Understanding Higher Bottomonia in Unquenched Quark Model

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Based on: Y. Lu, MNA and B.-S Zou, Phys. Rev. D 94 (2016)

Outline

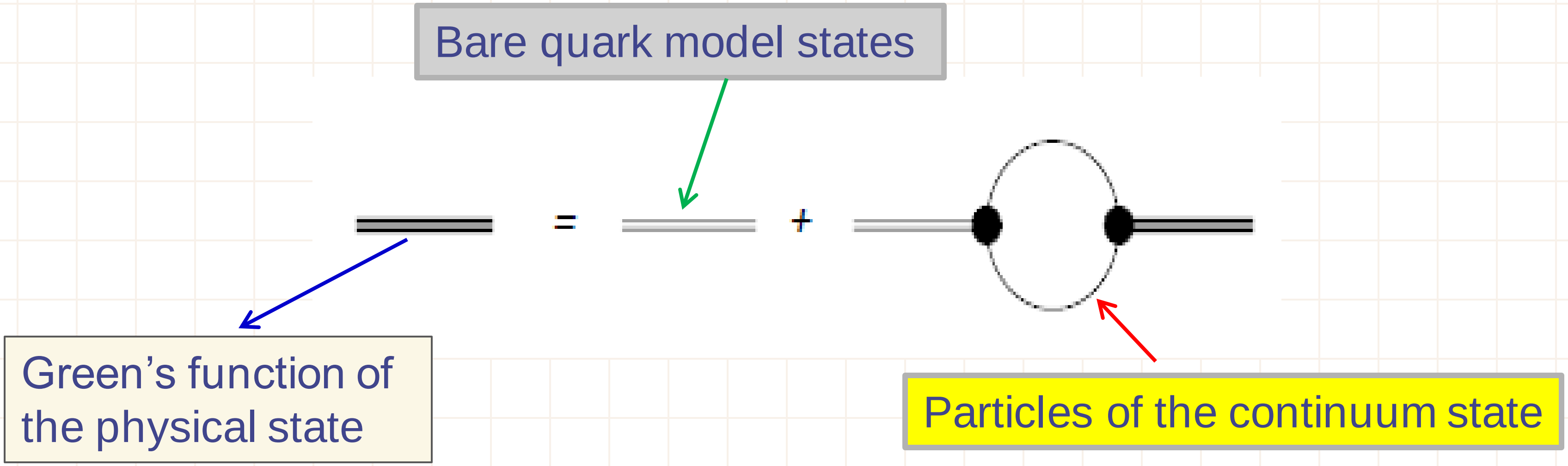
- Introduction & Motivation
- The Coupled-Channel Model
- Mass Shifts and Probabilities
- Open flavor strong decay widths
- S-D Mixing in Coupled-Channel Model (A new insight)
- Suggestions for Experiments
- Summary

The Unquenched Quark Model (UQM)

- Quenched approximation: A Draw back of the quark model
- Creation & annihilation of light $q\bar{q}$ pairs within hadrons can't easily be ignored
- Below threshold: Shift in the mass of the physical state + mixing b/w the states having the same quantum numbers
- Above threshold: Open-flavor strong decays + mass shifts + mixing b/w the states having the same quantum numbers
- Physical mass = bare mass + shift (overall mass shifts are –ive in this study but its not universal, it might be +ive)
- Probabilities of heavy $q\bar{q}$ component (bare state) can be worked out

Dyson Equation: An Illustration

- Quark model states get coupled each other via meson loops



Hammer, Hanhart & Nefediev arXiv:1607.06971 [hep-ph]

- In principle, sum over all the intermediate states
- Δm = bare mass – intermediate mass, if Δm goes larger it contributes less to the mass shift
- States closer to the thresholds have larger probabilities of multiquark components

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Heavy $c\bar{c}$ and $b\bar{b}$ quarkonium states and unitarity effects

Pioneer Extension of QM to UQM

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UQM with realistic potential

TABLE IV. Hadronic mass shifts and the OZI-rule-allowed decay widths (NRSO).

	Physical mass (MeV)	Bare mass (MeV)	ΔM (MeV)	Γ (MeV) experiment (Ref. 3)	Γ (MeV) theory
$\psi(1S)$	3096.9	3286.8	−189.9		0
$\psi(2S)$	3686	3837.3	−151.3		0
$\psi(1D)$	3770	3932.1	−162	25±3	10.9
$\psi(3S)$	4030	4127.6	−97.6	52±10	60.1
$\psi(2D)$	4159	4262.3	−103.2	78±20	71.5
$\psi(4S)$	4415	4439.0	−24	43±20	19.6
$\eta_c(1S)$	2981	3167.7	−186.7		0
$\eta_c(2S)$	3599	3728.1	−134.1		0
$\eta_c(3S)$		(4017.0)	−84		72
$\eta_c(4S)$		(4338.0)	−23		29
$\Upsilon(1S)$	9459.7	9489.5	−30		0
$\Upsilon(2S)$	10016	10066	−50		0
$\Upsilon(1D)$	(10130)	(10184)	−54		0
$\Upsilon(3S)$	10347	10409	−62		0
$\Upsilon(2D)$	(10415)	(10469)	−54		0
$\Upsilon(4S)$	10569	10644	−75	14±5	13.9[a]
$\Upsilon(3D)$	(10650)	(19661)	−11		48.3[a]
$\Upsilon(5S)$					
$\eta_b(1S)$		(9489.5)	−30		0
$\eta_b(2S)$		(10066)	−50		0
$\eta_b(3S)$		(10409)	−62		0
$\eta_b(4S)$		(10634)	−65		0

