



# Hadronic Contributions to $(g-2)_\mu$

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8<sup>th</sup> Workshop on Hadron Physics in China and Opportunities Worldwide  
8<sup>th</sup> Aug.~11<sup>th</sup> Aug. 2016 Wuhan

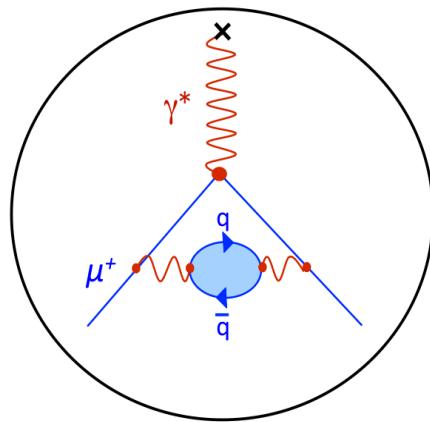
# Muon Anomaly: $a_\mu = (g-2)_\mu / 2$

- **Experimental:**  $a_{\mu}^{\text{exp}} = 1165\ 920\ 8.9\ (6.3) \times 10^{-10}$  (0.54 ppm)  
[BNL-E821: PRD 73 072003]
- **Standard Model prediction:**  $a_{\mu}^{\text{SM}} = a_{\mu}^{\text{QED}} + a_{\mu}^{\text{weak}} + a_{\mu}^{\text{had}}$

Contribution	in units $10^{-10}$	
QED( $\gamma$ +lepton)	$1165\ 847\ 1.8951 \pm 0.0080$	Kinoshita et.al. (2012)
EW	$15.36 \pm 0.10$	Gnendiger, Stöckinger, Stöckinger-kim (2013)
HVP,LO	$692.3 \pm 4.2$	Davier et. al. (2011)
HVP,NLO	$-9.84 \pm 0.07$	Hagiwara et al. (2009)
HLbL	$11.6 \pm 4.0$	Jegerlehner, Nyffler (2009)
Total	$1165\ 918\ 1.3 \pm 5.8$	

# Hadronic Contributions

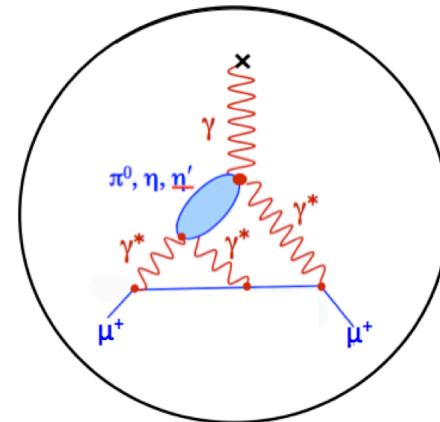
Hadronic Vacuum Polarization



$692.3 \pm 4.2$

[Davier et. al. (2011)]

Hadronic Light-by-Light

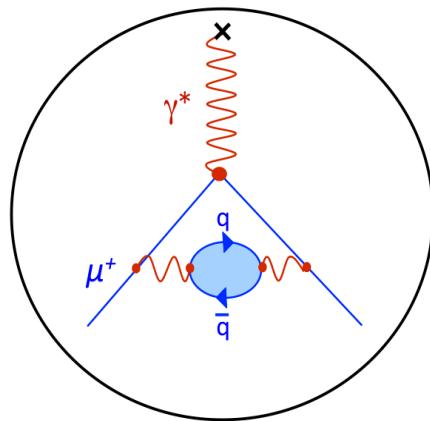


$11.6 \pm 4.0$

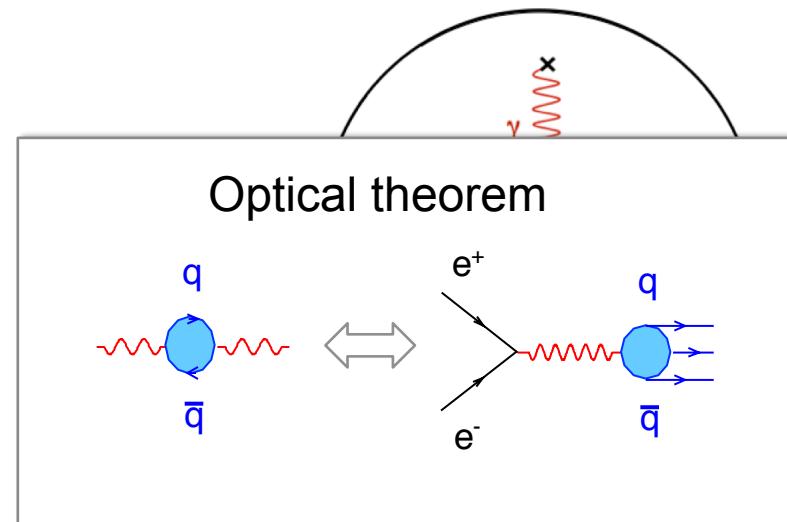
Jegerlehner, Nyffler (2009)

# Hadronic Contributions

Hadronic Vacuum Polarization



Hadronic Light-by-Light



Dispersion integral

$$a_{\mu,LO}^{\text{HVP}} = \frac{1}{4\pi^3} \int_{m_{\pi^0}}^{\infty} ds K(s) \sigma_{\text{had}}(s)$$

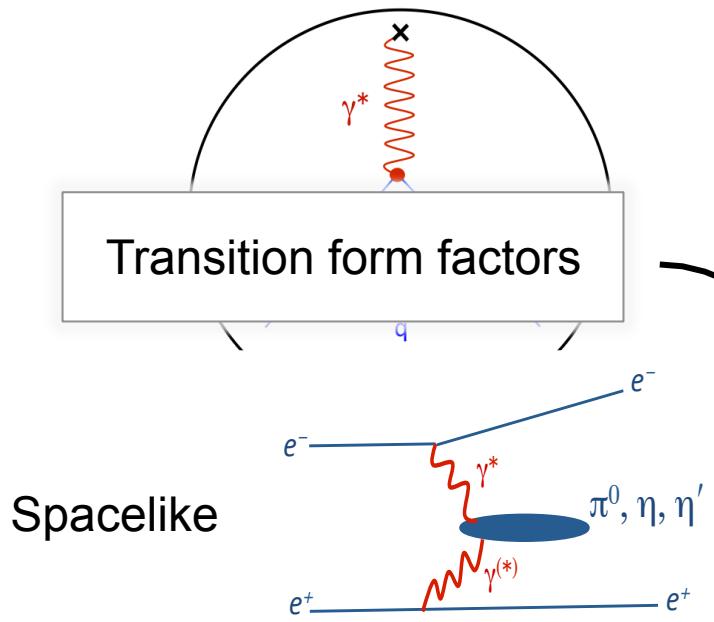
Kernel function  $\sim 1/s$

$$\sigma_{\text{had}} = \sigma(e^+e^- \rightarrow \text{hadrons})$$

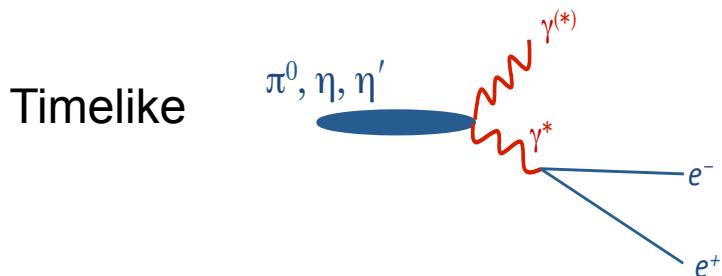
$\sim 1/s \rightarrow$  Low energy contributions important!

# Hadronic Contributions

## Hadronic Vacuum Polarization

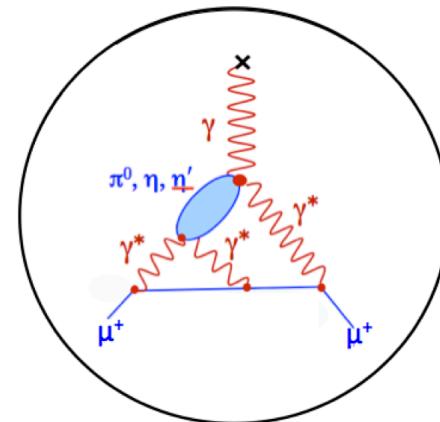


Spacelike



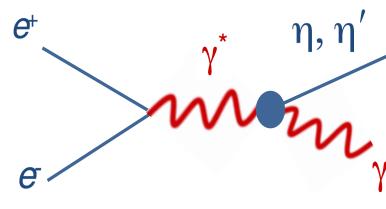
Timelike

## Hadronic Light-by-Light



- Only model calculations so far
- Data-driven approach been developed

[Colangelo et al '14; Pauk, Vanderhaeghen '14]

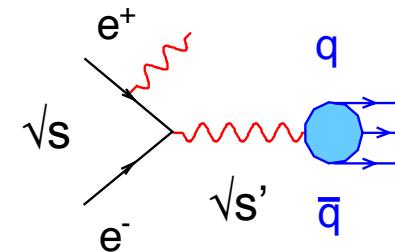
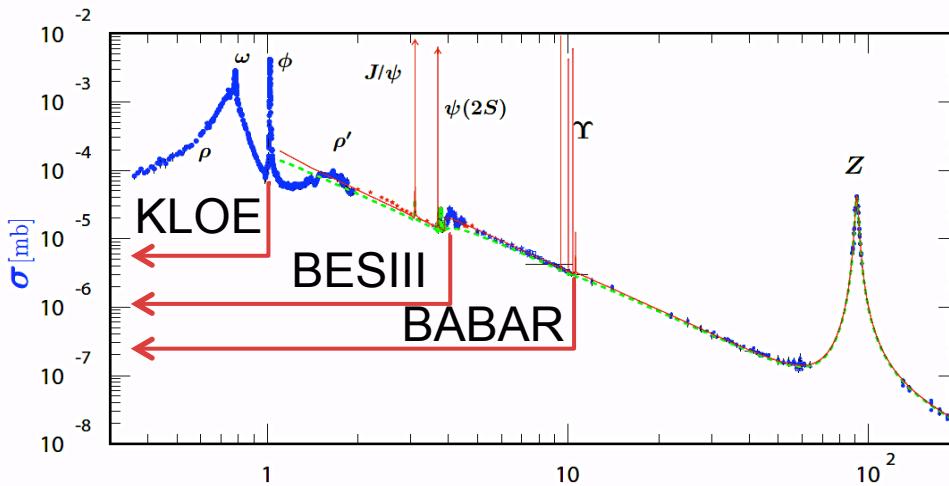


# **Hadronic Cross Section**

$\sigma_{\text{had}} (\text{e}^+ \text{e}^- \rightarrow \text{hadrons})$

# Hadronic Cross Section

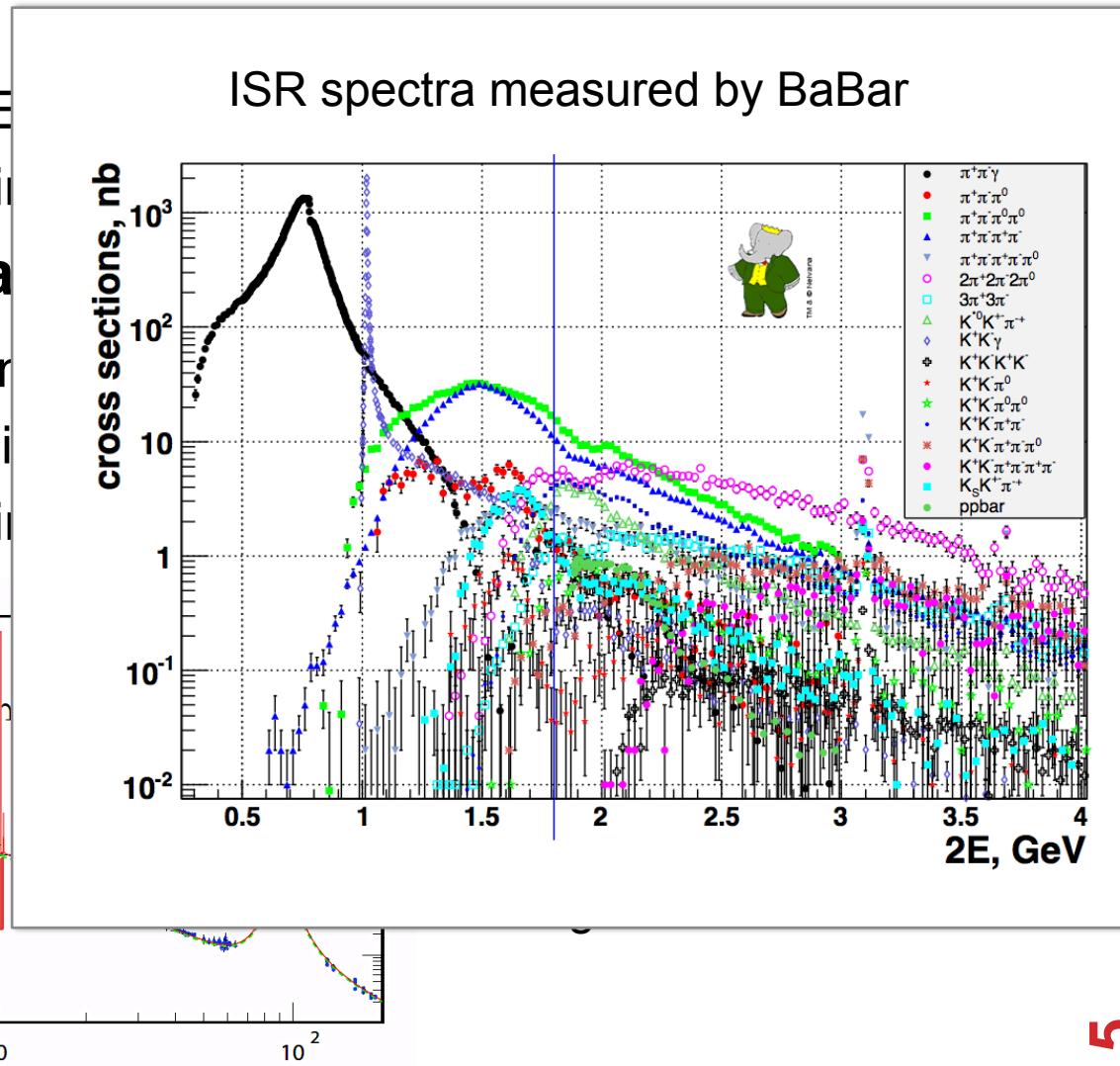
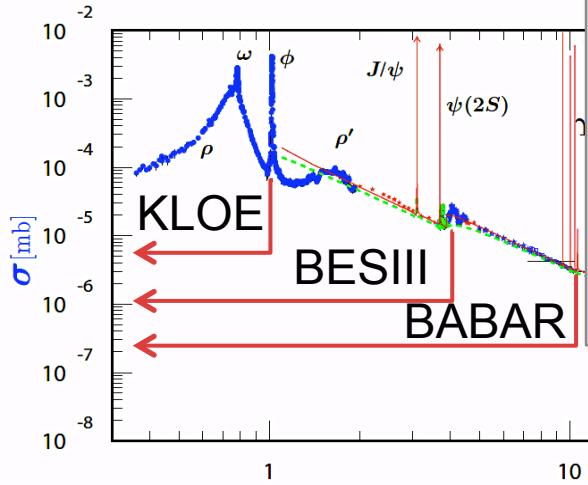
- Energy Scan:
  - CMD & SND at VEPP-2M & VEPP-2000 in Novosibirsk
  - BESIII at BEPCII in Beijing
- Initial State Radiation:
  - KLOE at DA $\phi$ NE in Frascati
  - BABAR at PEP-II in Stanford
  - BESIII at BEPCII in Beijing



