$\bar{B} \to X_s\gamma$ at NNLO

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hep-ph/0609232

The weak radiative $\bar{B}$-meson decay branching ratio:

$$\mathcal{B}(\bar{B} \to X_s\gamma)_{E_\gamma > 1.6\ \mathrm{GeV}} = \mathcal{B}(\bar{B} \to X_c e\bar{\nu})_{\mathrm{exp}} \left[\frac{\Gamma(b\to s\gamma)}{\Gamma(b\to ce\bar{\nu})}\right]_{\mathrm{LO\ EW}} f\left(\frac{\alpha_s(M_W)}{\alpha_s(m_b)}\right) \times$$

$$\times \left\{ 1 + \mathcal{O}(\alpha_s) + \mathcal{O}(\alpha_s^2) + \mathcal{O}(\alpha_{\mathrm{em}}) + \mathcal{O}\left(\frac{\Lambda^2}{m_b^2}\right) + \mathcal{O}\left(\frac{\Lambda^2}{m_c^2}\right) + \mathcal{O}\left(\frac{\Lambda}{m_b}\alpha_s\right) \right\}$$

NLO **NNLO**

$\sim 30\%$ $\sim 10\%$ $\sim 4\%$ $\sim 1\%$ $\sim 3\%$ $\sim 5\%$

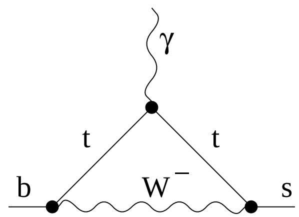
perturbative corrections

non-perturbative corrections
(methods: Optical Theorem,
Operator Product Expansion,
Heavy Quark Effective Theory)

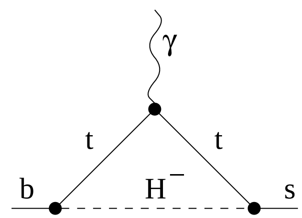
The current experimental world average: $\left(3.55 \pm 0.24\ {}^{+0.09}_{-0.10} \pm 0.03\right) \times 10^{-4}$
(HFAG, hep-ex/0603003)

Combined error: $\simeq 7.4\%$ $\Rightarrow$ **need for the NNLO.**

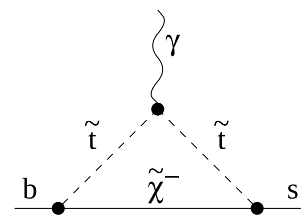
Sample LO EW diagrams:



SM

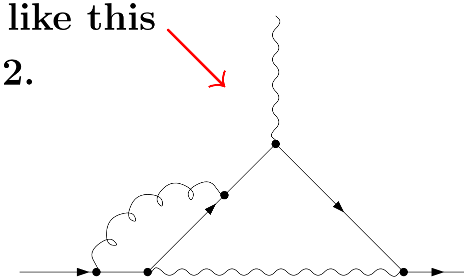


2HDM



MSSM

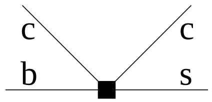
LO QCD effects that originate from two-loop diagrams like this enhance the $\bar{B} \to X_s\gamma$ rate by more than a factor of 2.
The function $f\left(\alpha_s(M_W)/\alpha_s(m_b)\right)$ arises from resummation of $\left(\alpha_s \ln M_W^2/m_b^2\right)^n$ using the renormalization group techniques.

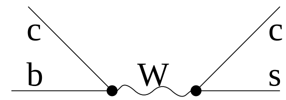


Two loops at the LO
$\Rightarrow$ Four loops at the NNLO

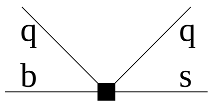
Resummation of $\left(\alpha_s \ln M_W^2/m_b^2\right)^n$ **is most conveniently performed in the framework of an effective theory that arises from the SM after decoupling of the heavy electroweak bosons and the top quark. The Lagrangian of such a theory reads:**

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm QCD \times QED}(u,d,s,c,b) + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{8} C_i(\mu)\, O_i + \left(\begin{array}{c} \text{higher-dimensional,} \\ \text{on-shell vanishing,} \\ \text{evanescent} \end{array} \right).$$

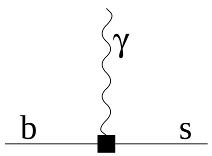
$O_{1,2}$ = 

$= (\bar{s}\Gamma_i c)(\bar{c}\Gamma'_i b)$, **from** 

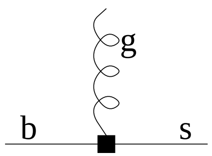
, $\quad |C_i(m_b)| \sim 1$

$O_{3,4,5,6}$ = 

$= (\bar{s}\Gamma_i b)\sum_q (\bar{q}\Gamma'_i q)$, $\quad |C_i(m_b)| < 0.07$

O_7 = 

$= \frac{e m_b}{16\pi^2}\, \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}$, $\quad C_7(m_b) \simeq -0.3$