[a]Using the recently reported experimental B-meson mass (Ref. 39) (instead of our guess of 5280 MeV), the $\Upsilon(4S)$ width increases to 19.8 MeV and $\Upsilon(3D)$ decreases (because of the node structure) to 40 MeV.

Motivations

- For the creation of light $q\bar{q}$ pairs widely used formalism is ${}^3P_0$ model but most of its calculations are in SHO approximation
- There was a lack of the method to deal the wave functions precisely

- There are few studies from Segovia *et al.* with GEM and ${}^3P_0$ model where they computed the spectrum and open flavor strong decays of light mesons and some specific charmonia

- UQM effects are considered by the same group with GEM and ${}^3P_0$ model but only for X(3872) and D_s mesons
- So far, the precise method of evaluating UQM effects is still missing

- In this study we try to fill this gap by discussing interesting UQM effects using GEM and ${}^3P_0$ model for $b\bar{b}$ sector and predict some important results on the radiative decays of vector $b\bar{b}$ mesons

Bare Mass Spectrum

- Bare Hamiltonian is

$$H_0 = 2m_b + \frac{p^2}{m_b} + V(r) + V_s(r)$$

- We consider the Cornell potential to compute the bare masses

$$V(r) = -\frac{4}{3}\frac{\alpha}{r} + \lambda r + c$$

- Spin dependent potential is

$$\tilde{\delta}(r) = (\sigma/\sqrt{\pi})^3 e^{-\sigma^2 r^2}$$

$$V_s(r) = \left(\frac{2\alpha}{m_b^2 r^3} - \frac{\lambda}{2m_b^2 r}\right)\vec{L}\cdot\vec{S} + \frac{32\pi\alpha}{9m_b^2}\tilde{\delta}(r)\vec{S}_b\cdot\vec{S}_{\bar{b}} + \frac{4\alpha}{m_b^2 r^3}\left(\frac{\vec{S}_b\cdot\vec{S}_{\bar{b}}}{3} + \frac{(\vec{S}_b\cdot\vec{r})(\vec{S}_{\bar{b}}\cdot\vec{r})}{r^2}\right)$$

- Solve Schrödinger equation by Numerov's method with parameters

$\alpha = 0.34$	$\lambda = 0.22\ \text{GeV}^2$	$c = 0.435\ \text{GeV}$
$m_b = 4.5\ \text{GeV}$	$m_u = m_d = 0.33\ \text{GeV}$	$m_s = 0.5\ \text{GeV}$
$\sigma = 3.838\ \text{GeV}$	$\gamma = 0.205$	

- Parameters are chosen to fit the dielectric decay widths of $\Upsilon(nS)$ with $n = 1, 2, 3$

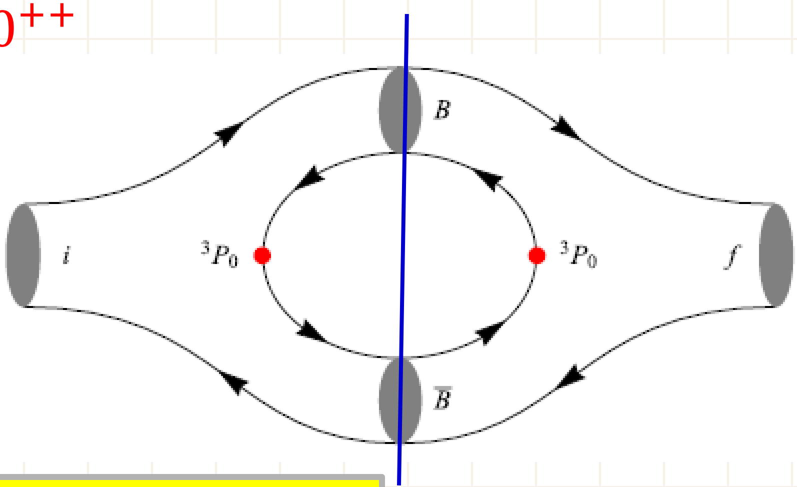
The Coupled-Channel Model (CCM)

