Proton spin decomposition from lattice QCD

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August 2016

Proton Spin decomposition

The quantities in the light cone gauge

R. L. Jaffe and A. V. Manohar, Nucl. Phys. B 337, 509 (1990).

$$J = \frac{1}{2}\vec{\Sigma}_q + \vec{L}_q + \Delta G + L_g$$

quark spin

quark OAM

$$= \int d^3x \, \frac{1}{2} \, \overline{\psi} \, \vec{\gamma} \gamma^5 \, \psi + \int d^3x \psi^\dagger \left\{\vec{x} \times (i\vec{\nabla})\right\} \psi$$

$$+ \int d^3x 2\mathrm{Tr}[\vec{E} \times \vec{A}] + \int d^3x 2\mathrm{Tr}[E^i \vec{x} \times \vec{\nabla} A^i]$$

glue helicity

glue OAM

From the non-symmetric canonical energy momentum tensor.
Directly related to the experiment, but can't be calculated on lattice, except the quark spin

Proton Spin decomposition

Frame independent decomposition

X.D. Ji., Phys. Rev. Lett. 78, 610–613 (1997).

$$\vec{J} = \int d^3x\, \bar{\psi} \left\{ \vec{x} \times \tfrac{i}{2}(\gamma^4 \vec{D} + \vec{\gamma} D^4) \right\} \psi + \int d^3x 2\{\vec{x} \times \mathrm{Tr}[\vec{E} \times \vec{B}]\}$$

quark AM — *glue AM*

$$= \int d^3x\, \tfrac{1}{2}\, \overline{\psi}\, \vec{\gamma} \gamma^5\, \psi + \int d^3x \psi^\dagger \left\{ \vec{x} \times (i\vec{D}) \right\} \psi + \int d^3x 2\{\vec{x} \times \mathrm{Tr}[\vec{E} \times \vec{B}]\}$$

quark spin — *quark OAM* — *glue AM*

From the symmetric energy momentum tensor, gauge invariant, frame independent, and well defined on the lattice

In this talk,

I will focus on:

- *The quark/glue angular momentum in proton*

$$\vec{J} = \int d^3x\, \bar{\psi}\, \{\vec{x} \times \tfrac{i}{2}(\gamma^4 \vec{D} + \vec{\gamma} D^4)\}\, \psi + \int d^3x 2\{\vec{x} \times \mathrm{Tr}[\vec{E} \times \vec{B}]\}$$

- *The glue spin and helicity*

$$S_g = \int d^3x 2\mathrm{Tr}[\vec{E} \times \vec{A^{phys}}], \quad \lim_{P^z \to \infty} S_g^z \to \Delta G$$

Outline

- *The proton spin decomposition*
  Two types of decompositions.
- ***The quark/glue angular momentum in proton***
  The preliminary results and the perturbative matching to the MS-bar scheme.
- *The glue spin and helicity*
  The glue spin based on Chen's decomposition and connection to the glue helicity.

Proton Spin decomposition

Calculation through the EMT form factors

X.D. Ji., Phys. Rev. Lett. 78, 610–613 (1997).

Ji's angular momentum (AM) can be written in terms of the symmetrized energy momentum tensor (EMT) as,

$$J^{q,g} = \langle p, s| \int d^3x \, x \times \mathcal{T}^{\{0i\}q,g} |p, s\rangle, \qquad \mathcal{T}^{\{0i\}q} = \frac{1}{4}\bar{\psi}\gamma^{(0}\overleftrightarrow{D}^{i)}, \ \mathcal{T}^{\{0i\}g} = \vec{E} \times \vec{B}.$$

, with the form factors of the off-diagonal part of EMT defined by,

$$(p', s'|\mathcal{T}^{\{0i\}q,g}|p, s) = \left(\frac{1}{2}\right) \bar{u}(p', s') \left[T_1(q^2)(\gamma^0\bar{p}^i + \gamma^i\bar{p}^0) + \frac{1}{2m} T_2(q^2) \left(\bar{p}^0(i\sigma^{i\alpha}) + \bar{p}^i(i\sigma^{0\alpha})\right) q_\alpha + \frac{1}{m} T_3(q^2) q^0 q^i \right]^{q,g} u(p, s),$$

Ji's quark and glue AM correspond to the forward limit of the form factor combination,

$$J^{q,g} = \frac{1}{2}\left[T_1(0) + T_2(0)\right]^{q,g}$$

Proton Spin decomposition

The lattice ensembles

Smaller lattice spacing

L ~ 4.3 fm
m_π ~ 170 MeV
32^3 x 64, a =0.143 fm

L ~ 2.8 fm
m_π ~ 330 MeV
24^3 x 64, a =0.111 fm

L ~ 5.6 fm
m_π ~ 140 MeV
48^3 x 96, a =0.114 fm

Larger Volume

Lighter sea quark

L ~ 2.8 fm
m_π ~ 300 MeV
32^3 x 64, a =0.086 fm

L ~ 2.0 fm
m_π ~ 370 MeV
32^3 x 64, a =0.063 fm

2+1 flavor DWF configurations (RBC-UKQCD)

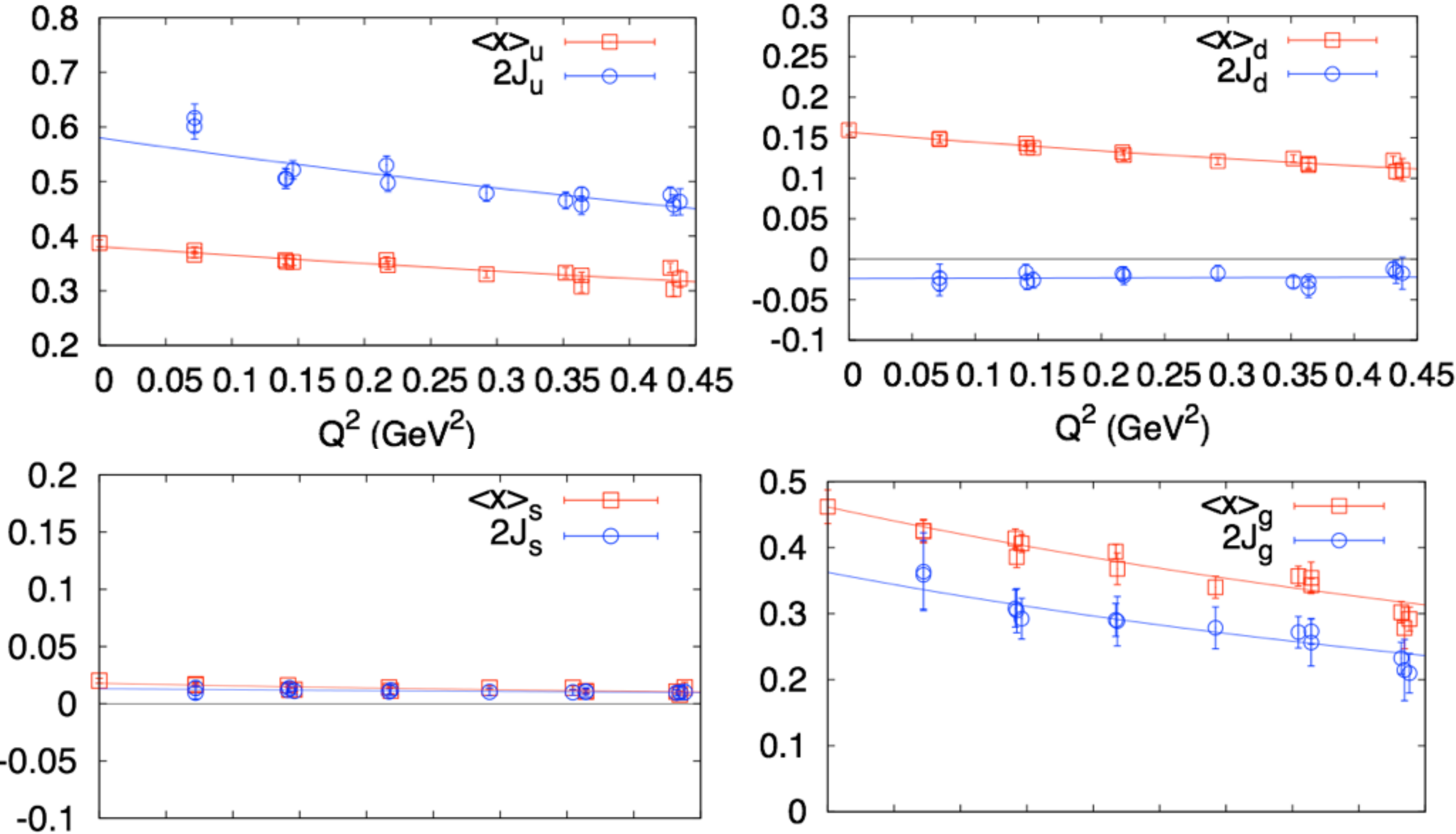
T. Blum et al. (RBC, UKQCD), Phys. Rev. D93, 074505 (2016)