- Needs **no** systematic variation of beam energy
- High statistics thanks to high integrated luminosities

# Hadronic Cross Section

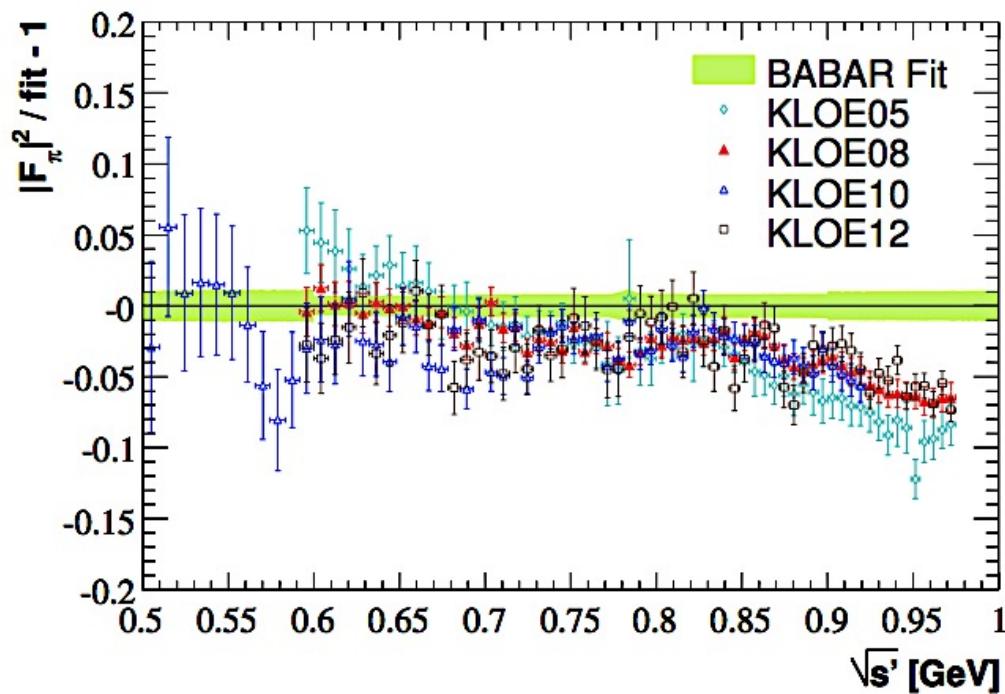
- Energy Scan:
  - CMD & SND at VEPP-2M
  - BESIII at BEPCII in progress
- Initial State Radiation
  - KLOE at DA $\phi$ NE in progress
  - BABAR at PEP-II in progress
  - BESIII at BEPCII in progress



# Most Relevant Channel:



- KLOE and BABAR dominate the world average
- Both with uncertainties smaller than 1%
- Relatively large systematic differences, especially above  $\rho$  peak
- Knowledge of  $a_\mu^{\text{had}}$  dramatically limited due to this difference

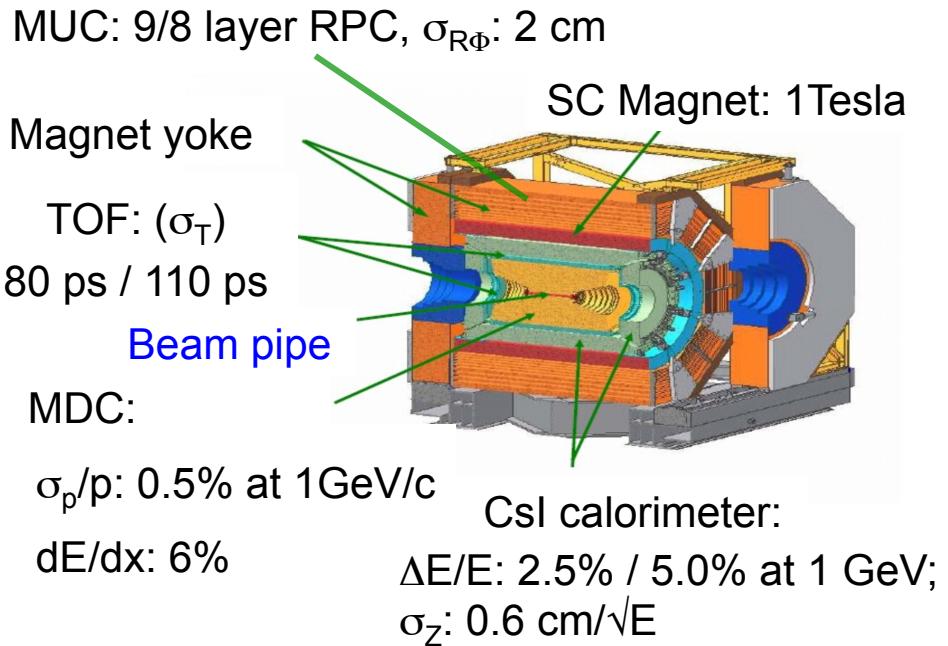


Note: KLOE05 super-  
seded by KLOE08

# Beijing Electron Positron Collider-II



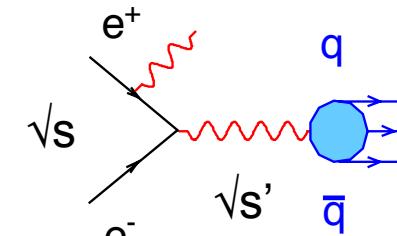
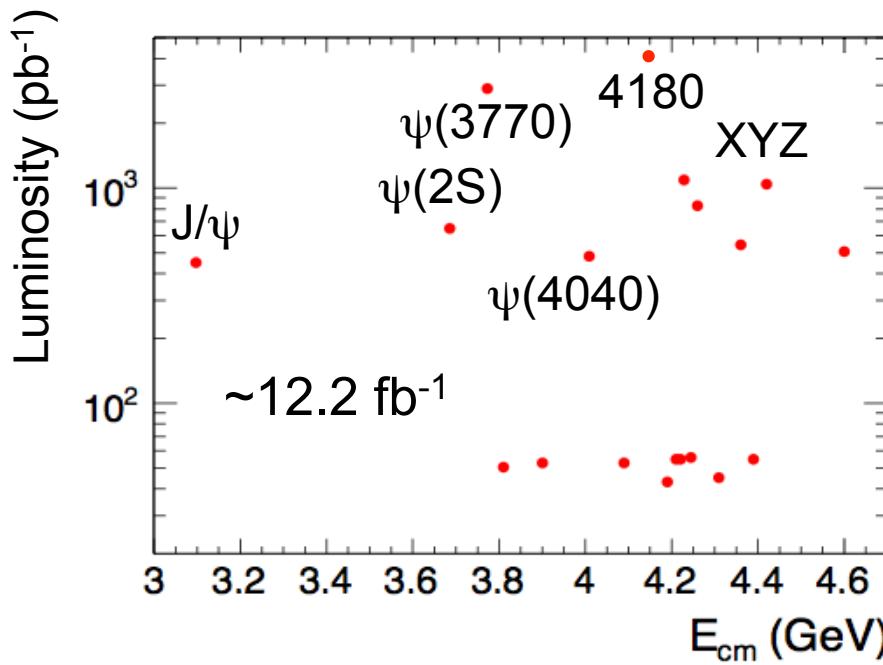
# Beijing Electron Positron Collider-II



BEPCII:  $\tau$ -charm factory  
Beam energy:  
1-2.3 GeV  
Design luminosity:  
 $1 \times 10^{33} \text{ cm}^{-2}\text{s}^{-1}$   
Data taking from 2009 to present

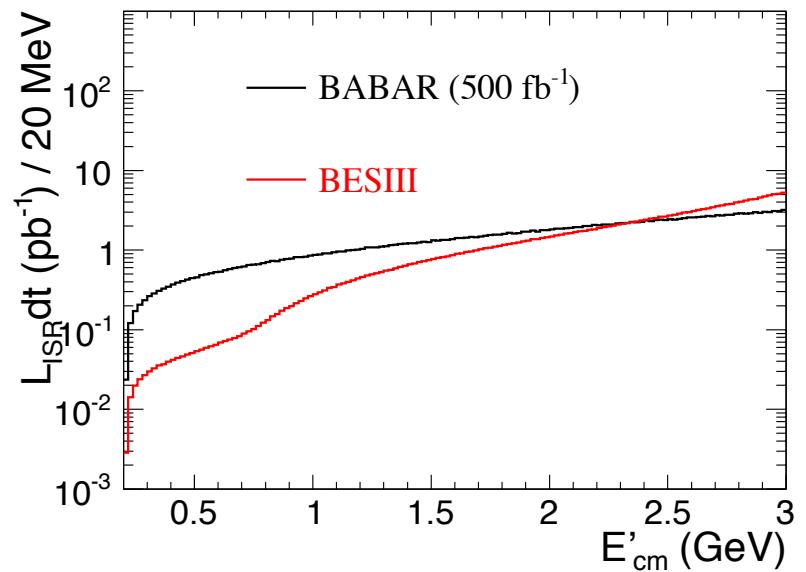


# Data Samples for ISR Study



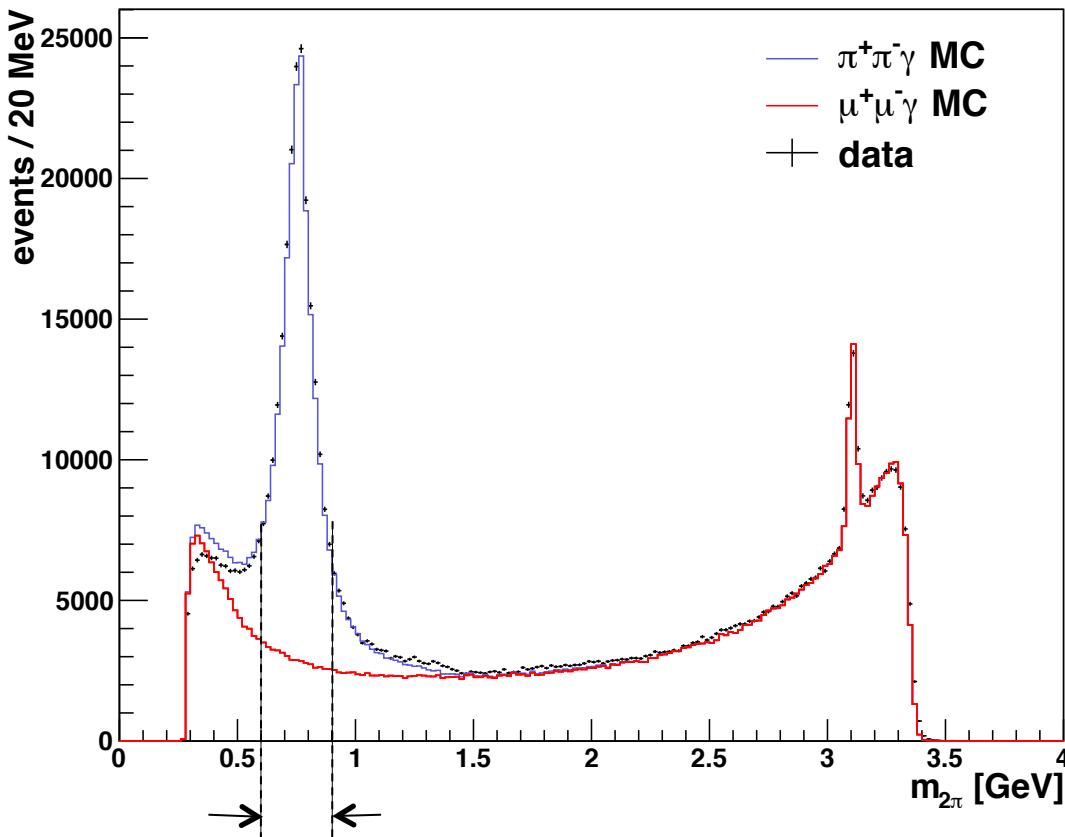
$$\text{ISR luminosity} = L dt \times H_{\text{rad}}$$

