Relating partonic information to lattice QCD calculation

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Based on works done with Tomomi Ishikawa, Jian-Wei Qiu and Shinsuke Yoshida

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Outline

I. Introduction to PDFs

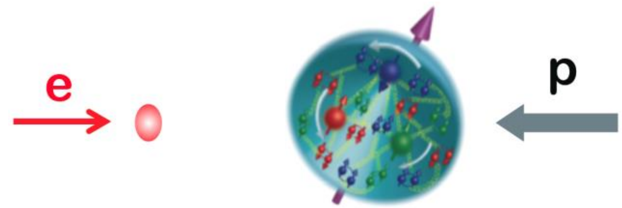
II. New ideas to calculate PDFs on lattice

III. A case study: Quasi PDFs

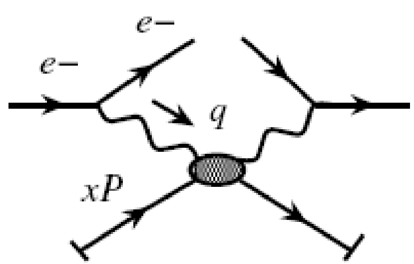
IV. Summary

QCD factorization

- The key and a first principle method to relate experimental data to QCD theory
- Electron-hadron:



$\sigma_{\text{tot}}^{\text{DIS}}$ =



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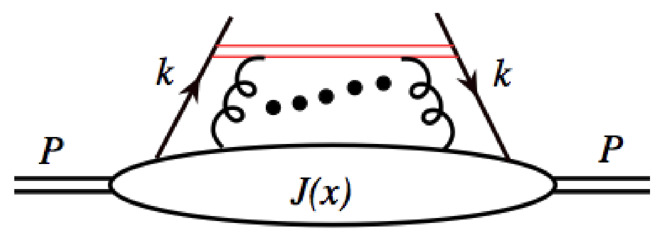


$+ O(\frac{1}{QR})$

Hard-part Probe	Parton-distribution Structure	Correction Approximation

Operator definition of PDFs

- Spin-averaged quark distribution

$$f_{q/p}(x,\mu^2) = \int \frac{d\xi_-}{4\pi} e^{-ix\xi_- P_+} \langle P|\overline{\psi}(\xi_-)\, \gamma_+ \exp\left\{-ig\int_0^{\xi_-} d\eta_- A_+(\eta_-)\right\} \psi(0)|P\rangle$$

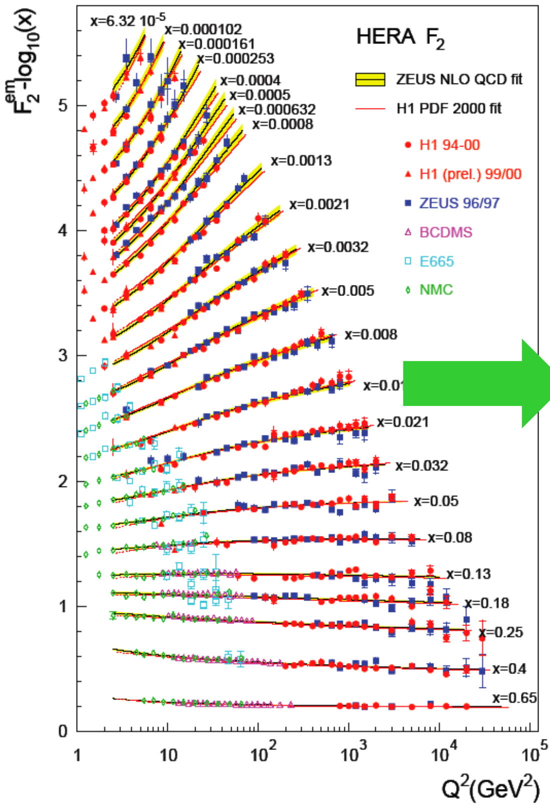
  - Simplest of all parton correlation functions
  - Not direct physical observable, like cross section; but well defined in QCD

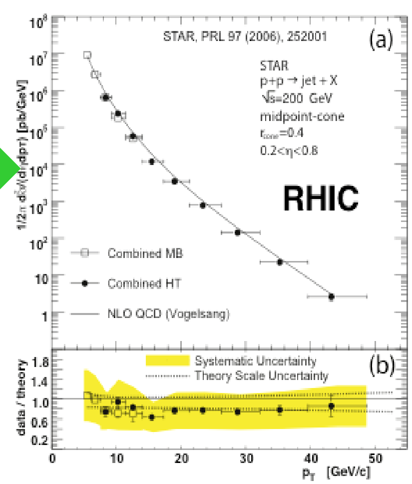
- Boost invariant along "+" direction
- Parton interpretation emerges in $A_+ = 0$ gauge
- Logarithmic UV divergent, renormalizable
- Time dependent!

