Evaluation of the Hadronic Vacuum Polarisation Contribution to $(g-2)_\mu$

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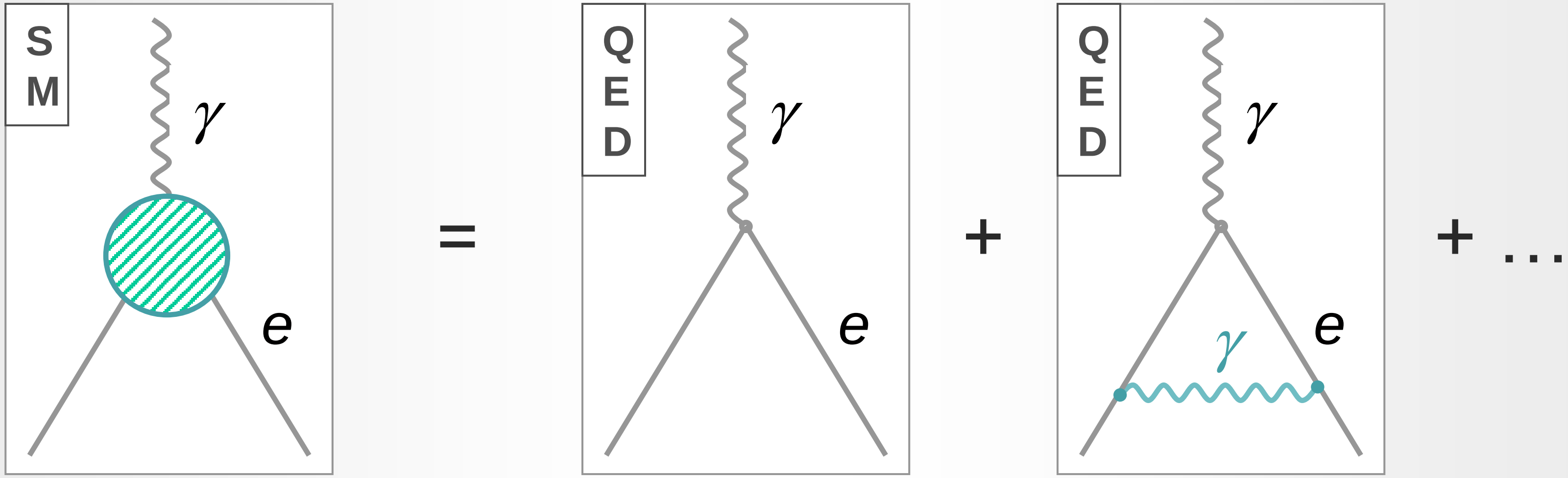
Flavour in the era of the LHC workshop, October 9–11, 2006, CERN, Switzerland

The *Anomalous* Magnetic Moment ...of the electron

- Dirac's gyromagnetic factor corresponds to the lowest order QED graph
- However, there are corrections to it:

Schwinger 1948
(Nobel price 1965)



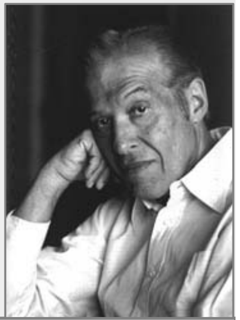
$(g–2)_e \neq 0$ (full Standard Model)

$(g–2)_e = 0$ (Dirac)

coupling to virtual fields:
$(g–2)_e \neq 0$ (1st order QED)

- Quantum fluctuations shift the gyromagnetic ratio

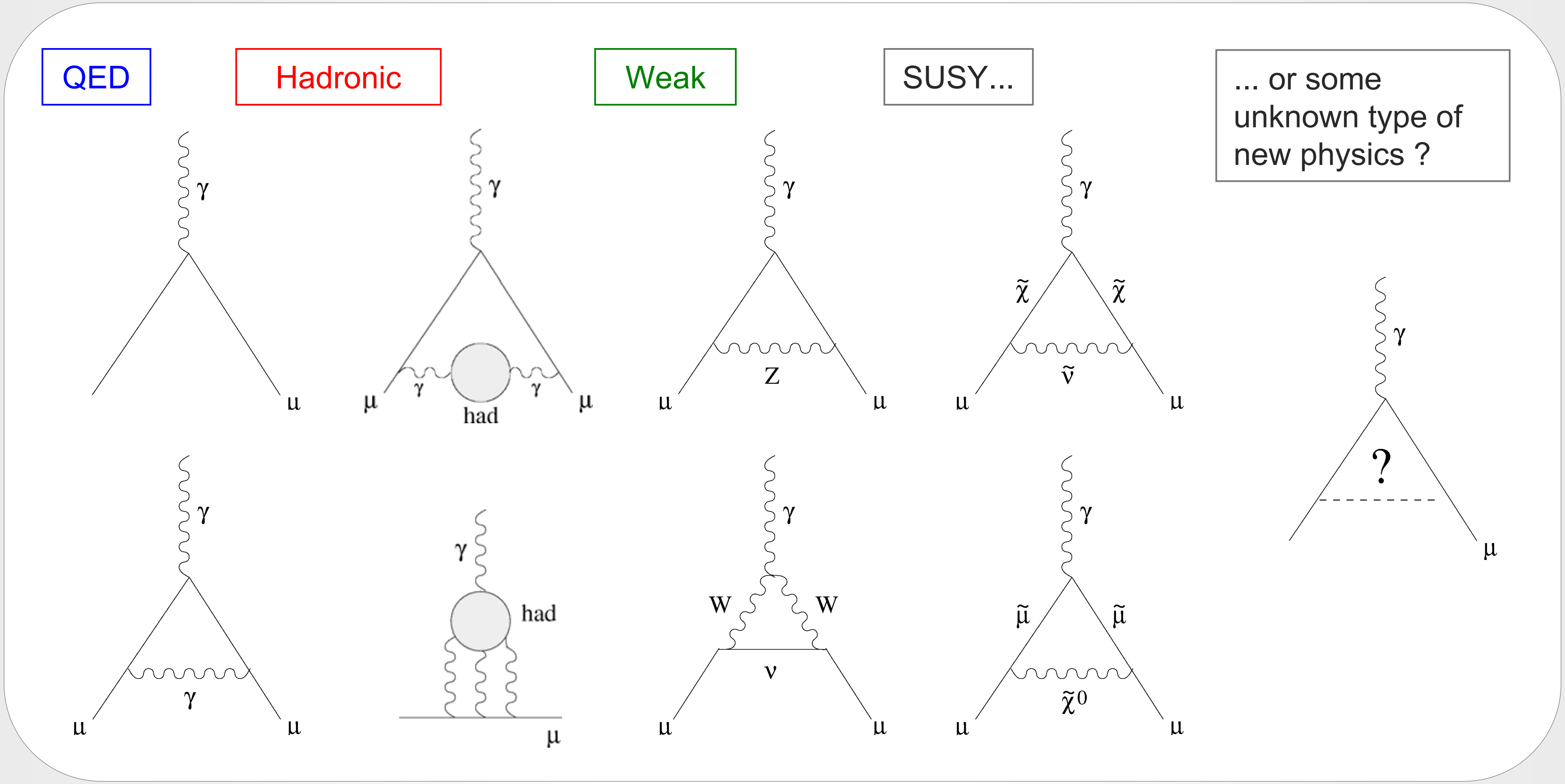
$$a_\ell \equiv \frac{g_\ell - 2}{2} = \frac{\alpha}{2\pi} + \ldots = \frac{1}{2\pi}\underbrace{\frac{e^2}{4\pi}}_{1/137.036\ldots} + \ldots = 0.001161\ldots$$



Schwinger,
finding QED easy

The *Muon* Anomalous Magnetic Moment

- Diagrams contributing to the magnetic moment



QED

Hadronic

Weak

SUSY...

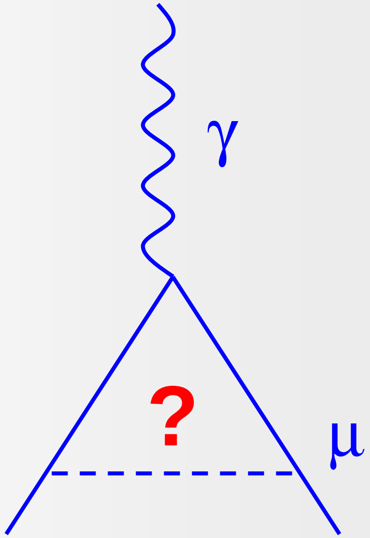
... or some unknown type of new physics ?

The Quest for "New Physics"

- The experimental precision for a_μ will be worse than for a_e, so why do it ?

- From chiral symmetry expect the New Physics (NP) effects to scale ~ $m^2(e/\mu)$:

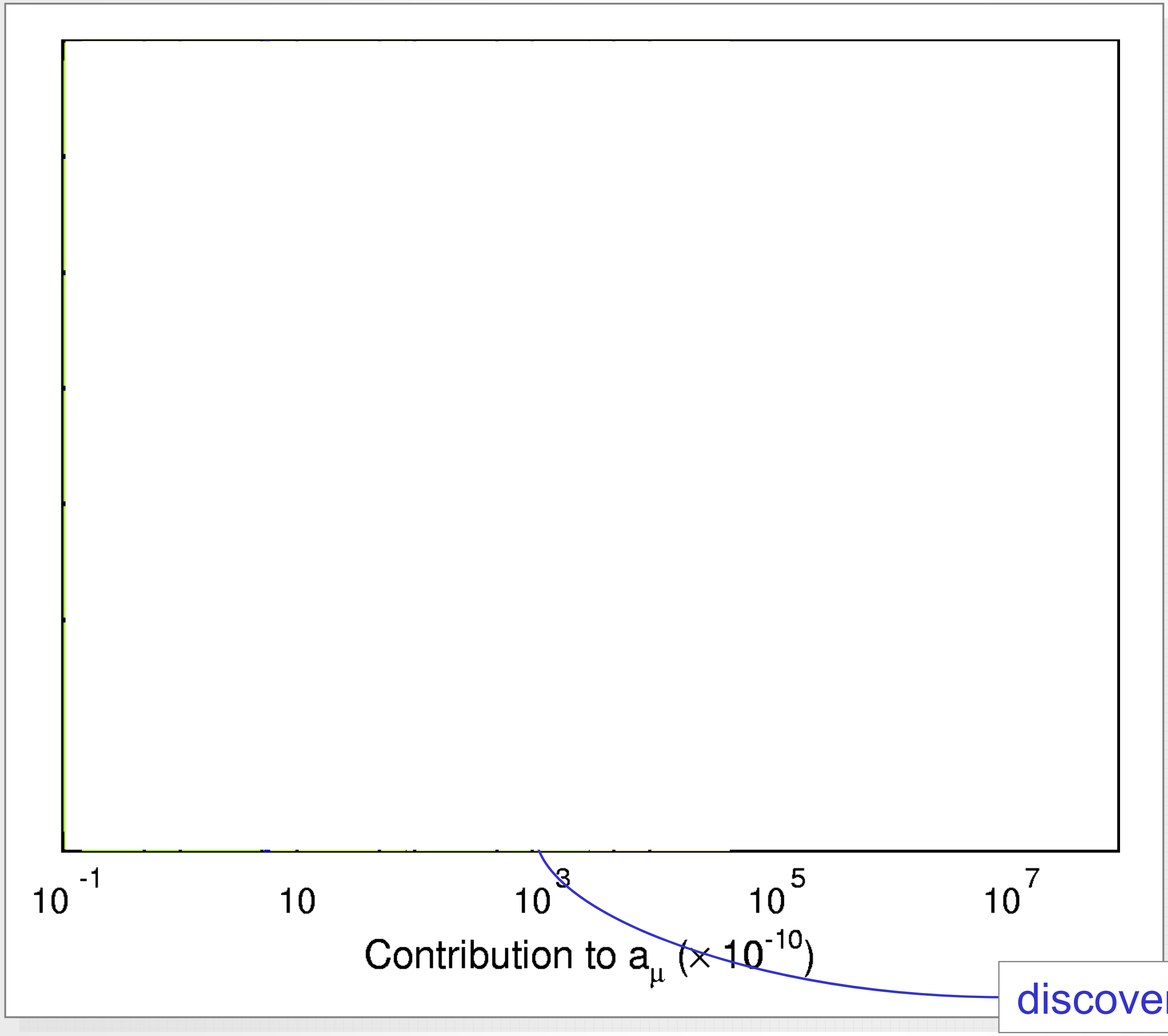
$$a_\ell^{\mathrm{NP}}(\Lambda_{\mathrm{NP}}) \propto \mathcal{O}\left(\frac{m_\ell^2}{\Lambda_{\mathrm{NP}}^2}\right) \Longrightarrow \frac{a_\mu^{\mathrm{NP}}}{a_e^{\mathrm{NP}}} \propto \mathcal{O}\left(\frac{m_\mu^2}{m_e^2}\right) \approx 42{,}000$$



- Expect to loose about a factor of 200 in experimental precision

→ a_μ should be roughly 200 times more sensitive to NP than a_e !

