NNLO VERTEX CORRECTIONS IN QCD FACTORIZATION

[G. Bell - in preparation]

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LMU

OUTLINE

$B \to \pi\pi$ DECAYS IN QCD FACTORIZATION

STATUS OF NNLO CALCULATION

NNLO VERTEX CORRECTIONS

QCD Factorization

[Beneke, Buchalla, Neubert, Sachrajda 99,01]

systematic expansion in $\alpha_s(m_b)$ and Λ_{QCD}/m_b

$$\langle M_1 M_2 | Q_i | \bar{B} \rangle =$$

- Q_i effective weak Hamiltonian (current-current, QCD/ew penguins, . . .)
- M_1 recoil meson (picks up spectator antiquark)
- M_2 emission meson

QCD Factorization

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$$\langle M_1 M_2 | Q_i | \bar{B} \rangle = F^{B \to M_1}(0) \int du \; T_i^I(u) \; \phi_{M_2}(u)$$

- Q_i effective weak Hamiltonian (current-current, QCD/ew penguins, ...)
- M_1 recoil meson (picks up spectator antiquark)
- M_2 emission meson

- $F^{B \to M_1}$ heavy-to-light form factor (maximum recoil $q^2 = 0$)
- T_i^I perturbatively calculable hard-scattering kernel
- ϕ_{M_2} light-cone distribution amplitude

QCD Factorization

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$$\langle M_1 M_2 | Q_i | \bar{B} \rangle = F^{B\to M_1}(0) \int du \; T_i^I(u)\, \phi_{M_2}(u) + \int d\omega\, du\, dv \; T_i^{II}(\omega,u,v)\, \phi_B(\omega)\, \phi_{M_1}(v)\, \phi_{M_2}(u) + \mathcal{O}\left(\frac{\Lambda_{QCD}}{m_b}\right)$$

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- ϕ_B, ϕ_{M_1} light-cone distribution amplitudes

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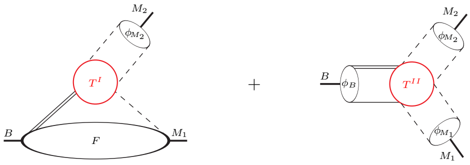
strong phases (from final state interactions)



→ perturbatively calculable in QCD Factorization

→ predicted to be small $\sim \mathcal{O}(\alpha_s)$, $\mathcal{O}(1/m_b)$

Topological amplitudes

$$\sqrt{2}\,\mathcal{A}(B^- \to \pi^-\pi^0) = V_{ub}V_{ud}^* \Big[\alpha_1 + \alpha_2 + \ldots\Big] A_{\pi\pi}$$

$$\mathcal{A}(\bar{B}^0 \to \pi^+\pi^-) = \Big\{V_{ub}V_{ud}^* \Big[\alpha_1 + \alpha_4^u\Big] + V_{cb}V_{cd}^*\,\alpha_4^c + \ldots\Big\} A_{\pi\pi}$$

$$-\,\mathcal{A}(\bar{B}^0 \to \pi^0\pi^0) = \Big\{V_{ub}V_{ud}^* \Big[\alpha_2 - \alpha_4^u\Big] - V_{cb}V_{cd}^*\,\alpha_4^c + \ldots\Big\} A_{\pi\pi}$$

α_1 color-allowed tree α_2 color-suppressed tree $\alpha_4^{u,c}$ QCD penguins

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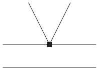
Tree amplitudes in QCDF $(C_1 \sim 1.1,\ C_2 \sim -0.2)$

$$\alpha_1 = C_1 + \frac{C_2}{N_c}$$

$$\alpha_2 = C_2 + \frac{C_1}{N_c}$$



Naive factorization

Topological amplitudes

$$\begin{aligned}
\sqrt{2}\,\mathcal{A}(B^- \to \pi^-\pi^0) &= V_{ub}V_{ud}^* \Big[\alpha_1 + \alpha_2 + \ldots\Big] A_{\pi\pi} \\
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\alpha_1 &= C_1 + \frac{C_2}{N_c} + \frac{\alpha_s}{4\pi} \left[C_2\, V^{(1)} \right. \\
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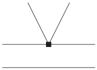

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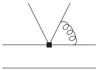

Vertex corrections

Topological amplitudes

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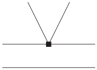

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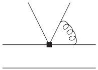

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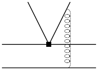

Hard spectator interactions

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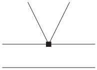

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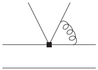
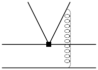


Vertex corrections



Hard spectator interactions

current effort !