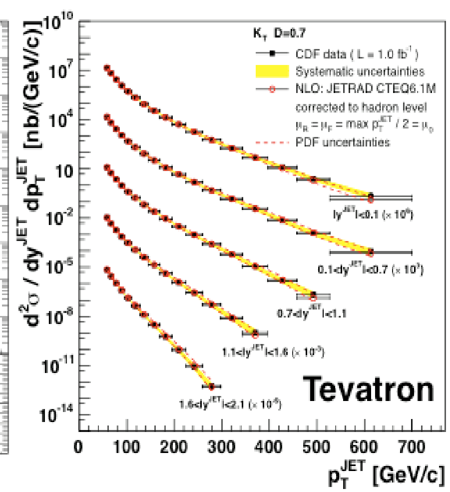
Extract PDFs by fitting data

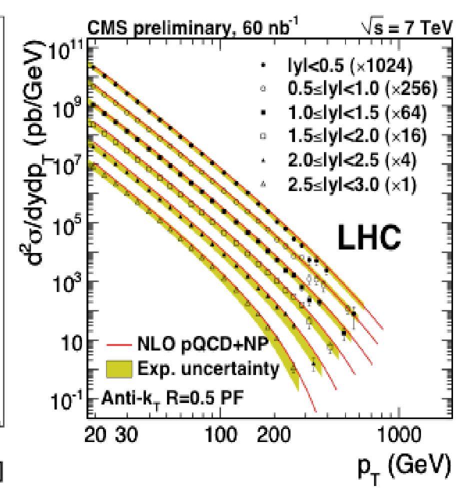
- **Successful**

Measure e-p at 0.3 TeV (HERA)
Predict p-p at 0.2, 1.96, and 7 TeV







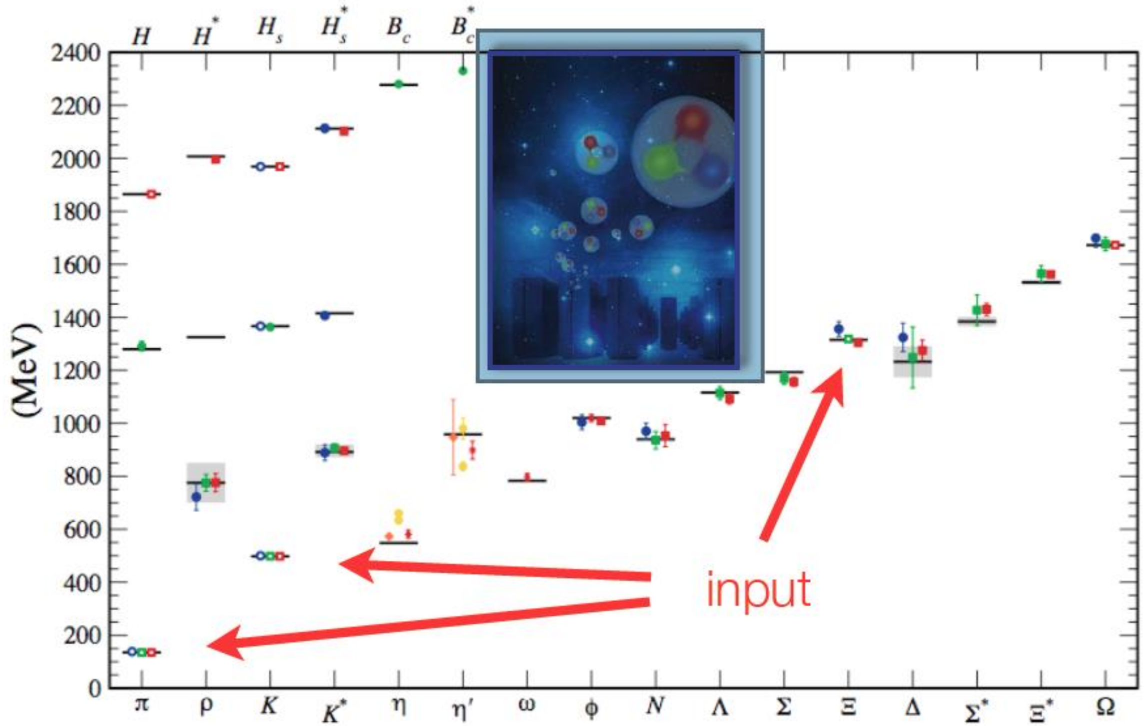


Question

Is it possible to determine PDFs nonperturbatively from first principle?