O_8 = 

$= \frac{g m_b}{16\pi^2}\, \bar{s}_L \sigma^{\mu\nu} T^a b_R G^a_{\mu\nu}$, $\quad C_8(m_b) \simeq -0.15$

Three steps of the calculation:

Matching: **Evaluating** $C_i(\mu_0)$ **at** $\mu_0 \sim M_W$ **by requiring equality of the SM and the effective theory Green functions.**

Mixing: **Deriving the effective theory Renormalization Group Equations** $(C_j^{\rm bare} = C_i Z_{ij})$ **and evolving** $C_i(\mu)$ **from** μ_0 **to** $\mu_b \sim m_b$**.**

Matrix elements: **Evaluating the on-shell amplitudes at** $\mu_b \sim m_b$**.**

Contributions to the NNLO analysis:

Three-loop matching for O_7 and O_8:

M. Steinhauser, MM, NPB 683 (2004) 277, hep-ph/0401041

Three-loop mixing in the (O_1,...,O_6) and (O_7, O_8) sectors:

M. Gorbahn, U. Haisch, NPB 713 (2005) 291, hep-ph/0411071

M. Gorbahn, U. Haisch, MM, PRL 95 (2005) 102004, hep-ph/0504194

Four-loop mixing (O_1,...,O_6) $\longrightarrow$ (O_7, O_8):

M. Czakon, U. Haisch, MM: O_7 **finished,** O_8 **in progress.**

Two-loop matrix elements of O_7 and O_8 (and bremsstrahlung):

K. Bieri, C. Greub, M. Steinhauser, PRD 67 (2003) 114019, hep-ph/0302051 (large β_0)

I. Blokland, A. Czarnecki, MM, M. Ślusarczyk, F. Tkachov, PRD 72 (2005) 033014, hep-ph/0506055

H.M. Asatrian, T. Ewerth, C. Greub, T. Hurth, A. Hovhannisyan, V. Poghosyan, NPB 749 (2006) 325, hep-ph/0605009

K. Melnikov, A. Mitov, PLB 620 (2005) 69, hep-ph/0505097

H.M. Asatrian, T. Ewerth, A. Ferroglia, P. Gambino, C. Greub, hep-ph/0607316

Three-loop matrix elements of O_1 and O_2 (and bremsstrahlung):

K. Bieri, C. Greub, M. Steinhauser, PRD 67 (2003) 114019, hep-ph/0302051 (large β_0)

M. Steinhauser, MM, hep-ph/0609241 (interpolation in m_c)

Perturbative expansion of the Wilson coefficients:

$$C_i(\mu) = C_i^{(0)}(\mu) + \frac{\alpha_s(\mu)}{4\pi} C_i^{(1)}(\mu) + \left(\frac{\alpha_s(\mu)}{4\pi}\right)^2 C_i^{(2)}(\mu) + \ldots$$

Branching ratio:

$$\mathcal{B}(\bar{B} \to X_s \gamma)_{E_\gamma > E_0} = \mathcal{B}(\bar{B} \to X_c e \bar{\nu})_{\rm exp} \left| \frac{V_{ts}^* V_{tb}}{V_{cb}} \right|^2 \frac{6\alpha_{\rm em}}{\pi\, C} \left[\underbrace{P(E_0)}_{\text{pert.}} + \underbrace{N(E_0)}_{\text{non-pert.}} \right]$$

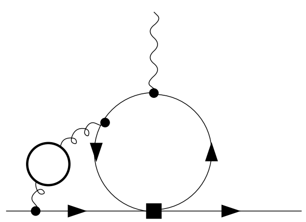
$$\frac{\Gamma[b \to X_s \gamma]_{E_\gamma > E_0}}{|V_{cb}/V_{ub}|^2\, \Gamma[b \to X_u e \bar{\nu}]} = \left| \frac{V_{ts}^* V_{tb}}{V_{cb}} \right|^2 \frac{6\alpha_{\rm em}}{\pi} P(E_0), \qquad C = \left| \frac{V_{ub}}{V_{cb}} \right|^2 \frac{\Gamma[\bar{B} \to X_c e \bar{\nu}]}{\Gamma[\bar{B} \to X_u e \bar{\nu}]}$$

Perturbative expansion of $P(E_0)$:

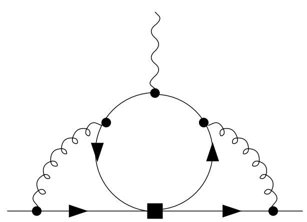
$$P = P^{(0)} + \frac{\alpha_s}{4\pi}\left(P_1^{(1)} + P_2^{(1)}(r)\right) + \left(\frac{\alpha_s(\mu)}{4\pi}\right)^2 \left(P_1^{(2)} + P_2^{(2)}(r) + P_3^{(2)}(r)\right)$$

$$P_1^{(1)}, P_3^{(2)} \sim C_i^{(0)} C_j^{(1)}, \qquad P_2^{(1)}, P_2^{(2)} \sim C_i^{(0)} C_j^{(0)}, \qquad P_1^{(2)} \sim \left(C_i^{(0)} C_j^{(2)},\ C_i^{(1)} C_j^{(1)}\right)$$