- We use $^3P_0$ model to produce light $q\bar{q}$ within bottomonium
- Interaction Hamiltonian of the model reads as

$$H_I = 2m_q\gamma \int d^3x \bar{\psi}_q \psi_q$$

- Quantum numbers of $q\bar{q}$ are $J^{PC} = 0^{++}$
- Hamiltonian for Physical State is

$$H = H_0 + H_{BC} + H_I$$



- Physical state defined as

$$|A\rangle = c_0|\psi_0\rangle + \sum_{BC} \int d^3p \, c_{BC}(p)|BC;p\rangle$$

Bare state

Higher Fock components added by hand as perturbation

- Above threshold $^3P_0$ model have ability to reproduce most of the observed open flavor strong decay widths

Formalism…

- With these definitions we obtain the coupled channel equation

$$\begin{aligned} H_0|\psi_0\rangle &= M_0|\psi_0\rangle \\ H_0|BC;p\rangle &= 0 \\ H_{BC}|\psi_0\rangle &= 0 \\ H_{BC}|BC;p\rangle &= E_{BC}|BC;p\rangle \end{aligned}$$

$$E_{BC} = \sqrt{m_B^2 + p^2} + \sqrt{m_C^2 + p^2}$$

- By solving the Schrödinger-like equation

$$H|A\rangle = M|A\rangle$$

With

$$M = M_0 + \Delta M$$

- Relation for mass shift is

$$\Delta M = \sum_{BC} \int d^3p \frac{|\langle BC;p|H_I|\psi_0\rangle|^2}{M - E_{BC} - i\epsilon}$$

- Using the normalization condition

$$|c_0|^2 + \int d^3p |c_{BC}|^2 = 1$$

- Probabilities of $b\bar{b}$ component is

$$P_{b\bar{b}} := |c_0|^2 = 1 / \left(1 + \sum_{BCLS} \int_0^\infty dp \frac{p^2 |\mathcal{M}^{LS}|^2}{(M - E_{BC})^2}\right)$$

$$|\mathcal{M}^{LS}|^2 = \int d\Omega_B \, |\langle BC; P_B | H_I | \psi_0 \rangle|^2$$

Differences b/w GEM & SHO

- In GEM the position space wave function is

$$\psi_{NLM}(r) = \left(\sum_{i=1}^{n} c_i \beta_i^{L+\frac{3}{2}} e^{-\frac{1}{2}\beta_i^2 r^2} r^L \right) Y_L^M(\theta, \varphi)$$

- c_i is corresponding coeff. and β_i is oscillatory parameter

- n is number of Gaussian basis for $b\bar{b}$ states $n = 5 - 20$ and for B mesons $n = 5$ (min $n = 3$ for ground states)

- ${}^3P_0$ model's calculation are easy to do in momentum space

- GEM's wave function is invariant under Fourier transformation with $\beta \rightarrow 1/\beta$

$$\psi_{NLM}(p) = \left(\sum_{i=1}^{n} c_i \beta_i^{-(L+\frac{3}{2})} e^{-\frac{p^2}{2\beta_i^2}} p^L \right) Y_L^M(\theta, \varphi)$$

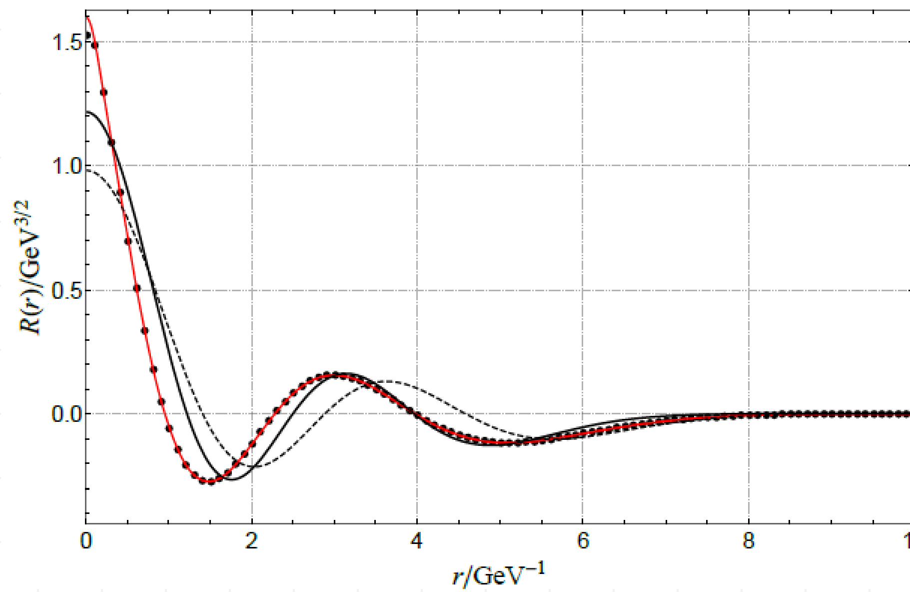


FIG. 2. Comparison of $\Upsilon(4S)$'s spatial wave function. Numerical values and GEM fit are denoted by black dots and red solid curve, respectively. Black dashed and solid curve represent single SHO approximation by matching $\langle r \rangle$ and maximizing wave function overlap, respectively.