Lattice QCD

- The main nonperturbative approach to solve QCD
- Predict the hadron mass



- An intrinsically Euclidean time: $\tau = i\,t$

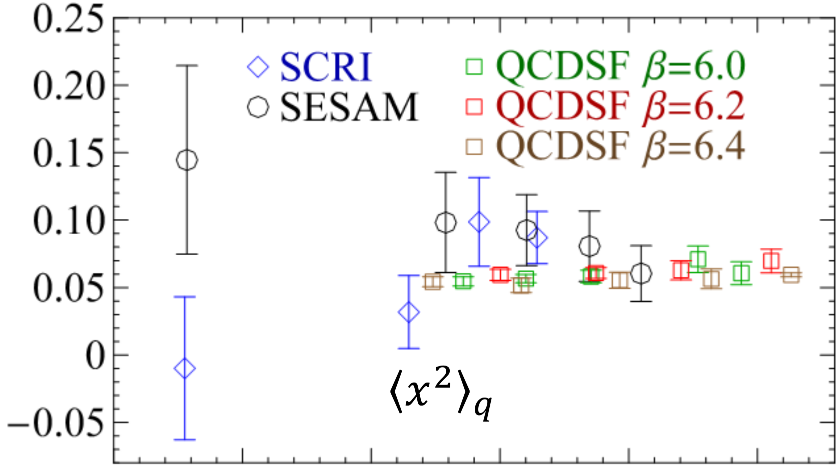
Cannot calculate PDFs directly

PDFs from lattice QCD

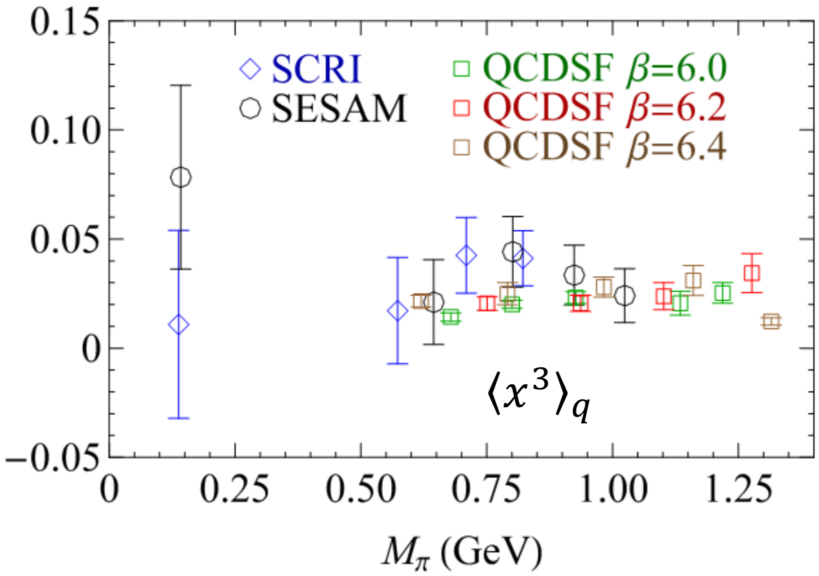
- **Moments: matrix elements of local operators**

$$\langle x^n(\mu^2)\rangle_q \equiv \int_0^1 dx\, x^n f_{q/p}(x,\mu^2)$$

- **Works, but only for limited moments**



Dolgov et al., hep-lat/0201021



Gockeler et al., hep-ph/0410187

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Ji's approach

➢ Quasi distribution:

Ji, 1305.1539, 1404.6680

$$\tilde{f}_{q/p}(x,\mu^2,P_z) = \int \frac{d\xi_z}{4\pi} e^{-ix\xi_z P_z} \langle P|\overline{\psi}(\xi_z)\,\gamma_z \exp\left\{-ig\int_0^{\xi_z} d\eta_z A_z(\eta_z)\right\}\psi(0)|P\rangle$$

➢ Features of quasi PDFs

- Fields separated along the z-direction
- No time dependence: calculable using standard lattice method
- Using OPE: quasi PDFs → normal PDFs, as $P_z \to \infty$.

➢ Proposed matching at finite P_z

$$\tilde{q}(x,\mu^2,P_z) = \int_x^1 \frac{dy}{y} Z\left(\frac{x}{y},\frac{\mu}{P_z}\right) q(y,\mu^2) + \mathcal{O}\left(\frac{\Lambda^2}{P_z^2},\frac{M^2}{P_z^2}\right)$$

Ma-Qiu's approach

YQM, Qiu, 1404.6860, 1412.2688

- "Lattice cross section": $\tilde{\sigma}_{\mathrm{E}}^{\mathrm{Lat}}(\tilde{x}, 1/a, P_z)$
  - Hadronic matrix element
  - $P_z \leftrightarrow \sqrt{s}$: "collision energy"
  - $1/a \leftrightarrow Q$: hard scale, resolution
  - $\tilde{x} \leftrightarrow x$: parameter

- Condition for a good "lattice cross section"