Experimental Progress from CERN to BNL

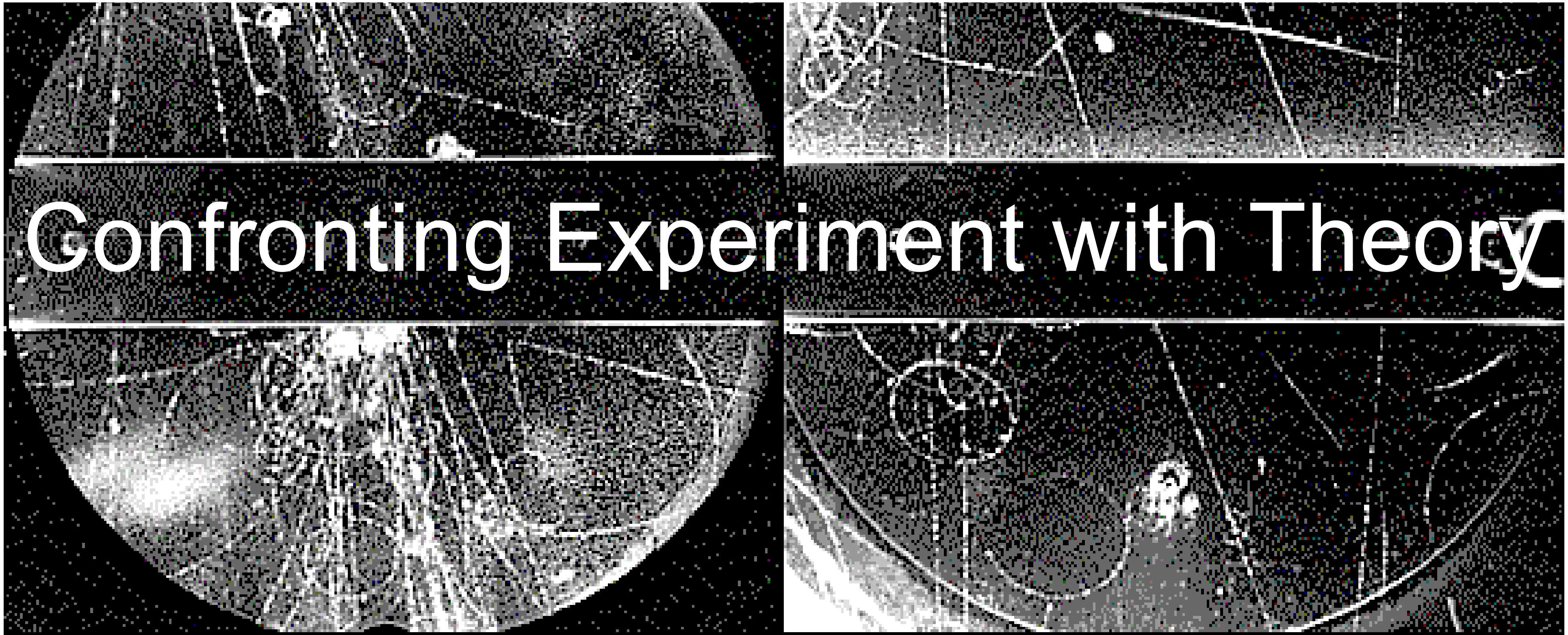


Experimental progress on precision of $(g-2)_\mu$

Outperforms theory precision on hadronic contribution

discovery of magic γ

J. Bailey et al., NP B150 1 (1979)



Confronting Experiment with Theory

The Standard Model prediction of a_μ is decomposed in its main contributions:

$$a_\mu^{\mathrm{SM}} \equiv \left(\frac{g-2}{2}\right)_\mu = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{had}} + a_\mu^{\mathrm{weak}}$$

of which the hadronic contribution has the largest uncertainty

The Muon Magnetic Anomaly in the Standard Model

QED contribution

Computed up to 4th order
(5th order estimated)

$$a_\mu^{\text{QED}} \approx \begin{pmatrix} 11{,}614{,}098.1 + 41{,}321.8 \\ + \; 3{,}014.2 + 38.1 + \underbrace{0.6}_{\text{estimated}} \end{pmatrix} \times 10^{-10} = 11{,}658{,}472.1(0.1) \times 10^{-10}$$

Kinoshita-Nio, PRD 70, 113001 (2005)

Electroweak contribution

Computed up to 2nd order

a_μ^{weak} suppressed by $\dfrac{\alpha}{\pi}\dfrac{m_\mu^2}{m_W^2} \sim 10^{-9}$ (!)

$$\underbrace{a_\mu^{\text{weak}}}_{\text{1-loop}} = \frac{G_\mu m_\mu^2}{8\sqrt{2}\pi^2}\left(\frac{5}{3} + \frac{1}{3}\left(1 - 4\sin^2\theta_W\right) + \mathcal{O}\left(\frac{m_\mu^2}{m_W^2}\right) + \mathcal{O}\left(\frac{m_\mu^2}{m_H^2}\right)\right) = +19.5 \times 10^{-10}$$

Czarnecki *et al.*,
PRD 52, 2619 (1995);
PRL 76, 3267 (1996)

2nd order contribution surprisingly large: $\underbrace{a_\mu^{\text{weak}}}_{\text{2-loop}} = -4.1(0.2) \times 10^{-10}$

(due to large logs: $\ln[m_Z/m_\mu]$)

Note that between a_μ and a_e, the same sensitivity factor as for “new physics” applies here

The Hadronic Contribution to $(g-2)_\mu$

$$a_\mu^{\mathrm{SM}} = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{had}} + a_\mu^{\mathrm{weak}}$$

The Situation 1995

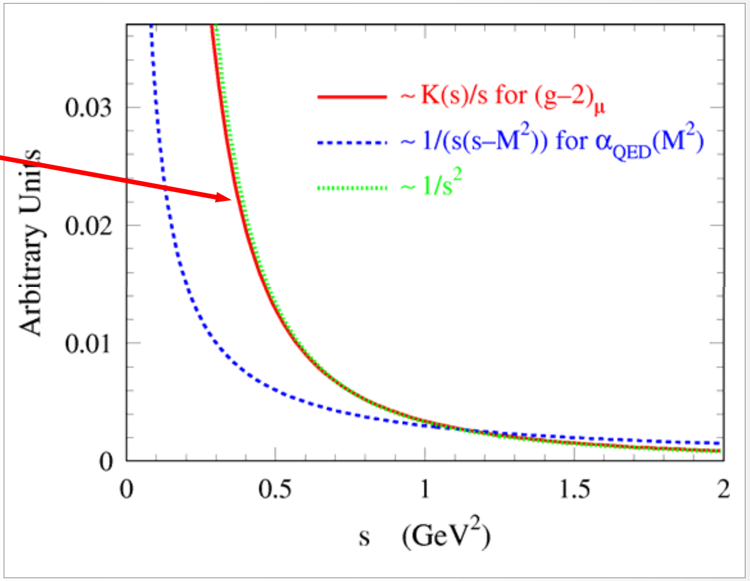
Source	$\sigma(a_\mu)$	Reference
QED	$\sim 0.1 \times 10^{-10}$	[Schwinger ’48 & others]
Hadrons	$\sim (15 \oplus 4) \times 10^{-10}$	[Eidelman-Jegerlehner ’95 & others]
***Z, W* exchange**	$\sim 0.2 \times 10^{-10}$	[Czarnecki *et al.* ‘95 & others]

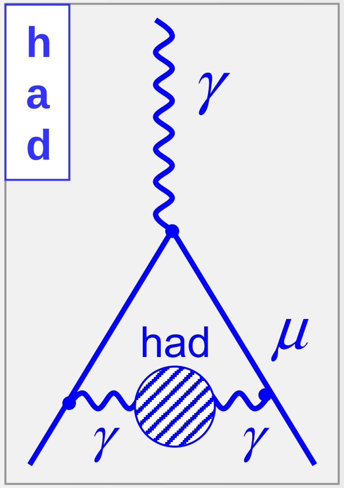
Dominant uncertainty from lowest order hadronic piece. Cannot be calculated from QCD (“first principles”) – but: we can use experiment (!)

$$a_\mu^{\mathrm{had,LO}} = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^{\infty} ds \, \frac{K(s)}{s} R(s)$$

Dispersion relation, uses unitarity (optical theorem) and analyticity

... *(see digression)*





digression: Vacuum polarization … and the running of α_{QED}

Define: photon vacuum polarization function $\Pi_\gamma(q^2)$

$$i\int d^4x\, e^{iqx}\langle 0|TJ^{\mu}_{em}(x)\left(J^{\nu}_{em}(0)\right)^{\dagger}|0\rangle = -\left(g^{\mu\nu}q^2 - q^{\mu}q^{\nu}\right)\Pi_\gamma(q^2)$$

Ward identities: only vacuum polarization modifies electron charge

$$\alpha(s) = \frac{\alpha(0)}{1-\Delta\alpha(s)}$$

with: $\Delta\alpha(s) = -4\pi\alpha\,\mathrm{Re}\left[\Pi_\gamma(s)-\Pi_\gamma(0)\right]$

Leptonic $\Delta\alpha_{lep}(s)$ calculable in QED. However, quark loops are modified by long-distance hadronic physics, cannot (yet) be calculated within QCD (!)

Born: $\sigma^{(0)}(s) = \sigma(s)\left(\alpha/\alpha(s)\right)^2$

Way out: Optical Theorem (*unitarity*) ...

$$12\pi\,\mathrm{Im}\Pi_\gamma(s) = \frac{\sigma^{(0)}[e^+e^- \to \text{hadrons}]}{\sigma^{(0)}[e^+e^- \to \mu^+\mu^-]} \equiv R(s)$$

... and the subtracted dispersion relation of $\Pi_\gamma(q^2)$ (*analyticity*)

$$\Pi_\gamma(s)-\Pi_\gamma(0) = \frac{s}{\pi}\int_0^\infty ds'\frac{\mathrm{Im}\Pi_\gamma(s')}{s'(s'-s)-i\varepsilon}$$

➡

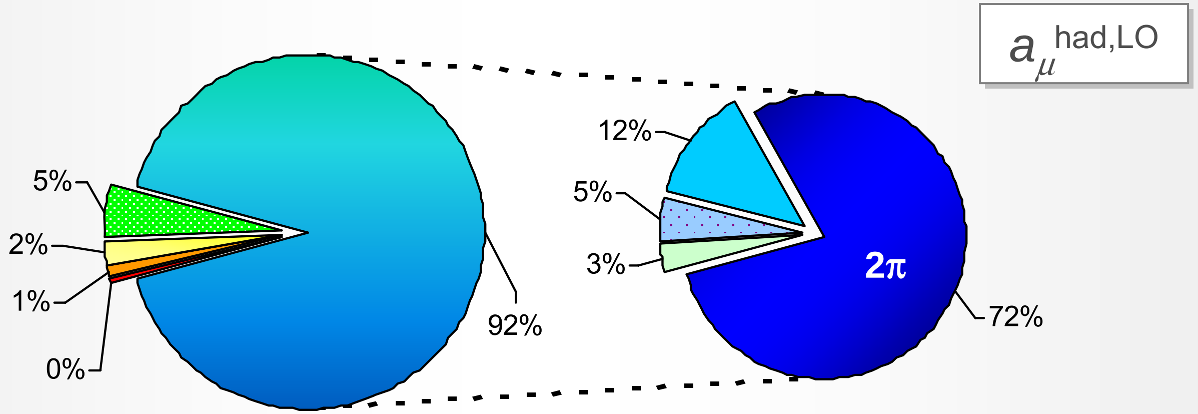
$$\Delta\alpha_{had}(s) = -\frac{\alpha s}{3\pi}\mathrm{Re}\int_0^\infty ds'\frac{R(s')}{s'(s'-s)-i\varepsilon}$$

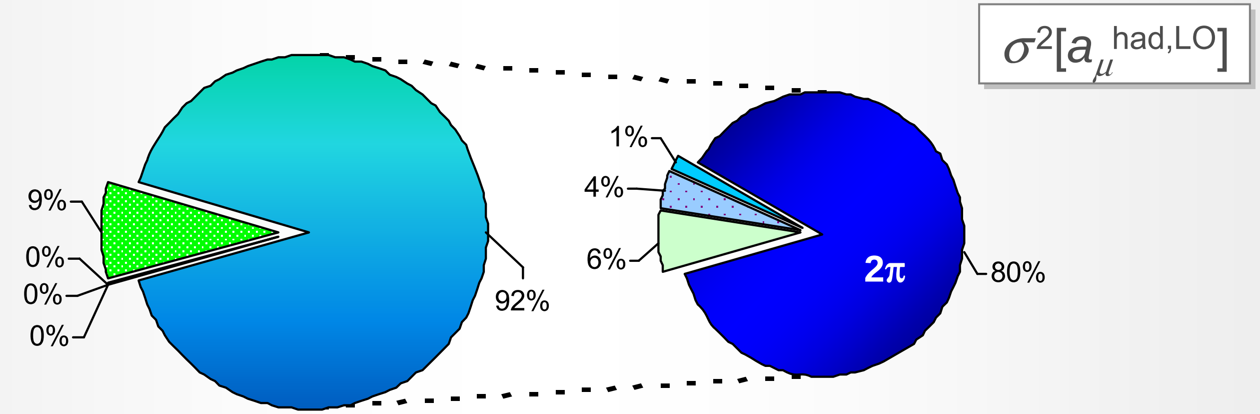
... and equivalently for a_μ[had]

Contributions to the Dispersion Integrals

2π	3π $(+\omega,\phi)$	4π	$> 4\pi$ (+KK)	1.8 - 3.7	3.7 - 5 $(+J/\psi, \psi)$	5 - 12 $(+\Upsilon)$	12 - ∞

< 1.8 GeV





Hadronic Contribution to $(g-2)_\mu$: Roadmap

Eidelman-Jegerlehner, Z.Phys. C67, 585 (1995)

Since 1995, improved determination of the dispersion integral:

- better data
- extended use of QCD

Energy [GeV]	Input 1995	Input after 1998
$2m_\pi$ - 1.8	Data	Data (e^+e^- & τ) (+ QCD)
1.8 – ψ(3770)	Data	QCD
J/ψ - ϒ	Data	Data (+ QCD)
ϒ - 40	Data	QCD
40 - ∞	QCD	QCD

Improvement in 4 Steps:

- Inclusion of precise τ data using SU(2) (CVC)

  Alemany-Davier-H.'97, Narison'01, Trocóniz-Ynduráin'01, + later works (!)