- In SHO approximation there are two ways to fit β, 1st is by matching $< r >$ with the initial state
- 2nd is maximize the overlap of SHO with the numerical/realistic wave function

Results

From: Phys. Rev. D 94 (2016)
arXiv:1606.06927 [hep-ph]

Mass Shifts

States	M_0	$-\Delta M$				M_{theory}				M_{exp}
		GEM	SHO	Ref. [22]	Ref. [23]	GEM	SHO	Ref. [22]	Ref. [23]	
$\Upsilon(1^3S_1)$	9482.0	22.8	22.8	58.2	69	9459.2	9459.2	9460.3	9489	9460.3
$\Upsilon(2^3S_1)$	10054.9	43.8	42.8	68.0	108	10011.2	10012.1	10026.2	10022	10023.3
$\Upsilon(3^3S_1)$	10433.4	60.0	53.5	68.2	146	10373.4	10379.9	10351.9	10358	10355.2
$\Upsilon(4^3S_1)$	10746.7	92.6	28.7	76.3		10654.2	10718.0	10602.7		10579.4
$\Upsilon(5^3S_1)$	11024.3	25.7	27.2	84.2		10998.6	10997.1	10819.9		10876.0
$\Upsilon(6^3S_1)$	11278.2	13.5	45.9	85.5		11264.8	11232.3	11022.6		11019.0
$h_b(1^1P_1)$	9921.7	35.8	37.3	85.7	115	9885.9	9884.4	9915.5	9885	9899.3
$h_b(2^1P_1)$	10315.4	53.1	52.7	78.8	146	10262.3	10262.7	10259.1	10247	10259.8
$h_b(3^1P_1)$	10637.9	77.9	69.4	79.8	114	10560.1	10568.5	10523.2	10591	

- Largest mass shift is for Y(10580) agrees with the pioneer work

- We found all the ΔMs are –ive in this study
- Closer to the threshold arises larger ΔM

- In our case ΔM is maximum for $h_b(3P)$, but for Ref. [22]'s case largest ΔM is for $h_b(1P)$ and Ref. [23] for $h_b(2P)$ state

$^3P_0$ model with SHO: Other Studies

- We compare our results with recent studies on the topic

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Regular Article - Theoretical Physics

Ref. [22]

Bottomonium spectrum with coupled-channel effects

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Ref. [23]

Higher mass bottomonia

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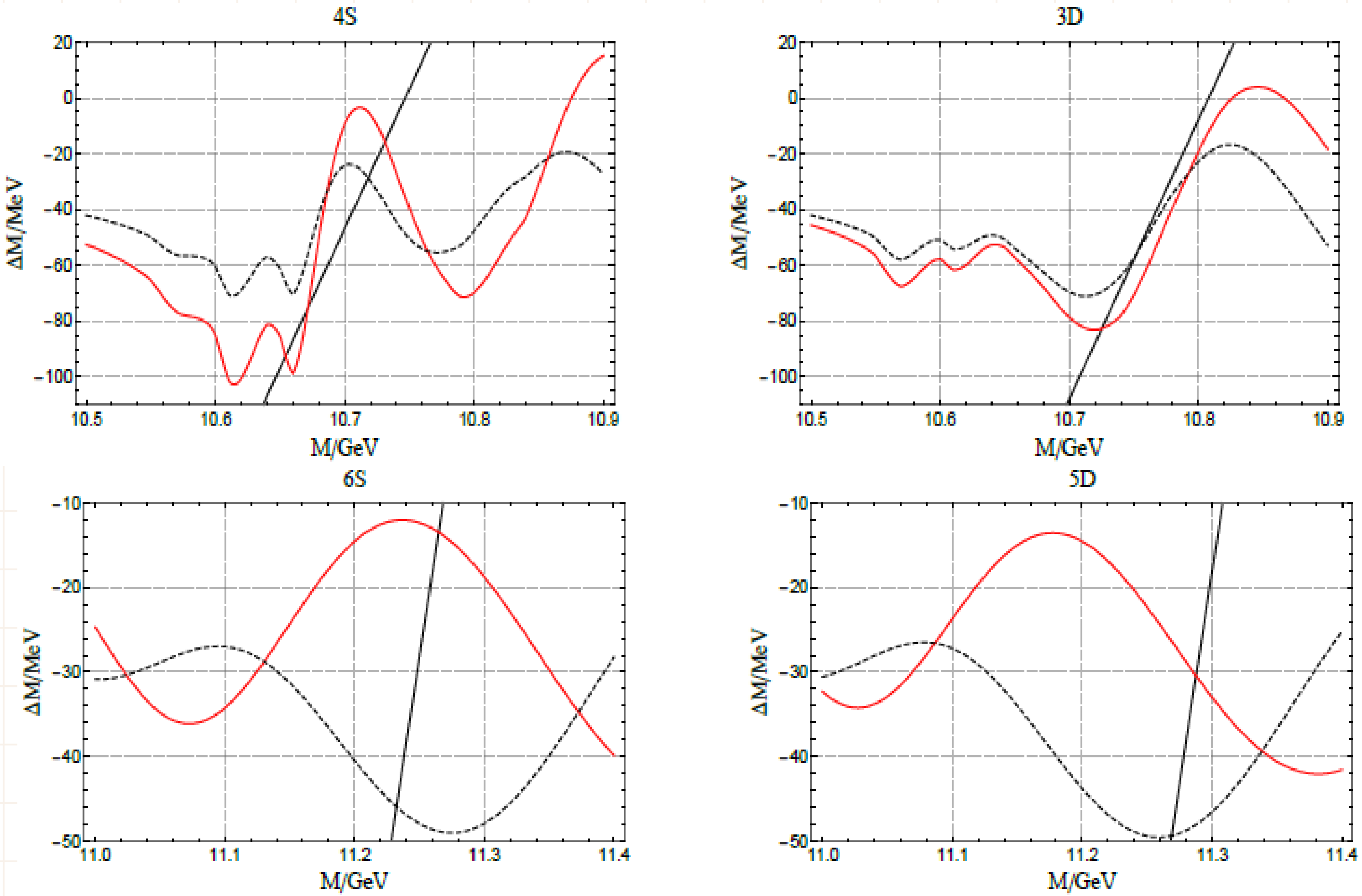
Mass Shifts for Individual Channels

