New Measurement of the Electron Magnetic Moment and the Fine Structure Constant

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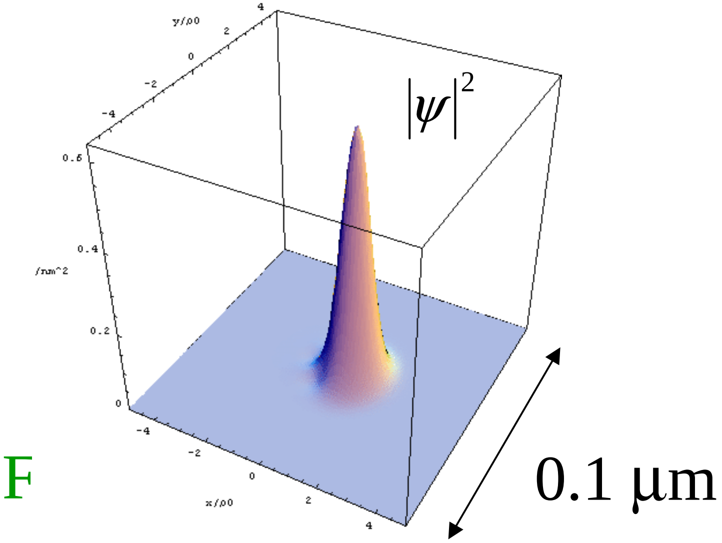
Almost finished student: David Hanneke
Earlier contributions: Brian Odom,
Brian D'Urso,
Steve Peil,
Dafna Enzer,
Kamal Abdullah
Ching-hua Tseng
Joseph Tan

2006 DAMOP Thesis Prize Winner

20 years
6.5 theses



N$F

Why Does it take Twenty Years and 6.5 Theses?

Explanation 1: Van Dyck, Schwinberg, Dehemelt did a good job in 1987!
Phys. Rev. Lett. **59**, 26 (1987)

Explanation 2a: We do experiments much too slowly

Explanation 2b: Takes time to develop new ideas and methods needed to measure with 7.6 parts in 10^{13} uncertainty

first measurement with these new methods

- One-electron quantum cyclotron
- Resolve lowest cyclotron states as well as spin
- Quantum jump spectroscopy of spin and cyclotron motions
- Cavity-controlled spontaneous emission
- Radiation field controlled by cylindrical trap cavity
- Cooling away of blackbody photons
- Synchronized electrons identify cavity radiation modes
- Trap without nuclear paramagnetism
- One-particle self-excited oscillator

The New Measurement of Electron g

U. Michigan	**U. Washington**	**Harvard**	
beam of electrons	one electron	one electron	
spins precess with respect to cyclotron motion	observe spin flip	quantum cyclotron motion	100 mK
	thermal cyclotron motion	resolve lowest quantum levels	self-excited oscillator
		cavity-controlled radiation field (cylindrical trap)	inhibit spontan. emission
			cavity shifts
Crane, Rich, …	Dehmelt, Van Dyck		

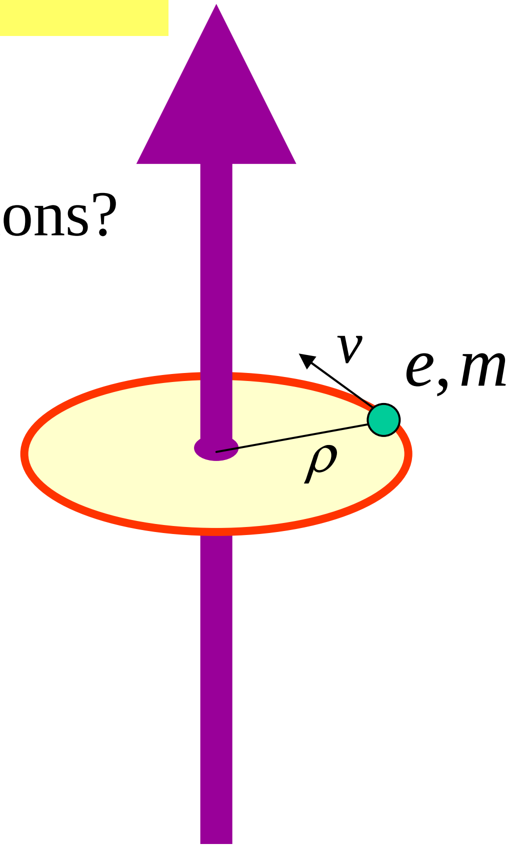
Magnetic Moments, Motivation and Results

Magnetic Moments

magnetic moment $\vec{\mu} = g\mu_B \frac{\vec{L}}{\hbar}$ ← angular momentum

Bohr magneton $\frac{e\hbar}{2m}$

e.g. What is g for identical charge and mass distributions?



$$\mu = IA = \frac{e}{\left(\frac{2\pi\rho}{v}\right)}(\pi\rho^2) = \frac{ev\rho}{2}\frac{L}{mv\rho} = \frac{e}{2m}L = \underset{\underset{\mu_B}{\uparrow}}{\frac{e\hbar}{2m}}\frac{L}{\hbar}$$

$$\rightarrow \quad g = 1$$

Magnetic Moments

magnetic moment

$$\vec{\mu} = g\mu_B \frac{\vec{S}}{\hbar}$$

angular momentum

Bohr magneton $\frac{e\hbar}{2m}$

$g = 1$ identical charge and mass distribution

$g = 2$ spin for Dirac point particle

$g = 2.002\ 319\ 304\ \ldots$ simplest Dirac spin, plus QED

(if electron g is different → electron has substructure)

Why Measure the Electron Magnetic Moment?

1. **Electron g - basic property of simplest of elementary particles**
2. **Determine fine structure constant – from measured g and QED (May be even more important when we change mass standards)**
3. **Test QED – requires independent α**
4. **Test CPT – compare g for electron and positron → best lepton test**
5. **Look for new physics beyond the standard model**
   - **Is g given by Dirac + QED? If not → electron substructure (new physics)**
   - **Muon g search needs electron g measurement**

New Measurement of Electron Magnetic Moment

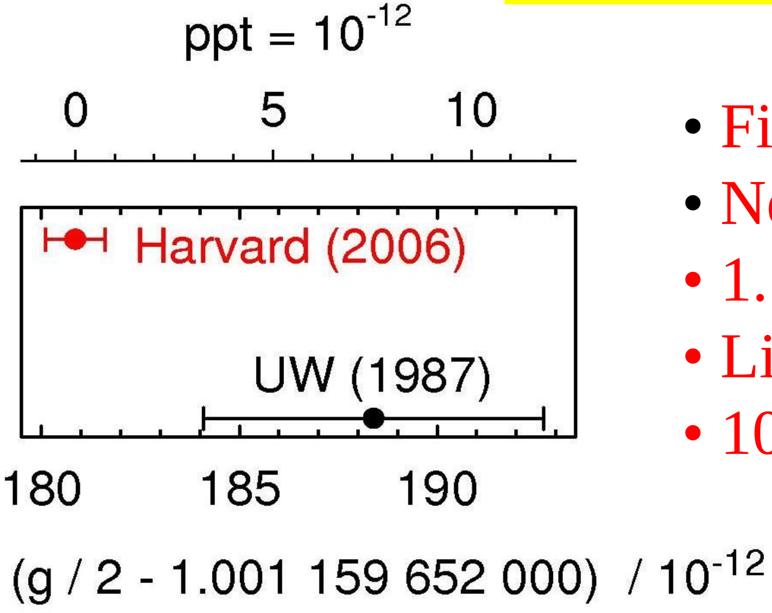
magnetic moment $$\vec{\mu} = g \mu_B \frac{\vec{S}}{\hbar}$$

spin

Bohr magneton $\frac{e\hbar}{2m}$

$$g/2 = 1.001\,159\,652\,180\,85$$

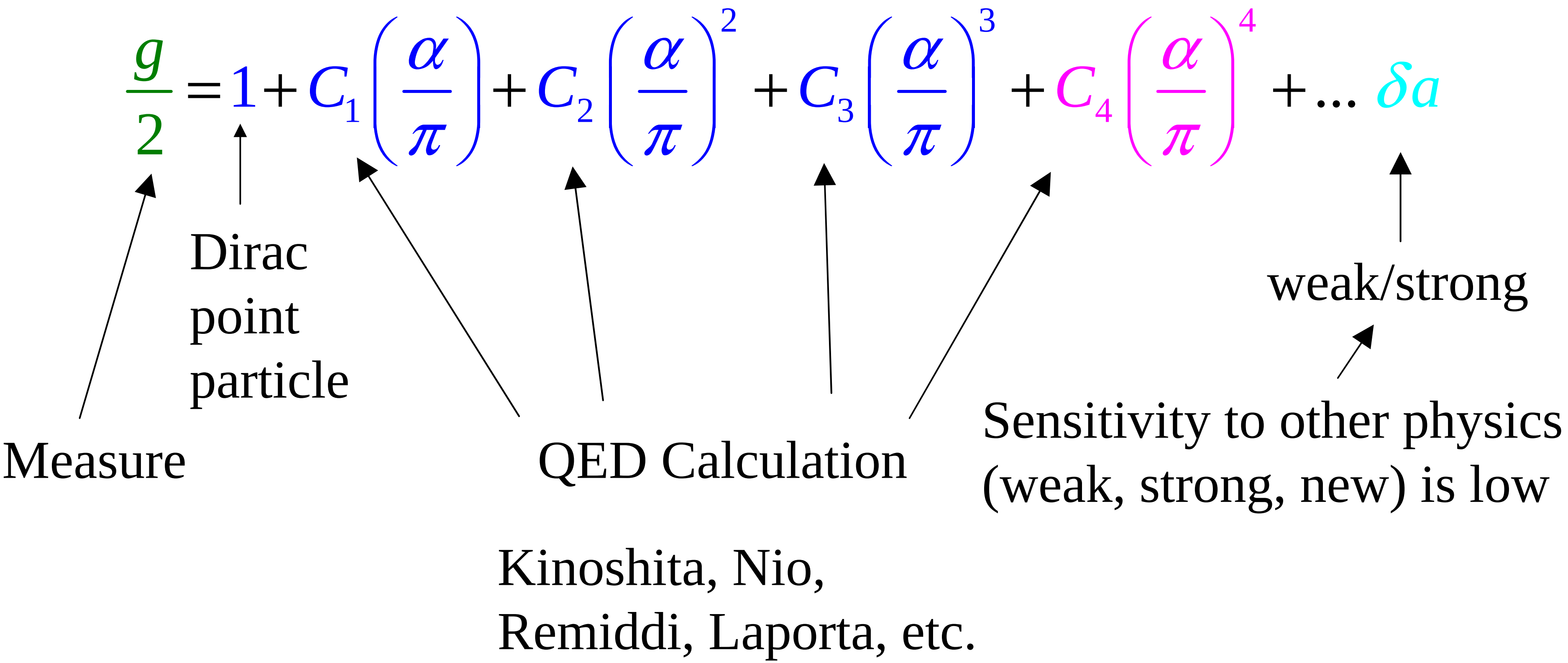
$$\pm 0.000\,000\,000\,000\,76 \qquad 7.6 \times 10^{-13}$$



- First improved measurement since 1987
- Nearly six times smaller uncertainty
- 1.7 standard deviation shift
- Likely more accuracy coming
- 1000 times smaller uncertainty than muon g

B. Odom, D. Hanneke, B. D'Urso and G. Gabrielse, Phys. Rev. Lett. **97**, 030801 (2006).

Dirac + QED Relates Measured g and Measured α



$$\frac{g}{2} = 1 + C_1\left(\frac{\alpha}{\pi}\right) + C_2\left(\frac{\alpha}{\pi}\right)^2 + C_3\left(\frac{\alpha}{\pi}\right)^3 + C_4\left(\frac{\alpha}{\pi}\right)^4 + \ldots\ \delta a$$

Dirac
point
particle

weak/strong

Measure

QED Calculation

Sensitivity to other physics
(weak, strong, new) is low

Kinoshita, Nio,
Remiddi, Laporta, etc.

1. **Use measured g and QED to extract fine structure constant**
2. **Wait for another accurate measurement of α → Test QED**

Basking in the Reflected Glow of Theorists

$$\frac{g}{2} = 1 + C_1\left(\frac{\alpha}{\pi}\right) + C_2\left(\frac{\alpha}{\pi}\right)^2 + C_3\left(\frac{\alpha}{\pi}\right)^3 + C_4\left(\frac{\alpha}{\pi}\right)^4 + C_5\left(\frac{\alpha}{\pi}\right)^4 + \ldots\ \delta a$$

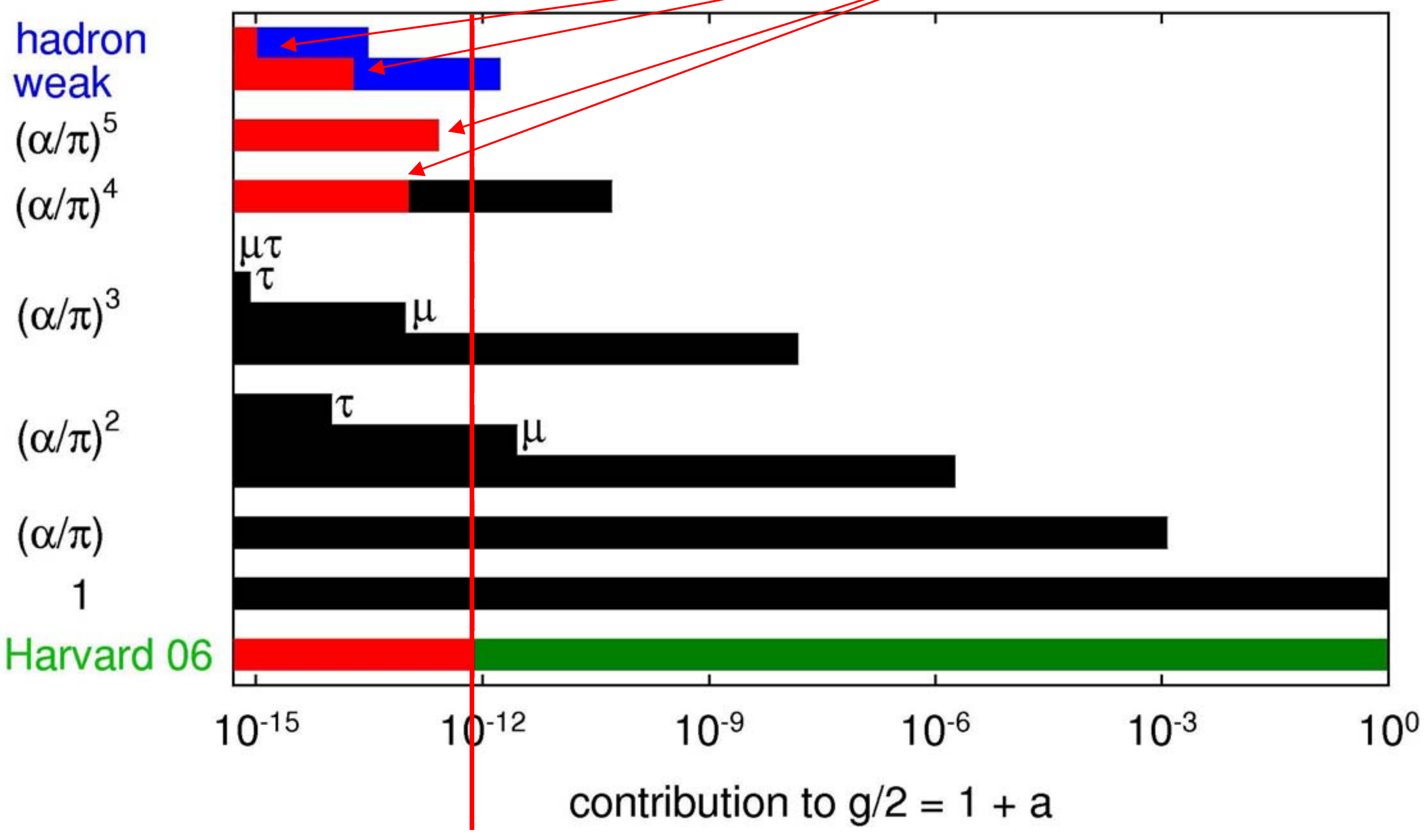
2004

Remiddi Kinoshita G.G

Gabrielse

$$\frac{g}{2} = 1 + C_1\left(\frac{\alpha}{\pi}\right) + C_2\left(\frac{\alpha}{\pi}\right)^2 + C_3\left(\frac{\alpha}{\pi}\right)^3 + C_4\left(\frac{\alpha}{\pi}\right)^4 + \ldots\ \delta a$$

theoretical uncertainties



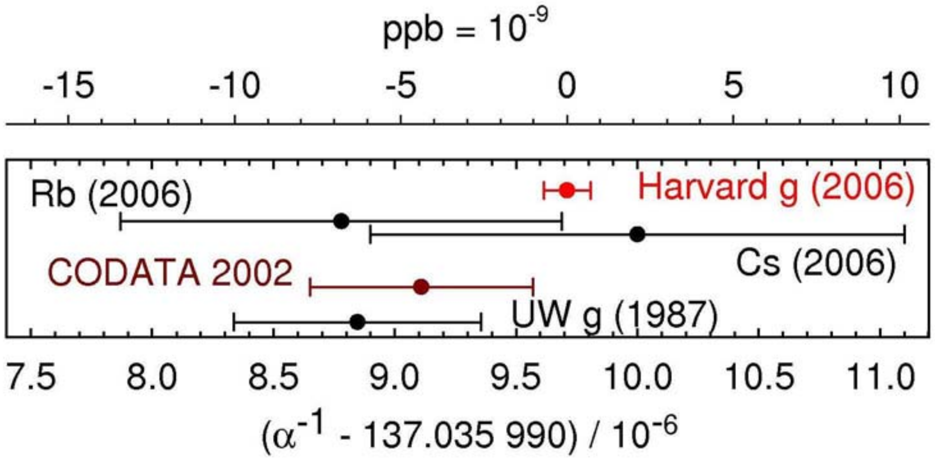
experimental uncertainty

New Determination of the Fine Structure Constant

$$\alpha = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{\hbar c}$$

- Strength of the electromagnetic interaction
- Important component of our system of fundamental constants
- Increased importance for new mass standard

$$\alpha^{-1} = 137.035\,999\,710$$
$$\pm \quad 0.000\,000\,096 \qquad 7.0\times10^{-10}$$



- First lower uncertainty since 1987
- Ten times more accurate than atom-recoil methods

G. Gabrielse, D. Hanneke, T. Kinoshita, M. Nio, B. Odom,
Phys. Rev. Lett. 97}, 030802 (2006).

Next Most Accurate Way to Determine α (use Cs example)

Combination of measured Rydberg, mass ratios, and atom recoil

$$\alpha \equiv \boxed{\frac{1}{4\pi\varepsilon_0}\frac{e^2}{hc}} \quad \leftarrow \quad R_\infty \equiv \frac{1}{(4\pi\varepsilon_0)^2}\frac{e^4 m_e c}{2h^3c^2}$$

Haensch, …

$$\alpha^2 = \frac{2R_\infty}{c}\frac{h}{m_e}$$

Pritchard, …

Chu, …

$$= \frac{2R_\infty}{c}\frac{h}{M_{Cs}}\frac{M_{Cs}}{M_p}\frac{M_p}{m_e} \quad \leftarrow \quad \frac{h}{M_{Cs}} = 2c^2\frac{f_{recoil}}{(f_{D1})^2}$$

Haensch, …
Tanner, …

$$\alpha^2 = 4\boxed{R_\infty c\frac{f_{recoil}}{(f_{D1})^2}\frac{M_{Cs}}{M_{12C}}\frac{M_{12C}}{m_e}}$$

Werthe, Quint, … (also Van Dyck)

Biraben, …

- Now this method is 10 times less accurate
- We hope that will improve in the future → test QED

(Rb measurement is similar except get h/M[Rb] a bit differently)