① Calculable on Euclidean lattice QCD

② UV and IR safe perturbatively (renormalizable)

③ CO divergence: factorizable (similar to DIS cross section)

$$\tilde{\sigma}_{\mathrm{M}}(\tilde{x}, \tilde{\mu}^2, P_z) \approx \sum_i \int_0^1 \frac{dx}{x} f_{i/h}(x, \mu^2)\, \mathcal{C}_i(\frac{\tilde{x}}{x}, \tilde{\mu}^2, \mu^2, P_z) + O(1/\mu^2)$$

Ji's approach V.S. Ma-Qiu's approach

➢ For quasi PDFs:

- **Ji's approach: large momentum effective field theory**

$$\tilde{q}(x,\mu^2,P_z) = \int_x^1 \frac{dy}{y} Z\left(\frac{x}{y},\frac{\mu}{P_z}\right) q(y,\mu^2) + \mathcal{O}\left(\frac{\Lambda^2}{P_z^2},\frac{M^2}{P_z^2}\right)$$

- **Ma-Qiu's approach: QCD factorization (if it is possible)**

$$\tilde{\sigma}_{\mathrm{M}}(\tilde{x},\tilde{\mu}^2,P_z) \approx \sum_i \int_0^1 \frac{dx}{x} f_{i/h}(x,\mu^2)\, \mathcal{C}_i(\frac{\tilde{x}}{x},\tilde{\mu}^2,\mu^2,P_z) + O(1/\mu^2)$$

Ji's approach V.S. Ma-Qiu's approach cont.

- **Ma-Qiu's approach has relaxed condition, beyond quasi-PDFs**

  - Any quantity calculated on lattice can be used to determine PDFs, as far as its CO structure can be expanded by PDFs
  - The quantity is not demanded to go to PDF in any limit

  Factorization is the essential question!

- **Ma-Qiu's approach is a generalization of Ji's**

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Quasi PDFs

- "Quasi quark" PDF as an example

$$\tilde{f}_{q/p}(x,\mu^2,P_z)=\int\frac{d\xi_z}{4\pi}e^{-ix\xi_z P_z}\langle P|\overline{\psi}(\xi_z)\,\gamma_z\exp\left\{-ig\int_0^{\xi_z}d\eta_z A_z(\eta_z)\right\}\psi(0)|P\rangle$$

- A good "lattice cross section"?

  ✓ No time dependence: calculable on Euclidean lattice

  ✓ IR divergence: cancelled by unitarity YQM, Qiu, 1404.6860, 1412.2688

  ? UV safe perturbatively: renormalizable?

  ? CO divergence: factorizable?

➢ Exploratory studies

Lin et al. 1402.1462
Alexandrou et al. 1504.07455

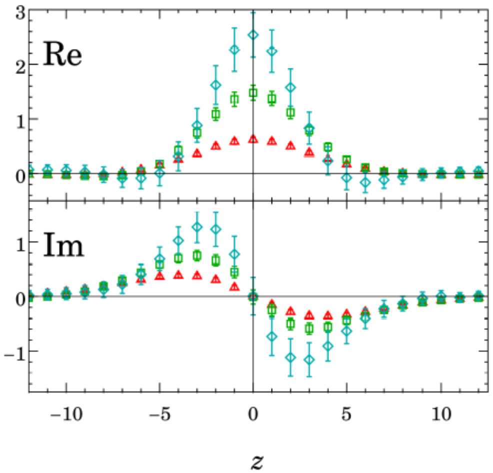


FIG. 1. The real (top) and imaginary (bottom) parts of the nonlocal isovector matrix element h of Eq. 3 computed on a lattice with the nucleon momentum P_z (in units of $2\pi/L$) = 1 (red triangles), 2 (green squares), 3 (cyan diamonds).

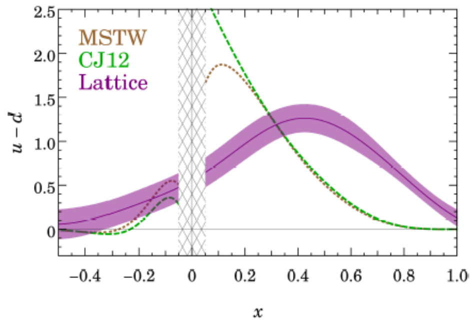


FIG. 2. The unpolarized isovector quark distribution $u(x) - d(x)$ computed on the lattice (purple band), compared with the global analyses by MSTW [13] (brown dotted line), and CTEQ-JLab (CJ12, green dashed line) [14] with medium nuclear correction near $(1.3\text{GeV})^2$. The negative x region is the sea quark distribution with $\bar{q}(x) = -q(-x)$.