Radiator function, well known



# $e^+e^- \rightarrow \gamma_{\text{ISR}} \pi^+\pi^-$ at BESIII

Event yield after preliminary selection



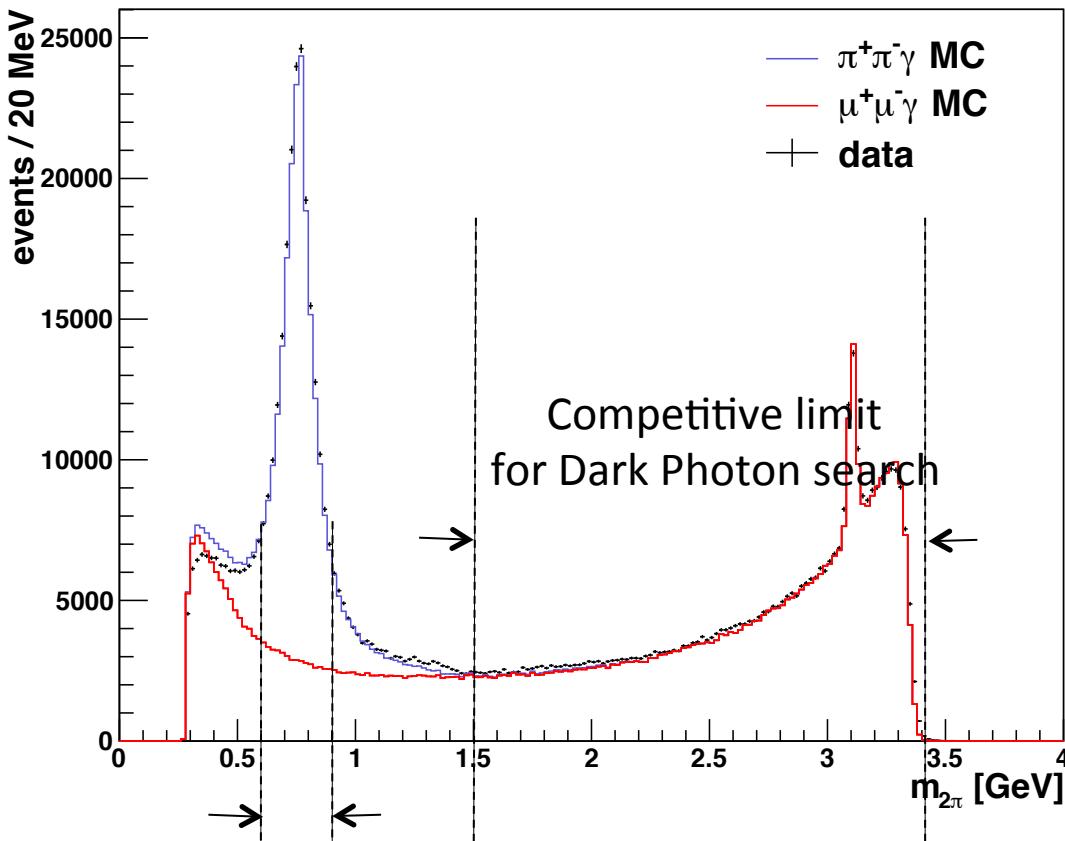
Initial publication  
600 – 900 MeV

[Phys. Lett. B753 (2016) 629]

- $\psi(3770)$  data only ( $2.9 \text{ fb}^{-1}$ )
- Tag ISR photon
- No dedicated background subtraction
- $e^+e^- \rightarrow \gamma\pi^+\pi^-$ : large statistics
- $e^+e^- \rightarrow \gamma\mu^+\mu^-$ : dominate background
- Data - MC differences visible

# $e^+e^- \rightarrow \gamma_{\text{ISR}} \pi^+\pi^-$ at BESIII

Event yield after preliminary selection



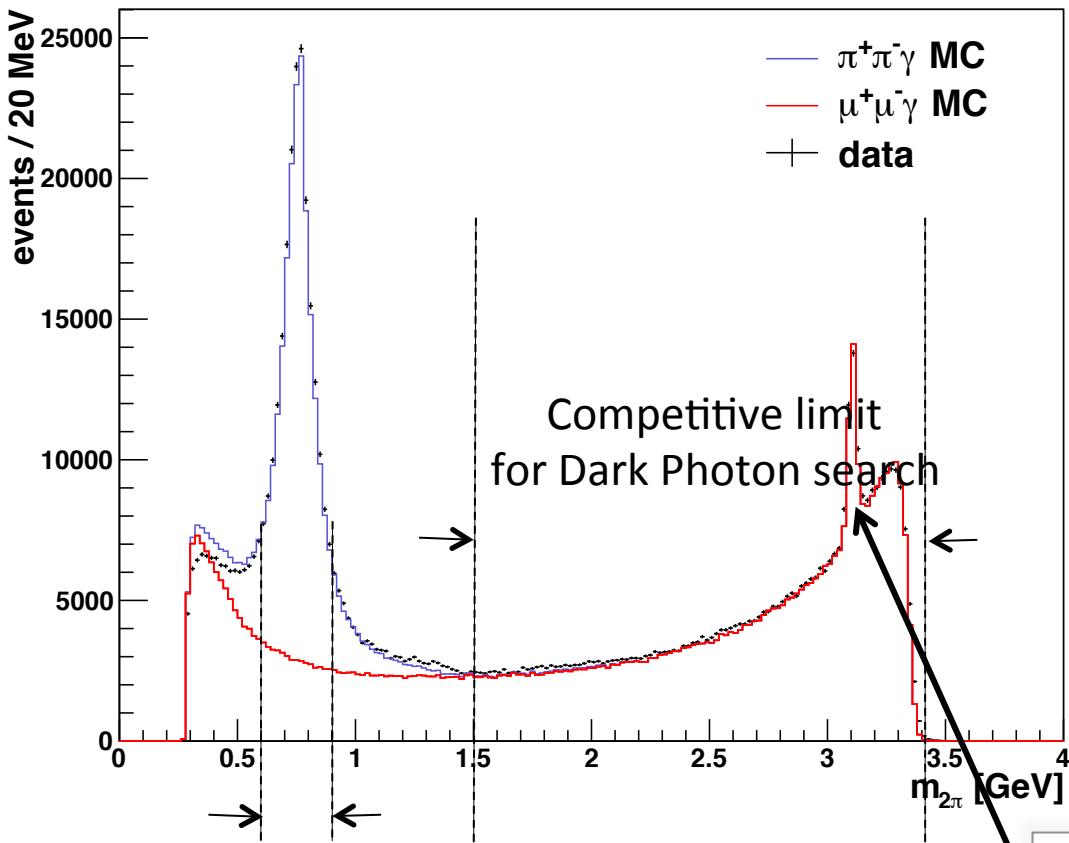
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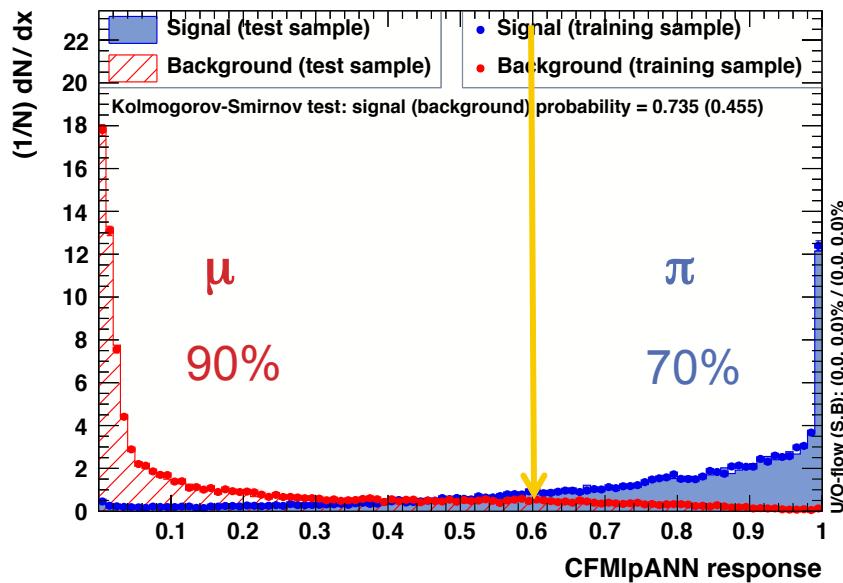
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- Data - MC differences visible

World's best measurement  
of  $\Gamma_{ee}$  of  $J/\psi$

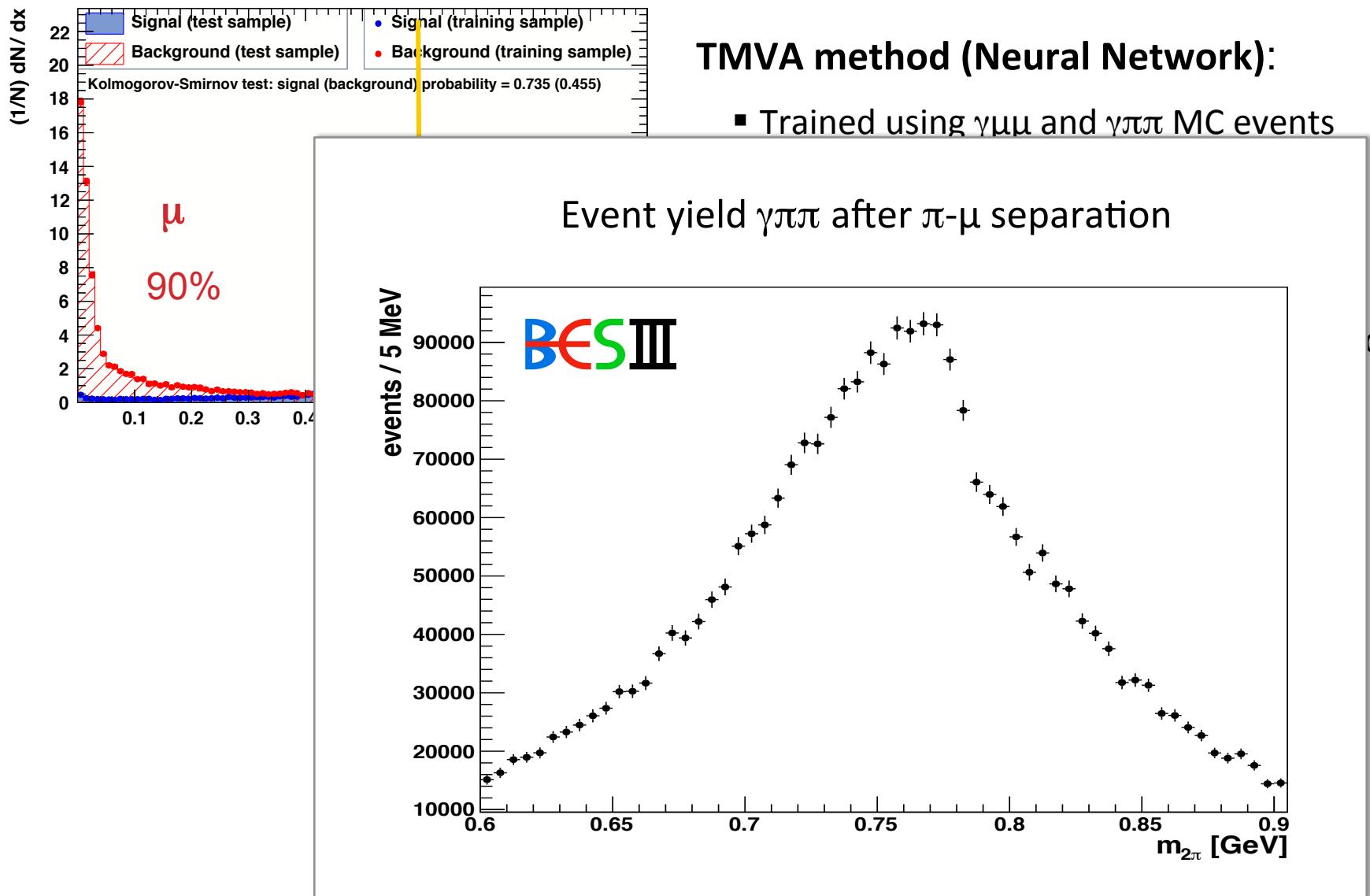
# $e^+e^- \rightarrow \gamma_{ISR} \pi^+\pi^-$ : $\pi\text{-}\mu$ separation



