Charm contributions to $b \to sl^+l^-$ transitions

Alexander Khodjamirian (University of Siegen)

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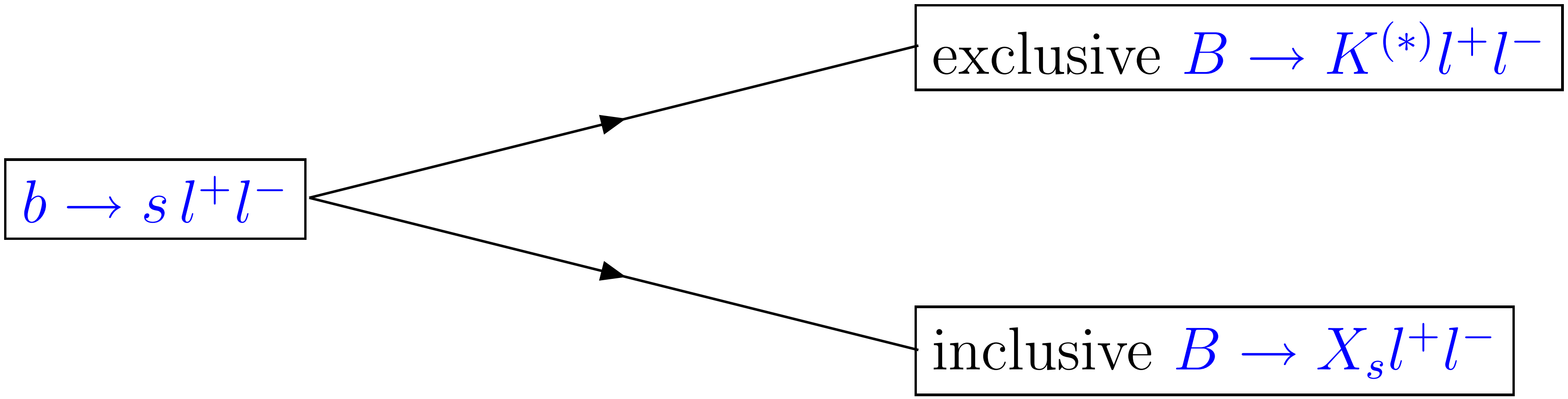


• a very promising FCNC transition,
observables: $q^2 = (p_{l^+} + p_{l^-})^2$ distribution, CP- , forward-backward asymm.,...

• exclusive channels: measured by BABAR,Belle, good perspectives for LHC,
hadronic matrix elements (form factors): effective theories, LCSR, lattice QCD

• inclusive: more difficult to measure,
theory: OPE, (heavy quark expansion) at NNLO

The problem of $c\bar{c}$ states in $b \to sl^+l^-$

$*$ a chain of “routine” electroweak decays mimics $b \to sl^+l^-$, e.g.,

$$BR(B^0 \to J/\psi K^0) \times BR(J/\psi \to l^+l^-) =$$
$$= [(8.5 \pm 0.5) \times 10^{-4}] \times 5.9\% \simeq 5 \times 10^{-5}$$

$$BR(B^0 \to \psi(2S) K^0) \times BR(\psi(2S) \to l^+l^-) =$$
$$= [(6.2 \pm 0.7) \times 10^{-4}] \times [7.4 \times 10^{-3}] \simeq 4.5 \times 10^{-6}$$