States	$B\bar{B}$		$B\bar{B}^* + \text{H.c.}$		$B^*\bar{B}^*$		$B_s\bar{B}_s$		$B_s\bar{B}_s^* + \text{H.c.}$		$B_s^*\bar{B}_s^*$	
	GEM	SHO	GEM	SHO	GEM	SHO	GEM	SHO	GEM	SHO	GEM	SHO
$\Upsilon(1^3S_1)$	1.4	1.4	5.4	5.4	9.2	9.2	0.6	0.6	2.3	2.3	3.9	3.9
$\Upsilon(2^3S_1)$	3.0	2.9	11.4	11.1	18.9	18.5	0.9	0.9	3.5	3.5	5.9	5.9
$\Upsilon(3^3S_1)$	4.8	4.2	17.2	15.2	27.1	24.3	1.0	0.9	3.7	3.4	6.1	5.6
$\Upsilon(4^3S_1)$	−0.7	3.7	−2.4	16.0	85.4	−0.6	1.0	1.0	3.6	3.3	5.7	5.2
$\Upsilon(5^3S_1)$	−0.5	2.8	2.8	6.8	17.8	10.1	0.8	0.7	1.7	2.7	3.1	4.0
$\Upsilon(6^3S_1)$	1.5	3.5	2.4	14.2	1.5	21.2	0.6	0.6	2.8	2.3	4.7	4.1
$h_b(1^1P_1)$	0	0	13.5	14.0	13.0	13.4	0	0	4.8	5.0	4.6	4.8
$h_b(2^1P_1)$	0	0	21.9	21.6	20.3	20.2	0	0	5.6	5.6	5.3	5.3
$h_b(3^1P_1)$	0	0	38.0	33.5	29.5	26.3	0	0	5.4	5.0	5.0	4.6
$\chi_{b0}(1^3P_0)$	4.1	4.3	0	0	21.4	22.2	1.3	1.4	0	0	7.8	8.1
$\chi_{b0}(2^3P_0)$	9.3	9.0	0	0	31.1	31.0	2.1	2.1	0	0	8.4	8.5
$\chi_{b0}(3^3P_0)$	25.5	22.4	0	0	40.7	36.9	2.3	2.0	0	0	7.6	7.2

- "0" shows that the process is forbidden
- For simplicity we multiplied each ΔM with "–"
- Few channels have positive mass shifts, but the overall contribution is –ive