Moreover: $P_2^{(2)} = A\, n_f + B = -\frac{3}{2}(11 - 2/3 n_f) A + \frac{33}{2} A + B = P_2^{(2)\beta_0} + P_2^{(2)\rm rem}$



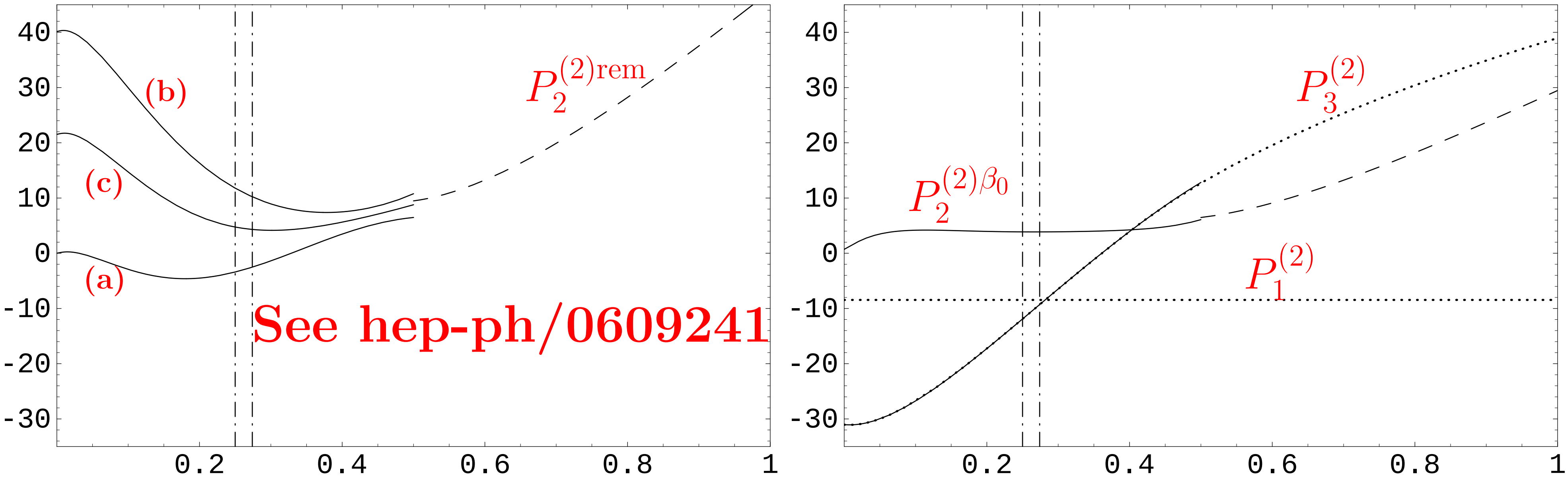
$P_2^{(2)\beta_0}$ known for all r



$$r = \frac{m_c(m_c)}{m_b^{1S}}$$

The complete $P_2^{(2)}$ has been calculated only for $r \gg \frac{1}{2}$.

The NNLO corrections $P_k^{(2)}$ as functions of $r = m_c(m_c)/m_b^{1S}$



Dotted: exact, **Solid: small-r expansions,** **Dashed: leading large-r asymptotics.**

Interpolation:

$$P_2^{(2)\mathrm{rem}}(r) = x_1 + x_2\, P_2^{(1)}(r) + x_3\, r\, \frac{d}{dr} P_2^{(1)}(r) + x_4\, P_2^{(2)\beta_0}(r) + x_5 |A_{\mathrm{NLO}}(r)|^2$$

The coefficients x_k are determined from the asymptotic behaviour at large r

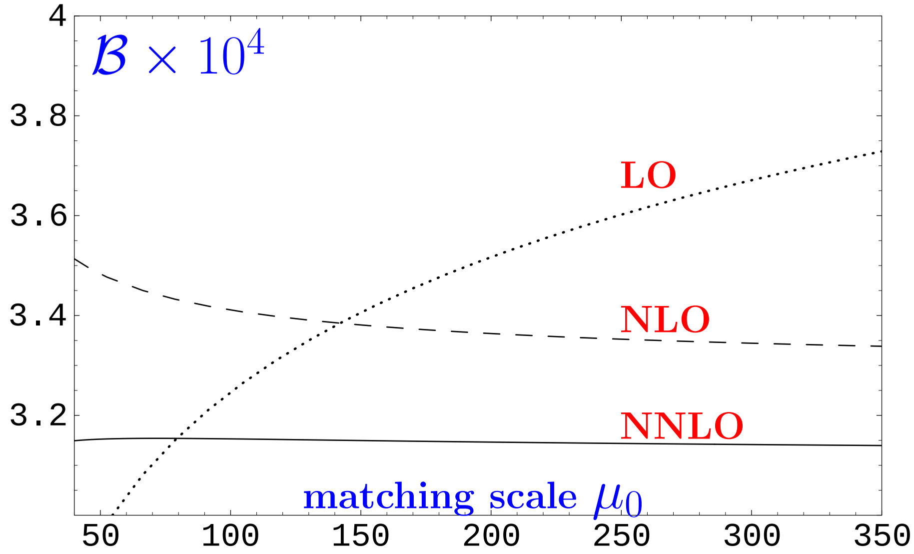
and from the requirement that either **(a)** $P_2^{(2)\mathrm{rem}}(0) = 0$,

