Progress on the lattice QCD calculation of the rare kaon decays $K^+ \to \pi^+ \nu \bar{\nu}$

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Frontiers in high energy physics

Discovery of Higgs boson ⇒ Nobel prize to **Englert & Higgs**

⇒ It is not a happy ending but a beautiful start!

Three frontiers to search for Physics Beyond Standard Model

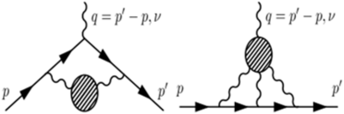
- **Cosmic frontier**
  ⇒ detect dark matter, energy and cosmically-produced new particles
- **High-energy frontier**
  ⇒ increase collision energy, directly produce new particles
- **High-intensity frontier**
  ⇒ precisely measure rare processes, look for discrepancies with SM:
  long baseline neutrino, muon g-2, rare kaon decay, $\cdots$

This requires the precise predication from Standard Model

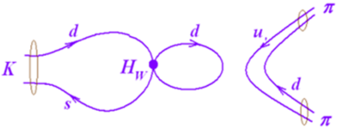
Look at rare processes in lattice QCD

Selected projects

$g_\mu - 2 = 0.00116592089(63)$

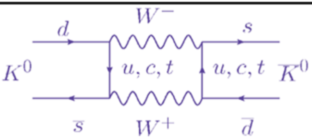


Direct CP violation $K \to \pi\pi$
$|\epsilon'| = 3.70(53) \times 10^{-6}$

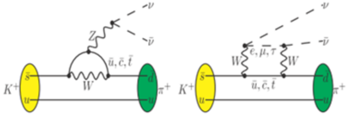


Indirect CP violation $K \to \pi\pi$
$\epsilon = 0.002228(11)$

$m_{K_L} - m_{K_S} = 3.2(1.0) \times 10^{-12}$ MeV

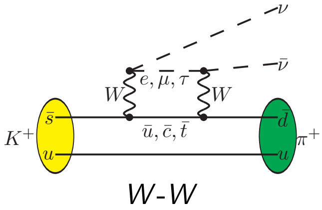


$K^+ \to \pi^+ \nu \bar{\nu}$: BR=$1.73^{+1.15}_{-1.05} \times 10^{-10}$



$K^+ \to \pi^+ \nu\bar{\nu}$: Experiment vs Standard model

As FCNC process, $K \to \pi\nu\bar{\nu}$ decay through second-order weak interaction



SM effects highly suppressed in the second order → ideal probes for NP

Past experimental measurement is 2 times larger than SM prediction

$$\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})_{\mathrm{exp}} = 1.73^{+1.15}_{-1.05} \times 10^{-10}$$ arXiv:0808.2459

$$\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})_{\mathrm{SM}} = 9.11 \pm 0.72 \times 10^{-11}$$ arXiv:1503.02693

but still consistent with > 60% exp. error

New experiments

New generation of experiment: NA62 at CERN aims at

- observation of $O(100)$ events in 2-3 years
- 10%-precision measurement of $\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})$

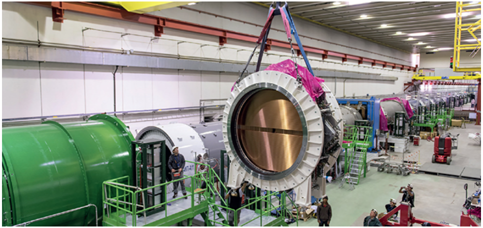
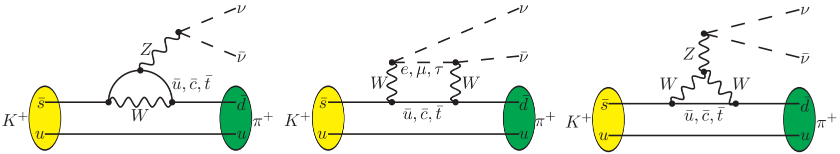


Fig: 09/2014, the final straw-tracker module is lowered into position in NA62

$K_L \to \pi^0 \nu\bar{\nu}$

- even more challenging since all the particles involved are neutral
- only upper bound was set by KEK E391a in 2010
- new KOTO experiment at J-PARC designed to observe K_L decays

$K^+ \to \pi^+ \nu \bar{\nu}$ in the Standard Model



Factors of $\frac{1}{M_W^4}$ or $\frac{1}{M_W^2 M_Z^2}$ implies quadratic GIM mechanism