Y. Yang, et al, χQCD Collaboration, in progress.

Quark and glue angular momentums

m_π=400 MeV

The bare results on lattice

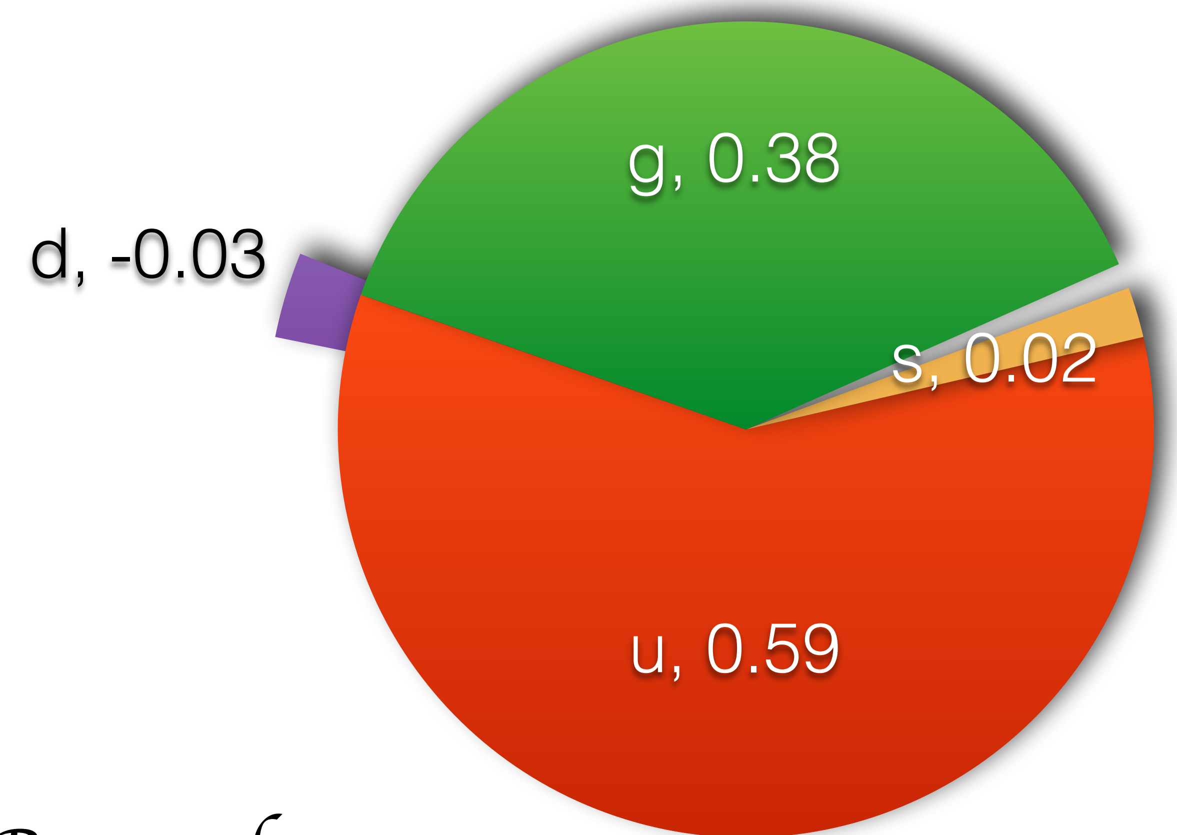


Next steps?

- *Repeat the calculation on the other ensembles to access the systematic uncertainty (lattice spacing, volume, sea quark mass, etc.)*

Costly but the framework has been set up.

- ***Matching the lattice bare results to that under MS-bar scheme at 2GeV.***



Bare values

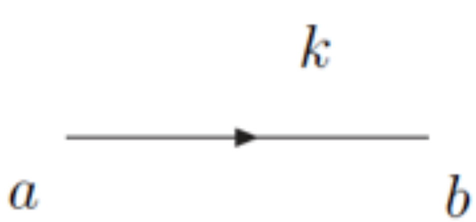
A non-trivial lattice perturbative calculation (will be addressed in the following a few pages).

The Feynman rules of LatPT with the extra vertices

See
S. Capitani, Phys.Rept. 382 (2003) 113-302,
as example

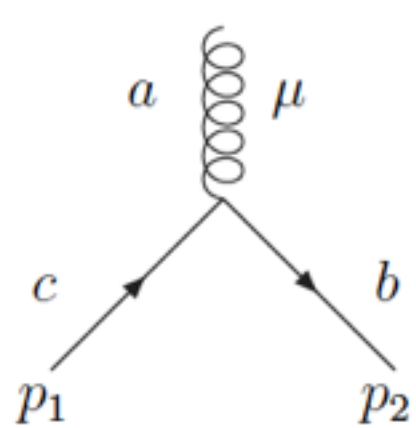
Taking the simplest Wilson fermion as example,

$$S_W = a^4 \sum_x \Big[-\frac{1}{2a} \sum_\mu \big[\overline{\psi}(x)(r-\gamma_\mu)U_\mu(x)\psi(x+a\hat{\mu}) + \overline{\psi}(x+a\hat{\mu})(r+\gamma_\mu)U_\mu^\dagger(x)\psi(x) \big] + \overline{\psi}(x)\left(m_0 + \frac{4r}{a}\right)\psi(x) \Big]$$



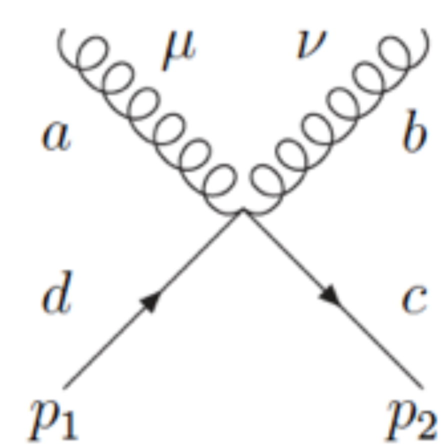
$$\delta^{ab} \cdot a \frac{-\mathrm{i}\sum_\mu \gamma_\mu \sin ak_\mu + am_0 + 2r\sum_\mu \sin^2 \frac{ak_\mu}{2}}{\sum_\mu \sin^2 ak_\mu + \left(2r\sum_\mu \sin^2 \frac{ak_\mu}{2} + am_0\right)^2}$$

