm \nu_a f_c$; $\nu_a = a\gamma$

$h_2 = h_1/2$, or $h_1/4$, ...

$V_{RF1} \sim 15 \div 20$ MV/turn

$V_{RF2} \ll V_{RF1}$

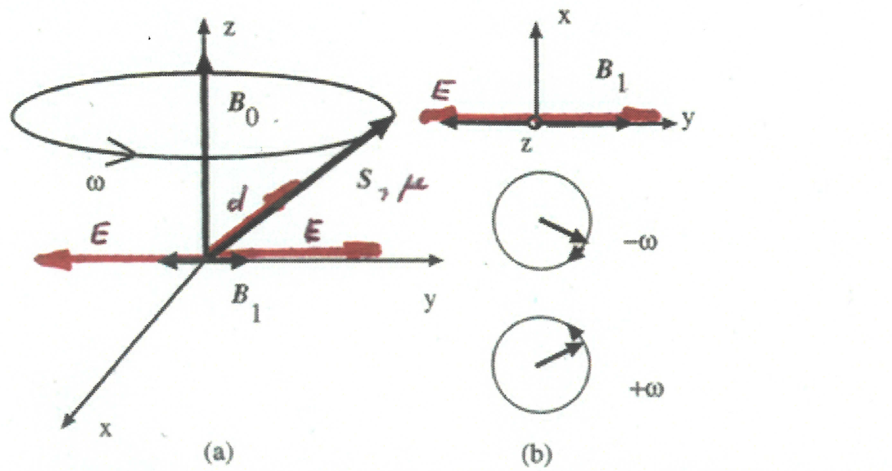


Fig. 3.1a,b. Classical vector model of a spin-1/2 system precessing in a magnetic field. (a) A static field is applied along the z axis, and an oscillating field can be applied along the y axis. (b) The oscillating field can be represented as two counter-rotating fields

EDM resonance idea:

In the d_D-case, resonant oscillations of E in the rest frame come mostly from the deuteron velocity oscillations,

$$\vec{E} = \gamma\, \vec{v} \times \vec{B} = \gamma \left\{ \vec{v}_0 \times \vec{B} + \Delta\vec{v}(t) \times \vec{B} \right\}$$

$$\Delta\vec{v}(t) = \Delta\vec{v}_0 \cdot \cos(\omega_{g-2}\, t)$$

If $d_D \neq 0$, then, on the average, we will observe a slow spin rotation in the vertical plane (in addition to the usual (g-2) rotations in the x-y plane).

THE VELOCITY OF EVERY PARTICLE IS A CLASSICAL SUPERPOSITION:

$$v = v_0 + (\Delta v)_{free}\cos(\omega_L t + \alpha_L) + (\Delta v)_F \cos(\omega_a t + \varphi_a).$$

AS FOR THE SPIN,

$$s_L = s_{L0}\cos(\omega_a t + \varphi_a)\ \text{(WHERE } \omega_a \neq \omega_L\text{'s)}.$$

THUS, THE VELOCITY PARTIALLY OSCILLATES IN RESONANCE WITH THE s_L IN THE PRESENCE OF EDM, SO:

$$\frac{ds_v}{dt} = \omega_e s_L = -\eta\frac{eB_V}{2mc}s_{L0}\cos(\omega_a t+\varphi_a)\cdot\left[(\Delta v)_F\cos(\omega_a t+\varphi_a) + (\Delta v)_{free}\cos(\omega_L t+\alpha_L) + v_0\right]$$

$$= -\eta\frac{eB_V}{4mc}s_{L0}(\Delta v)_F + \text{NON-RESONANCE TERMS},$$

THUS,

$$s_V - s_{V0} \approx -\eta\frac{eB_V}{4mc}s_{L0}(\Delta v)_F t + \ldots$$

(IN THE DEUTERON EDM RING, $(\Delta v)_F / v_0 \sim 0.01$; $B_V = 2T$; $p \sim 1.5GeV/c$; $R \sim 2.5m$. THE GOAL IS $\delta d_D \sim 10^{-29}$, $\delta\eta \sim 2\times10^{-15}$.)