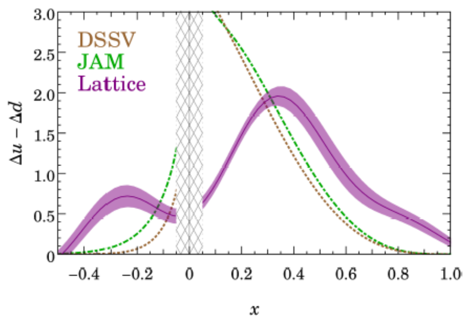


FIG. 3. (top) The isovector helicity distribution $\Delta u(x) - \Delta d(x)$ (purple band) computed on the lattice, along with selected global polarized analyses by JAM [19] (green dot-dashed) and DSSV09 [3] (brown dotted line). The corresponding sea-quark distributions are $\Delta\bar{q}(x) = \Delta q(-x)$.

- Works, good convergence
- Not consistent with experimental data
- Any problem with quasi PDFs?

Possible problems with quasi PDFs

➢ Renormalization

- Power UV divergent, nonlocal operator, renormalizable?
- Whether mixing with operators that has t dependence under renormalization? If yes, cannot calculate on lattice

➢ "Bad" large $\tilde{x}$ behavior

$$\tilde{f}_{i/p}(\tilde{x}, \tilde{\mu}^2, P_z) \approx \sum_j \int_0^1 \frac{dx}{x} \mathcal{C}_{ij}\left(\frac{\tilde{x}}{x}, \tilde{\mu}^2, \mu^2, P_z\right) f_{j/p}(x, \mu^2)$$

$$\text{as } x \to 0,\ f_{j/p}(x, \mu^2) \to x^{-\alpha} \text{ with } 1 < \alpha < 2$$

$$\mathcal{C}_{ij}^{(1)}(\tilde{x}/x, \tilde{\mu}^2, \mu^2, P_z) \text{ has } x/\tilde{x} \text{ behaviour as } x \to 0 \text{ in DR}$$

- Integration divergent, ruin factorization

Renormalization in coordinate space

➢ Coordinate space definition

$$\tilde{F}_{q/p}(\xi_z,\tilde{\mu}^2,P_z)=\langle h(P)|\overline{\psi}(\xi_z)\,\frac{\gamma_z}{2}\Phi^{(f)}_{n_z}(\{\xi_z,0\})\,\psi(0)|h(P)\rangle$$

➢ Why coordinate space

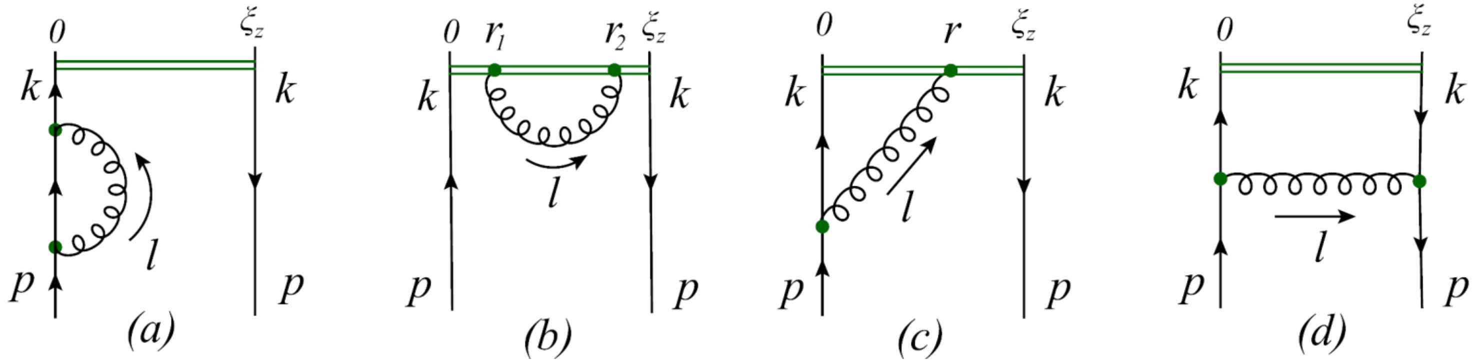
- "Bad" large x behavior corresponds to "bad" small ξ_z behavor: quasi PDFs are ill-defined as $\xi_z \to 0$
- No problem with finite ξ_z

➢ Go back to momentum space

- Further subtraction needed, like $\ln(\xi_z)$

Renormalize power UV div.