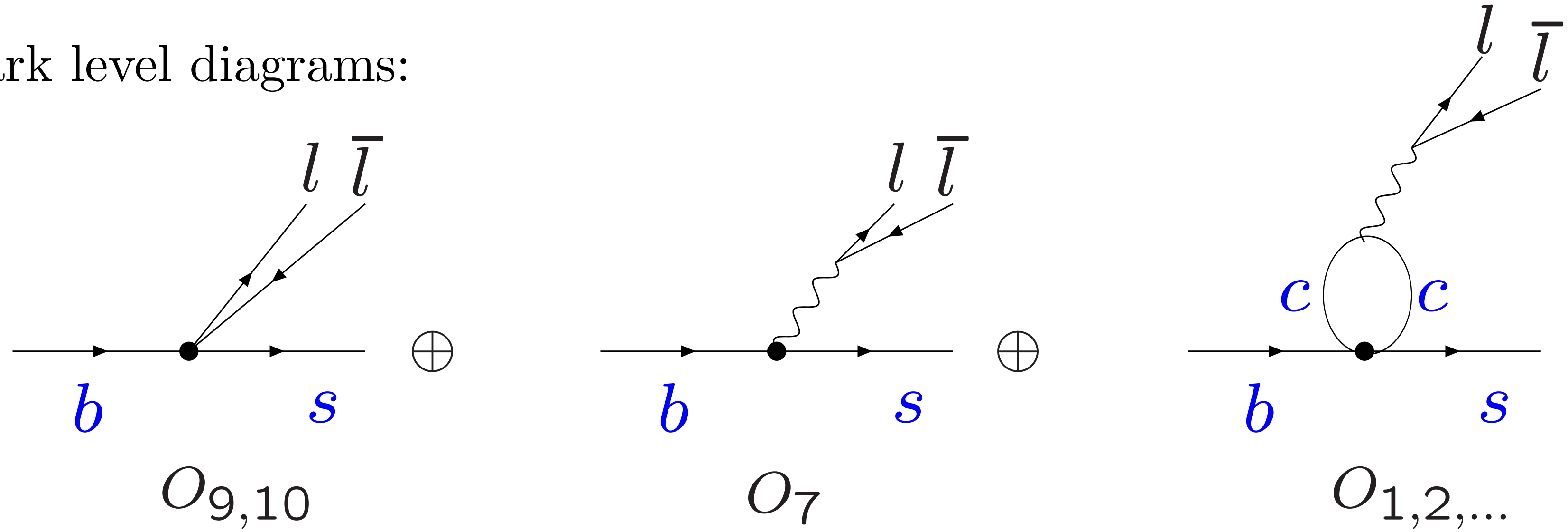
$*$ “direct” decay via $b \to sl^+l^-$ transition
(subtracting the intervals of q^2 near J/ψ and $\psi(2S)$)

$BR(B^0 \to K^0 l^+l^-) = (3.12 \pm 1.0) \times 10^{-7}$ [HFAG average, August 2006]

how large is the effect of $c\bar{c}$ states outside $J/\psi, \psi(2S)$ regions ?

“Anatomy” of $b \to s l^+ l^-$ transitions

Quark level diagrams:



Effective Hamiltonian: $H_{eff} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i O_i$

$O_{9(10)} = (\bar{s}b)_{V-A}(\bar{l}l)_{V(A)}$, $O_7 = (\bar{s}\Sigma_{\mu\nu}b)F^{\mu\nu}$, $O_1^c = (\bar{s}c)_{V-A}(\bar{c}b)_{V-A}$, O_2^c,...

Effects of 4-quark $O_{1,..,6}$ operators in $b \to s l^+ l^-$

a simplified setup:

- neglecting u-quark loops from $O^u_{1,2}$ (CKM suppressed for $b \to s$),
  caution: light-quark resonances at small masses, $B \to \pi(\rho) l^+ l^-$

- neglecting $O_{3,4,5,6}$ with small Wilson coeff. $C_{3,4,5,6}$
  include them later, analogous to $O_{1,2}$