$d\theta_v/dt \sim 10^{-3}\ \mu rad/s$

This is the first basic idea of the resonance method.

THE EXISTENCE OF SUCH RESONANCE HAS BEEN CONFIRMED BY SIMULATIONS (Y.S.).

Forming coherent synchrotron oscillations of $\Delta v/v$ by using strong nonlinearities + a resonance.

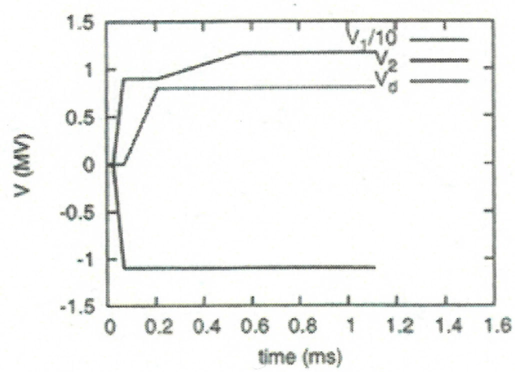


FIG. 3: Voltage amplitudes versus time with $h_1 = 50$ and $h_d + Q_c = 25.1788$.

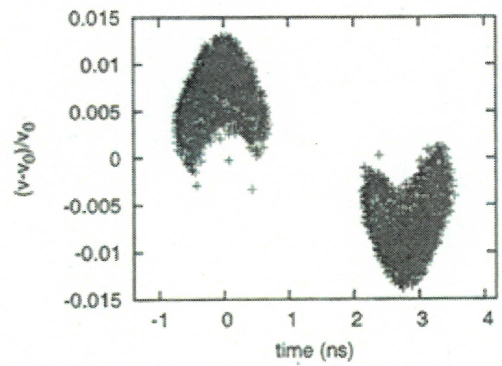


FIG. 4: Phase space plots on a single turn for two adjacent bunches.

From M. Blaskiewicz

Spin dynamics systematics

1. The usual spin resonances, with no reletions to our EDM resonance.

T. Roser, BNL

From Courant

Thomas-BMT equation with azimuthal coordinate θ as independent variable and the fields expressed in terms of the particle coordinates:

$$\frac{d\psi}{d\theta} = -\frac{i}{2}\begin{bmatrix} G\gamma & -\xi \\ -\xi^* & -G\gamma \end{bmatrix}\psi \tag{1}$$

$$\xi = -\rho y''(1+G\gamma)$$
$$-i\left[(1+G\gamma)y' - \rho(1+G)\left(\frac{y}{\rho}\right)'\right]$$

where ρ is the bending radius. Resonance strength is [1]

$$\epsilon_K = \frac{1}{2\pi}\oint \xi \exp(-iK\theta)\, d\theta \tag{2}$$

K is a combination of frequencies possessed by vertical oscillations, $y(\theta)$.

A spin resonance happens when $a\gamma = K$.

The most part of these res's must be avoided. With which accuracy? What exactly means "most"? → INVESTIGATE

Control bunches with NO EDM

8

Alternate bunches can have different velocities (for controlling systematic errors)

Deuteron bunch

Different control bunches have different phase differences between spin and Δv/v oscillations.

How many different Δφ's are needed??