- Extended use of (dominantly) perturbative QCD

  Martin-Zeppenfeld'95, Davier-H.'97, Kühn-Steinhauser'98, Erler'98, + others

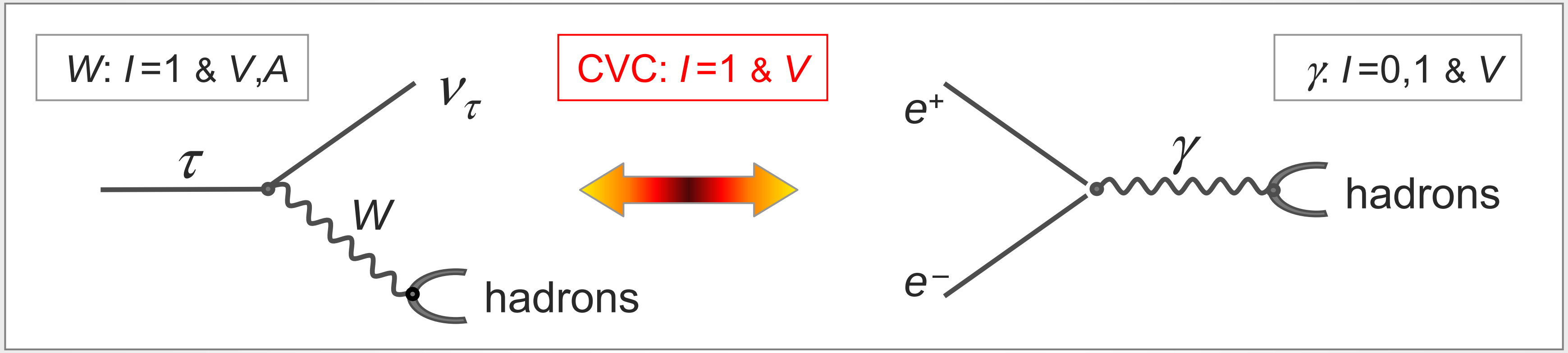
- Theoretical constraints from QCD sum rules

  Groote-Körner-Schilcher-Nasrallah'98, Davier-H.'98, Martin-Outhwaite-Ryskin'00, Cvetič-Lee-Schmidt'01, Jegerlehner et al'00, Dorokhov'04 + others

- Better data for the $e^+e^- \to \pi^+\pi^-$ cross section

  CMD-2'02, KLOE'04, SND'05 (!)

Using also Tau Data through CVC – SU(2)



Hadronic physics factorizes in Spectral Functions :

Isospin symmetry connects $I=1$ e^+e^- cross section to vector τ spectral functions:

$$\sigma^{(I=1)}\left[e^+e^- \to \pi^+\pi^-\right] = \frac{4\pi\alpha^2}{s}\,\upsilon\left[\tau^- \to \pi^-\pi^0\nu_\tau\right]$$

Experimentally: τ and e^+e^- data are complementary with resp. to normalisation and shape uncertainties

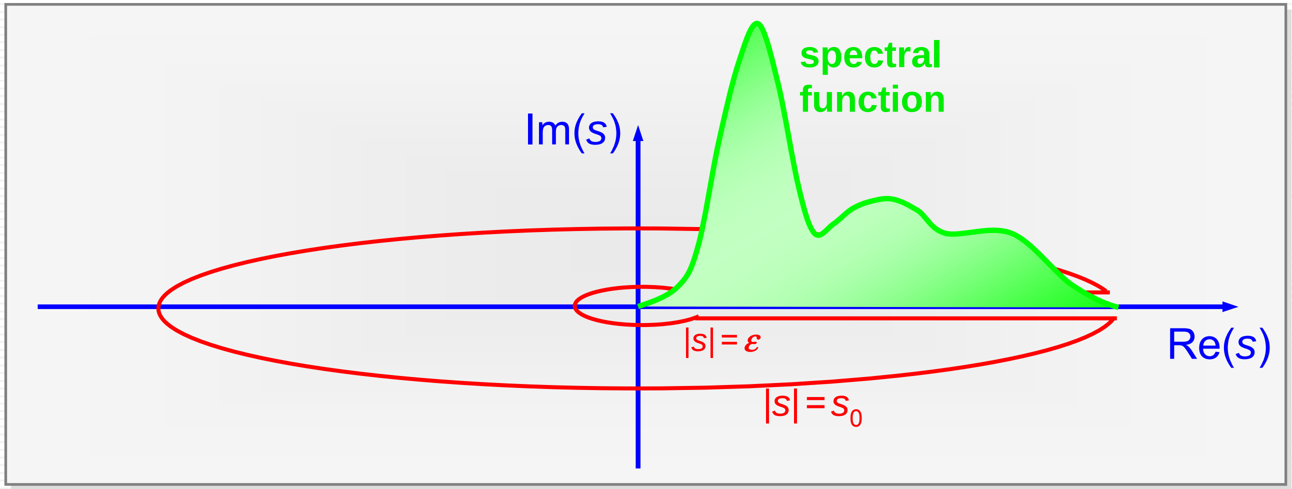
$$\upsilon\left[\tau^- \to \pi^-\pi^0\nu_\tau\right] \propto \underbrace{\frac{\mathrm{BR}\left[\tau^- \to \pi^-\pi^0\nu_\tau\right]}{\mathrm{BR}\left[\tau^- \to e^-\bar{\nu}_e\nu_\tau\right]}}_{\text{branching fractions}} \; \underbrace{\frac{1}{N_{\pi\pi^0}}\frac{dN_{\pi\pi^0}}{ds}}_{\text{mass spectrum}} \; \underbrace{\frac{m_\tau^2}{\left(1-s/m_\tau^2\right)^2\left(1+s/m_\tau^2\right)}}_{\text{kinematic factor (PS)}}$$

digression: Tau Spectral Functions and QCD

(1) Optical theorem $\upsilon(s) \propto \mathrm{Im}\,\Pi(s)$

(2) Apply Cauchy’s theorem for “save” (*i.e.*, sufficiently large) s_0:

$$R(s_0) \propto \int_0^{s_0} ds\, \underbrace{f(s)}_{\text{kinematic factor}}\, \mathrm{Im}\,\Pi(s) \quad \Leftrightarrow \quad -\frac{1}{2i}\oint_{|s|=s_0} ds\, f(s)\,\Pi(s)$$

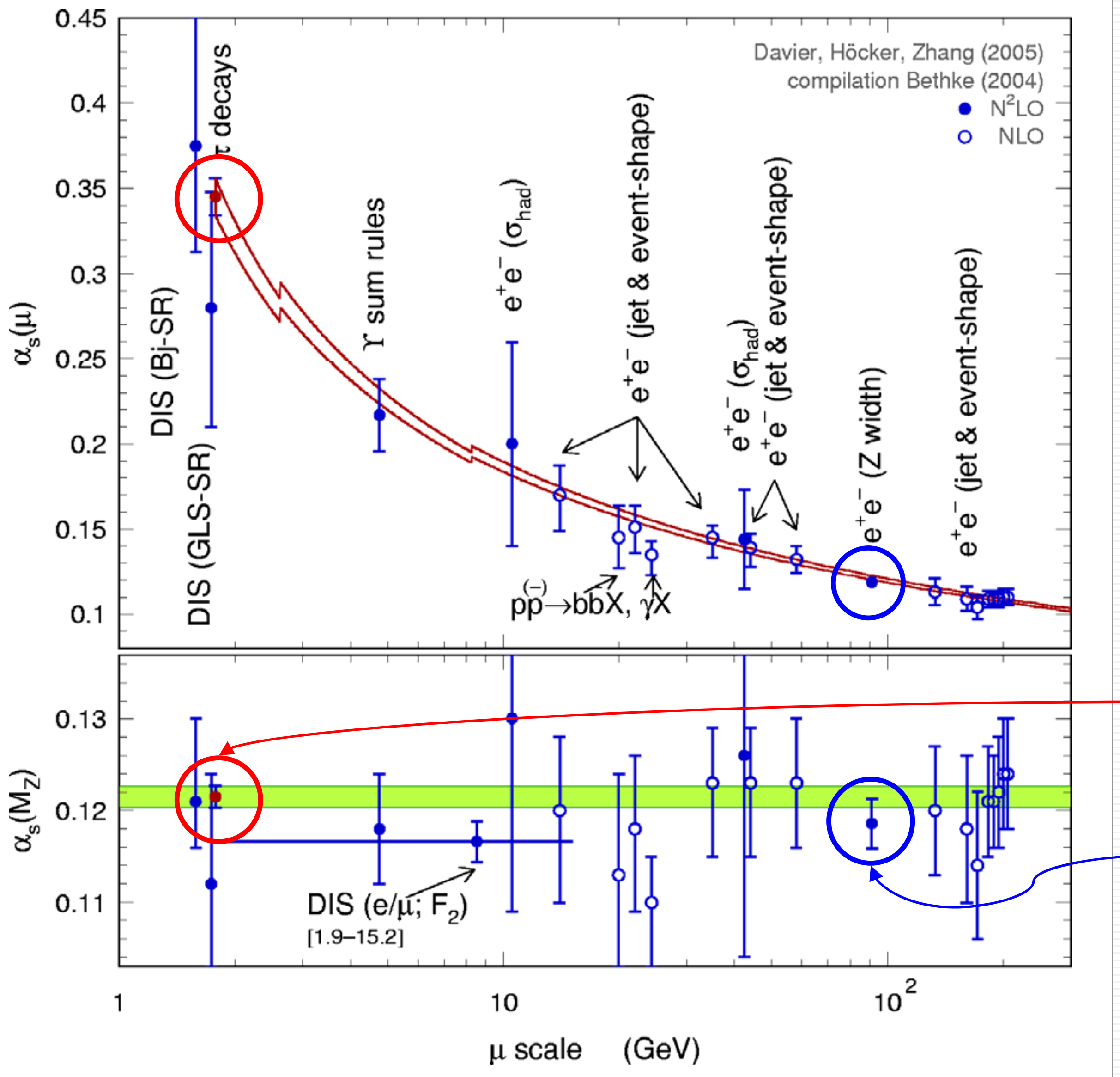


(3) Use the Adler function to remove unphysical subtractions: $D(s) = s\,\frac{d\,\Pi(s)}{ds}$

(4) Use global quark-hadron duality in the framework of the Operator Product Expansion (OPE) to predict: $D(s) \sim D_{\text{pert}}(s) + D_{\text{q-mass}}(s) + D_{\text{non-pert}}(s)$

(5) Use analytical moments $f_n(s) = f(s)\cdot\text{poly}_n(s)$ to fix non-perturbative parameters of the OPE ... and then fit $\alpha_s(m_\tau)$

digression: QCD Results from Hadronic Tau Decays



Evolution of $\alpha_s(m_\tau)$, measured using τ decays, to M_Z using RGE (4-loop QCD β-function & 3-loop quark flavor matching) shows the excellent compatibility of τ result with EW fit !

Davier-Höcker-Zhang
hep-ph/0507078

$\alpha_s(M_Z)$ = 0.1215 ± 0.0012

(DHZ’05, theory dominated)

$\alpha_s(M_Z)$ = 0.1186 ± 0.0027

(LEP’00, exp. dominated)

LEP EW WG,
LEPEWWG 2002-01

SU(2) Breaking

Electromagnetism does not respect isospin and hence we have to consider isospin breaking when dealing with an experimental precision of 0.5%

Corrections for SU(2) breaking applied to τ data for dominant $\pi^-\pi^+$ contribution:

- Electroweak radiative corrections:
  - dominant contribution from short distance correction S_{EW} to effective 4-fermion coupling $\propto (1 + 3\alpha(m_\tau)/4\pi)(1+2\langle Q\rangle)\log(M_Z/m_\tau)$
  - subleading corrections calculated and small
  - long distance radiative correction $G_{EM}(s)$ calculated [add FSR to the bare cross section in order to obtain $\pi^-\pi^+(\gamma)$]

Marciano-Sirlin' 88

Braaten-Li' 90

Cirigliano-Ecker-Neufeld' 02

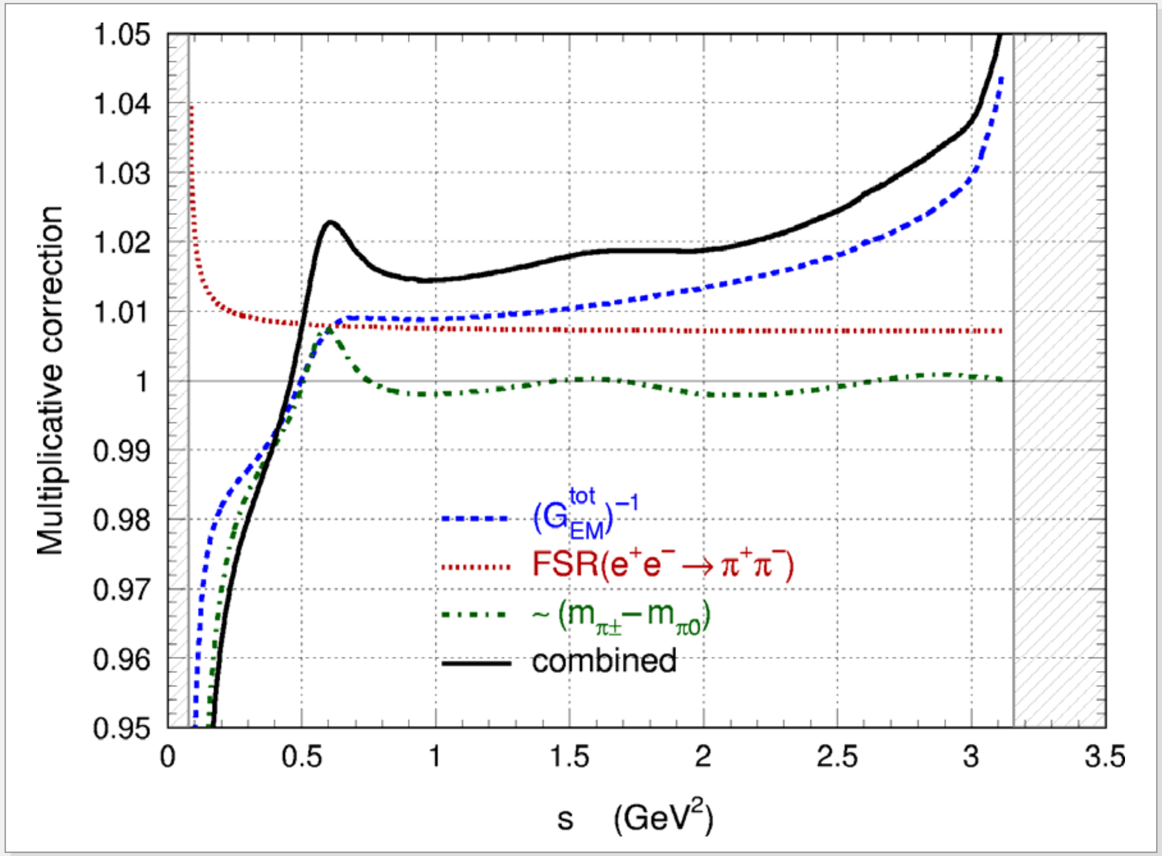
- Charged/neutral mass splitting:
  - $m_{\pi^-} \neq m_{\pi^0}$ leads to phase space (cross sec.) and width (FF) corrections
  - ρ-ω mixing (EM $\omega \to \pi^-\pi^+$ decay) corrected using FF model
  - $m_{\rho^-} \neq m_{\rho^0}$ and $\Gamma_{\rho^-} \neq \Gamma_{\rho^0}$ [not corrected !]