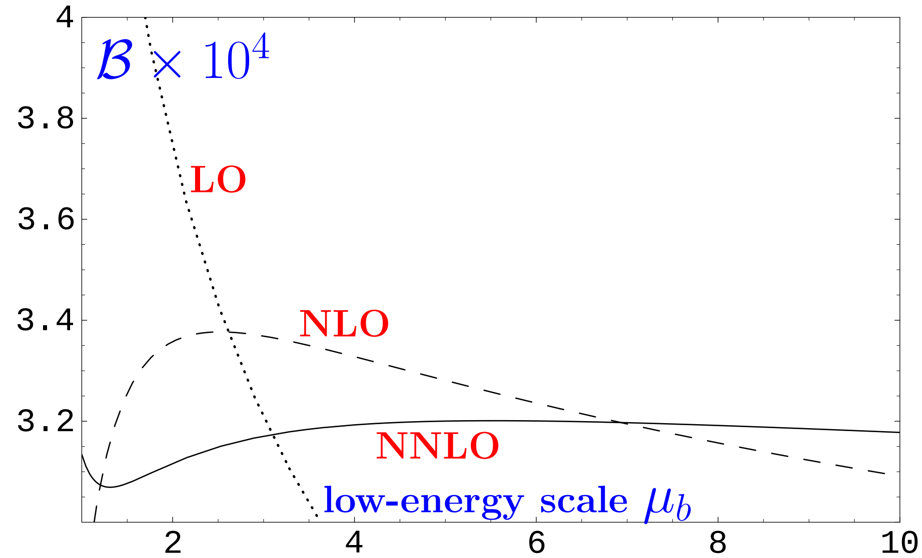
or **(b)** $P_1^{(2)} + P_2^{(2)\mathrm{rem}}(0) + P_3^{(2)}(0) = 0$,

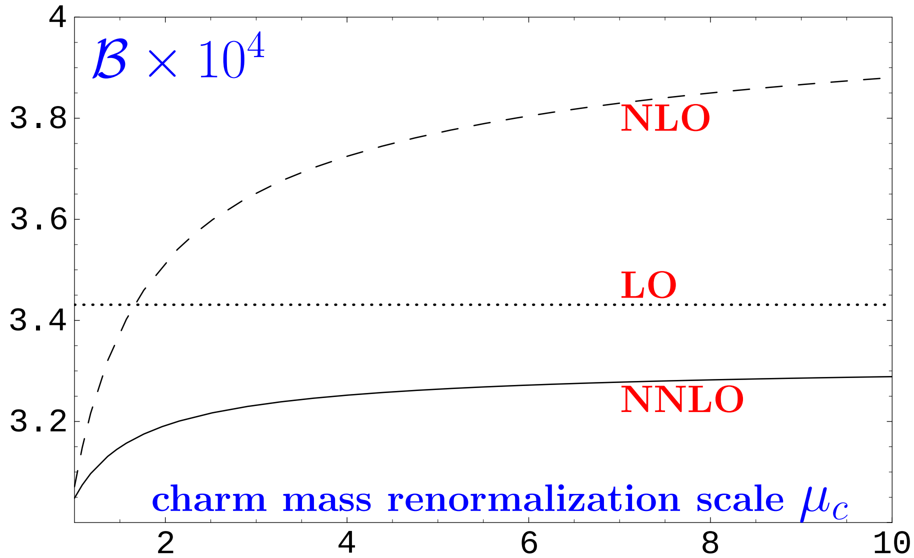
or **(c)** $P_2^{(2)\mathrm{rem}}(0) = \left[P_2^{(2)\mathrm{rem}}(0) \right]_{77}$.

The average of (a) and (b) is chosen to determine the central value of the NNLO branching ratio.
The difference between these two cases is used to estimate the interpolation ambiguity.

