Charm contributions to $b \rightarrow s l^+ l^-$ transitions

Alexander Khodjamirian (University of Siegen)
A very promising FCNC transition, observables: $q^2 = (p_{l+} + p_{l-})^2$ distribution, CP-, forward-backward asymm.,...

- exclusive channels: measured by BABAR, Belle, good perspectives for LHC, hadronic matrix elements (form factors): effective theories, LCSR, lattice QCD

- inclusive: more difficult to measure, theory: OPE, (heavy quark expansion) at NNLO
The problem of \( c\bar{c} \) states in \( b \rightarrow sl^+l^- \)

* a chain of “routine” electroweak decays mimics \( b \rightarrow sl^+l^- \), e.g.,

\[
\text{BR}(B^0 \rightarrow J/\psi K^0) \times \text{BR}(J/\psi \rightarrow l^+l^-) = \\
= [(8.5 \pm 0.5) \times 10^{-4}] \times 5.9\% \simeq 5 \times 10^{-5}
\]

\[
\text{BR}(B^0 \rightarrow \psi(2S)K^0) \times \text{BR}(\psi(2S) \rightarrow l^+l^-) = \\
= [(6.2 \pm 0.7) \times 10^{-4}] \times [7.4 \times 10^{-3}] \simeq 4.5 \times 10^{-6}
\]

* “direct” decay via \( b \rightarrow sl^+l^- \) transition
(subtracting the intervals of \( q^2 \) near \( J/\psi \) and \( \psi(2S) \))

\[
\text{BR}(B^0 \rightarrow K^0l^+l^-) = (3.12 \pm 1.0) \times 10^{-7} \quad \text{[HFAG average, August 2006]}
\]

how large is the effect of \( c\bar{c} \) states outside \( J/\psi, \psi(2S) \) regions?
“Anatomy” of $b \to s l^+ l^-$ transitions

Quark level diagrams:

\[ O_{9,10} \]

\[ O_7 \]

\[ O_{1,2,\ldots} \]

Effective Hamiltonian: \[ H_{eff} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i O_i \]

\[ O_{9(10)} = (\bar{s}b)_{V-A} (\bar{l}l)_{V(A)}, \quad O_7 = (\bar{s} \Sigma_{\mu \nu} b) F^{\mu \nu}, \quad O_{1}^c = (\bar{s}c)_{V-A} (\bar{c}b)_{V-A}, \quad O_{2}^c, \ldots \]
Effects of 4-quark $O_{1,\ldots,6}$ operators in $b \to s l^+ l^-$

a simplified setup:

- neglecting $u$-quark loops from $O_{1,2}^u$ (CKM suppressed for $b \to s$),
  caution: light-quark resonances at small masses, $B \to \pi(\rho) l^+ l^-$

- neglecting $O_{3,4,5,6}$ with small Wilson coeff. $C_{3,4,5,6}$
  include them later, analogous to $O_{1,2}$

- main effect from $c$-quark loops, due to
  \[
  C_1 O_1^c + C_2 O_2^c = \left( \frac{C_1}{3} + C_2 \right) (\bar{c}c)_{V-A}(\bar{s}b)_{V-A} + 2C_1 (\bar{c}t^a c)_{V-A}(\bar{s}t^a b)_{V-A}
  \]

  at $\mu \sim m_b$ \quad $C_1 \sim 1$, $C_2 \sim -0.3$, \quad $C_1/3 + C_2 \ll C_1$
\( q^2 \ll 4m_c^2, \) \( c \)-quarks form a highly virtual loop \( (2m_c \gg \Lambda_{QCD}) \)

LO simple loop, \hspace{1cm} NLO \( O(\alpha_s) \), \hspace{1cm} soft gluons (nonfactoriz.)

factorizable to \( C_9^{eff} \cdot O_9 \)

[Bobeth,Misiak,Urban(2000); Asatryan,Greub,Walker(2001)]
c-quark loop effects at $q^2 \ll 4m_c^2$ in terms of OPE

$$i \int d^4 x e^{i q x} T \{ \bar{c} \gamma_\nu c(x) [C_1 O^c_1(0) + C_2 O^c_2(0)] \}$$

$$= [(C_1/3 + C_2) g(q^2, m_c^2) + C_1 \alpha_s g'(q^2, m_c^2)](\bar{s} \gamma_\nu b)$$

$$+ C_1 h_{\nu \rho \alpha \beta}(q^2, m_c^2)(\bar{s} \Gamma_\rho g_s G_{\alpha \beta} b) + ...$$

- $g, g', h, ...$ - calculable Wilson coefficients, $h \sim 1/m_c^2$,

NLO worked out mainly for inclusive $B \rightarrow X_s \gamma$ decays

- new effective operator $\bar{s} g_s \Gamma_\rho G_{\alpha \beta} b$

[Voloshin (1997); A.K., Rückl, Stoll, Wyler (1997); Ligeti, Randall, Wise; Grant, Morgan, Nussinov, Peccei (1997); Buchalla, Isidori, Rey (1998)]

- are there other important operators? is the local expansion efficient?