Alemany-Davier-Höcker' 97, Czyż-Kühn' 01

- Electromagnetic decays, like: $\rho \to \pi\pi\gamma$, $\rho \to \pi\gamma$, $\rho \to \eta\gamma$, $\rho \to l^+l^-$
- Quark mass difference $m_u \neq m_d$ generating "second class currents" (negligible)

Mass Dependence of SU(2) Breaking

Multiplicative SU(2) corrections applied to $\tau^- \to \pi^-\pi^0\nu_\tau$ spectral function:



Only β^3 and EW short-distance corrections applied to 4π spectral functions

e^+e^- Radiative Corrections

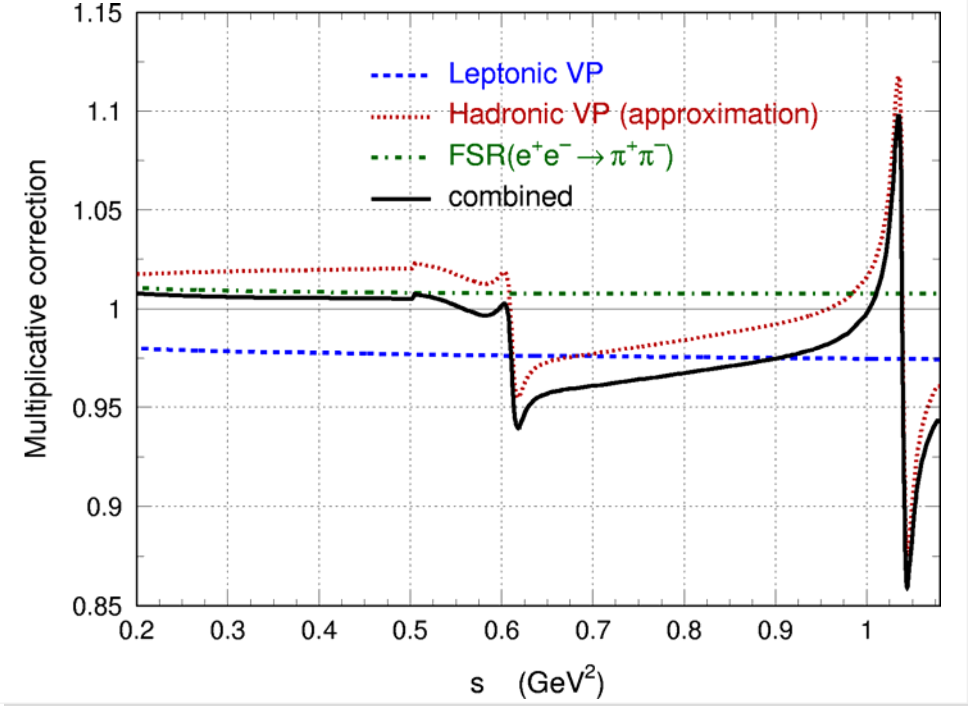
Multiple radiative corrections are applied on measured e^+e^- cross sections

Situation often unclear: whether or not and if - which corrections were applied

- Vacuum polarization (VP) in the photon propagator:
  - leptonic VP in general corrected for
  - hadronic VP correction not applied, but for CMD-2 (in principle: iterative procedure)

- Initial state radiation (ISR)
  - corrected by experiments

- Final state radiation (FSR) [need $e^+e^- \to$ hadrons (γ) in disp. integral]
  - usually, experiments obtain bare cross section so that FSR has to be added "by hand"; done for CMD-2, (supposedly) not done for others



Inputs to the Analyses

Motivation for published 2002-2003 re-analyses:

- New high precision e^+e^- results (0.6% sys. error) around ρ from CMD-2 (Novosibirsk)
- New τ results from ALEPH using full LEP1 statistics
- New R results from BES between 2 and 5 GeV
- New theoretical analysis of SU(2) breaking
- **discrepancy between e^+e^- and τ input !**

CMD-2 PL B527, 161 (2002)

ALEPH Phys. Rep. 2005

BES PRL 84 594 (2000); PRL 88, 101802 (2002)

Cirigliano-Ecker-Neufeld
JHEP 0208 (2002) 002

Davier-Eidelman-Höcker-Zhang
Eur.Phys.J. C27 (2003) 497; C31 (2003) 503

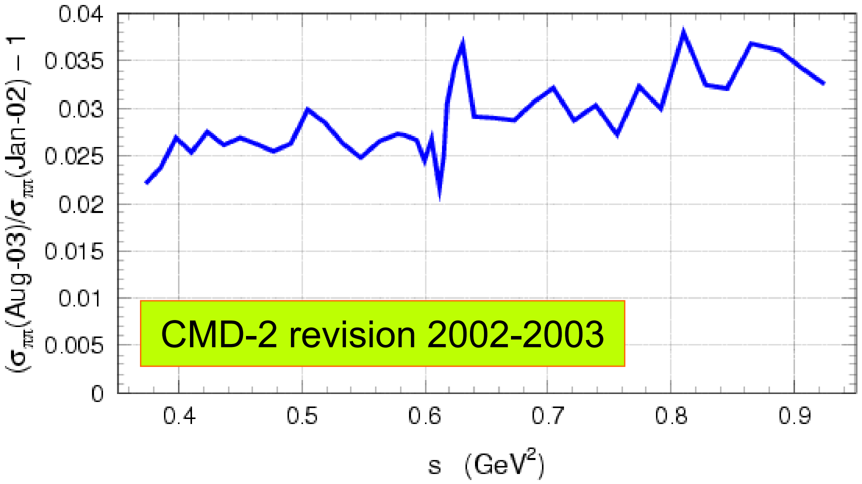
Hagiwara-Martin-Nomura-Teubner,
Phys.Rev. D69 (2004) 093003 (no τ data)

Jegerlehner,
hep-ph/0312372 (no τ data)

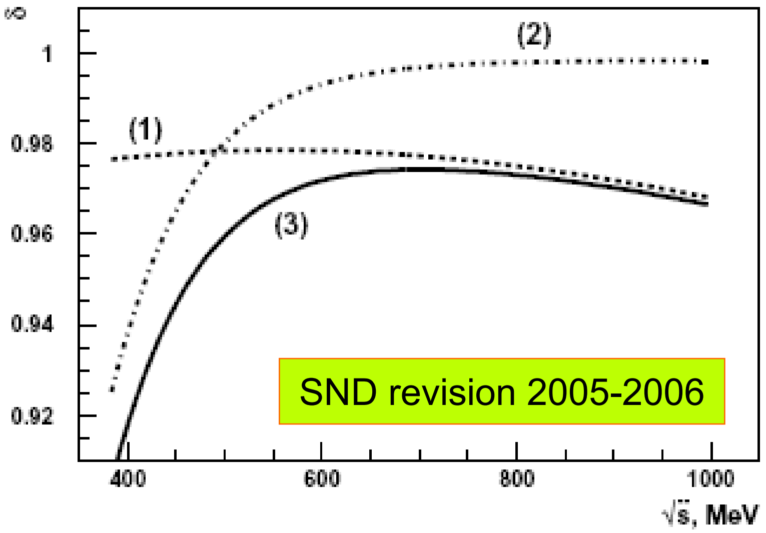
New input:

- KLOE 2004, SND 2005 (revised 2006), CMD-2 2006 (still preliminary)
- reduction of VP+FSR additional uncertainties
- BABAR exclusive channels $2\pi\pi^0$, 4π, 6π, $4\pi2\pi^0$
- QCD in 1.8–3.7 GeV

e^+e^- Radiative Corrections Revised



CMD-2 revision 2002-2003



SND revision 2005-2006

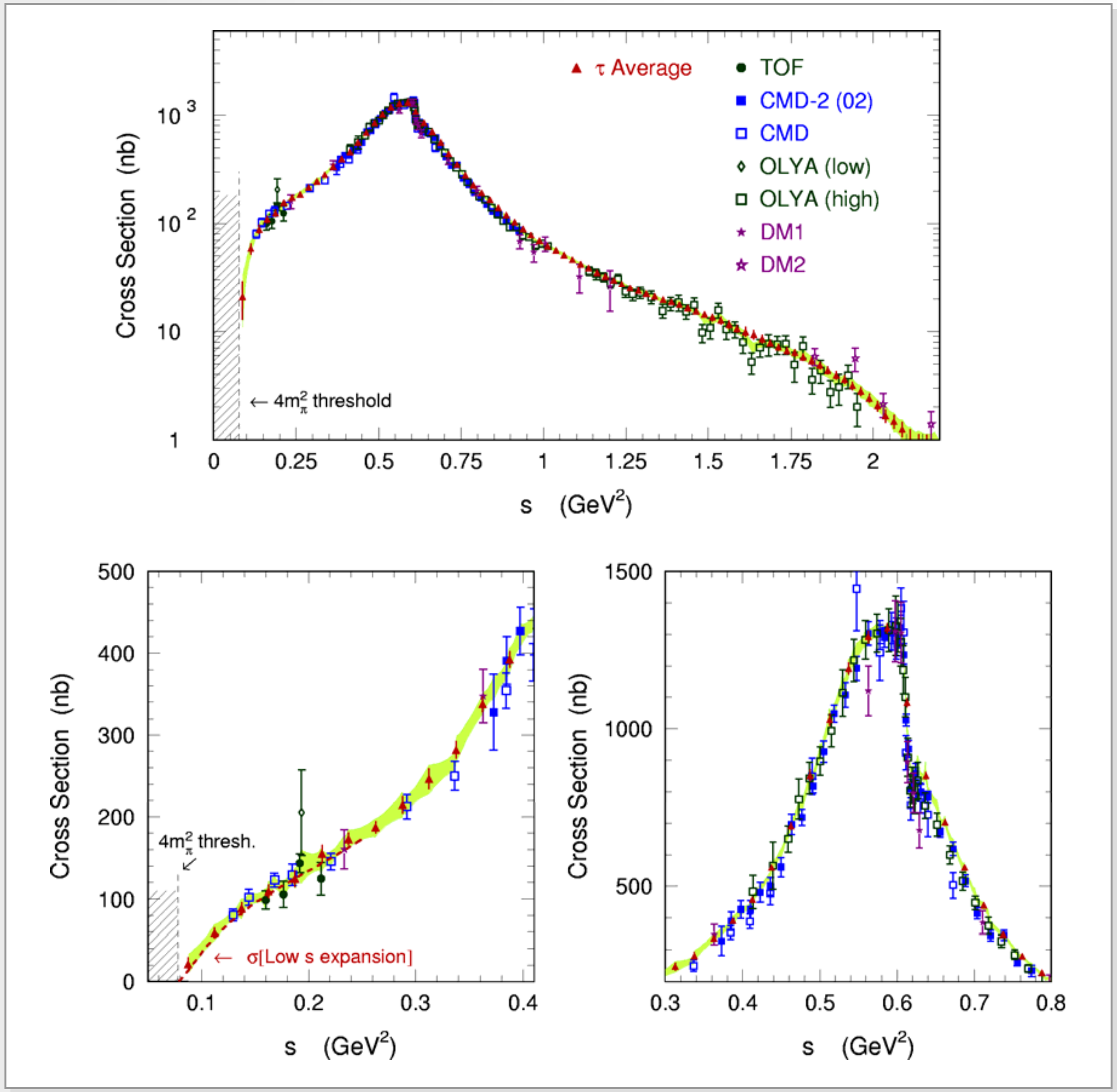
- 2.2-2.7% luminosity correction from change in σ_{Bhabha}
- 1.2-1.4% change in $\sigma_{\mu\mu}$
- Both changes affect *ee* / $\mu\mu$ / $\pi\pi$ separation

0.4% systematic error unchanged (0.6% total)

- Monte Carlo generators not correct for (1) $\pi\pi(\gamma)$ and (2) $\mu\mu(\gamma)$
- 3% change in $\sigma_{\pi\pi}$ (3)

0.2% systematic error unchanged (1.3% total)

Comparing $e^+e^- \to \pi^+\pi^-$ and $\tau \to \pi^-\pi^0\nu_\tau$



Correct τ data for missing ρ-ω mixing (taken from BW fit) and all other SU(2)-breaking sources

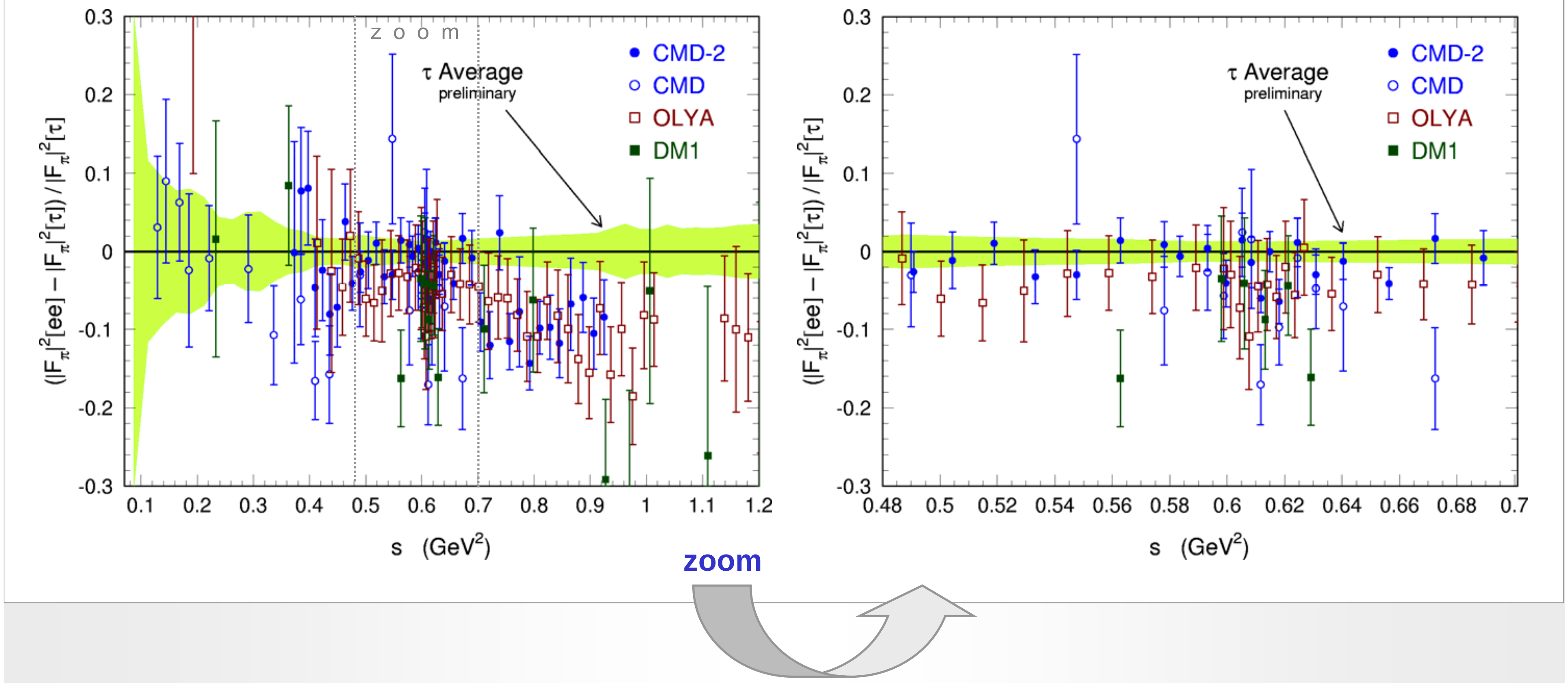
Remarkable agreement

But: not good enough...