## TMVA method (Neural Network):

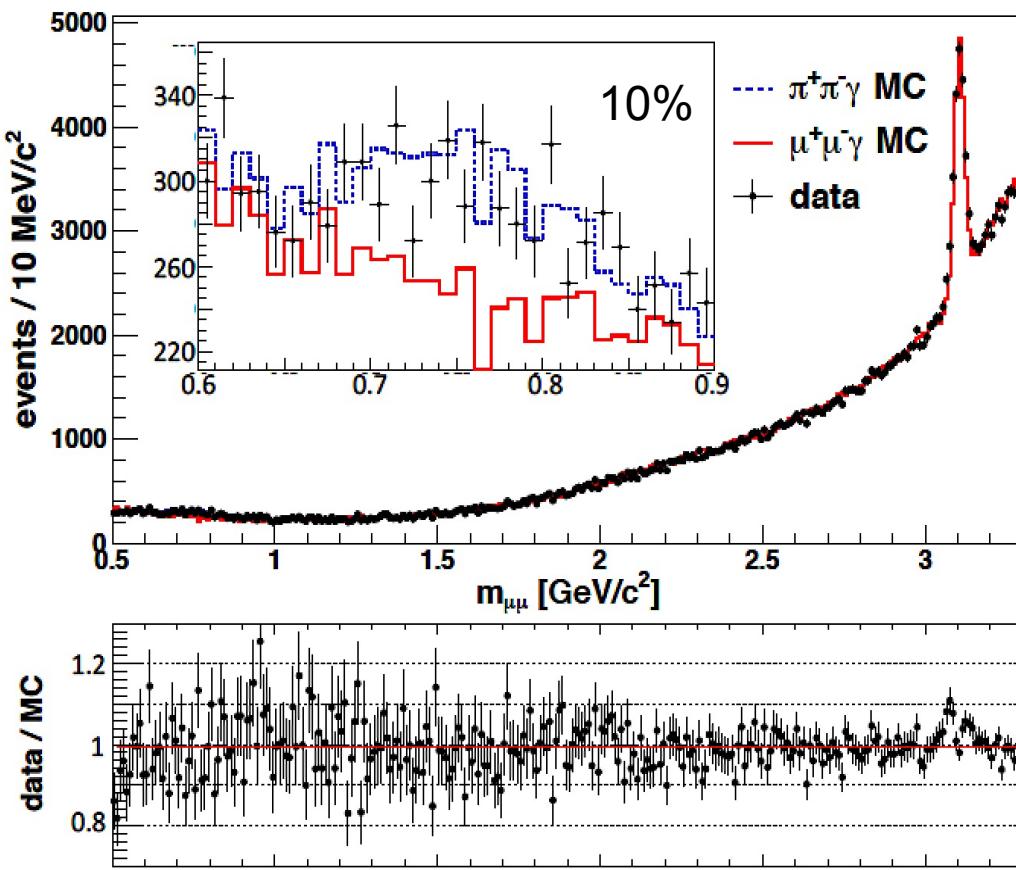
- Trained using  $\gamma\mu\mu$  and  $\gamma\pi\pi$  MC events
- Information based on track level
- Efficiency matrix ( $p, \Theta$ ) for data, MC
- Correct for data - MC differences
- Cross checked for different TMVA methods

# $e^+e^- \rightarrow \gamma_{\text{ISR}} \pi^+\pi^-$ : $\pi\text{-}\mu$ separation



# QED Test: $e^+e^- \rightarrow \gamma\mu^+\mu^-$

Event yield  $\gamma\mu\mu$  after  $\pi\text{-}\mu$  separation and all efficiency corrections



- Background from  $\gamma\pi\pi$  small
- PHOKHARA uncertainty < 0.5%
- Luminosity measurement based on Bhabha events, 1.0% accuracy

$$\Delta(\text{MC/QED-data}) - 1 = (1.0 \pm 0.3_{\text{stat}} \pm 0.9_{\text{syst}}) \%$$

- Excellent agreement with QED
- Accuracy on 1% level as needed to be competitive !

# $e^+e^- \rightarrow \pi^+\pi^-$ Cross section

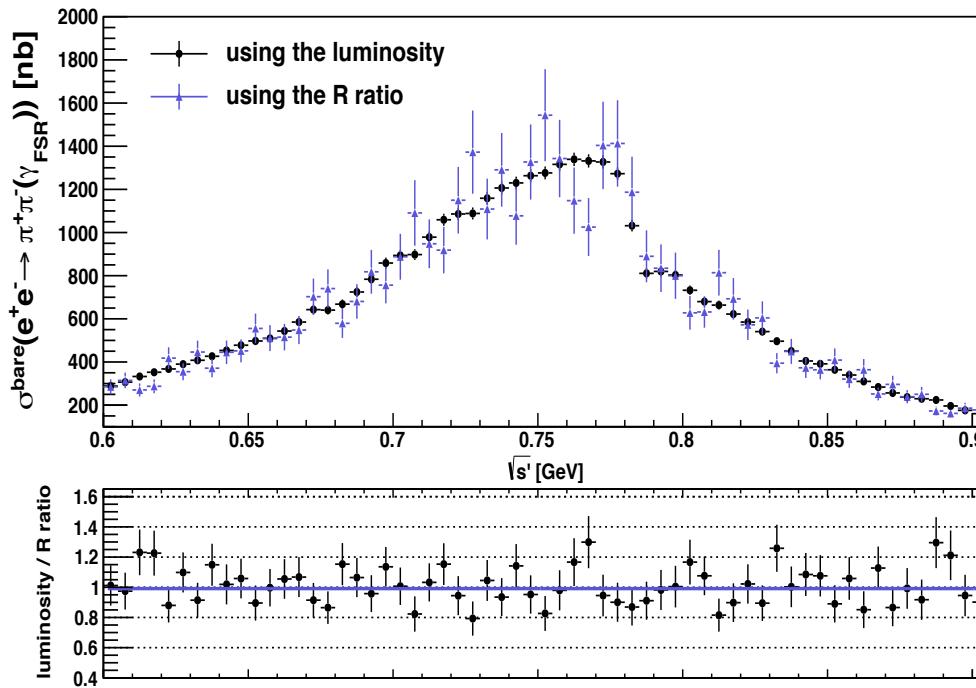
## 2 normalization methods:

- Normalization to  $L_{int}$  (obtained from Bhabha events)

$$\sigma_{bare}(e^+e^- \rightarrow \pi^+\pi^-) = \frac{N_{\pi\pi\gamma}}{L_{int} \cdot H_{rad} \cdot \delta_{vac} \cdot (1 + \delta_{FSR})}$$

- Normalization to  $\gamma\mu\mu$  events, i.e. R ratio ( $\gamma\pi\pi/\gamma\mu\mu$ )

$L_{int}$ ,  $H_{rad}$ ,  $\delta_{vac}$  cancel in ratio



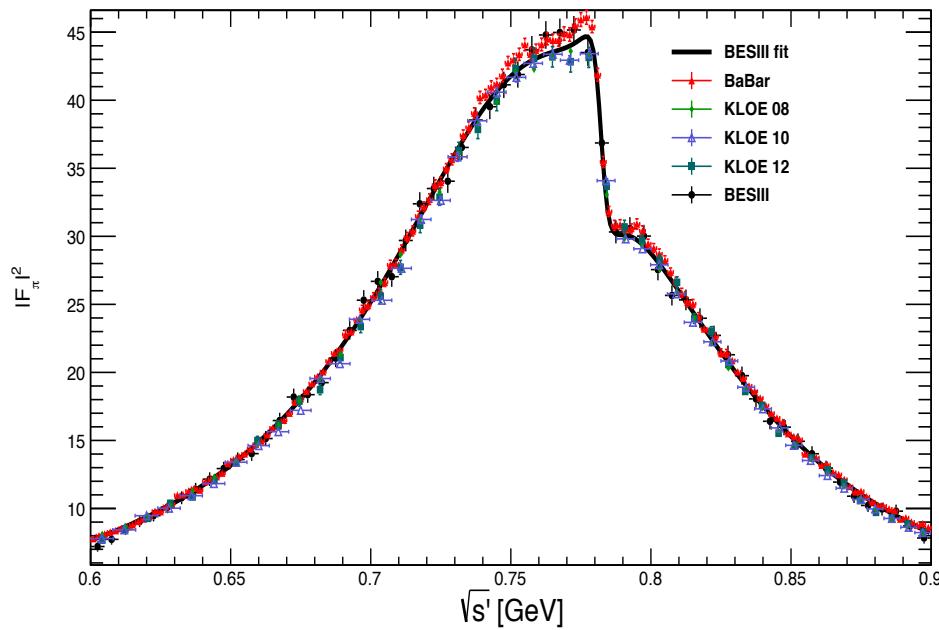
Good agreement between  
two methods

**luminosity / R ratio -1  
=  $(0.85 \pm 1.68) \%$**

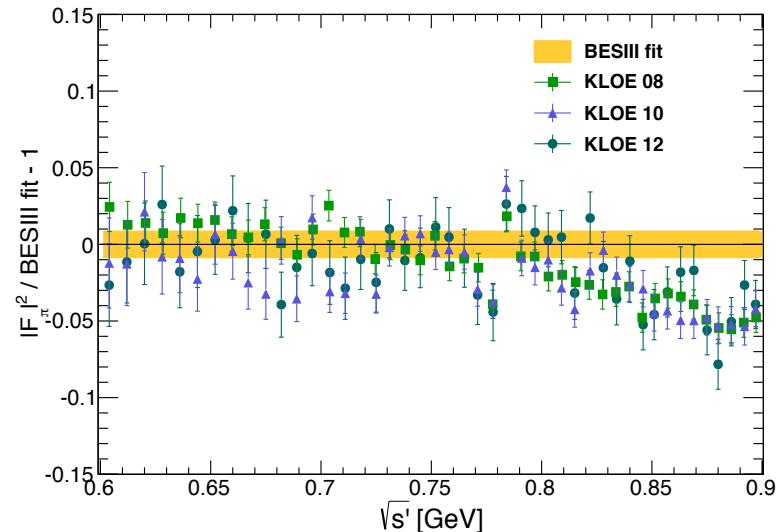
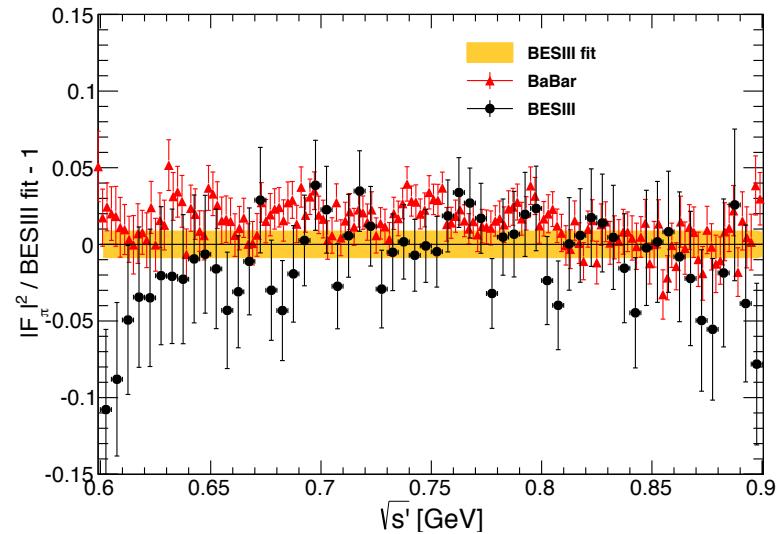
limited by low  $\gamma\mu\mu$  statistics