EDM effect:

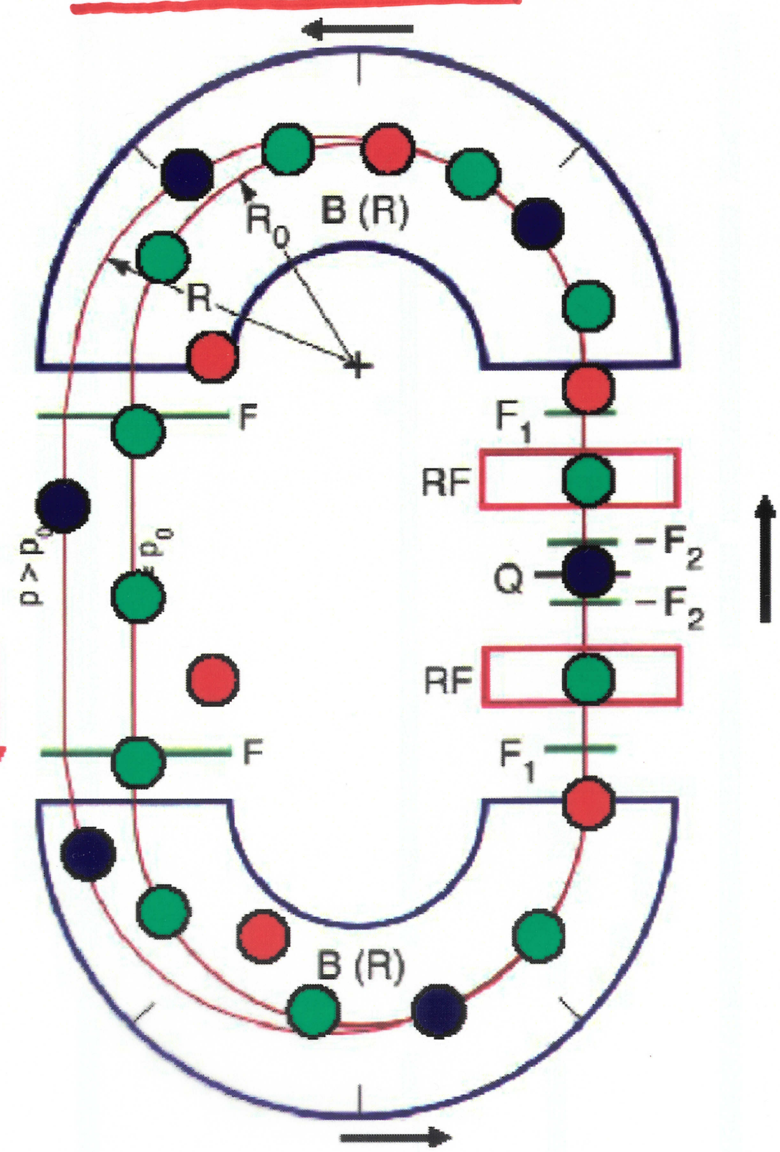
Spin Up

Spin Down

No EDM effect

Spin direction

Velocity direction



Spin dynamics systematics

2. High modes of f_a, kf_a, in the (g-2) Fourier spectrum.

$$\boxed{\text{We need } \alpha_p = 1}$$

$\Delta L/L = \alpha_p \Delta p/p$ (by definition)

Then,

$$\omega_a = \langle \frac{e}{mc} a B \rangle = \frac{e}{mc} a \gamma \omega_c = 2\pi \frac{e}{mc} a \frac{\gamma v}{L}$$

$$\frac{\gamma v}{L} = \frac{p}{L} = \frac{p_0}{L_0} \frac{1+\Delta p/p}{1+\Delta L/L}$$

ω_a does not depend on p, $\omega_a = \omega_{a0}$, only if $\Delta L/L = \Delta p/p$, i.e. $\alpha_p = 1$.

In our experiment, $\Delta p/p$ oscillates with that very ω_a frequency, so, if $\alpha_p \neq 1$, then

$$\sin \omega_a t = \sin \omega_{a0} t + \sum_{k>1} \xi_k \sin(k\omega_{a0} t + \varphi_k)$$

$\Rightarrow$ many additional

$\Rightarrow$ spin resonances!

$\alpha_p = 1$, consequences.

If $n=1$, then $D_{max} \sim 15\,m$. If $\frac{\Delta v}{v} \sim 1\%$, then $\Delta p/p = \gamma^2 \Delta v/v \sim 2\%$, and then $x_{max} = D_{max}\,\Delta p/p \sim 30\,cm \rightarrow$ dipole pole at that end must be $> 60\,cm$ wide!