Ordinary vertices



$$-g_0(T^a)^{bc}\left(\mathrm{i}\gamma_\mu \cos\frac{a(p_1+p_2)_\mu}{2} + r\sin\frac{a(p_1+p_2)_\mu}{2}\right)$$

Extra vertex

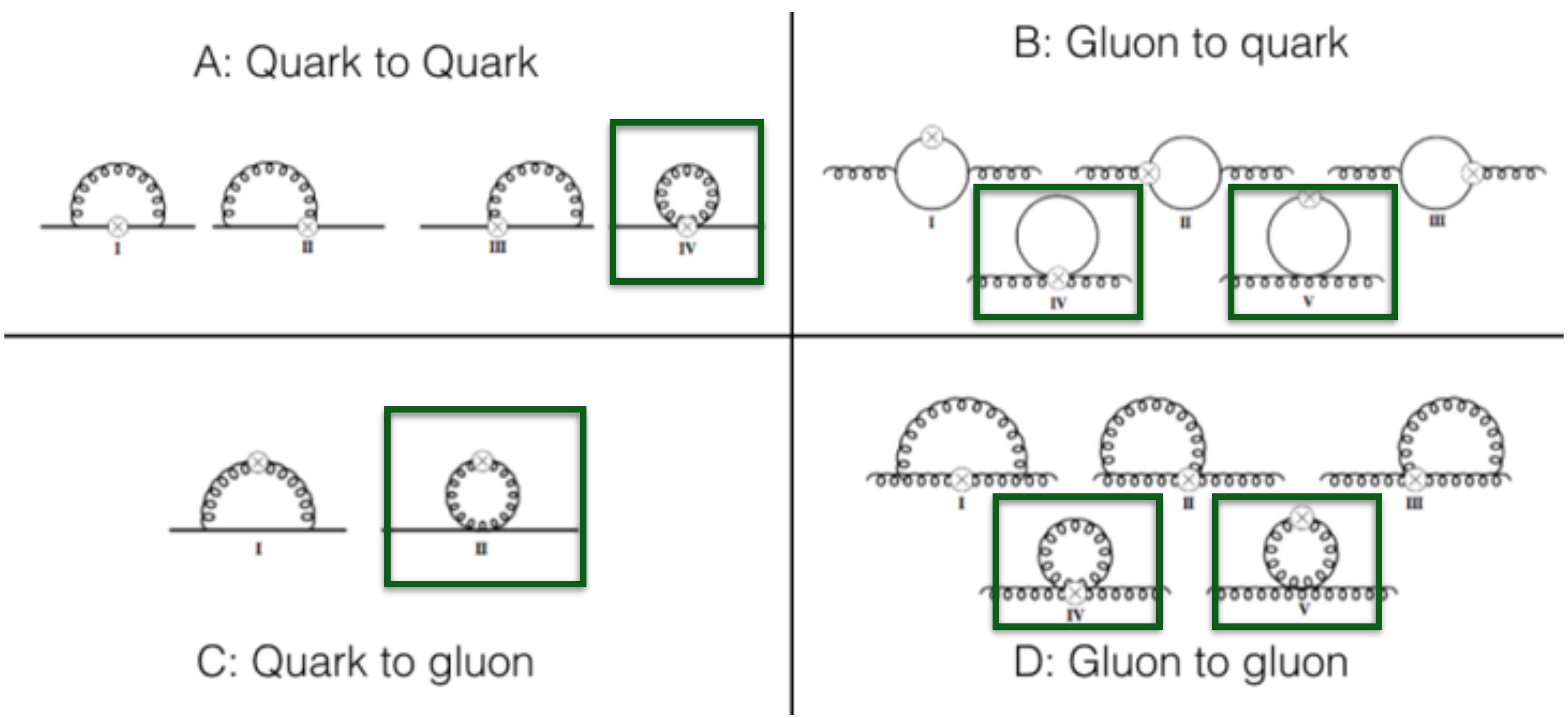


$$-\frac{1}{2}ag_0^2\,\delta_{\mu_1\mu_2}\left(\frac{1}{N_c}\delta^{ab} + d^{abe}T^e\right)^{cd} \left(-\mathrm{i}\gamma_\mu \sin\frac{a(p_1+p_2)_\mu}{2} + r\cos\frac{a(p_1+p_2)_\mu}{2}\right)$$

The renormalization of AM for quark and glue

See
S. Capitani and G. Rossi, Nuclear Physics B 433 (1995) 351–389, as example

Diagrams don’t exist or contribute in the continuum



The renormalization under MS-bar scheme
for the lattice bare quantities

The renormalization of the quark EMT $\mathcal{T}^{\{0i\}q} = \frac{1}{4}\bar{\psi}\gamma^{(0}\overleftrightarrow{D}^{i)}$ with the lattice regularization and under RI-MOM scheme is,

$$Z_L^{MOM} = 1 - \frac{g^2 C_F}{16\pi^2}[\frac{8}{3}\log(a^2p^2) + B_{QQ} + \xi] + O(g^4),$$

where B_{QQ} with $B_{QQ}|_{a\to 0} \neq 0$ is the gauge independent finite piece which is sensitive to the lattice quark and gluon actions.

The continuum field renormalization with the dimensional regularization and under RI-MOM and $\overline{MS}$ scheme is,

$$\begin{aligned} Z_{DR}^{\overline{MS}} &= 1 + \frac{g^2 C_F}{16\pi^2}[\frac{8}{3}\frac{1}{\epsilon}] + O(g^4), \\ Z_{DR}^{MOM} &= 1 + \frac{g^2 C_F}{16\pi^2}[\frac{8}{3}\frac{1}{\epsilon} + \frac{8}{3}\log(\mu^2/p^2) + \frac{40}{9} - \xi] + O(g^4). \end{aligned}$$

So the final renormalization under $\overline{MS}$ scheme for the lattice quantity is,

$$\begin{aligned} Z_L^{\overline{MS}}(a,\mu) &= \frac{Z_{DI}^{\overline{MS}}}{Z_{DI}^{MOM}} Z_L^{MOM}(a,\mu) \\ &= 1 - \frac{g^2 C_F}{16\pi^2}[\frac{8}{3}\log(a^2\mu^2) + \frac{40}{9} + B_{QQ}] + O(g^4). \end{aligned}$$

The renormalization of AM

the formulas

From the lattice bare quantities to that under the MOM scheme,

$$\begin{pmatrix} O^{MOM}_{Q,(1)} \\ O^{MOM}_{G,(1)} \end{pmatrix} = \begin{pmatrix} 1-\frac{g^2C_F}{16\pi^2}[\frac{8}{3}\log(a^2p^2)+B_{QQ}+\xi] & +\frac{g^2N_f}{16\pi^2}[\frac{2}{3}\log(a^2p^2)+B_{GQ}] \\ +\frac{g^2C_F}{16\pi^2}[\frac{8}{3}\log(a^2p^2)+B_{QG}] & 1-\frac{g^2N_f}{16\pi^2}[\frac{2}{3}\log(a^2p^2)+B^f_{GG}]-\frac{g^2N_c}{16\pi^2}[B_{GG}+2\xi-\frac{\xi^2}{4}] \end{pmatrix} \begin{pmatrix} O^{L}_{Q,(1)} \\ O^{L}_{G,(1)} \end{pmatrix}$$
$$+O(g^2)O_{E.O.M.}+O(g^2)O_{G.V.}+O(g^4)$$