Since $m_t = 173$ GeV, $m_c = 1.3$ GeV, $m_u = 2.3$ MeV, rough estimate yields

- top quark contribution dominates $\sim \lambda_t \frac{m_t^2}{M_W^2}$
- SD charm quark contribution subdominates $\sim \lambda_c \frac{m_c^2}{M_W^2} \ln \frac{m_c^2}{M_W^2}$
  - although suppressed by $\frac{m_c^2}{m_t^2}$, but enhanced by $\frac{\lambda_c}{\lambda_t}$. Here $\lambda_q = V_{qs}^* V_{qd}$
- remaining LD contribution $\sim \lambda \frac{m_c^2}{M_W^2}$, $\lambda \frac{\Lambda_{\mathrm{QCD}}^2}{M_W^2}$

Branching ratio

Branching ratio for $K^+ \to \pi^+ \nu \bar{\nu}$ [Buras et.al. JHEP11(2015)033]

$$\text{Br} = \kappa_+ (1 + \Delta_{\text{EM}}) \cdot \Big[\big(\underbrace{\frac{\text{Im}\,\lambda_t}{\lambda^5} X(x_t)}_{0.270 \times 1.481(9)} \big)^2 + \big(\underbrace{\frac{\text{Re}\,\lambda_c}{\lambda} P_c}_{-0.974 \times 0.405(23)} + \underbrace{\frac{\text{Re}\,\lambda_t}{\lambda^5} X(x_t)}_{-0.533 \times 1.481(9)} \big)^2 \Big]$$

- $X(x_t)$: top quark contribution; P_c: charm and LD contribution

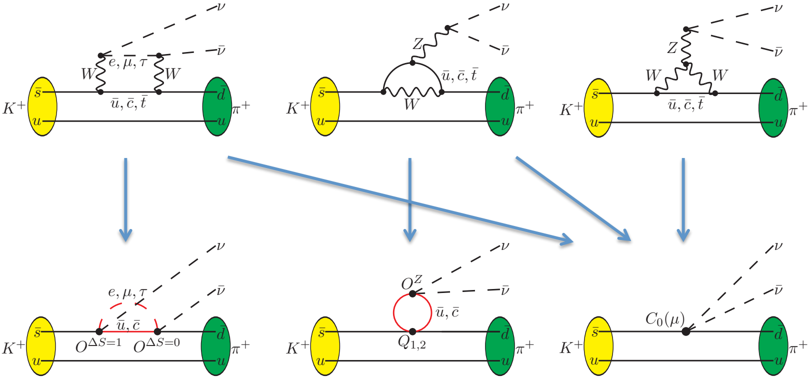
Without P_c, branching ratio is 50% smaller

Uncertainty budget

- dominant uncertainty from CKM factor λ_t
- once fixing CKM factor, then P_c dominates the uncertainty
  - P_c's uncertainty mainly come from LD

Important to determine the LD contribution to P_c accurately

OPE: integrate out the heavy fields, Z, W, t, $\cdots$



Local SD contribution

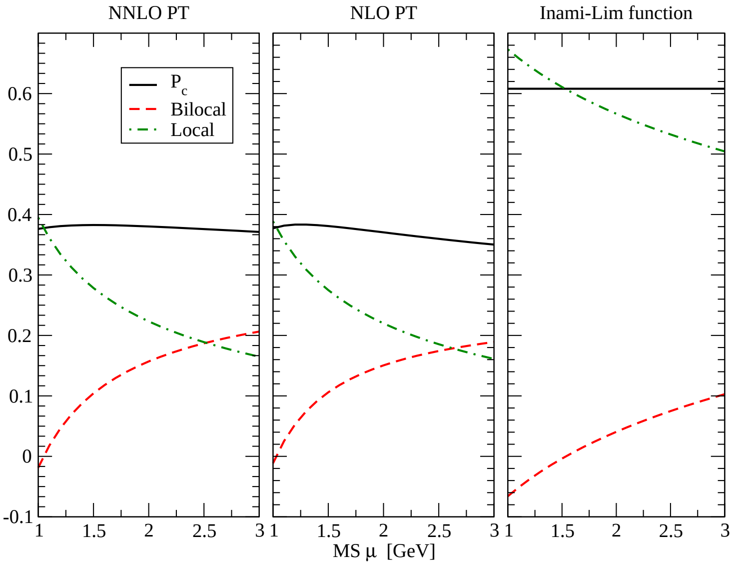
Bilocal LD contribution

Hadronic part known: $\langle \pi^+ | V_\mu | K^+ \rangle$

$\langle \pi^+ \nu \bar{\nu} | Q_A(x) Q_B(0) | K^+ \rangle$: need lattice QCD

Bilocal contribution vs local contribution

Bilocal $C_A^{\overline{\mathrm{MS}}}(\mu)\, C_B^{\overline{\mathrm{MS}}}(\mu)\, r_{AB}^{\overline{\mathrm{MS}}}(\mu)$ vs Local $C_0^{\overline{\mathrm{MS}}}(\mu)$, hep-ph/0603079