Renormalization scale dependence of $\mathcal{B}(\bar{B} \to X_s\gamma)_{E_\gamma > 1.6\ \mathrm{GeV}}$







"Central" values:

$$\mu_0 = 160\ \textbf{GeV}$$

$$\mu_b = 2.5\ \textbf{GeV}$$

$$\mu_c = 1.5\ \textbf{GeV}$$

Final result of the current analysis:

$$\mathcal{B}(\bar{B} \to X_s\gamma)^{\text{NNLO}}_{E_\gamma > 1.6 \text{ GeV}} = (3.15 \pm 0.23) \times 10^{-4}$$

Contributions to the total uncertainty:

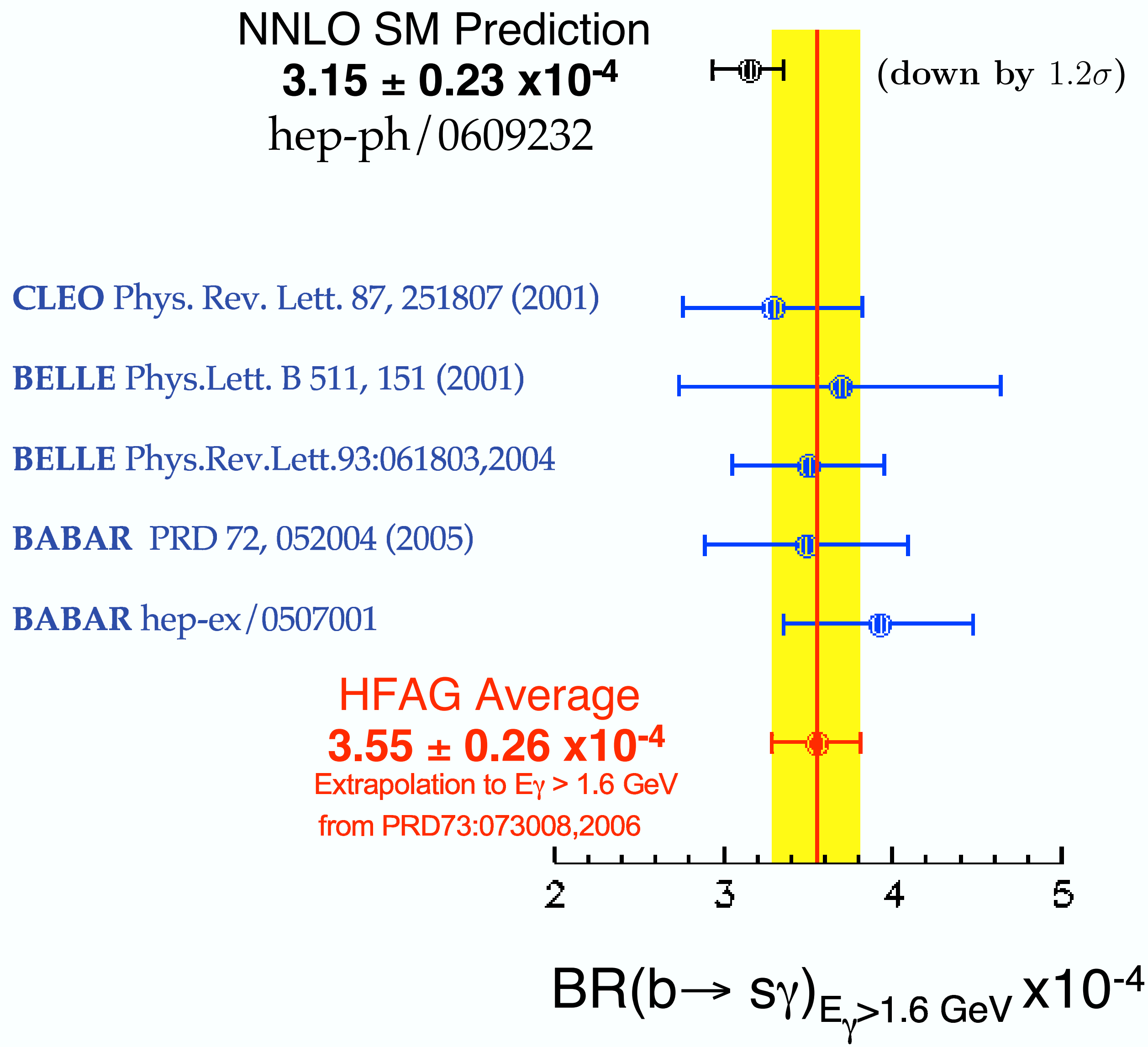
5% non-perturbative $\mathcal{O}\left(\alpha_s \frac{\Lambda}{m_b}\right)$ $\Rightarrow$ **Dedicated analysis necessary (see Lee, Neubert, Paz, hep-ph/0609224)**

3% parametric $(\alpha_s(M_Z),\ \mathcal{B}^{\text{exp}}_{\text{semileptonic}},\ m_c,\ \ldots)$

2.0% ($\alpha_s(M_Z)$), **1.6%** ($\mathcal{B}^{\text{exp}}_{\text{semileptonic}}$), **1.1%** ($m_c$)