(beyond BBNS 01)

Status of NNLO calculation

$$\alpha_i \Big|_{\text{NNLO}} = \frac{\alpha_s^2}{(4\pi)^2} \left[C_i\, V_1^{(2)} + C_{i\pm1}\, V_2^{(2)} + N_H \left(C_i\, H_1^{(2)} + C_{i\pm1}\, H_2^{(2)} \right) \right]$$

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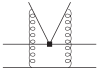
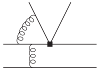
$H_i^{(2)}$: NLO hard spectator interactions (→ talk by S. Jäger, 2nd meeting)

1-loop calculation

2 perturbative scales $(m_b, \sqrt{\Lambda_{\text{QCD}} m_b})$

Status: tree amplitudes calculated [Beneke, Jäger 05; Kivel 06; Pilipp 06 (prel)]

(first results for penguin amplitudes) [Beneke, Jäger 06 (prel)]

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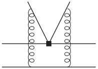
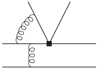

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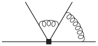
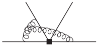
$V_i^{(2)}$: NNLO vertex corrections (→ this talk)

2-loop calculation

1 perturbative scale (m_b)

Status: imaginary part of tree amplitudes calculated [GB 06 (prel)]

Status of NNLO calculation

$$\alpha_i\big|_{\text{NNLO}} \;=\; \frac{\alpha_s^2}{(4\pi)^2}\;\left[C_i\,V_1^{(2)} + C_{i\pm1}\,V_2^{(2)} + N_H\left(C_i\,H_1^{(2)} + C_{i\pm1}\,H_2^{(2)}\right)\right]$$

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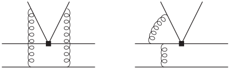

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2-loop calculation

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Normalization: $N_H \;=\; 4\pi^2\,\dfrac{f_B f_{M_1}}{N_c m_B \lambda_B F_+^{B\to M_1}(0)} \;\sim\; 0.5 - 1.5$

Input parameters and power-corrections

Non-perturbative input

- heavy-to-light form factor $F^{B\to M_1}(0)$
- distribution amplitude of light mesons

$$\phi_M(u;\mu) \;=\; 6u\bar{u}\;\left[1 + a_1^M(\mu)\,C_1^{(3/2)}(2u-1) + a_2^M(\mu)\,C_2^{(3/2)}(2u-1) + \ldots\right]$$

- *B* meson distribution amplitude

$$\frac{1}{\lambda_B(\mu)} \;=\; \int_0^\infty \frac{d\omega}{\omega}\;\phi_B(\omega;\mu) \qquad\qquad \sigma_n(\mu) \;=\; \lambda_B(\mu)\int_0^\infty \frac{d\omega}{\omega}\;\ln^n\frac{\mu}{\omega}\;\phi_B(\omega;\mu)$$

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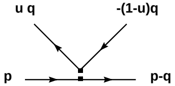
Power-corrections

- parametrically small $\sim \mathcal{O}\left(\frac{\Lambda_{QCD}}{m_b}\right)$
- in general non-factorizable $\rightarrow$ not accessible with same systematics
- BBNS include $\rightarrow$ fact. corrections — calculable in QCDF
  $\rightarrow$ non-fact. corrections — model-dependent estimate (X_H, X_A)

NNLO vertex corrections

Kinematics : 4-point function, only 2 linearly independent momenta



$$p^2 = m_b^2$$
$$q^2 = (p-q)^2 = 0$$

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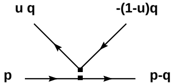

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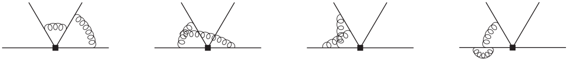
Examples:



$\sim$ 75 2-loop diagrams, 6 propagators (0-3 massive)