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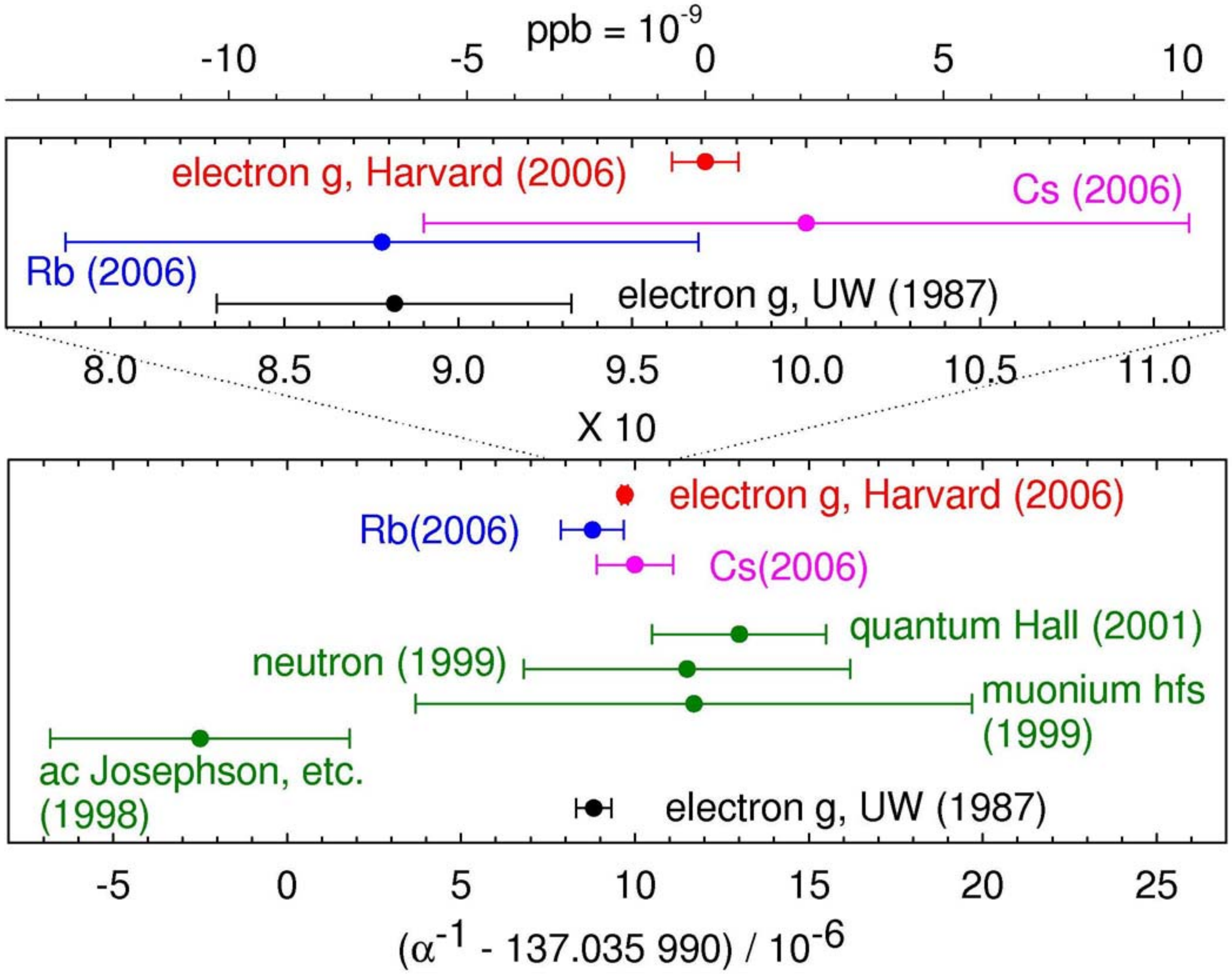
Earlier Measurements
Require Larger Uncertainty Scale

ten times
larger scale
to see larger
uncertainties



Test of QED

Most stringent test of QED: Comparing the measured electron g to the g calculated from QED using an independent α

$$\delta g < 15 \times 10^{-12}$$

- The uncertainty does not comes from g and QED
- All uncertainty comes from α[Rb] and α[Cs]
- With a better independent α could do a ten times better test

From Freeman Dyson – One Inventor of QED

Dear Jerry,

... I love your way of doing experiments, and I am happy to congratulate you for this latest triumph. Thank you for sending the two papers.

Your statement, that QED is tested far more stringently than its inventors could ever have envisioned, is correct. As one of the inventors, I remember that we thought of QED in 1949 as a temporary and jerry-built structure, with mathematical inconsistencies and renormalized infinities swept under the rug. We did not expect it to last more than ten years before some more solidly built theory would replace it. We expected and hoped that some new experiments would reveal discrepancies that would point the way to a better theory. And now, 57 years have gone by and that ramshackle structure still stands. The theorists … have kept pace with your experiments, pushing their calculations to higher accuracy than we ever imagined. And you still did not find the discrepancy that we hoped for. To me it remains perpetually amazing that Nature dances to the tune that we scribbled so carelessly 57 years ago. And it is amazing that you can measure her dance to one part per trillion and find her still following our beat.

With congratulations and good wishes for more such beautiful experiments, yours ever, Freeman.

Direct Test for Physics Beyond the Standard Model

$$g = 2 + 2a_{QED}(\alpha) + \delta g_{SM:Hadronic+Weak} + \delta g_{New\,Physics}$$

Is g given by Dirac + QED? If not → electron substructure

Does the electron have internal structure? Brodsky, Drell, 1980

$$m^* > \frac{m}{\sqrt{\delta g/2}} = 130\ GeV/c^2$$

limited by the uncertainty in independent α values

$$m^* > \frac{m}{\sqrt{\delta g/2}} = 600\ GeV/c^2$$

if our g uncertainty was the only limit

Not bad for an experiment done at 100 mK, but LEP does better

$m^* > 10.3\ TeV$ LEP contact interaction limit

Muon Test for Physics Beyond the Standard Model Needs Measured Electron g

less accurately measured
than we measure electron g
by a factor of 1000

expected to be bigger
than for electron
by ~40,000

$$g = 2 + 2a_{QED}(\alpha) + \delta g_{SM:Hadronic+Weak} + \delta g_{New\,Physics}$$

big contribution
must be subtracted out

→ need α

→ need test the QED calculation
of this large contribution

→ Muon search for new physics
needs the measurement of the electron g and α

Can We Check the 3σ Muon Disagreement between Measurement and "Calculation"?

$$g = 2 + 2a_{QED}(\alpha) + \delta g_{SM:Hadronic+Weak} + \delta g_{New\,Physics}$$

$(m_\mu/m_e)^2 \sim 40{,}000$ ← muon more sensitive to "new physics"

÷1,000 ← how much more accurately we measure

÷ 3 ← 3σ effect is now seen

→ If we can improve the electron g uncertainty
by an additional factor of 13
should be able to see the 3σ effect (or not)

(also need improved calculations, of course)

Not impossible to imagine, but may be impossible in practice

How Does One Measure the Electron g to 7.6 parts in 10^{13}?

How to Get an Uncertainty of 7.6 parts in 10^{13}

first measurement with these new methods

- One-electron quantum cyclotron
- Resolve lowest cyclotron as well as spin states
- Quantum jump spectroscopy of cyclotron and spin motions
- Cavity-controlled spontaneous emission
- Radiation field controlled by cylindrical trap cavity
- Cooling away of blackbody photons
- Synchronized electrons probe cavity radiation modes
- Elimination of nuclear paramagnetism
- One-particle self-excited oscillator

Make a “Fully Quantum Atom” for the electron

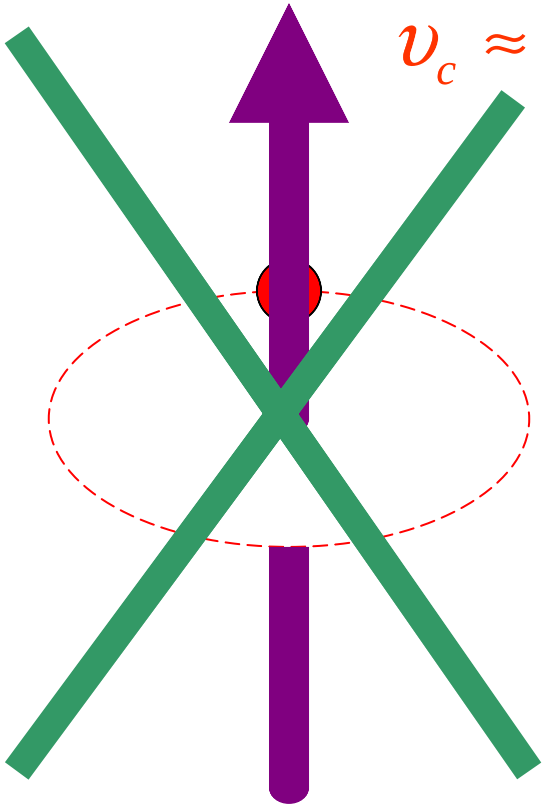
Challenge: An elementary particle has no internal states to probe or laser-cool

→ Give introduction to some of the new and novel methods

Basic Idea of the Measurement

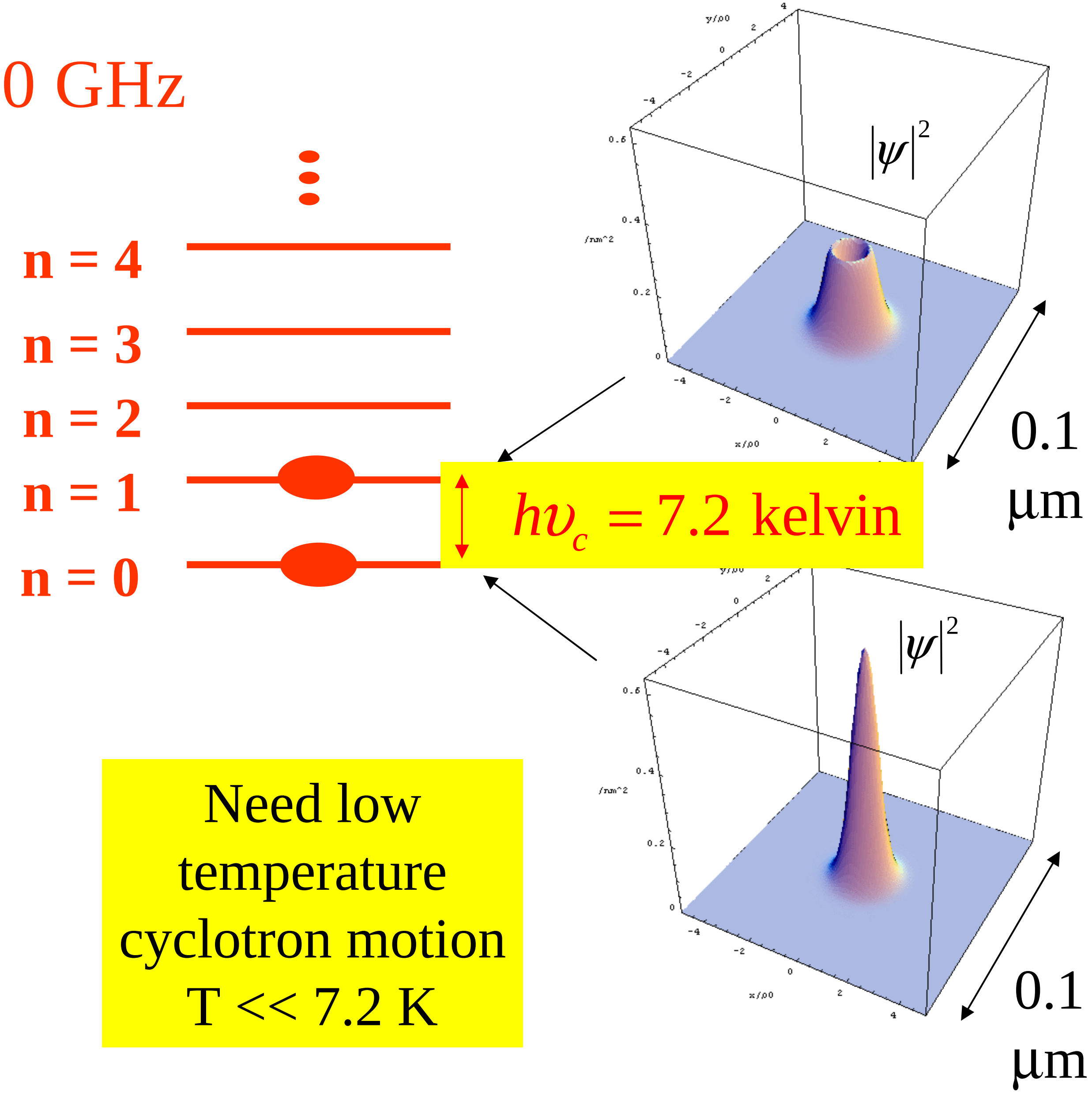
Quantum jump spectroscopy
of lowest cyclotron and spin levels
of an electron in a magnetic field

One Electron in a Magnetic Field



$\nu_c \approx 150$ GHz

B $\approx$ 6 Tesla



n = 4

n = 3

n = 2

n = 1

n = 0

$h\nu_c = 7.2$ kelvin

0.1 µm

0.1 µm

Need low temperature cyclotron motion

T << 7.2 K

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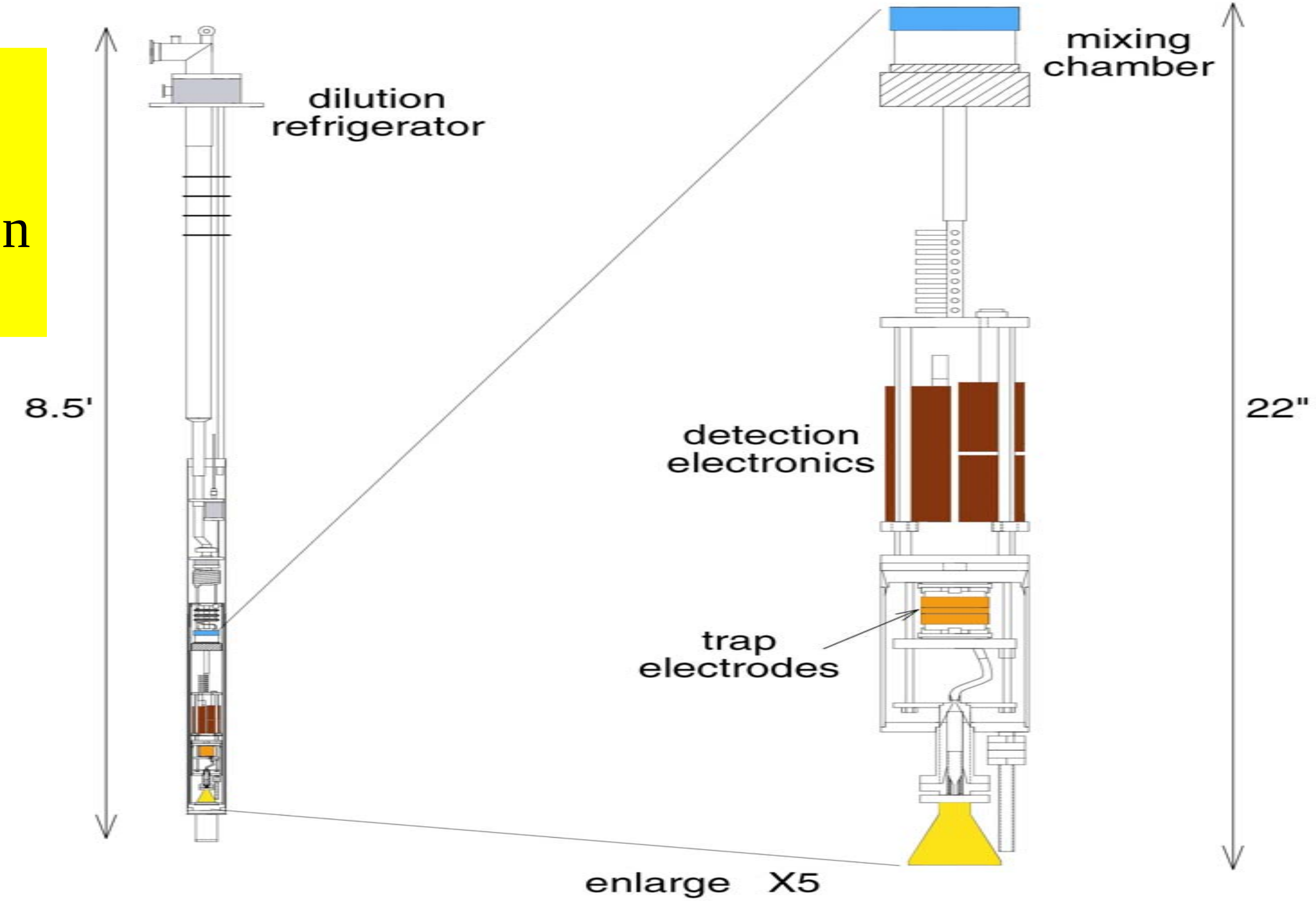
First Penning Trap Below 4 K → 70 mK