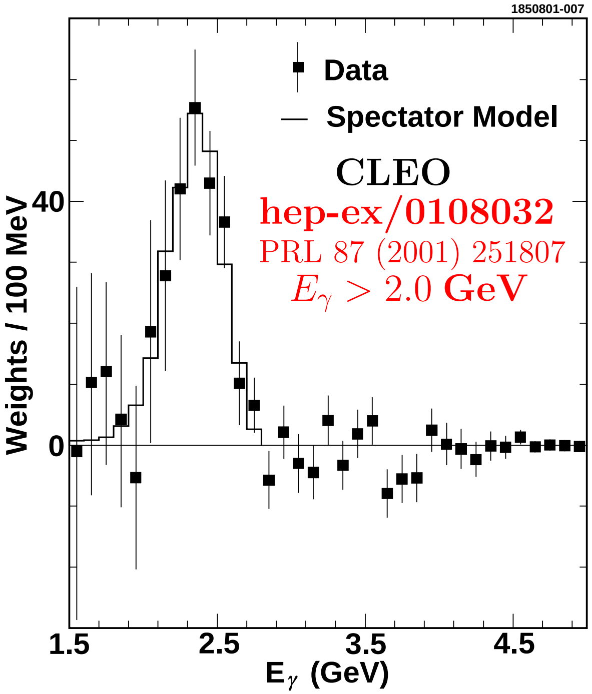
3% m_c-interpolation ambiguity $\Rightarrow$ **Complete three-loop on-shell matrix element calculation even for $m_c = 0$ would help a lot.**

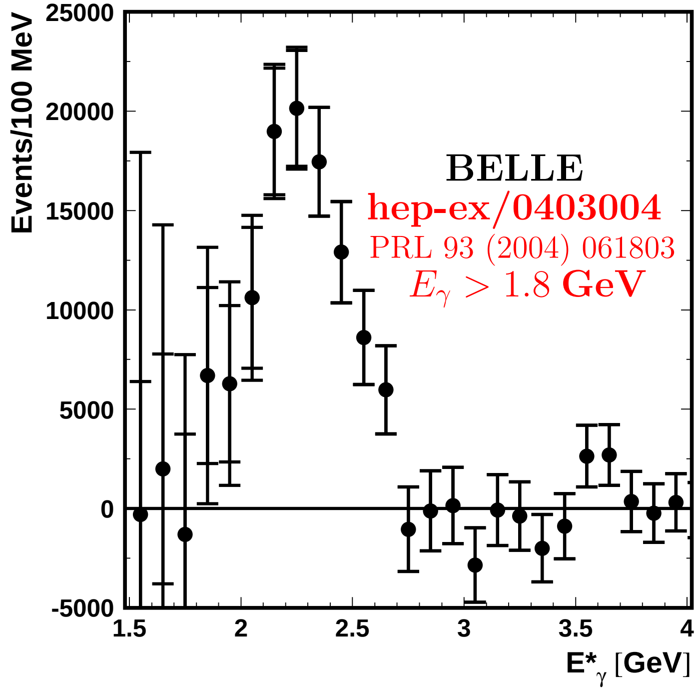
3% higher order $\mathcal{O}(\alpha_s^3)$ $\Rightarrow$ **This uncertainty will stay with us.**