(infinite tower of operators → light-cone expansion)
Hadronic matrix elements in $B \rightarrow K^{(*)} l^+ l^-$

quark/gluons $\rightarrow$ hadrons

$O_i \rightarrow \langle K^{(*)} | O_i | B \rangle$

- $\langle K^{(*)} | \bar{s} \gamma_\nu b | B \rangle$ $B \rightarrow K^{(*)}$ form factors accessible with lattice QCD, LCSR, QCDF, SCET etc., uncertainty 10-15%

- $\langle K^{(*)} | \bar{s} \Gamma_\rho g_s G_{\alpha\beta} b | B \rangle$, estimates at $q^2 = 0$ (for $B \rightarrow K^* \gamma$):
  - local QCD sum rules [A.K., G.Stoll, D.Wyler (1997)];
  - LCSR [P.Ball, R.Zwicky (2006)], see talk by R.Zwicky

small $O(1\%)$ corrections with large $O(50\%)$ uncertainties
Increasing the lepton pair mass $q^2$

With the local OPE answer for $\bar{c}c$ - effects valid at $q^2 \ll 4m_c^2 \sim 6 \text{ GeV}^2$: 

- how large is $q_{max}^2 < m_{J/\psi}^2 = 9.6 \text{ GeV}^2$

- are $q^2 > m_{\psi(2S)}^2 = 13.6 \text{ GeV}^2$ accessible?

$(\bar{m}_c(m_c) = 1.25 \text{ GeV})$
Models of “long-distance” contributions in $b \to s l^+ l^-$

- use the LO $\bar{c}c$ loop everywhere (with Im-part above $4m_c^2$):
  too crude for the $q^2$-distribution, nonfactorizable effects ignored

- add a bunch of $\psi$ resonances to the $\bar{c}c$ loop:
  double counting of quark and hadronic d.o.f

- the method avoiding double counting of quark and hadron d.o.f.
  [Krüger, Sehgal(1997) (for $B \to X_s l^+ l^-$)]
  dispersion relation (analyticity, unitarity) with $\psi$ resonances $\oplus$ open charm
  continuum matched to a LO $\bar{c}c$ loop at small $q^2$

  but: nonfactorizable $O(\alpha_s), O(1/m_c^2)$ contributions not taken into account
  (see discussion in [Buchalla, Isidori, 1998])

  analogous method used in $K \to \pi l^+ l^-$ [Okun, Shifman, Vainshtein, Zakharov, (1976)]
Is the region $q^2 < m_{J/\psi}^2$ directly accessible with OPE?

- Series of effective soft-gluon operators diverge at $q^2 \to 4m_c^2$

- QCD sum rules for charmonium: the OPE is valid at $q^2 = 0$ and matches dispersion relation with $\psi'$s, but the gluon condensate effects grow very fast at $q^2 \to 4m_c^2$

- $B \to J/\psi K^(*)$, $B \to \psi(2S) K^(*)$ decay amplitudes extracted from the measured BR’s cannot be approximated by factorization, nonfactorizable effects are at $O(100\%)$ level
Use of dispersion relation in $J/\psi$ channel

• match OPE to dispersion relation at small $q^2$, where both are valid:

$$\langle K^{(*)} | i \int d^4x e^{i q \cdot x} T \{ c \gamma_\nu \bar{c}(x) (C_1 O_1^c(0) + C_2 O_2^c(0)) | B \rangle$$

$$= \sum_{\psi=J/\psi,...} \frac{f_\psi A_\nu(B \to \psi K^{(*)})}{m_\psi^2 - q^2 - im_\psi \Gamma_\psi} + \text{open charm}$$

• use dispersion relation to predict the $q^2$ distribution below $\psi(3S)$
• very preliminary crude estimate for $B \to Kl^+l^-$: (work in progress)

$$C_{9}^{\text{eff},O_{1,2}}(q^2) = C_9 + \delta_c(q^2)$$

the charm effect not exceeding $5\%$ ($10\%$) of $C_9$ in at $q^2 \leq 7 \text{ GeV}^2$ ($\leq 8 \text{ GeV}^2$)

• a potential problem with $q^2$ above $\psi(3S)$: broad resonances, dependence on the model for the widths
Summary and final comments

- local OPE provides a reliable framework for estimating $\bar{c}c$ effects at small $q^2$ more work needed to understand $(1/m_c^2)^n$ suppressed contributions

- matching with the dispersion relation helps to avoid double counting and unaccounted nonperturbative effects

- optimistically, charm effects in $B \to K^{(*)}l^+l^-$ add $\leq$ few $\%$ to $c_9$ below $J/\psi$

- light-quark loops need a parallel/similar analysis for the region $q^2 > m_{\rho}^2, m_{\phi}^2$, (broad resonances?)

- not clear if the same analysis can be simply translated to $B \to X_s l^+l^-$ (a specific problem of “hidden” background due to $J/\psi \to l^+l^-X$ with a lepton-pair mass $< m_{J/\psi}$)