# Compare with Existing Data

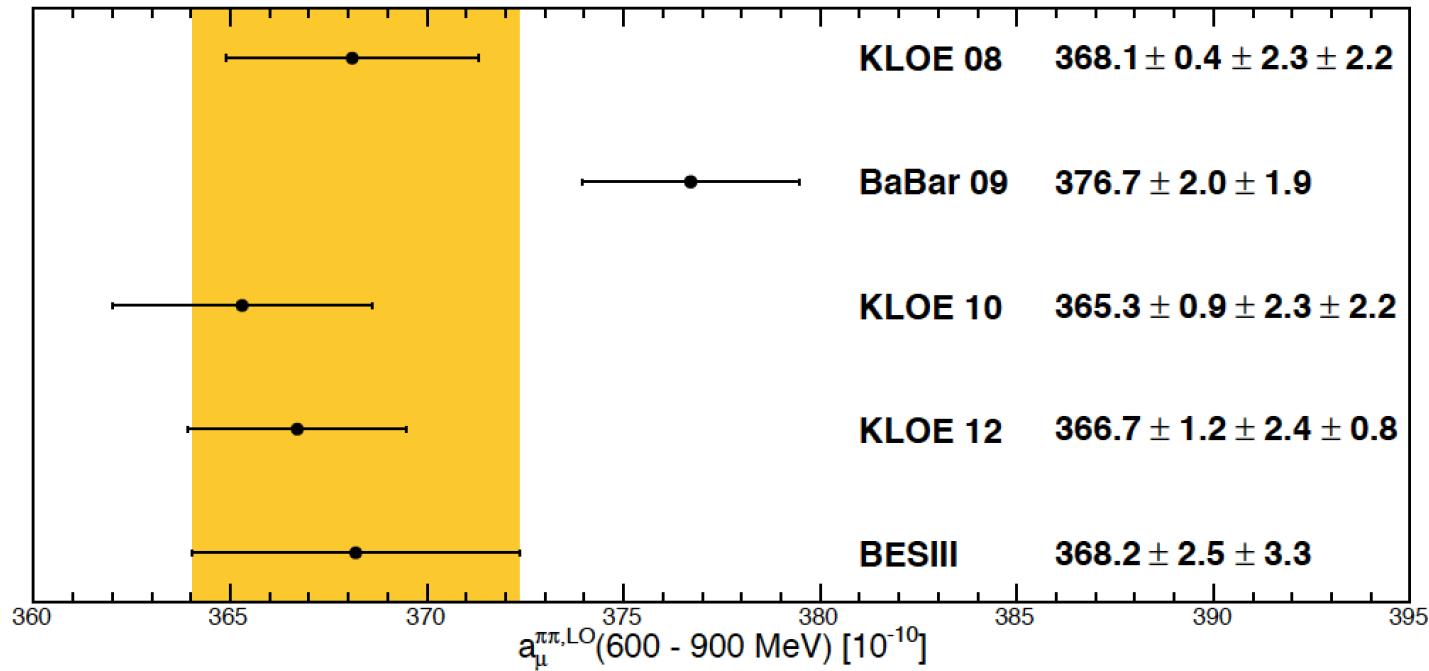
## Pion Form Factor $F_\pi$



- Gounaris and Sakurai parameterization
- 0.9 % accuracy (dominated by theory)
- Normalization to luminosity  $\times$  radiator function

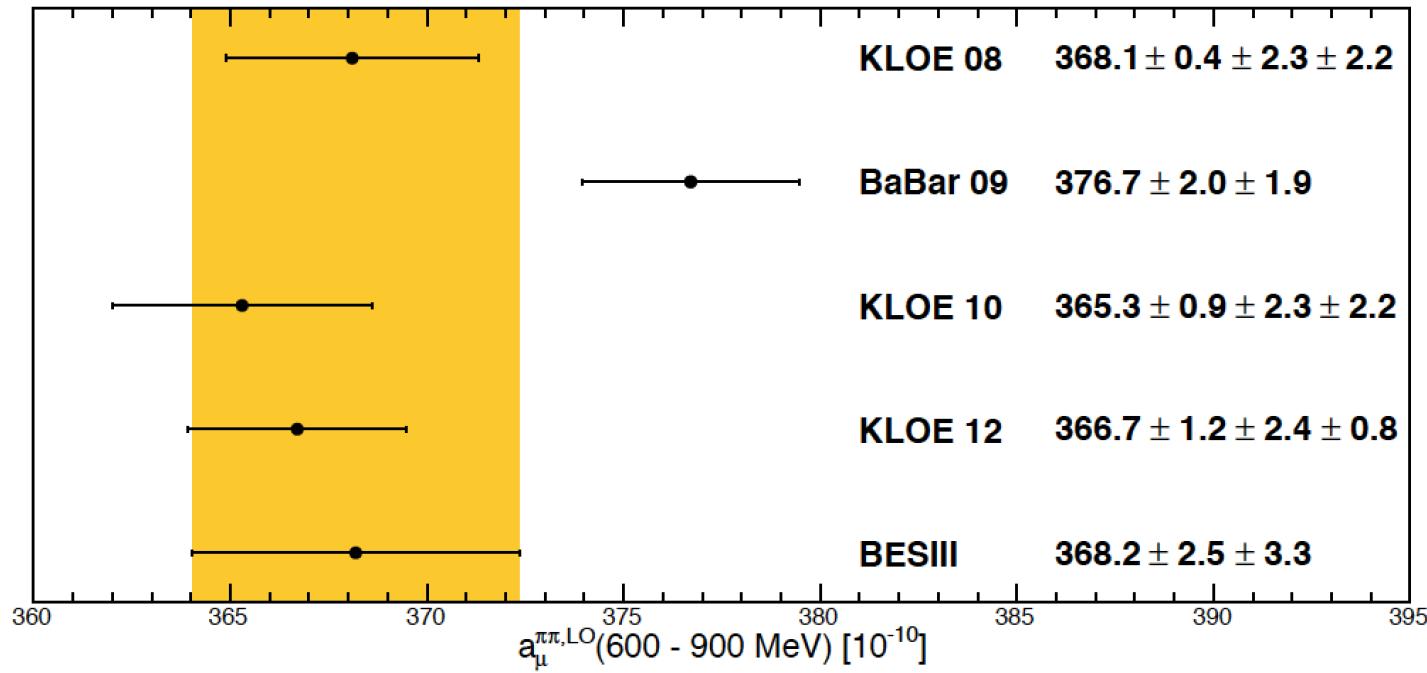


# Impact on $a_{\mu}^{\text{HVP}}$

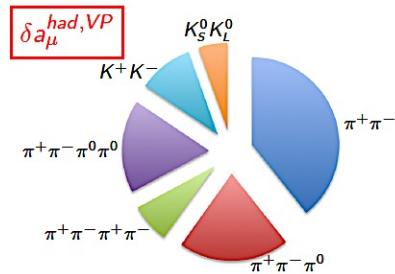


Deviation on  $(g-2)_{\mu}$  between experimental and SM has been confirmed

# Impact on $a_\mu^{\text{HVP}}$



Deviation on  $(g-2)_\mu$  between experimental and SM has been confirmed



Study of  $\pi^+\pi^-\pi^0$  and  $\pi^+\pi^-\pi^0\pi^0$  processes undergoing at BESIII

# Energy Scan from 2.0 to 4.6 GeV

World's best measurement from BES/BESII with 5% ~ 8% total uncertainty (statistical uncertainty: 3% ~ 5%)

## BESIII: aim at systematic accuracy: 3.0%

151 energy points  $>10^5$  hadronic events each → statistical error negligible

Energy region	Energy points	Note
2.400~3.400	4	Mini-scan
3.800~4.590	104	Fine-scan heavy charm resonant
2.000~3.080	21	R&QCD-scan
3.050~3.120	16	J/ $\psi$ -scan
3.542~3.600	5	$\tau$ -scan
3.650,3.671	2	$\psi(3686)$ -scan

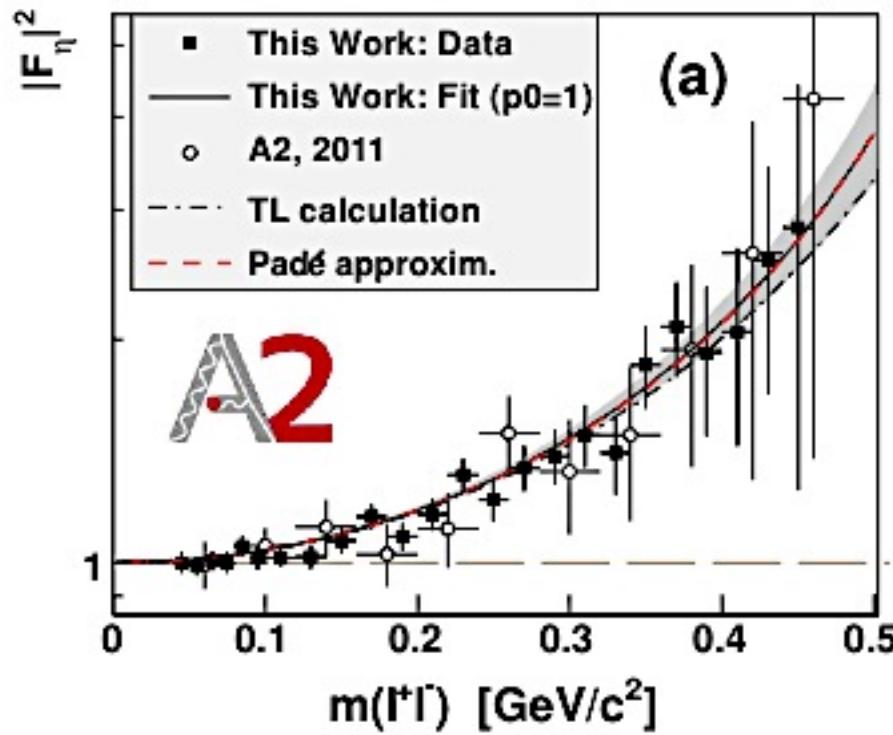
Reducing the uncertainty of  $\alpha_{\text{em}}(M_Z^2)$

→ A new quantity of electroweak precision fits

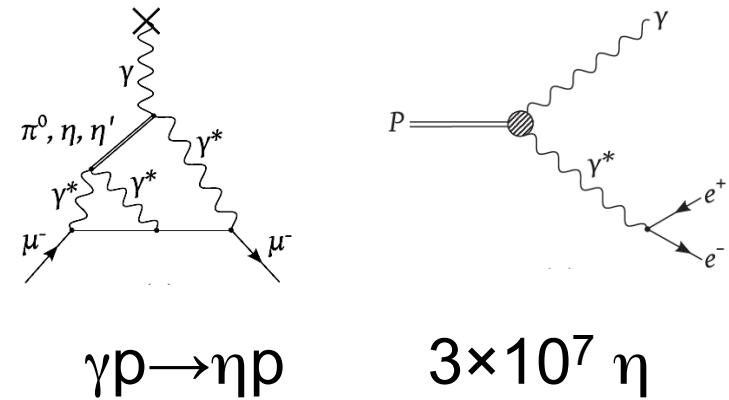
# **Meson Transition Form Factor $|F(Q^2)|$**

# Timelike Transition FFs

$$\frac{d\Gamma(\eta \rightarrow l^+l^-\gamma)}{dm_{ll}\Gamma(\eta \rightarrow \gamma\gamma)} = [QED] \cdot |F_\eta(m_{ll})|^2$$



[Phys. Rev. C89 (2014) 044608]



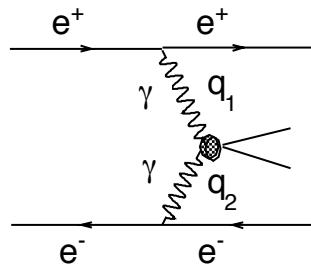
Vector Meson Dominance Model:  
single-pole:

$$F(q) = (1-q^2/\Lambda^2)^{-1}$$

$$\Lambda^{-2} = (1.95 \pm 0.15 \pm 0.10) \text{ GeV}^{-2}$$

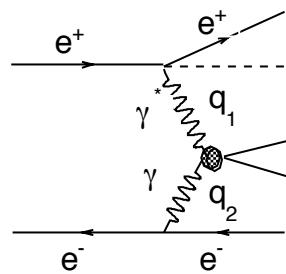
In good agreement with previous measurements!

# Spacelike Transition FFs



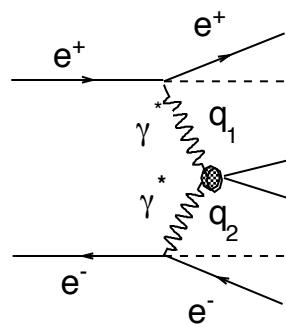
Untag:

- Only tag the hadron products,  $P_t$ -balance
- $Q_i^2 \sim 0 \text{ GeV}^2$ , quasi-real photon



Single tag:

- Tag the hadron products
- Tag only one lepton, missing momentum direction
- $Q_1^2 \sim 0 \text{ GeV}^2$ ,  $Q_1^2 = -q_2^2 \text{ GeV}^2$ ; highly virtual photon



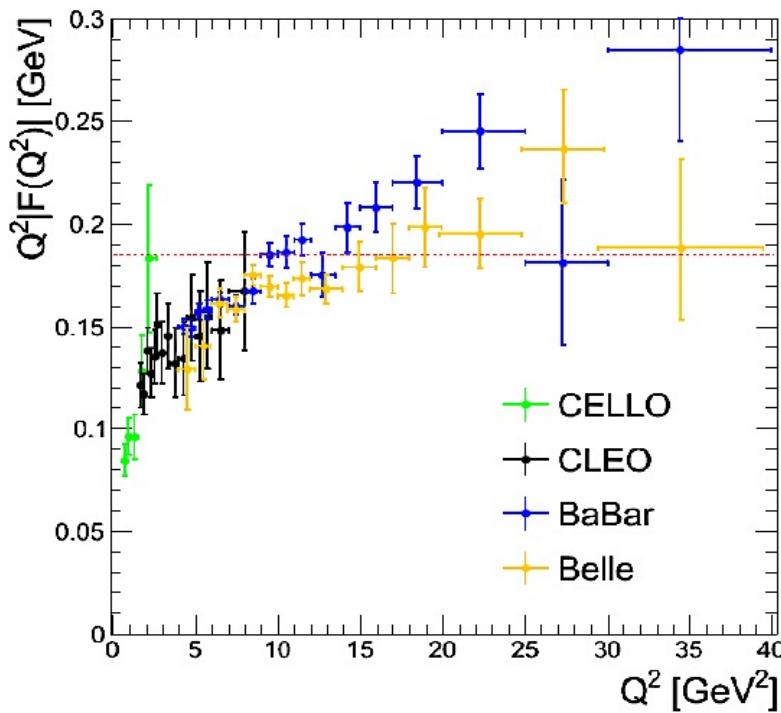
Double tag:

- Tag the hadron products
- Tag both leptons
- Both photons are virtual

Input for data-driven  
approach

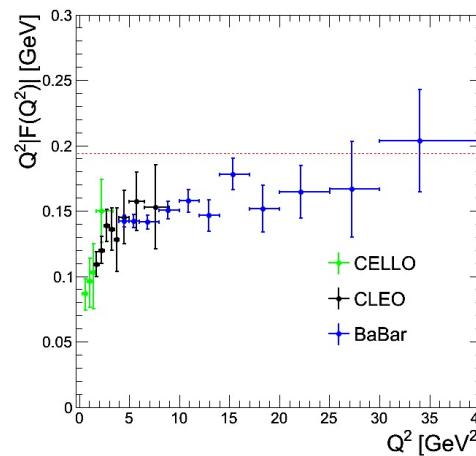
# Existing Data on Spacelike TFFs

$e^+e^- \rightarrow e^+e^- \pi^0$

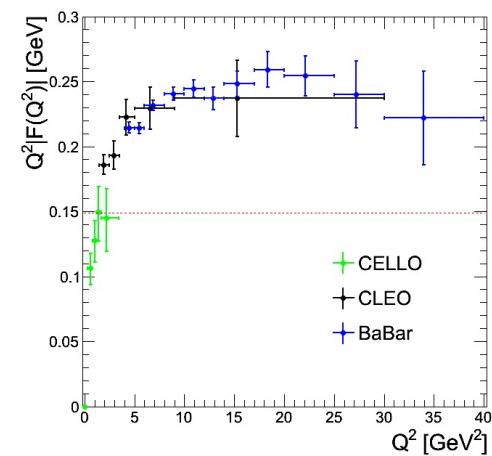


[CELLO: Z. Phys. C 49 401 (1991)]  
 [CLEO: Phys. Rev. D57 33 (1998)]  
 [BaBar: Phys. Rev. D80 052002 (2009)]  
 [Belle: Phys. Rev. D86 092007 (2012)]

$e^+e^- \rightarrow e^+e^- \eta$



$e^+e^- \rightarrow e^+e^- \eta'$

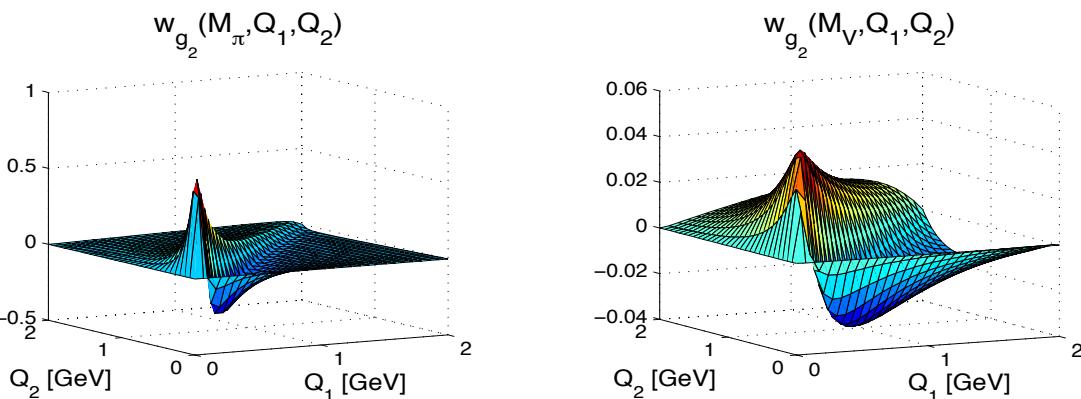
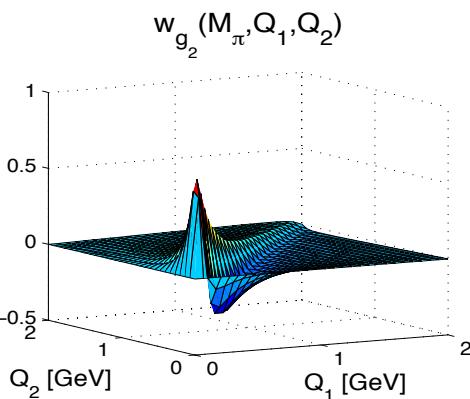
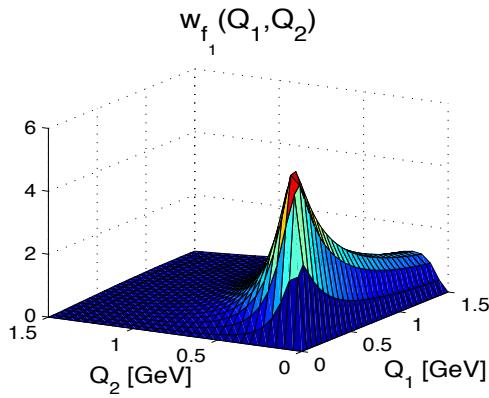


- Recent results from BABAR and BELLE:  
 $Q^2 > 4 \text{ GeV}^2$
- CLEO:  $Q^2 > 1.5 \text{ GeV}^2$
- CELLO:  $Q^2 < 1.5 \text{ GeV}^2$ , very poor accuracy

Low  $Q^2$  range not covered/precise

# Relevant $Q^2$ Region

$$a_\mu^{\text{HLBL};\pi^0} = \int_0^\infty dQ_1 \int_0^\infty dQ_2 \sum_i w_i(Q_1, Q_2) f_i(Q_1, Q_2)$$



Form factor dependent

Universal weight functions

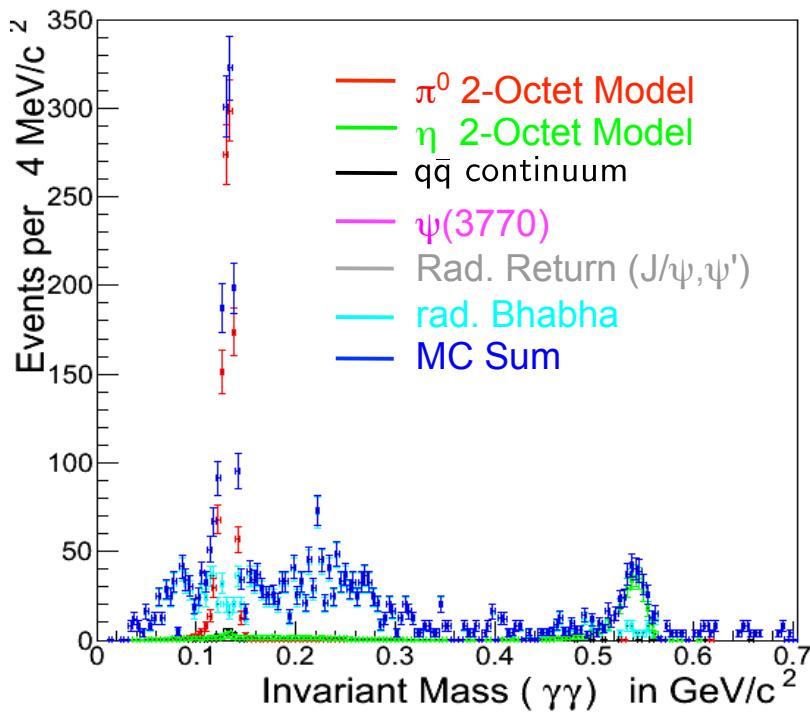
Relevant  $Q^2$  region:  
 $<1.5 \text{ GeV}^2$

[M. Knecht and A. Nyffeler: Phys. Rev. D 65, 073034 (2002)]

# $e^+e^- \rightarrow e^+e^- \pi^0$ at BESIII

$L_{\text{int}}$ : 927 pb $^{-1}$  Tagged lepton:  $e^-$

MC only, part of full statistics



## Event Selection:

- Exactly one lepton candidate
- At least two, max four photons
- Helicity angle  $\cos \theta_H > 0.8$
- Kinematic cuts to reject ISR background
- Cut on angle of missing momentum

**Strategy:**

Count  
 $\pi^0$  yield in  
bins of  $Q^2$

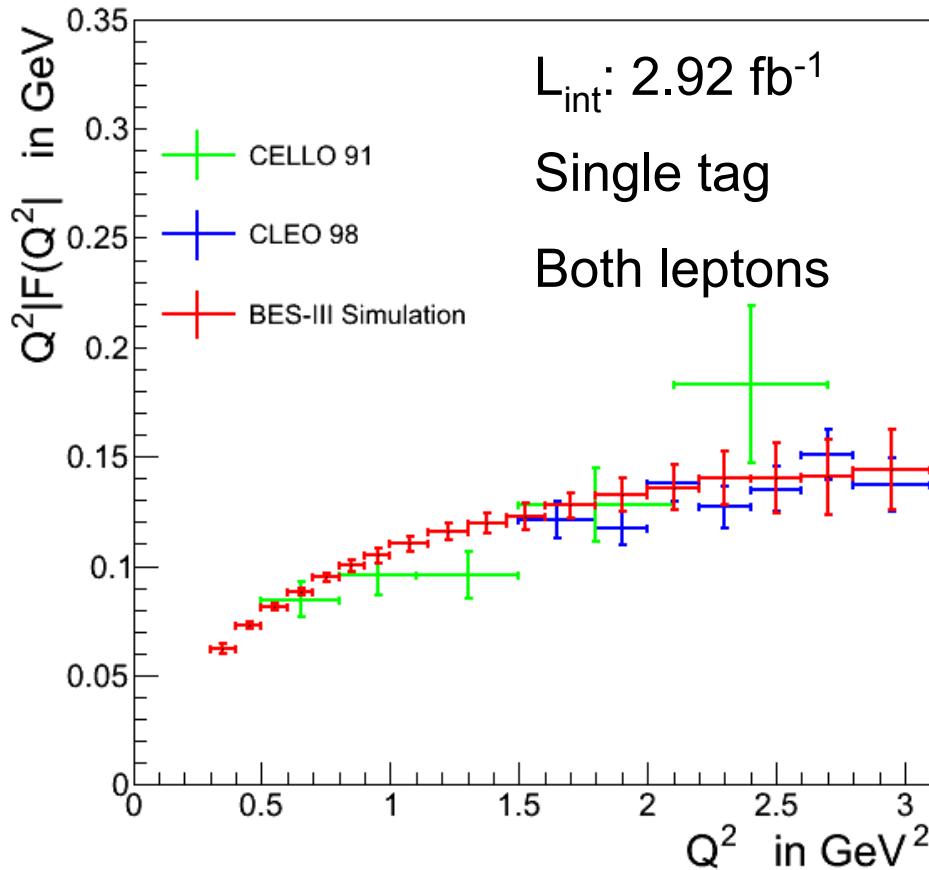


$d\sigma/dQ^2$



Form factor  
 $F(Q^2)$

# Spacelike transition FFs: $\pi^0$



[CELLO: Z. Phys. C 49 401 (1991)]  
[CLEO: Phys. Rev. D57 33 (1998)]

MC only, red error bars  
corresponding to BESIII statistics

Extract TFF for:

$$0.3 \leq Q^2[\text{GeV}^2] \leq 3.1$$

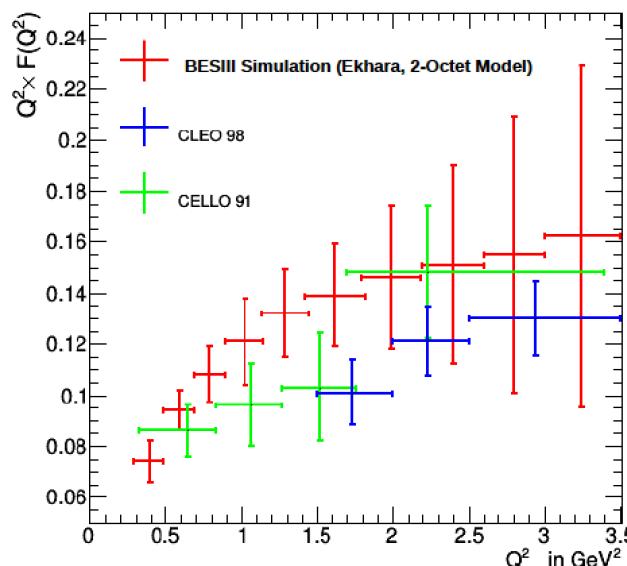
Significantly improves and  
extends data set below  $Q^2 =$   
 $1.5 \text{ GeV}^2$

Input for  $(g-2)_\mu$ !