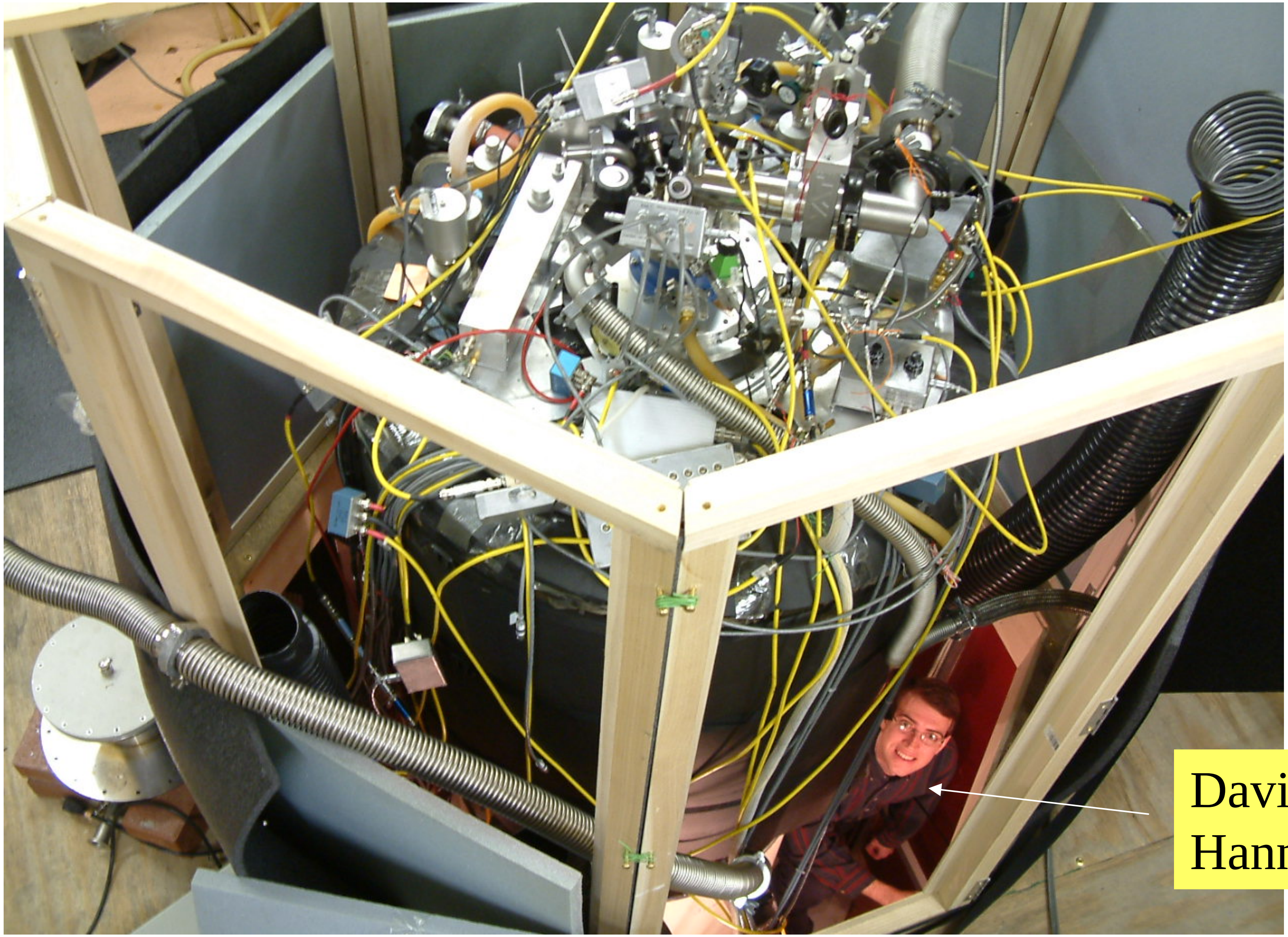
Need low temperature cyclotron motion

T << 7.2 K

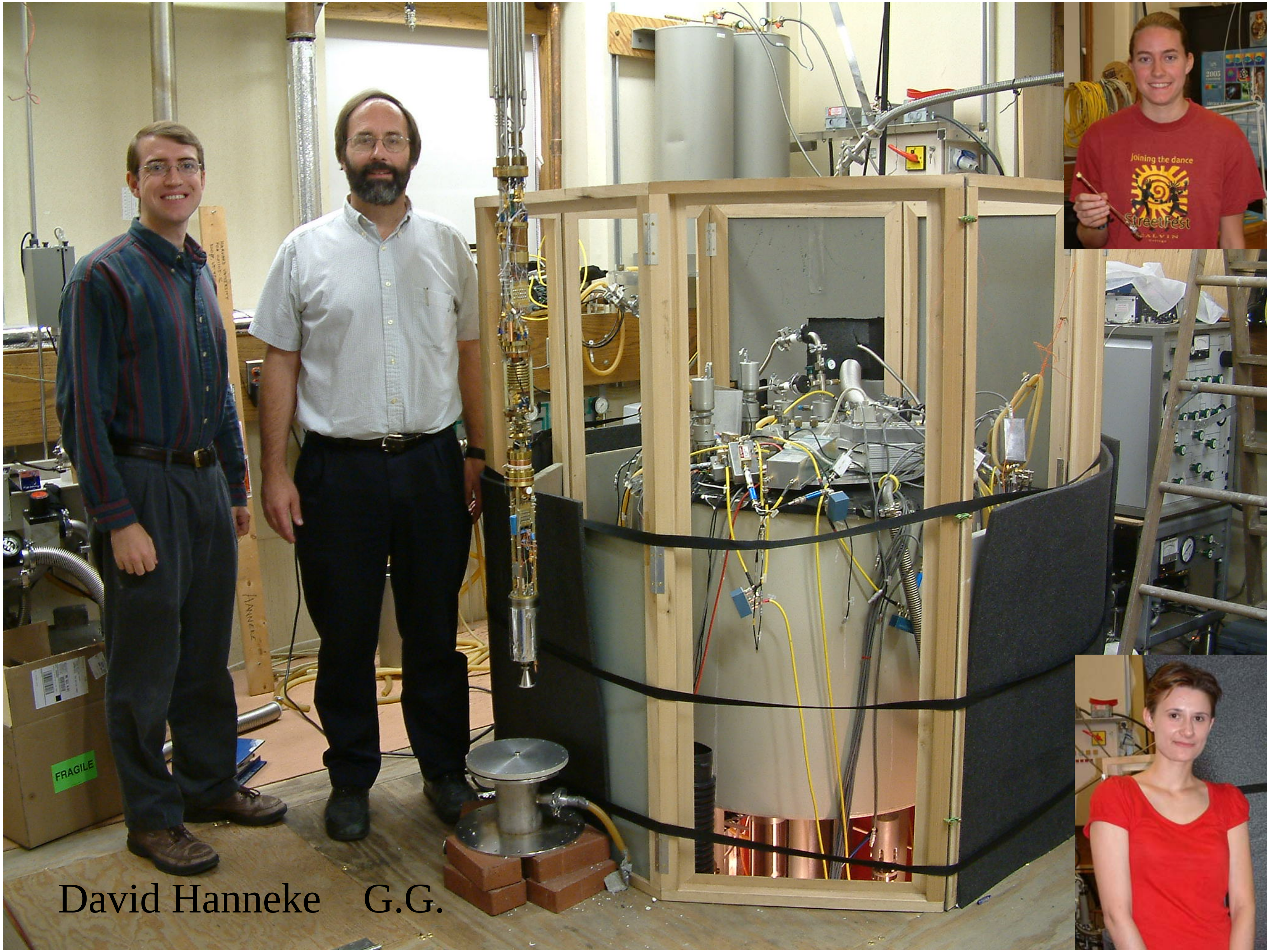


A Tabletop Experiment …

if you have a high ceiling

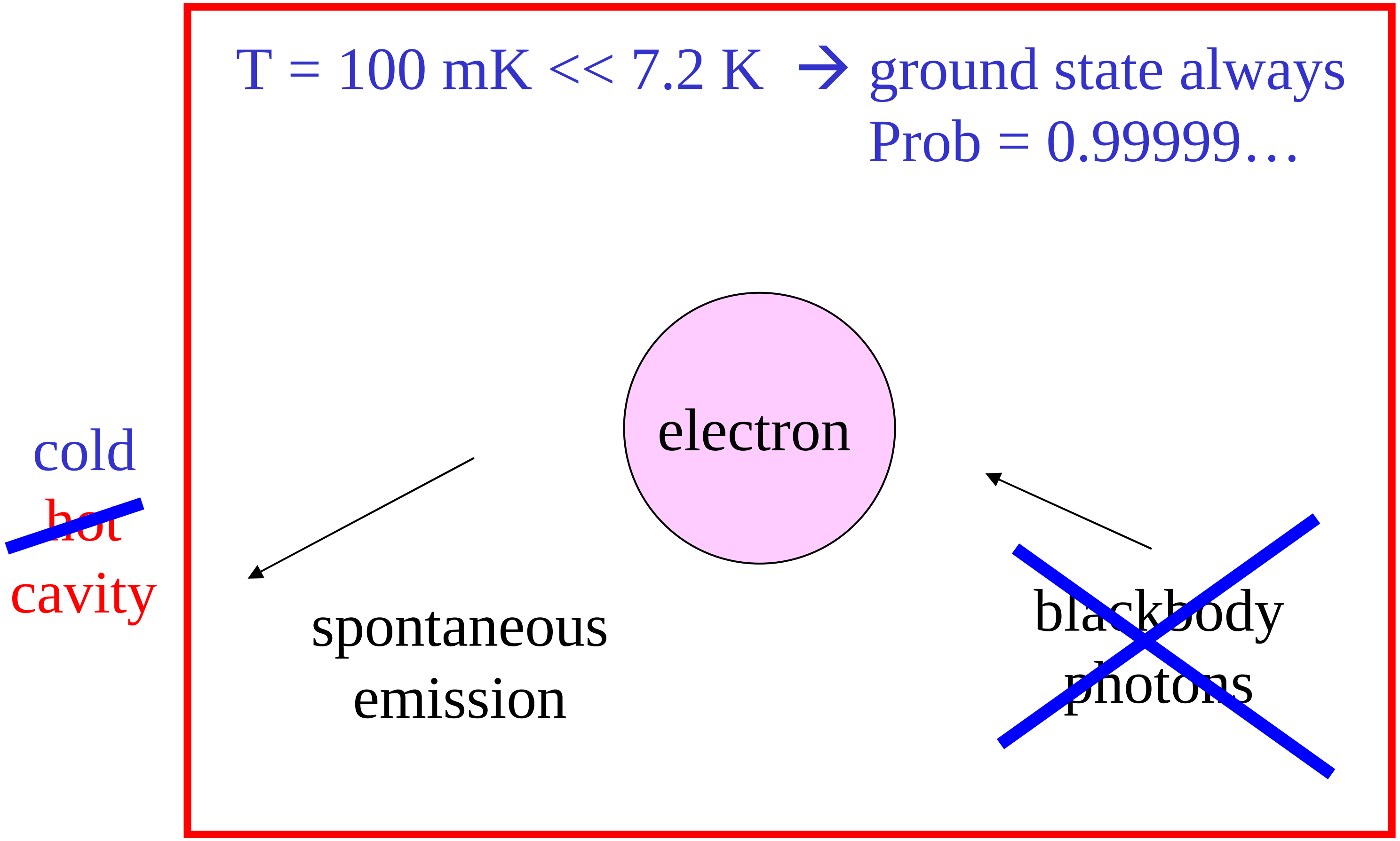


David Hanneke



David Hanneke G.G.

Electron Cyclotron Motion Comes Into Thermal Equilibrium



T = 100 mK << 7.2 K → ground state always

Prob = 0.99999…

cold

~~hot~~

cavity

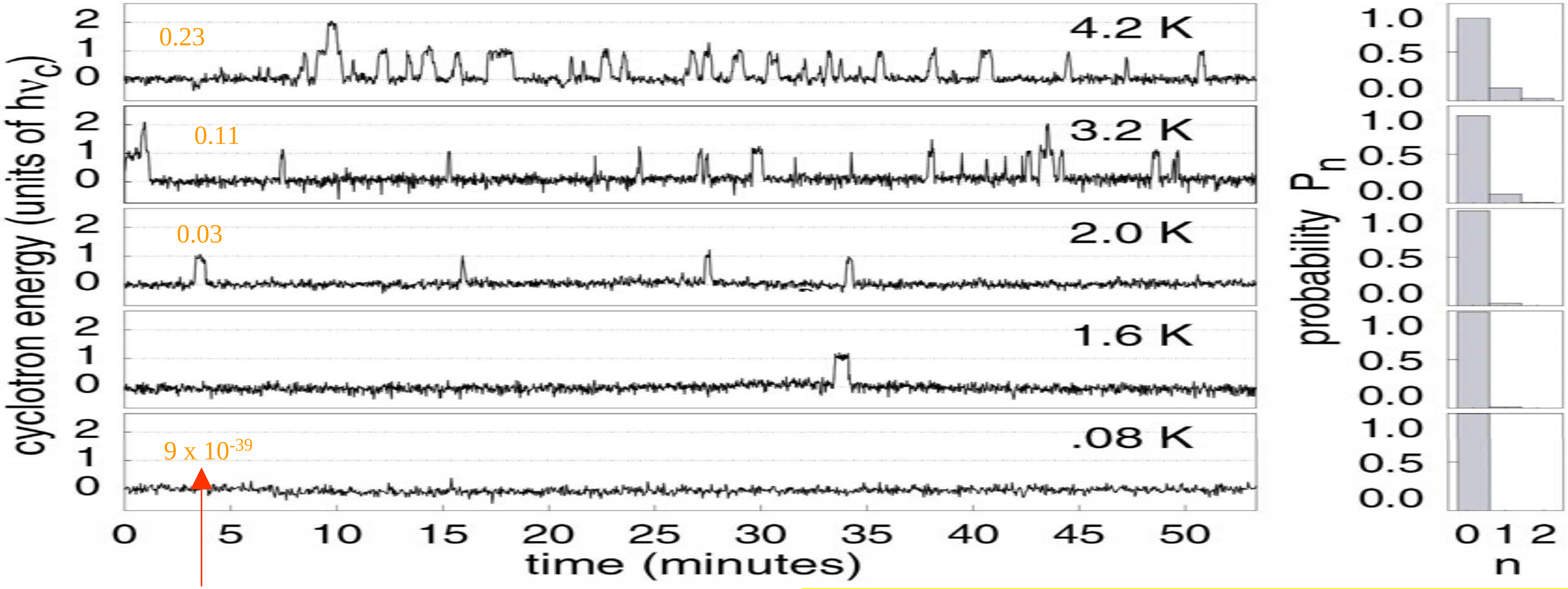
electron

spontaneous emission

~~blackbody photons~~

Electron in Cyclotron Ground State

QND Measurement of Cyclotron Energy vs. Time



average number of blackbody photons in the cavity

On a short time scale

→ in one Fock state or another

Averaged over hours

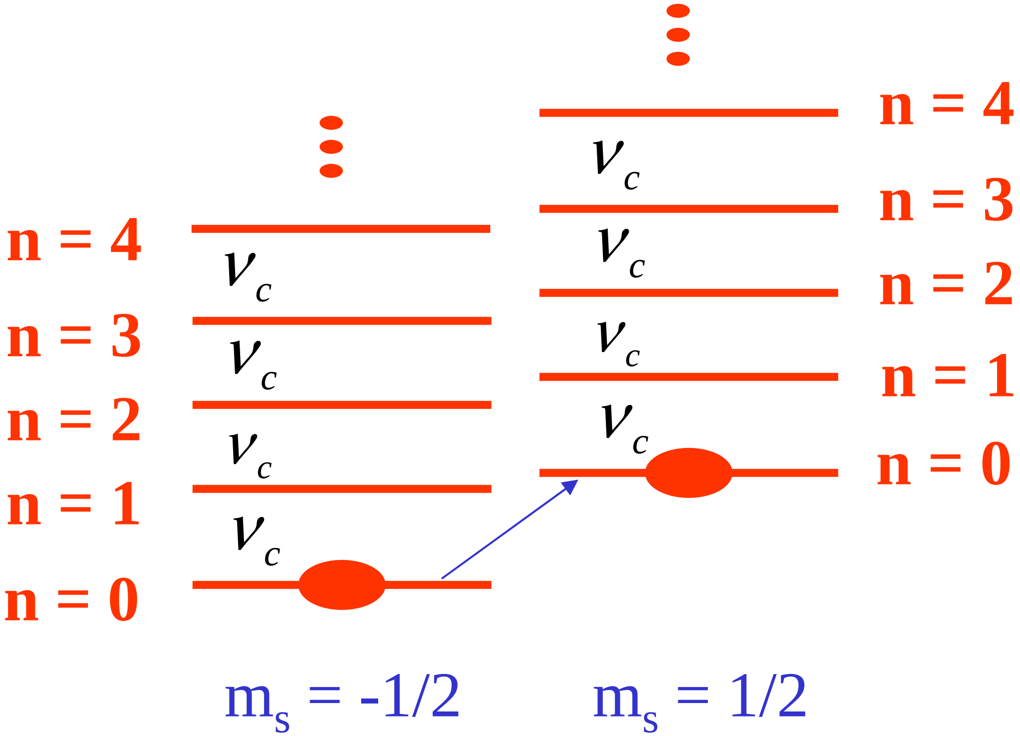
→ in a thermal state

S. Peil and G. Gabrielse, Phys. Rev. Lett. **83**, 1287 (1999).

Spin → Two Cyclotron Ladders of Energy Levels

Cyclotron frequency:

$$\nu_c = \frac{1}{2\pi}\frac{eB}{m}$$



n = 4

n = 3

n = 2

n = 1

n = 0

ν_c

ν_c

ν_c

ν_c

$m_s = -1/2$

n = 4

n = 3

n = 2

n = 1

n = 0

ν_c

ν_c

ν_c

ν_c

$m_s = 1/2$

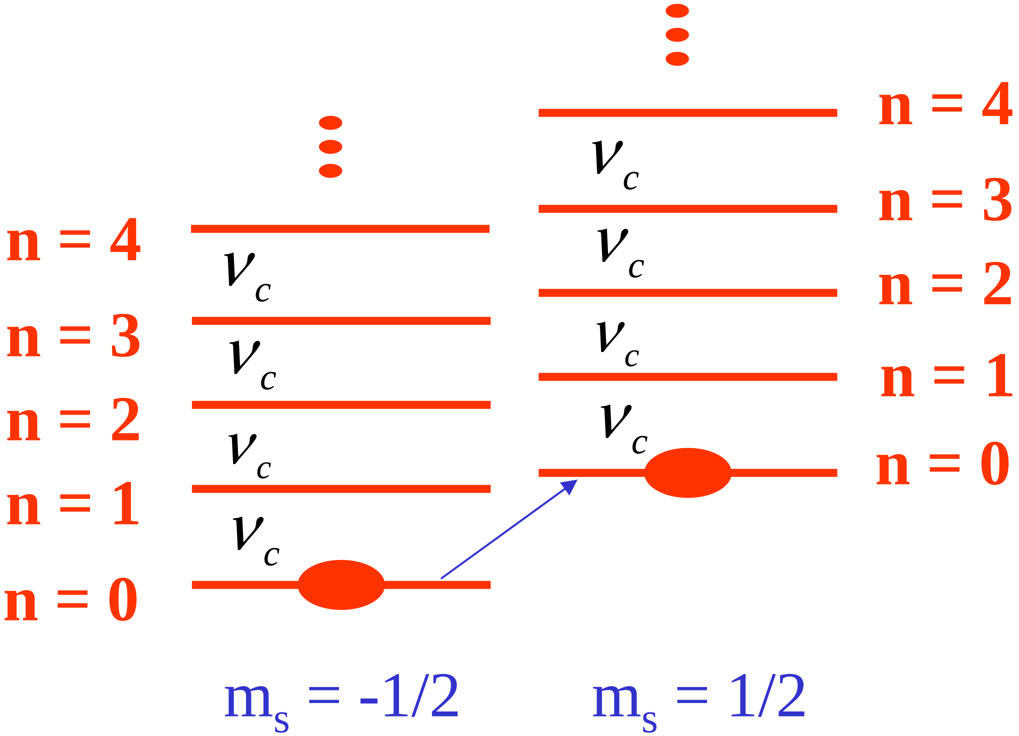
Spin frequency:

$$\nu_s = \frac{g}{2}\nu_c$$

Basic Idea of the Fully-Quantum Measurement

Cyclotron frequency:

$$\nu_c = \frac{1}{2\pi}\frac{eB}{m}$$



Spin frequency:

$$\nu_s = \frac{g}{2}\nu_c$$

Measure a ratio of frequencies: $\frac{g}{2} = \frac{\nu_s}{\nu_c} = 1 + \frac{\nu_s - \nu_c}{\nu_c}$

□ 10^{-3}

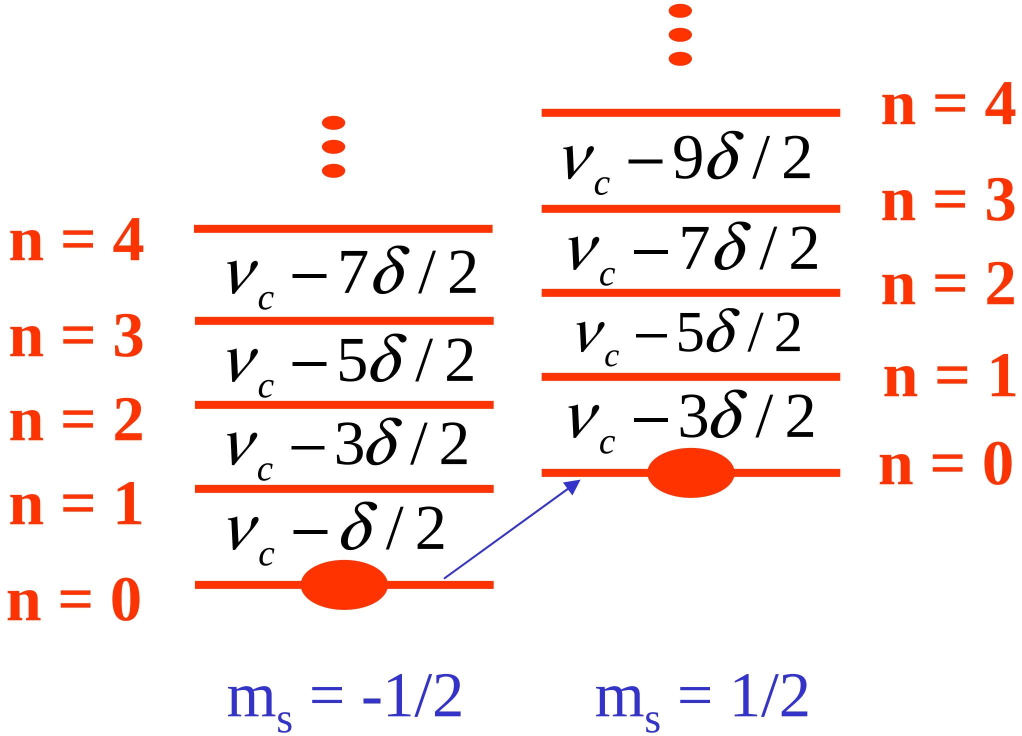
B in free space

- almost nothing can be measured better than a frequency
- the magnetic field cancels out (self-magnetometer)