From the MOM scheme to the MS-bar scheme,

$$\begin{pmatrix} O^{\overline{MS}}_{Q,(1)} \\ O^{\overline{MS}}_{G,(1)} \end{pmatrix} = \begin{pmatrix} 1-\frac{g^2C_F}{16\pi^2}[\frac{8}{3}\log(\mu^2/p^2)+\frac{40-9\xi}{9}] & +\frac{g^2N_f}{16\pi^2}[\frac{2}{3}\log(\mu^2/p^2)+\frac{4}{9}] \\ +\frac{g^2C_F}{16\pi^2}[\frac{8}{3}\log(\mu^2/p^2)+\frac{22}{9}] & 1-\frac{g^2N_f}{16\pi^2}[\frac{2}{3}\log(\mu^2/p^2)+\frac{10}{9}]-\frac{g^2N_c}{16\pi^2}(\frac{4}{3}-2\xi+\frac{\xi^2}{4}) \end{pmatrix} \begin{pmatrix} O^{MOM}_{Q,(1)} \\ O^{MOM}_{G,(1)} \end{pmatrix}$$
$$+O(g^2)O_{E.O.M.}+O(g^2)O_{G.V.}+O(g^4)$$

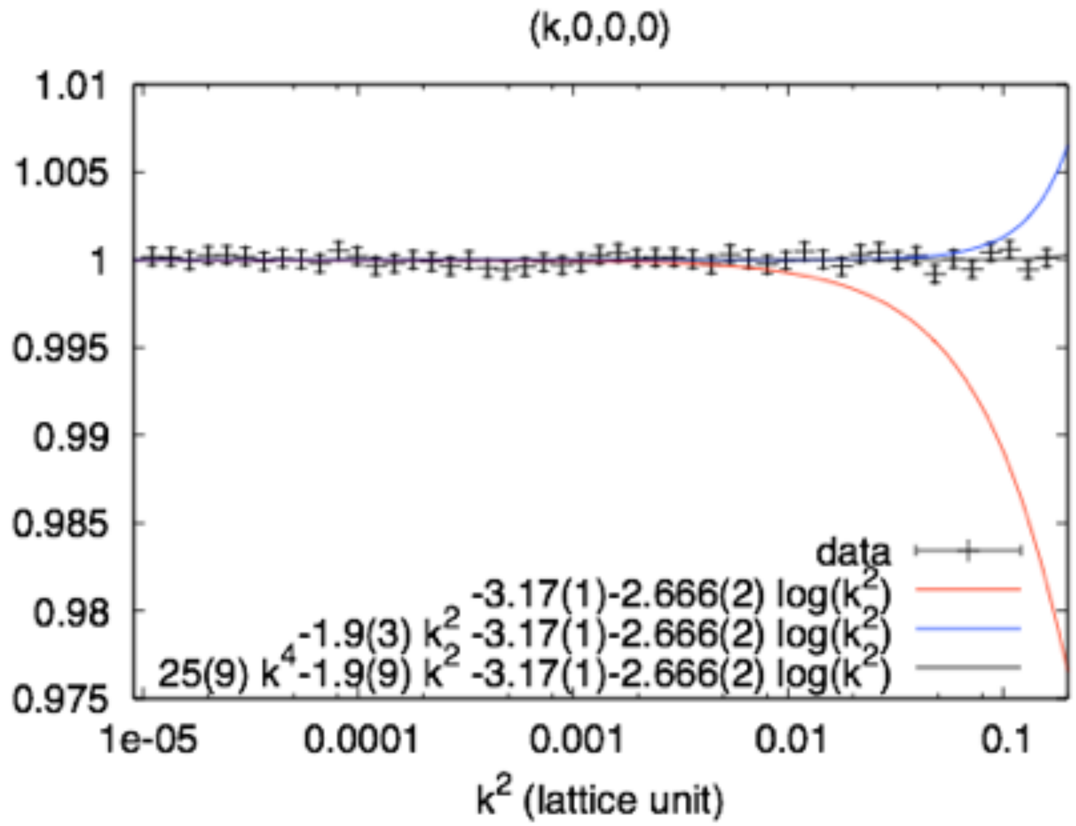
Based on Package-X described in H. H. Patel, Comput.Phys.Commun. 197 (2015) 276-290

B_{XY} are sensitive to the fermion and gauge action, but ξ independent. One can focus on the case under the Feynman gauge to simplify the calculation.

Do the loop integration numerically…

$$Z_L^{MOM} = 1 - \boxed{\frac{g^2 C_F}{16\pi^2}}\left[\frac{8}{3}\log(a^2p^2) + B_{QQ}\right] + O(g^4)$$

~0.02



The ratio of the fit v.s. the numerical integration on different $k^2 = a^2p^2$

- With the higher order of a^2p^2, the larger a^2p^2 region can be well described with the constant part unchanged.
- $B_{QQ}|_{a\to 0}$=3.17(1) is precise enough given our statistical error in the simulation.
- We will focus on the constant part of B_{XY} in the following discussions.

The finite pieces

with kinds of actions

$$Z_L^{\overline{MS}}(a,\mu) = \frac{Z_{DI}^{\overline{MS}}}{Z_{DI}^{MOM}} Z_L^{MOM}(a,\mu) = (1 - \frac{g^2 C_F}{16\pi^2}[\frac{8}{3}\log(a^2\mu^2) + \frac{40}{9} + B_{QQ}] + O(g^4)$$

B_{QQ}	Wilson	Iwasaki	Iwasaki^{HYP}
wilson	-3.17	-2.59	-1.53
overlap	-34.90	-18.83	-4.89
D_c	-42.10	-24.25	-8.63

The gluon actions

The quark actions

- The values are sensitive to both the quark and gluon actions.
- The values with the unimproved Wilson glue action can be very large.
- The HYP smearing can make the values smaller and become less sensitive to the quark action.

The renormalization of AM

the results

From the lattice bare quantities with the chiral fermion and HYP smeared Iwasaki gluon to that under the MS-bar scheme,
at a scale μ=1/a,

$$\begin{pmatrix} O^{\overline{MS}}_{Q,(1)} \\ O^{\overline{MS}}_{G,(1)} \end{pmatrix} = \begin{pmatrix} 1-\frac{g^2C_F}{16\pi^2}[\frac{40}{9}-8.63(1)] & +\frac{g^2N_f}{16\pi^2}[\frac{4}{9}+0.20(1)] \\ +\frac{g^2C_F}{16\pi^2}[\frac{22}{9}+3.56(1)] & 1-\frac{g^2N_f}{16\pi^2}[\frac{10}{9}-3.52(1)] \\ & -\frac{g^2N_c}{16\pi^2}[\frac{4}{3}+1.54(1)+V.T.] \end{pmatrix} \begin{pmatrix} O^{L}_{Q,(1)} \\ O^{L}_{G,(1)} \end{pmatrix} + O(g^4)$$

$$\xrightarrow[g^2\sim 3]{} \begin{pmatrix} 1.1060(2) & 0.0122(2)N_f \\ 0.1521(2) & 0.8363(6)+0.0458(2)N_f-0.0570V.T.] \end{pmatrix} \begin{pmatrix} O^{L}_{Q,(1)} \\ O^{L}_{G,(1)} \end{pmatrix} + O(g^4)$$

V.T.: The 4-gluon vertex tadpole contribution, in progress.