➢ Quasi quark PDFs in coordinate space

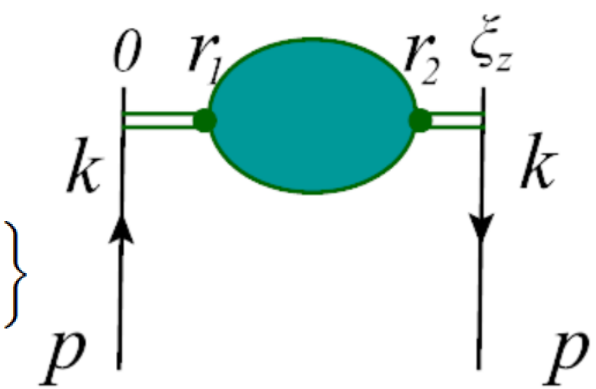


- **Power divergence: diagram (b)**

$$\int_0^{\xi_z} dr_1 \int_{r_1}^{\xi_z} dr_2 \frac{1}{(r_2-r_1)^2}$$

$$\int_0^{\xi_z-a} dr_1 \int_{r_1+a}^{\xi_z} dr_2 \frac{1}{(r_2-r_1)^2} = \frac{\xi_z}{a} - \ln\frac{\xi_z}{a} - 1 \;\Rightarrow\; \exp\left\{-c\frac{\xi_z}{a}\right\}$$

One loop **All order**



- **Well known, mass renormalization of test particle** **Dotsenko, Vergeles, NPB (1980)**

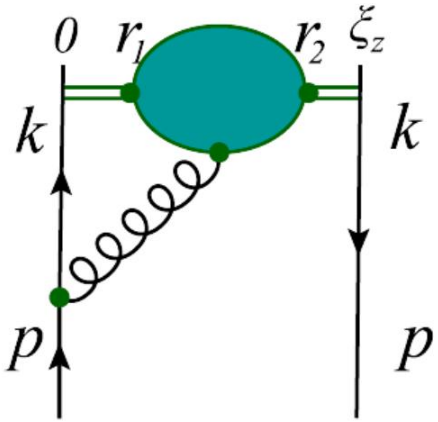
Log UV div.

- **Using DR after power divergence removed**

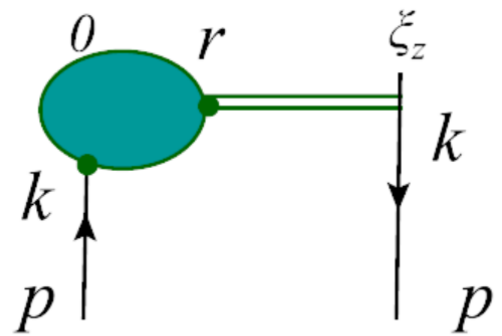
$$\int_0^{\xi_z} dr_1 \int_{r_1}^{\xi_z} dr_2 \frac{1}{(r_2-r_1)^{2-2\epsilon}} = \frac{1}{-2\epsilon}\frac{1}{1-2\epsilon}\xi_z^{2\epsilon} = \frac{1}{-2\epsilon} - \frac{1}{2}\ln(\xi_z^2) + O(\epsilon)$$

- Left over log divergence: come from end points of gauge link, independent of ξ_z

- **Additional log UV divergence**



renormalization of QCD Lagrangian



come from end points of gauge link

Renormalize log UV div.

- After power UV div. subtracted, and using renormalized QCD Lagrangian:
  - All UV divergences come from endpoints of gauge link: local

    nonlocal UV div. for normal PDFs
  - Renormalizable by a local UV counterterm

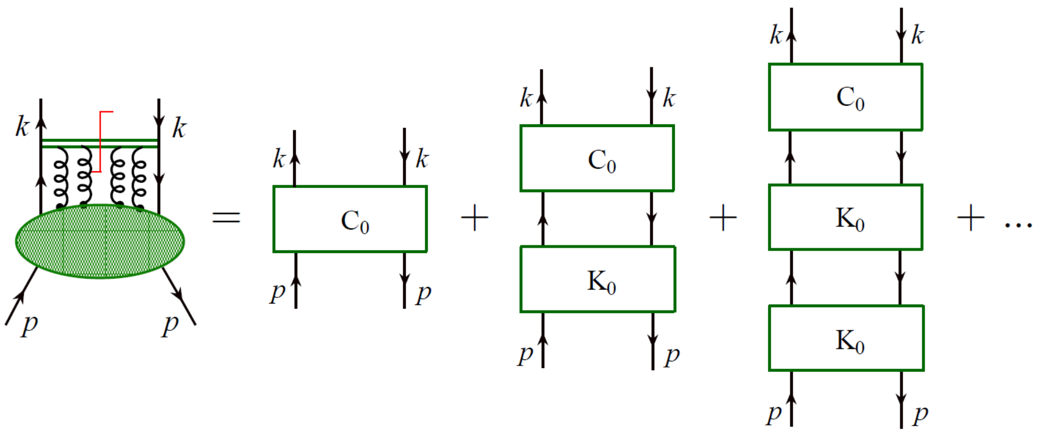
$$\tilde{F}^{R}_{i/p}(\xi_z, \tilde{\mu}^2, P_z) = Z_i \tilde{F}^{b}_{i/p}(\xi_z, \tilde{\mu}^2, P_z)$$