Mass Shift Plot



- Gap b/w the curves depend on the node structure of wave function

Probabilities of $b\bar{b}$ component

States	$B\bar{B}$		$B\bar{B}^* + \text{H.c.}$		$B^*\bar{B}^*$		$B_s\bar{B}_s$		$B_s\bar{B}_s^* + \text{H.c.}$		$B_s^*\bar{B}_s^*$		$P_{b\bar{b}}$	
	GEM	SHO	GEM	SHO	GEM	SHO	GEM	SHO	GEM	SHO	GEM	SHO	GEM	SHO
$h_b(1P)$	0	0	1.22	1.24	1.12	1.14	0	0	0.35	0.37	0.33	0.34	96.99	96.91
$h_b(2P)$	0	0	3.51	3.24	2.96	2.76	0	0	0.59	0.56	0.52	0.5	92.43	92.94
$h_b(3P)$	0	0	19.75	18.19	9.04	7.7	0	0	0.67	0.54	0.54	0.45	70.0	73.12
$\chi_{b0}(1P)$	0.45	0.46	0	0	1.74	1.77	0.11	0.12	0	0	0.52	0.55	97.18	97.1
$\chi_{b0}(2P)$	1.85	1.68	0	0	4.13	3.88	0.26	0.25	0	0	0.77	0.75	92.98	93.45
$\chi_{b0}(3P)$	34.08	38.84	0	0	8.07	6.21	0.31	0.22	0	0	0.62	0.48	56.92	54.26
$\chi_{b1}(1P)$	0	0	1.03	1.06	1.27	1.29	0	0	0.28	0.3	0.38	0.4	97.03	96.95
$\chi_{b1}(2P)$	0	0	3.38	3.11	3.0	2.81	0	0	0.53	0.51	0.56	0.54	92.53	93.04
$\chi_{b1}(3P)$	0	0	21.9	20.1	7.54	6.44	0	0	0.64	0.51	0.54	0.46	69.38	72.5

- State with lowest probability is $\chi_{b0}(3P)$ i.e. 57%

$h_b(3P)$, $\chi_{bn}(3P)$ with $n = 0,1,2$ states

State	Mass	Open channel	Threshold	Mass Diff.
$h_b(3P)$	10560	BB^*	10604.46	44
$\chi_{b0}(3P)$	10532.6	BB	10558.52	**26**
$\chi_{b1}(3P)$	10555.5	BB^*	10604.46	49
$\chi_{b2}(3P)$	10567.6	BB	10558.52	-9

- Mass goes closer to the open channel threshold results the highly suppression of $b\bar{b}$ component's probability
- $\chi_b(3P)$ system have mass closer to the corresponding threshold therefore its understood to have important continuum components

Ferretti, Galata & Santopinto PRD90 054010 (2014)

- Our $\chi_{b2}(3P)$ mass is above $B\bar{B}$ threshold, so its not possible to compute the probabilities for this state

Open flavor Strong Decay Widths

- We compute the open channel decay widths of Υ(10580), Υ(10860) and Υ(11020) by considering them pure S and D wave states

State	$B\bar{B}$		$B\bar{B}^* + h.c.$		$B^*\bar{B}^*$		$B_s\bar{B}_s$		$B_s\bar{B}_s^* + h.c.$		$B_s^*\bar{B}_s^*$		Γ_{theory}		Γ_{exp}
	GEM	SHO	GEM	SHO	GEM	SHO	GEM	SHO	GEM	SHO	GEM	SHO	GEM	SHO	
$4S$	21.1	12.5	0	0	0	0	0	0	0	0	0	0	21.1	12.5	20.5 ± 2.5
$3D$	34.1	24.2	0	0	0	0	0	0	0	0	0	0	34.1	24.2	
$5S$	5.1	3.5	4.8	11.1	1.9	4.1	0.9	0.2	0.6	0.4	4.5	0.5	17.9	19.7	55 ± 28
$4D$	10.8	7.2	4.0	5.4	18.1	18.1	1.21	0.3	0.3	0.2	2.8	0.9	37.3	32.1	
$6S$	2.9	1.3	3.4	6.4	0.1	6.5	0.3	0.0	1.0	0.1	0.2	0.2	7.8	14.5	79 ± 16
$5D$	6.5	3.0	2.9	3.3	9.2	10.1	0.4	0.0	0.4	0.1	1.1	0.2	20.4	16.8	

- Disagreement for Υ(11020) is obvious, we neglect excited B mesons
- Υ(11020)→ BB_1 is roughly 50% (40 MeV)

Segovia *et al.* PRD93 074027 (2016)

- Result for Υ(10580) considering it as pure 4S state with GEM agrees with PDG and also with Segovia *et al.* ⟶ Same formalism $^3P_0$ + GEM
- For the state Υ(10860) we both are away from the experimental data
- One possible reason is that it is not a pure S wave state
- Is it D wave dominant state??? Or admixture???