...

The e^+e^--τ Problem

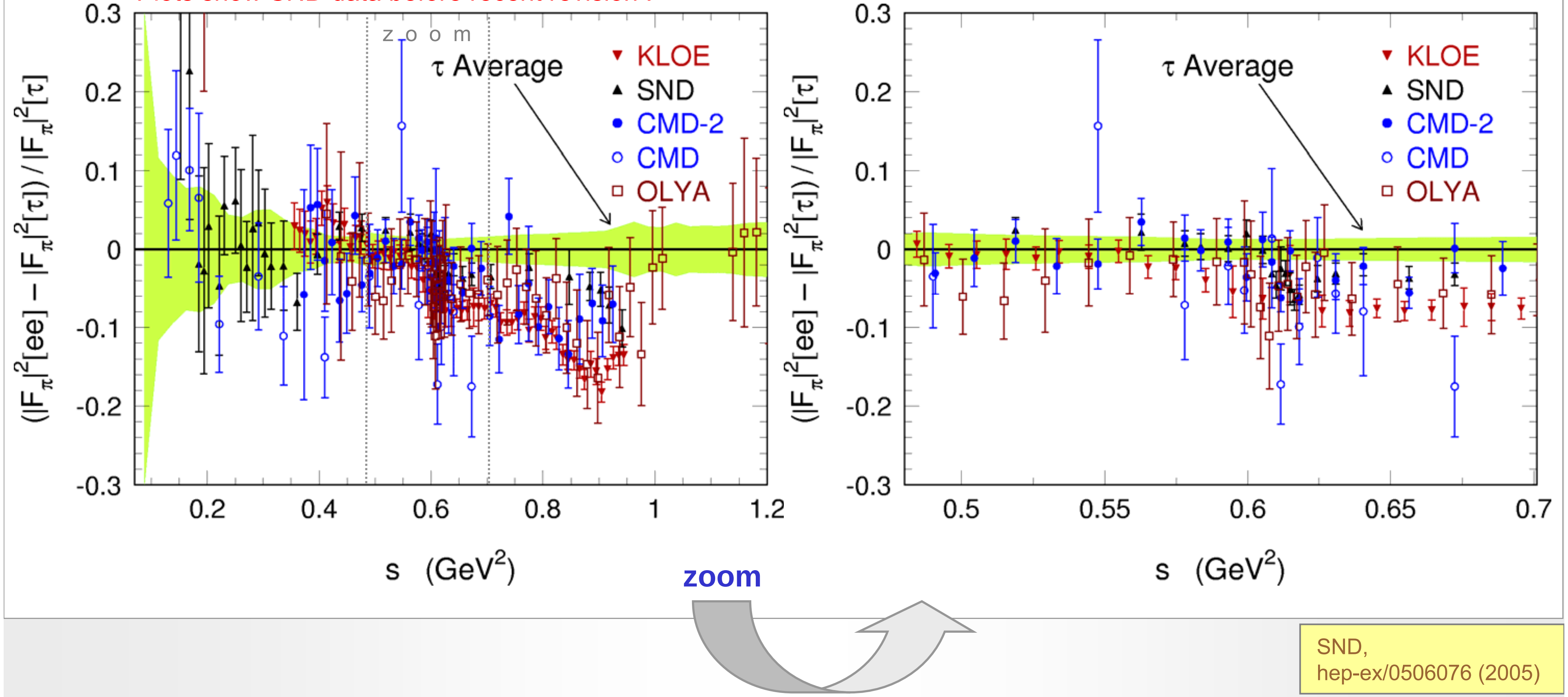
Relative difference between τ and e^+e^- data:



New $e^+e^- \to \pi^+\pi^-$ Data from KLOE ("radiative return") & SND

Relative difference between τ and e^+e^- data:

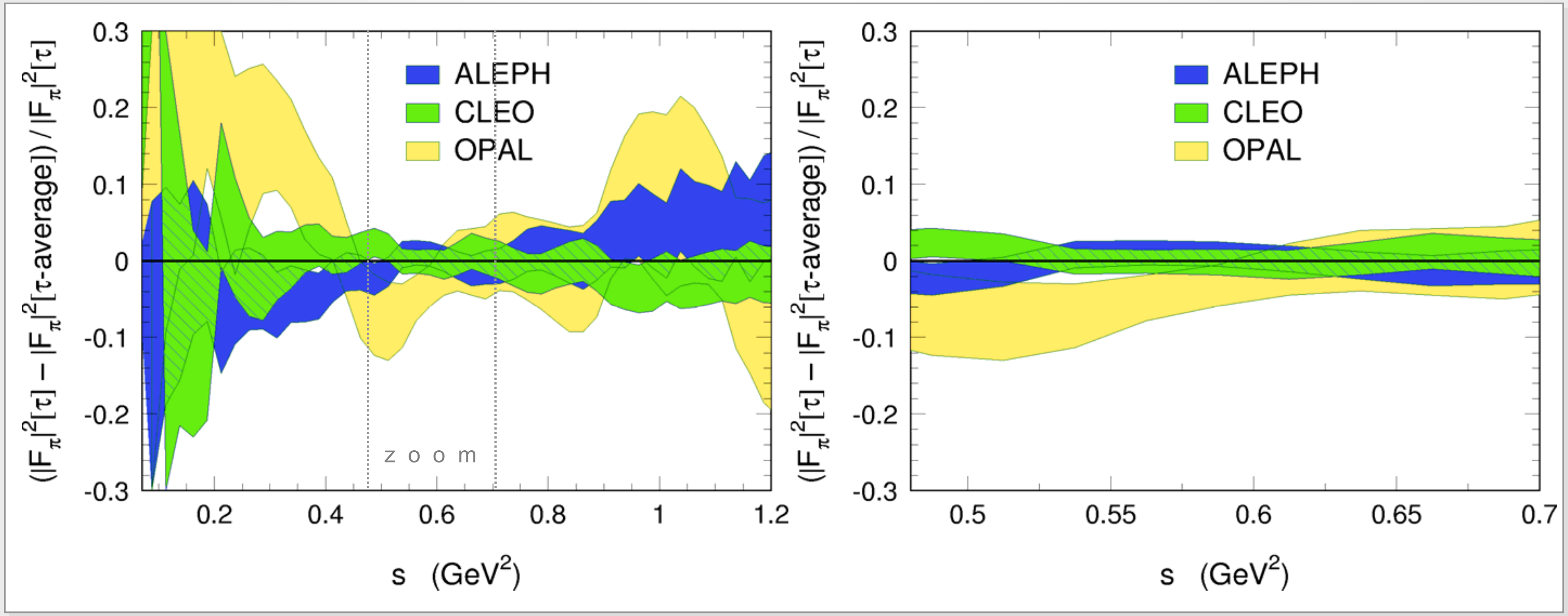
Plots show SND data before recent revision !



SND,
hep-ex/0506076 (2005)

KLOE,
PL B606, 12 (2005)

$\tau^- \to \pi^-\pi^0\nu_\tau$: Comparing ALEPH, CLEO, OPAL



- Good agreement observed between ALEPH and CLEO
- ALEPH more precise at low *s*
- CLEO better at high *s*

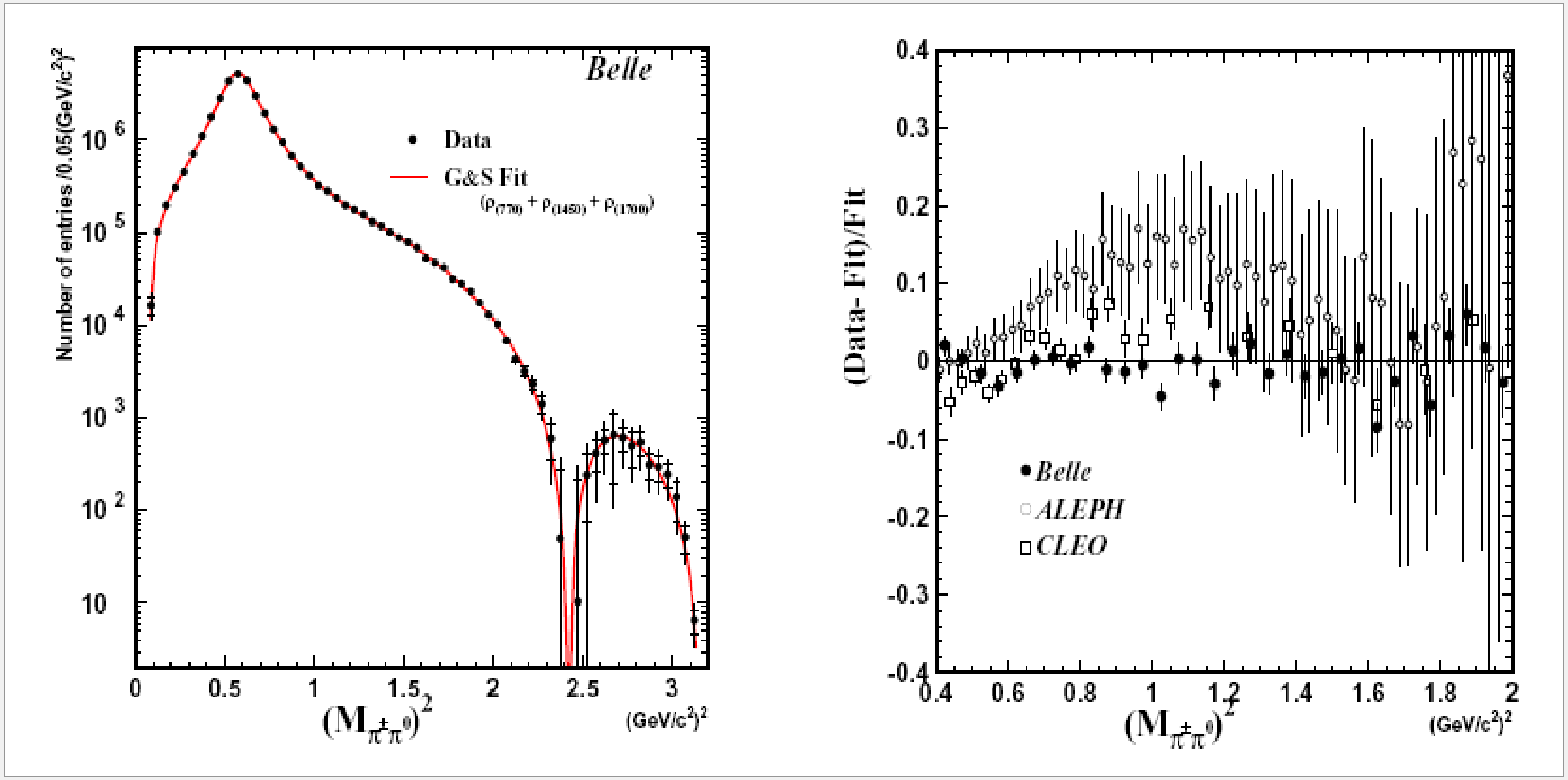
ALEPH,
Phys. Rept. 421, 191 (2005)

CLEO,
PRD 61, 112002 (2000)

OPAL,
EPJ C35, 437 (2004)