At $\mu = 2.5$ GeV, 50% charm quark contribution from bilocal term

Lattice methodology

Exponential contamination at large Euclidean time

Hadronic matrix element for the 2nd weak interaction

$$\int_{-T}^{T} dt \, \langle \pi^+ \nu \bar{\nu} | T\left[Q_A(t) Q_B(0) \right] | K^+ \rangle$$

$$= \sum_n \left\{ \frac{\langle \pi^+ \nu \bar{\nu} | Q_A | n \rangle \langle n | Q_B | K^+ \rangle}{M_K - E_n} + \frac{\langle \pi^+ \nu \bar{\nu} | Q_B | n \rangle \langle n | Q_A | K^+ \rangle}{M_K - E_n} \right\} \left(1 - e^{(M_K - E_n)T} \right)$$

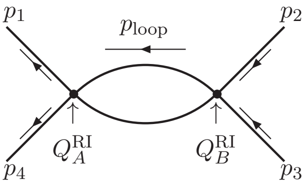
- For $E_n > M_K$, the exponential terms exponentially vanish at large T
- For $E_n < M_K$, the exponentially growing terms must be removed
- $\sum_n$: principal part of the integral replaced by finite-volume summation
  - possible large finite volume correction when $E_n \to M_K$

    [N. Christ, XF, G. Martinelli, C. Sachrajda, arXiv:1504.01170]

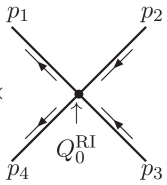
New SD divergence appears in $Q_A(x)Q_B(0)$ when $x \to 0$

- Introduce a counter term $X \cdot Q_0$ to remove the SD divergence

$\langle\{Q_A Q_B\}^{\mathrm{RI}}\rangle\Big|_{p_i^2=\mu_0^2} =$



$- \; X(\mu_0, a) \times$



$= \; 0$

  The coefficient X is determined in the RI/(S)MOM scheme

- The bilocal operator in the $\overline{\mathrm{MS}}$ scheme can be written as

$$\left\{\int d^4x\, T[Q_A^{\overline{\mathrm{MS}}}(x)\, Q_B^{\overline{\mathrm{MS}}}(0)]\right\}^{\overline{\mathrm{MS}}}$$

$$= Z_A Z_B \left\{\int d^4x\, T[Q_A^{\mathrm{lat}}\, Q_B^{\mathrm{lat}}]\right\}^{\mathrm{lat}} + \left(-X^{\mathrm{lat}\to\mathrm{RI}} + Y^{\mathrm{RI}\to\overline{\mathrm{MS}}}\right) Q_0(0)$$

  - $X^{\mathrm{lat}\to\mathrm{RI}}$ is calculated using NPR and $Y^{\mathrm{RI}\to\overline{\mathrm{MS}}}$ calculated using PT

Lattice results

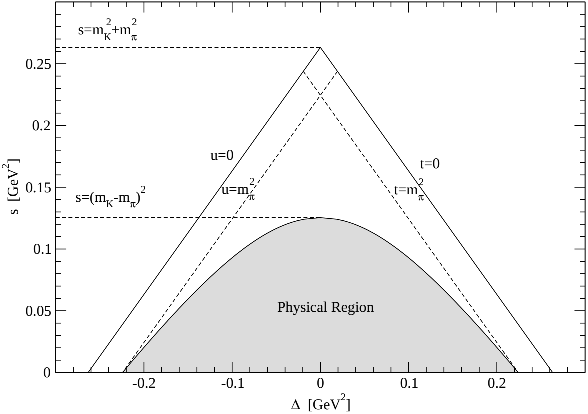
Scalar amplitude

All results are given as scalar amplitudes

$$\int d^4x \, \langle \pi^+ \nu \bar{\nu} | T[Q_A(x) Q_B(0)] | K^+ \rangle = F(s, \Delta) \cdot \bar{u}(p_\nu) \not{p}_K (1 - \gamma_5) v(p_{\bar{\nu}})$$

where s and Δ are Lorentz invariant variables

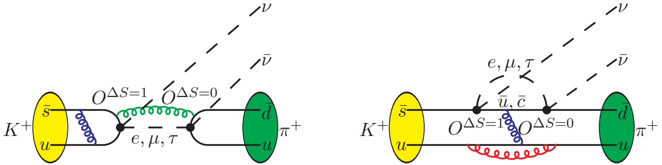
$$s = (p_K - p_\pi)^2, \quad \Delta = (p_K - p_\nu)^2 - (p_K - p_{\bar{\nu}})^2$$