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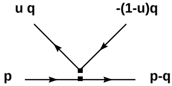

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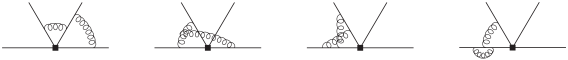

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Strategy:

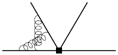
- general tensor decomposition $\longrightarrow$ scalar integrals
- automatized reduction algorithm

  $\sim$ 6.000 scalar integrals $\longrightarrow$ $\sim$ 30 Master Integrals
- calculation of Master Integrals

NNLO vertex corrections (c'td)

Characterization:

- $+$ only two physical scales
- $-$ complicated topologies



- $-$ combinatorics $\longrightarrow$ large number of MIs
- $-$ IR-structure $\sim$ $1/\varepsilon_{IR}^4$ (and $1/\varepsilon_{UV}^2$) $\longrightarrow$ calculate 5 coeffs of MIs

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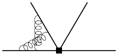

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Simplification: focus on imaginary part ($\rightarrow$ strong phases)

- extraction at level of Master Integrals
- less diagrams ($\sim$ 40), less Master Integrals (12)
- $e^{i\pi\varepsilon} = 1 + i\pi\varepsilon + \ldots$ $\rightarrow$ $i\pi/\varepsilon_{IR}^3$ $\rightarrow$ calculate 4 coeffs of MIs

NNLO vertex corrections (c'td)

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+ only two physical scales
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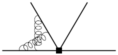

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Reminder:

$$\text{Im}[\alpha_1] = \frac{\alpha_s}{4\pi} \; \text{Im}\left\{ C_2 V^{(1)} + \frac{\alpha_s}{4\pi} \left[C_1 V_1^{(2)} + C_2 V_2^{(2)} + N_H \left(C_1 H_1^{(2)} + C_2 H_2^{(2)} \right) \right] \right\}$$

$$\text{Im}[\alpha_2] = \frac{\alpha_s}{4\pi} \; \text{Im}\left\{ C_1 V^{(1)} + \frac{\alpha_s}{4\pi} \left[C_2 V_1^{(2)} + C_1 V_2^{(2)} + N_H \left(C_2 H_1^{(2)} + C_1 H_2^{(2)} \right) \right] \right\}$$

$\rightarrow$ NLO complexity, inverted role of $C_1 \sim 1.1, C_2 \sim -0.2$

Reduction to Master Integrals

scalar integral in Dim Reg

$$I(p_i) = \int d^d k \; d^d l \quad \frac{\mathcal{S}_1^{m_1} \ldots \mathcal{S}_s^{m_s}}{\mathcal{P}_1^{n_1} \ldots \mathcal{P}_p^{n_p}} \qquad\qquad m_i \geq 0, \quad n_i \geq 1$$

defines unique topology (= interconnection of propagators and external momenta)

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- Integration by parts – identities [Chetyrkin, Tkachov 81]

$$\int d^d k \; d^d l \quad \frac{\partial}{\partial v^\mu} \; \frac{\mathcal{S}_1^{m_1} \ldots \mathcal{S}_s^{m_s}}{\mathcal{P}_1^{n_1} \ldots \mathcal{P}_p^{n_p}} = 0 \qquad\qquad v \in \{k, l\}$$

Reduction to Master Integrals

scalar integral in Dim Reg

$$I(p_i) = \int d^d k \; d^d l \quad \frac{\mathcal{S}_1^{m_1} \dots \mathcal{S}_s^{m_s}}{\mathcal{P}_1^{n_1} \dots \mathcal{P}_p^{n_p}} \qquad\qquad m_i \geq 0, \quad n_i \geq 1$$

defines unique topology (= interconnection of propagators and external momenta)

- Integration by parts – identities [Chetyrkin, Tkachov 81]

$$\int d^d k \; d^d l \quad \frac{\partial}{\partial v^\mu} \; \frac{\mathcal{S}_1^{m_1} \dots \mathcal{S}_s^{m_s}}{\mathcal{P}_1^{n_1} \dots \mathcal{P}_p^{n_p}} = 0 \qquad\qquad v \in \{k, l\}$$

- Lorentz invariance – identities [Gehrmann, Remiddi 00]