$\tau^- \to \pi^-\pi^0\nu_\tau$: Preliminary Results from Belle

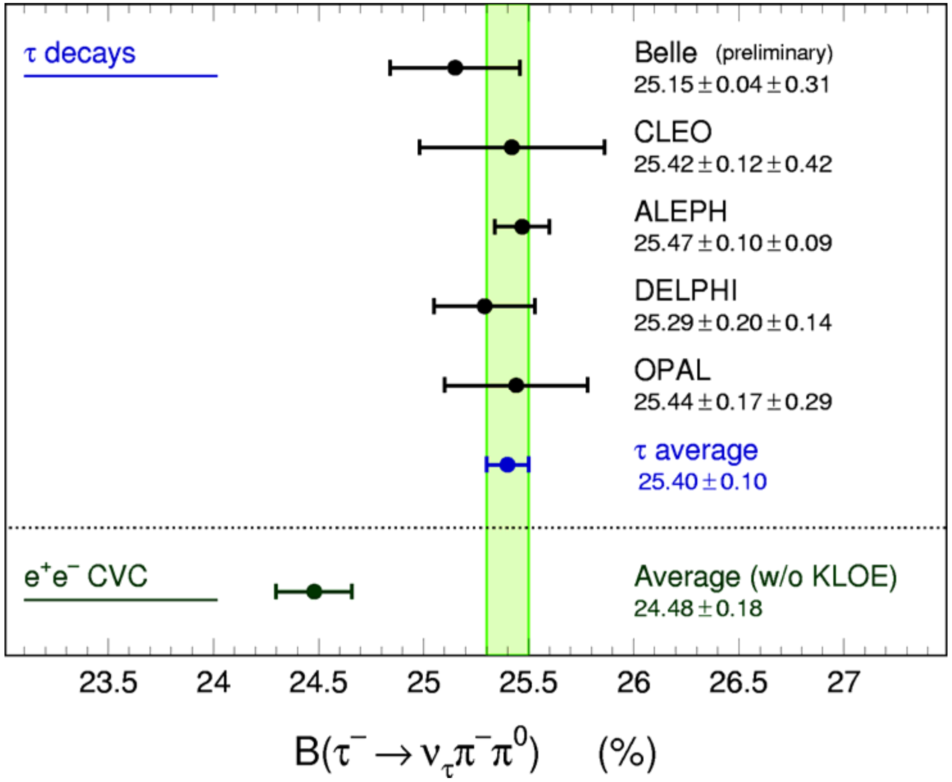
- Preliminary spectral function presented by Belle at EPS 2005
- High statistics: see significant dip at 2.4 GeV^2 for first time in τ data !
- Discrepancies with ALEPH/CLEO at large mass and with e^+e^- data at low mass



Another Way to Look at the Data

Infer τ branching fractions (more robust than spectral functions) from e^+e^- data:

$$\mathrm{BR}_{\mathrm{CVC}}\left(\tau^- \to \pi^-\pi^0\nu_\tau\right) = \frac{6\pi\,|V_{ud}|^2\,S_{EW}}{m_\tau^2}\int_0^{m_\tau^2} ds\ \mathrm{kin}(s)\cdot \upsilon^{\mathrm{SU(2)\text{-}corrected}}(s)$$



Difference: BR[τ] – BR[e^+e^- (CVC)]:

Mode	$\Delta(\tau - e^+e^-)$	"Sigma"
$\tau^- \to \pi^-\pi^0\nu_\tau$	+ 0.92 ± 0.21	4.5
$\tau^- \to \pi^- 3\pi^0\nu_\tau$	– 0.08 ± 0.11	0.7
$\tau^- \to 2\pi^-\pi^+\pi^0\nu_\tau$	+ 0.91 ± 0.25	3.6

e^+e^- data on $\pi^-\pi^+\pi^0\pi^0$ not satisfactory

Final Remarks on Main $\pi^+\pi^-$ Contribution

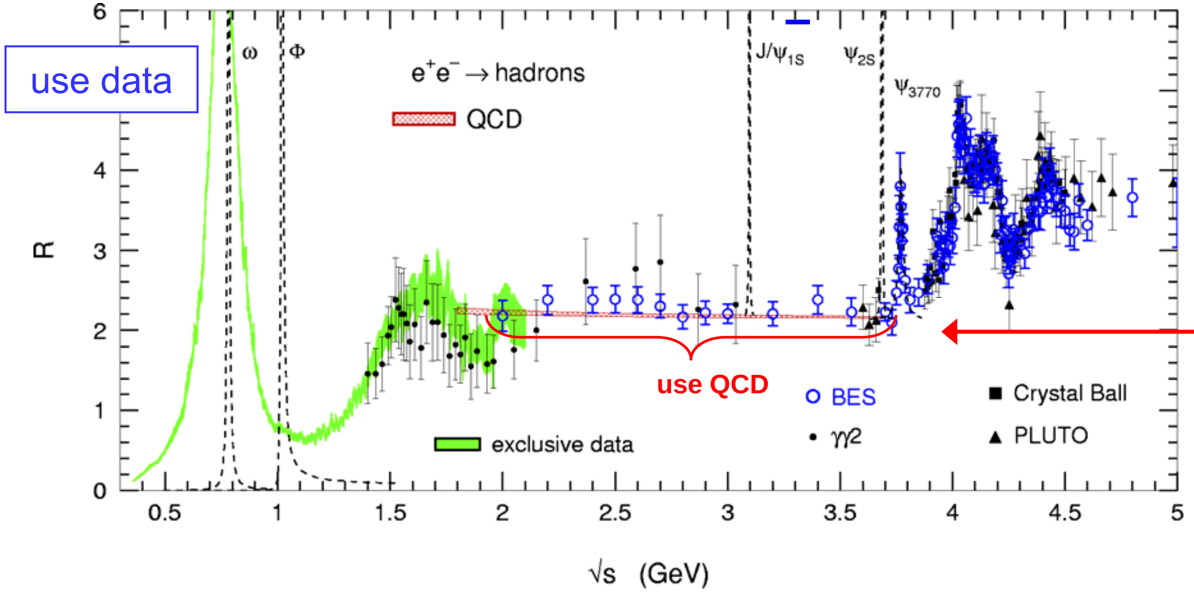
The problem of the $\pi^+\pi^-$ contribution:

- Experimental situation:
  - revised SND results in agreement with CMD-2
  - τ data without $m(\rho)$ and $\Gamma(\rho)$ corr. in strong disagreement with both data sets
  - ALEPH, CLEO and OPAL τ data in ok agreement, preliminary Belle less so
  - *e^+e^- spectral functions have now reached the precision of τ data*
- Concerning the remaining line shape discrepancy (0.7- 0.9 GeV^2):
  - SU(2) corrections: basic contributions identified and stable since long; overall correction applied to τ is (–2.2 ± 0.5) %, dominated by uncontroversial short distance piece; additional long-distance corrections found to be small
  - ρ lineshape corrections can improve the situation, but cannot account for the difference above 0.7 GeV^2

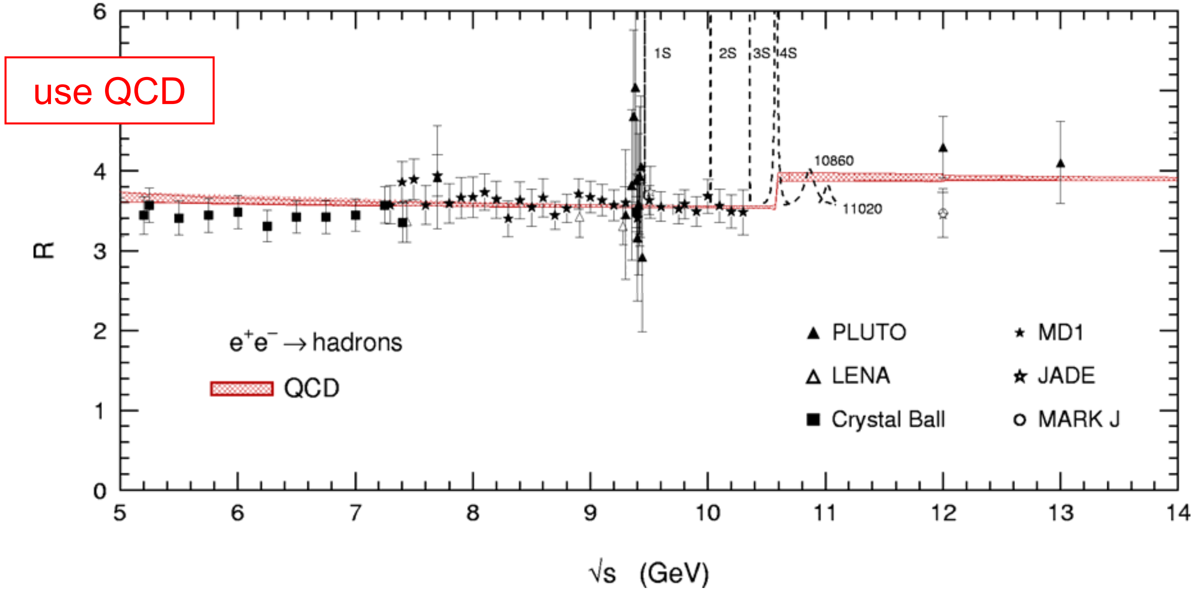
- The agreement between SND and CMD-2 invalidates the use of τ data until a better understanding of the discrepancies is achieved (an interesting question as such)
- Discrepancy between KLOE and CMD-2/SND results: not safe to take advantage of decreased error when including KLOE

Evaluating the Dispersion Integral

use data



use QCD



$$a_\mu^{\text{had,LO}} = \frac{\alpha^2}{3\pi^2}\int_{4m_\pi^2}^{\infty} ds\,\frac{K(s)}{s}R(s)$$

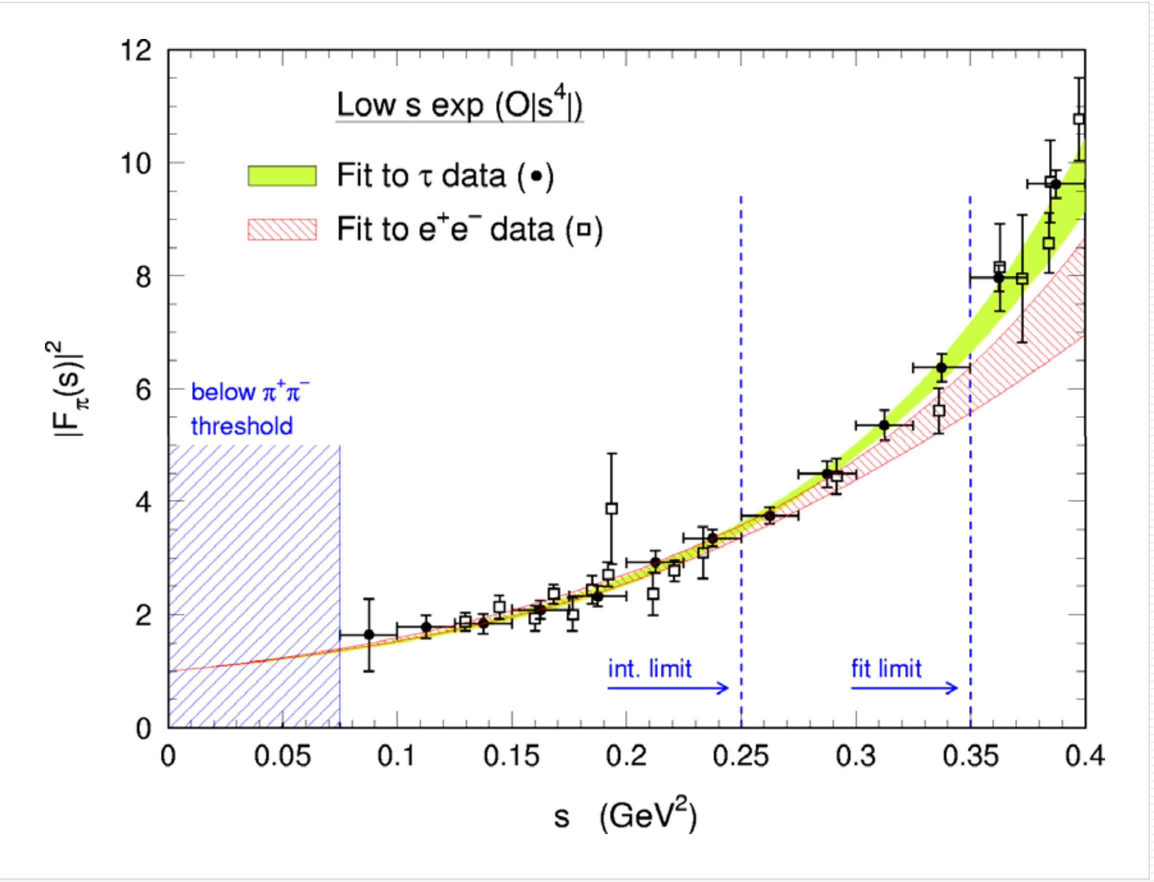
Agreement between Data (BES) and pQCD (within *correlated* systematic errors)

Better agreement between exclusive and inclusive (γγ2) data than in 1997-1998 analyses

digression: Specific Contributions: Threshold

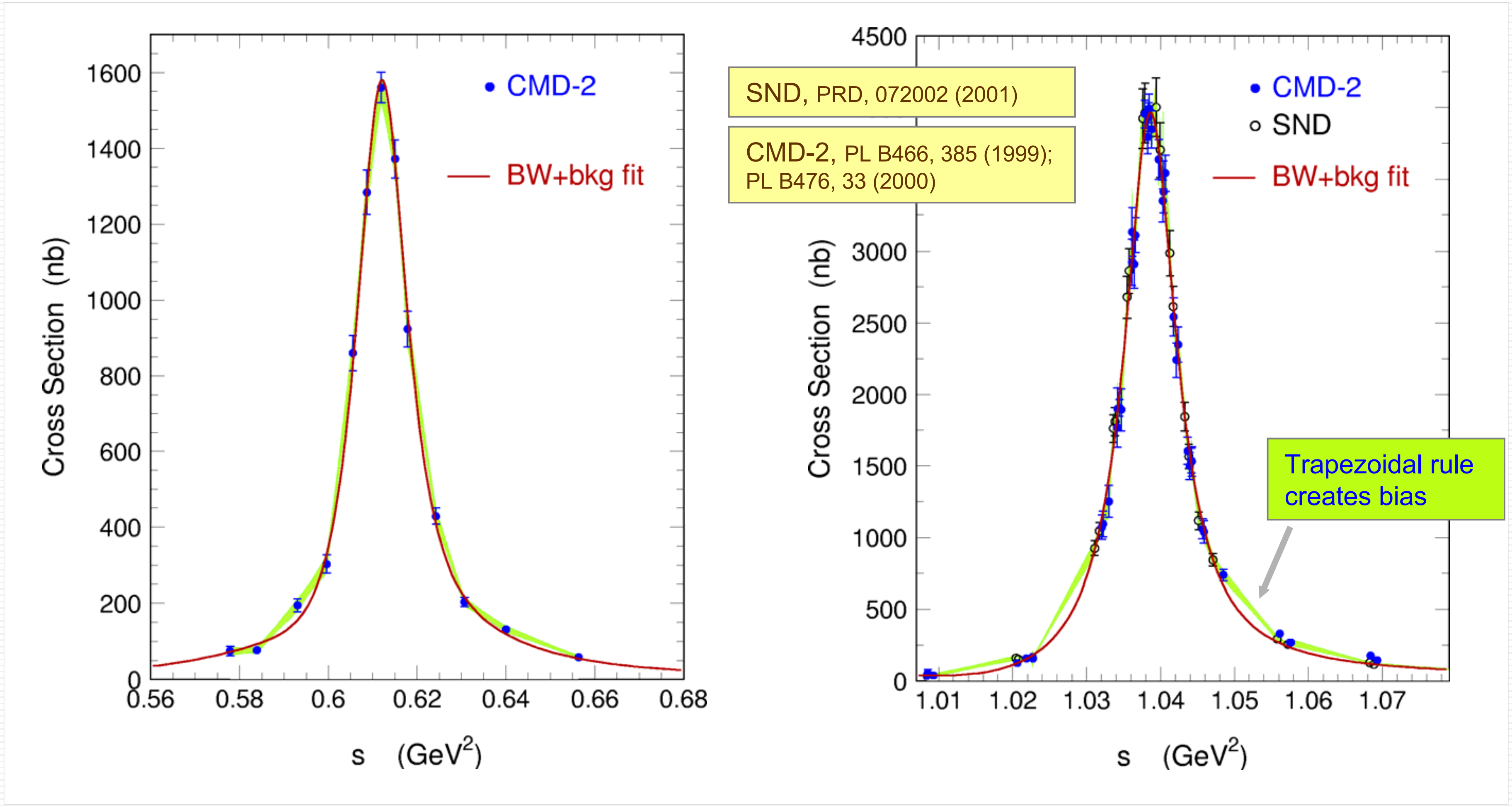
Use expansion for $\pi^+\pi^-$ threshold inspired by chiral perturbation theory:

$$\sigma_{\pi\pi} = \frac{\pi\alpha^2\beta^3}{3s}|F_\pi|^2 \qquad \text{and :} \qquad F_\pi = 1 + \frac{1}{6}\langle r^2\rangle_\pi s + c_1 s^2 + c_2 s^3 + O(s^4)$$



digression: Specific Contributions: Narrow Light Resonances

Use direct data integration for $\omega(782)$ and $\phi(1020)$ to account for non-resonant contributions.



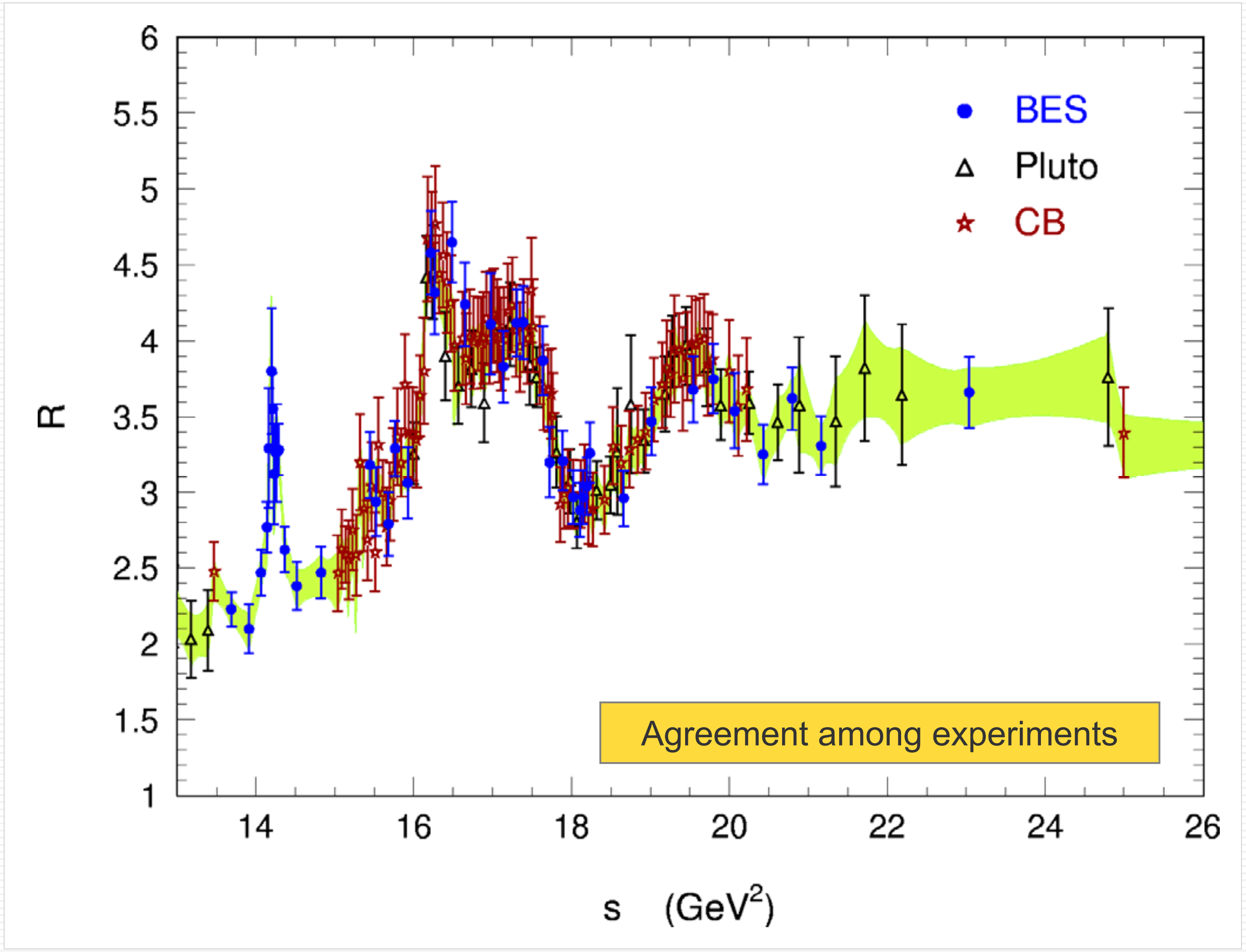
SND, PRD, 072002 (2001)

CMD-2, PL B466, 385 (1999); PL B476, 33 (2000)

Trapezoidal rule creates bias

digression: Specific Contributions: Charm Threshold

New precise BES data improve $c\bar{c}$ resonance region:



Agreement among experiments

Results: the Compilation (including newest data)

Contributions to $a_\mu^{\text{had,LO}}$ [in 10^{-10}] from the different energy domains:

Modes	Energy [GeV]	e^+e^-	τ
Low s expansion	$2m_\pi$ – 0.5	$55.6 \pm 0.8 \pm 0.1_{\text{rad}}$	$56.0 \pm 1.6 \pm 0.3_{\text{SU(2)}}$
$\pi^+\pi^-$ (+SND+CMD-2)	0.5 – 1.8	$449.0 \pm 3.0 \pm 0.9_{\text{rad}}$	$464.0 \pm 3.0 \pm 2.3_{\text{SU(2)}}$
$\pi^+\pi^- 2\pi^0$	$2m_\pi$ – 1.8	$16.8 \pm 1.3 \pm 0.2_{\text{rad}}$	$21.4 \pm 1.3 \pm 0.6_{\text{SU(2)}}$
$2\pi^+ 2\pi^-$ (+BABAR)	$2m_\pi$ – 1.8	$13.1 \pm 0.4 \pm 0.0_{\text{rad}}$	$12.3 \pm 1.0 \pm 0.4_{\text{SU(2)}}$
ω(782)	0.3 – 0.81	$38.0 \pm 1.0 \pm 0.3_{\text{rad}}$	–
ϕ(1020)	1.0 – 1.055	$35.7 \pm 0.8 \pm 0.2_{\text{rad}}$	–
Other excl. (+BABAR)	$2m_\pi$ – 1.8	$24.3 \pm 1.3 \pm 0.2_{\text{rad}}$	–
J/ψ, ψ(2S)	3.08 – 3.11	$7.4 \pm 0.4 \pm 0.0_{\text{rad}}$	–
R [QCD]	1.8 – 3.7	$33.9 \pm 0.5_{\text{theo}}$	–
R [data]	3.7 – 5.0	$7.2 \pm 0.3 \pm 0.0_{\text{rad}}$	–
R [QCD]	5.0 – ∞	$9.9 \pm 0.2_{\text{theo}}$	–
Sum (w/o KLOE)	$2m_\pi$ – ∞	$690.8 \pm 3.9 \pm 1.9_{\text{rad}} \pm 0.7_{\text{QCD}}$	$710.1 \pm 5.0 \pm 0.7_{\text{rad}} \pm 2.8_{\text{SU(2)}}$

The Full Hadronic Contribution

- Hadronic leading order contribution

$$a_\mu^{\text{had,LO}}[e^+e^-] = (693.4 \pm 5.3 \pm 3.5_{\text{rad}}) \times 10^{-10}$$

Electroweak: $(15.4 \pm 0.2) \times 10^{-10}$

QED: $(11\,658\,471.9 \pm 0.1) \times 10^{-10}$

- Hadronic next-to-leading order contributions
  - Vacuum polarization (1-loop) + additional photon or VP insertion:
    - computed akin to LO part via dispersion integral with modified kernel function
    - $a_\mu^{\text{had,NLO}} = -9.8(0.1) \times 10^{-10}$
  - Light-by-light scattering :
    - dispersion relation approach not possible (4-point function)
    - no first-principle calculation yet (*e.g.*, on the lattice)
    - model calculations using short dist. quark loops, π^0, $\eta^{(\prime)}$, … pole insertions and $\pi^\pm$ loops in the large-N_C limit
    - $a_\mu^{\text{had,LBL}} = +12.0(3.5) \times 10^{-10}$

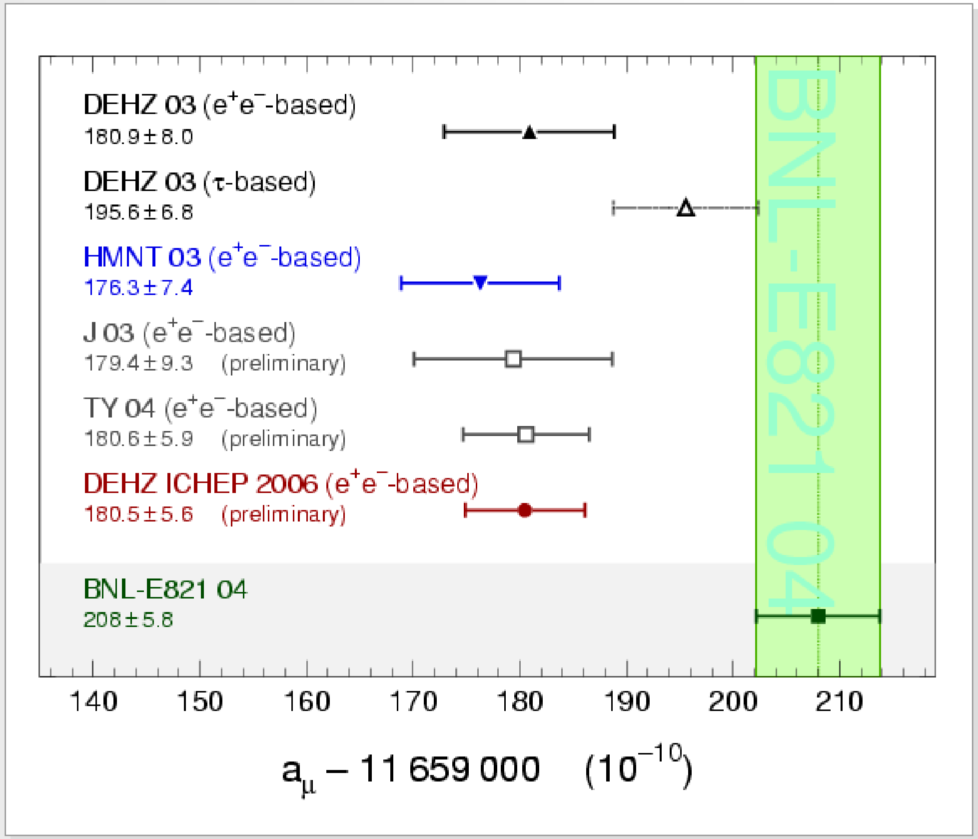
Knecht-Nyffeler, Phys.Rev.Lett. 88 (2002) 071802

Melnikov-Vainshtein, PRD 70, 113006 (2004)

And the Complete Result

$$a_\mu^{SM}[e^+e^-] = (11\,659\,180.5 \pm 4.4_{had,LO} \pm 3.5_{LBL} \pm 0.2_{QED+weak})\times 10^{-10}$$

DEHZ (Tau 2006)



BNL E821 (2004):
a_μ^{exp} = (11 659 208.0 ± 6.3) 10^{-10}

Observed Difference with Experiment:

$$a_\mu^{exp} - a_\mu^{SM} = (27.5 \pm 8.4) \times 10^{-10}$$

➡ 3.3 ”standard deviations“

conclusions

and perspectives

conclusions

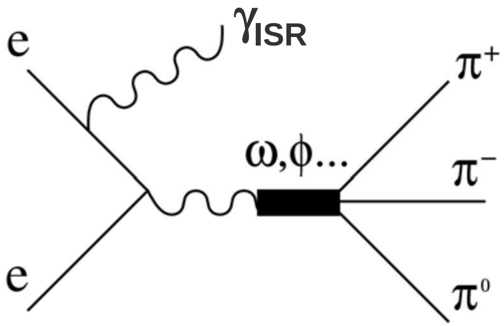
- Phenomenal experimental progress from BNL (E821) $g-2$ measurement
- Improved theory prediction due to new CMD-2 and SND data
- Hadr. part dominates SM uncertainty (5.6), but more precise than experiment (6.3)
- Disagreement between SND/CMD-2 and KLOE data sets; so far KLOE not incl.
- Tau data in agreement (but Belle); revised SND data confirm τ / e^+e^- discrepancy
- What is behind the 4.5σ τ / e^+e^- discrepancy of the CVC BR ?
- KLOE will publish cross sections based on pion/muon ratios
- BABAR ISR: $\pi^+\pi^-$ spectral function over full mass range, multihadron channels
- Difference between experiment and $\mathrm{SM}_{[e+e-]}$ within range of possible New Physics

appendix

the BABAR ISR programme

Radiative Return Cross Section Results from BABAR

- The *Radiative Return*: benefit from huge luminosities at *B* and ϕ Factories to perform continuous cross section measurements



$$\boxed{\frac{d\sigma(s,x)}{dx\,d(\cos\theta)} = H(s,x,\theta)\cdot\ \sigma_0(s(1-x))}$$