Special Relativity Shift the Energy Levels δ

Cyclotron frequency:

$$2\pi\nu_c = \frac{eB}{m}$$



n = 4

n = 3

n = 2

n = 1

n = 0

$\nu_c - 7\delta/2$

$\nu_c - 5\delta/2$

$\nu_c - 3\delta/2$

$\nu_c - \delta/2$

$\nu_c - 9\delta/2$

$\nu_c - 7\delta/2$

$\nu_c - 5\delta/2$

$\nu_c - 3\delta/2$

n = 4

n = 3

n = 2

n = 1

n = 0

m_s = -1/2

m_s = 1/2

Spin frequency:

$$\nu_s = \frac{g}{2}\nu_c$$

Not a huge relativistic shift,
but important at our accuracy

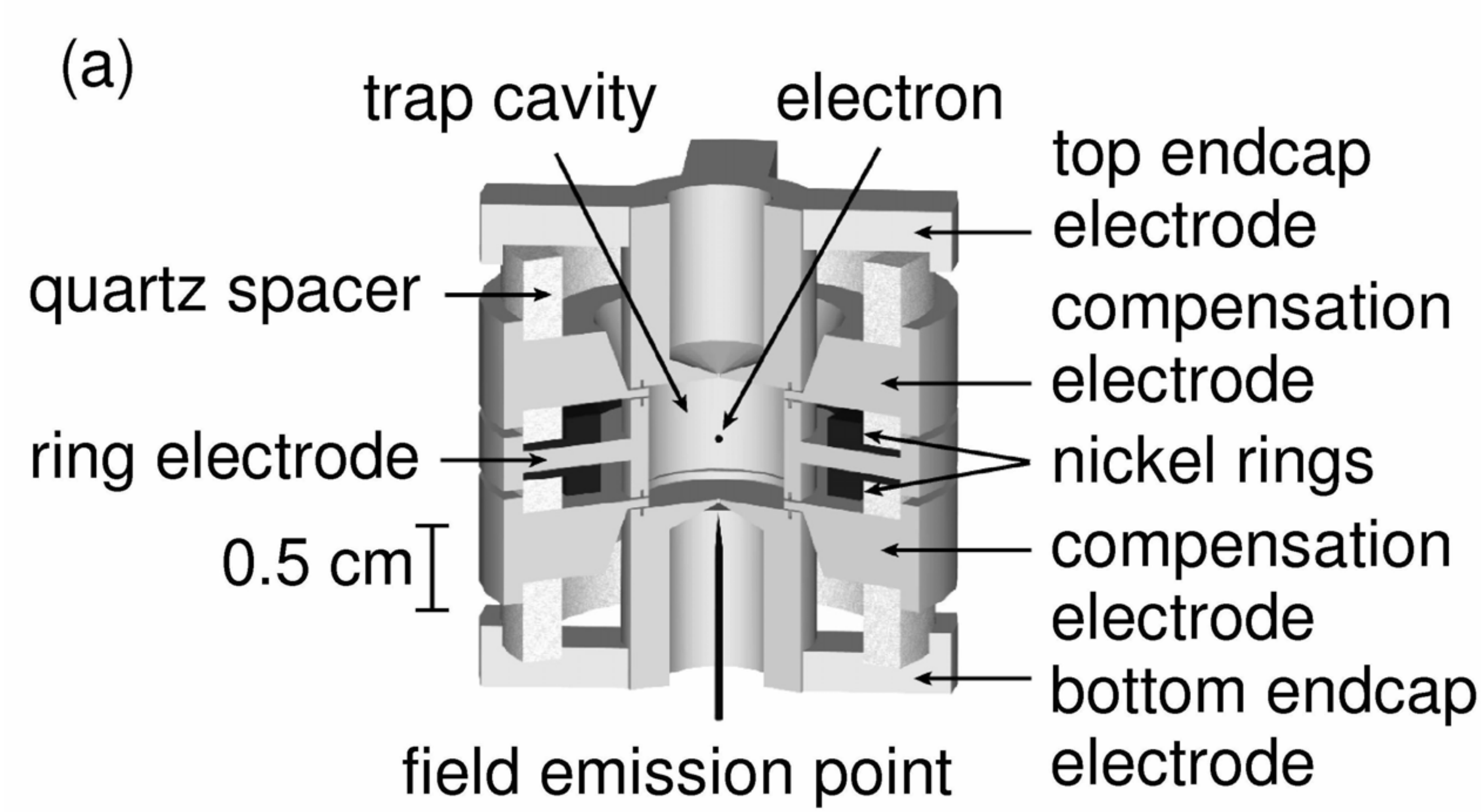
$$\frac{\delta}{\nu_c} = \frac{h\nu_c}{mc^2} \approx 10^{-9}$$

Solution: Simply correct for δ if we fully resolve the levels

(superposition of cyclotron levels would be a big problem)

Cylindrical Penning Trap

$$V \propto 2z^2 - x^2 - y^2$$



- Electrostatic quadrupole potential → good near trap center
- Control the radiation field → inhibit spontaneous emission by 200x

(Invented for this purpose: G.G. and F. C. MacKintosh; Int. J. Mass Spec. Ion Proc. ***57***, 1 (1984))

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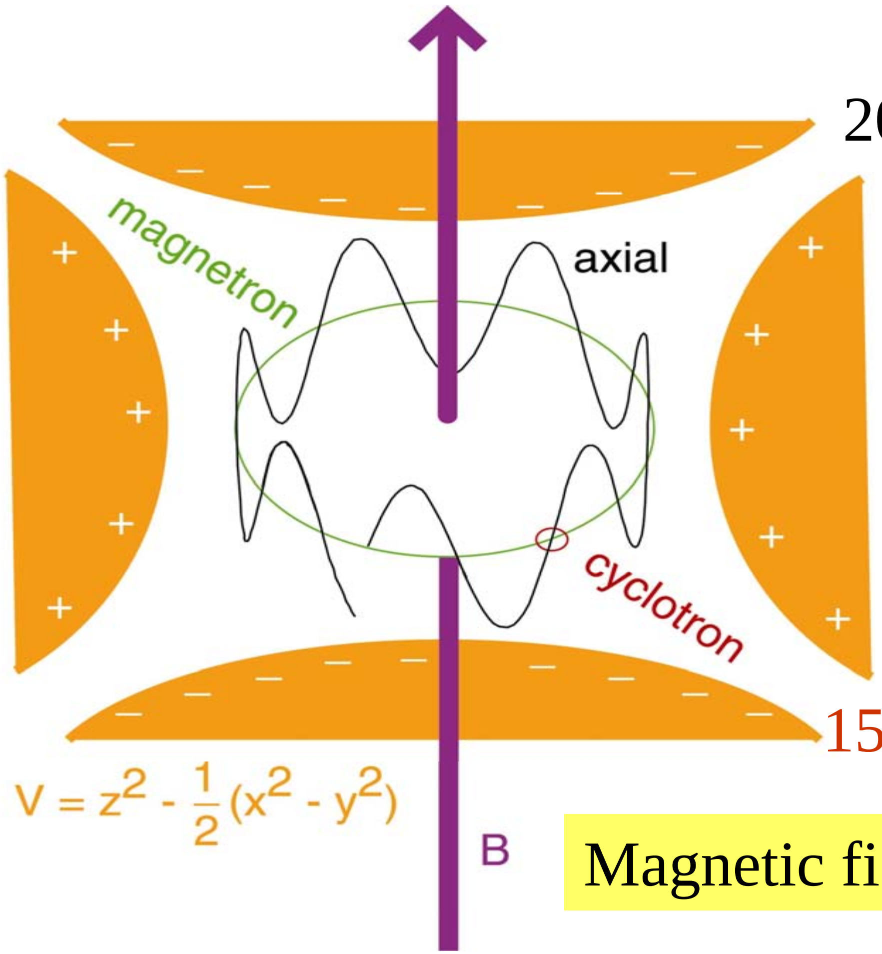
One Electron in a **Penning Trap**

- very small accelerator
- designer atom



cool 12 kHz

200 MHz detect

153 GHz

need to measure for g/2

Electrostatic quadrupole potential

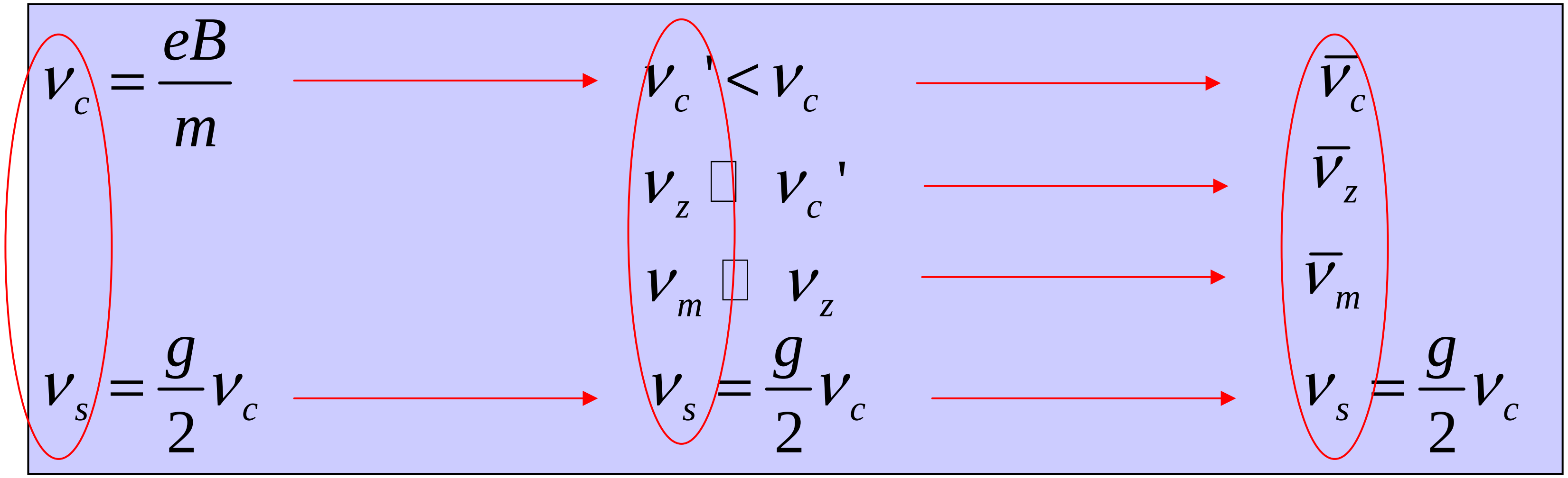
Magnetic field

Frequencies Shift

B in Free Space

Perfect Electrostatic Quadrupole Trap

Imperfect Trap
- **tilted B**
- **harmonic distortions to V**



$$\nu_c = \frac{eB}{m} \longrightarrow \nu_c' < \nu_c \longrightarrow \bar{\nu}_c$$

$$\nu_z \,\square\, \nu_c' \longrightarrow \bar{\nu}_z$$

$$\nu_m \,\square\, \nu_z \longrightarrow \bar{\nu}_m$$

$$\nu_s = \frac{g}{2}\nu_c \longrightarrow \nu_s = \frac{g}{2}\nu_c \longrightarrow \nu_s = \frac{g}{2}\nu_c$$

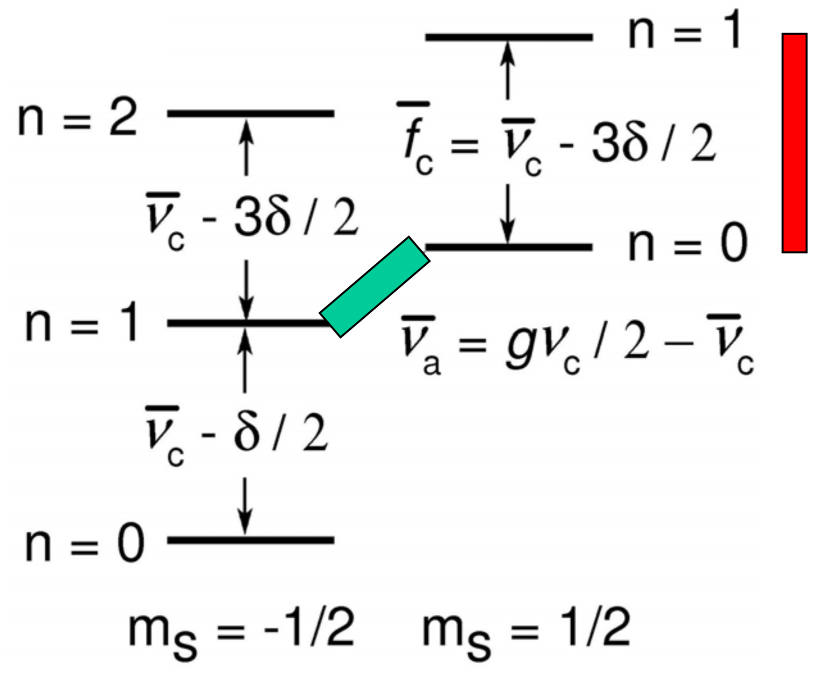
Problem: $\frac{g}{2} = \frac{\nu_s}{\nu_c}$ ← not a measurable eigenfrequency in an imperfect Penning trap

Solution: Brown-Gabrielse invariance theorem

$$\nu_c = \sqrt{(\bar{\nu}_c)^2 + (\bar{\nu}_z)^2 + (\bar{\nu}_m)^2}$$

Spectroscopy in an Imperfect Trap

- one electron in a Penning trap
- lowest cyclotron and spin states



$$\frac{g}{2} = \frac{\nu_s}{\nu_c} = \frac{\bar{\nu}_c + (\nu_s - \bar{\nu}_c)}{\nu_c} = \frac{\bar{\nu}_c + \bar{\nu}_a}{\nu_c}$$

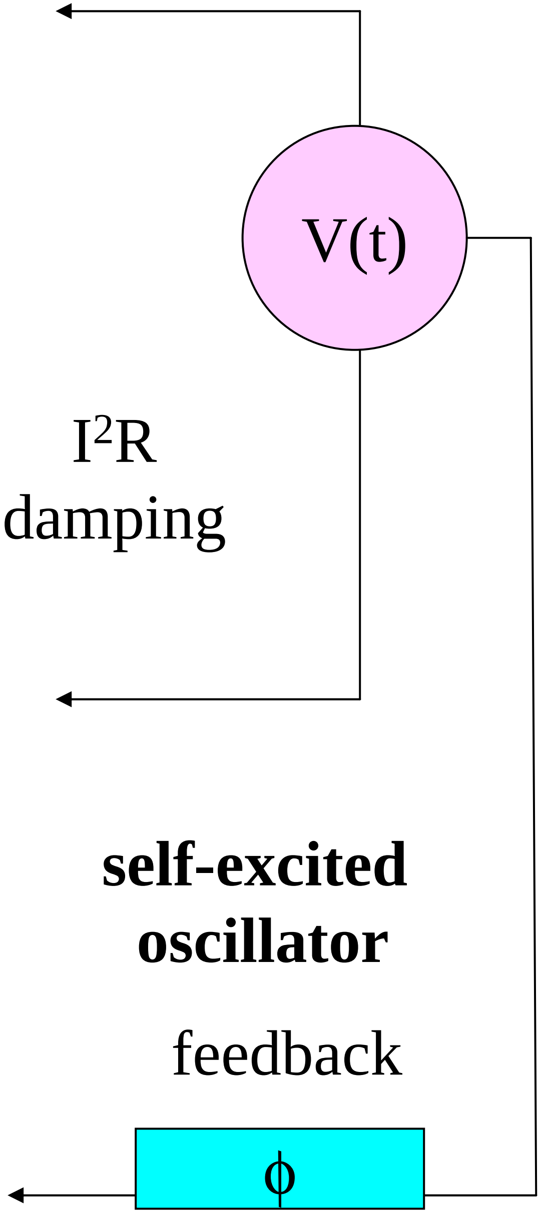
$$\frac{g}{2} \approx 1 + \frac{\bar{\nu}_a - \frac{(\bar{\nu}_z)^2}{2\bar{\nu}_c}}{\bar{f}_c + \frac{3\delta}{2} + \frac{(\bar{\nu}_z)^2}{2\bar{\nu}_c}}$$

expansion for $\bar{\nu}_c \gg \bar{\nu}_z \gg \bar{\nu}_m \gg \delta$

To deduce g → measure only three eigenfrequencies
of the imperfect trap

Detecting and Damping Axial Motion

Axial motion
200 MHz
of
trapped electron



measure voltage

V(t)

I^2R
damping

self-excited
oscillator

feedback

ϕ

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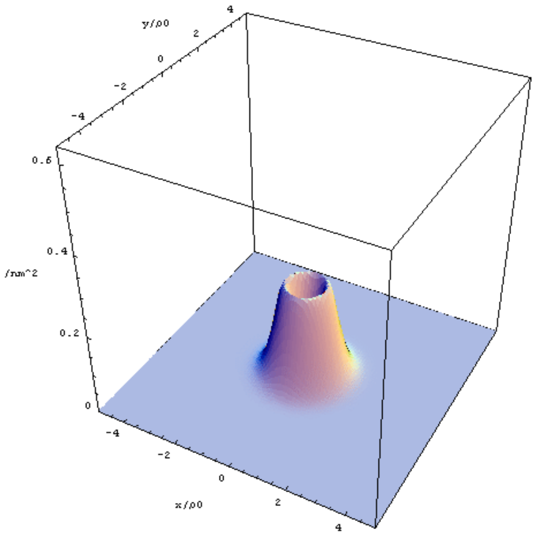
QND Detection of One-Quantum Transitions

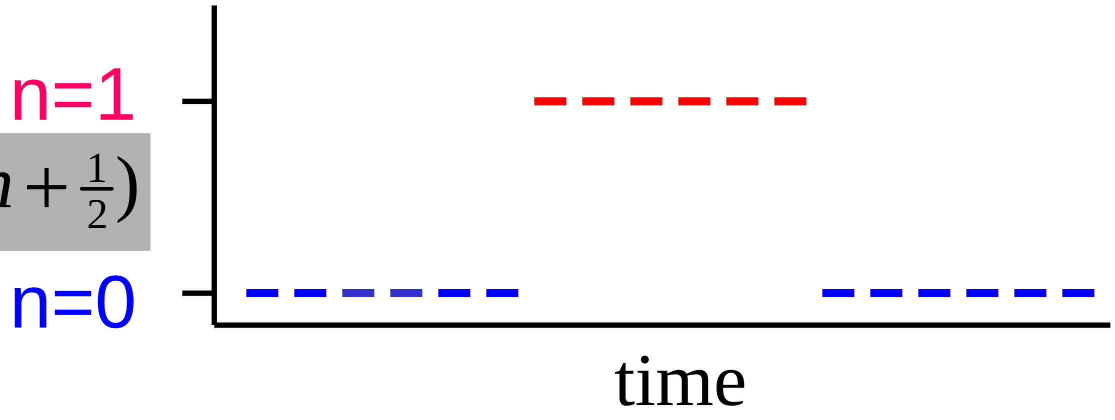
one-electron self-excited oscillator



$$\Delta\vec{B} \sim B_2\, z^2 \quad \rightarrow \quad H = \frac{1}{2} m\omega_z{}^2 z^2 - \mu B_2 z^2$$

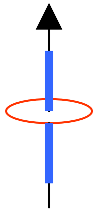
n=0 cyclotron ground state	n=1 cyclotron excited state	n=0 cyclotron ground state

$$freq \sim E_{cyclotron} = hf_c(n+\tfrac{1}{2})$$



QND

Quantum Non-demolition Measurement



B

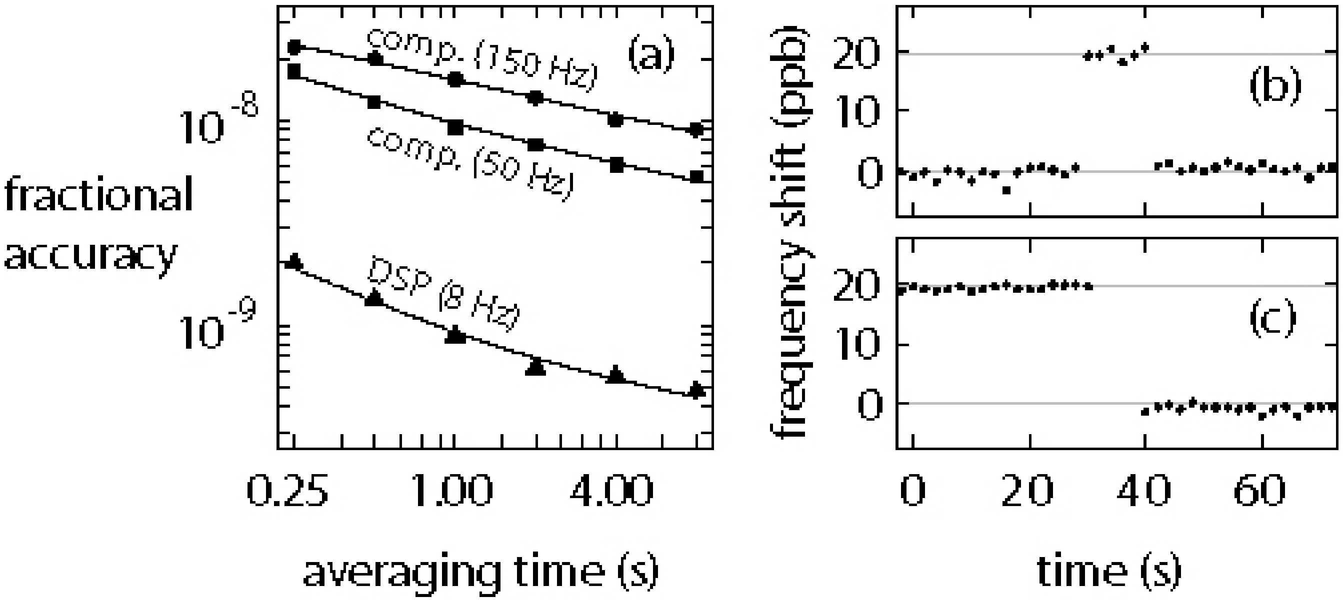
$$H = H_{cyclotron} + H_{axial} + H_{coupling}$$

$$[\ H_{cyclotron}, H_{coupling}\ ] = 0$$

QND condition

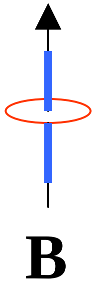
QND: **Subsequent time evolution of cyclotron motion is not altered by additional QND measurements**

Observe Tiny Shifts of the Frequency of a One-Electron Self-Excited Oscillator



one quantum
cyclotron
excitation

spin flip



Unmistakable changes in the axial frequency signal one quantum changes in cyclotron excitation and spin

"Single-Particle Self-excited Oscillator"
B. D'Urso, R. Van Handel, B. Odom and G. Gabrielse
Phys. Rev. Lett. **94**, 113002 (2005).