We can force the sum rule of the momentum fractions to avoid the calculation of V.T., and the final normalization factor for the gluon operator is ***~0.8.***

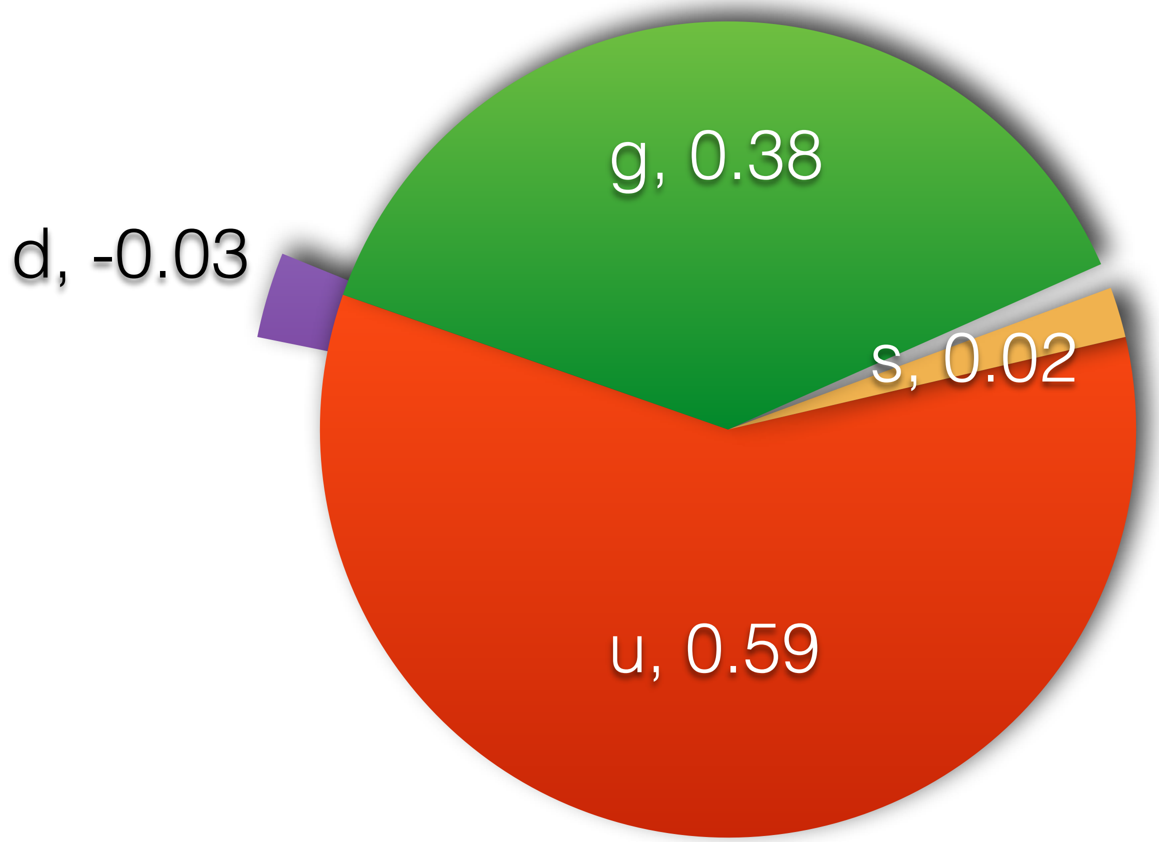
Y. Yang, M Glatzmaier, et al, χQCD Collaboration, in progress.

The pie charts

of the quark and gluon AM in proton

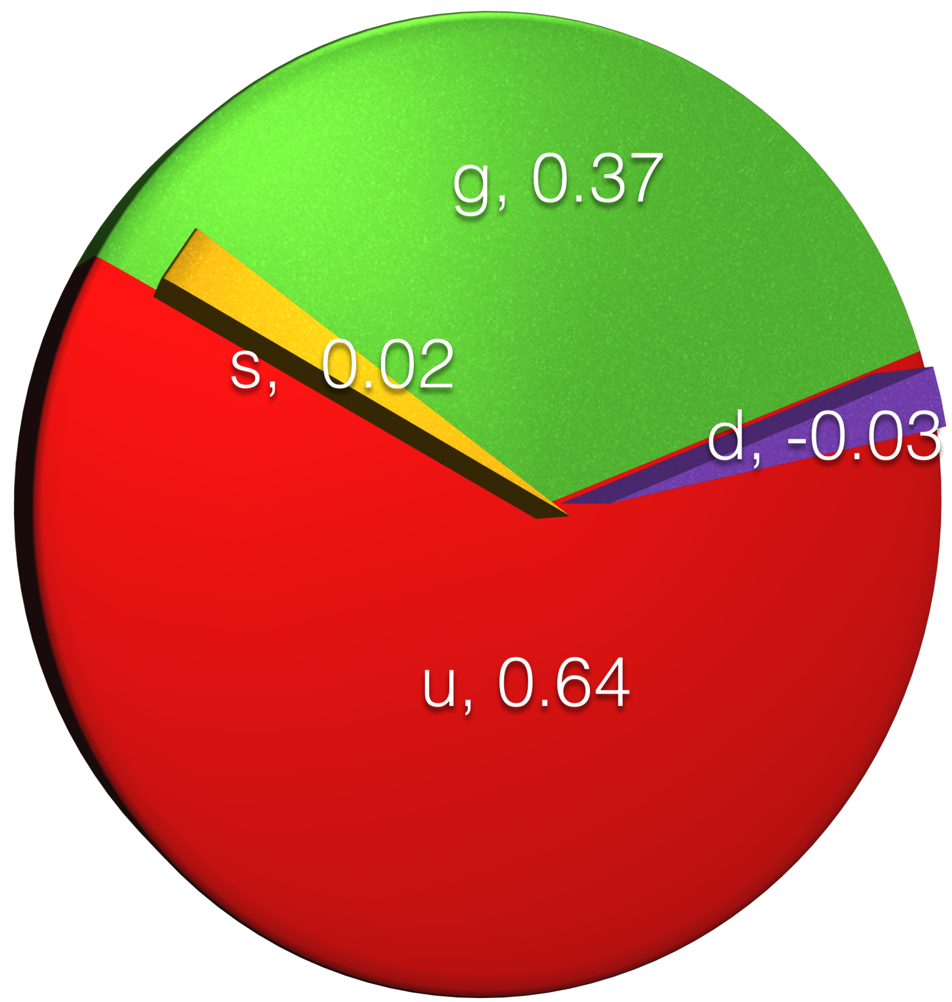
Y. Yang, et al,
χQCD Collaboration,
in progress.

m_π=400 MeV, preliminary



Bare values

1-loop renormalized values



The percentage of the angular momentum in proton

Outline

- *The proton spin decomposition*

  Two types of decompositions.