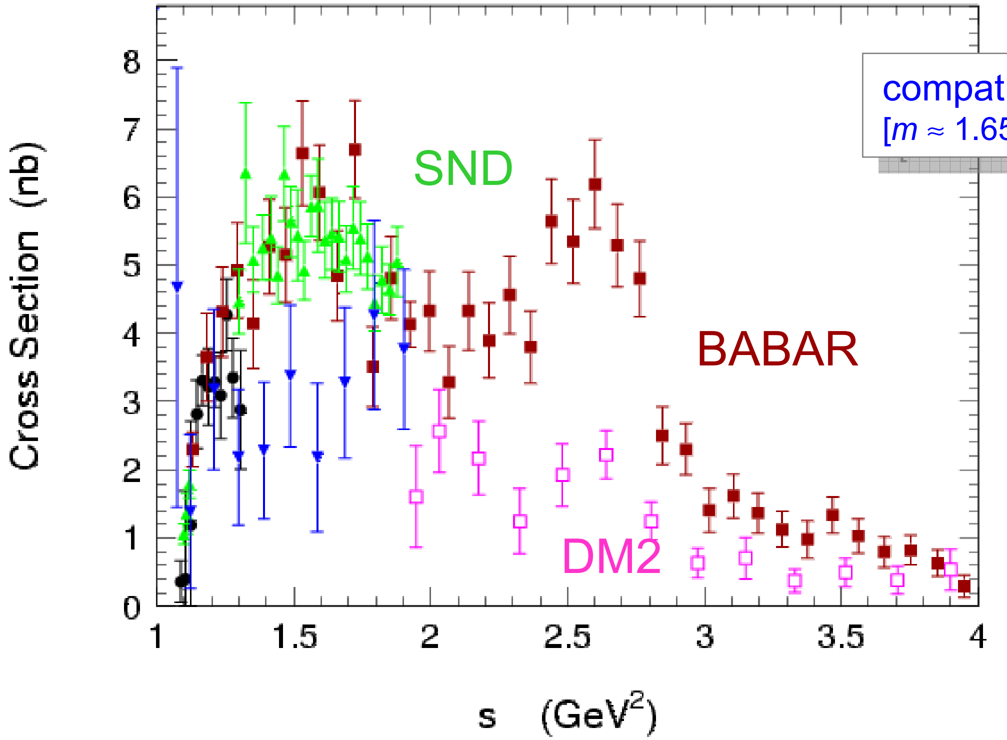
$$H(s,x,\theta) = \frac{\alpha}{\pi x}\left(\frac{2-2x+x^2}{\sin^2\theta} - \frac{x^2}{2}\right),\quad x = \frac{2E_\gamma}{\sqrt{s}}$$

H is radiation function

- High PEP-II luminosity at √s = 10.58 GeV ⟶ precise measurement of the e^+e^- cross section σ_0 at low c.m. energies with BABAR
- Comprehensive program at BABAR
- Results for $\pi^+\pi^-\pi^0$, preliminary results for $2\pi^+2\pi^-$, $K^+K^-\pi^+\pi^-$, $2K^+2K^-$ from 89.3 fb^{-1}

The $e^+e^- \to \pi^+\pi^-\pi^0$ Cross Section

- Cross section above ϕ resonance : DM2 missed a resonance **!**



compatible with $\omega(1650)$
[$m \approx 1.65$ GeV, $\Gamma \approx 0.22$ GeV]

contribution to a_μ^{had} (<1.8 GeV) :

all before BABAR [10^{-10}]	2.45 ± 0.26 ± 0.03
all + BABAR [10^{-10}]	2.79 ± 0.19 ± 0.01
all + BABAR – DM2 [10^{-10}]	3.25 ± 0.09 ± 0.01

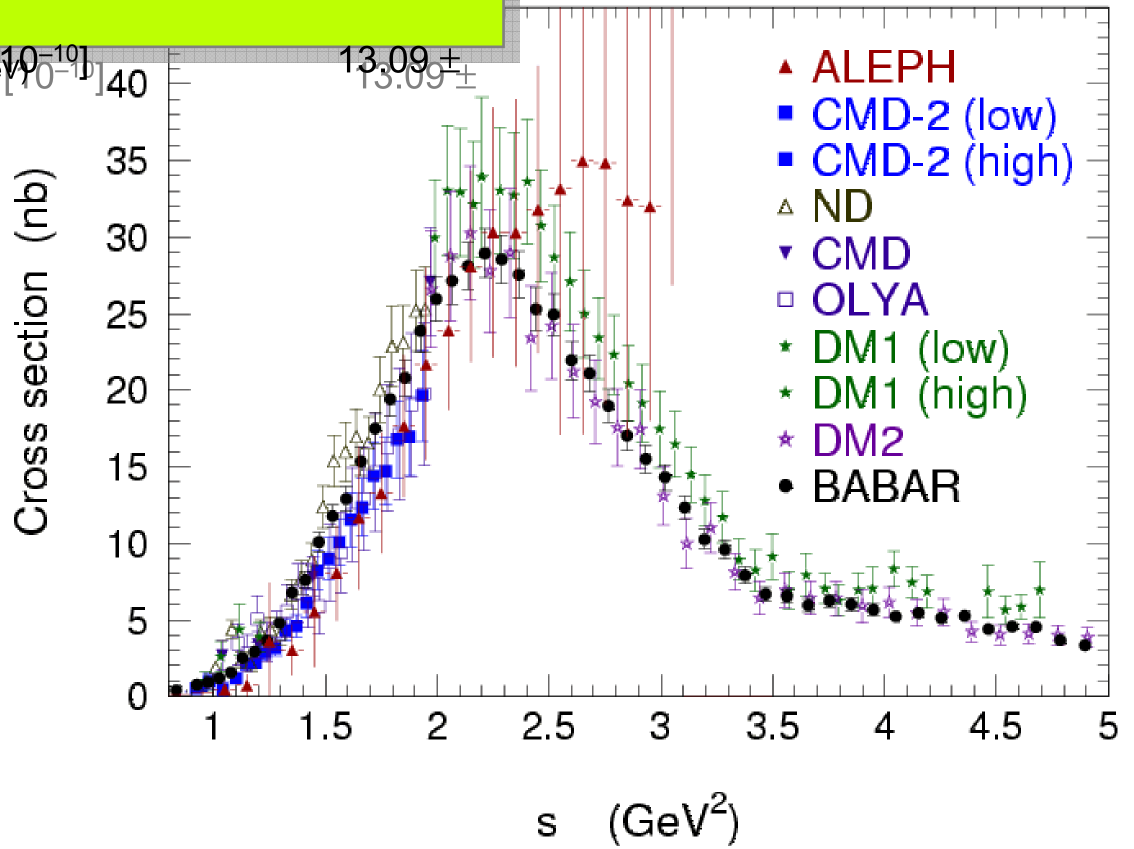
all + BABAR – DM2 [10^{-10}] 3.25 ± 0.09 ± 0.01

The $e^+e^- \to 2\pi^+2\pi^-$ Cross Section

- all before BABAR [10^{-10}] 14.20 ± 0.87 ± 0.24
- all + BABAR [10^{-10}] 13.09 ± 0.44 ± 0.00

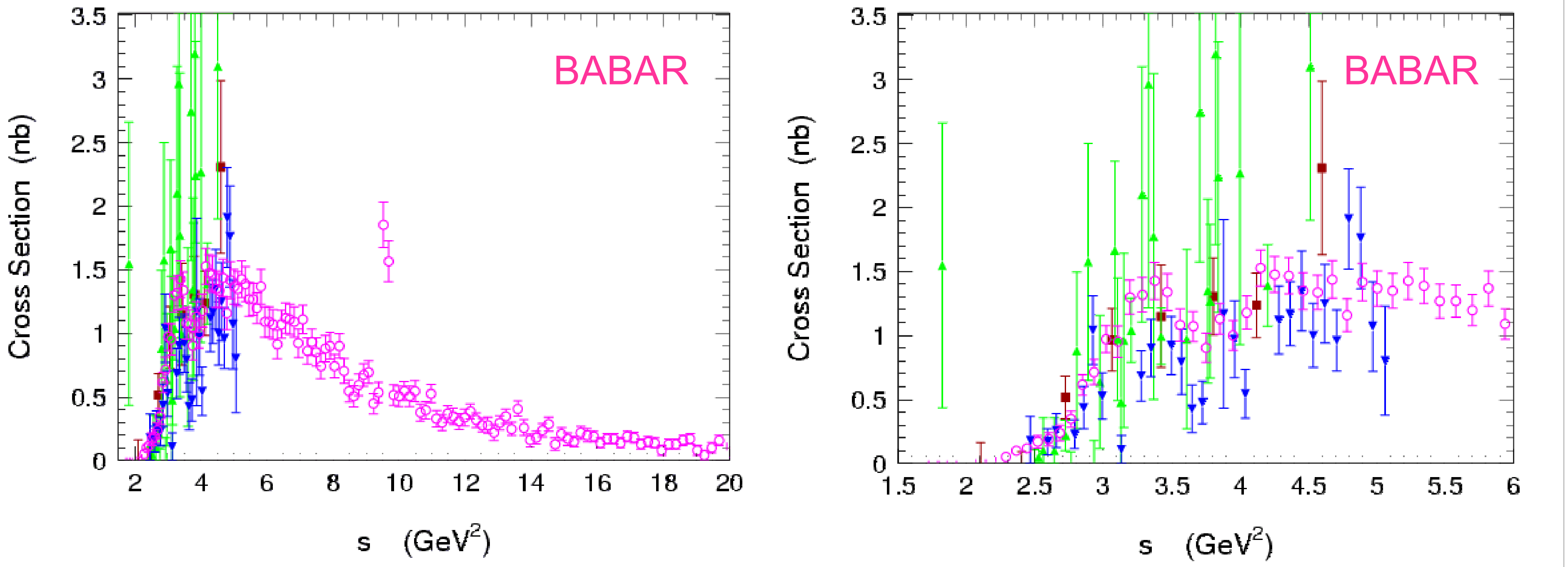
all + BABAR [10^{-10}] 13.09 ± 0.44 ± 0.00

contribution to $a_\mu^{\rm had}$ (<1.8 GeV)



- Good agreement with direct e^+e^- measurements
- Most precise result above 1.4 GeV

BaBar ISR: $3\pi^+3\pi^-$

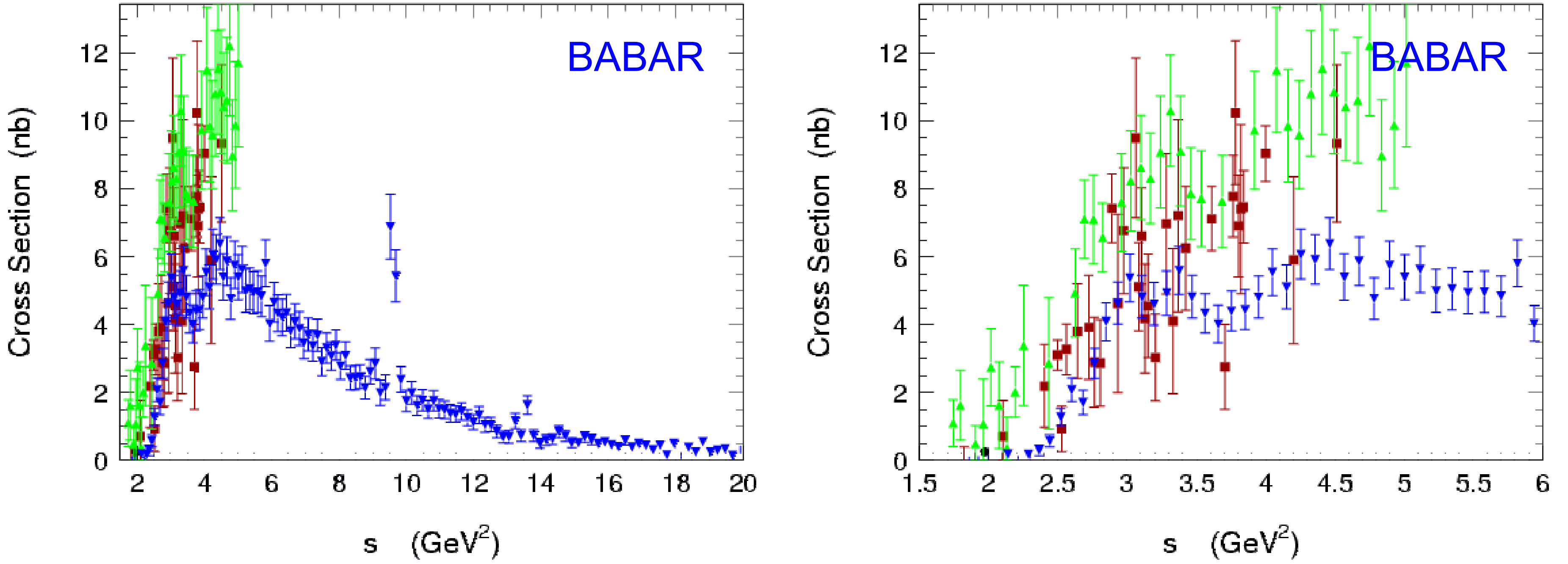


contribution to a_μ^{had} (<1.8 GeV) :

- all before BABAR [10^{-10}] 0.10 ± 0.10
- all + BABAR [10^{-10}] 0.108 ± 0.016

- all + BABAR [10^{-10}] 0.108 ± 0.016

BABAR ISR: $2\pi^+2\pi^-2\pi^0$



contribution to a_μ^{had} (<1.8 GeV) :

- all before BABAR [10^{-10}] 1.42 ± 0.30 ± 0.03
- all + BABAR [10^{-10}] 0.89 ± 0.09

- all + BABAR [10^{-10}] 0.89 ± 0.09