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$\rightarrow$ identities relate $I(p_i)$ to simpler and more complicated integrals

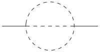
increasing m_i and n_i $\rightarrow$ identities $\gg$ integrals [Laporta 00]

here: $\mathcal{O}(10.000)$ identities $\rightarrow$ systematic solution (e.g. Laporta-algorithm)

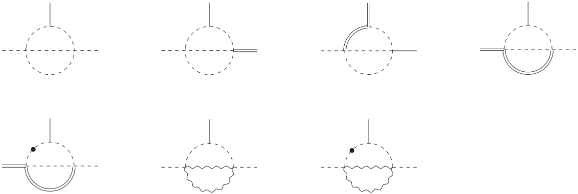
List of Master Integrals (Im part)

$t = 2$

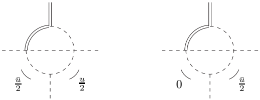
$t = 3$



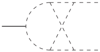
$t = 4$



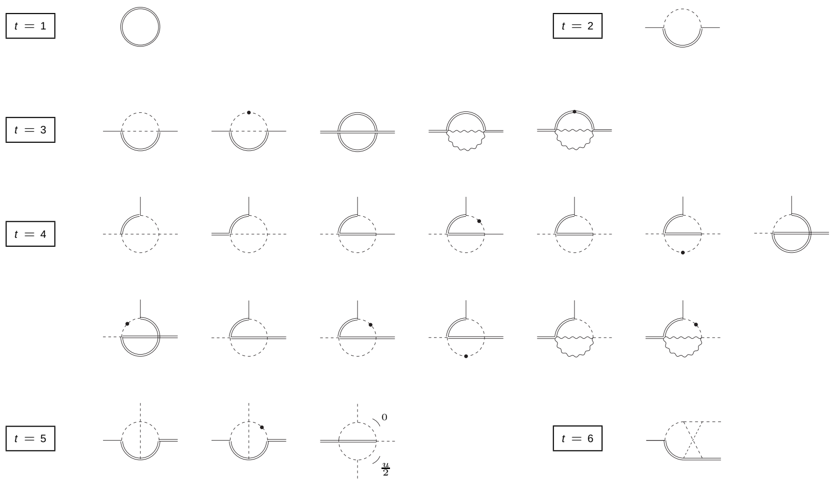
$t = 5$



$t = 6$



List of Master Integrals (Re part)



Calculation of Master Integrals

- Method of differential equations [Kotikov 91; Remiddi 97]

$$\frac{\partial}{\partial u} MI_i(u) = \int d^dk\, d^dl \quad \frac{\partial}{\partial u} \frac{\mathcal{S}_1^{m_1} \ldots \mathcal{S}_s^{m_s}}{\mathcal{P}_1^{n_1} \ldots \mathcal{P}_p^{n_p}}$$

$$\stackrel{red}{=} a(u;d)\, MI_i(u) + \sum_{j \neq i} b_j(u;d)\, MI_j(u)$$

inhomogeneity: MIs of simpler topologies

ansatz: $MI_i(u) = \sum_j c_{ij}(u)\, \varepsilon^j$ $\longrightarrow$ solve order by order in ε

main task: inhomogeneous equation $\longrightarrow$ Harmonic Polylogarithms

[Remiddi, Vermaseren 00]

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[Remiddi, Vermaseren 00]

- Calculation of boundary condition

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$\rightarrow$ Mellin-Barnes techniques [Smirnov 99; Tausk 99]

- Check with numerical method

$\rightarrow$ Method of sector decomposition [Binoth, Heinrich 04]

Example: 4-topology MI

$$\equiv \quad (m^2)^{-1-2\varepsilon} \left\{ \sum_{i=-2}^{2} c_i^{(45)} \, \varepsilon^i + \mathcal{O}(\varepsilon^3) \right\}$$