We need a smaller n!

But then we will have smaller straight sections + we will loose the advantages of $n=1$.

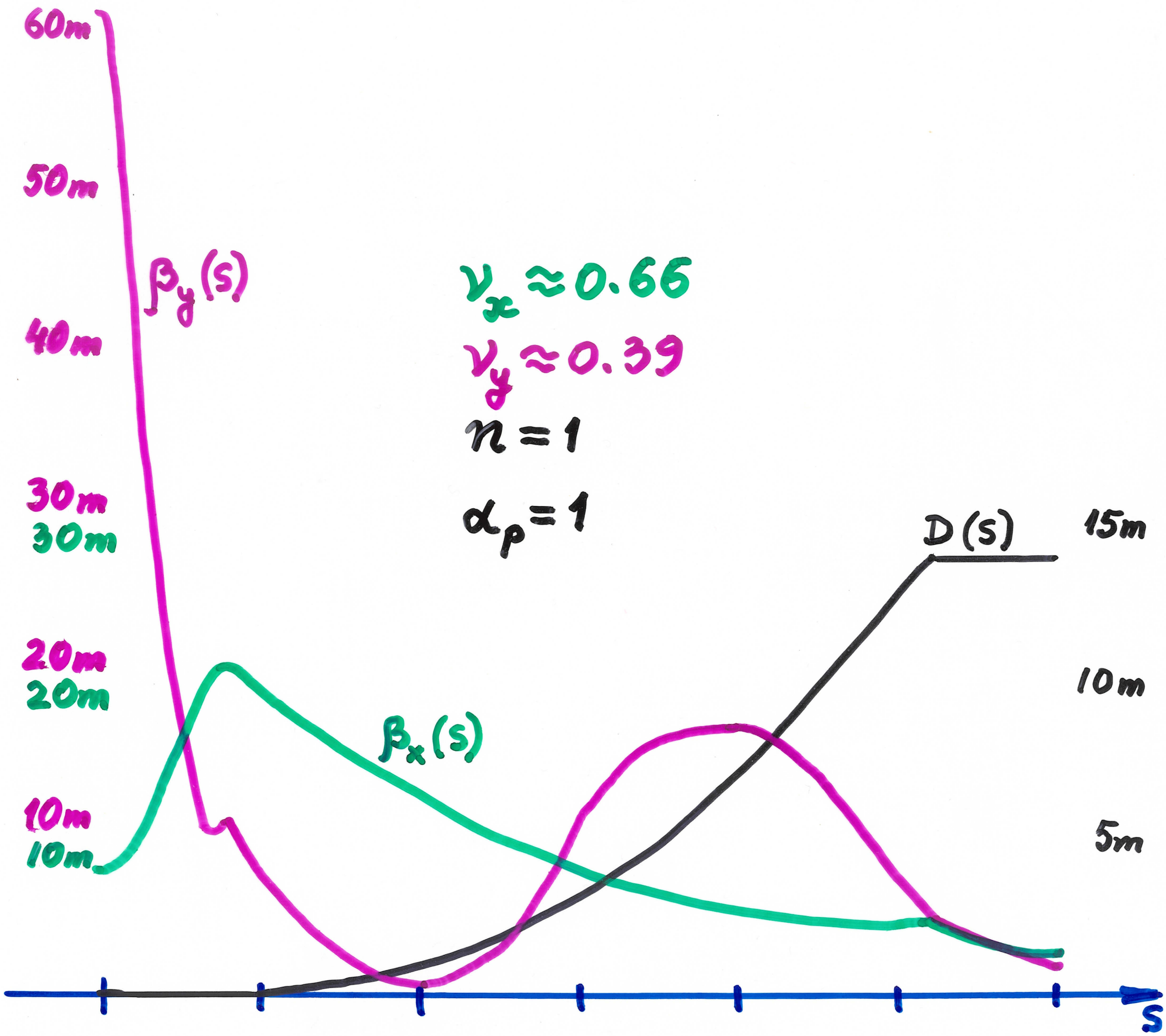
Advantages of $n=1$:

(1) $\langle B_R \rangle$, $\langle B_L \rangle$ do not depend on $\Delta p/p$ oscillations.