- *The quark/glue angular momentum in proton*

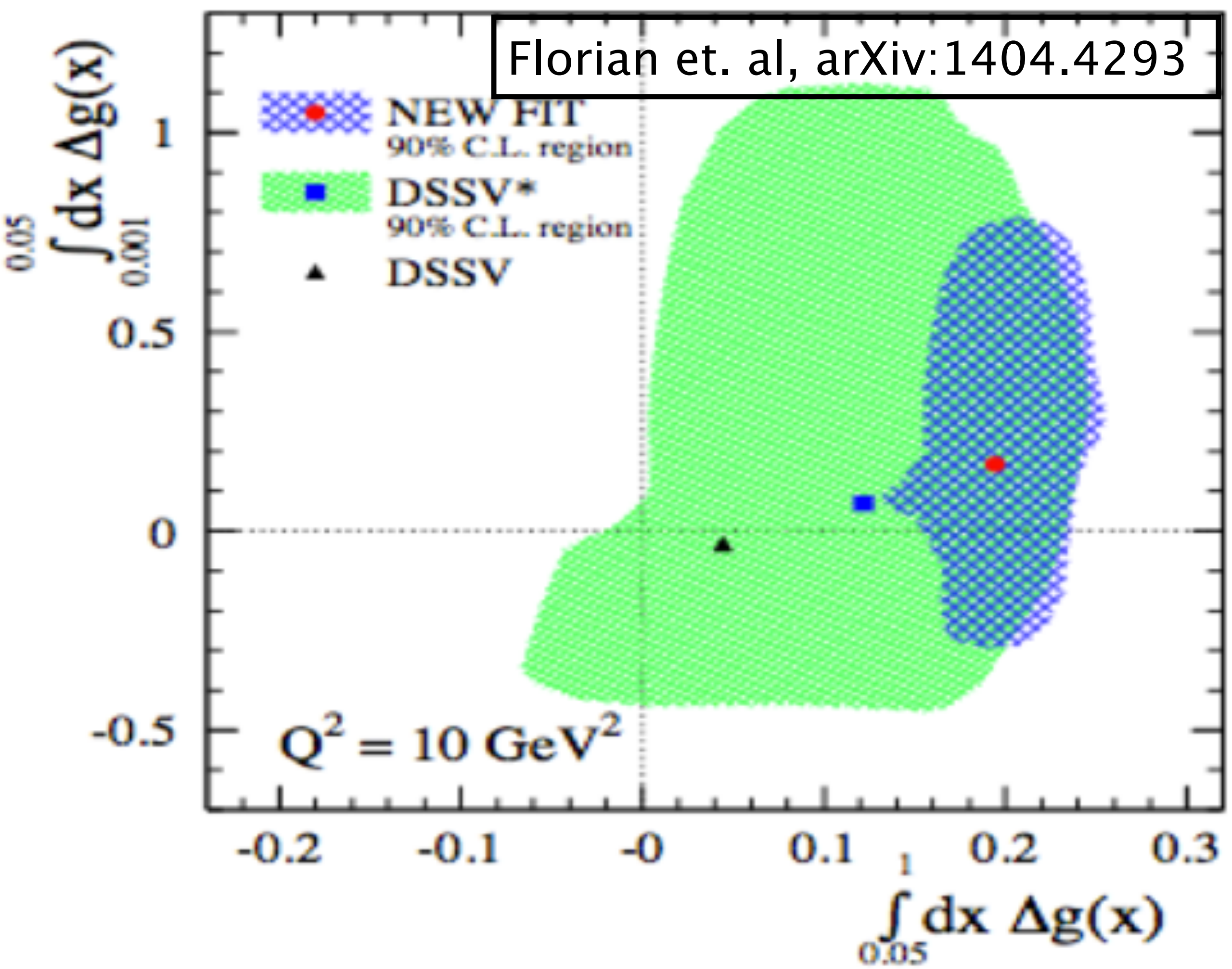
  The preliminary results and the perturbative matching to the MS-bar scheme.

- ***The glue spin and helicity***

  The glue spin based on Chen's decomposition and connection to the glue helicity.

Glue spin

The glue helicity



The global fit based on recent experimental data (2009 RHIC) shows evidence of nonzero polarization of gluon in the proton. The glue helicity is defined as,

$$\Delta G = \int_0^1 \Delta g(x) dx$$

$$= \int dx \frac{i}{2xP+} \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} \langle PS|F_a^{+\alpha}(\xi^-)\mathcal{L}^{ab}(\xi^-,0)\tilde{F}^+_{\alpha,b}(0)|PS\rangle$$

A. V. Manohar, Phys. Lett. B255, 579 (1991)

After integrating the longitudinal momentum x,

$$O_{\Delta G} = \left[\vec{E}^a(0) \times (\vec{A}^a(0) - \frac{1}{\nabla^+}(\vec{\nabla} A^{+,b})\mathcal{L}^{ba}(\xi^-,0))\right]$$

Y. Hatta, Phys. Rev. D84, 041701 (2011),
X. Ji, J.H. Zhang, and Y. Zhao, Phys. Rev. Lett. 111 112002 (2013)

Proton Spin decomposition

Decomposition targets to the IMF quantities

X. –S. Chen et al., Phys. Rev. Lett. 100, 232002 (2008).
X. –S. Chen et al., Phys. Rev. Lett. 103, 062001 (2009).

quark spin — *quark OAM*

$$\vec{J} = \int d^3x \, \frac{1}{2} \, \overline{\psi} \, \vec{\gamma} \gamma^5 \, \psi + \int d^3x \psi^\dagger \, \{\vec{x} \times (i\vec{D}^{pure})\} \, \psi$$

$$+ \int d^3x 2\mathrm{Tr}[\vec{E} \times \vec{A}^{phys}] + \int d^3x 2\mathrm{Tr}[E^i \vec{x} \times \overrightarrow{D^{pure}} A^{i,phys}]$$

glue spin — *glue OAM*

Gauge invariant but frame dependent

What is A^{phys}?

The "pure" gauge part, A_μ^{pure} is defined to follow the same gauge transformation as A_μ and does not give rise to a field tensor by itself,

$$\begin{aligned} A_\mu^{pure} &\to A_\mu^{\prime pure} = g(x) A_\mu^{pure} g^{-1}(x) + \frac{i}{g_0} g(x) \partial_\mu g^{-1}(x), \\ F_{\mu\nu}^{pure} &= \partial_\mu A_\nu^{pure} - \partial_\nu A_\mu^{pure} + i g_0 [A_\mu^{pure}, A_\nu^{pure}] = 0. \end{aligned}$$

Thus $A_\mu^{phys} = A_\mu - A_\mu^{pure}$ transforms homogeneously as

$$A_\mu^{phys} \to A_\mu^{\prime phys} = g(x) A_\mu^{phys} g^{-1}(x)$$

and a non-Abelian transverse condition

$$D_i A_i^{phys} = \partial_i A_i^{phys} - i g_0 [A_i, A_i^{phys}] = 0$$

is applied on that to have a unique solution.

X. -S. Chen et al., Phys. Rev. Lett. 103, 062001 (2009).

When boosting the glue spin operator $\vec{S}_g = E \times A^{phys}$ to IMF, the non-Abelian transverse condition corresponds to the light-cone gauge fixing condition $A_+^{phys} = 0$ and the forward matrix element of the longitudinal glue spin operator corresponds to the glue helicity, ΔG.

X. Ji, J.H. Zhang, and Y. Zhao, Phys. Rev. Lett. 111 112002 (2013)

Proton Spin decomposition

Two decompositions

X.D. Ji., Phys. Rev. Lett. 78, 610-613 (1997).

$$\vec{J} = \int d^3x \, \frac{1}{2} \, \overline{\psi} \, \vec{\gamma} \, \gamma^5 \, \psi + \int d^3x \psi^\dagger \, \{\vec{x} \times (i\vec{D})\} \, \psi \; + \; \int d^3x 2\{\vec{x} \times \mathrm{Tr}[\vec{E} \times \vec{B}]\}$$

quark spin

Different definitions of the quark OAM

$$\int d^3x \psi^\dagger \, \{\vec{x} \times \overrightarrow{A^{phys}}\} \, \psi$$

Further decomposition of the glue AM

$$\vec{J} = \int d^3x \, \frac{1}{2} \, \overline{\psi} \, \vec{\gamma} \, \gamma^5 \, \psi + \int d^3x \psi^\dagger \, \{\vec{x} \times (i\vec{D}^{pure})\} \, \psi$$

$$+ \int d^3x 2\mathrm{Tr}[\vec{E} \times \vec{A}^{phys}] \; + \; \int d^3x 2\mathrm{Tr}[E^i \vec{x} \times \overrightarrow{D^{pure}} A^{i,phys}]$$

glue spin — glue OAM

X. -S. Chen et al., Phys. Rev. Lett. 100, 232002 (2008).
X. -S. Chen et al., Phys. Rev. Lett. 103, 062001 (2009).

How to obtain S_g on the lattice?