Emboldened by the Great Signal-to-Noise

Make a one proton (antiproton) self-excited oscillator

→ try to detect a proton (and antiproton) spin flip

- Hard: nuclear magneton is 500 times smaller
- Experiment underway → Harvard

→ also Mainz and GSI (without SEO)
(build upon bound electron g values)

→ measure proton spin frequency

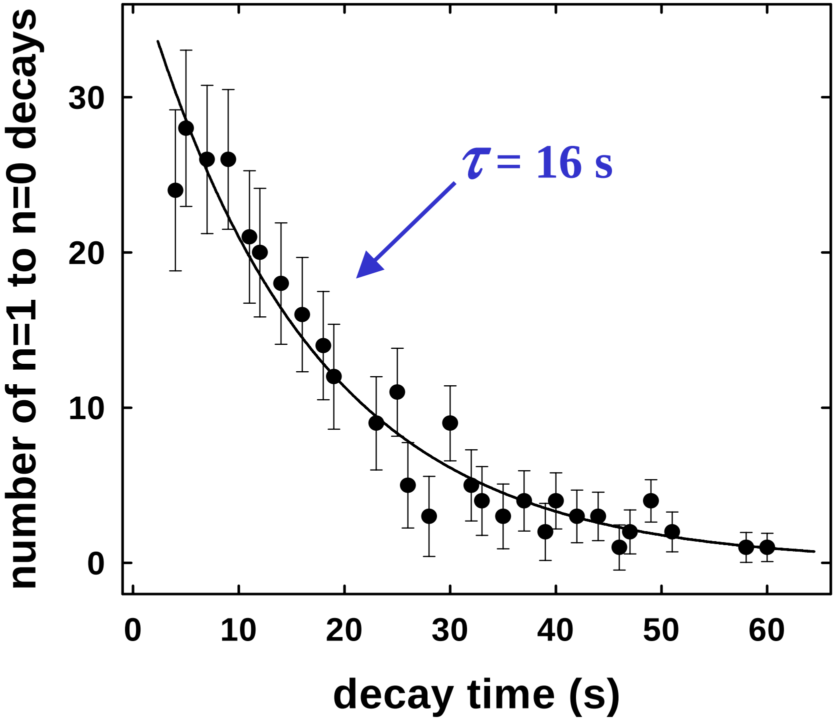
→ we already accurately measure antiproton cyclotron frequencies

→ get antiproton g value (Improve by factor of a million or more)

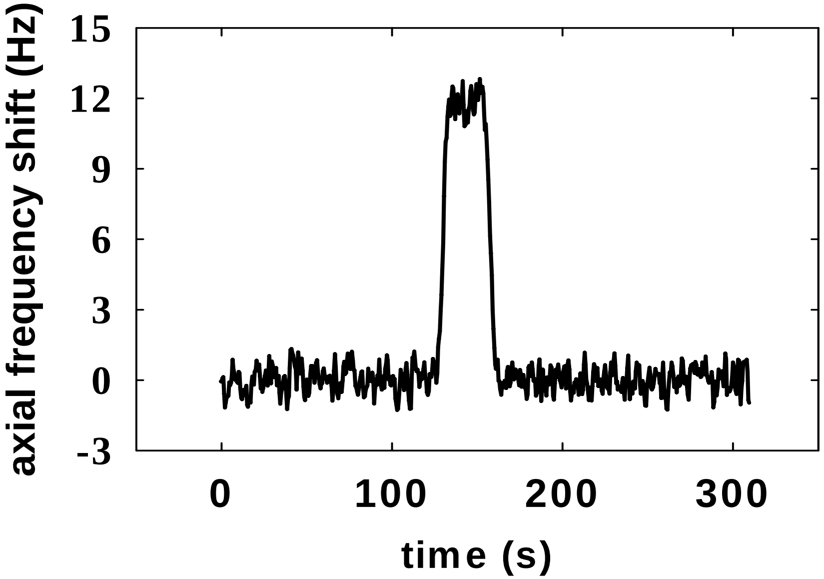
Need Averaging Time to Observe a One-quantum Transition

→ Cavity-Inhibited Spontaneous Emission

Application of Cavity QED

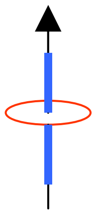


excite,
measure time in excited state



Cavity-Inhibited Spontaneous Emission

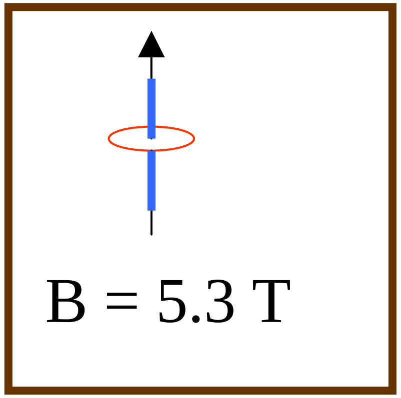
Free Space



B = 5.3 T

$$\gamma = \frac{1}{75\ ms}$$

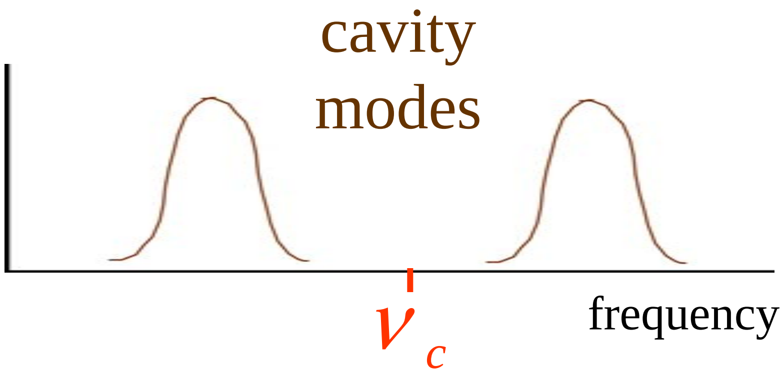
Within
Trap Cavity



B = 5.3 T

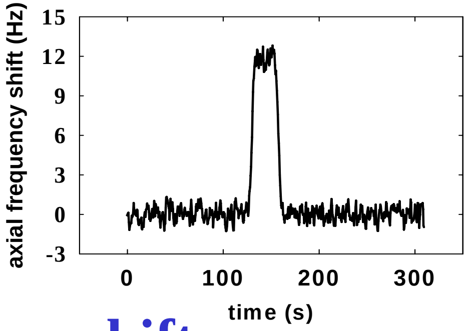
$$\gamma = \frac{1}{16 \text{ sec}}$$

Inhibited
By 210!

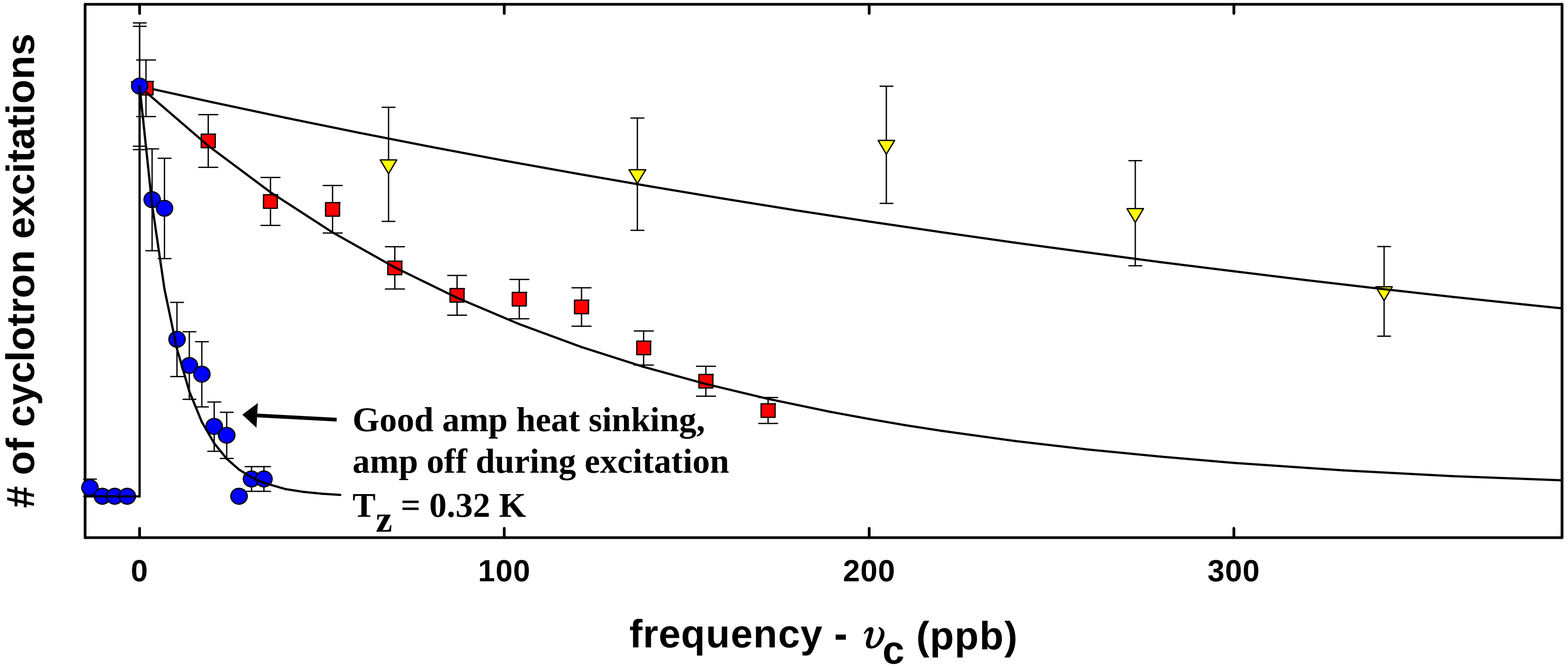


Purcell
Kleppner
Gabrielse and Dehmelt

“In the Dark” Excitation → Narrower Lines

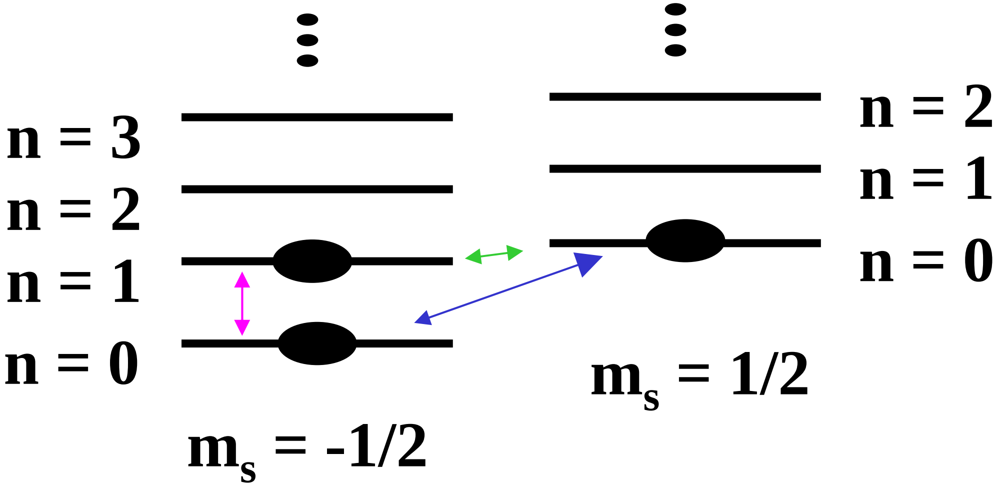


1. Turn FET amplifier off
2. Apply a microwave drive pulse of ~150 GH
   (i.e. measure “in the dark”)
3. Turn FET amplifier on and check for axial frequency shift
4. Plot a histograms of excitations vs. frequency



Good amp heat sinking,
amp off during excitation
$T_z = 0.32$ K

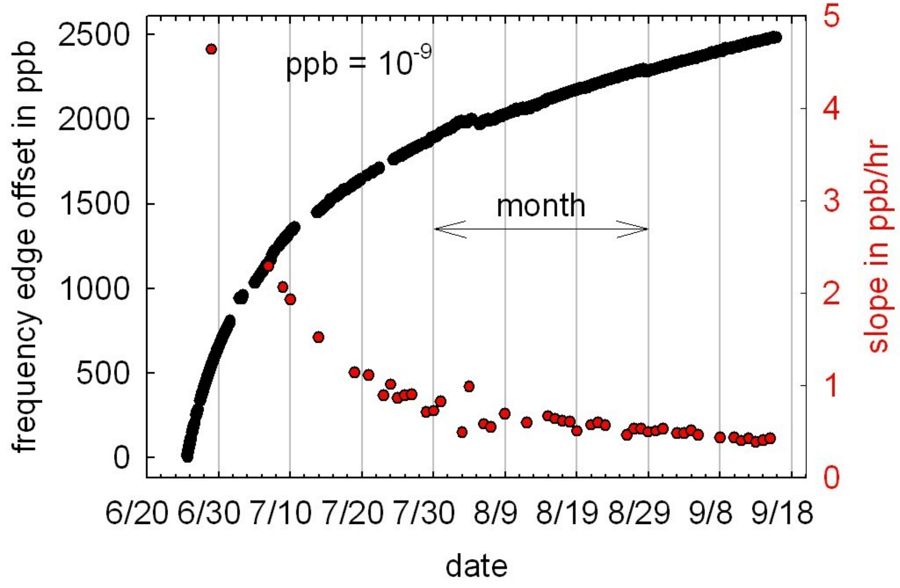
Big Challenge: Magnetic Field Stability



Magnetic field cancels out

$$\frac{g}{2}=\frac{\omega_s}{\omega_c}=1+\frac{\omega_a}{\omega_c}$$

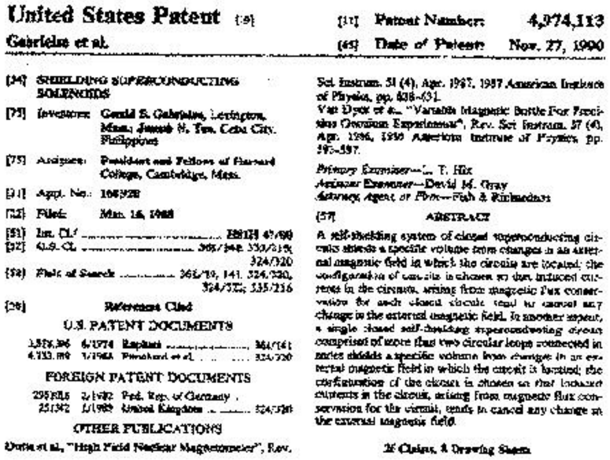
But: problem when B drifts during the measurement



Magnetic field take ~ month to stabilize

Self-Shielding Solenoid Helps a Lot

Flux conservation → Field conservation
Reduces field fluctuations by about a factor > 150



United States Patent [19]
Gabrielse et al.

[11] **Patent Number:** **4,974,113**
[45] **Date of Patent:** **Nov. 27, 1990**

[54] SHIELDING SUPERCONDUCTING SOLENOIDS

[75] Inventors: Gerald S. Gabrielse, Lexington, Mass.; Joseph N. Tan, Cebu City, Philippines

[73] Assignee: President and Fellows of Harvard College, Cambridge, Mass.

[21] Appl. No.: 168928

[22] Filed: Mar. 16, 1988

[51] Int. Cl.⁵ H01H 47/00
[52] U.S. Cl. 361/148; 335/216; 324/320
[58] Field of Search 361/19, 141; 324/320, 324/322; 335/216

[56] References Cited

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3,818,396 6/1974 Kaplan 361/141
4,733,189 3/1988 Punchard et al. 324/320

FOREIGN PATENT DOCUMENTS

2951815 2/1982 Fed. Rep. of Germany .
2183?? 1/1988 United Kingdom 324/320

OTHER PUBLICATIONS

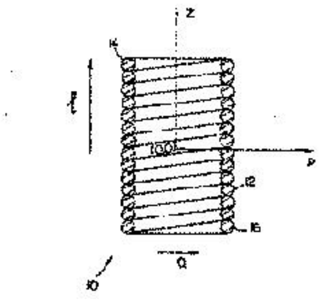
Dorri et al., "High Field Nuclear Magnetometer", Rev. Sci. Instrum. 58 (4), Apr. 1987, 1987 American Institute of Physics, pp. 628–631.
Van Dyck et al., "Variable Magnetic Bottle For Precision Geonium Experiments", Rev. Sci. Instrum. 57 (4), Apr. 1986, 1986 American Institute of Physics, pp. 593–597.

Primary Examiner—L. T. Hix
Assistant Examiner—David M. Gray
Attorney, Agent, or Firm—Fish & Richardson

[57] ABSTRACT

A self-shielding system of closed superconducting circuits shields a specific volume from changes in an external magnetic field in which the circuits are located; the configuration of circuits is chosen so that induced currents in the circuits, arising from magnetic flux conservation for each closed circuit, tend to cancel any change in the external magnetic field. In another aspect, a single closed self-shielding superconducting circuit comprised of more than two circular loops connected in series shields a specific volume from changes in an external magnetic field in which the circuit is located; the configuration of the circuit is chosen so that induced currents in the circuit, arising from magnetic flux conservation for the circuit, tends to cancel any change in the external magnetic field.