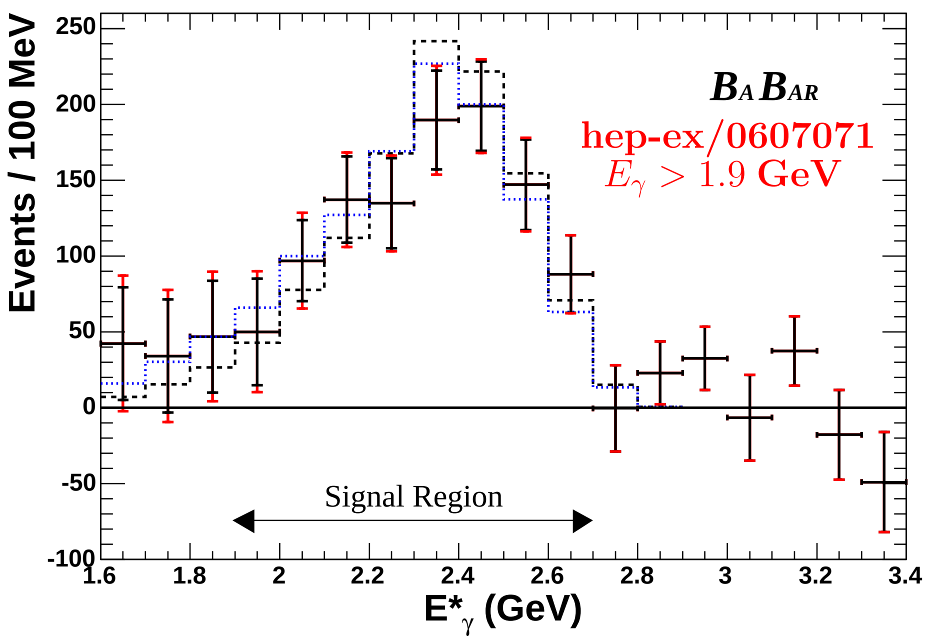


NNLO SM Prediction
3.15 ± 0.23 x10-4
hep-ph/0609232
(down by 1.2σ)
CLEO Phys. Rev. Lett. 87, 251807 (2001)
BELLE Phys.Lett. B 511, 151 (2001)
BELLE Phys.Rev.Lett.93:061803,2004
BABAR PRD 72, 052004 (2005)
BABAR hep-ex/0507001
HFAG Average
3.55 ± 0.26 x10-4
Extrapolation to Eγ > 1.6 GeV
from PRD73:073008,2006
2
3
4
5
BR(b→ sγ)Eγ>1.6 GeV x10-4

HFAG has combined experimental results with different photon energy cuts:





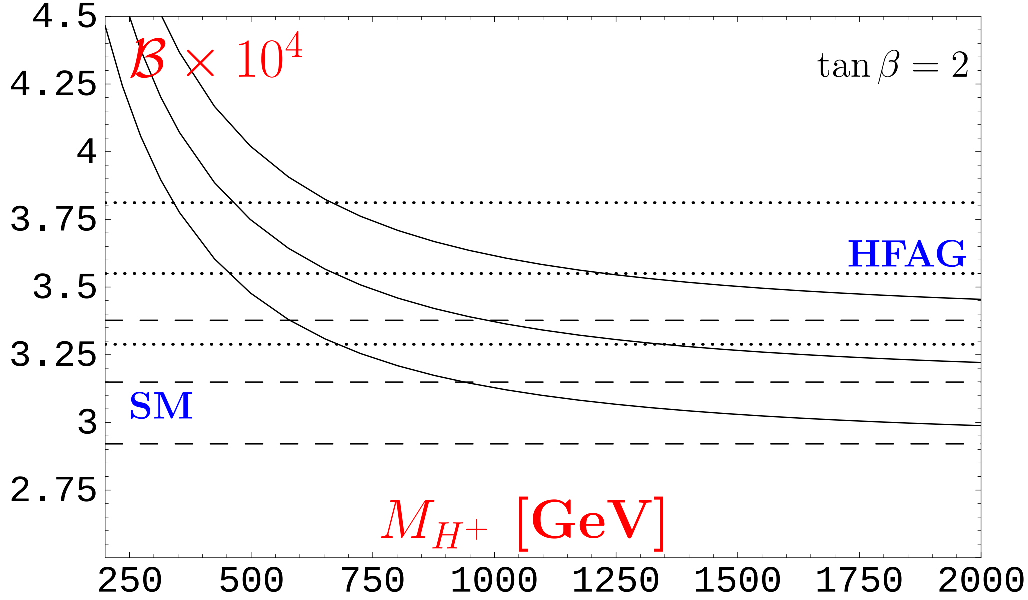


Since additional non-perturbative effects arise for high photon energy cuts E_0, combination of the three results and extrapolation to $E_0 = 1.6$ GeV are performed in the same step, to minimize model-dependence.

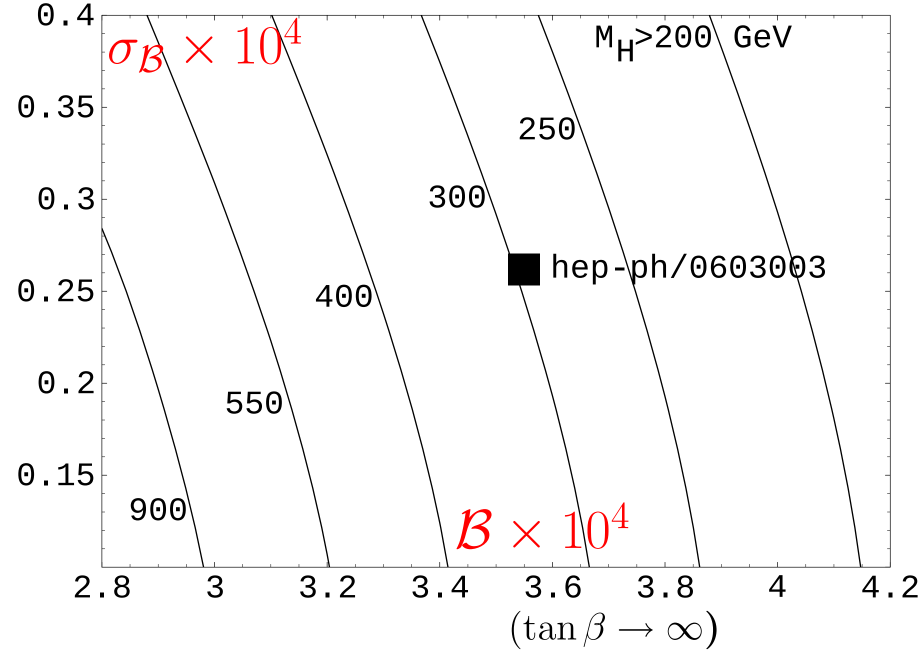
The cutoff-related effects die out for $E_0 \lesssim 1.6$ GeV [A. Kagan, M. Neubert, Eur. Phys. J. C7 (1999) 5].