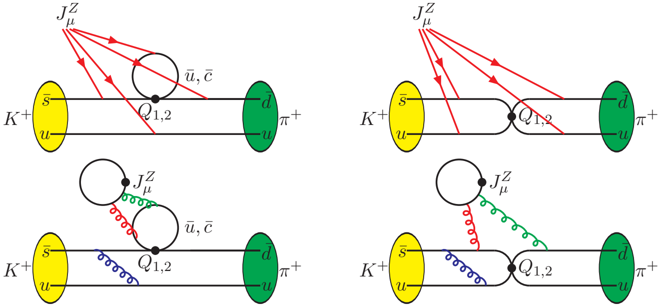
Summary of diagrams

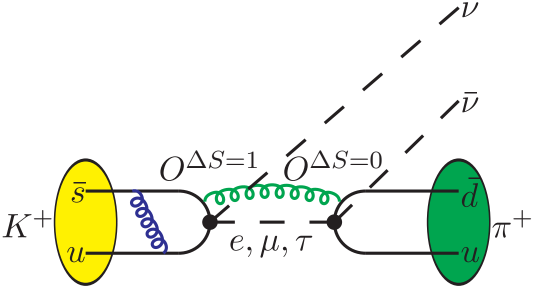
All diagrams are calculated

- W-W diagram:



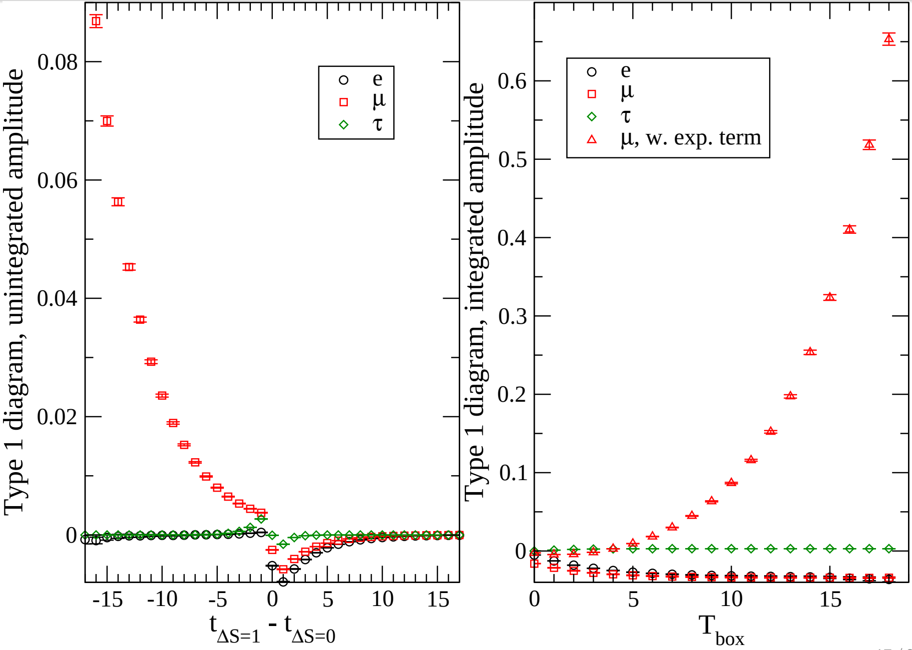
- Z-exchange diagram:





ν
$\bar{\nu}$
$O^{\Delta S=1}$
$O^{\Delta S=0}$
$\bar{s}$
$\bar{d}$
K^+
π^+
u
e, μ, τ
u

Time dependence of the unintegrated scalar amplitude



F_{WW} for type 1 diagram

F_{WW}	Type 1	model
e	$-1.685(47) \times 10^{-2}$	$-1.740(6) \times 10^{-2}$
μ	$-1.818(40) \times 10^{-2}$	$-1.822(6) \times 10^{-2}$
τ	$1.491(36) \times 10^{-3}$	$1.471(5) \times 10^{-3}$