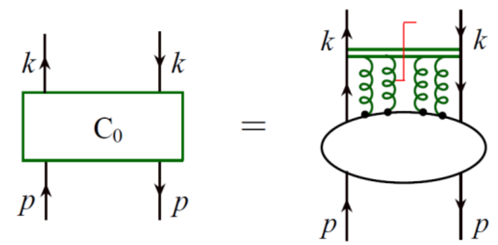
Z_i: constant

- Renormalization: multiplicative factor, no operator mixing
  - $\ln(\xi_z^2)$: divergent as $\xi_z \to 0$, freely subtracted

Ladder decomposition

➢ Generalized ladder diagrams decomposition





- C_0, K_0: **2PI kernels**
- **Ordering in virtuality** $p^2 \ll k^2 \sim \mu^2$

Factorization

- Using physical gauge, 2PI diagrams are finite

Ellis, Georgi, Machacek, Politzer, Ross, 1978, 1979

- Factorize the last kernel, and then recursively:

$\widehat{\mathcal{P}}$: pick up the singular part of integration

$$\tilde{f}_{q/p} = \lim_{m\to\infty} C_0 \sum_{i=0}^{m} K^i + \mathrm{UVCT}$$

$$= \lim_{m\to\infty} C_0 \left[1 + \sum_{i=0}^{m-1} K^i (1-\widehat{\mathcal{P}})K \right]_{\mathrm{ren}} + \tilde{f}_{q/p}\,\widehat{\mathcal{P}}\,K$$

$$= \lim_{m\to\infty} C_0 \left[1 + \sum_{i=1}^{m} \left[(1-\widehat{\mathcal{P}})K\right]^i \right]_{\mathrm{ren}} + \tilde{f}_{q/p}\,\widehat{\mathcal{P}}\,K\,,$$

$$\tilde{f}_{q/p} = \left[C_0 \frac{1}{1-(1-\widehat{\mathcal{P}})K} \right]_{\mathrm{ren}} \left[\frac{1}{1-\widehat{\mathcal{P}}K} \right]$$

Normal PDFs

All CO divergences of quasi PDF

Finite

$$\tilde{\sigma}_{\mathrm{M}}(\tilde{x}, \tilde{\mu}^2, P_z) \approx \sum_i \int_0^1 \frac{dx}{x} f_{i/h}(x, \mu^2)\, \mathcal{C}_i\!\left(\frac{\tilde{x}}{x}, \tilde{\mu}^2, \mu^2, P_z\right) + O(1/\mu^2)$$

- Modified qusi PDFs: good “lattice cross section”

One loop example: quark→quark

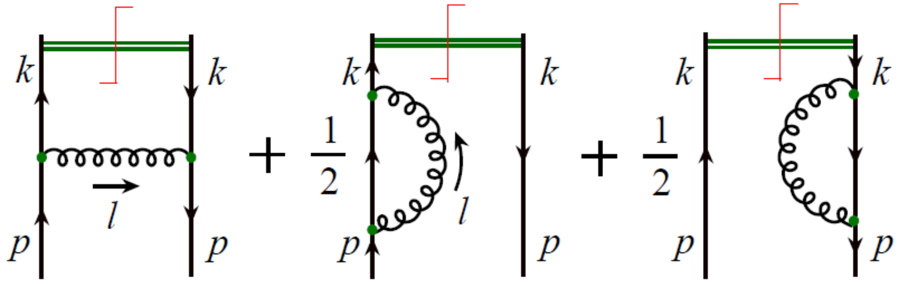
- **Expand the factorization formula**

$$\tilde{f}^{(1)}_{q/q}(\tilde{x}) = f^{(0)}_{q/q}(x) \otimes \mathcal{C}^{(1)}_{q/q}(\tilde{x}/x) + f^{(1)}_{q/q}(x) \otimes \mathcal{C}^{(0)}_{q/q}(\tilde{x}/x)$$

$$\Rightarrow \quad \mathcal{C}^{(1)}_{q/q}(t, \tilde{\mu}^2, \mu^2, P_z) = \tilde{f}^{(1)}_{q/q}(t, \tilde{\mu}^2, P_z) - f^{(1)}_{q/q}(t, \mu^2)$$

- **Feynman diagrams**

  Same diagrams for both, but with different gauge



- **Gauge choice**

$A_z = 0$ for $\tilde{f}_{q/q}$

Gluon propagator:

$$\tilde{d}^{\alpha\beta}(l) = -g^{\alpha\beta} + \frac{l^\alpha n_z^\beta + n_z^\alpha l^\beta}{l_z} - \frac{n_z^2 l^\alpha l^\beta}{l_z^2}$$