S-D Mixing in Coupled-Channel Model

S-D Mixing in CCM

- Meson coupling to $B\bar{B}$ continuum can be written as

$$\begin{pmatrix} M_0 & \int d^3p\langle\psi_0|H_I|BC\rangle \\ \langle BC|H_I|\psi_0\rangle & E_{BC} \end{pmatrix}\begin{pmatrix} c_0 \\ c_{BC} \end{pmatrix} = M\begin{pmatrix} c_0 \\ c_{BC} \end{pmatrix}$$

- Generalization of this matrix for S-D mixing case is

$$\begin{pmatrix} M_S^0 & H_T & \int d^3p\langle\psi_S|H_I|BC\rangle \\ H_T & M_D^0 & \int d^3p\langle\psi_D|H_I|BC\rangle \\ \langle BC|H_I|\psi_S\rangle & \langle BC|H_I|\psi_D\rangle & E_{BC} \end{pmatrix}\begin{pmatrix} c_S \\ c_D \\ c_{BC} \end{pmatrix} = M\begin{pmatrix} c_S \\ c_D \\ c_{BC} \end{pmatrix}$$

- Mixing induce by H_T is so mall i.e. 0.8^o so can be neglected $H_T = 0$
- S-D matrix become

Badalian, Bakker and Danilkin arXiv:0903.3643 [hep-ph]

$$\begin{pmatrix} M_S^0 + \Delta M_S & \Delta M_{SD} \\ \Delta M_{DS} & M_D^0 + \Delta M_D \end{pmatrix}\begin{pmatrix} c_S \\ c_D \end{pmatrix} = M\begin{pmatrix} c_S \\ c_D \end{pmatrix}$$

- With

$$\Delta M_f = \int d^3p \frac{|\langle\psi_f|H_I|BC\rangle|^2}{M - E_{BC} - i\epsilon} \qquad (f = S, D).$$

$$\Delta M_{SD} = \Delta M_{DS}^* = \int d^3p \frac{\langle\psi_S|H_I|BC\rangle\langle BC|H_I|\psi_D\rangle}{M - E_{BC} - i\epsilon}$$

S-D Mixing…

- For below threshold states if the mass is known the probability can be generalize to

$$|c_S|^2 + |c_D|^2 + \sum_{BC} \int d^3p \frac{1}{(M - E_{BC})^2} (|c_S|^2 H^2_{S,BC} + |c_D|^2 H^2_{D,BC} + 2\mathrm{Re}[c_S c^*_D H_{S,BC} H_{BC,D}]) = 1$$

- S-D mixed Decay width can be obtained by using

$$\Gamma_{SD} = 2\Big(|c_S|^2 \mathrm{Im}(\Delta M_S) + |c_D|^2 \mathrm{Im}(\Delta M_D) + 2\mathrm{Re}\big(c^*_S c_D \mathrm{Im}(\Delta M_{SD})\big)\Big)$$

Results of S-D Mixing in CCM

		2S	1D	3S	2D	4S	3D	5S	4D	6S	5D
	M_0	10.055	10.182	10.433	10.516	10.747	10.808	11.024	11.073	11.278	11.318
GEM	M_{pure}	10.011	10.136	10.373	10.454	10.654	10.725	10.999	11.058	11.265	11.288
	M_{comp}	10.011	10.136	10.373	10.454	10.651 +0.047i	10.731 +0.032i	10.999 +0.047i	11.058 +0.01i	11.265 +0.012i	11.288 +0.005i
	M_{real}	10.011	10.136	10.373	10.454	10.653	10.734	10.999	11.058	11.265	11.288
	$c_S/c_D(\text{comp})$	5482	0.0	524	−0.005	2.55 +2.63i	2.10 −1.16i	32.37 +12.65i	−0.03 −0.002i	10.55 −40.41i	−0.01 +0.02i
	$c_S/c_D(\text{real})$	5482	0.0	524	−0.005	6.19	0.97	41.8	−0.03	77.2	−0.005
	θ°	0.01	0.02	0.11	0.27	9.18	44.1	1.37	1.79	0.74	0.3

- $M_{\text{comp}} = M_{\text{BW}} + i\Gamma_{SD}/2$ is the exact solution of the equation

$$\begin{pmatrix} M_S^0 + \Delta M_S & \Delta M_{SD} \\ \Delta M_{DS} & M_D^0 + \Delta M_D \end{pmatrix} \begin{pmatrix} c_S \\ c_D \end{pmatrix} = M \begin{pmatrix} c_S \\ c_D \end{pmatrix}$$

- M_{real} is the real part of M_{comp} which is the solution of pot. Model
- $c_S/c_D\,(\text{real})$ is directly related to the mixing angle
- $c_S/c_D\,(\text{comp})$ contains all information about Γ_{ee} or Γ_γ
- Largest mixing angle is found for Υ(10580) state i.e. 9^{o} and 44.1^{o} for S and D wave respectively using GEM

$\Gamma_{ee}/\Gamma_{ee}(1S)$ in different models

- Ratios for Υ(10580) and Υ(11020) are highly suppressed experimentally
- Considering these as pure S wave states a large mixing angle can be introduce as arXiv:0903.3643 [hep-ph]
- Why there is big gap b/w the predictions of pot. Model and CCM for Υ(10580)???
- Neglecting the only open channel ($B\bar{B}$) contribution, Prob. = 67% means $\frac{2}{3}$ of the pot. Model as Fig. 6 shows
- Large central values of Υ(10860) is in favor of small mixing angle but due to large error bar large mixing angle can be introduce as arXiv:0903.3643 [hep-ph]
- Disagreement for Υ(11020) is quite obvious due to the neglect of excited B mesons