- Vacuum saturation approximation assumes only single-lepton contribution in the intermediate state

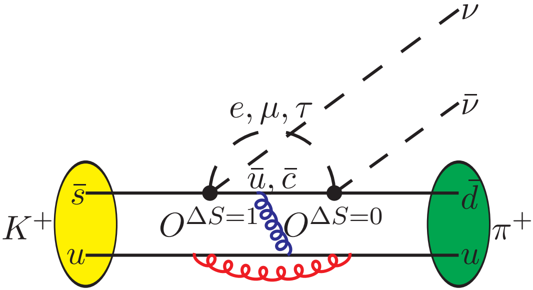
$$-f_K \bar{u}(p_\nu) \not{p}_K (1-\gamma_5) \frac{\not{q}}{q^2 - m_\ell^2} \not{p}_\pi (1-\gamma_5) v(p_{\bar{\nu}}) f_\pi$$

$$= -f_K f_\pi \frac{2q^2}{q^2 - m_\ell^2} \bar{u}(p_\nu) \not{p}_K (1-\gamma_5) v(p_{\bar{\nu}})$$

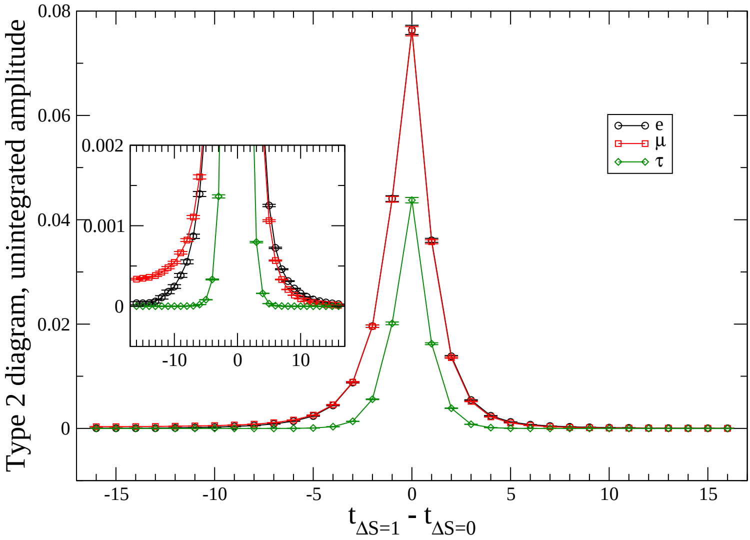
with $q = p_K - p_\nu = p_\pi + p_{\bar{\nu}}$

- Lattice vs model suggests small contribution from excited states

Type 2 diagram



Time dependence of the unintegrated scalar amplitude



Unintegrated transition amplitude for the Type 2 diagram

Summary results for W-W diagram

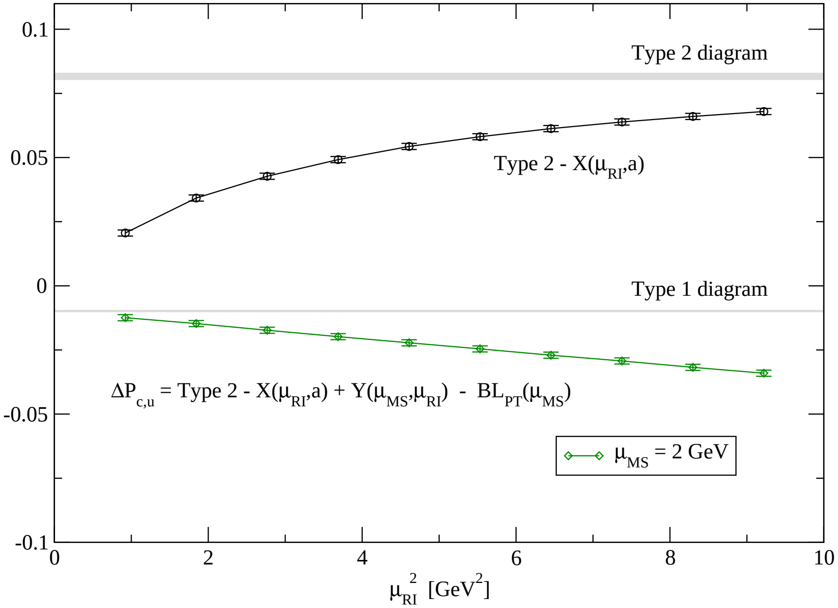
Scalar amplitude for W-W diagram

F_{WW}	Type 1	model	Type 2
e	$-1.685(47) \times 10^{-2}$	$-1.740(6) \times 10^{-2}$	$1.123(17) \times 10^{-1}$
μ	$-1.818(40) \times 10^{-2}$	$-1.822(6) \times 10^{-2}$	$1.194(18) \times 10^{-1}$
τ	$1.491(36) \times 10^{-3}$	$1.471(5) \times 10^{-3}$	$4.690(77) \times 10^{-2}$