$A_+ = 0$ for $f_{q/q}$

$$d^{\alpha\beta}(l) = -g^{\alpha\beta} + \frac{l^\alpha n^\beta + n^\alpha l^\beta}{l_+}$$

One-loop expression

- After the integration of energy component by using residue theory

$$\tilde{f}^{(1)}_{q/q}(\tilde{x},\tilde{\mu}^2,P_z) = C_F\frac{\alpha_s}{2\pi}\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}\int_0^{\tilde{\mu}^2}\frac{dl_\perp^2}{l_\perp^{2+2\epsilon}}\int_{-\infty}^{+\infty}\frac{dl_z}{P_z}\left[\delta\left(1-\tilde{x}-y\right)-\delta\left(1-\tilde{x}\right)\right]\left\{\frac{1}{y}\left(1-y+\frac{1-\epsilon}{2}y^2\right)\right.$$

$$\left.\times\left[\frac{y}{\sqrt{\lambda^2+y^2}}+\frac{1-y}{\sqrt{\lambda^2+(1-y)^2}}\right]+\frac{(1-y)\lambda^2}{2y^2\sqrt{\lambda^2+y^2}}+\frac{\lambda^2}{2y\sqrt{\lambda^2+(1-y)^2}}+\frac{1-\epsilon}{2}\frac{(1-y)\lambda^2}{[\lambda^2+(1-y)^2]^{3/2}}\right\}$$

where $y = l_z/P_z,\ \lambda^2 = l_\perp^2/P_z^2,\ C_F = (N_c^2-1)/(2N_c)$

- Cancellation of CO divergence

$$\frac{y}{\sqrt{\lambda^2+y^2}}+\frac{1-y}{\sqrt{\lambda^2+(1-y)^2}} = 2\theta(0<y<1)-\left[\mathrm{Sgn}(y)\frac{\sqrt{\lambda^2+y^2}-|y|}{\sqrt{\lambda^2+y^2}}+\mathrm{Sgn}(1-y)\frac{\sqrt{\lambda^2+(1-y)^2}-|1-y|}{\sqrt{\lambda^2+(1-y)^2}}\right]$$

Only the first term is CO divergent for, which is the same as normal PDF - necessary!

One-loop coefficient functions

➢ $\overline{MS}$ **scheme for normal PDF**

$$\mathcal{C}^{(1)}_{q/q}(t,\tilde{\mu}^2,\mu^2,P_z)=\tilde{f}^{(1)}_{q/q}(t,\tilde{\mu}^2,P_z)-f^{(1)}_{q/q}(t,\mu^2)$$

$t=\tilde{x}/x$

$$\frac{\mathcal{C}^{(1)}_{q/q}(t)}{C_F\frac{\alpha_s}{2\pi}}=\left[\frac{1+t^2}{1-t}\ln\frac{\tilde{\mu}^2}{\mu^2}+1-t\right]_+ +\left[\frac{t\Lambda_{1-t}}{(1-t)^2}+\frac{\Lambda_t}{1-t}\right.$$

$$\left.+\frac{\mathrm{Sgn}(t)\Lambda_t}{\Lambda_t+|t|}-\frac{1+t^2}{1-t}\left[\mathrm{Sgn}(t)\ln\left(1+\frac{\Lambda_t}{2|t|}\right)+\mathrm{Sgn}(1-t)\ln\left(1+\frac{\Lambda_{1-t}}{2|1-t|}\right)\right]\right]_N$$

where $\Lambda_t=\sqrt{\tilde{\mu}^2/P_z^2+t^2}-|t|$, $\mathrm{Sgn}(t)=1$ if $t\geq 0$, and -1 otherwise

Coefficient functions for all partonic channels are free of CO div.

To do list

- **Additional matching:**

$$\tilde{\sigma}_{\mathrm{E}}^{\mathrm{Lat}}(\tilde{x}, 1/a, P_z) \stackrel{\mathcal{Z}}{\longleftrightarrow} \tilde{\sigma}_{\mathrm{E}}(\tilde{x},\tilde{\mu}^2, P_z)$$

$$\Updownarrow$$

$$\tilde{\sigma}_{\mathrm{M}}(\tilde{x},\tilde{\mu}^2, P_z) \stackrel{\mathcal{C}}{\longleftrightarrow} f_{i/h}(x, \mu^2)$$

- **Lattice perturbation theory**
- **Nonperturbative matching**

In progress

Summary

- "Lattice cross section" = hadronic matrix elements that are calculabe on lattice QCD + factorizable to partonic structure functions

  E.g. Modified quasi PDFs

- Find more good "Lattice cross section", PDFs extracted by global fit of these data

- Find good "lattice cross section" for other nonperturbative quantities: GPDs, TMDs, ...

- Lattice QCD can calculate partonic structure functions now, but, more works are needed!

Thank you!