with

$$\begin{aligned}
c_{-2}^{(45)} &= -\frac{1}{u}, \\
c_{-1}^{(45)} &= -\frac{1}{u}\Big(2 - \ln u + i\pi\Big), \\
c_{0}^{(45)} &= \frac{1}{u}\left[\frac{\pi^2}{6} - 4 - \frac{1-2u}{2\bar{u}}\ln^2(u) + 2\ln u - i\pi\Big(2 - \frac{1-2u}{\bar{u}}\ln u\Big)\right], \\
c_{1}^{(45)} &= \frac{1}{u}\bigg\{\frac{\pi^2}{3} - 8 - 5\zeta_3 + \frac{1}{\bar{u}}\bigg[(1-2u)\Big(\mathrm{Li}_3(u) - \ln u \,\mathrm{Li}_2(u) - \frac{1}{2}\ln^2 u \ln \bar{u} - \ln^2(u)\Big) \\
&\quad + \frac{1-6u}{6}\ln^3(u) + \Big(4\bar{u} - \frac{(1-10u)\pi^2}{6}\Big)\ln u + \zeta_3\bigg] - i\pi\bigg[4 + \frac{1-6u}{2\bar{u}}\ln^2(u) \\
&\quad - \frac{1-2u}{\bar{u}}\Big(\mathrm{Li}_2(u) + \ln u \ln \bar{u} + 2\ln u - \frac{\pi^2}{6}\Big)\bigg]\bigg\},
\end{aligned}$$

$$\begin{aligned}
c_{2}^{(45)} &= \frac{1}{u}\bigg\{\frac{2\pi^2}{3} + \frac{17\pi^4}{60} - 16 - 10\zeta_3 + \frac{1}{\bar{u}}\bigg[2\zeta_3 - \frac{17\pi^4}{40} + 2(1+7u)\mathrm{Li}_4(u) - \frac{1-22u}{24}\ln^4(u) \\
&\quad - 3(1+2u)\ln u \,\mathrm{Li}_3(u) + (2-u)\ln^2(u)\mathrm{Li}_2(u) + \frac{(1-36u)\pi^2}{12}\ln^2(u) + (3-4u) \\
&\quad \Big(\mathrm{S}_{2,2}(u) - \ln u \,\mathrm{S}_{1,2}(u) + \ln \bar{u}(\mathrm{Li}_3(u) - \ln u \,\mathrm{Li}_2(u)) - \frac{1}{4}\ln^2(u)\ln^2 \bar{u} - \zeta_3 \ln \bar{u}\Big) \\
&\quad + 2(1-2u)\Big(\mathrm{Li}_3(u) - \ln u \,\mathrm{Li}_2(u) + \frac{5}{12}\ln^3(u)\ln \bar{u} - \frac{1}{2}\ln^2(u)\ln \bar{u} - \ln^2(u)\Big) \\
&\quad - \frac{(7-16u)\pi^2}{6}\Big(\mathrm{Li}_2(u) + \ln u \ln \bar{u}\Big) + \Big(8\bar{u} - \frac{(1-10u)\pi^2}{3} + 4(1+u)\zeta_3\Big)\ln u \\
&\quad + \frac{1-6u}{3}\ln^3(u)\bigg] + i\pi\bigg[4\zeta_3 - 8 - \frac{2\pi^2}{3} + \frac{1}{\bar{u}}\Big(3(1+2u)\mathrm{Li}_3(u) - 2(2-u)\ln u \,\mathrm{Li}_2(u) \\
&\quad + (3-4u)\Big(\mathrm{S}_{1,2}(u) + \ln \bar{u}\,\mathrm{Li}_2(u) + \frac{1}{2}\ln u \ln^2(\bar{u}) - \frac{\pi^2}{6}\ln \bar{u}\Big) + \frac{1-22u}{6}\ln^3(u) \\
&\quad + (1-2u)\Big(2\mathrm{Li}_2(u) - \frac{5}{2}\ln^2(u)\ln \bar{u} + 2\ln u \ln \bar{u} + 4\ln u\Big) - (1-6u)\ln^2(u) \\
&\quad + \frac{(1-8u)\pi^2}{6}\ln u + \frac{\pi^2}{3} - 8\zeta_3\Big)\bigg]\bigg\}.
\end{aligned}$$

Renormalization and IR-subtractions

- Renormalization

  counterterms: *QCD* and $\mathcal{H}_{eff}$

  subtleties: evanescent operators (NLO: standard, NNLO: complicated)

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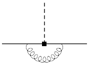
- IR-subtractions

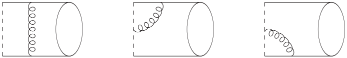
$$\langle Q_i \rangle^{ren} = F\ T_i * \phi$$

$$\to\ F^{(0)}\ T_i^{(2)} * \phi^{(0)} + F^{(1)}\ T_i^{(1)} * \phi^{(0)} + F^{(0)}\ T_i^{(1)} * \phi^{(1)} \qquad \text{IR divergent}$$

$F^{(1)}$ form factor subtraction

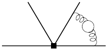


$\phi^{(1)}$ wave function subtraction



$\to$ cancelation of all UV- and IR-divergencies provides important cross-check!