If one can find a gauge transformation g_c to make the gauge potential after the rotation

$$A_{c,\mu} = g_c^{-1} A_\mu g_c + \frac{i}{g_0} g_c \partial_\mu g_c^{-1}, \text{ or equivalently } A_\mu = g_c A_{c,\mu} g_c^{-1} + \frac{i}{g_0} g_c \partial_\mu g_c^{-1}$$

to satisfy the condition $\partial \cdot A = 0$. Then it is easy to confirm that the decomposition defined by,

$$A_\mu^{pure} = \frac{i}{g_0} g_c \partial_\mu g_c^{-1}, \; A_\mu^{phys} = g_c A_{c,\mu} g_c^{-1}.$$

can satisfy all the requirement of the decomposition defined by Chen et.al. In the other word, Chen et.al's decomposition is equivalent to the gauge invariant extension of the Coulomb gauge.

C. Lorce, et al. Phys.Rev. D85 (2012) 114006
Yong Zhao, Keh-Fei Liu, Yibo Yang, Phys.Rev. D93 (2016) 054006

On the lattice, such a gauge transformation g_c can be obtained numerically with $O(a)$ corrections. So the glue spin operator on the lattice can be simply defined on the Coulomb gauge fixed configuration,

$$\vec{S}_g = \int d^3x \; 2\mathrm{Tr}(\vec{E} \times g_c \vec{A}_c g_c^{-1}) = \int d^3x \; 2\mathrm{Tr}(\vec{E}_c \times \vec{A}_c)$$

with E_c and A_c are the lattice version of the electric field and gauge potential.

Glue Spin

Lattice setup

- Overlap valence quark on 2+1 Domain wall fermion configuration.

Symbol	$L^3 \times T$	$a(fm)$	$m_\pi^{(s)}$MeV	N_{cfg}
32ID	$32^3 \times 64$	0.1431(7)	170	200
48I	$48^3 \times 96$	0.1141(2)	140	81
24I	$24^3 \times 64$	0.1105(3)	330	203
32I	$32^3 \times 64$	0.0828(3)	300	309
32If	$32^3 \times 64$	0.0627(3)	370	238

T. Blum et al. (RBC, UKQCD), Phys. Rev. D93, 074505 (2016)

- 2pt: grid smear source, loop over t, with the low mode substitution.
- glue: clover operator based on 5 HYP smeared configuration.

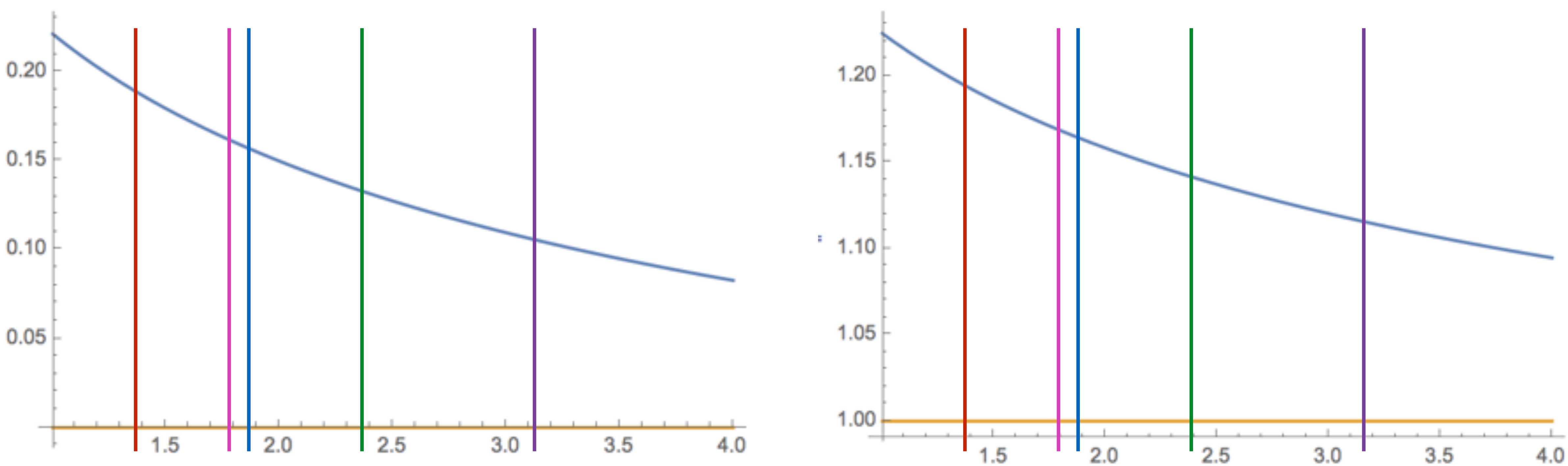
Glue spin

Y. Yang, Y. Zhao, in progress.

The renormalization and mixing

$$S^{\overline{MS}}_{G,(1)} = \left(1 - \frac{g^2 N_f}{16\pi^2}\left[\frac{2}{3}\log(\mu^2 a^2) + \frac{10}{9} - 3.52(1)\right] + \frac{g^2 C_A}{16\pi^2}\left[\frac{4}{3}\log(\mu^2 a^2) + C_{GG}\right]\right) S^L_{G,(1)}$$
$$+ \frac{g^2 C_F}{16\pi^2}\left[\frac{5}{3}\log(\mu^2 a^2) + 3.1921 + 1.72(1)\right] \sum_{q=u,d,s\ldots} \Delta_q^{L,(1)} + O(g^4),$$