26 Claims, 8 Drawing Sheets



U.S. Patent Nov. 27, 1990 Sheet 6 of 8 **4,974,113**

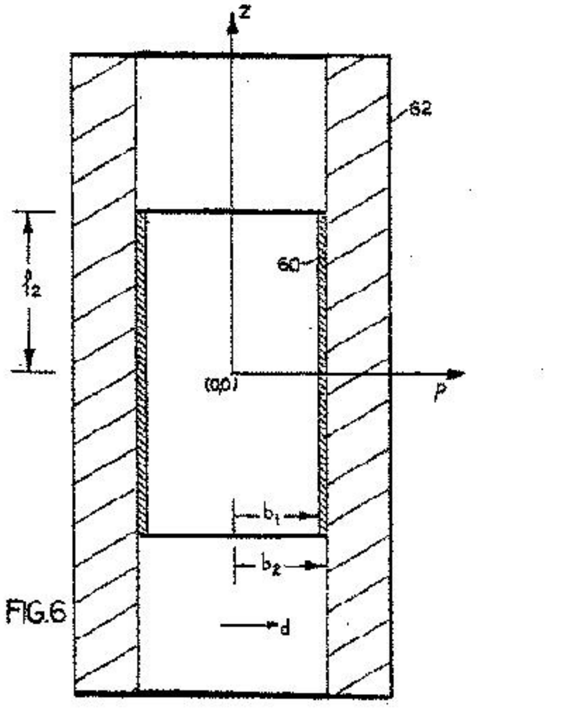


FIG.6

"Self-shielding Superconducting Solenoid Systems",
G. Gabrielse and J. Tan, J. Appl. Phys. **63**, 5143 (1988)

Gabrielse

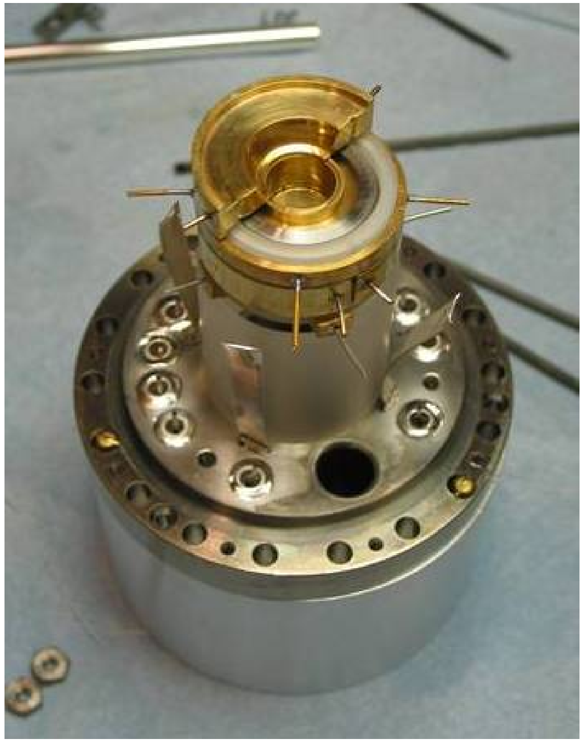
Eliminate Nuclear Paramagnetism

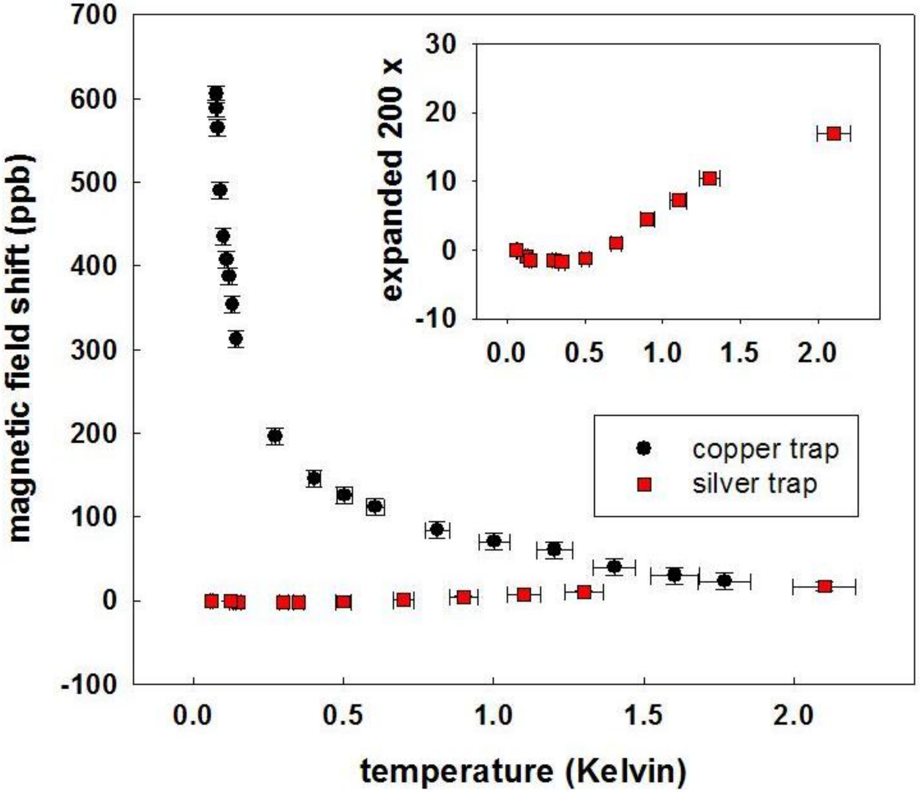
Deadly nuclear magnetism of copper and other “friendly” materials

→ Had to build new trap out of silver

→ New vacuum enclosure out of titanium

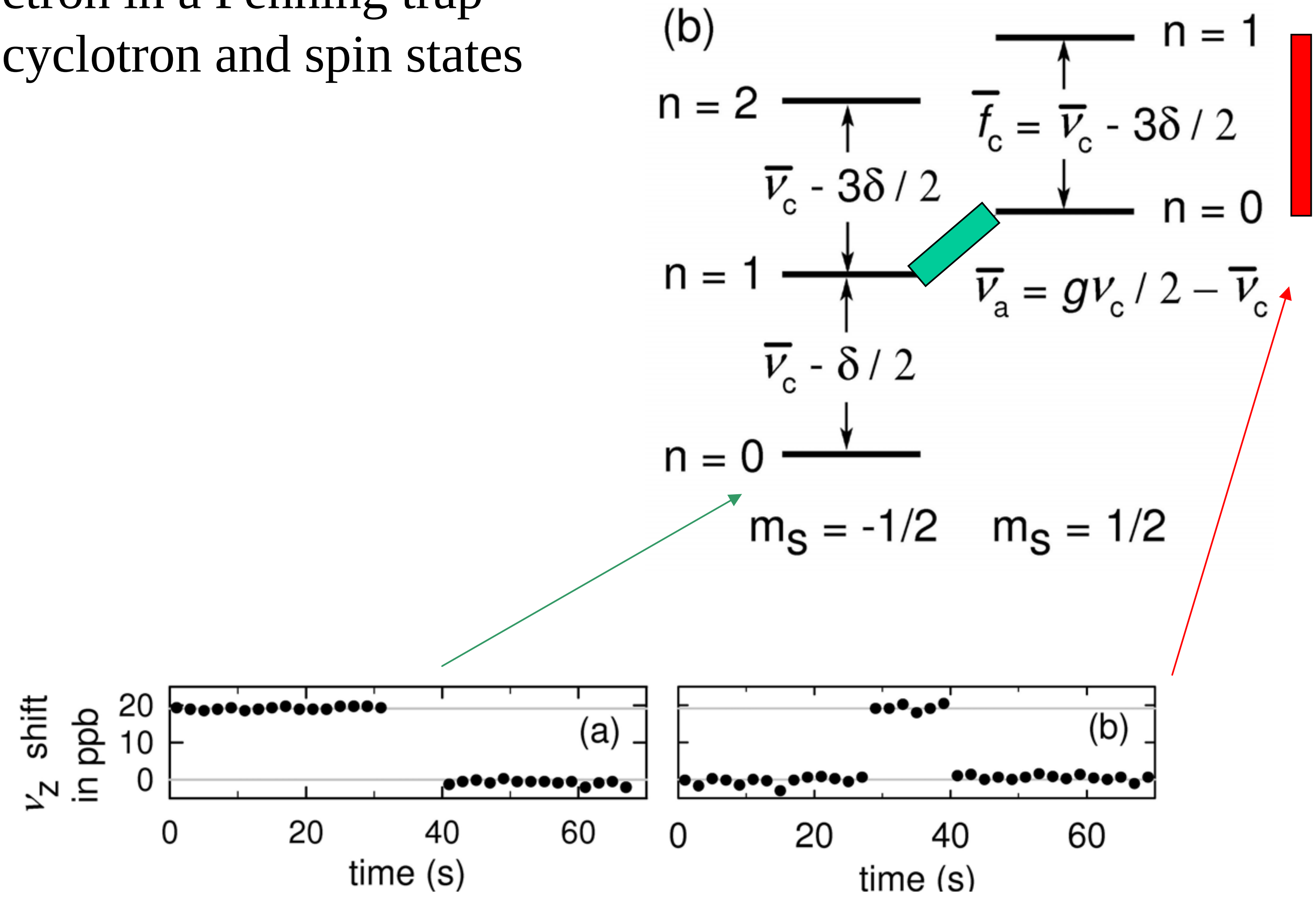
~ 1 year setback





Quantum Jump Spectroscopy

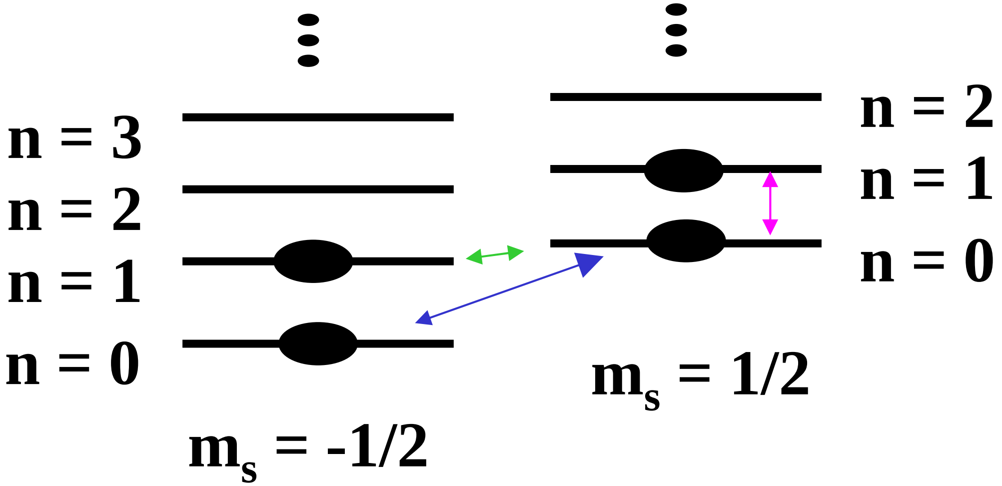
- one electron in a Penning trap
- lowest cyclotron and spin states



Measurement Cycle

$$\frac{g}{2}=\frac{\omega_s}{\omega_c}=1+\frac{\omega_a}{\omega_c}$$

simplified



3 hours

1. Prepare n=0, m=1/2 → measure anomaly transition
2. Prepare n=0, m=1/2 → measure cyclotron transition

0.75 hour

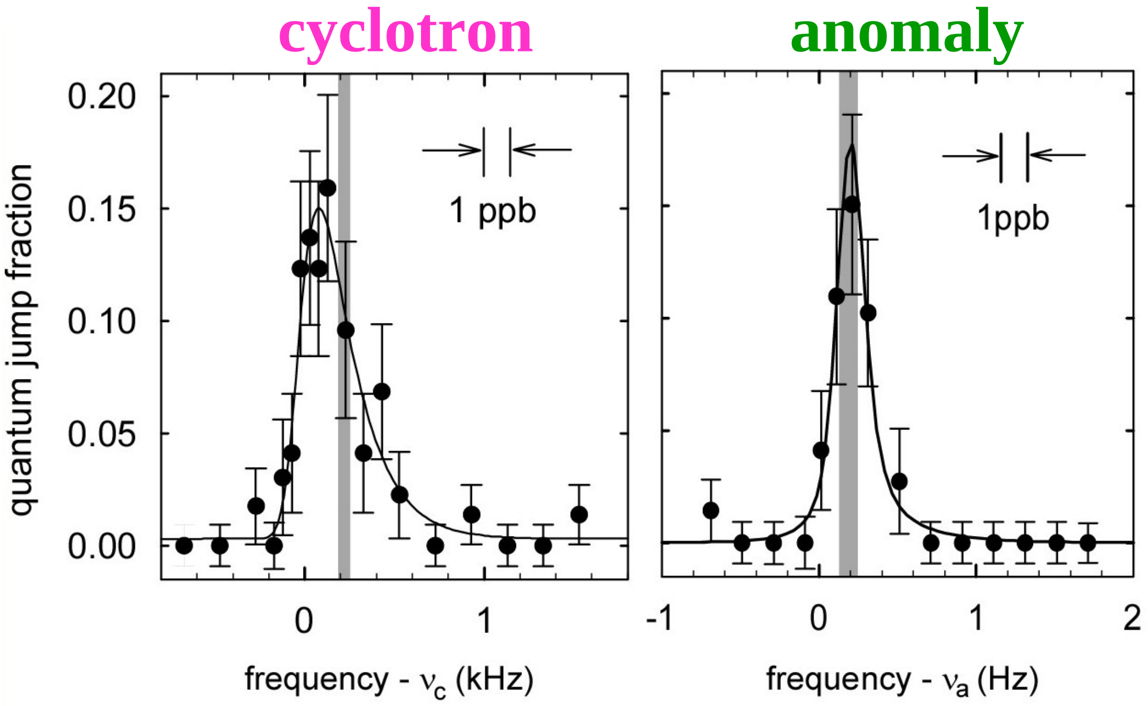
3. Measure relative magnetic field

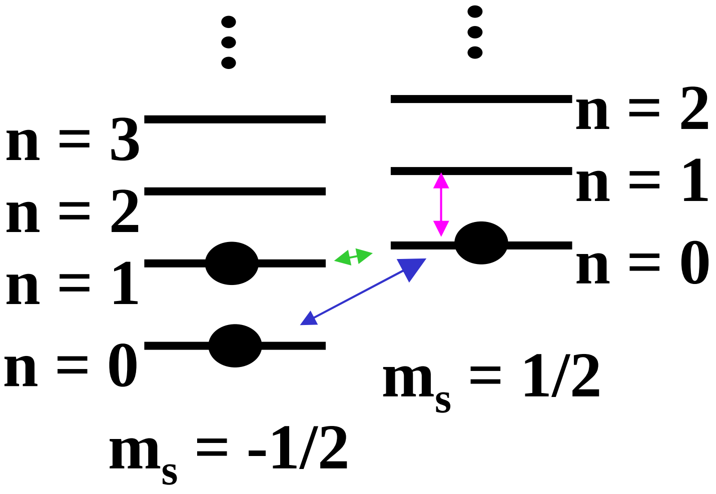
Repeat during magnetically quiet times

Measured Line Shapes for g-value Measurement

It all comes together:

- Low temperature, and high frequency make narrow line shapes
- A highly stable field allows us to map these lines



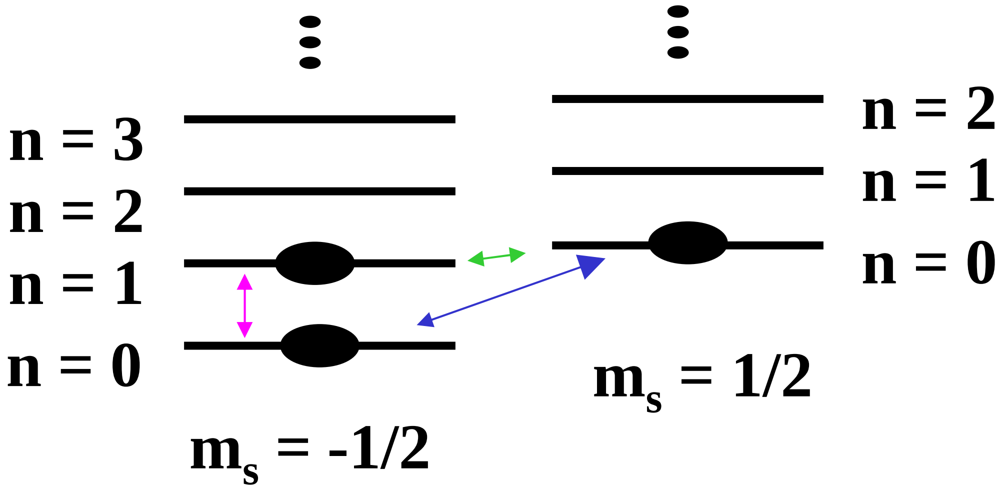


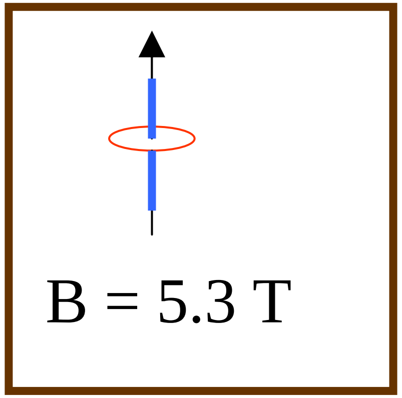
Precision:

Sub-ppb line splitting (i.e. sub-ppb precision of a *g*-2 measurement) is now “easy” after years of work