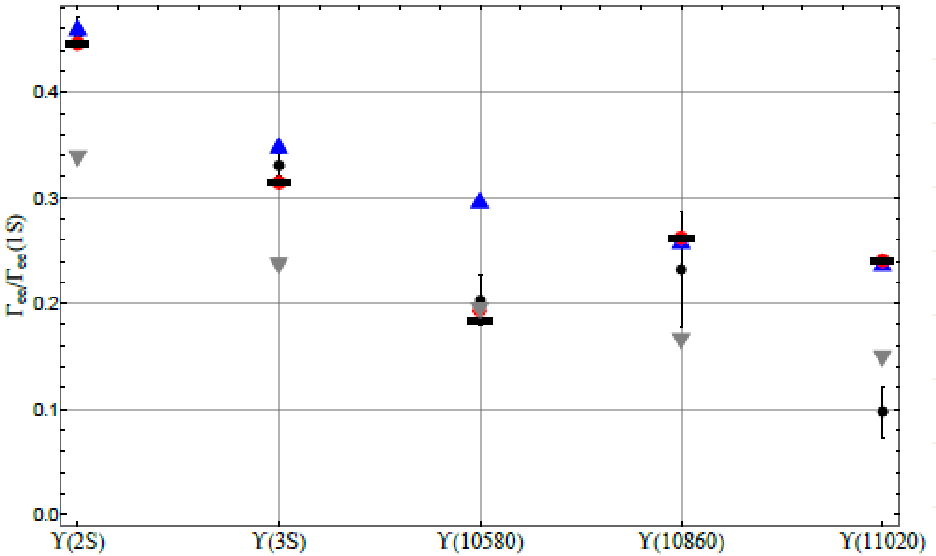


FIG. 6. Comparison of $\Gamma_{ee}/\Gamma_{ee}(1S)$ between different models. Results of the Cornell potential with our parameters and parameters of Ref. [22] are respectively represented by blue regular and gray inverted triangles. Red dots and black rectangles respectively denote the predictions of Eq. (24) with and without neglecting the imaginary part. Black dots with error bars are values taken from PDG [64].

- For of Υ(10860) & Υ(11020) all ground state B channel are open, so can't compute Prob. And no suppression of Γ_{ee} from continuum coupling

Suggestions for the Experiments

- Radiative decays can distinguish the suppression due to S-D mixing and B meson continuum

$$r_\gamma(S) := \frac{\Gamma(\Upsilon(S) \to \chi_{b2}(1P)+\gamma)}{\Gamma(\Upsilon(S) \to \chi_{b0}(1P)+\gamma)} = 5\left(\frac{E_{\gamma 2}}{E_{\gamma 0}}\right)^3,$$

- Below threshold we have

$$r_\gamma(D) := \frac{\Gamma(\Upsilon_1(D) \to \chi_{b2}(1P)+\gamma)}{\Gamma(\Upsilon_1(D) \to \chi_{b0}(1P)+\gamma)} = \frac{1}{20}\left(\frac{E_{\gamma 2}}{E_{\gamma 0}}\right)^3$$

$r_\gamma(2S)$	$r_\gamma(1D)$	$r_\gamma(3S)$	$r_\gamma(2D)$	
1.57	0.0157	3.6	0.036	← Theoretical Predictions
1.91 ± 0.29		3.82 ±1.05		← Measurements PDG

- Small mixing below threshold agrees with the exp. observations i.e. Y(2S) and Y(3S) are pure S wave states
- Considering Y(10580) and Y(11020) as pure S wave, Γ_{ee} require large mixing angle which highly suppress $r_\gamma(S)$
- Mixing caused by multiquark component have no effect on $r_\gamma(S)$
- Predicting small mixing angle we expect large $r_\gamma(S)$
- Unfortunately no data on r_γ available for Y(10580), Y(10860) and Y(11020)

Summary

- S-D mixing is not the only mechanism to suppress Γ_{ee}, coupling to the meson continuum can also suppress it

- Precise measurements on the radiative decays can distinguish these two effects

- We suggest BaBar & Belle to make precise measurements on Γ_{γ} which is a key to understand the internal structure

- Υ(4S)'s $\Gamma_{\rm GEM} \approx 2\ \Gamma_{\rm SHO}$ and mass shift with GEM ≈ 3 SHO
- Deviations b/w GEM and SHO is not negligible which leads that SHO is not a good approximation, in many cases it ruin the results

- Essential to treat wave functions more accurately near thresholds states

Thanks for Your Attention