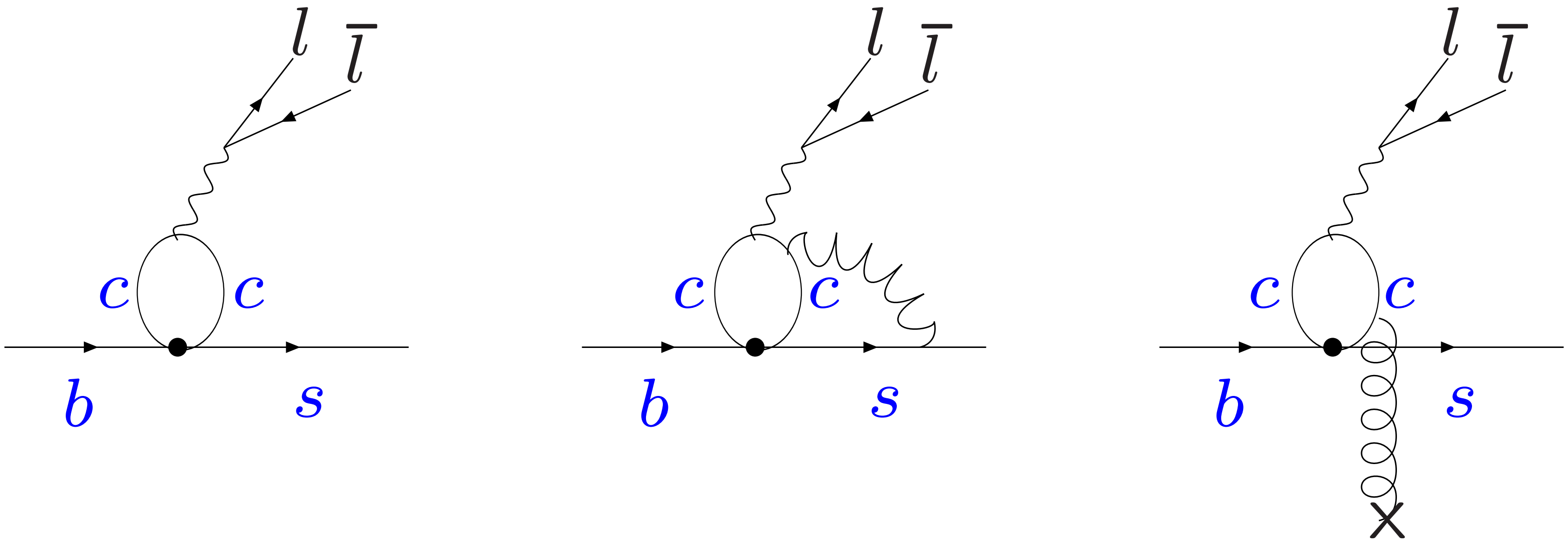
- main effect from c-quark loops, due to

$$C_1 O_1^c + C_2 O_2^c = \left(\frac{C_1}{3} + C_2 \right) (\bar{c}c)_{V-A} (\bar{s}b)_{V-A} + 2C_1 (\bar{c}t^a c)_{V-A} (\bar{s}t^a b)_{V-A}$$

at $\mu \sim m_b \quad C_1 \sim 1, C_2 \sim -0.3, \quad C_1/3 + C_2 \ll C_1$

$c\bar{c}$-loop effects in $b \to s l^+ l^-$

$q^2 \ll 4m_c^2$, c-quarks form a highly virtual loop ($2m_c \gg \Lambda_{QCD}$)



LO simple loop, NLO $O(\alpha_s)$, soft gluons (nonfactoriz.)

. factorizable to $C_9^{eff}\, O_9$

[Bobeth,Misiak,Urban(2000); Asatryan,Greub,Walker(2001)]

Short-distance OPE in $\alpha_s(2m_c)$, $1/m_c^2$

c-quark loop effects at $q^2 \ll 4m_c^2$ in terms of OPE

$$i\int d^4x e^{iqx} T\{\bar{c}\gamma_\nu c(x)[C_1\, O_1^c(0) + C_2\, O_2^c(0)]\}$$
$$= [(C_1/3 + C_2)g(q^2, m_c^2) + C_1\alpha_s g'(q^2, m_c^2)](\bar{s}\gamma_\nu b)$$
$$+C_1 h_{\nu\rho\alpha\beta}(q^2, m_c^2)(\bar{s}\Gamma_\rho g_s G_{\alpha\beta} b) + ...$$

- $g, g', h, ...$ -calculable Wilson coefficients, $h \sim 1/m_c^2$,

NLO worked out mainly for inclusive $B \to X_s\gamma$ decays

- new effective operator $\bar{s}g_s\Gamma_\rho G_{\alpha\beta}b$

[Voloshin (1997); A.K., Rückl, Stoll, Wyler(1997);
Ligeti,Randall,Wise;Grant,Morgan,Nussinov,Peccei(1997); Buchalla,Isidori,Rey (1998)]

- are there other important operators? is the local expansion efficient ?