Type 2 contribution is much larger than type 1, but

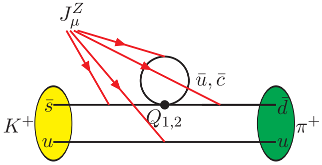
Type 2 diagram contain the large lattice cutoff effects due to SD divergence

Contribution from W-W diagram

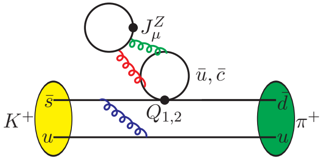


Summary of Z-exchange diagrams

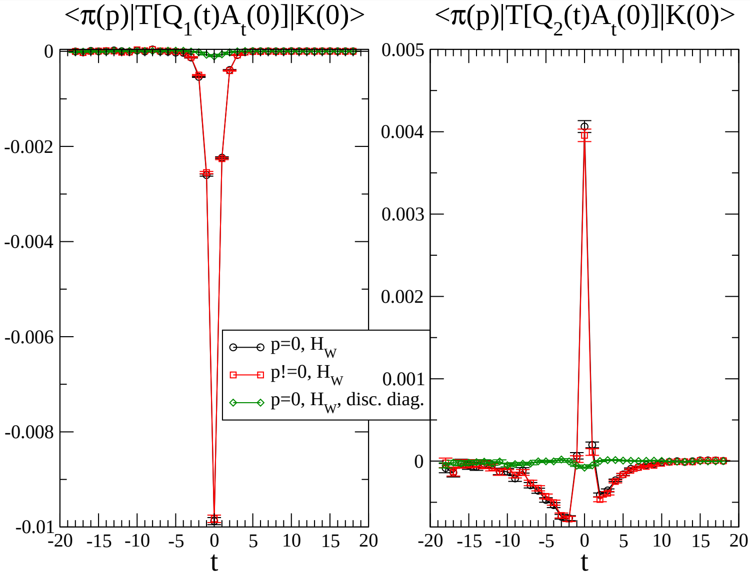
Connected diagrams, J_μ^Z can be inserted into all the possible quark line



Disconnected diagrams (difficult since they are noisy)

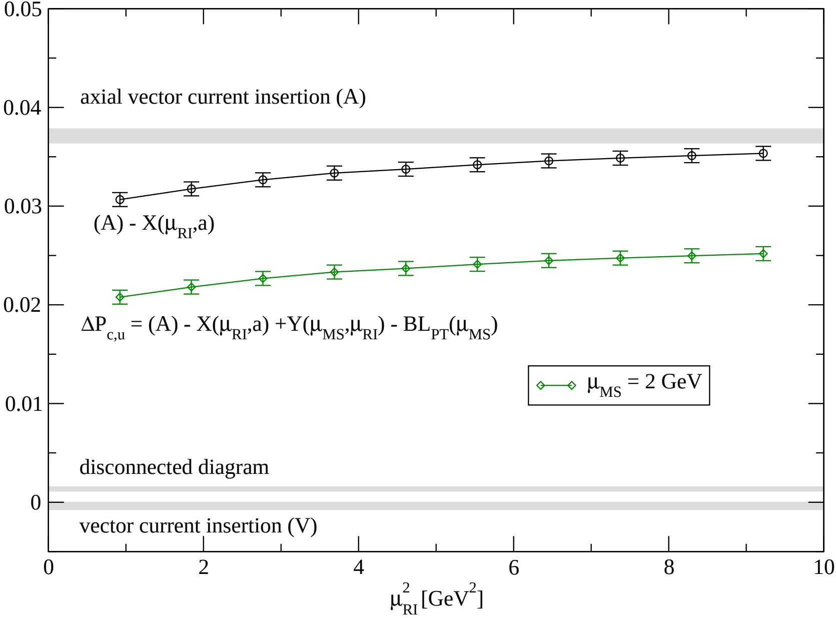


Z-exchange diagram: unintegrated matrix element



Unintegrated matrix element for Z-exchange diagram

Contribution from Z-exchange diagram



Results for charm quark contribution

Charm quark contribution P_c

$$P_c = P_c^{\mathrm{SD}} + \delta P_{c,u}$$

NNLO PT [Buras et.al, hep-ph/0603079]:

$$P_c^{\mathrm{SD}} = 0.365(12)$$

Phenomenological ansatz [Isidori et.al, hep-ph/0503107]:

$$\delta P_{c,u} = 0.040(20)$$

Preliminary Lattice results

$$\Delta P_{c,u} = \underbrace{-0.007(2)}_{WW:-0.032(1),Z:+0.025(1)} \left(\begin{array}{c} +7 \\ -11 \end{array}\right)_{\mathrm{RI}} \left(\begin{array}{c} +5 \\ -21 \end{array}\right)_{\mathrm{MS}}$$

$$\Delta P_{c,u} = \mathrm{Lattice} - X(\mu_{\mathrm{RI}}, a) + Y(\mu_{\overline{\mathrm{MS}}}, \mu_{\mathrm{RI}}) - \mathrm{Bilocal}_{\mathrm{PT}}(\mu_{\overline{\mathrm{MS}}})$$

How far we are from the destination?

Premature to make a direct comparison:

- current lattice calculation: $16^3 \times 32$, $m_\pi = 420$ MeV, $m_c = 860$ MeV
- physical pion and charm quark mass $\Rightarrow$ large volume
- control both $\mu_{\overline{\rm MS}}$ and $\mu_{\rm RI}$ dependence

Rare kaon decay: accessible to LQCD $\Rightarrow$ control systematic effects

Next steps

- USQCD project: use a larger volume $32^3 \times 64$ with $m_\pi = 170$ MeV
  - awarded 27 million BGQ core hours, data collected, now being analyzed
- Move to $1/a = 2.38$ GeV, $64^3 \times 128$ and physical m_π and m_c
  - currently a USQCD Incite proposal: 100 million BGQ core hours for 3 years

Outlook

- Calculation of the non-local matrix element is highly non-trivial
- Our exploratory study sheds light on the feasibility of lattice calculation of $K^+ \to \pi^+ \nu\bar{\nu}$
- Other interesting bilocal system
  - K^0-$\bar{K}^0$ and D^0-$\bar{D}^0$ mixing
  - other rare decays: $K_S \to \pi^0 \ell^+ \ell^-$, $B \to K^* \ell^+ \ell^-$, $\cdots$
  - electromagnetic correction to hadron mass and leptonic decay width
  - nucleon double beta decay: $0\nu\beta\beta$
  - ...

Bilocal system: an exciting and new area for lattice QCD!

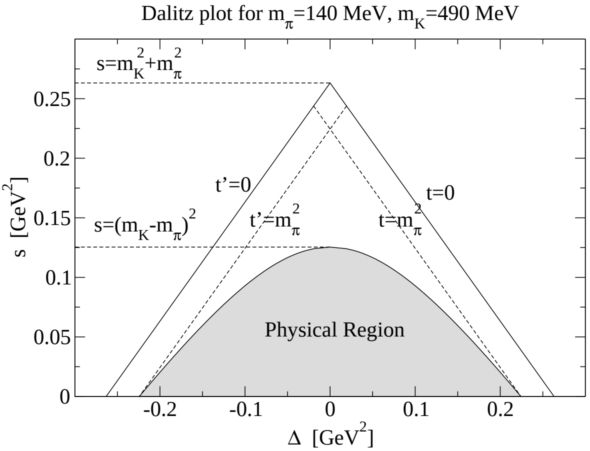
Backup slides

Dalitz plot

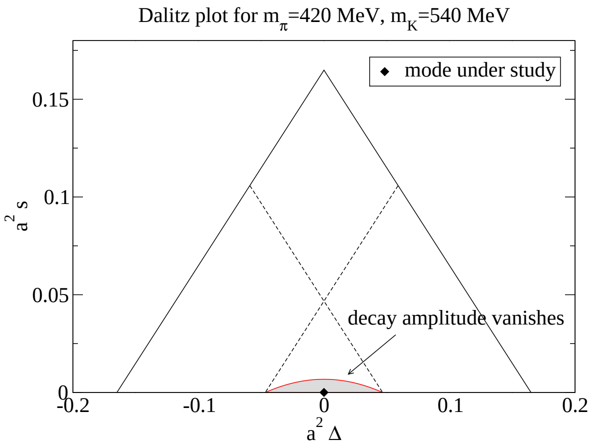
Three Lorentz invariants s, t, t'

$$s = (p_K - p_\pi)^2 = (p_\nu + p_{\bar\nu})^2, \quad t = (p_K - p_\nu)^2 = (p_{\bar\nu} + p_\pi)^2$$
$$t' = (p_K - p_{\bar\nu})^2 = (p_\nu + p_\pi)^2, \quad s + t + t' = m_K^2 + m_\pi^2$$