Very recently, a $\sim -3\%$ cutoff-related effect was announced for $E_0 = 1.6$ GeV [T. Becher, M. Neubert, hep-ph/0610067]. This effect is not included here.

Constraints on the charged Higgs boson mass in the Two-Higgs-Doublet Model II:



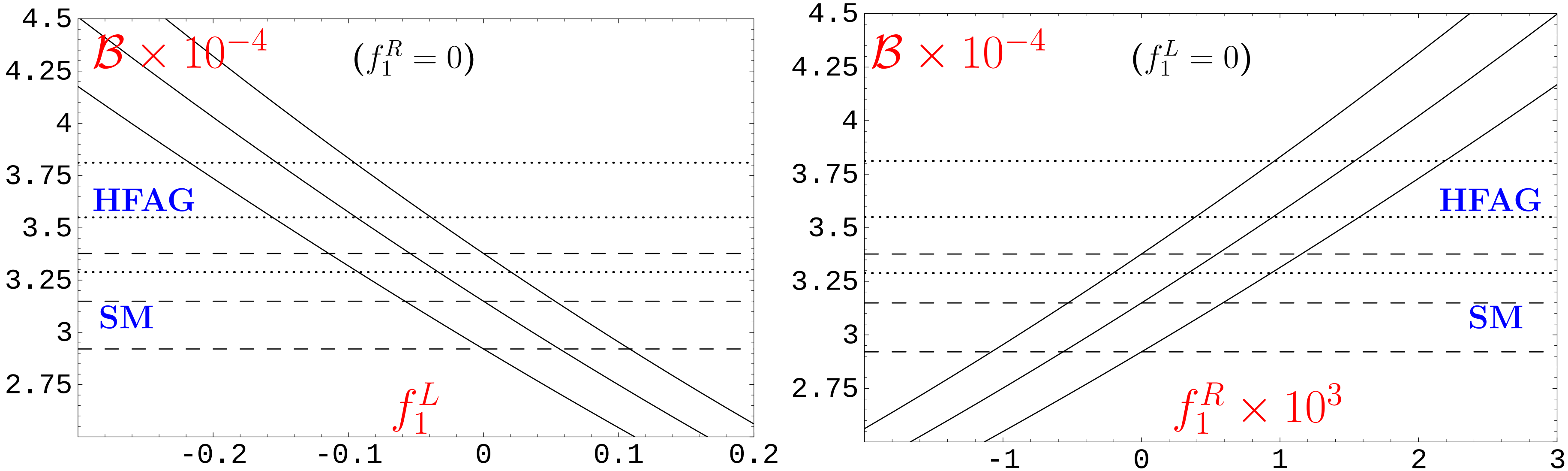
The data favour $M_{H^+} \sim 650$ GeV.



$M_{H^+} > 295$ **GeV @95% C.L.**

Constraints on dimension-four anomalous $W\bar{t}b$ couplings:

$$\mathcal{L}_4 = -\frac{g}{\sqrt{2}} \left[(V_{tb} + f_1^L)(\bar{t}_L \gamma^\mu b_L) \; + \; f_1^R (\bar{t}_R \gamma^\mu b_R) \right] W_\mu^+ \quad + \quad \text{h.c.}$$



$\Rightarrow$ **Bounds on f_1^L and f_1^R are of order 10^{-1} and 10^{-3}, respectively.**

Summary and Outlook

- **The accuracy of the experimental world average for $\mathcal{B}(\bar{B} \to X_s\gamma)$ is already a little better than the accuracy of the SM prediction at the NLO in QCD. Including the NNLO QCD corrections reverses the situation.**

- **The 3-loop matrix elements are found using an interpolation in m_c. The interpolation relies on the assumption that the large-β_0 approximation is accurate in the $m_c \to 0$ limit. A verification of this assumption by an explicit calculation would be more than welcome.**

- **It is essential to perform a dedicated study of non-perturbative corrections to the matrix elements of Q_1 and Q_2 that arise at $\mathcal{O}(\alpha_s)$, and scale like Λ/m_b in the $m_c \to 0$ limit. In the opposite limit ($m_c > m_b/2$), they scale like Λ^2/m_b^2 and can be neglected.**

- **Constraints on the charged Higgs boson mass and on the anomalous top quark couplings remain very stringent.**