Comparison with NLO and β_0-limit (preliminary)

- β_0-limit:  $n_f \longrightarrow -\frac{3}{2}\beta_0$ [Neubert, Pecjak 02; Burrell, Williamson 05]

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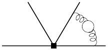
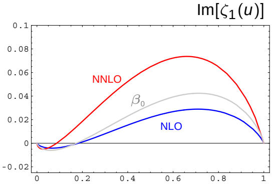
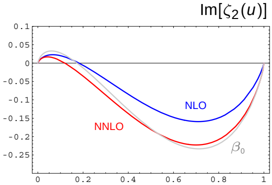
- β_0-limit:  $n_f \to -\frac{3}{2}\beta_0$ [Neubert, Pecjak 02; Burrell, Williamson 05]

- qualitative result (without NLO spectator scattering)

  shown as $\zeta_i(u) = C_j \; T_{ji}(u) \; \phi(u), \quad \to \quad \alpha_i = \int du \;\; \zeta_i(u),$





$\to$ $V^{(2)}$ adds construcively to NLO results, substantial contribution

$\to$ β_0-limit: fine for $\text{Im}[\alpha_2]$, fails for $\text{Im}[\alpha_1]$

remind: $\text{Im}[\alpha_1] \sim C_2$ in NLO!

Full NNLO result (preliminary)

input parameters:

$\Lambda^{\overline{\text{MS}}(5)}$	$m_b(m_b)$	$m_c(m_b)$	m_b^{pole}	f_B	$F_+^{B\to\pi}(0)$	λ_B	a_2^π (2 GeV)
0.225	4.2	1.3 ± 0.2	4.8	0.2 ± 0.03	0.28 ± 0.05	0.35 ± 0.15	0.1 ± 0.2

NNLO result:

	$V^{(1)}$	$V^{(2)}$	$H^{(2)}$	NNLO
	[BBNS 01]	[GB 06]	[BJ 05]	

$$\text{Im}[\alpha_1(\pi\pi)] = 0.012$$
$$\text{Im}[\alpha_2(\pi\pi)] = -0.077$$

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	$V^{(1)}$ [BBNS 01]		$V^{(2)}$ [GB 06]	$H^{(2)}$ [BJ 05]	NNLO
$\text{Im}[\alpha_1(\pi\pi)] =$	0.012	+	0.026		
$\text{Im}[\alpha_2(\pi\pi)] =$	−0.077	−	0.042		

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$\text{Im}[\alpha_1(\pi\pi)]$	$=$	0.012	$+$	0.026	$-$	0.028	$=$	0.010 ± 0.030
$\text{Im}[\alpha_2(\pi\pi)]$	$=$	-0.077	$-$	0.042	$+$	0.043	$=$	-0.076 ± 0.046

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- NNLO important, but accidental cancelation between $V^{(2)}$ and $H^{(2)}$
- dominant errors: a_2^π and λ_B [no estimate of power-corrections]

Conclusion

- on-going effort to calculate NNLO corrections in exclusive charmless B decays
- imaginary part of tree amplitudes completed: (preliminary)

$$\begin{aligned} \text{Im}[\alpha_1(\pi\pi)] &= 0.010 \pm 0.030 \\ \text{Im}[\alpha_2(\pi\pi)] &= -0.076 \pm 0.046 \end{aligned}$$

  essentially no correction to BBNS result due to a cancelation of $V^{(2)}$ and $H^{(2)}$

- no evidence for large strong phases
- individual NNLO contributions sizeable

  $\rightarrow$ motivates NNLO analysis of penguin amplitudes which are particularly important for CP violating observables