Cavity Shifts of the Cyclotron Frequency

$$\frac{g}{2} = \frac{\omega_s}{\omega_c} = 1 - \frac{\omega_a}{\omega_c}$$

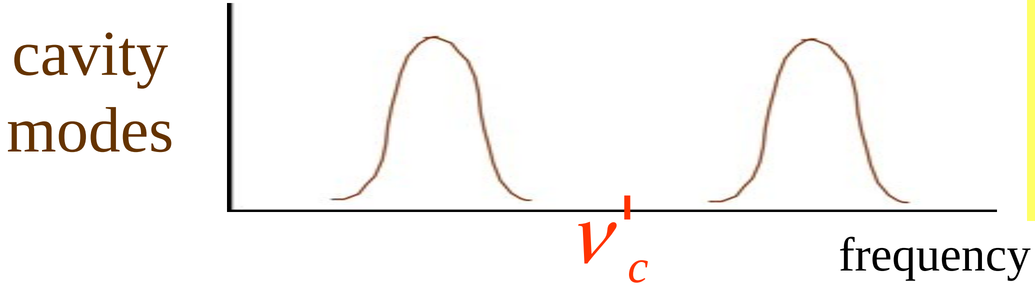




$$\gamma = \frac{1}{16\ \text{sec}}$$

spontaneous emission inhibited by 210

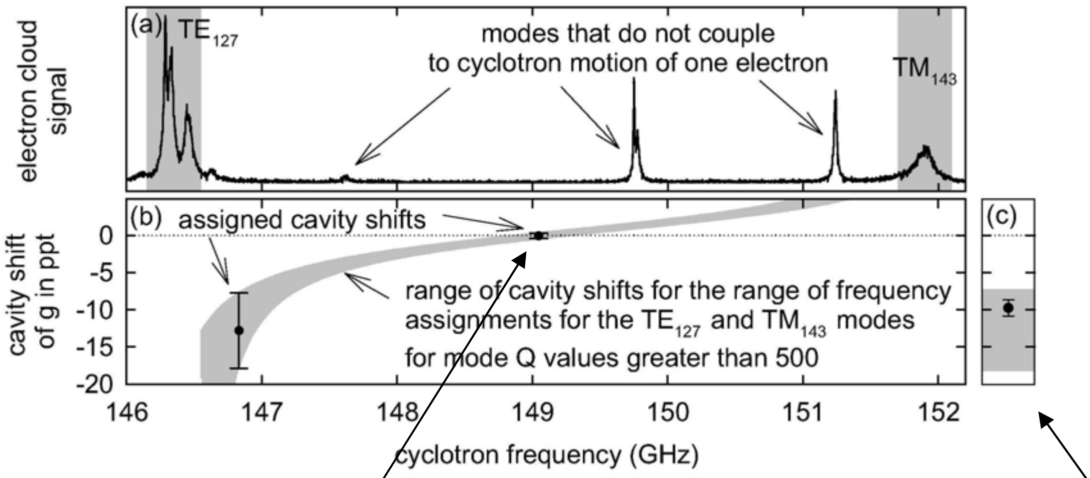
Within a Trap Cavity

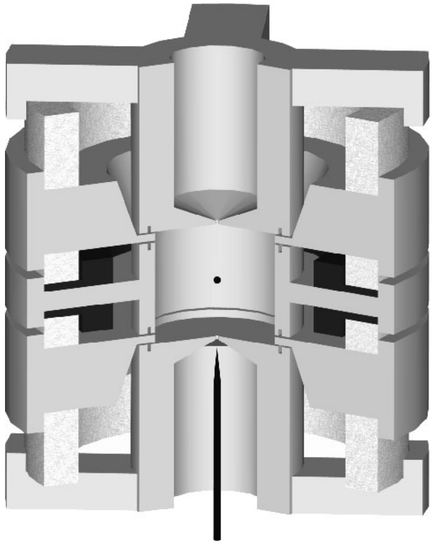


cyclotron frequency is shifted by interaction with cavity modes

Cavity modes and Magnetic Moment Error

use synchronization of electrons to get cavity modes





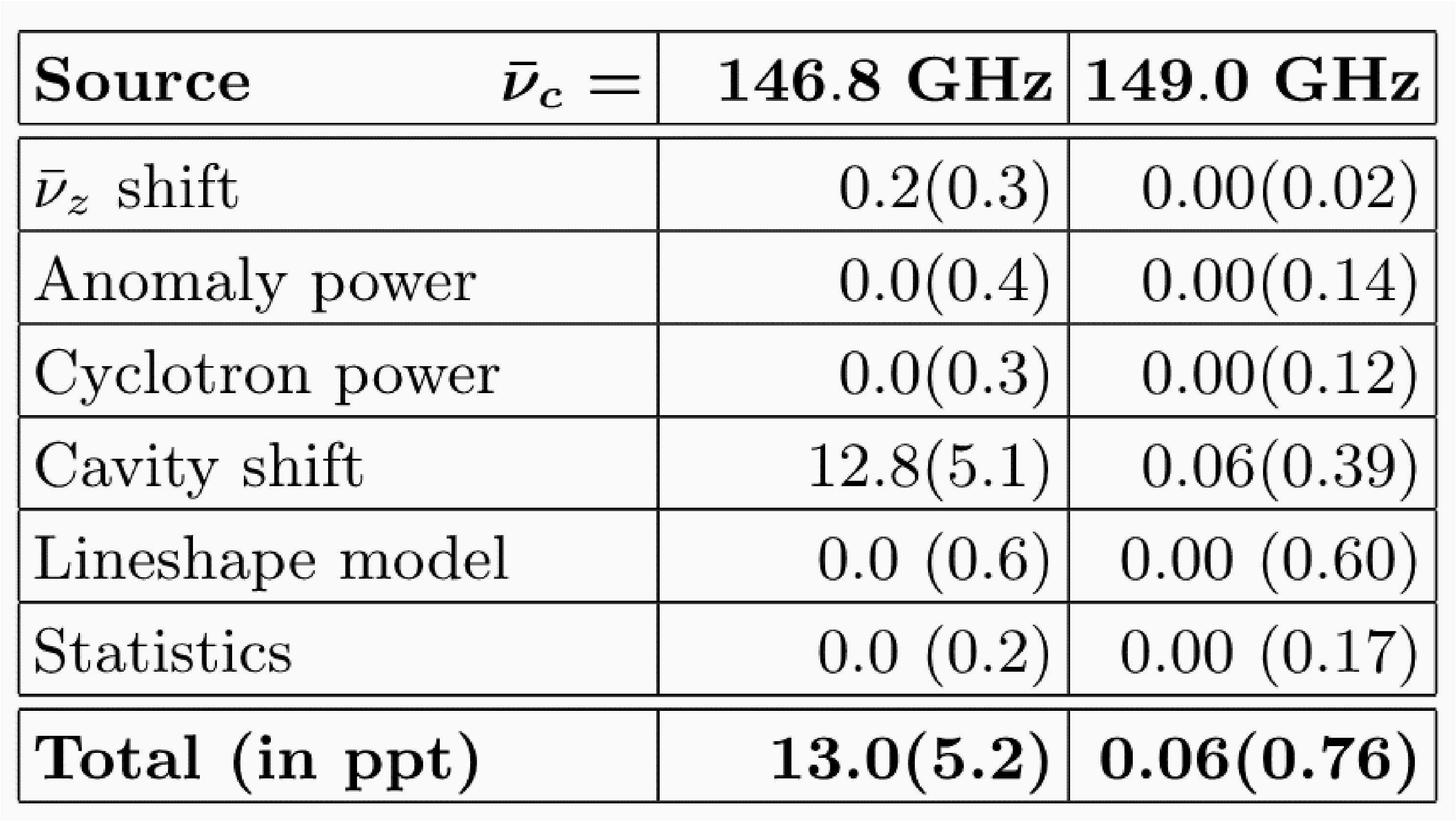
Operating between modes of cylindrical trap
where shift from two cavity modes
cancels approximately

first measured
cavity shift of g

Summary of Uncertainties for g (in ppt = 10^{-12})

Test of cavity shift understanding

Measurement of g-value



Source $\bar{\nu}_c =$	**146.8 GHz**	**149.0 GHz**
$\bar{\nu}_z$ shift	0.2(0.3)	0.00(0.02)
Anomaly power	0.0(0.4)	0.00(0.14)
Cyclotron power	0.0(0.3)	0.00(0.12)
Cavity shift	12.8(5.1)	0.06(0.39)
Lineshape model	0.0 (0.6)	0.00 (0.60)
Statistics	0.0 (0.2)	0.00 (0.17)
Total (in ppt)	**13.0(5.2)**	**0.06(0.76)**

Attempt Started to Measure g for Proton and Antiproton

- Improve proton g by more than 10
- Improve antiproton g by more than 10^6

- Compare g for antiproton and proton – test CPT

Current Proton g Last Measured in 1972

CODATA 2002: g_p=5.585 694 701(56) (10 ppb)

$$g_p = g_e \frac{\mu_p(H)}{\mu_e(H)} \frac{g_e(H)}{g_e} \frac{g_p}{g_p(H)} \frac{m_p}{m_e}$$

electron g-factor,
measured to
< 0.001 ppb
(Harvard)

bound magnetic moment ratio,
measured to 10 ppb
(MIT: P.F. Winkler, D. Kleppner,
T. Myint, F.G. Walther,
Phys. Rev. A 5, 83-114 (1972))

bound / free corrections,
calculated to < 1 ppb
(Breit, Lamb, Lieb, Grotch, Faustov,
Close, Osborn, Hegstrom, Persson,
others)

proton-electron mass ratio,
measured to < 1 ppb
(Mainz)

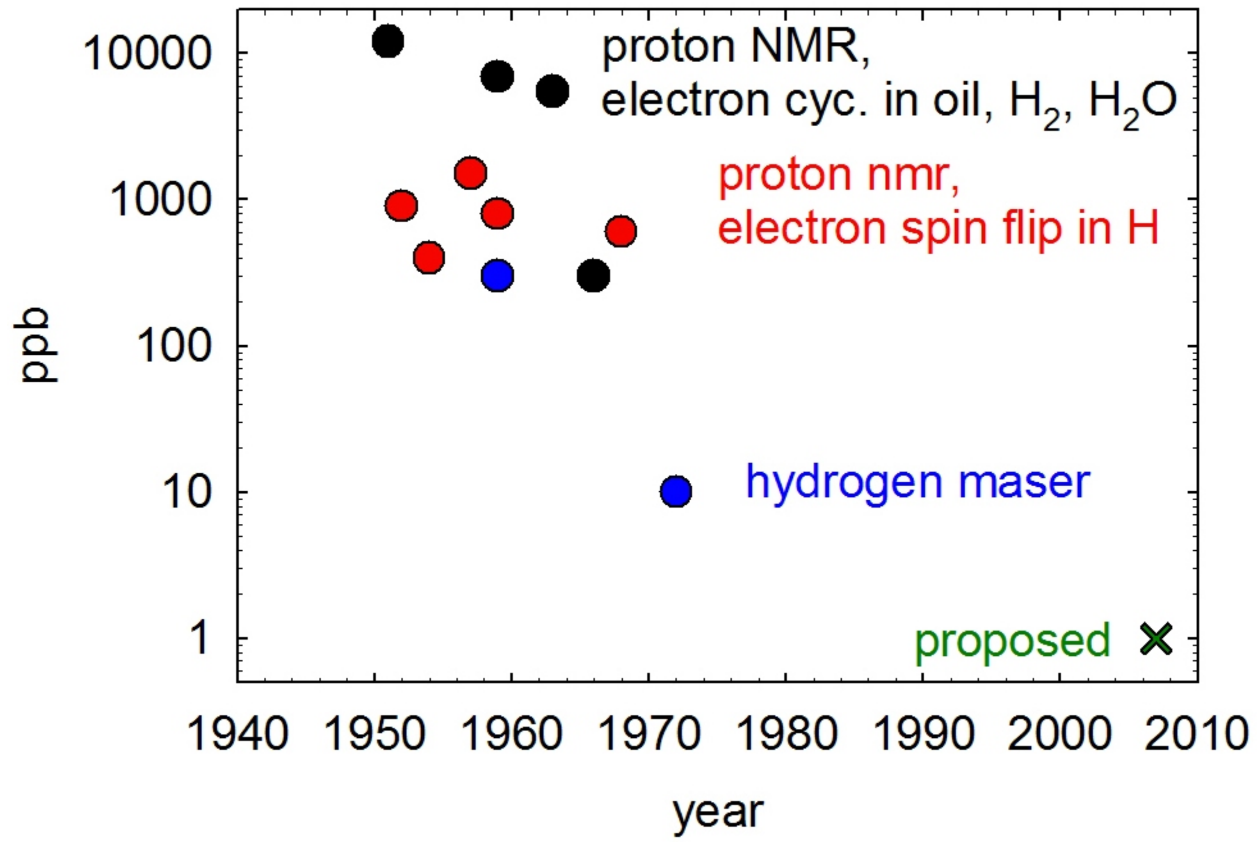
$$\frac{g_e(H)}{g_e} = 1 - \frac{1}{3}(Z\alpha)^2 - \frac{1}{12}(Z\alpha)^4 + \frac{1}{4}(Z\alpha)^2\left(\frac{\alpha}{\pi}\right) + \frac{1}{2}(Z\alpha)^2\left(\frac{m_e}{m_p}\right) + \dots$$

$$= 1 - 17.7053 \times 10^{-6}$$

$$\frac{g_p(H)}{g_p} = 1 - \frac{1}{3}Z\alpha^2 + \frac{1}{6}Z\alpha^2\left(\frac{m_e}{m_p}\right)\left(\frac{3+4a_p}{1+a_p}\right) + \dots$$

$$= 1 - 17.7328 \times 10^{-6}$$

History of Measurements of Proton g



(from bound measurements of μ_p/μ_e,
with current values of g_e, m_e/m_p and theory)

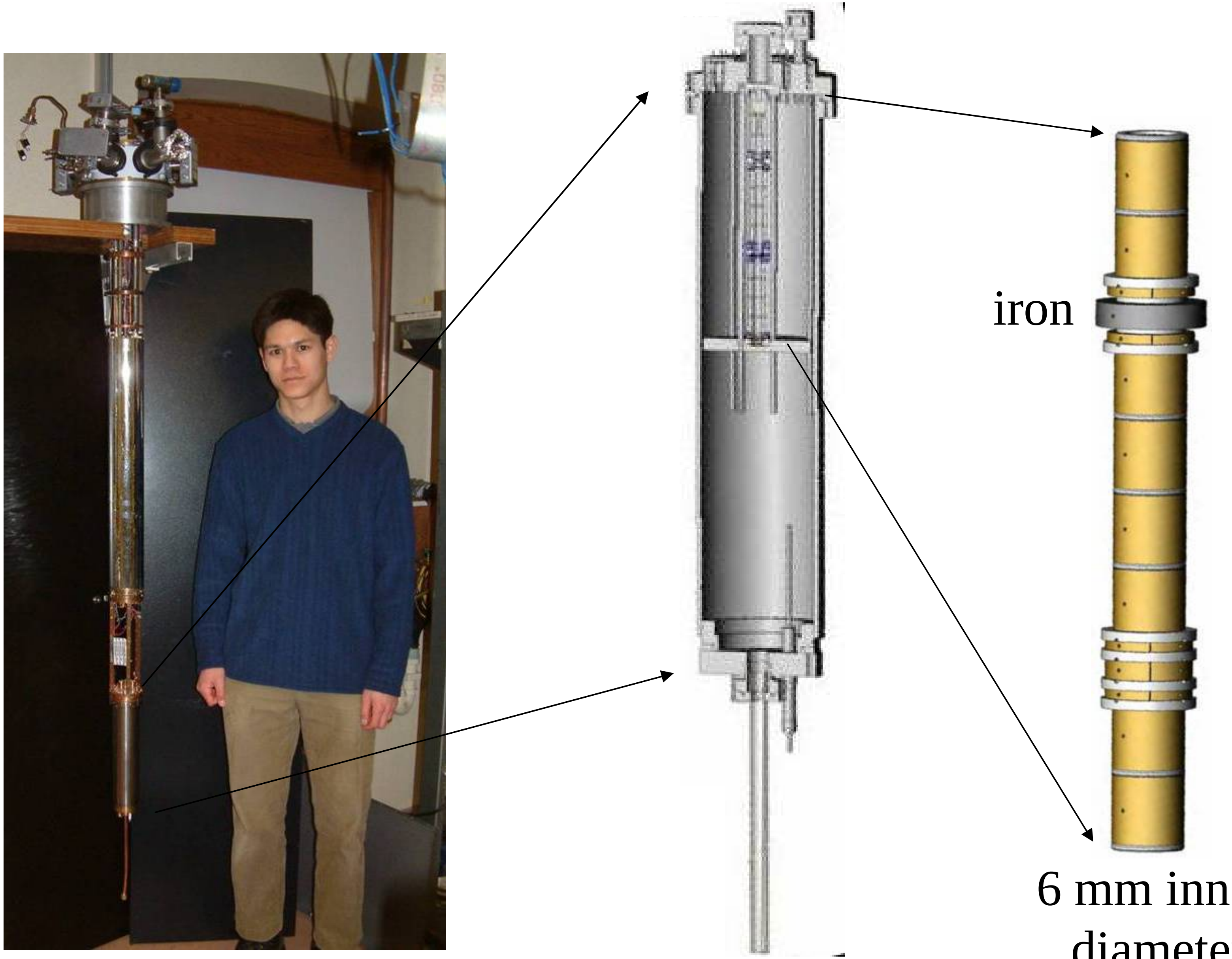
Antiproton g-factor

Antiproton g-factor is known to less than a part per *thousand*

$$g_{\bar{p}} = 5.601(18)$$

We hope to do roughly one million times better.

Apparatus Built, Not Yet Tried



iron

detect spin flip

make spin flip

6 mm inner diameter

Nick Guise

Summary and Conclusion

Summary

How Does One Measure g to 7.6 Parts in 10^{13}?

→ Use New Methods

first measurement with
these new methods

- One-electron quantum cyclotron
- Resolve lowest cyclotron as well as spin states
- Quantum jump spectroscopy of lowest quantum states
- Cavity-controlled spontaneous emission
- Radiation field controlled by cylindrical trap cavity
- Cooling away of blackbody photons
- Synchronized electrons probe cavity radiation modes
- Trap without nuclear paramagnetism
- One-particle self-excited oscillator

New Measurement of Electron Magnetic Moment

magnetic
moment

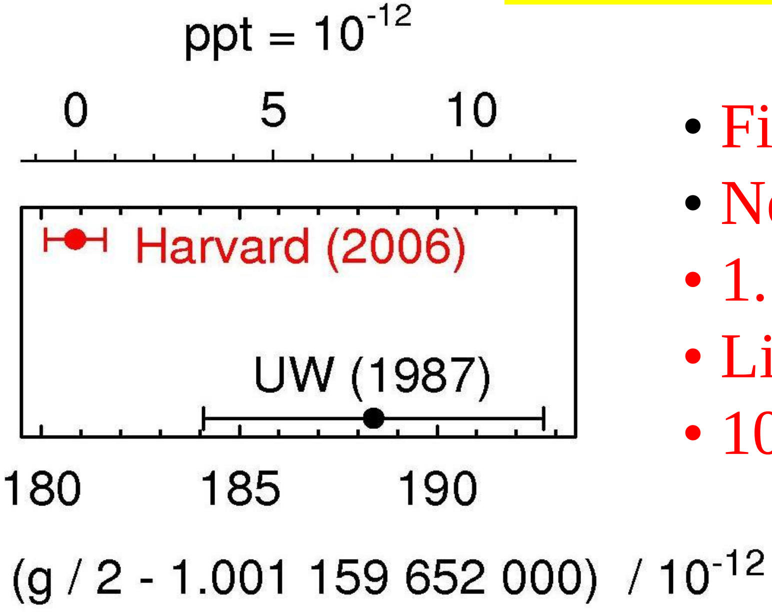
$$\vec{\mu} = g\mu_B \frac{\vec{S}}{\hbar}$$

spin

Bohr magneton $\frac{e\hbar}{2m}$

$$g/2 = 1.001\,159\,652\,180\,85$$

$$\pm 0.000\,000\,000\,000\,76 \qquad 7.6\times10^{-13}$$



- First improved measurement since 1987
- Nearly six times smaller uncertainty
- 1.7 standard deviation shift
- Likely more accuracy coming
- 1000 times smaller uncertainty than muon g

B. Odom, D. Hanneke, B. D’Urso and G. Gabrielse,
Phys. Rev. Lett. **97**, 030801 (2006).

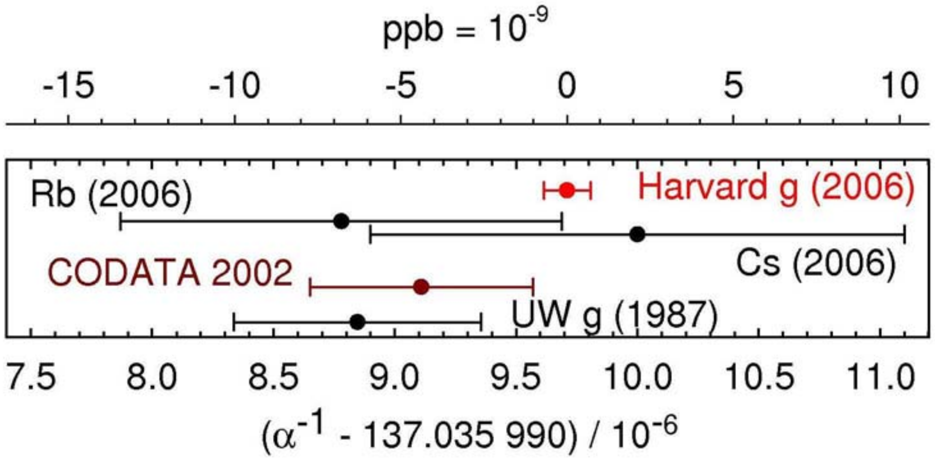
New Determination of the Fine Structure Constant

$$\alpha = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{\hbar c}$$

- Strength of the electromagnetic interaction
- Important component of our system of fundamental constants
- Increased importance for new mass standard

$$\alpha^{-1} = 137.035\ 999\ 710$$

$$\pm \quad 0.000\ 000\ 096 \qquad 7.0\times10^{-10}$$



- First lower uncertainty since 1987
- Ten times more accurate than atom-recoil methods

G. Gabrielse, D. Hanneke, T. Kinoshita, M. Nio, B. Odom,
Phys. Rev. Lett. 97}, 030802 (2006).

We Intend to do Better

Stay Tuned – The new methods have just been made to work all together

- With time we can utilize them better
- Some new ideas are being tried (e.g. cavity-sideband cooling)
- Lowering uncertainty by factor of 13 → check muon result (hard)

Spin-off Experiments

- Use self-excited antiproton oscillator to measure the antiproton magnetic moment → million-fold improvement?