(2) It is easier to cancel spin decoherence.

All this means that we must investigate 2-3 versions, say, $n=1$, $n=0.5$, $n=0.2$.

Also, what is tolerance for $\delta(\alpha_p - 1)$??



$\frac{1}{2}$ of the ring

Spin dynamics systematics

3. x-y coupling

Example: a skew quadrupole effect in the area $D \neq 0$.

Our $\Delta v/v$ oscillations, $f = f_a$, $\varphi = \varphi_a \Rightarrow$
x-oscillations, $x = D\Delta p/p$, $f = f_a$, $\varphi = \varphi_a \Rightarrow$
y-oscillations, $y = D_y \Delta p/p$, $f = f_a$, $\varphi = \varphi_a$
(only in presence of any x-y coupling).

Now in the Courant formula for the spin resonance strength, we have the term with $K = f_a/f_c = a\gamma$.

The tolerance for the rotation ange, θ_{sk}, of the quadrupole at $D = D_{max}$,

$$\theta_{sk} < 10^{-12}.$$

Our tools (to be further investigated) include:

I. Corrections of false edm spin resonance signals based on observations of spin itself.

Example: $d_D = 10^{-29}\, e\cdot cm$ corresponds to $\Delta \dot{S}_v \sim 10^{-9}/s$. If we already know that $d_D < 10^{-26}$, then any observed $\Delta \dot{S}_v > 10^{-6}/s$ can be boldly corrected, with some errors. In the zero-method described below, there are no difference between "natural" and these, induced errors.

Tools:

II. Introduction of very slow oscillations of Courant's β-functions, $\beta = \beta_0 + \Delta\beta \cos ft$, say, $f = 10^3$ Hz. $\Delta\beta = \Delta\beta(s)$ depends on where and how you perturb $\partial B/\partial x$. Different, mutually independent (in a certain sense) $\Delta\beta = \Delta\beta(s)$ will oscillate with different f's. Then we Fourier-analyse the Signal, with respect to the chosen f's.

How this helps.

Signal $\sigma = \sigma_{edm} + \sum_{n=1}^{N} a_n \sigma_n$,

$\sigma_{edm} = (dS_v/dt)_{edm}$; $\sigma_n = \langle \sigma_n \rangle + \text{random} + \dots$

σ_n is contribution from imperfection # n.

Our main concern is a constant in time

$\langle \sigma_n \rangle$.

$$a_n = a_{n0} + \Delta a_n^j \cos f_j t,$$

Δa_n^j is known from lattice and from how we produce perturbations oscillating with this frequency f_j. If, now, at some time t,

$$\sigma_j = \int_0^t dt\, \sigma \cdot \cos f_j t = 0, \quad j = 1, 2, \dots, \underline{\underline{M \geq N}},$$

then

$$\sum_n^N \Delta a_n^j \langle \sigma_n \rangle + \text{some terms to be investigate} = 0$$

If determinant $\| a_n^j \| \neq 0$ for different sets of j's, etc, then, at this time t,

$$\langle \sigma_n \rangle = 0, \quad n = 1, \dots, N$$

and $\qquad \sigma(t) = \sigma_{edm}$

How to produce such a_n^j's

We may try to use only lenses, not any dipole sections (though it is not forbidden). There are more than 5 lenses in Fig. 5. There are $2^5-1=31$ combinations of them, singles, pairs, triplets, quadruples, and all five, which can be used for 31 different frequencies.

However, we want not to disturb betatron tunes. This condition decreses the number of combinations.