Two independent variables: s and $\Delta = t' - t$



Momentum mode under study

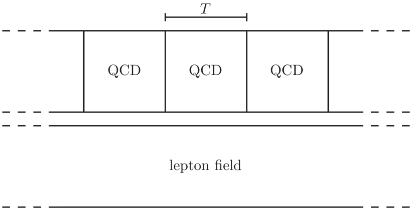


- Allowed momentum region highly suppressed at $m_\pi = 420$ MeV
- On-shell massless neutrinos → modulus of decay amplitude vanishes at the edge of the Dalitz plot
- Away from edge $(\Delta, s) = (0, 0) \quad \Rightarrow \quad \vec{p}_\nu = \vec{p}_{\bar{\nu}},\ \vec{p}_\pi = -\vec{p}_\nu - \vec{p}_{\bar{\nu}}$

Evaluation of non-local matrix element

$$\int dt \langle \pi^+ \nu \bar{\nu} | T\{O^{\Delta S=1}(t) O^{\Delta S=0}(0)\} | K^+ \rangle$$

- Construct 4-point correlator $\langle \phi_\pi(t_\pi) O^{\Delta S=1}(t_1) O^{\Delta S=0}(t_0) \phi_K^\dagger(t_K) \rangle$
- Perform time translation average → statistical error reduced by $\sqrt{T}$
  - propagators generated on all time slices, quite a lot of cost
  - use low-mode deflation w. 100 low-lying eigenvectors to accelerate CG
  - time required to generate light quark propagators is reduced to 10%
- Use overlap fermion for lepton propagator
  - time extent for lepton is infinite



Evaluation of non-local matrix element

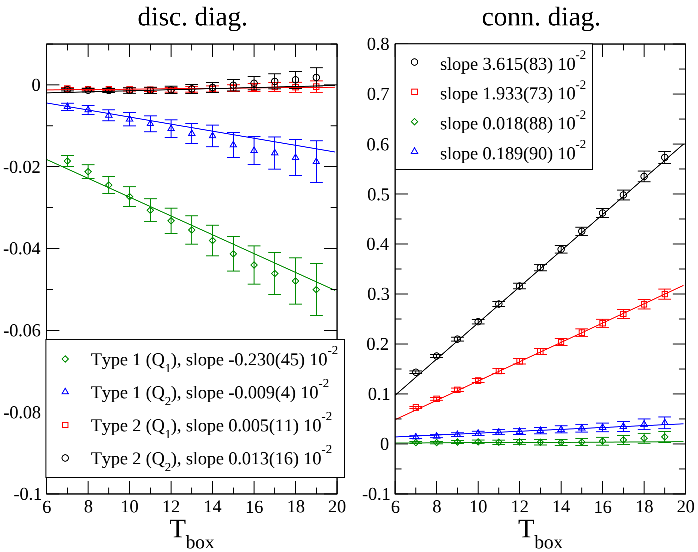
$$T_\mu^Z = \int dt \, \langle \pi^+ | T\{Q_{1,2}(t) J_\mu^Z(0)\} | K^+ \rangle$$

- Z-exchange diagrams do not require on-shell neutrinos
  - we use $\vec{p}_K = \vec{p}_\pi = 0, \quad J_\mu^Z, \ \mu = t$
- Hadronic current J_μ^Z has vector and axial vector component
  - for the vector current, according to Ward identity (WI), we have

$$T_\mu^{Z,V} = F^{Z,V}(q^2)\left(q^2(p_K + p_\pi)_\mu - (m_K^2 - m_\pi^2)q_\mu\right), \quad q = p_K - p_\pi$$

  - with $\vec{p}_K = \vec{p}_\pi = 0 \quad \Rightarrow \quad q^2(p_K + p_\pi)_\mu - (m_K^2 - m_\pi^2)q_\mu = 0$
  - WI suggests $T_\mu^{Z,V} = 0$, this is confirmed by our numerical calculation
- In the following, I will present the results for axial vector current

Integrated matrix element for Z-exchange



Disc. diag. is relatively noisy, but its contribution is small. Adding the disc. part does not affect the conn. part significantly