- Compare positron and electron g-values to make best test of CPT for leptons

- Measure the proton-to-electron mass ration directly

Gabrielse

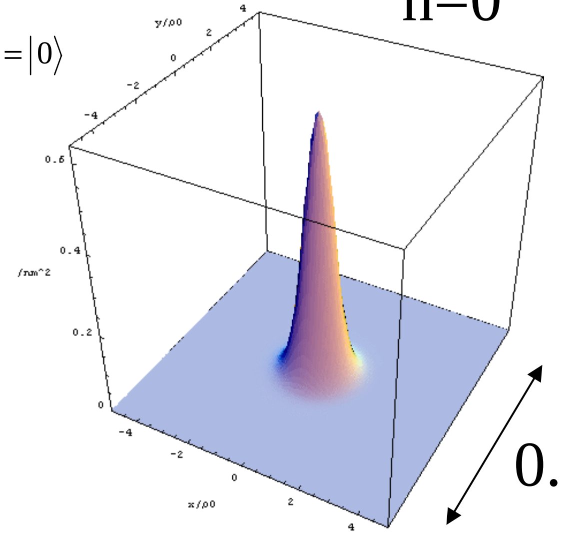
For Fun: Coherent State

Eigenfunction of the lowering operator: $a|\alpha\rangle = \alpha|\alpha\rangle$

Fock states do not oscillate

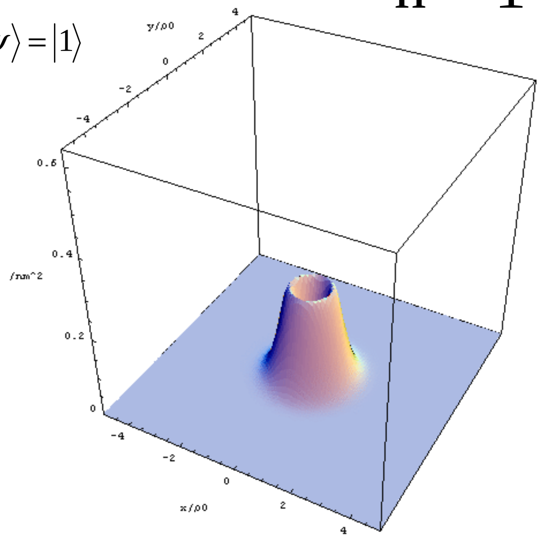
n=0

$|\psi\rangle = |0\rangle$



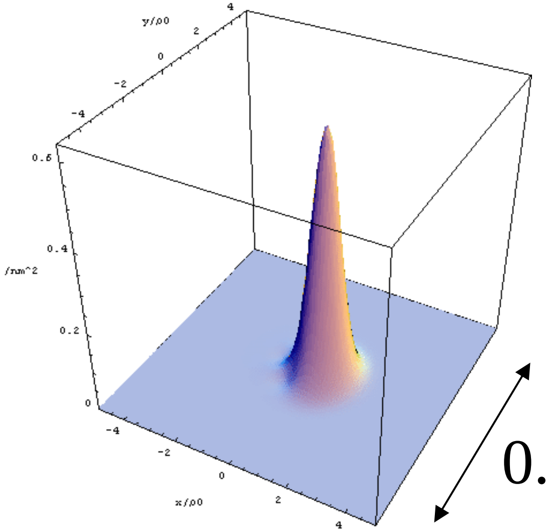
n = 1

$|\psi\rangle = |1\rangle$



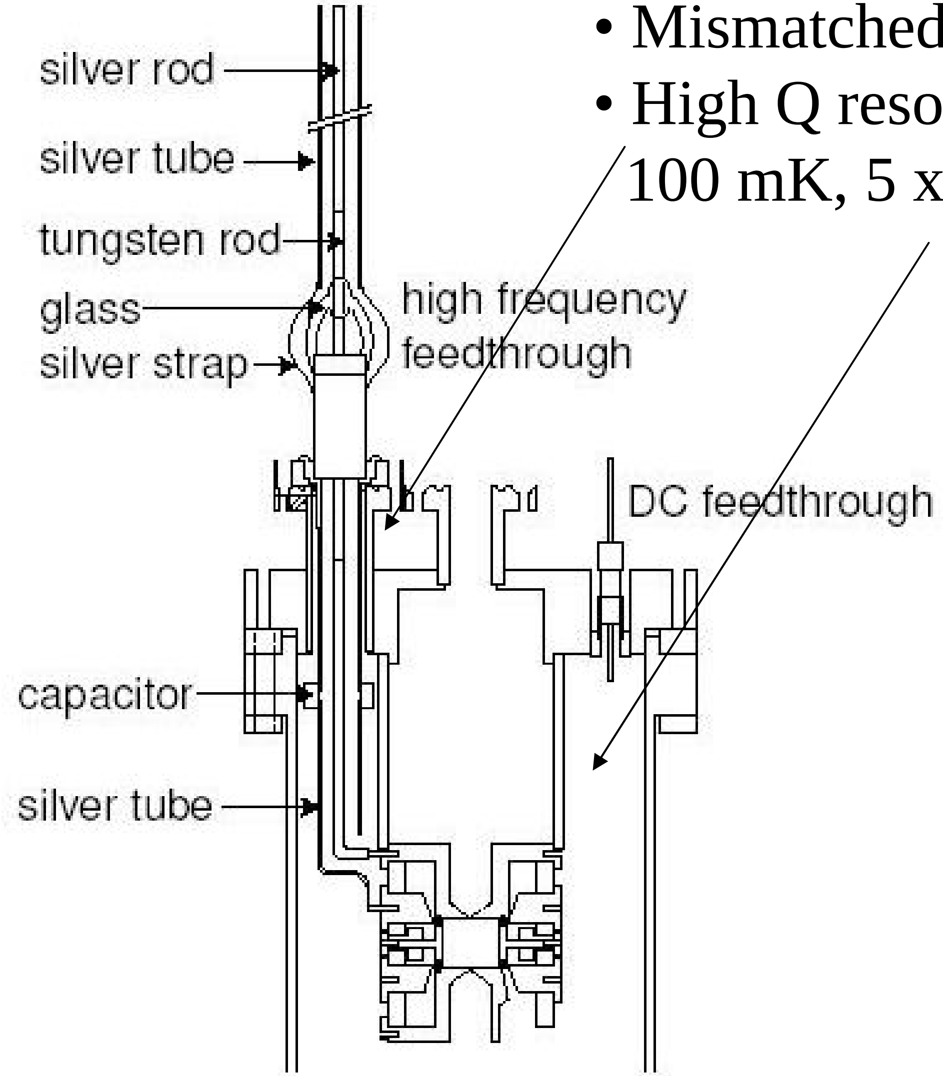
Coherent state with $\bar{n} = 1$

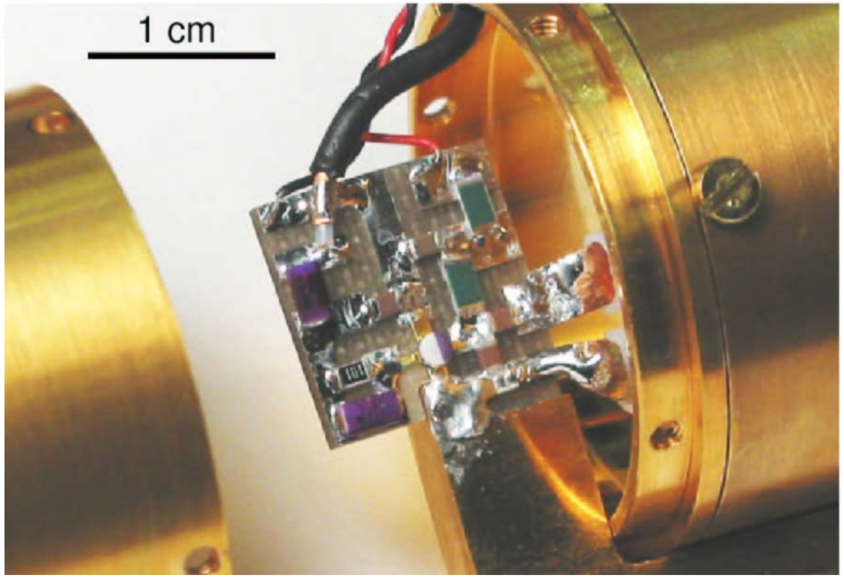
$$|\psi\rangle = e^{-\bar{n}/2} \sum_{n=0}^{n=\infty} \frac{\sqrt{\bar{n}}\, e^{i\beta}}{\sqrt{n!}} e^{-in\omega_c' t} |n\rangle,$$



200 MHz Detection of Axial Oscillation

- Turn off during sensitive times in experiment
- Mismatched, current-starved HEMPT
- High Q resonant feedthrough into 100 mK, 5 x 10^{-17} Torr vacuum enclosure





First One-Particle Self-Excited Oscillator

Feedback eliminates damping

Oscillation amplitude must be kept fixed
  Method 1: comparator
  Method 2: DSP (digital signal processor)

"Single-Particle Self-excited Oscillator"
B. D'Urso, R. Van Handel, B. Odom and G. Gabrielse
Phys. Rev. Lett. **94**, 113002 (2005).

Use Digital Signal Processor → DSP

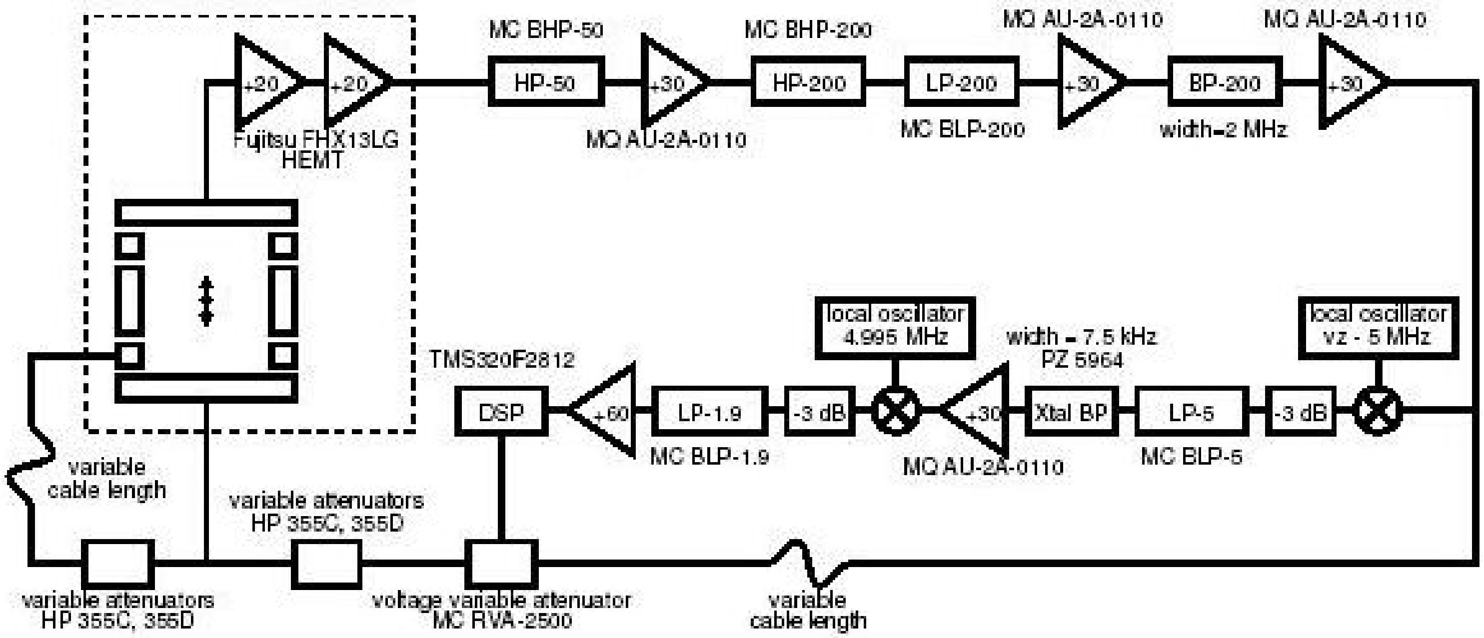
- Real time fourier transforms
- Use to adjust gain so oscillation stays the same

MANUFACTURERS

MC = minicircuits
MQ = miteq
PZ = piezotech
HP = hewlett packard

FILTERS

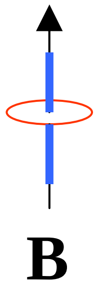
LP = low pass
HP = high pass
BP = band pass



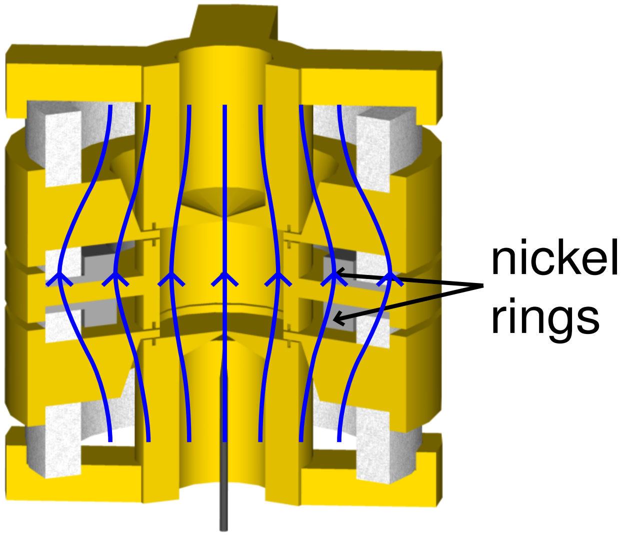
Detecting the Cyclotron State

cyclotron frequency	$\nu_C = 150$ GHz	**too high to detect directly**
axial frequency	$\nu_Z = 200$ MHz	**relatively easy to detect**

Couple the axial frequency ν_Z to the cyclotron energy.



Small measurable shift in ν_Z indicates a change in cyclotron energy.



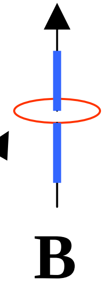
$$B_z = B_0 + B_2 z^2$$

Couple Axial Motion and Cyclotron Motion

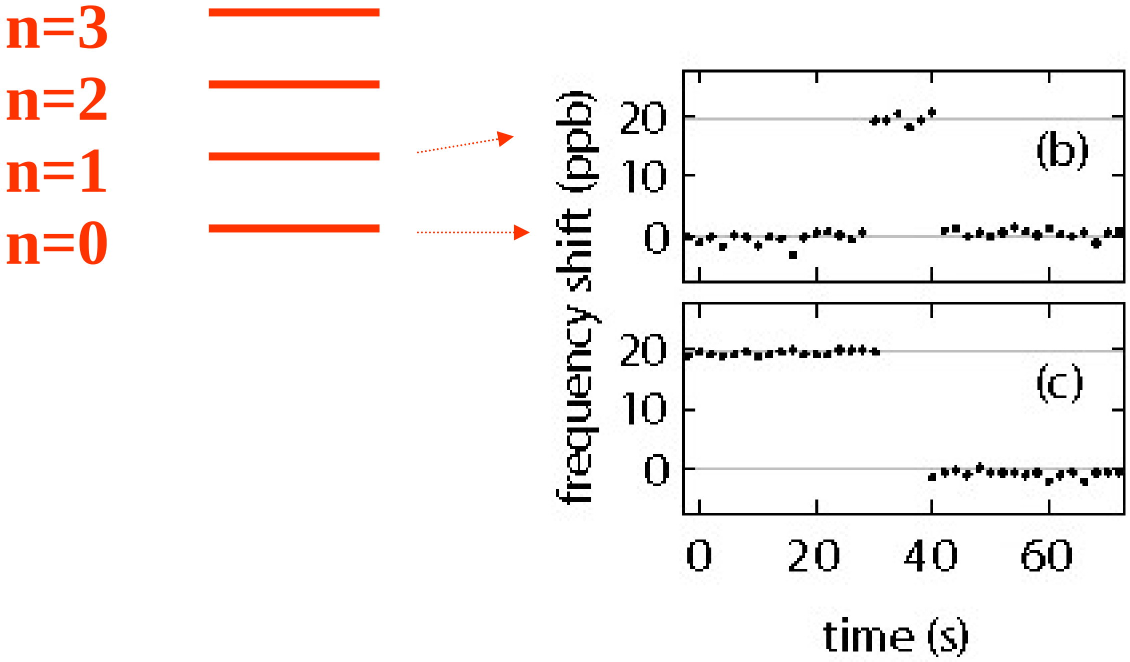
Add a “magnetic bottle” to uniform B

$$\Delta\vec{B} = B_2[(z^2 - \rho^2/2)\hat{z} - z\vec{\rho}]$$

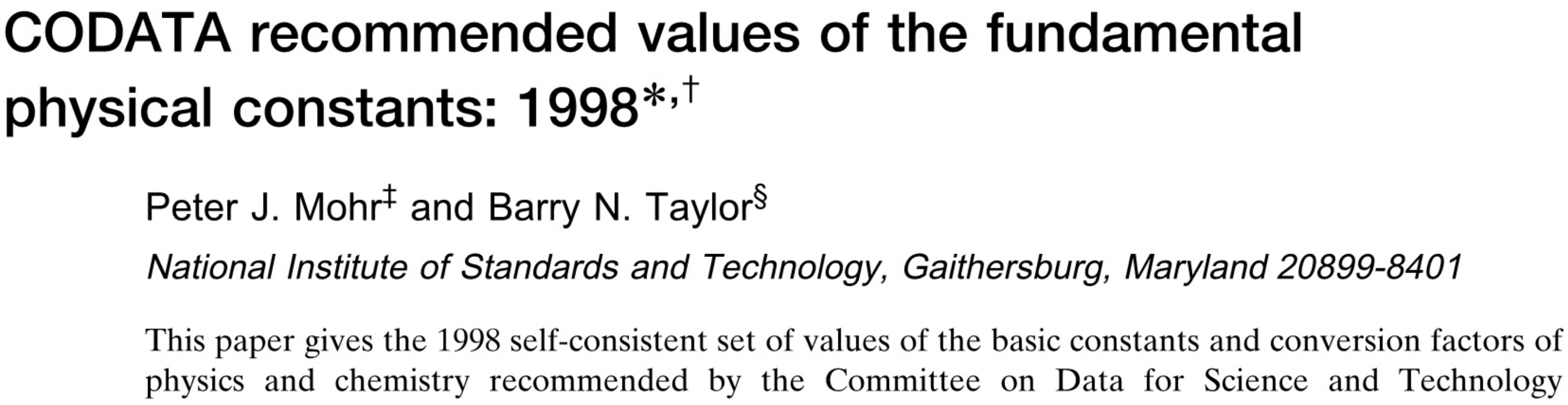
$$H = \frac{1}{2}m\omega_z{}^2 z^2 - \mu B_2 z^2$$



change in μ
changes effective ω_z



spin flip
is also a change in μ

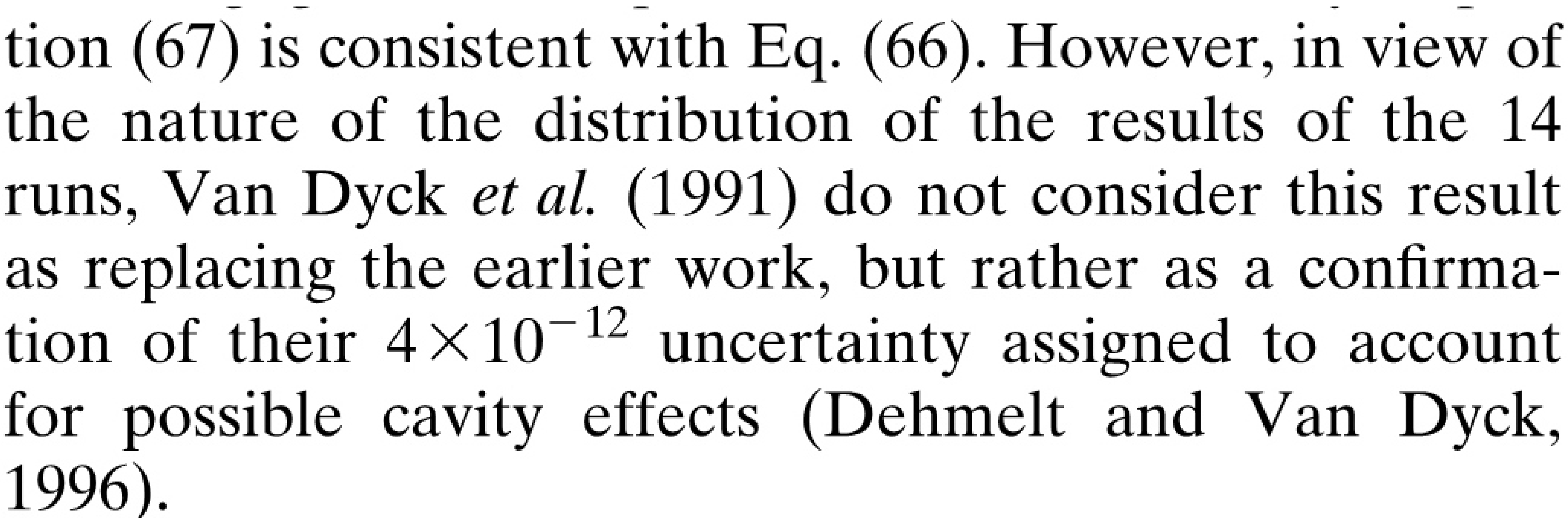


CODATA recommended values of the fundamental physical constants: 1998*,†

Peter J. Mohr‡ and Barry N. Taylor§

National Institute of Standards and Technology, Gaithersburg, Maryland 20899-8401

This paper gives the 1998 self-consistent set of values of the basic constants and conversion factors of physics and chemistry recommended by the Committee on Data for Science and Technology



tion (67) is consistent with Eq. (66). However, in view of the nature of the distribution of the results of the 14 runs, Van Dyck *et al.* (1991) do not consider this result as replacing the earlier work, but rather as a confirmation of their 4×10^{-12} uncertainty assigned to account for possible cavity effects (Dehmelt and Van Dyck, 1996).

What About Measurements After 1987?

There was one – Dehmelt and Van Dyck used a lossy trap
to see if cavity-shifts were problem for 1987 result

Not used by CODATA because
- there was a non-statistical distribution of measurements that was not understood
- the authors said that this result should be regarded as a confirmation of the assigned cavity shift uncertainty

Before we released our measurement, Van Dyck expressed the same point of view to me