# Spacelike transition FFs: $\eta$ / $\eta'$

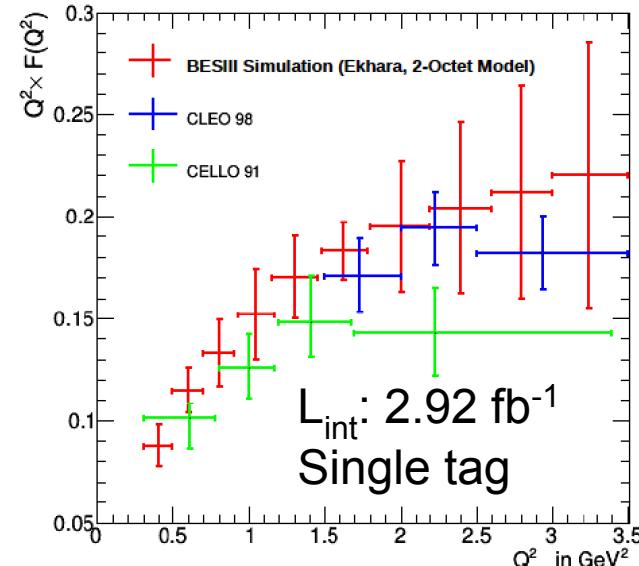
MC only, red error bars corresponding to BESIII statistics

$$F_{\eta,\gamma,\gamma^*}(Q^2)$$



$$\eta \rightarrow \pi^+ \pi^- \pi^0 \rightarrow \pi^+ \pi^- \gamma \gamma$$

$$F_{\eta',\gamma,\gamma^*}(Q^2)$$

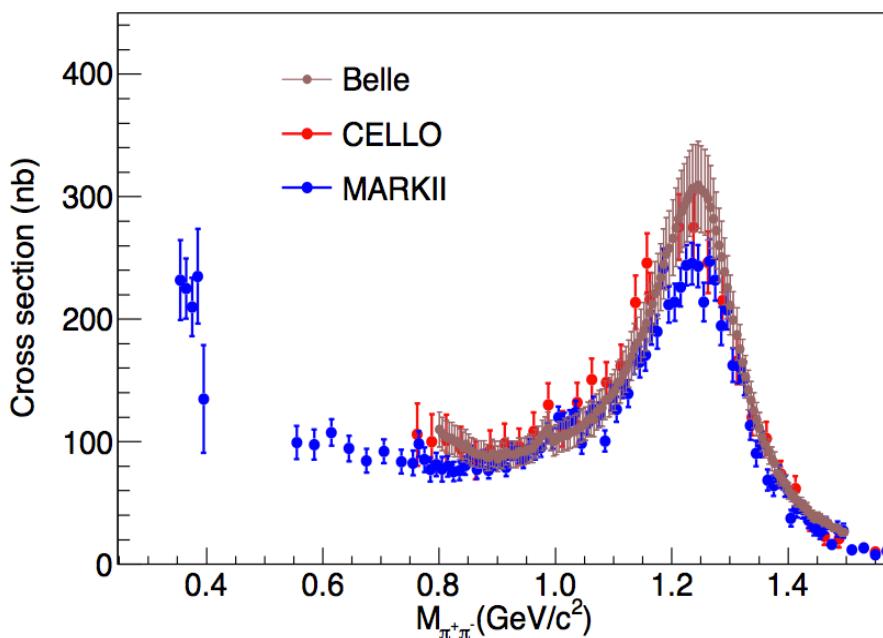


$$\eta' \rightarrow \pi^+ \pi^- \eta \rightarrow \pi^+ \pi^- \gamma \gamma$$

- Results competitive to previous measurement
- More data and more decay modes → order of magnitude improvement

# $\gamma \gamma^* \rightarrow \pi^+ \pi^-$ at BESIII

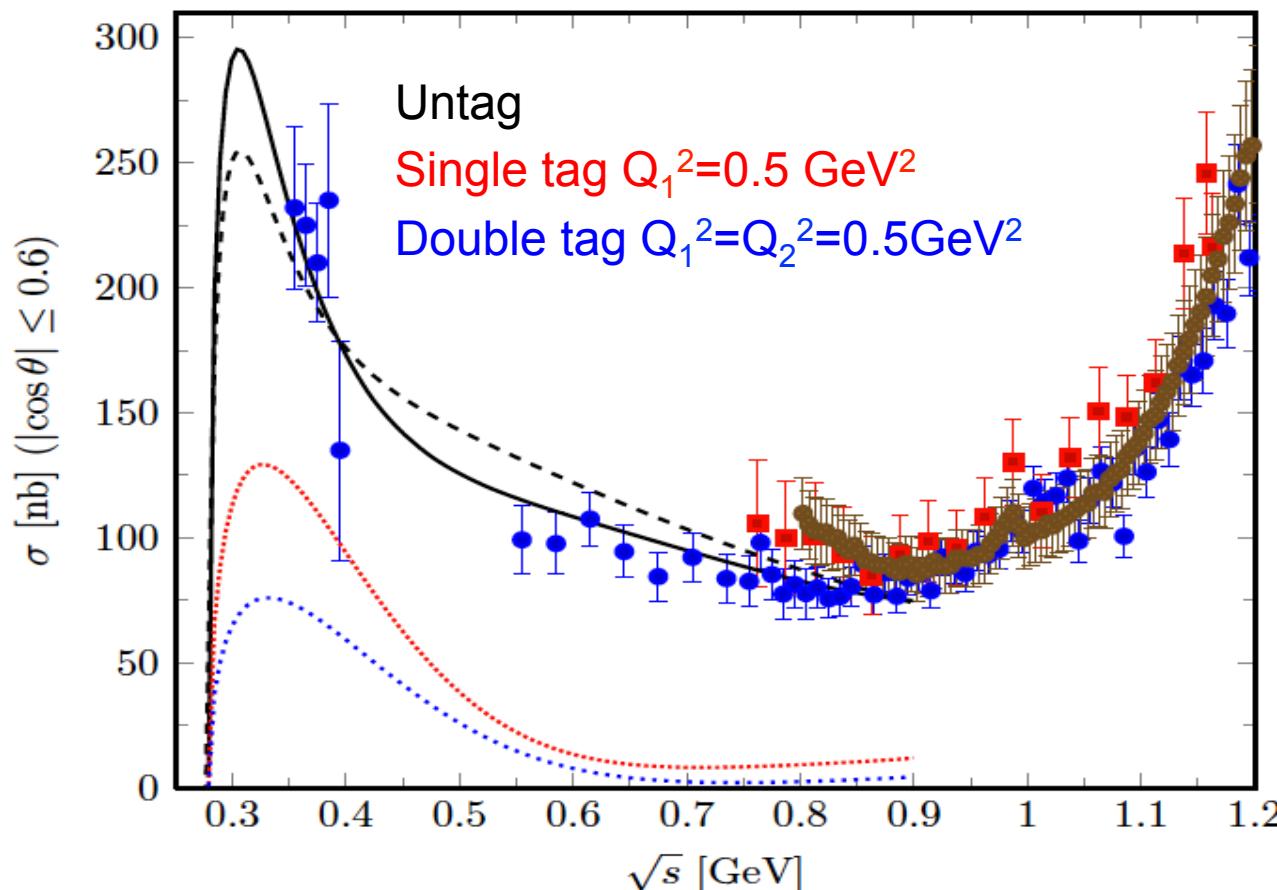
- Hadron form factor
- Resonance parameters for  $0^{\pm\pm}$ ,  $2^{++}$  states
- Pion polarizability, probe the structure of pion
- Re-scattering effect study at low mass region



- All in two real photon case:  
 $\gamma\gamma \rightarrow \pi^+\pi^-$
- In low mass region, only measurement come from MarkII

# $\gamma \gamma^* \rightarrow \pi^+ \pi^-$ at BESIII

N. Asmussen, P. Masjuan, M. Vanderhaeghen



Access to:  $Q^2$  region:  $0.2 \sim 2.0 \text{ GeV}^2$

$M_{\pi\pi}$  : threshold  $\sim 2.0 \text{ GeV}$

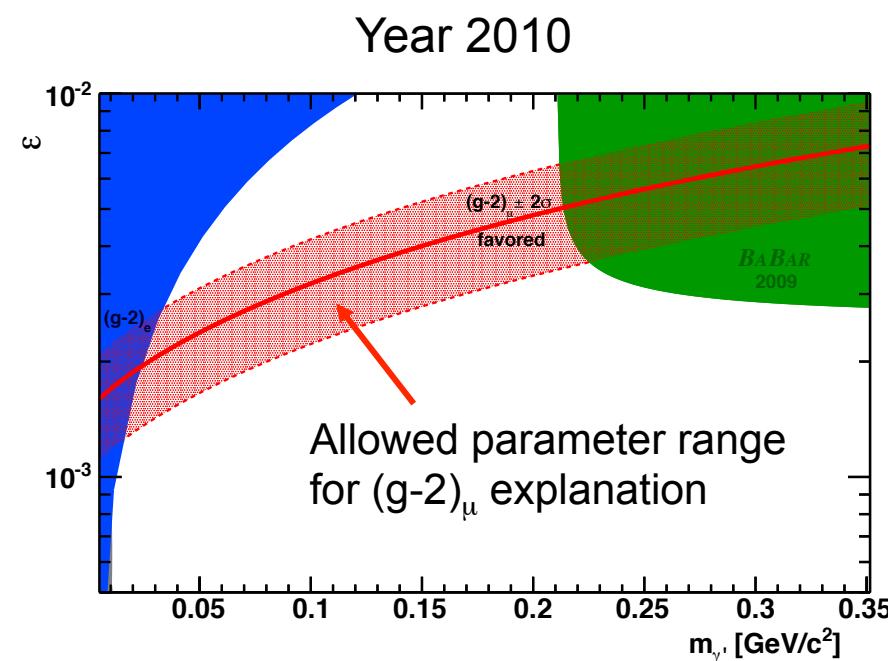
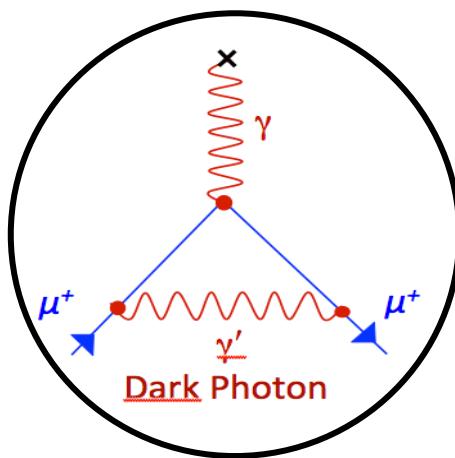
# Conclusion and Outlook

- Important results (to be expected) from BESIII for SM prediction of  $(g-2)_\mu$ 
  - HVP: precision inclusive and exclusive measurements
  - HLbL: spacelike form factors measurement in relevant region
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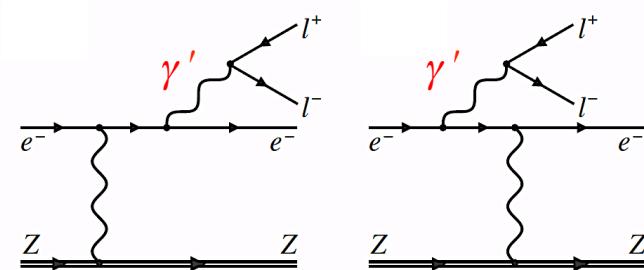
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### A1/MAMI: Electron Scattering

- $E_{\text{beam}}$  180 - 855 MeV
- Stack of Ta targets
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[Phys. Rev. L112 (2014) 221802]

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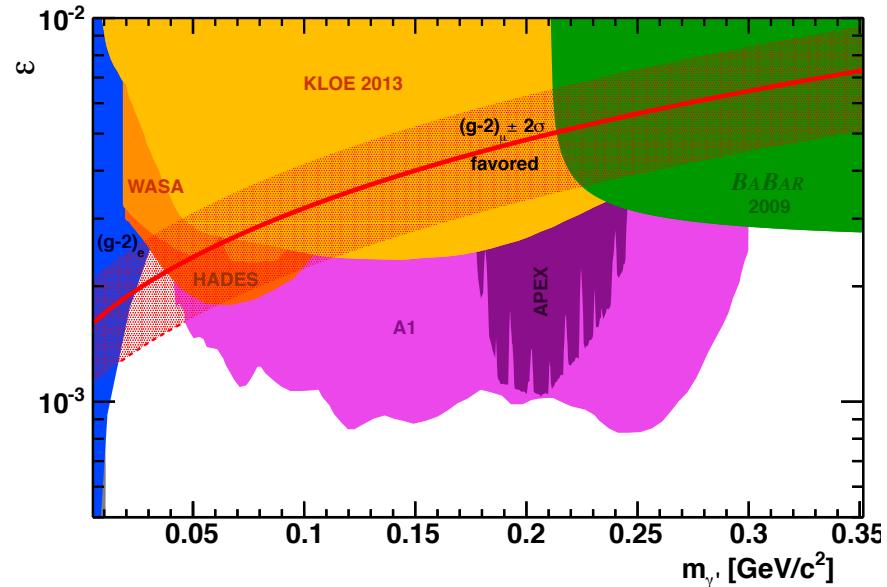
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Year 2014