(infinite tower of operators$\to$ light-cone expansion)

Hadronic matrix elements in $B \to K^{(*)} l^+ l^-$

quark/gluons $\longrightarrow$ hadrons
$O_i \to \langle K^{(*)} | O_i | B \rangle$

- $\langle K^{(*)} | \bar{s} \gamma_\nu b | B \rangle$ $\quad$ $B \to K^{(*)}$ form factors

accessible with lattice QCD, LCSR, QCDF, SCET etc., uncertainty 10-15 %

- $\langle K^{(*)} | \bar{s} \Gamma_\rho g_s G_{\alpha\beta} b) | B \rangle$, $\quad$ estimates at $q^2 = 0$ (for $B \to K^* \gamma$):

-local QCD sum rules [A.K.,G.Stoll, D.Wyler (1997)];
-LCSR [P.Ball,R.Zwicky(2006)], see talk by R.Zwicky

small $O(1\%)$ corrections with large $O(50\%)$ uncertainties

Increasing the lepton pair mass q^2

With the local OPE answer for $\bar{c}c$ - effects
valid at $q^2 \ll 4m_c^2 \sim 6$ GeV^2: $(\overline{m}_c(m_c) = 1.25$ GeV$)$

- how large is $q^2_{max} < m^2_{J/\psi} = 9.6\text{GeV}^2$

- are $q^2 > m^2_{\psi(2S)} = 13.6$ GeV^2 accessible ?

Models of “long-distance” contributions in $b \to s l^+ l^-$

- use the LO $\bar{c}c$ loop everywhere (with Im-part above $4m_c^2$):

too crude for the q^2-distribution, nonfactorizable effects ignored

- add a bunch of ψ resonances to the $\bar{c}c$ loop:

double counting of quark and hadronic d.o.f

- the method avoiding double counting of quark and hadron d.o.f.

[Krüger, Sehgal(1997) (for $B \to X_s l^+ l^-$)]
dispersion relation (analyticity, unitarity) with ψ resonances $\oplus$ open charm continuum matched to a LO $\bar{c}c$ loop at small q^2

but: nonfactorizable $O(\alpha_s), O(1/m_c^2)$ contributions not taken into account
(see discussion in [Buchalla,Isidori,1998])

analogous method used in $K \to \pi l^+ l^-$ [Okun,Shifman, Vainshtein, Zakharov,(1976)]

Is the region $q^2 < m^2_{J/\psi}$ directly accessible with OPE ?

- series of effective soft-gluon operators diverge at $q^2 \to 4m_c^2$

- QCD sum rules for charmonium: the OPE is valid at $q^2 = 0$ and matches dispersion relation with ψ's, but the gluon condensate effects grow very fast at $q^2 \to 4m_c^2$

- $B \to J/\psi K^{(*)}$, $B \to \psi(2S)K^{(*)}$ decay amplitudes extracted from the measured BR's cannot be approximated by factorization, nonfactorizable effects are at O(100%) level

Use of dispersion relation in J/ψ channel

- match OPE to dispersion relation at small q^2, where both are valid:

$$\langle K^{(*)}|i\int d^4x e^{iqx} T\{c\gamma_\nu \bar{c}(x)(C_1\, O_1^c(0) + C_2\, O_2^c(0))|B\rangle$$

$$= \sum_{\psi=J/\psi,\dots} \frac{f_\psi A_\nu(B\to\psi K^{(*)})}{m_\psi^2 - q^2 - i m_\psi \Gamma_\psi} + open\ charm$$

- use dispersion relation to predict the q^2 distribution below $\psi(3S)$
- very preliminary crude estimate for $B \to K l^+ l^-$: (work in progress)

$$C_9^{eff,O_{1,2}}(q^2) = C_9 + \delta_c(q^2)$$

the charm effect not exceeding 5% (10%) of C_9 in at $q^2 \leq 7$ GeV^2 (≤ 8 GeV^2)

- a potential problem with q^2 above $\psi(3S)$:

broad resonances, dependence on the model for the widths

Summary and final comments

• local OPE provides a reliable framework for estimating $\bar{c}c$ effects at small q^2
more work needed to understand $(1/m_c^2)^n$ suppressed contributions

• matching with the dispersion relation helps to avoid double counting and unaccounted nonperturbative effects

• optimistically, charm effects in $B \to K^{(*)} l^+ l^-$ add $\leq$ few % to c_9 below J/ψ

• light-quark loops need a parallel/similar analysis for the region $q^2 > m_\rho^2, m_\phi^2$, (broad resonances?)

• not clear if the same analysis can be simply translated to $B \to X_s l^+ l^-$
(a specific problem of "hidden" background due to $J/\psi \to l^+ l^- X$ with a lepton-pair mass $< m_{J/\psi}$)