Equations:

$$0=\cos 2\pi\nu_x-\cos 2\pi\nu_{x0}=\frac{1}{2}\sin 2\pi\nu_{x0}\int_s^{s+C}\beta\,\Delta K\,ds;$$

and similar for ν_y, and, for β_y,

$$\frac{\Delta\beta(s)}{\beta(s)}=\frac{1}{2\sin 2\pi\nu}\sum(\Delta K\ell)_i\,\beta(s_i)\left[\cos(2|\phi(s)-\phi(s_i)|-2\pi\nu)\right],$$

$K\propto\partial B/\partial x$.

Spin dynamics systematics

4. CCW $\longleftrightarrow$ CW

Imitation of time reversal:

$t \to -t$, $v \to -v$, $B \to -B$, $E \to E$

The accuracy of the f_j-method depends on $\Delta a_n^j / a_{n0}$, which is $\ll 1$, so the additional cancellation by CW–CCW may be needed. It will work, since our main concern is a constant in time $\langle \sigma_n \rangle$.

Obviousely, all this tools need to be further investigated.

About spin coherence time

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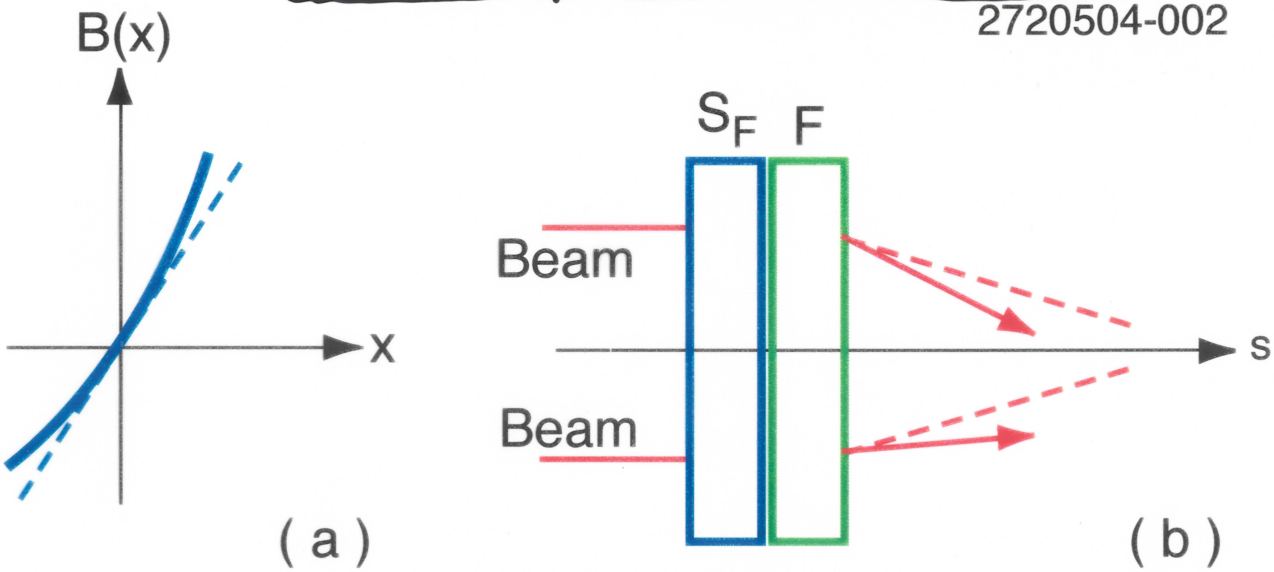
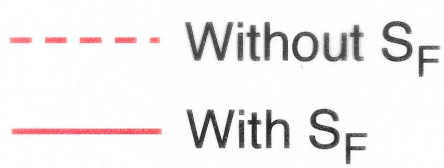


Fig. 6.1.1.4



In this example, the sextupole lense changes the average radius of the particle, individually:

$$\Delta R = -\xi x^2.$$

Therefore, it corrects the average B_v-field met by this particle. (The picture is more copmpicated due to involvement of synchrotron oscillations.)