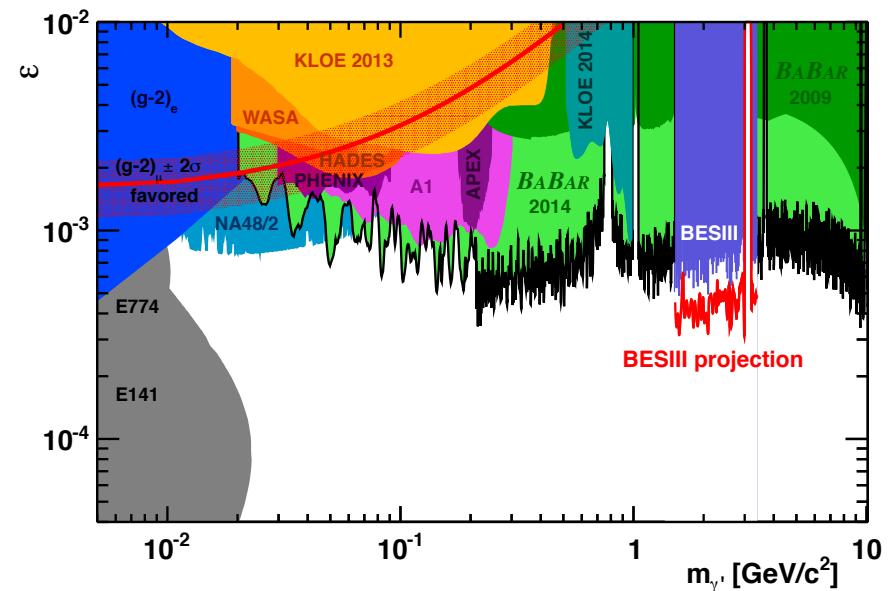


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## Dark photon and $(g-2)_\mu$

Limit ruling out major part of the parameter range motivated by  $(g-2)_\mu$

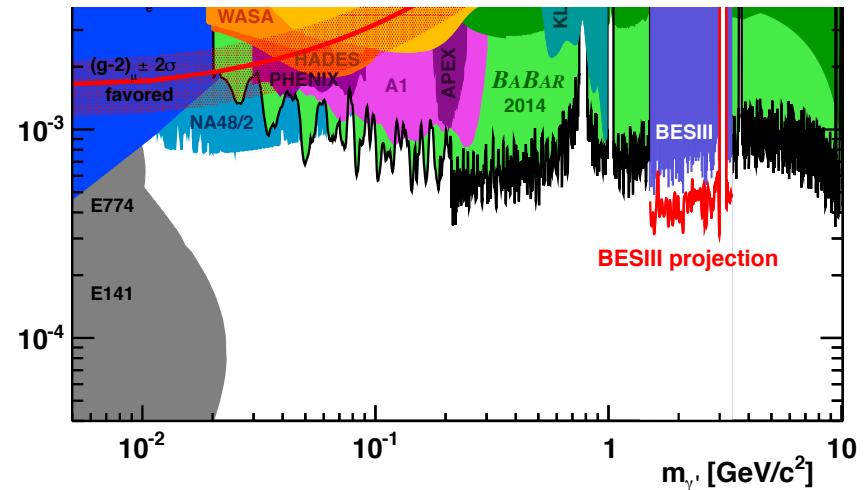


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## THANK YOU FOR YOUR ATTENTION!

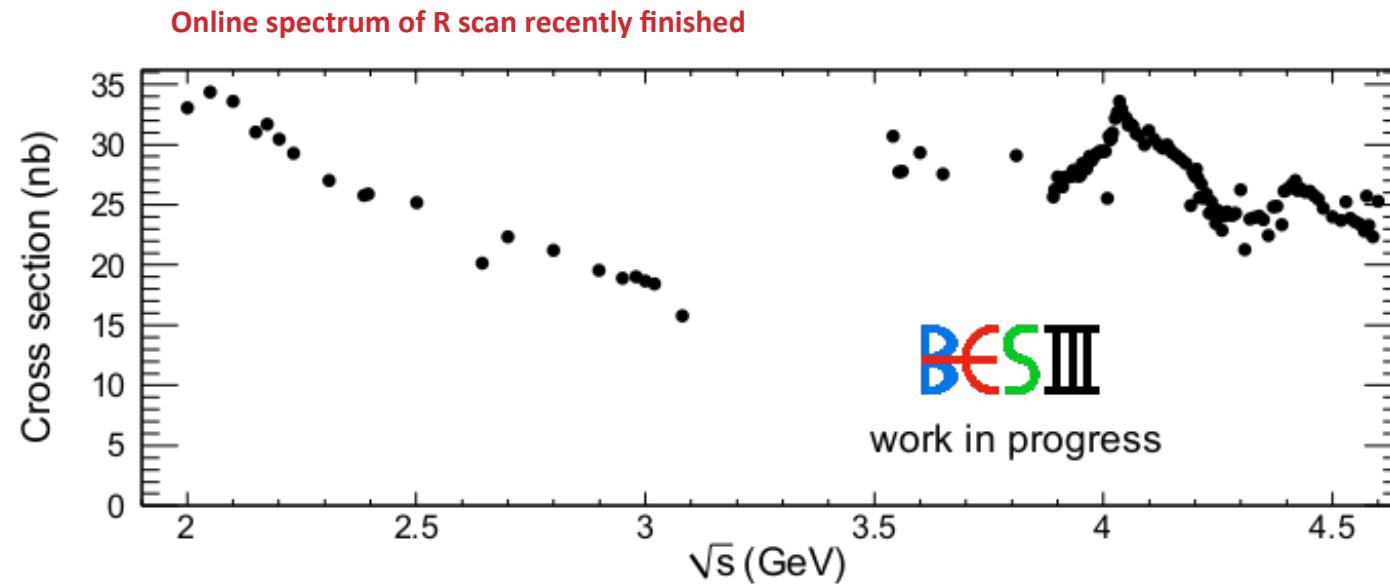
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# Energy Scan from 2.0 to 4.6 GeV

Reducing the uncertainty of  $\alpha_{\text{em}}(M_Z^2)$  by a factor of 2  
→ A new quantity of electroweak precision fits

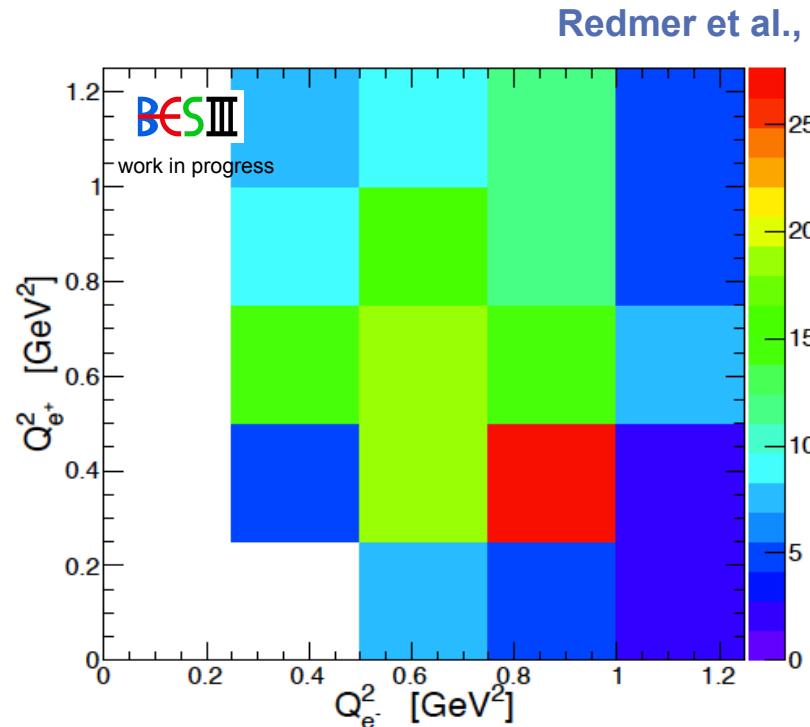
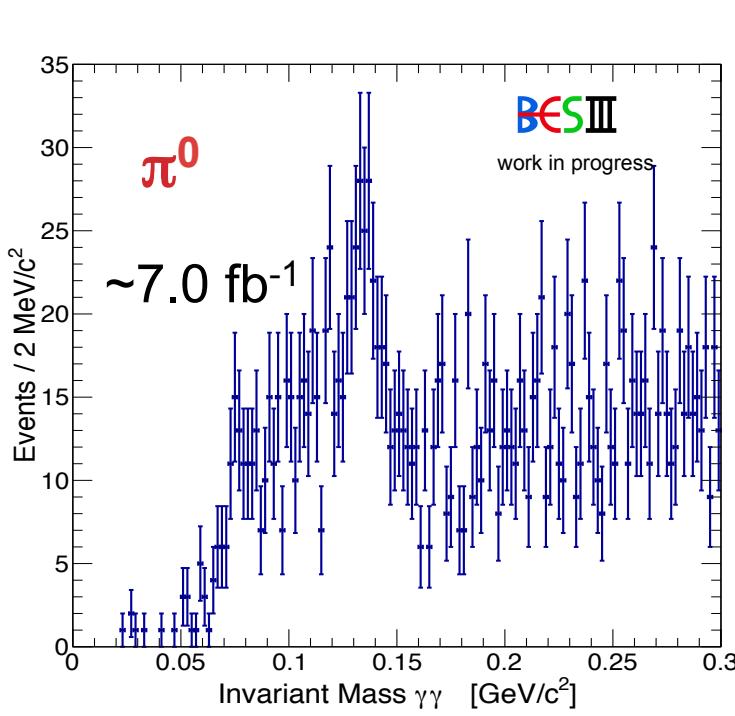


$R = \sigma_{\text{had}}/\sigma_{\mu\mu}$  with targeted systematic accuracy: 3.0%

- 125 scan points with  $>10^5$  hadronic events each → statistical error negligible
- World's best measurement so far from BES /BESII with 5 ... 8 % total error (with 3 ... 5% statistical error)

# Spacelike transition FFs

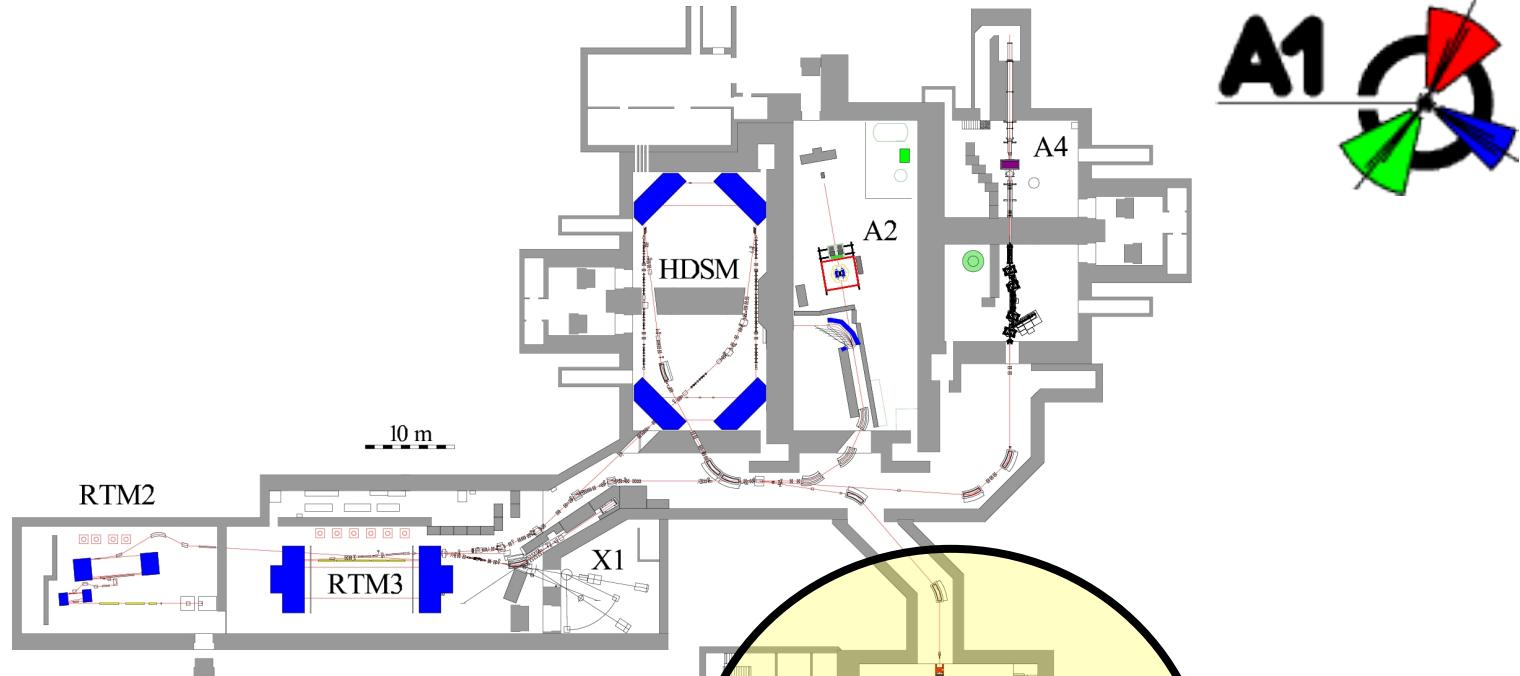
Exploratory first double tag measurement:  $\gamma^* \gamma^* \rightarrow \pi^0$



Redmer et al.,

- Preliminary study shows feasible in most of the parameter space
- Further background suppression using multivariate analysis tool

# Experiment A1: High-Resolution Spectrometers



## Experiment A1: Electron Scattering

- 4 magnetic focussing spectrometers
- Resolution:  $\delta p/p < 10^{-4}$
- Angular acceptance: <30 mrad