The scale used by the experiment for the glue helicity is ***μ²=10 GeV***

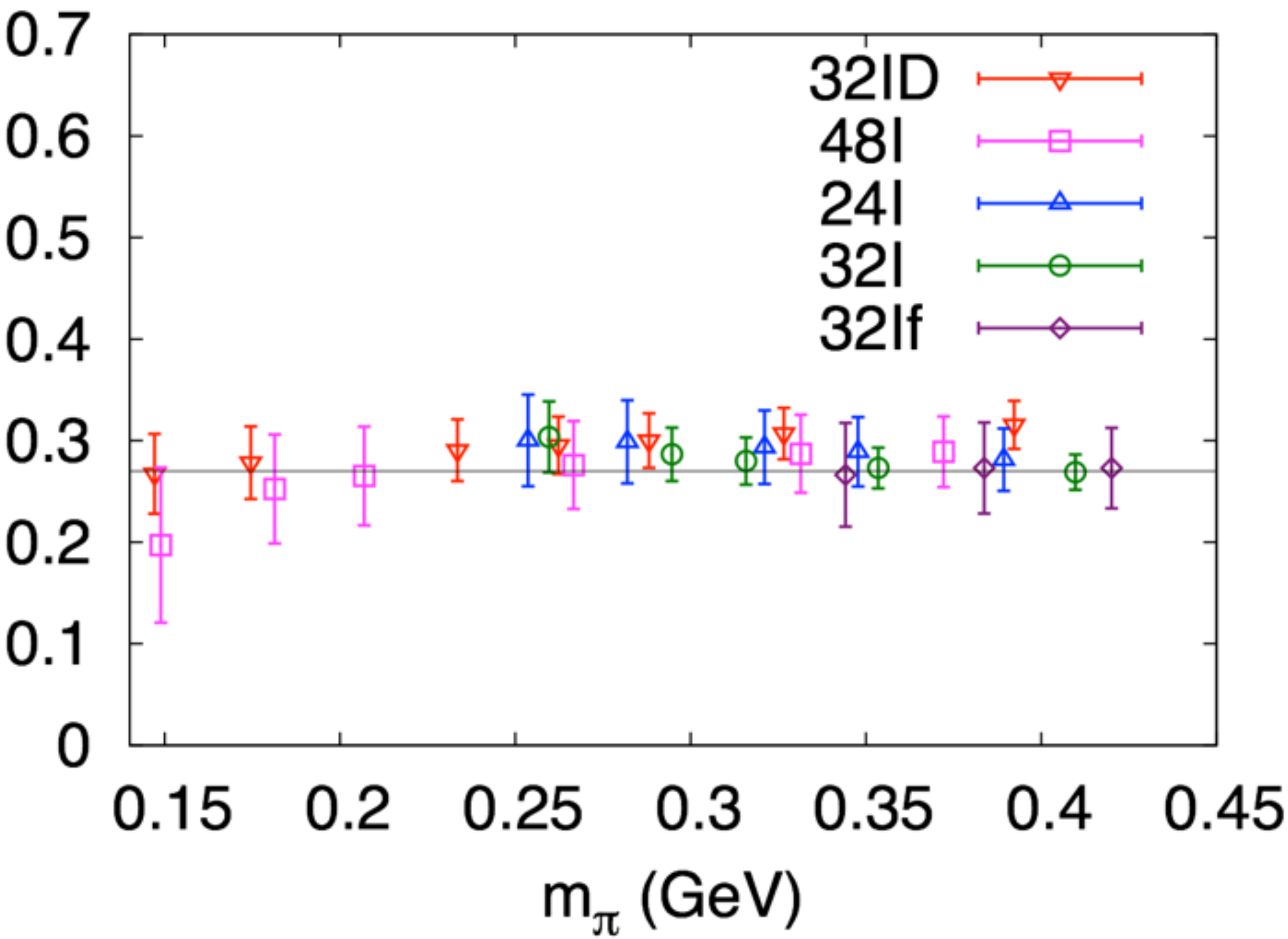


mixing a^{-1} (GeV) *renormalization*

The dependence

Y. Yang, R. S. Sufian, et al, χQCD Collaboration, in progress.

of m_π, a, and V



μ^2=10 GeV

In the rest frame, *the pion mass (both valence and sea), lattice spacing and volume dependences are mild.*

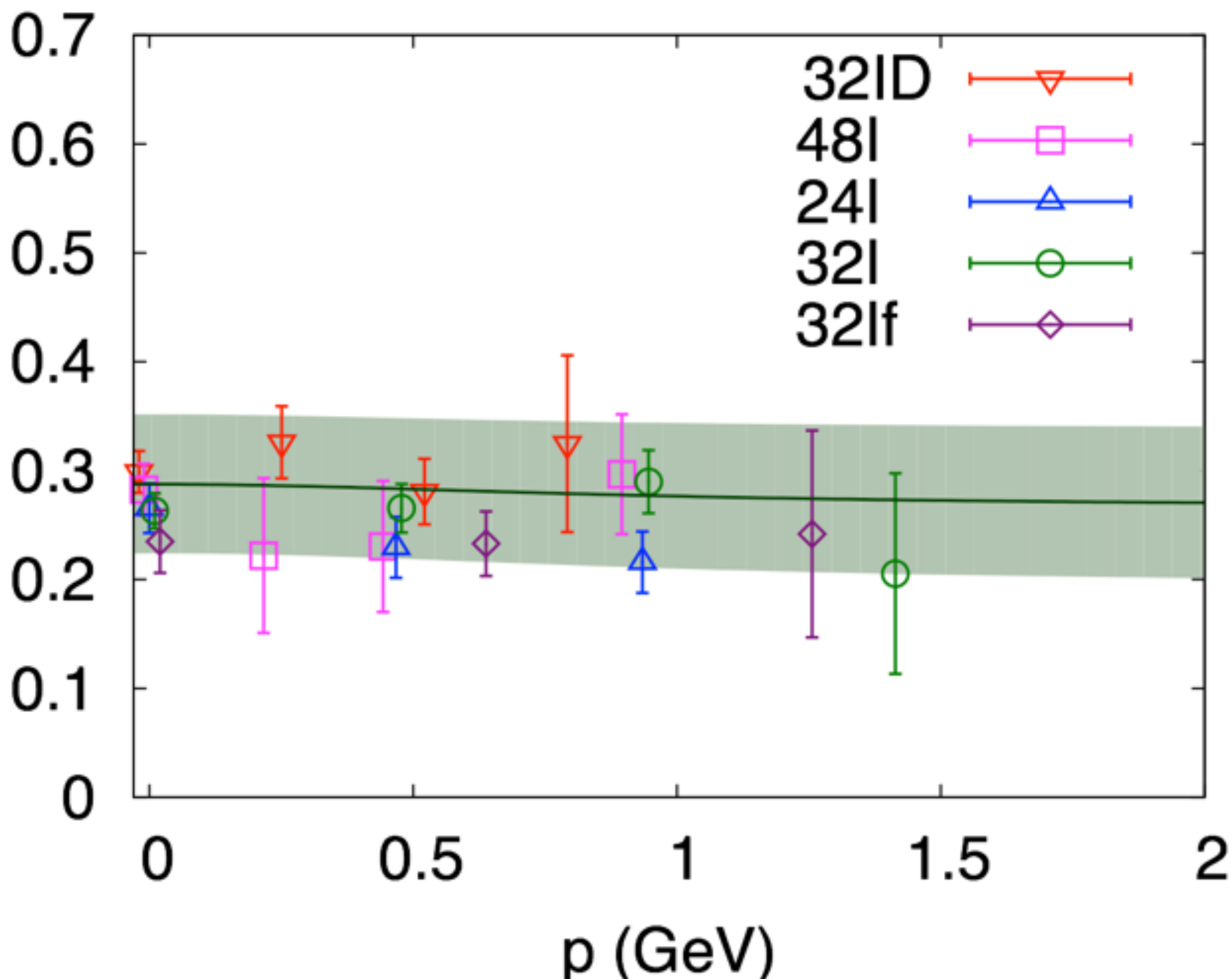
Glue spin

Y. Yang, R. S. Sufian, et al, χQCD Collaboration, in progress.

The frame dependence

$$S_G(p^2) = S_G(\infty) + \frac{C_1}{M^2 + p^2} + C_2 m^2_{\pi,vv} + C_3 m^2_{\pi,ss} + C_4 a,$$

$$M = 0.939 \text{ GeV}$$



The glue spin at the large momentum limit for the renormalized value at μ^2=10GeV^2:

S_G=0.27(7)

Large-momentum

effective field theory?

$$\Delta G(\mu) = \frac{g^2 C_F}{16\pi^2}[\frac{4}{3}\log\frac{(p^z)^2}{\mu^2} - 5.2627]\Delta\Sigma$$
$$+(1 + \frac{g^2 C_A}{16\pi^2}[\frac{7}{3}\log\frac{(p^z)^2}{\mu^2} - 10.2098])S_G(P^z, \mu)$$
$$+O(g^4) + O(\frac{1}{(P^z)^2}).$$

X. Ji, J.-H. Zhang, and Y. Zhao, Phys. Lett. B743, 180 (2015)

- **We don't observed such a strong frame dependence. The higher order correction would be also large to cancel it.**
- **The above result is based on the on-shell scheme and then is not directly applicable to the lattice calculation which is off-shell in general.**
- **So we skipped this step at the present stage.**

Summary

- **The proton spin decomposition based on Ji's scheme is in progress.**
- ***1. The perturbative matching is almost done and the simulation is ongoing,***
- ***2. The proton spin is dominated by the U quark (~60%) and gluon (~40%).***
- ***3. Our goal is to obtain the final results with all the systematic uncertainty (lattice spacing, volume, physical pion mass etc.) under control.***

- **The glue spin in the finite momenta <2 GeV^2 have been obtained at several lattice spacing and volumes, with the chiral extrapolation.**
- ***1. The glue spin and helicity at 10 GeV^2 are ~0.27(7),***
- ***2. The perturbative matching is ongoing.***